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Barbosa, António

ISCTE - Instituto Universitário de Lisboa

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António Barbosa*

Department of Finance

ISCTE-IUL - Instituto Universitário de Lisboa

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Abstract

In this paper I show how the existence of short-term trading causes a divergence between the average price and the average expectation of the fundamental value by embedding higher-order expectations –expectations of expectations of expectations...– into prices. Short-term trading arises when investors receive private information and (i) either net supply mean reverts or (ii) the release of additional information related to existing information is combined with residual uncertainty. Mean-reversion of net supply, brings the average expectation closer to the fundamental value than the average price after the release of private information. By the contrary, residual uncertainty and an incoming release of information brings the average price closer to the fundamental value than the average expectation before the new information is released. When both (i) and (ii) are present, the average expectation tends to be closer to the fundamental value than the average price in the periods immediately after information releases, but the opposite happens in the periods immediately before information releases.

*ISCTE Business School, Av. Forças Armadas, 1649-026 Lisbon, Portugal. Phone: +351 21 790 39 16. E-mail: antonio.barbosa@iscte.pt

1 Introduction

In a static setting, the equilibrium price of a risky asset equals the average expectation of market participants about its terminal payoff, adjusted for a risk premium. If the asset is, on average, in zero net supply, there is no risk premium and the average price coincides with the average expectation. However, this is not necessarily true in a dynamic setting. If investors engage in short-term trading, they care about intermediate prices, which embeds higher-order expectations – expectations of expectations of expectations...– into prices.[1] This creates a wedge between the average price and the average first-order expectation of the liquidation value because the law of iterated expectations fails for average expectations (Allen et al., 2006).

Allen et al. (2006) show that, in a dynamic economy populated by short-term investors, with no residual uncertainty and i.i.d. noise trade, short-term trading leads to over-reliance on public information when investors have private information.[2] This means that the average expectation of the liquidation value are closer to the fundamental value than the average price. In their own words,

> "Now suppose that the individual is asked to guess what the average expectation of the asset's payoff is. Since he knows that others have also observed the same public signal, the public signal is a better predictor of average opinion; he will put more weight on the public signal than on the private signal. Thus if individuals' willingness to pay for an asset is related to their expectations of the average opinion, then we will tend to have asset prices overweighting public information relative to the private information."

and go as far as saying that (my emphasis)

> "Thus *any* model where higher-order beliefs play a role in pricing assets will deliver the conclusion that there is an excess reliance on public information."

Cespa and Vives (2012) show that, contrary to Allen et al.'s bold claim, over-reliance on public information is not an universal result when higher-order beliefs are embedded in prices. This is true even for an economy populated by short-run investors as in Allen et al., 2006 if noise trade is not i.i.d.. In an economy populated by long-run

[1]Higher-order expectation are imbedded into prices because the price at the second to last trading date depends on the average expectation of the price at the last trading date, which in turn depends on the average expectation of the terminal payoff, and so on. Thus, prices exhibit beauty contest features, as envisioned by Keynes (1936).

[2]There is residual uncertainty when the market as a whole does not have enough information to reveal the terminal payoff.

investors, Cespa and Vives (2012) claim that whether over-reliance or under-reliance on public information obtains, depends on the parameters of the model. Specifically, over-reliance on public information is obtained when residual uncertainty is low, and net supply changes are strongly correlated. Otherwise, under-reliance on public information is obtained.

My contribute to this recent literature is to identify the crucial role of the information environment on over- and under-reliance on public information when investors have long-run investment horizons. Cespa and Vives (2012) fail to identify the role of the information environment for two reasons: first, they always assume that investors observe private exogenous information at all periods; second, they consider only two trading dates.[3] By relaxing the first assumption, I show that under-reliance on public information occurs only if additional exogenous information related to existing private information is known to be released in the future. By expanding the model to allow more than two trading dates, I show that over- and under-reliance on public information can arise in the same economy at different dates: over-reliance on public information is most likely to occur in the periods immediately after the release of exogenous information; in turn, under-reliance on public information tends to occur in the periods immediately preceding the release of additional exogenous information.

The intuition is the following. On average, investors believe that the price reaction to the initial release of *private* exogenous information is due not only to a change in informed demand, but also to a change in the liquidity driven demand away from zero. If noise trading is persistent, future price changes are unpredictable, and there is no short-term trading. In this case the average price and expectation coincide. But if noise trading mean reverts, investors can forecast future net supply levels and prices. This creates a short-term opportunity to profit from liquidity traders as they exit the market, and makes investors less eager to build their long-run position immediately. That is, short-term trading diverts attentions from the long-run. As a result, less of the private information makes its way into prices, which causes the average price to be further away from the fundamental value than the average expectation of the liquidation value. That is, prices over-rely on public information.

In turn, when investors expect the release of additional exogenous information related to their private information, they use their private information more aggressively to bet on the price impact of the incoming information. This impounds more of the existing information into prices, bringing the average price closer to the fundamental

[3]Endogenous information is that obtained from the observation of prices. Exogenous information is information obtained from any other source.

value than the average expectation. In other words, prices under-rely on public information. Using Allen et al.'s intuition, on the one hand, all investors *observed* the same public information (the common prior), and a private signal. Although investors know what the public information was, they can only make an inaccurate estimate of what other investors' private signals might have been. Hence the tendency to over-rely on public information. But, on the other hand, all investors *will observe* another private signal related to the private signal already observed. Since the incoming private signal can be forecasted with the previously observed private signal, investors will rely more heavily on their own private signal and less on the public signal when forming beliefs about others' beliefs. Allen et al. (2006) do not obtain this result because in their model (i) there is no residual uncertainty and (ii) there is mean-reversion of net supply.

When net supply mean reverts *and* there is an additional release of information, the effect of the former tends to dominate in the periods closer to the initial relase of private information, because this is when the net supply level is believed to be further away from its unconditional mean; whereas the effect of the latter tends to dominate in the periods closer to the relase of the new information, for that is when expected price impact of the new information is believed to be strongest. This means that prices tend to over-rely on public information in the periods following information releases and under-rely on public information in the periods leading to information releases.

In addition, I show that over-reliance on public information is one-to-one with the average expectation being closer to the fundamental value than the average price only when all exogenous information is based on the same underlying signal. When that is not the case, it is possible for the average price to be the best predictor of the fundamental value even though prices over-rely on public information.

Finally, I point out that the price pattern preceding the release of additional exogenous information resembles the leakage of inside information, and that prices tend to be more informative about the fundamental value in periods of high price volatility. The latter backs the traditional view of a more volatility market as one where more information is gathered (e.g. Admati and Pfleiderer, 1988) and it is in contrast with the results of Cespa (2002).

This paper is closely related to Barbosa (2011) in that they share the same model. However, the set of questions analyzed are clearly distinct. In Barbosa (2011) I study the time-series of average prices and expectations following the release of private exogenous information. Here, I study how the average price compares to the average expectation at any given point in time.

This paper is organized as follows. I describe the model in Section 2 and solve for the equilibrium price and demand functions in Section 3. In Section 4 I study how the average price diverges from the average expectation of the liquidation value and from the fundamental value when there is a single signal underlying all releases of exogenous information. In Section 5 I analyze how different underlying signals change those results. In Section 6 I discuss additional implications of the model, and Section 7 concludes. All proofs and additional discussion are provided in the Appendix.

2 The Model

I will use the same model developed in Barbosa (2011). For convenience, I will describe it here once again. There is one risky asset and one riskless asset traded at dates $1, 2, ..., T-1$. The riskless asset has a perfectly elastic supply and a zero net rate of return. The risky asset is liquidated at date T, paying $v \sim N(0, \sigma_v^2)$. The date t risky asset's per capita net supply (θ_t) is random, due to noise/liquidity driven demand, and follows the $AR(1)$ process

$$\theta_t = \rho\theta_{t-1} + \varepsilon_{\theta,t},\ \varepsilon_{\theta,t} \sim N\left(0, \sigma_\theta^2\right),\ t = 1, 2, ..., T-1 \tag{1}$$

with $0 \leq \rho \leq 1$ and $\theta_0 = 0$.

There is a continuum of risk averse investors of measure 1, indexed by i. All investors have CARA preferences over their terminal wealth (W_T^i) with the same coefficient of risk aversion (α). At every date t, investor i chooses the risky asset demand (X_t^i) that maximizes his expected utility conditional on the information currently available to him ($\mathcal{F}_t^i$) by solving

$$\max_{X_t^i} \mathbb{E}\left[-e^{-\alpha W_T^i} | \mathcal{F}_t^i\right] \quad s.t. \quad W_{t+1}^i = W_t^i + X_t^i \left(P_{t+1} - P_t\right), \tag{2}$$

where P_t is the date t risky asset's price. Investors have homogeneous prior beliefs, $v \sim N(0, \sigma_v^2)$ and $\theta_0 = 0$, denoted by $\mathcal{F}_0$.

There are n underlying signals for the liquidation value v,

$$\boldsymbol{s} = v\mathbf{1}_{(n\times 1)} + \boldsymbol{\varepsilon}_s,\ \boldsymbol{\varepsilon}_s \sim N\left(\mathbf{0}_{(n\times 1)}, \Sigma_s\right), \tag{3}$$

where $\mathbf{1}_{(n\times 1)}$ and $\mathbf{0}_{(n\times 1)}$ denote $(n \times 1)$-dimensional vectors of ones and zeros, respectively. (To distinguish vector-valued variables and parameters from scalar ones, I use bold-faced letters and numbers throughout the paper to denote the former.) I allow for correlation among signals meaning that Σ_s is not necessarily a diagonal matrix.

Investors do not observe these underlying signals directly. Instead, at every date t investors observe a private version of those underlying signals:

$$\tilde{\boldsymbol{s}}_t^i = \boldsymbol{s} + \boldsymbol{\varepsilon}_t^i,\ \boldsymbol{\varepsilon}_t^i \sim N\left(\mathbf{0}_{(n\times 1)}, \Sigma_{\tilde{s},t}\right), \tag{4}$$

with private noise $\boldsymbol{\varepsilon}_t^i$ independent across i and t and with independent components (i.e. $\Sigma_{\tilde{s},t}$ is restricted to be diagonal). The set of covariance matrices $\{\Sigma_{\tilde{s},t} : 1 \leq t < T\}$ defines the timing of exogenous information release: the j-th diagonal entry of $\Sigma_{\tilde{s},t}$ is equal to ∞ if there is no signal for the j-th underlying signal at date t; and less than ∞ otherwise.

The total information available to investor i and the common information available to all investors are defined as

$$\mathcal{F}_t^i = \left\{\mathcal{F}_0, \tilde{\boldsymbol{s}}_\tau^i, P_\tau : \tau = 1, ..., t\right\},\ \mathcal{F}_t^c = \left\{\mathcal{F}_0, P_\tau : \tau = 1, ..., t\right\},$$

respectively. Unless it is explicitly stated otherwise, all random variables are independent of each other and across time.

With the model laid down, I can now define three concepts that I will use throughout the paper: residual uncertainty, exogenous information and endogenous information. Residual uncertainty is the amount of uncertainty that would persist after the direct observation of the underlying signals. Therefore residual uncertainty is a function of the covariance matrix Σ_s. If at least one diagonal entry of Σ_s is zero, the corresponding underlying signal coincides with the liquidation value, and there is no residual uncertainty.

The second concept, exogenous information, is defined as the information directly obtained from the observation of $\tilde{\boldsymbol{s}}_t^i$. I say that there is exogenous information at date t if at least one of the diagonal entries of $\Sigma_{\tilde{s},t}$ is less than ∞.

Finally, endogenous information is the information that is obtained from the observation of the series of equilibrium prices. Information is endogenously produced whenever investors have previously observed private exogenous information about at least one underlying signal, and the net supply mean reverts. The mechanism by which endogenous information is produced is described in detail in Barbosa (2011). But in short, upon the observation of exogenous information and the equilibrium price, investors form a belief about the underlying signal and net supply level (the two unknowns that influence prices). Mean-reversion of net supply makes price changes predictable. And this allows investors to correct their initial beliefs when they detect a discrepancy between their forecast and the realized price change. Therefore, investors extract endogenously produced information from the observation of the sequence of prices that follow the initial release of private exogenous information.

3 Equilibrium

The full details on how to determine the equilibrium price and demand functions are provided in Barbosa (2011). To avoid unnecessary repetition, here I will only present the price and demand functions without further details on how to derive them.

Theorem 1. *In a linear equilibrium, the price function is given by*

$$P_t = \left(\hat{K} - \boldsymbol{p}_t\right) \mathbb{E}\left[\boldsymbol{s}|\mathcal{F}_t^c\right] + \boldsymbol{p}_t \boldsymbol{s} + p_{\theta,t}\theta_t = \left(\hat{K} - \boldsymbol{p}_t\right) \mathbb{E}\left[\boldsymbol{s}|\mathcal{F}_t^c\right] + \xi_t \tag{5}$$

where $\xi_t \equiv \boldsymbol{p}_t \boldsymbol{s} + p_{\theta,t}\theta_t$ *and the demand function is given by*

$$X_t^i = \frac{1}{\alpha} Q_t \boldsymbol{\psi}_t,\ t = 1, 2, ..., T-1 \tag{6}$$

where $\boldsymbol{\psi}_t$ *is a vector of state variables defined as*

$$\boldsymbol{\psi}_t \equiv \begin{bmatrix} 1 \\ \mathbb{E}\left(\boldsymbol{s}|\mathcal{F}_t^i\right) \\ \mathbb{E}\left(\boldsymbol{s}|\mathcal{F}_t^c\right) \\ \mathbb{E}\left(\theta_t|\mathcal{F}_t^i\right) \end{bmatrix}.$$

The demand of investor i is a linear function of the expectations of the vector of underlying signals $\boldsymbol{s}$ and the net supply level θ_t conditional on his information set $\mathcal{F}_t^i$. This implies that, once investors observe private exogenous information about one underlying signal, and until that underlying signal is revealed to them, every investor will demand a different amount of the risky asset. As shown in theorem 2 of Barbosa (2011), expectations conditional on $\mathcal{F}_t^i$ are a linear function of prior beliefs, information extracted from prices and investor i's private exogenous information. Since the private noise in exogenous information ($\boldsymbol{\varepsilon}_t^i$) is normally distributed, it follows that expectations conditional on $\mathcal{F}_t^i$ are also normally distributed across investors. And so is investor's demand.

Corollary 2. *The demand for the risky asset is conditionally normally distributed across investors. Investors whose private information is more optimistic demand a larger quantity of the risky asset than those whose private information is more pessimistic.*

Market clearing requires that the average demand matches the per capita supply for the asset, $\int_i X_t^i = \theta_t \, \forall t$. The existence of a continuum of investors with conditionally normally distributed demands implies that there is one investor whose demand is exactly equal to the average demand. I refer to this investor as the average investor (AI). It follows that the market clears if and only if the demand of the AI equals the per capita net supply. This allows us to simplify the analysis in the next sections by focusing on this representative investor. The next corollary identifies the AI, at date t, as the investor whose all past and current private signals exactly coincide with the corresponding underlying signals.

Corollary 3. *The market clears at date t if and only if the demand of the investor who observes $\tilde{\boldsymbol{s}}_\tau^i = \boldsymbol{s}$, $\forall \tau \leq t$, the average investor (AI), matches the per capita asset's net supply. In addition, the beliefs of the AI coincide with the average beliefs across investors, i.e.* $\mathbb{E}\left(v|\mathcal{F}_t^{AI}\right) = \mathbb{E}_i\left[\mathbb{E}\left(v|\mathcal{F}_t^i\right)\right]$.

The demand function (6), although parsimonious, is not particularly intuitive. The next lemma tells us that the risky asset demand can be written as a linear function of the expectations about all future price changes; or as a linear function of the short-term myopic demand and all expected future non-myopic demands (the hedging demand).

Lemma 4. *The demand function can also be written as*

$$X_t^i = \frac{1}{\alpha}\sum_{\tau=1}^{T-t} \chi_{\tau,t}^Q \mathbb{E}\left[\Delta P_{t+\tau}|\mathcal{F}_t^i\right], \tag{7}$$

$$X_t^i = \frac{1}{\alpha}\sum_{\tau=1}^{T-t} \xi_{\tau,t} \mathbb{E}\left[P_{t+\tau} - P_t|\mathcal{F}_t^i\right], \tag{8}$$

$$X_t^i = \phi_{1,t}^Q \widetilde{X}_t^i + \sum_{\tau=1}^{T-t-1} \phi_{\tau,t}^Q \mathbb{E}\left[X_{t+\tau}|\mathcal{F}_t^i\right], \tag{9}$$

where $\Delta P_{t+\tau} \equiv P_{t+\tau} - P_{t+\tau-1}$ is the one-period return and $\widetilde{X}_t^i \equiv \frac{\mathbb{E}\left(\Delta P_{t+1}|\mathcal{F}_t^i\right)}{\alpha Var\left(\Delta P_{t+1}|\mathcal{F}_t^i\right)}$ is the short-term myopic demand. Moreover, $\chi_{1,t}^Q > 0$ and $\phi_{1,t}^Q > 0$. All parameters are defined in Appendix A.2.

Investors trade the asset not only to profit from their expectations about the price change over the next period, but also to hedge their expected positions at future dates. Hence the dependence of demand on the entire series of expected price changes. As discussed in more detail in Appendix C, typically the mean reversion of supply shocks makes the next period expected return negatively correlated with all subsequent expected returns. In that case, hedging future positions requires the investor to take today a position of the same sign of their expected future short-term position (i.e., the same sign of the expected return), and so $\chi^Q_{\tau,t} > 0$ for $\tau \geq 1$. Expected returns may become positively correlated, leading to $\chi^Q_{\tau,t} < 0$, in the period before exogenous information is released. But only if that exogenous information significantly reduces the uncertainty about the liquidation value.

The following assumption relates the magnitude of short-term demand with that of hedging demand.

Assumption 5. *The following holds:* $\chi^Q_{1,t} > \sum_{\tau=2}^{T-t} \chi^Q_{\tau,t} \, \forall t$ *. That is, demand is strictly positive (negative) whenever the expected price change over the next period is positive (negative) and all future expected prices are never below (above) the current price.*

This means that if the price change over the next period has the same magnitude but the opposite sign of the price change from the next period all the way to the liquidation date, then short-term demand dominates the hedging demand and the overall demand has the same sign of the short-term demand. Note that, in this case, the asset has only upside or only downside potential, and so it is expectable that investors take advantage of it with a short or long position in the asset, respectively. However intuitive this assumption might be, it does not hold true for generic return processes. In Appendix D I discuss the validity of this assumption for the endogenous return process implied by the model used in this paper.

4 Discrepancy between Average Prices and Expectations: The Case of a Single Underlying Signal

I will now use the model introduced in the previous sections to study in which circumstances and in which direction the average price differs from the average expectation of the liquidation value. In this section, the focus will be on the case of a single underlying signal s. Note that, even though there is a single underlying signal, investors may observe several private signals based on that same underlying signal at different dates. The case where investors observe private signals based on multiple underlying signals is addressed in the next section.

For convenience, from now on I will loosely use the expression "prices are closer to fundamentals than expectations" or vice-versa as meaning that "the average (over net supply level) price is closer to the fundamental value than the average (over net supply level and investors) expectation of the liquidation value". I will also refer to the average expectation of the liquidation value simply as the average expectation.

This setup with a single underlying signal is the closest to the one used by Cespa and Vives (2012). The main difference relative to their work is that I relax the assumption that investors observe exogenous information for s in every period (i.e. I allow $\Sigma_{\tilde{s},t} = \infty$ at some dates t), and consider more than 2 trading dates ($T > 3$). The former allows me to identify the existence of additional exogenous information, following an initial release of private exogenous information, as a crucial factor to have prices closer to fundamentals than expectations. In turn, the latter allows me to find that the prices can be closer to fundamentals in some periods, while expectations are closer to fundamentals in other periods.

I show that, following an initial release of private exogenous information, either (i) mean-reversion of net supply or (ii) a combination of residual uncertainty with an additional release of exogenous information create a discrepancy between prices and expectations. In both cases, the root cause for this discrepancy between prices and expectations is the existence of short-term speculative trading opportunities. Moreover, I show that prices differ from expectations in a different way depending on whether (i), (ii) or both occur. Mean-reversion of net supply tends to bring expectations closer to fundamentals than prices *after* the release of private exogenous information. By the contrary, residual uncertainty and an incoming release of exogenous information tends to make prices closer to fundamentals than expectations *before* the new information is released. Figure 1 previews these results.

Without loss of generality, I assume henceforth that the initial private exogenous information is released at date 1. Since there is only one underlying signal, the fundamental value of the risky asset (FV) is defined as the expected liquidation value given the direct observation of the underlying signal, and is independent of t. The next lemma provides the expressions for the fundamental value, average price ($\mathbb{E}_{\theta}\left(P_t\right)$) and average expectation of the liquidation value ($\mathbb{E}_{\theta,i}\left[\mathbb{E}\left(v|\mathcal{F}_t^i\right)\right] \equiv \mathbb{E}_{\theta}\left[\mathbb{E}\left(v|\mathcal{F}_t^{AI}\right)\right]$). All averages are over the net supply level and investors, and conditional on the underlying signal s.

Lemma 6. *When there is a single underlying signal s and exogenous information about that signal is released at date 1, the date t fundamental value, average price and average*

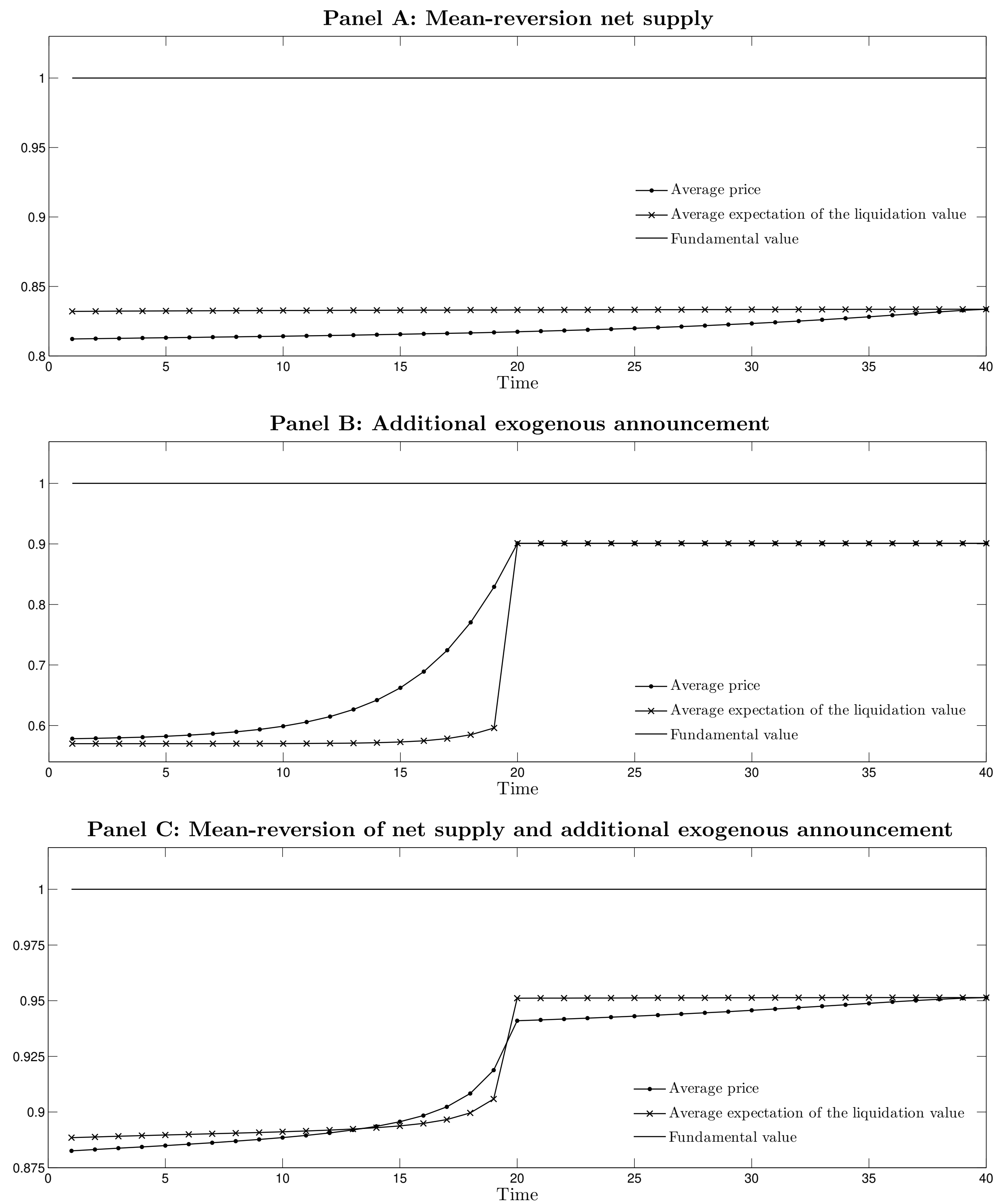


Figure 1: Discrepancy between the average price, the average expectation of the liquidation value and the fundamental value. This figure shows the discrepancy between the average price, the average expectation of the liquidation value and the fundamental value when: there is mean-reversion of net supply (panel A); there is an incoming release of additional exogenous information (panel B); and the previous two occur simultaneously (panel C). In all cases, private exogenous information is released at date 1, and there is residual uncertainty. In panel A, $\rho = 0.9$ and $\Sigma_{\tilde{s},20} = 10^{10}$; in panel B, $\rho = 1$ and $\Sigma_{\tilde{s},20} = 0.1$; and in panel C, $\rho = 0.9$ and $\Sigma_{\tilde{s},20} = 0.1$. The rest of the parametrization is common to all panels: $T = 41$, $n = 1$, $\sigma_v^2 = 0.25$, $\Sigma_s = 0.5$, $\Sigma_{\tilde{s},1} = 1$, $\Sigma_{\tilde{s},t} = 10^{10}\,\forall t \setminus \{1, 20\}$, $\sigma_\theta^2 = 0.1$, $\alpha = 2$, $s = 3$. The case of a negative underlying signal is the symmetric.

expectation of the liquidation value conditional on s *are given by*

$$\begin{aligned} FV &= \hat{K}s \\ \mathbb{E}_\theta\left(P_t\right) &= \left(\hat{K}-p_t\right)\mathbb{E}_\theta\left[\mathbb{E}\left(s|\mathcal{F}_t^c\right)\right]+p_t s \\ \mathbb{E}_{\theta,i}\left[\mathbb{E}\left(v|\mathcal{F}_t^i\right)\right] &= \hat{K}\left(1-\hat{\Gamma}_t\right)\mathbb{E}_\theta\left[\mathbb{E}\left(s|\mathcal{F}_t^c\right)\right]+\hat{K}\hat{\Gamma}_t s \end{aligned}$$

where $\hat{K}=\frac{\sigma_v^2}{\sigma_v^2+\sigma_s^2}$ *and* $\hat{\Gamma}_t = Var\left(v|\mathcal{F}_t^i\right)\sum_{k=1}^{t}\frac{1}{\sigma_{i,k}^2}$, $\sigma_{i,t}^2\equiv\Sigma_{\tilde{s},t}$.

Furthermore, the following holds:

(i) The average price and the average expectation of the liquidation value are biased toward the prior belief about the liquidation value;

(ii) The average price is closer to (further away from) the fundamental value than the average expectation of the liquidation value is if and only if prices exhibit under-(over-)reliance on public information relatively to the optimal statistical weight, i.e., $\hat{K}-p_t<(>)\,\hat{K}\left(1-\hat{\Gamma}_t\right)$.

This result establishes the one to one link between the (relative) under-reliance of prices on public information and prices being the best estimator of the fundamental value when there is a single underlying signal. The less weight prices put on public information, i.e. $\mathbb{E}\left(\boldsymbol{s}_t|\mathcal{F}_t^c\right)$, the less they overweight the prior belief about v, and the closer they are to the fundamental value, compared to average expectations. A similar result is provided by Cespa and Vives (2012). However, this result is specific to the case of a single underlying signal.

4.1 The Base Case: Exogenous Information at a Single Date and No Mean-Reversion of Net Supply

In order to understand why, and in which direction, the average price deviates from the average expectation of the liquidation value, I start by considering the simplest case: private exogenous information is available *only* at date 1 and there is no mean-reversion of net supply ($\rho=1$).

In this stripped down version of the model, the average price always coincides with the average expectation. And the reason is that investors do not expect prices to change in the periods before the liquidation date, and so do not engage in short-term trading. As we will see in more detail in the next subsections, short-term trading is crucial to “deviate investors’ attentions” from the discrepancy between the average price and the average expectation, allowing this discrepancy to persist. This is easy to grasp by looking at equation (8). When price changes are not expected in the periods before the liquidation date, then demand depends only on the expectation of the liquidation value and the current price, that is

$$X_t^i = \frac{1}{\alpha}\xi_{T-t,t}\mathbb{E}\left(v - P_t|\mathcal{F}_t^i\right),$$

just like in the static model (in this case $\xi_{T-t,t} = Var\left(v|\mathcal{F}_t^i\right)^{-1}$)

In the absence of the arrival of additional exogenous information, the only event with impact on prices are changes in net supply. And although prices do change every period in response to changes in net supply, they do not change in a predictable direction because there is no mean-reversion of net supply change. Therefore, the expected change in net supply and prices is zero. This means that, in every period, investors trade as if they hold their position until the liquidation date. Since, on average, the net supply level is zero, this means that the AI (the investor whose demand has to match the net supply level for the market to clear) has to demand zero. But this only happens if the price matches the AI's expectation of the liquidation value, that is, if the price coincides with the average expectation of the liquidation value,

$$\begin{aligned}
& \mathbb{E}_\theta\left(X_t^{AI}\right) = \frac{1}{\alpha}\xi_{T-t,t}\mathbb{E}_\theta\left[\mathbb{E}\left(v - P_t|\mathcal{F}_t^{AI}\right)\right] = \mathbb{E}_\theta\left(\theta_t\right) \\
\Leftrightarrow\ & \frac{1}{\alpha}\xi_{T-t,t}\mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}_t^{AI}\right) - P_t\right] = 0 \\
\Leftrightarrow\ & \mathbb{E}_\theta\left(P_t\right) = \mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}_t^{AI}\right)\right] = \mathbb{E}_{\theta,i}\left[\mathbb{E}\left(v|\mathcal{F}_t^{i}\right)\right].
\end{aligned}$$

Therefore, to create a discrepancy between average price and the average expectation of the liquidation value investors need to engage in short-term trading. This can be achieved either by introducing mean-reversion of net supply, which leads to endogenous production of information, or by allowing the arrival of additional exogenous information combined with residual uncertainty. This is the plan for the next two subsections and, as we will see, these two changes to the base case will cause the average price to deviated from the average expectation of the liquidation value in different directions.

4.2 Mean-Reversion of Net Supply: Endogenous Information

The first modification to the base case is the introduction of mean-reversion in the net supply ($\rho < 1$). In this setting, investors expect the net supply to change in the direction of its unconditional mean. And, from price function (5), this means that investors expect prices to change as well, which leads to short-term trading. The only exception is when investors believe that the current supply level is at its unconditional mean.

Obviously, when averaging over supply shocks, the net supply equals its unconditional mean and there is no net supply changes. But, in this case, do investors believe that net supply is equal to its unconditional mean? The answer is no. And this is the reason why, on average, investors expect price changes that lead them to engage in short-term trading which, in turn, allows the average price to diverge from the average expectation.

To understand why, on average, investors expect net supply (and thus prices) to change when in fact it does not, we need to understand what happens when investors observe private exogenous information. To simplify the exposition, let us focus the attention on the average investor (AI). Like any other investor, the AI has two pieces of information: his private signal for s, $\tilde{s}_1^i = s + \varepsilon_1^i$; and the informational equivalent to the contemporaneous equilibrium price, $\xi_1 = s + \frac{p_{\theta,1}}{p_1}\theta_1$.[4] However, there are three unknowns to the AI: the underlying signal s; the idiosyncratic error of his signal ε_1^i; and the current net supply level θ_1. This means that there is an infinite number of combinations of these three unknowns that can generate the two observations: for a given $\tilde{s}^i$, the larger the underlying signal, the smaller the error in the signal (i.e. the less positive or more negative the error is); and for a given price, the larger the underlying signal, the larger the net supply level.[5] Therefore, it is impossible to learn the value of s and θ_1 with certainty at date 1. Instead, the AI has to estimate the value of these three unknowns.

The AI always observe $\tilde{s}_1^{AI} = s$, although he does not know that (since he does not know that he is the AI). And, when averaging over net supply shocks, he, like any other investor, observes $\xi_1 = s$. However, because both signals for s are noisy, the AI gives some weight to his prior belief when forming his posterior belief about s. This means that, on average, his belief about s is biased toward the prior, i.e. $\left|\mathbb{E}_\theta\left[\mathbb{E}\left(s|\mathcal{F}_1^{AI}\right)\right]\right| < |s|$. This is evident from the expression for the posterior belief about s derived for a 3-period model without residual uncertainty (see Barbosa, 2011),

$$\mathbb{E}\left(s|\mathcal{F}_1^i\right) = \frac{\frac{1}{\sigma_v^2}0 + \frac{1}{\alpha^2\sigma_{i,1}^4\sigma_\theta^2}\xi_1 + \frac{1}{\sigma_{i,1}^2}\tilde{s}_1^i}{\frac{1}{\sigma_v^2} + \frac{1}{\alpha^2\sigma_{i,1}^4\sigma_\theta^2} + \frac{1}{\sigma_{i,1}^2}}, \; \mathbb{E}_{\theta,i}\left[\mathbb{E}\left(s|\mathcal{F}_1^i\right)\right] = \frac{\frac{1}{\alpha^2\sigma_{i,1}^4\sigma_\theta^2} + \frac{1}{\sigma_{i,1}^2}}{\frac{1}{\sigma_v^2} + \frac{1}{\alpha^2\sigma_{i,1}^4\sigma_\theta^2} + \frac{1}{\sigma_{i,1}^2}} s$$

where $\sigma_{i,1}^2 \equiv \Sigma_{\tilde{s},1}$. As a consequence, when averaging over net supply shocks, the

[4] When there is a single underlying signal it is convenient use this definition of ξ_t, in which case the price function (5) becomes $P_t = \left(\hat{K} - p_t\right) E\left[s|\mathcal{F}_t^c\right] + p_t\xi_t$.

[5] This follows from $\frac{p_{\theta,1}}{p_1} < 0$, which gives us the expected negative relation between supply and prices.

AI believes that the net supply was different from its unconditional mean of zero, $\mathbb{E}_\theta\left[\mathbb{E}\left(\theta_1|\mathcal{F}_1^{AI}\right)\right] \neq \mathbb{E}(\theta) = 0$.[6] Specifically, in the 3-period example of Barbosa (2011),

$$\mathbb{E}_{\theta,i}\left[\mathbb{E}\left(\theta_1|\mathcal{F}_1^i\right)\right] = -\frac{Var\left(v|\mathcal{F}_1^i\right)}{\alpha\sigma_{i,1}^2\sigma_v^2}s.$$

However, $\mathbb{E}_\theta\left[\mathbb{E}\left(\theta_1|\mathcal{F}_1^{AI}\right)\right] \neq \mathbb{E}(\theta) = 0$ only if: (i) the underlying signal s does not coincide with its prior belief ($s \neq 0$); (ii) the prior belief about s is informative ($\sigma_v^2 < \infty$); (iii) the exogenous information received at date 1 is informative ($\sigma_{i,1}^2 < \infty$); and (iv) the underlying signal is not learned exactly at date 1 ($Var\left(v|\mathcal{F}_1^i\right) > 0$, which requires that $\alpha > 0$, $\sigma_\theta^2 > 0$ and $\sigma_{i,1}^2 > 0$, i.e. the initial exogenous information has to be private).

So, because on average the AI expects a non-zero net supply level when exogenous information is released, he expects the net supply to mean-revert in the following periods, which has an impact on future prices. For example, following the release of exogenous information based on a positive underlying signal ($s > 0$), the AI believes that the net supply was negative. Then he expects the net supply to increase in the following periods as it converges to its unconditional mean of zero. And, as a result, he expects prices to decrease in the short-run. In the 3-period example,

$$\begin{aligned}
\mathbb{E}_{\theta,i}\left[\mathbb{E}\left(\theta_2-\theta_1|\mathcal{F}_1^i\right)\right] &= (1-\rho)\frac{Var\left(v|\mathcal{F}_1^i\right)}{\alpha\sigma_{i,1}^2\sigma_v^2}s > 0 \\
\mathbb{E}_{\theta,i}\left[\mathbb{E}\left(\Delta P_2|\mathcal{F}_1^i\right)\right] &= -\frac{Var\left(v|\mathcal{F}_2^i\right)Var\left(v|\mathcal{F}_1^i\right)\left(1-\rho+\alpha^2\sigma_\theta^2\sigma_{i,1}^2\right)\alpha^2\sigma_\theta^2}{\sigma_v^2+\alpha^2\sigma_\theta^2\sigma_{i,1}^2\left(\sigma_v^2+\sigma_{i,1}^2+\alpha^2\sigma_\theta^2\sigma_{i,1}^2\sigma_v^2\right)}(1-\rho)s < 0.
\end{aligned}$$

To profit from this expectation, the AI takes a short position. However, since the net supply is in fact zero, his demand has to be zero as well, otherwise the market does not clear. This requires that his hedging demand exactly offsets his short-term speculative demand. But that cannot happen if the price equals his expectation of the liquidation value. This is very easy to see in a 3-period model. Using equation (8) we have

$$X_t^i = \frac{1}{\alpha}\xi_{1,1}\mathbb{E}\left(P_2-P_1|\mathcal{F}_t^i\right)+\frac{1}{\alpha}\xi_{2,1}\mathbb{E}\left(v-P_1|\mathcal{F}_t^i\right).$$

[6] Another way to see this is the following. Consider the realized scenario $\left(s,\varepsilon_1^i,\theta_1\right) = (s,0,0)$ and that, without loss of generality, $s > 0 = \mathbb{E}(v)$. This scenario implies a deviation of s from its prior mean of 0, but no deviation of ε_1^i and of θ_1 from their prior means. All three variables have a normal prior distribution and, as we know, the only point at which the probability density function of a normal distribution is zero is at its mean. This means that the alternative scenario $\left(s-\epsilon,\epsilon,\frac{p_1}{p_{\theta,1}}\epsilon\right)$, $\epsilon \gtrapprox 0$ is marginally more likely than the realized scenario $(s,0,0)$ and so, on average, the AI believes the price increase that was exclusively due to $s > 0$ was in fact partly explained by decrease in the net supply.

When there is no residual uncertainty it can be shown that $\xi_{1,1}$ and $\xi_{2,1}$ are strictly positive (see Appendix A.4). The first term corresponds to the short-term demand, which in our example is negative, and the second to the hedging demand. It is clear that the market clears only if the hedging demand is positive, which means that the date 1 price has to be smaller than the expectation of the liquidation value,

$$\begin{aligned}\mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}_t^{AI}\right)-P_1\right] &= -\frac{\xi_{1,1}}{\xi_{2,1}}\mathbb{E}_\theta\left[\mathbb{E}\left(P_2-P_1|\mathcal{F}_t^{AI}\right)\right] \\ &= \frac{Var\left(v|\mathcal{F}_2^i\right)Var\left(v|\mathcal{F}_1^i\right)\left[\sigma_v^2+\alpha^2\sigma_\theta^2\sigma_{i,1}^2\left(\sigma_{i,1}^2+\rho\sigma_v^2\right)\right]}{\sigma_{i,1}^2\sigma_v^2\left[\sigma_v^2+\alpha^2\sigma_\theta^2\sigma_{i,1}^2\left(\sigma_v^2+\sigma_{i,1}^2+\alpha^2\sigma_\theta^2\sigma_{i,1}^2\sigma_v^2\right)\right]}\left(1-\rho\right)s>0.\end{aligned}$$

When there are more than 3 periods, the intuition remains the same. The hedging demand has to be large enough to offset the short-term speculative demand. But once again, that does not happen if the current price equals the AI's expectation of the liquidation value. If that were the case, the AI would expect some of the future prices to be below (above) the current price, but none above it, if $s>0$ ($s<0$). For the AI to demand the market clearing zero quantity, he would have to ignore the short-term trading opportunities. Therefore, under assumption 5, the current price has to be below (above) the AI's expectation of the liquidation value when $s>0$ ($s<0$), so that the hedging demand offsets the short-term demand. In other words, conditional on s, expectations will have to be closer to fundamentals than prices.

By now it is clear that the AI anticipates short-term trading opportunities only if he expects the current net supply level to differ from its unconditional mean. As we just saw, this is the case at the date of release of private exogenous information. But since there is endogenous production of information, is this still the case in the periods that follow? On average, yes. This is so because endogenous information allows the AI to learn about the liquidation value only gradually, without ever knowing exactly its value. If investors were to learn exactly the liquidation value, they would be able to deduce the level of net supply from prices. And then, on average, they would see that the net supply is zero and would not expect short-term trading opportunities.

But in reality, this is never the case. Each period investors obtain an additional noisy signal for the liquidation value from the contemporaneous price (ξ_t, which on average is unbiased i.e. $\mathbb{E}_\theta\left(\xi_t\right)=v$), by comparing their forecast with the realized price. Forecast errors can originate from wrong previous expectations, reason why investors can extract additional information from each successive price. However, they can also be the result of a contemporaneous supply shock, which introduces noise. Because all signals observed are noisy, investors always put some weight into their prior beliefs. This

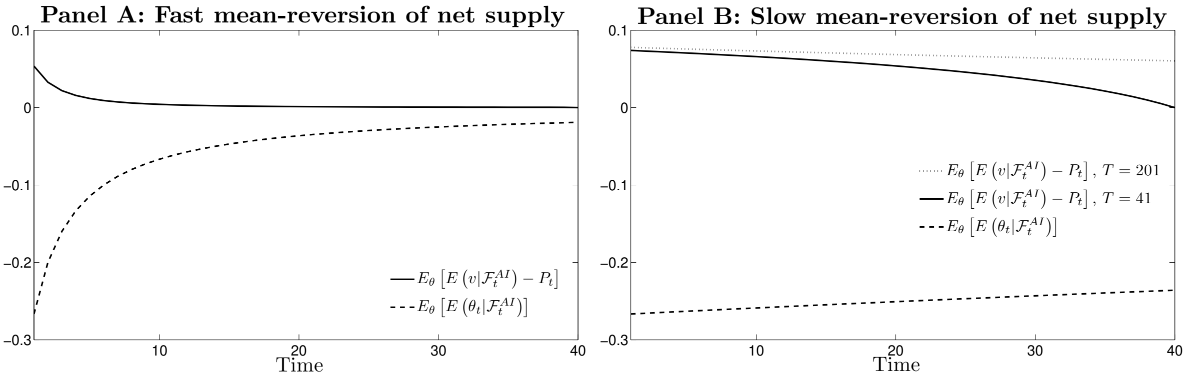


Figure 2: The average expectation of the net supply level and the discrepancy between the average price and the average expectation of the liquidation value. This figure illustrates the relation between the the average expectation of the net supply level and the difference between the average expectation of the liquidation value and the average price when there is mean-reversion of net supply. In both panels, $s > 0$, and so positive values of $\mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}_t^{AI}\right) - P_t\right]$ mean that expectations are closer to fundamentals than prices. In panel A the mean-reversion of net supply is fast ($\rho = 0$), and lots of endogenous information is produced, whereas in panel B the mean-reversion is slow ($\rho = 0.9$). The rest of the parametrization is common to both panels: $T = 41$, $n = 1$, $\sigma_v^2 = 0.25$, $\Sigma_s = 0$, $\Sigma_{\tilde{s},1} = 1$, $\Sigma_{\tilde{s},t} = 10^{10}$ $(1 < t \leq T-1)$, $\sigma_\theta^2 = 0.1$, $\alpha = 2$, $s = 1$. The case of a negative underlying signal ($s < 0$) is the symmetric of the case depicted.

means that, on average, AI's beliefs about the liquidation value remain biased toward prior beliefs. And this implies that the AI always believes that the contemporaneous net supply level is negative.[7] However, the relative weight investors put on prior beliefs when forming posterior beliefs declines as time passes and more prices are observed. Thus, beliefs about the underlying signal and the contemporaneous net supply level become more accurate as time passes, without ever being perfectly accurate. In the 3-period example,

$$\begin{aligned}
\mathbb{E}_\theta\left[\mathbb{E}\left(s|\mathcal{F}_2^i\right)\right] &= \frac{\frac{1}{\alpha^2\sigma_{i,1}^4\sigma_\theta^2} + \frac{1}{\sigma_{i,1}^2} + \frac{(1-\rho)^2}{\alpha^2\sigma_{i,1}^4\sigma_\theta^2}}{\frac{1}{\sigma_v^2} + \frac{1}{\alpha^2\sigma_{i,1}^4\sigma_\theta^2} + \frac{1}{\sigma_{i,1}^2} + \frac{(1-\rho)^2}{\alpha^2\sigma_{i,1}^4\sigma_\theta^2}} s > \mathbb{E}_\theta\left[\mathbb{E}\left(s|\mathcal{F}_2^i\right)\right] \\
\mathbb{E}_\theta\left[\mathbb{E}\left(\theta_2|\mathcal{F}_2^i\right)\right] &= -\frac{Var\left(v|\mathcal{F}_2^i\right)}{\alpha\sigma_{i,1}^2\sigma_v^2} s > \mathbb{E}_\theta\left[\mathbb{E}\left(\theta_1|\mathcal{F}_1^i\right)\right].
\end{aligned}$$

The gradual but incomplete convergence of beliefs to the truth implies that the AI always expects short-term trading opportunities to exist, even though the expected profitability of these short-term trading opportunities decreases as time passes. Moreover, the riskiness of these short-term trading opportunities increases as the liquidation

[7] On average, $\mathbb{E}_\theta(\xi_t) = v$. Since ξ_t is adapted to $\mathcal{F}_t^i$, we have $\xi_t = \mathbb{E}\left(\xi_t|\mathcal{F}_t^i\right)$ and so $\mathbb{E}_\theta\left[\mathbb{E}\left(v + \frac{p_{\theta,t}}{p_t}\theta_t|\mathcal{F}_t^i\right)\right] = v \Leftrightarrow \mathbb{E}_\theta\left[\mathbb{E}\left(\theta_t|\mathcal{F}_t^i\right)\right] = \frac{p_t}{p_{\theta,t}}\left\{v - \mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}_t^i\right)\right]\right\}$. If $v > 0$ and $v - \mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}_t^i\right)\right] > 0$, it then follows that $\mathbb{E}_\theta\left[\mathbb{E}\left(\theta_t|\mathcal{F}_t^i\right)\right] < 0$ since $\frac{p_t}{p_{\theta,t}} < 0$.

date comes closer, since the chance that a supply shock will not revert completely until the liquidation date increases. Therefore, although average prices are always worse predictors of the fundamental value compared to average expectations, the gap between these two decreases as time passes. This is illustrated in panel A of figure 1 and in figure 2, for the case of a positive underlying signal (the case of a negative underlying signal is the symmetric).

In panel A of figure 2, the mean-reversion of net supply is fast, which generates lots of endogenous information. As a result, there is little uncertainty in the periods close to the liquidation value. In this case, it is the convergence of the average expectation of the net supply level to zero, as opposed to the increased riskiness of short-term trading, the most important factor in the decreased attractiveness of short-term trading as the liquidation date approaches. In panel B the mean-reversion of net supply is low, and the opposite is true: the increase in riskiness is the dominant force. As we can see, the difference between the average expectation and the average price (solid line) converges to zero much faster than it would if the liquidation date took place in a more distant future (dotted line), and much faster than the AI's expectation of the net supply converges to zero (dashed line).

So, we already know that mean-reversion of net supply is crucial to obtain a discrepancy between the average price and the average expectation when there is no residual uncertainty. But how do different speeds of mean-reversion and different levels of residual uncertainty affect this result? Figure 3 provides the answer. Panel A shows that an increase in the speed of mean-reversion has a non-monotonic impact in the difference between the average expectation and the average price. This happens because, on the one hand, for a given expectation of the current net supply level, a faster mean-reversion increases the expected profitability of the short-term trading opportunities. But, on the other hand, the faster the mean-reversion, the more endogenous information is produced. The latter means that the expectation of the current net supply level will be closer to zero at all dates, which decreases the expected profitability of the short-term trading. This is specially true in the periods more distant from the initial release of exogenous information, since more endogenous information has been accumulated. The first effect, which tends to increase the divergence between the average expectation and the average price, dominates only when ρ is close to 1, otherwise the second effect, which tend to decrease that divergence, dominates.

In turn, panel B shows us that the existence of residual uncertainty has no effect on the qualitative results discussed previously. However, because residual uncertainty increases the riskiness of short-term trading, the difference between the average expec-

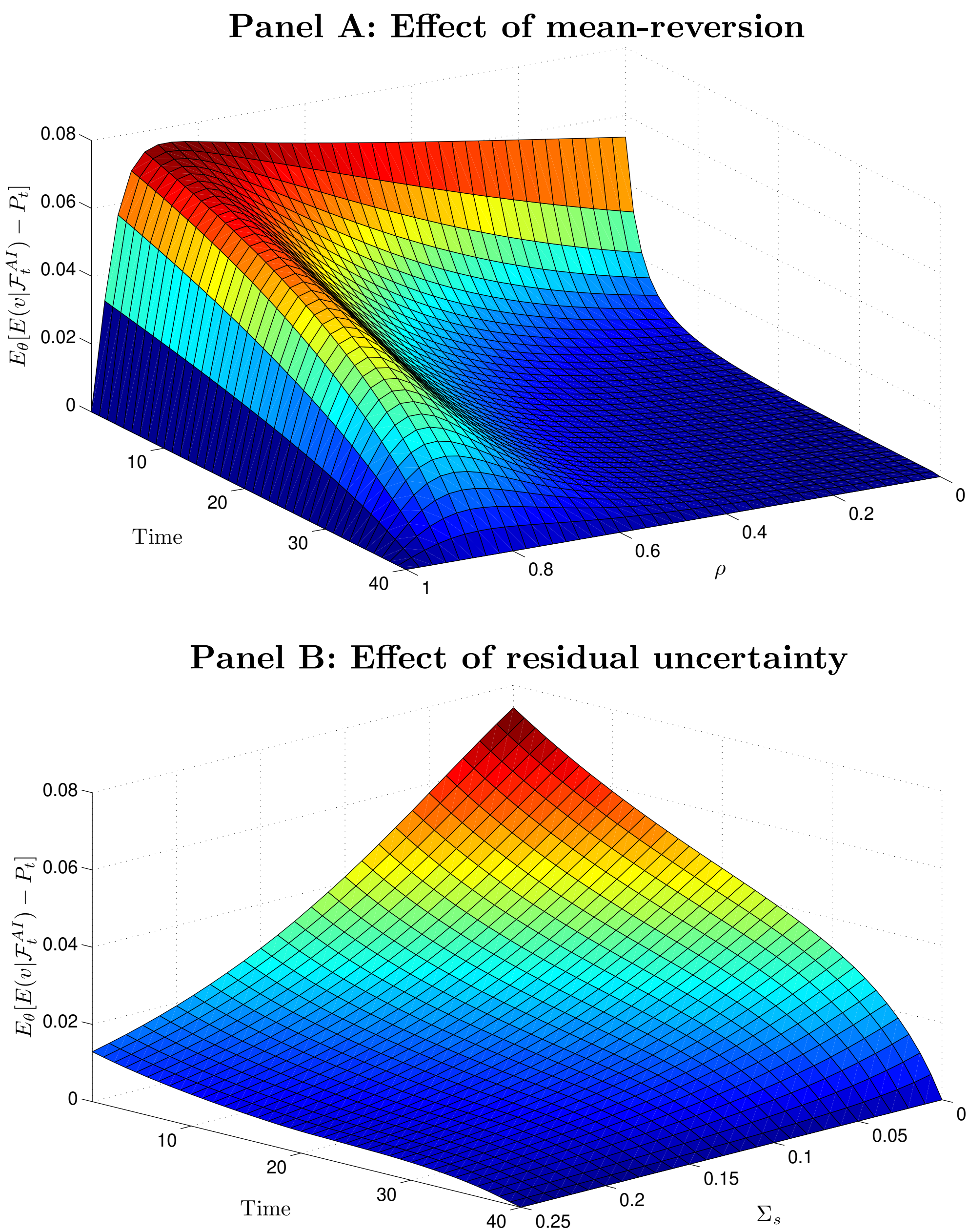


Figure 3: Comparative statics on the discrepancy between the average price and the average expectation of the liquidation value when there is mean-reversion of net supply. This figure shows how the speed of mean-reversion of net supply (panel A) and the level of residual uncertainty (panel B) impacts the difference between the average expectation of the liquidation value and the average price when the underlying signal is positive (the negative case is the symmetric) and there is mean-reversion of net supply. Positive values mean that expectations are closer to fundamentals than prices. In panel A $\rho \in [0,1]$, $\Sigma_s = 0$ and $s = 1$. In panel B $\rho = 0.8$,$\Sigma_s \in [0, 0.25]$ and s is adjusted so that the fundamental value remains unchanged as the level of residual uncertainty changes (higher residual uncertainty requires higher s). The remaining parametrization is common to both panels: $T = 41$, $n = 1$, $\sigma_v^2 = 0.25$, $\Sigma_{\tilde{s},1} = 1$, $\Sigma_{\tilde{s},t} = 10^{10}$ $(1 < t \leq T-1)$, $\sigma_\theta^2 = 0.1$, $\alpha = 2$.

tation and the average price decreases with the level of residual uncertainty.

To sum up, the release of private exogenous information based on an underlying signal that differs from investors' prior beliefs ($s \neq 0$) and the mean-reversion of net supply are the two key ingredients to create a discrepancy between the average price and the average expectation of the liquidation value. The former makes the average investor believe that the price movement contemporaneous to the release of exogenous information was partly driven by noise/liquidity demand. And the latter makes the average investor expect liquidity traders to gradually exit the market. This creates an opportunity to profit from liquidity traders on the short-run, which diverts attentions from the long-run. As a consequence, the average price differs from the average expectation of the liquidation value: the latter is always closer to the fundamental value than the former.

An alternative way of looking at this is the following. When investors expect to trade in the short-term, they care not only about the final liquidation value, but also about intermediate prices. Since prices reflect the opinions of all other investors about the liquidation value, each investor has to forecast others' opinions. These opinions reflect both the common prior information (which is public information) and the private signal. As Allen et al. (2006) put it:

> "Now suppose that the individual is asked to guess what the average expectation of the asset's payoff is. Since he knows that others have also observed the same public signal, the public signal is a better predictor of average opinion; he will put more weight on the public signal than on the private signal. Thus if individuals' willingness to pay for an asset is related to their expectations of the average opinion, then we will tend to have asset prices overweighting public information relative to the private information."

This over-reliance on public information then causes the average price to deviate more from the fundamental value than the average expectation of the liquidation value.

Finally, it is worth noting that, although it is not endogenous information per se that makes average prices worse predictors of the fundamental value than average expectations, the latter only happens when there is endogenous production of information. In this sense we can link over-reliance on public information to the existence of endogenous information.

4.3 Release of Additional Exogenous Information

The second modification to the base case is the release of additional exogenous information, which I consider here in place of the mean-reversion of net supply. I will analyze a version of the model incorporating these two features simultaneously in the next subsection. I assume that there are two dates at which exogenous information is released. The generalization to a larger number of information release dates is straightforward. In addition, I assume that the second date of information release is the last trading date ($T-1$). This is without loss of generality since, as we saw in Section 4.1, when there is no additional exogenous information to be released in the future and no mean-reversion of net supply, the average price and average expectation of the liquidation date coincide at all dates. Therefore, exogenous information is assumed to be released at dates 1 and $T-1$.

As we saw previously, investors need to engage in short-term trading for the average price to diverge from the average expectation of the liquidation value. For the former to happen, though, investors need to expect short-term price movements. In this setting, the only thing that can generate predictable short-term price changes is the incoming release of exogenous information. So, let us examine the expected impact of additional exogenous information... on investors' demands. I will get to prices later on.

Every investor anticipates two effects of the release of additional information: (i) the beliefs about the liquidation value will become more homogeneous which, for a given price, tends to make investors' demands more homogeneous as well; (ii) beliefs about the liquidation value become more precise, which leads investors to trade more aggressively based on their expectations, thus making their demands more heterogeneous. Therefore, whether the release of additional information makes demands more homogeneous or more heterogeneous, depends on which of these two effects dominate.

Curiously, when there is no residual uncertainty, the two effects exactly offset each other. That is, every investor expects the release of additional exogenous information to have no effect on the demand of every other investor. Investors may expect prices to change though,

$$\mathbb{E}\left(\Delta P_{t+\tau}|\mathcal{F}_t^i\right) = \alpha\left[Var\left(v|\mathcal{F}_{t+\tau-1}^i\right) - Var\left(v|\mathcal{F}_{t+\tau}^i\right)\right] X_t.$$

But since investors do not expect their demand to change, they will not take advantage of this expected price change, which means that they *never* engage in short-term speculative trading. Actually, it is easy to show that the demand function in this case coincides with the demand function we obtain in a static model or in the base case, that is,

$$X_t^i = \frac{\mathbb{E}\left(v|\mathcal{F}_t^i\right) - P_t}{\alpha Var\left(v|\mathcal{F}_t^i\right)}. \tag{10}$$

Focusing now on our AI, averaging over net supply shocks it follows that, $\forall\tau$

$$\mathbb{E}_{\theta,i}\left[\mathbb{E}\left(\Delta P_{t+\tau}|\mathcal{F}_t^i\right)\right] = \alpha\left[Var\left(v|\mathcal{F}_{t+\tau-1}^{AI}\right) - Var\left(v|\mathcal{F}_{t+\tau}^{iAI}\right)\right]\mathbb{E}_{\theta,i}\left(X_t\right) = 0$$

and so

$$\mathbb{E}_{\theta,i}\left[\mathbb{E}\left(v|\mathcal{F}_t^i\right)\right] = \mathbb{E}_{\theta}\left(P_t\right).$$

Therefore, when there is no residual uncertainty, the average price equals the average expectation of the liquidation value at all dates.

When there is residual uncertainty, though, the first effect always dominates the second, meaning that investors expect demands to become more homogeneous in response to the release of additional exogenous information. This happens because investors receive information about the underlying signal and not about the liquidation value. When there is residual uncertainty, the former is only a noisy signal for the latter. What this means is that, in relative terms, the new information resolves less uncertainty about the liquidation value than it resolves uncertainty about the underlying signal. As a consequence, the trading aggressiveness increases less than it would increase if there were no residual uncertainty and the underlying signal coincided with the liquidation value. Hence, the first effect dominates. But the simplest way to illustrate why residual uncertainty leads to more homogeneous demands, is to consider what happens when the incoming exogenous information resolves all uncertainty about the underlying signal. In this case all investors will share the same beliefs. When there is residual uncertainty, the asset remains risky, and so in equilibrium every investor demands the same quantity. Obviously, demands become more homogeneous. In contrast, if there is no residual uncertainty, then the asset becomes riskless. This makes investors indifferent between demanding any quantity in equilibrium, and so they can keep their previous demand unchanged and the market still clears.

So, what is the implication of expecting more homogeneous demands? The answer is: short-term speculative trading which leads to a divergence between the average price and the average expectation of the liquidation value. To show this, let us focus on the AI and on the average case. Also, to simplify the exposition, suppose that $s > 0$. As we saw in the previous subsection, following the initial release of private exogenous information at date 1, the AI will believe that the net supply level was negative, and form an expectation about the underlying signal that is biased toward his prior belief.

The latter means that he believes the idiosyncratic error of his signal was positive. That is the AI believes that the majority of investors received less optimistic signals and, in particular, that the average investor his somebody else who has observed a less optimistic signal. Even though there will be some endogenous production of information going on (more on this later), the AI will maintain these qualitative beliefs at all dates until additional exogenous information is released.

Let us start by considering what happens to the average price and average expectation at the date immediately before the release of new information, date $T-2$. As mentioned before, on average the AI believes that the current net supply level is negative. Because he expects demands to become more homogeneous following the release of new information at date $T-1$, he expects his own date $T-1$ demand to decrease and become negative (recall that market equilibrium requires that his demand be zero at date $T-2$). Since at date $T-1$ we are back to the base case, demands are as in the static model, and given by equation (10). Therefore, if the AI expects to demand a negative quantity, he has to expect a price above his current expectation of the liquidation value, that is, $\mathbb{E}_\theta\left[\mathbb{E}\left(\Delta P_T|\mathcal{F}^{AI}_{T-2}\right)\right] < 0$.

Market clearing at date $T-2$ requires the AI to demand zero. Thus, the short-term demand has to offset the hedging demand that results from the expectation of a negative demand at $T-1$. The sign of the hedging demand will depend on the precision of the exogenous information released at date $T-1$. If information is precise enough, the correlation between ΔP_{T-1} and ΔP_T will be positive, resulting in a positive hedging demand. But hedging demand can also be null or negative if information is not precise enough. Let us consider first the case of negative hedging demand. In this case, the short-term demand has to be positive, which means that $\mathbb{E}_\theta\left[\mathbb{E}\left(\Delta P_{T-1}|\mathcal{F}^{AI}_{T-2}\right)\right] > 0$. However, under assumption 5, the hedging demand offsets the short-term demand only if $\mathbb{E}_\theta\left[\mathbb{E}\left(\Delta P_{T-1}+\Delta P_T|\mathcal{F}^{AI}_{T-2}\right)\right] < 0$, and so we obtain that

$$\mathbb{E}_\theta\left(P_{T-2}\right) > \mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}^{AI}_{T-2}\right)\right].$$

The case of positive or null hedging demand is straightforward. In these cases, market clearing requires a weakly negative short-term demand, which implies that $\mathbb{E}_\theta\left[\mathbb{E}\left(\Delta P_{T-1}|\mathcal{F}^{AI}_{T-2}\right)\right] \leq 0$. It is then immediate that $\mathbb{E}_\theta\left[\mathbb{E}\left(\Delta P_{T-1}+\Delta P_T|\mathcal{F}^{AI}_{T-2}\right)\right] < 0$ and so $\mathbb{E}_\theta\left(P_{T-2}\right) > \mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}^{AI}_{T-2}\right)\right]$. Hence, in contrast to what happens when there is mean-reversion of net supply, the average price is closer to the fundamental value than the average expectation of the liquidation value in the date immediately before the release of additional exogenous information.

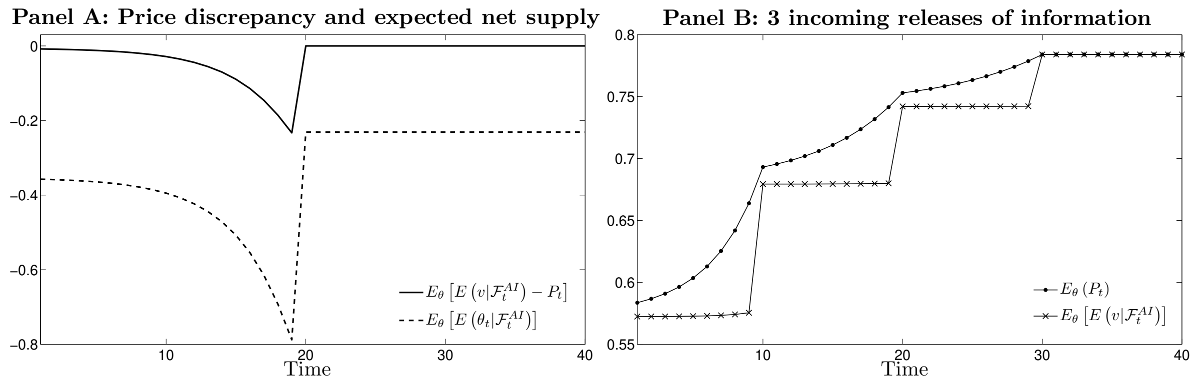


Figure 4: Discrepancy between the average price and the average expectation of the liquidation value. This figure shows how the average expectation of the liquidation value differs from the average price when there is an incoming release of information based on the same underlying signal and the underlying signal is positive (the negative case is the symmetric). Panel A shows the relation between that difference and the average expectation of the net supply level when new information is released at date 20, with $\Sigma_{\tilde{s},20} = 0.1$ and $\Sigma_{\tilde{s},t} = 10^{10}\,\forall t \setminus \{1, 20\}$. Negative values of $\mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}_t^{AI}\right) - P_t\right]$ mean that prices are closer to fundamentals than expectations. Panel B shows that difference when there is more than one incoming release of information, with $\Sigma_{\tilde{s},10} = \Sigma_{\tilde{s},20} = \Sigma_{\tilde{s},30} = 1$ and $\Sigma_{\tilde{s},t} = 10^{10}\,\forall t \setminus \{1, 10, 20, 30\}$. In both panels, the remaining parametrization is the same: $T = 41$, $n = 1$, $\rho = 1$, $\sigma_v^2 = 0.25$, $\Sigma_s = 0.5$, $\Sigma_{\tilde{s},1} = 1$, $\sigma_\theta^2 = 0.1$, $\alpha = 2$, $s = 3$.

This is also true at all dates between the two releases of information. However, the difference between the average price and the average expectation increases with the proximity to the date where the additional exogenous information is released (see panel B of figure 1). This is explained by the endogenous information that is produced by the speculative trading. As we just saw, investors trade on their expectation of the impact of new exogenous information on prices. Obviously that expectation is based on their own private information. Therefore, the increased trading aggressiveness that stems from short-term trading leaks some of investors' private information into prices every period. This means that, as the date of release of the new information approaches, the asset becomes less risky. Because the AI believes that the negative supply is negative, he expects this risk reduction to have a negative impact on price. However, unlike what the AI believes, on average the net supply level is in fact zero, and so the risk reduction has no impact on prices whatsoever. This means that the AI's price forecast always errs on the low side. The AI knows that only two things could have gone wrong: either he underestimated the previous period net supply level, which lead him to expect a stronger price adjustment in response to the risk reduction; or the contemporaneous supply shock was negative, offsetting the reduction in the riskiness of holding the asset with an increase in the magnitude of the quantity that needs to be held. Like in previous situations, the AI attributes his forecast error in part to each of these two factors. Thus, even though he corrects for an underestimation of the previous period net supply, he now believes that the net supply is even more negative than before.[8] Therefore, the AI now expects a larger reduction in his demand at date $T-1$ than before, which implies that he expects a larger discrepancy between P_{T-1} and $\mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}^{AI}_{T-\tau}\right)\right]$. And this translates into a bigger difference between $P_{T-\tau}$ and $\mathbb{E}_\theta\left[\mathbb{E}\left(v|\mathcal{F}^{AI}_{T-\tau}\right)\right]$. Figure 4 provides an illustration. In addition, panel B confirms that, as mentioned in the beginning of this subsection, all the results generalize to the case where there is more than one incoming release of information.

We just saw that the discrepancy between the average expectation and the average price stems from the combination of incoming release of exogenous information and residual uncertainty. The next question is how does that discrepancy change with the precision of incoming information and with the level of residual uncertainty. Obviously, the expected price impact of the incoming information increases with its precision. Thus, unsurprisingly the discrepancy between the average price and the average expectation increases with the precision of the incoming information, as we can see from panel A of figure 5.

In turn, as we can see from panel B, the increase in the level of residual uncertainty has a non-monotonic impact on the difference between the average expectation and the average price. The reason for this is that an increase in the level of residual uncertainty produces two effects on the profitability of short-term trading, and thus on the discrepancy between the average price and the average expectation. On the one hand, the higher the level of residual uncertainty, the more homogeneous the demands are expected to become after the release of the new information. As we saw, this tends to increase the expected profitability of short-term trading. But, on the other hand, as the residual uncertainty increases, the relevance of the incoming information decreases, since it resolves a smaller fraction of the overall uncertainty. Therefore, the expected price impact of the new information tends to decrease, which decreases the expected profitability of short-term trading. The first effect dominates only when the residual uncertainty is not too large, reason why initially the discrepancy between the average price and the average expectation increases with the level of residual uncertainty. But then after some point the second effect takes over and this difference starts decreasing with the level of residual uncertainty.

Summing up, the release of additional exogenous information based on the same

[8] This is the case because endogenous information is very noisy, and so investors attribute the bulk of the forecast error to a contemporaneous supply shock. If endogenous information were more accurate, investors would put more weight on the hypothesis of wrong beliefs, and would would make a stronger correction in their beliefs. In that case, the expectation of the net supply level could become less negative. However, numerical results suggest that this is never the case.

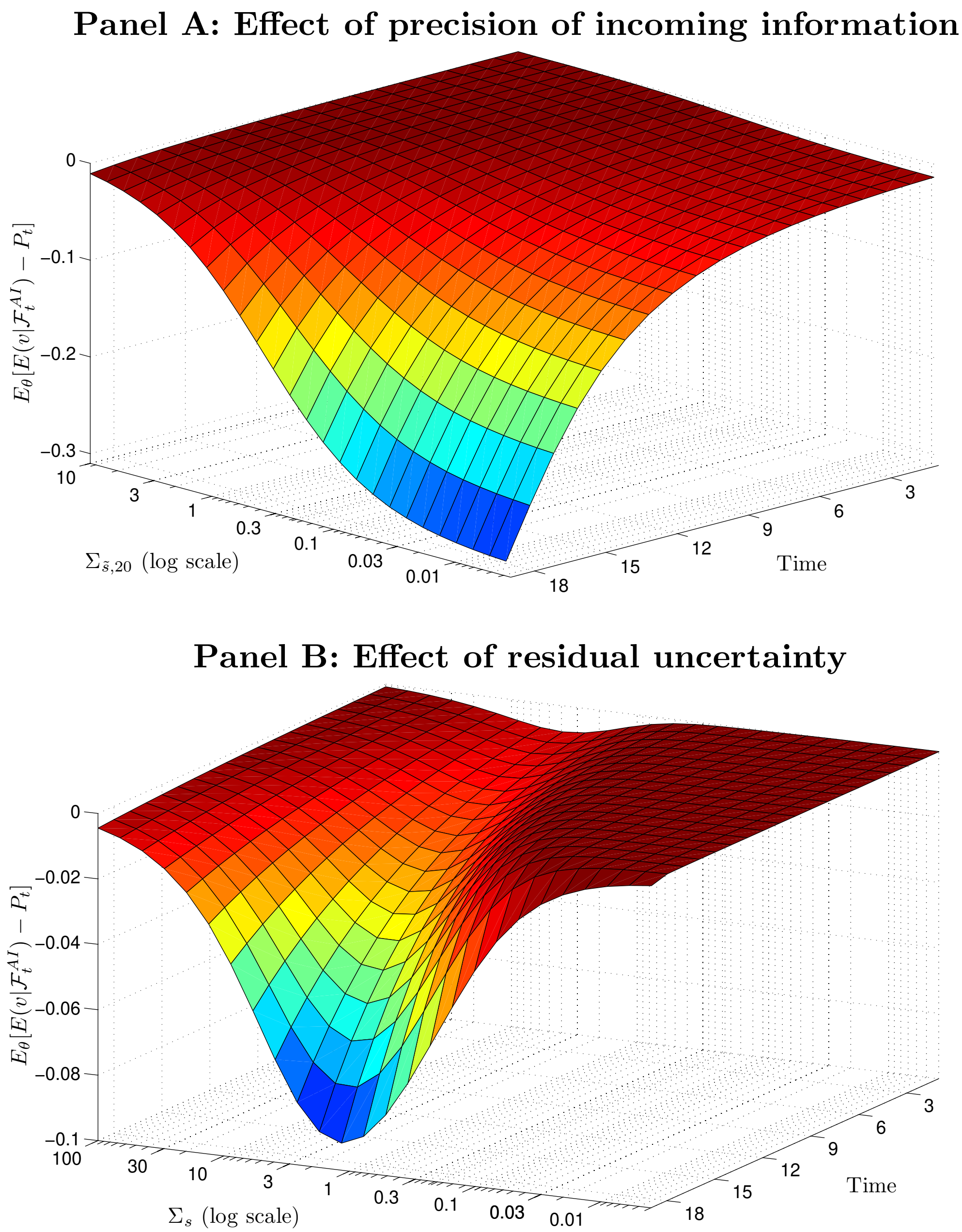


Figure 5: Comparative statics on the discrepancy between the average price and the average expectation of the liquidation value when there is an incoming release of exogenous information. This figure shows how the precision of incoming information (panel A) and the level of residual uncertainty (panel B) impacts the difference between the average expectation of the liquidation value and the average price when the underlying signal is positive (the negative case is the symmetric) and there is an incoming release of exogenous information. Negative values mean that prices are closer to fundamentals than expectations. In panel A $\Sigma_{\tilde{s},20} \in [0, 10]$, $\Sigma_s = 0.5$ and $s = 3$. I panel B: $\Sigma_s \in [0, 100]$, $\Sigma_{\tilde{s},20} = 1$ and s is adjusted adjusted so that the fundamental value remains unchanged as the level of residual uncertainty changes. The remaining parametrization is common to both panels: $T = 41$, $n = 1$, $\rho = 1$, $\sigma_v^2 = 0.25$, $\Sigma_s = 0.5$, $\Sigma_{\tilde{s}^i,1} = 1$, $\Sigma_{\tilde{s},t} = 10^{10}\, \forall t \setminus \{1, 20\}$, $\sigma_\theta^2 = 0.1$, $\alpha = 2$.

underlying signal as previously released *private* exogenous information, and residual uncertainty, are the two key ingredients to create a discrepancy between the average price and the average expectation, and bring the former closer to the fundamental value than the latter. Together, these two factors lead investors to speculate on the price impact of incoming information, engaging in short-term speculative trading. This diverts attentions from the long-run and allows a divergence in the average price and average expectation of the liquidation value to persist.

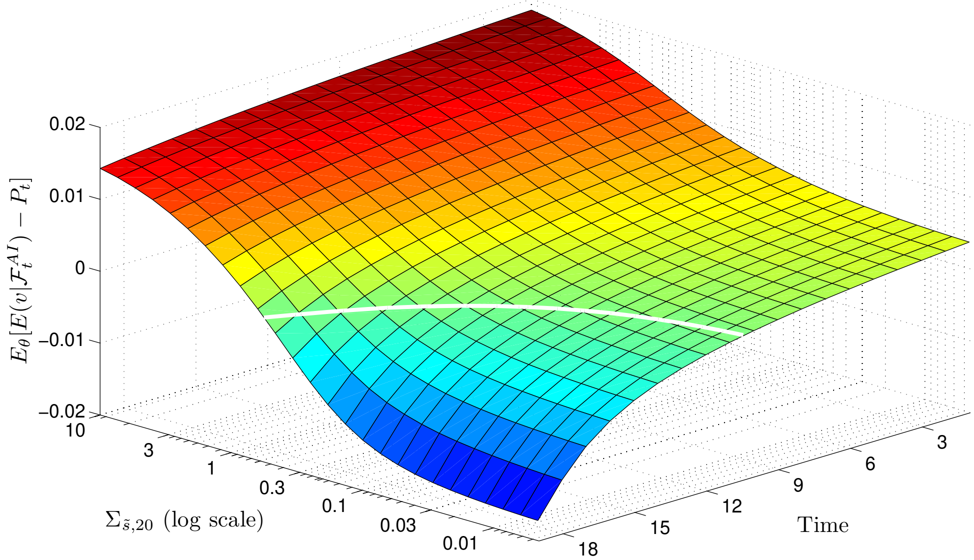
As in the case of mean-reversion of net supply (previous subsection), investors care not only about the liquidation value, but also about intermediate prices. For this reason, investors have to forecast the opinions of other investors. But in this case, forecasting the opinions of others does not call for overweighting public information. On the one hand, all investors observed the same public information (the common prior), and a private signal. Although any given investor knows what the public information was, he can only make an inaccurate estimate of what those private signals might have been, hence the tendency to over-rely on public information. But, on the other hand, all investors *will observe* another private signal related to the private signal already observed. Since the incoming private signal can be forecasted with the previously observed private signal, investors will overweight their private signal when forming their beliefs about others' beliefs. Therefore, prices will under-rely on public information.

This is what Allen et al. (2006) overlook when they asserted that short-term speculative trading *always* leads to over-reliance on public information. Even though in their model there is release of exogenous information at all dates, there is no residual uncertainty and net supply mean reverts. This is why they always obtain over-reliance but never under-reliance on public information.

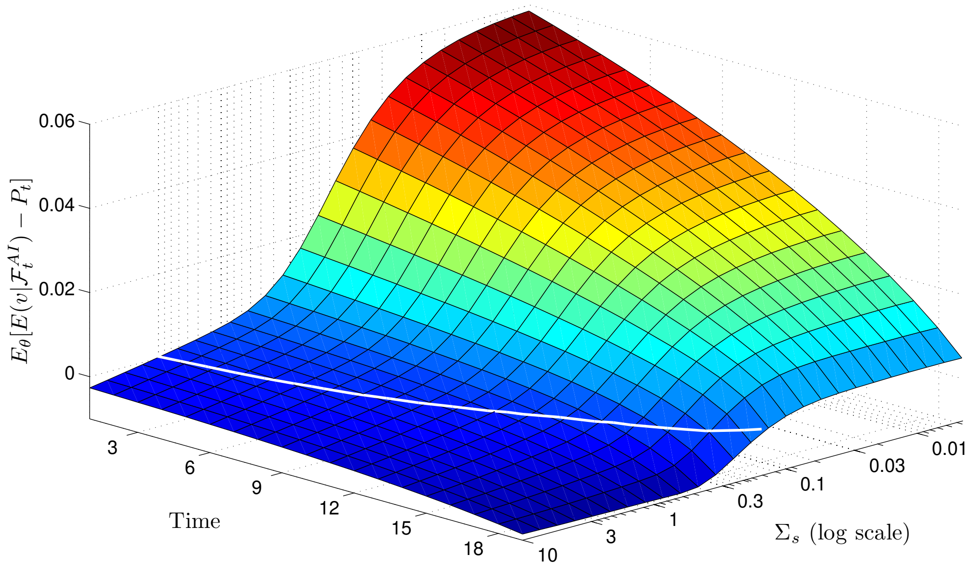
4.4 The General Case: Mean-Reversion of Net Supply and Release of Additional Exogenous Information

Now it is time to bring the two modifications to the base case considered in the previous subsections together. This corresponds to the setting in Cespa and Vives (2012), who never separate mean-reversion of net supply from the release of additional exogenous information. In a 3-period model, they find that date 1 prices over-rely on public information (that is prices are further away from fundamentals than expectations) when net supply quickly reverts to its mean and residual uncertainty is low, and that prices under-rely on public information otherwise. Since mean-reversion of net supply causes over-reliance on public information, and additional exogenous information coupled to

Panel A: Effect of precision of incoming information



Panel B: Effect of residual uncertainty



residual uncertainty causes under-reliance on public information, their result tells us the conditions in which each of the two effects dominates.

On the one hand, a persistent net supply means that the trading opportunities from changes in liquidity driven demand are bleak. In this case, and in the absence of additional exogenous information, the average price would be very close to the average expectation, specially in the dates far away from the release of the initial exogenous information. On the other hand, if relatively precise exogenous information is to be released in the future, and if residual uncertainty is relatively high, then trading opportunities based on the impact of the incoming information are substantial. In this case,

Panel C: Effect of mean-reversion of net supply

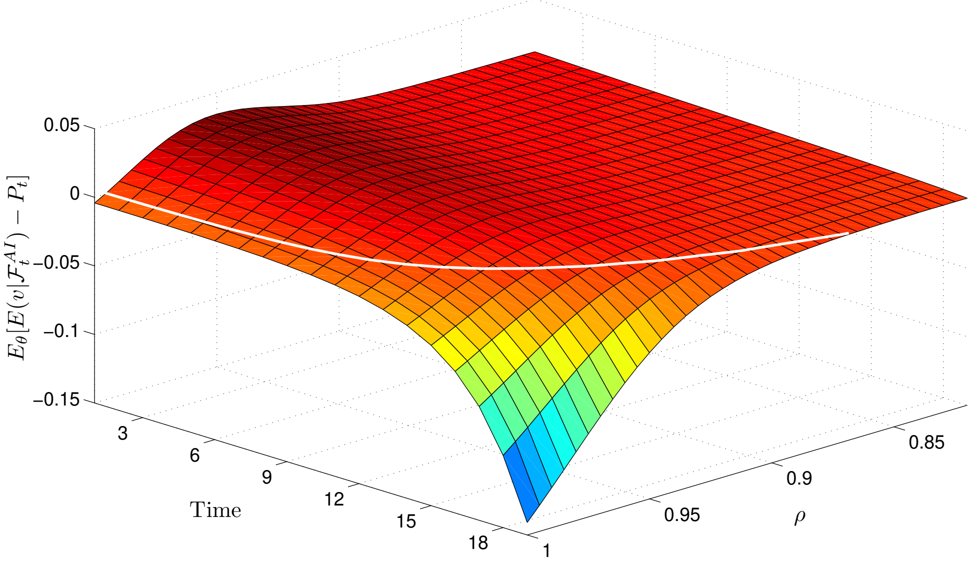


Figure 6: Comparative statics on the discrepancy between the average price and the average expectation of the liquidation value when there is mean-reversion of net supply and an incoming release of exogenous information. This figure shows how the precision of precision of incoming information (panel A), the level of residual uncertainty (panel B) and the speed of mean-reversion of net supply (panel C) impacts the difference between the average expectation of the liquidation value and the average price when there is an incoming release of exogenous information and mean-reversion of net supply, and the underlying signal is positive (the negative case is the symmetric). In panel A $\Sigma_{\tilde{s},20} \in [0, 10]$, $\Sigma_s = 0.5$ and $\rho = 0.9$. In panel B $\Sigma_s \in [0, 10]$, $\Sigma_{\tilde{s},20} = 0.2$ and $\rho = 0.9$. In panel C $\rho \in [0.82, 1]$, $\Sigma_s = 0.25$ and $\Sigma_{\tilde{s},20} = 0.2$. The remaining parametrization is the same in all panels: $T = 41$, $n = 1$, $\sigma_v^2 = 0.25$, $\Sigma_{\tilde{s},1} = 1$, $\Sigma_{\tilde{s}^i,t} = 10^{10}\,\forall t \setminus \{1, 20\}$, $\sigma_\theta^2 = 0.1$, $\alpha = 2$. In all panels, the white line corresponds to the intersection of the surface plot with the plane defined by the average price equal to the average expectation of the liquidation value. Positive (negative) values mean that prices are further away (closer) to fundamentals than expectations.

and if there is no mean-reversion of net supply, the average price differs considerably from the average expectation, specially in the dates immediately preceding the release of new information. Bringing these two effects together, it should not be a surprise that the latter dominates. Thus, the average price is closer to the fundamental value than the average expectation is or, in other words, prices under-rely on public information. This is the case specially as we move closer to the date where the new information is released, when the second effect is stronger, and away from the date where the initial information was released, where the first effect is stronger. When mean-reversion of net supply is fast and residual uncertainty is low and/or the information to be released is inaccurate, the opposite holds. But if the strength of the two effects that push the average price away from the average expectation of the liquidation value are balanced enough, we obtain under-reliance on public information in the dates closer to the release of new information, and over-reliance on public information in the dates closer to the release of the initial information. This is shown in panel C of figure 1.

Figure 6 shows how changing each of the three relevant parameters, precision of incoming information ($\Sigma_{\tilde{s},20}$), level of residual uncertainty (Σ_s) and speed of mean-reversion of net supply (ρ), can tilt the balance toward under-reliance or over-reliance on public information.

5 Discrepancy between Average Prices and Expectations: The Case of Multiple Underlying Signals

In contrast to the previous section this section, here I consider that different releases of exogenous information are based on different underlying signals. Provided that the two underlying signals are positively correlated, the qualitative results of Section 4.3 remain unchanged, as we can see from panel A of figure 7.

Note that the two underlying signals being perfectly positively correlated is equivalent to the underlying signal being the same. As the correlation between the two underlying signals weakens, the incoming information resolves increasingly more uncertainty. This implies that, even though demands will still become more homogeneous after the release of the new information, demands will converge less as that correlation weakens. In fact, when the two signals are uncorrelated, demands are not expected to change in response to the new information, exactly as in the case where there is no residual uncertainty. In that case, investors behave as in the static model and do not engage in short-term trading. As we discussed in Section 4.3, the expectation of more homogeneous demands generates short-term trading opportunities, which make prices closer to fundamentals than expectations. Therefore, as the positive correlation weakens, that difference between the average price and the average expectation shrinks.

As we move from uncorrelated signals, where the balance between the convergence of beliefs and the increasing trading aggressiveness that keeps demands unchanged is struck, to negatively correlated signals, demands become more *heterogeneous* after the release of information. Now the new information resolves so much uncertainty that the increase in trading aggressiveness dominates the convergence of beliefs. This case is basically the opposite of what we analyzed in Section 4.3, and we obtain that the average price is farther away from the fundamental value than the average expectation is, as we can see from panel B of figure 7. But there is a nuance. When the two signals are negatively correlated the average price starts closer to the fundamental value that the average expectation if the date of release of new information is distant enough,

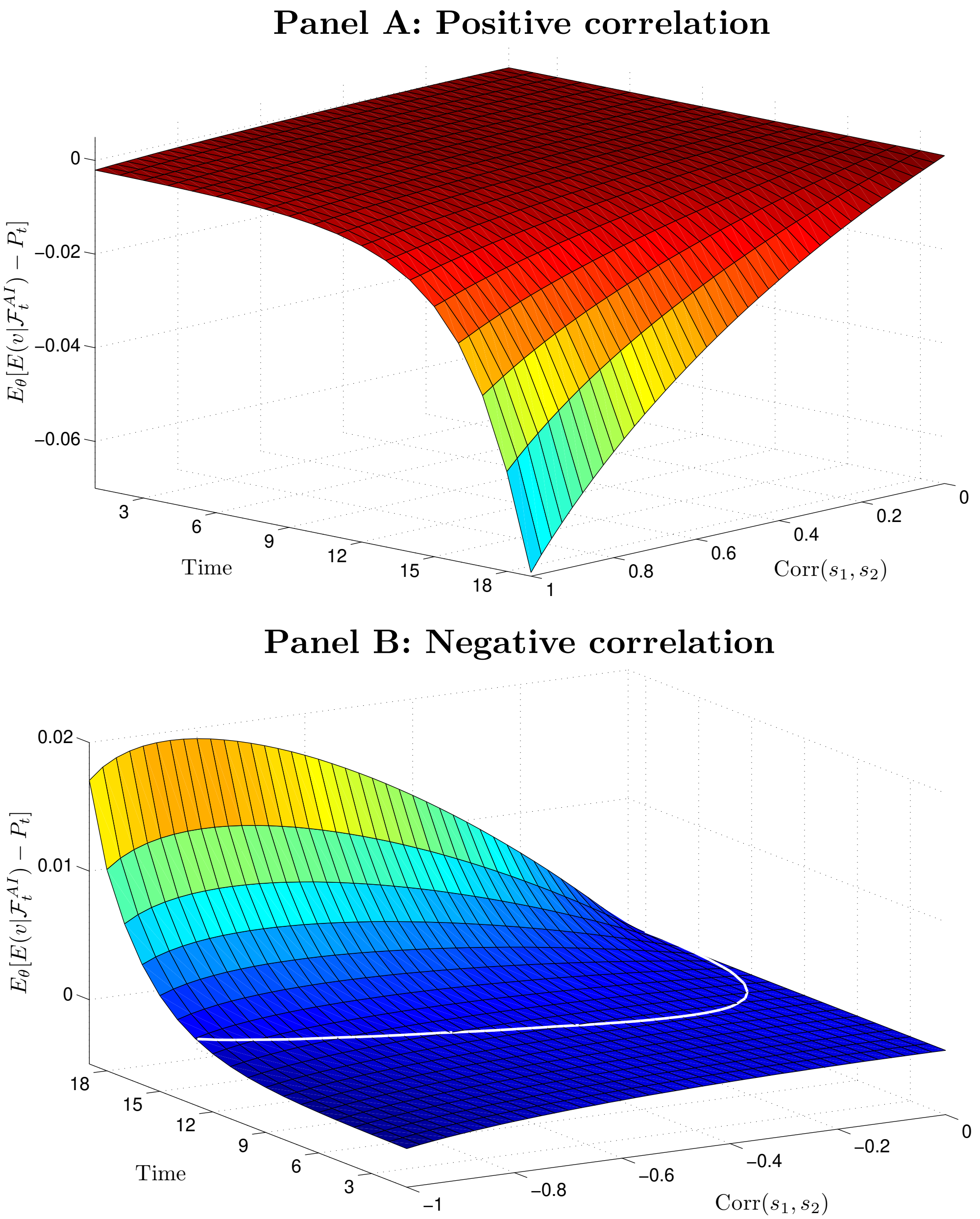


Figure 7: The correlation between underlying signals and the discrepancy between the average price and the average expectation of the liquidation value. This figures shows how the difference between the average expectation of the liquidation value and the average price changes with the correlation between the signal underlying the past release of information (s_1) and the signal underlying the incoming release of information (s_2), when there is no mean-reversion of net supply. In panel A the correlation ranges from 0 to 1, and in panel B from 0 to -1. In both panels the parametrization is as follows: $T = 41$, $n = 2$, $\rho = 1$, $\sigma_v^2 = 0.25$, $\Sigma_s = \begin{bmatrix} 0.25 & \sigma_{s_1,s_2} \\ \sigma_{s_1,s_2} & 0.25 \end{bmatrix}$, $\Sigma_{\tilde{s},1} = \begin{bmatrix} 1 & 0 \\ 0 & 1^{10} \end{bmatrix}$, $\Sigma_{\tilde{s},20} = \begin{bmatrix} 1^{10} & 0 \\ 0 & 0.2 \end{bmatrix}$, $\Sigma_{\tilde{s},t} = \begin{bmatrix} 1^{10} & 0 \\ 0 & 1^{10} \end{bmatrix}$ $\forall t \setminus \{1, 20\}$, $\sigma_\theta^2 = 0.1$, $\alpha = 2$, $s_1 = 1$, $s_2 = 1$. The white line corresponds to the intersection of the surface plot with the plane defined by the average price equal to the average expectation of the liquidation value. Positive (negative) values mean that prices are further away (closer) to fundamentals than expectations.

reversing later on. This result stems from a different correlation structure between expected price changes and the consequent impact on hedging demands.

In any case, the link between over-(under-)reliance on public information and prices being closer (further away) from fundamentals than expectations is preserved. When correlation is negative prices start by under-relying on public information and then after some point over-rely on public information

Now I switch the attention to how the existence of multiple underlying signals change the results of Section 4.2. As we saw in Section 4.2, when there is mean-reversion of net supply, prices under-rely on public information after the last release of exogenous information, and therefore prices are further away from fundamentals than expectations (see panel A and C of figure 1). However, when there are multiple underlying signals, this is no longer true. Even though prices under-rely on public information after the last information release, they may be either closer of further away from fundamentals than expectations. It all depends on the value of the underlying signals.

Before I proceed, I need to provide the definition of fundamental when there is more than one underlying signal. The date t fundamental value is now the expectation of the liquidation obtained from the direct observation of all underlying signals for which exogenous information was already released by date t. This means that the fundamental value changes whenever exogenous information for a new underlying signal is released for the first time.

From now on, let us focus on the case of two distinct underlying signals. Generalizing from lemma 6, we have that, after the release of the second and last exogenous information, the fundamental value, average price and average expectation of the liquidation value are given by

$$
\begin{aligned}
FV_t &= \hat{K}\boldsymbol{s} \\
&= a_1 s_1 + a_2 s_2 \\
\mathbb{E}_\theta\left(P_t\right) &= \hat{K}\boldsymbol{s} + \left(\hat{K} - \hat{\boldsymbol{p}}_t\right)\left\{\mathbb{E}_\theta\left[\mathbb{E}\left(\boldsymbol{s}|\mathcal{F}_t^c\right)\right] - \boldsymbol{s}\right\} \\
&= FV_t + b_{1,t}\left\{\mathbb{E}_\theta\left[\mathbb{E}\left(s_1|\mathcal{F}_t^c\right)\right] - s_1\right\} + b_{2,t}\left\{\mathbb{E}_\theta\left[\mathbb{E}\left(s_2|\mathcal{F}_t^c\right)\right] - s_2\right\} \\
\mathbb{E}_{\theta,i}\left[\mathbb{E}\left(v|\mathcal{F}_t^i\right)\right] &= \hat{K}\boldsymbol{s} + \hat{K}\left(I_2 - \hat{\Gamma}_t\right)\left\{\mathbb{E}_\theta\left[\mathbb{E}\left(\boldsymbol{s}|\mathcal{F}_t^c\right)\right] - \boldsymbol{s}\right\} \\
&= FV_t + c_{1,t}\left\{\mathbb{E}_\theta\left[\mathbb{E}\left(s_1|\mathcal{F}_t^c\right)\right] - s_1\right\} + c_{2,t}\left\{\mathbb{E}_\theta\left[\mathbb{E}\left(s_2|\mathcal{F}_t^c\right)\right] - s_2\right\}
\end{aligned}
$$

for some constants a_1, a_2, $b_{1,t}$, $b_{2,t}$, $c_{1,t}$ and $c_{2,t}$, and where I_2 is a 2-dimensional identity matrix. All three variables are a positive function of s_2. Generically, these functions have different slopes, because each variable puts different weights on prior information and underlying signals. Therefore, keeping s_1 fixed, if we plot these three variables as a function of s_2, we find that they will eventually cross with each other at some point. But, for prices to be further away from fundamentals than expectations in all scenarios, the three variables have to cross at the same point. This happens if and only if there is a s_2 that solves the following system of equations

$$\begin{cases} b_{1,t}\left\{\mathbb{E}_\theta\left[\mathbb{E}\left(s_1|\mathcal{F}_t^c\right)\right]-s_1\right\}+b_{2,t}\left\{\mathbb{E}_\theta\left[\mathbb{E}\left(s_2|\mathcal{F}_t^c\right)\right]-s_2\right\}=0 \\ c_{1,t}\left\{\mathbb{E}_\theta\left[\mathbb{E}\left(s_1|\mathcal{F}_t^c\right)\right]-s_1\right\}+c_{2,t}\left\{\mathbb{E}_\theta\left[\mathbb{E}\left(s_2|\mathcal{F}_t^c\right)\right]-s_2\right\}=0 \end{cases},$$

that is, we need $\frac{b_{1,t}}{b_{2,t}}=\frac{c_{1,t}}{c_{2,t}}$ or $\mathbb{E}_\theta\left[\mathbb{E}\left(s_1|\mathcal{F}_t^c\right)\right]-s_1=0$. However, generically the former does not hold, even though $b_{1,t}>c_{1,t}$ and $b_{2,t}>c_{2,t}$ (over-reliance on public information), and so the system of equations is solved only when both signals coincide with the prior belief on the liquidation value.[9] Therefore, if the first underlying signal does not coincide with its unconditional mean, then there are some values of the second underlying signal that bring the average price closer to the fundamental value than the average expectation. In other words over-reliance on public information is no longer a synonym of prices being further away from fundamentals than expectations.

Figure 8 illustrates the situation. In panel A the fundamental value is more sensitive to s_2 than the average expectation, and in turn the latter is more sensitive to s_2 than the average price. In this case there is a bounded region of s_2 values for which prices are closer to fundamentals than expectations. In contrast, in panel B the average expectation is more sensitive to s_2 than the average price and the fundamental value. Now there is an unbounded region of s_2 values for which prices are closer to fundamentals than expectations.

6 Other Implications

The model developed in this paper delivers two other implications worth noting. The first is the apparent leakage of inside information in the periods preceding the release of exogenous information, which we can see from panels B and C of figure 1. Prices appear to be move in anticipation and direction of the new information release. However, we know that in this model prices move in anticipation to the new information, but in the *direction of previous* information. Investors trade in anticipation to the impact of new information, and in doing so use their information more aggressively, which impounds more of the existing information into prices. Moreover, this is only

[9] Since posterior beliefs of the underlying signal are biased toward the prior belief, the only way to have $\mathbb{E}_\theta\left[\mathbb{E}\left(s_1|\mathcal{F}_t^c\right)\right]-s_1=0$ is for $s_1=\mathbb{E}\left(s_1\right)=0$.

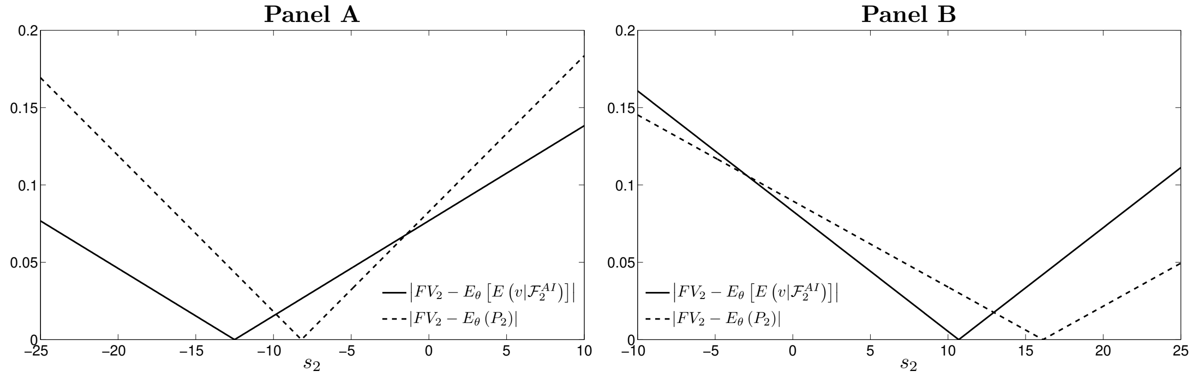


Figure 8: Distance of average expectation of the liquidation value and average price to the fundamental value, as a function of the second underlying signal. This plot shows the distance of the average expectation of the liquidation value and of the average price to the fundamental value as a function of the second underlying signal (s_2). These distances are computed at the date the information based on that underlying signal is released. The fundamental value, average expectation and average price are linear functions of s_2, and so cross at most once. In panel A, the average expectation and average price cross at the rightmost intersection of the difference lines, whereas in panel B they cross at the leftmost intersection. In panel A $\Sigma_{\tilde{s},2} = \begin{bmatrix} 1^{10} & 0 \\ 0 & 0.4 \end{bmatrix}$, and in panel B $\Sigma_{\tilde{s},2} = \begin{bmatrix} 1^{10} & 0 \\ 0 & 0.3 \end{bmatrix}$. The rest of the parametrization is identical in both panels: $T = 4$, $n = 2$, $\rho = 0$, $\sigma_v^2 = 0.25$, $\Sigma_s = \begin{bmatrix} 0.25 & 0 \\ 0 & 0.25 \end{bmatrix}$, $\Sigma_{\tilde{s},1} = \begin{bmatrix} 1 & 0 \\ 0 & 1^{10} \end{bmatrix}$, $\Sigma_{\tilde{s},3} = \begin{bmatrix} 1^{10} & 0 \\ 0 & 1^{10} \end{bmatrix}$, $\sigma_\theta^2 = 0.1$, $\alpha = 2$, $s_1 = 1$.

the case when investors have private information about an underlying signal that is positively correlated with that of the incoming information. Therefore, prices can and will sometimes move in the opposite direction of the new information. However, as long as the underlying signal of new and old information are positively correlated, good news tend to be folowed by good news and bad news by bad news. Thus, most of the time the price movement in anticipation to the release of new information is in the right direction, making it look like leakage of inside information.

A second implication is that prices tend to be more informative about the fundamental value when the price volatility is high. Using a model similar to the one in this paper, He and Wang (1995) show that price volatility increases in the periods leading to the release of exogenous information, public or private, when there is already private information based on the same underlying signal (more generally, the same pattern holds for correlated underlying signals). This is caused by investors trading more aggressively on their private information in anticipation to the effects of the release of additional information. As we saw, this impounds more of investors' information into prices, making prices better predictors of fundalmentals than average expectations. This result supports the common view of a more volatility market as one where more information is gathered (e.g. Admati and Pfleiderer (1988)); and it is in contrast with the results of Cespa (2002), who finds the opposite relation when the economy is populated by short-term investors.

7 Conclusion

In this paper I show how the existence of short-term trading causes a divergence between the average price and the average expectation of the fundamental value. When investors engage in short-term trading, they care about intermediate prices. This embeds higher-order expectations into prices which cause a discrepancy between prices and first-order expectations. In other words, short-term trading diverts investors' attentions from the long run, allowing a discrepancy between the average price and the average expectation to persist.

Short-term trading arises when investors receive private information and either (i) net supply mean reverts or (ii) the pending release of additional information based on the same underlying signal is combined with residual uncertainty. Mean-reversion of net supply creates the opportunity to profit from liquidity traders as they predictably exit the market. And the released of additional information creates the opportunity to trade in anticipation to its price impact.

However, (i) and (ii) cause the average price to diverge from the average expectation of the liquidation value in different directions. Mean-reversion of net supply, which produces endogenous information, tends to bring expectations closer to fundamentals than prices *after* the release of private exogenous information. By the contrary, residual uncertainty and an incoming release of exogenous information tends to make prices closer to fundamentals than expectations *before* the new information is released.

This paper extends Cespa and Vives (2012) results in two dimensions: (i) additional exogenous information is crucial for prices to be closer to fundamentals than average expectations; (ii) prices can be closer and further away from fundamentals than average expectations at different times in the same economy.

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A Proofs

A.1 Proof of Theorem 1

Theorem 1 is derived from theorems 2, 4 and 5 of Barbosa (2011). $\hat{K}$ is obtained from lemma 9 of Barbosa (2011), and $\hat{\Gamma}$ is obtained from equation (42) of Barbosa (2011).

A.2 Proof of Lemma 4

Theorem 3 of Barbosa (2011) establishes that

$$\Delta P_{t+1} = C_{t+1}\boldsymbol{\psi}_t + D_{t+1}\boldsymbol{\varepsilon}_{\Delta,t+1} \quad (11)$$

$$\boldsymbol{\psi}_{t+1} = F_{t+1}\boldsymbol{\psi}_t + G_{t+1}\boldsymbol{\varepsilon}_{\Delta,t+1} \quad (12)$$

From equation (11) we know that $\mathbb{E}\left(\Delta P_{t+1} \middle| \mathcal{F}_t^i\right) = C_{t+1}\boldsymbol{\psi}_t$. Recursive substitution of equation (12) into (11) implies that $\mathbb{E}\left(\Delta P_{t+\tau} \middle| \mathcal{F}_t^i\right) = C_{t+\tau}\prod_{j=1}^{\tau-1} F_{t+\tau-j}\boldsymbol{\psi}_t$. Thus, to prove equation (7) it suffices to show that Q_t in equation (6) is a linear function of $C_{t+\tau}\prod_{j=1}^{\tau-1} F_{t+\tau-j}$ for $\tau = 1, ..., T-t$.

We can rewrite equations (34) and (36) of Barbosa (2011) as

$$Q_t = \zeta_{1,t}^Q C_{t+1} + \zeta_{2,t}^Q H_{t+1}F_{t+1} \quad (13)$$

$$H_t = \zeta_{1,t}^H Q_t + \zeta_{2,t}^H H_{t+1}F_{t+1} \quad (14)$$

where

$$\underset{(1\times 1)}{\zeta_{1,t}^Q} \equiv \left(D_{t+1}\Xi_{t+1}D_{t+1}'\right)^{-1} \quad (15)$$

$$\underset{(1\times 2n+2)}{\zeta_{2,t}^Q} \equiv -\left(D_{t+1}\Xi_{t+1}D_{t+1}'\right)^{-1} D_{t+1}\Xi_{t+1}G_{t+1}' \quad (16)$$

$$\underset{(2n+2\times 1)}{\zeta_{1,t}^H} \equiv Q_t'D_{t+1}\Xi_{t+1}D_{t+1}' \quad (17)$$

$$\underset{(2n+2\times 2n+2)}{\zeta_{2,t}^H} \equiv F_{t+1}' - F_{t+1}'H_{t+1}G_{t+1}\Xi_{t+1}G_{t+1}'. \quad (18)$$

Starting at $t = T-1$, we have $Q_{T-1} = \zeta_{1,T-1}^Q C_T$ and $H_{T-1} = \zeta_{1,T-1}^H\zeta_{1,T-1}^Q C_T$. Using backward substitution, it is easy to verify that both Q_t and H_t are linear functions of $C_{t+\tau}\prod_{j=1}^{\tau-1} F_{t+\tau-j}$ for $\tau = 1, ..., T-t$,

$$Q_t = \sum_{\tau=1}^{T-t}\left(\chi_{\tau,t}^Q C_{t+\tau}\prod_{j=1}^{\tau-1} F_{t+\tau-j}\right)$$

$$H_t = \sum_{\tau=1}^{T-t}\left(\chi_{\tau,t}^H C_{t+\tau}\prod_{j=1}^{\tau-1} F_{t+\tau-j}\right)$$

where

$$
\begin{aligned}
\underset{(1\times 1)}{\chi^Q_{1,t}} &\equiv \zeta^Q_{1,t} && (19)\\
\underset{(1\times 1)}{\chi^Q_{\tau,t}} &\equiv \zeta^Q_{2,t}\chi^H_{\tau-1,t+1},\ \tau = 2,...,T-t && (20)\\
\underset{(2n+2\times 1)}{\chi^H_{1,t}} &\equiv \zeta^H_{1,t}\chi^Q_{1,t}\\
\underset{(2n+2\times 1)}{\chi^H_{\tau,t}} &\equiv \zeta^H_{1,t}\chi^Q_{\tau,t} + \zeta^H_{2,t}\chi^H_{\tau-1,t+1},\ \tau = 2,...,T-t.
\end{aligned}
$$

Finally, note that $\chi^Q_{1,t} = \left(D_{t+1}\Xi_{t+1}D'_{t+1}\right)^{-1} > 0$, since Ξ_{t+1} is a positive definite matrix (see Barbosa, 2011). This concludes the proof of equation (7).

With equation (7) as the starting point, it is straightforward to rewrite the demand function as equation (8). Simply rearrange the expected prices to obtain the expected price change from date t to date $\tau \in \{t+1, t+2, ..., T\}$ instead of price changes from date $\tau-1$ to τ, $\tau \in \{t+1, t+2, ..., T\}$. Proceeding in this way

$$
\begin{aligned}
X^i_t &= \frac{1}{\alpha}\sum_{\tau=1}^{T-t}\chi^Q_{\tau,t}\mathbb{E}\left[\Delta P_{t+\tau}|\mathcal{F}^i_t\right]\\
&= \frac{1}{\alpha}\sum_{\tau=1}^{T-t-1}\chi^Q_{\tau,t}\mathbb{E}\left[\Delta P_{t+\tau}|\mathcal{F}^i_t\right] + \frac{1}{\alpha}\chi^Q_{T-t,t}\mathbb{E}\left[P_T - P_{T-1}|\mathcal{F}^i_t\right]\\
&= \frac{1}{\alpha}\sum_{\tau=1}^{T-t-1}\chi^Q_{\tau,t}\mathbb{E}\left[\Delta P_{t+\tau}|\mathcal{F}^i_t\right] + \frac{1}{\alpha}\chi^Q_{T-t,t}\mathbb{E}\left[P_t - P_{T-1}|\mathcal{F}^i_t\right] + \frac{1}{\alpha}\chi^Q_{T-t,t}\mathbb{E}\left[P_T - P_t|\mathcal{F}^i_t\right]\\
&= \frac{1}{\alpha}\sum_{\tau=1}^{T-t-2}\chi^Q_{\tau,t}\mathbb{E}\left[\Delta P_{t+\tau}|\mathcal{F}^i_t\right] + \frac{1}{\alpha}\chi^Q_{T-t-1,t}\mathbb{E}\left[P_{T-1} - P_{T-2}|\mathcal{F}^i_t\right]\\
&\quad + \frac{1}{\alpha}\chi^Q_{T-t,t}\mathbb{E}\left[P_t - P_{T-1}|\mathcal{F}^i_t\right] + \frac{1}{\alpha}\chi^Q_{T-t,t}\mathbb{E}\left[P_T - P_t|\mathcal{F}^i_t\right]\\
&= \frac{1}{\alpha}\sum_{\tau=1}^{T-t-2}\chi^Q_{\tau,t}\mathbb{E}\left[\Delta P_{t+\tau}|\mathcal{F}^i_t\right] + \frac{1}{\alpha}\chi^Q_{T-t-1,t}\mathbb{E}\left[P_t - P_{T-2}|\mathcal{F}^i_t\right]\\
&\quad + \frac{1}{\alpha}\left(\chi^Q_{T-t-1,t} - \chi^Q_{T-t,t}\right)\mathbb{E}\left[P_{T-1} - P_t|\mathcal{F}^i_t\right] + \frac{1}{\alpha}\chi^Q_{T-t,t}\mathbb{E}\left[P_T - P_t|\mathcal{F}^i_t\right]\\
&= \frac{1}{\alpha}\sum_{\tau=1}^{T-t}\xi_{\tau,t}\mathbb{E}\left[P_{t+\tau} - P_t|\mathcal{F}^i_t\right]
\end{aligned}
$$

where

$$
\begin{aligned}
\xi_{\tau,t} &= \chi^Q_{\tau,t} - \chi^Q_{\tau+1,t}\ \forall\tau \in \{1, 2, ..., T-t-1\}, && (21)\\
\xi_{T-t,t} &= \chi^Q_{T-t,t}. && (22)
\end{aligned}
$$

To show that demand can be written as in equation (9) and derive the expressions for $\phi^Q_{\tau,t}$, I follow the same steps used above to derived the expressions for $\chi^Q_{\tau,t}$. Starting at $t = T-1$, equations (13) and (14) give us $Q_{T-1} = \zeta^{Q}_{1,T-1} C_T$ and $H_{T-1} = \zeta^{H}_{1,T-1} Q_{T-1}$. And at $t = T-2$ we have

$$\begin{aligned}
Q_{T-2} &= \zeta^{Q}_{1,T-2} C_{T-1} + \zeta^{Q}_{2,T-2} \zeta^{H}_{1,T-1} Q_{T-1} F_{T-1} \\
&= \phi^{Q}_{1,T-2} C_{T-1} + \phi_{2,T-2} Q_{T-1} F_{T-1} \\
H_{T-2} &= \zeta^{H}_{1,T-2} Q_{T-2} + \zeta^{H}_{2,T-2} \zeta^{H}_{1,T-1} Q_{T-1} F_{T-1} \\
&= \phi^{H}_{1,T-2} Q_{T-2} + \phi^{H}_{2,T-2} Q_{T-1} F_{T-1}.
\end{aligned}$$

Using backward substitution, it is easy to verify that

$$\begin{aligned}
Q_t &= \phi^{Q}_{1,t} C_{t+1} + \sum_{\tau=2}^{T-t} \phi^{Q}_{\tau,t} Q_{t+\tau-1} \prod_{j=1}^{\tau-1} F_{t+\tau-j} \qquad (23) \\
H_t &= \sum_{\tau=1}^{T-t} \phi^{H}_{\tau,t} Q_{t+\tau-1} \prod_{j=1}^{\tau-1} F_{t+\tau-j}
\end{aligned}$$

with

$$\begin{aligned}
\phi^{Q}_{1,t} &= \zeta^{Q}_{1,t} = \chi^{Q}_{1,t} \\
\phi^{Q}_{\tau,t} &= \zeta^{Q}_{2,t} \phi^{H}_{\tau-1,t+1} \\
\phi^{H}_{1,t} &= \zeta^{H}_{1,t} \\
\phi^{H}_{\tau,t} &= \zeta^{H}_{2,t} \phi^{H}_{\tau-1,t+1}.
\end{aligned}$$

Finally, notice that, from equation (11) we have $\mathbb{E}\left[\Delta P_{t+1} | \mathcal{F}^{i}_{t}\right] = C_{t+1} \boldsymbol{\psi}_t$. Moreover, taking the expectation of the demand function (6), using the law of iterated expectations and repeated substitution of equation (12) we obtain $\mathbb{E}\left[X^{i}_{t+\tau} | \mathcal{F}^{i}_{t}\right] = Q_{t+\tau} \left(\prod_{j=1}^{\tau} F_{t+\tau+1-j}\right) \boldsymbol{\psi}_t$. Therefore, from equation (23) we can write the demand function as equation (9).

A.3 Proof of Lemma 6

Generically, the fundamental value is defined as $FV_t = \mathbb{E}\left(v | \boldsymbol{s}^{o}\right)$, where $\boldsymbol{s}^{o}$ is the vector of underlying signals already observed by date t. We can use equation (28) of Barbosa (2011) to compute the expectation and obtain $FV_t = \hat{K} \boldsymbol{s}^{o}$. In Appendix A.2 Barbosa (2011) shows how the vector $\hat{K}$ can be computed. When there is a signal underlying signal s and exogenous information about it is released at date 1, it follows that $FV_t = \hat{K} s$.

The expression for the average price is obtained by averaging the price function (5) over the net supply, and using equation (14) of Barbosa (2011) to substitute for $p_t = \hat{K} - \hat{p}_t$.

Finally, the average expectation of the liquidation value is determined as

$$\begin{aligned}
\mathbb{E}_{\theta,i}\left[\mathbb{E}\left(v | \mathcal{F}^{i}_{t}\right)\right] &= \mathbb{E}_{\theta,i}\left\{\mathbb{E}\left[\mathbb{E}\left(v|s\right) | \mathcal{F}^{i}_{t}\right]\right\} \\
&= \hat{K} \mathbb{E}_{\theta,i}\left[\mathbb{E}\left(s | \mathcal{F}^{i}_{t}\right)\right] \\
&= \hat{K} \mathbb{E}_{\theta}\left[\hat{\Gamma}_t s + \left(1 - \hat{\Gamma}_t\right) \mathbb{E}\left(s | \mathcal{F}^{c}_{t}\right)\right] \\
&= \left(1 - \hat{\Gamma}_t\right) \mathbb{E}_{\theta}\left[\mathbb{E}\left(s | \mathcal{F}^{c}_{t}\right)\right] + \hat{K} \hat{\Gamma}_t s,
\end{aligned}$$

where the second equality follows from equation (28) of Barbosa (2011), and the third equality follows from equation (41) of Barbosa (2011). The expression for $\hat{\Gamma}_t$ is given by equation (42) of Barbosa (2011).

Averaging the expression for $\mathbb{E}\left(s|\mathcal{F}_t^c\right)$, equation (8) of Barbosa (2011), over the net supply, we can determine that $\mathbb{E}\left(s|\mathcal{F}_t^c\right)$ is a convex combination of the prior belief on v (which is zero) and s. This is obvious since investors form their beliefs by averaging over the signals they observe and their prior belief and, when averaging over net supply, all public signals for s are unbiased. In turn, we have that $\left\{\hat{K}, \hat{\Gamma}_t, p_t, \hat{p}_t\right\} \in (0,1)^4$. This is straightforward to prove for $\hat{K}$ and $\hat{\Gamma}_t$. But not for p_t and $\hat{p}_t$, since there is no closed form solution for the price function parameters when there is residual uncertainty. However, numerical results strongly suggest this is the case. With $\left\{\hat{K}, \hat{\Gamma}_t, p_t, \hat{p}_t\right\} \in (0,1)^4$ it then follows that both the fundamental value, the average price and the average expectation of the liquidation value are convex combinations of the prior on v and s. Since $\mathbb{E}\left(s|\mathcal{F}_t^c\right)$ gives some weight to the prior belief on v, the average price and the average expectation of the liquidation value are biased toward the prior belief on v. This proves point (i).

Obviously, the less weight is put on the public information $\mathbb{E}\left(s|\mathcal{F}_t^c\right)$, the smaller the bias toward the prior belief. Therefore, the average price is closer to the fundamental value than the average expectation of the liquidation value is whenever $\hat{p}_t < \hat{K}\left(1-\hat{\Gamma}_t\right)$, which proves point (ii).

A.4 Proof that $\xi_{1,1}$ and $\xi_{2,1}$ are Strictly Positive in a 3-Period Model without Residual Uncertainty

Equations (21) and (22) define the demand function coefficients $\xi_{1,1}$ and $\xi_{2,1}$ as a function of $\chi_{1,1}^Q$ and $\chi_{2,1}^Q$, which in turn are defined by equations (19) and (20). We can then write

$$\begin{aligned}\xi_{1,1} &= \chi_{1,1}^Q - \chi_{2,1}^Q = \chi_{1,1}^Q\left(1 + D_2\Xi_2 G_2' Q_2'\right) \\ \xi_{2,1} &= \chi_{2,1}^Q = -\chi_{1,1}^Q D_2 \Xi_2 G_2' Q_2'.\end{aligned}$$

Lemma 4 gives us that $\chi_{1,1}^Q > 0$. Therefore, $\xi_{1,1}$ and $\xi_{2,1}$ are strictly positive if and only if $-1 \leq D_2\Xi_2 G_2' Q_2' \leq 0$.

Using the results obtained in Appendix C of Barbosa (2011), where I solve a 3-period and no residual uncertainty version of the model, after long and tedious algebra we can determine that

$$D_2\Xi_2 G_2' Q_2' = -\frac{V_2^i\left(\frac{1}{\beta\sigma_{i,1}^2} + \frac{1-\beta\rho}{\alpha^2\beta^2\sigma_{i,1}^4\sigma_\theta^2}\right)\left(\alpha^2\beta\sigma_{i,1}^2\sigma_\theta^2 + \frac{V_1^i(1-\rho)(1-\beta\rho)}{\sigma_{i,1}^2}\right)}{1 + V_2^i\left(\frac{V_1^i(1-\rho)^2}{\sigma_{i,1}^4} + \frac{1-\beta}{\beta\sigma_{i,1}^2} + \alpha^2\sigma_\theta^2\right)} < 0$$

where, following the notation of Appendix C of Barbosa (2011), $V_t^i \equiv Var\left(v|\mathcal{F}_t^i\right)$ and $\Sigma_{\tilde{s},2} \equiv \sigma_{i,2}^2 \equiv \frac{\beta}{1-\beta}\sigma_{i,1}^2$, $0 \leq \beta \leq 1$. To determine that $D_2\Xi_2 G_2' Q_2' > -1$, let us write

$$D_2\Xi_2 G_2' Q_2' = -\frac{a}{1+b}.$$

Then,

$$D_2\Xi_2 G_2' Q_2' > -1 \Leftrightarrow b - a > -1.$$

After some algebra and simplification, using the definitions of V_t^i from Barbosa (2011), we determine the difference $b-a$ as

$$b-a=-(1-\rho)\frac{V_1^i}{\sigma_{i,1}^2}.$$

Since $0\leq\rho\leq 1$ and, by definition, $V_1^i\leq\sigma_{i,1}^2$, it follows that $b-a\geq -1$. This difference is strictly positive whenever there is mean-reversion of net supply ($\rho<1$) and/or the prior is informative (i.e. $\sigma_v^2>0\Rightarrow V_1^i<\sigma_{i,1}^2$). The latter is always assumed, and so $b-a>1$, which proves that $\xi_{1,1}$ and $\xi_{2,1}$ are strictly positive.

B Closed-Form Solution of the Model without Residual Uncertainty and without Mean-Reversion of Net Supply

Here I will solve a special case of the model, when there is no residual uncertainty nor mean-reversion of net supply, in closed form. This is similar to what was done in Appendix C of Barbosa (2011), where I solve a 3-period model without residual uncertainty in closed form. I will introduce here the same modifications were done in Barbosa (2011): I redefine $\xi_t\equiv s+\frac{p_{\theta,t}}{p_t}\theta_t$, $\varepsilon_{\Delta,t}\equiv\begin{bmatrix} s-\mathbb{E}\left(s|\mathcal{F}_t^i\right) & \varepsilon_{t+1}^i & \varepsilon_{\theta,t+1}\end{bmatrix}'$ and $\psi_t\equiv\begin{bmatrix} 1 & \mathbb{E}\left(s|\mathcal{F}_t^i\right)-\mathbb{E}\left(s|\mathcal{F}_t^c\right) & \mathbb{E}\left(\theta_t|\mathcal{F}_t^i\right)\end{bmatrix}'$. These modifications imply changes in C_{t+1}, F_{t+1}, G_{t+1}, Q_{t+1}, H_{t+1} that will be detailed below.

I start by defining

$$w_t\equiv\frac{1}{\sum_{k=1}^t\frac{1}{\sigma_{i,k}^2}},\ z_t\equiv\frac{1}{\sum_{k=1}^t\frac{1}{\sigma_{i,k}^4}} \tag{24}$$

Then, from corollary 2 of He and Wang (1995), we have

$$\frac{p_{\theta,t}}{p_t}=-\alpha w_t. \tag{25}$$

It is the knowledge of this ratio that allows us to solve for p_t and $p_{\theta,t}$ recursively from date $T-1$.

Conditional beliefs, and matrices K_t and K_t^c are obtained by applying Lemma 9 (Gaussian filtering) of Barbosa (2011), with x_t, y_t, A_t, B_t, $\Sigma_{x,t}$, $\Sigma_{y,t}$, $\mathbb{E}[x_0|\mathcal{F}_0]$ and $Var[x_0|\mathcal{F}_0]=\begin{bmatrix}\sigma_v^2 & 0\\ 0 & 0\end{bmatrix}$ exactly as defined in Appendix C of Barbosa (2011), but fixing $\rho=1$ in A_t. Using equation (25), after simplification we obtain

$$V_t^i\equiv Var\left(v|\mathcal{F}_t^i\right)=\frac{1}{\frac{1}{\sigma_v^2}+\frac{1}{w_t}+\frac{1}{\alpha^2\sigma_\theta^2 z_t}},\ V_t^c\equiv Var\left(v|\mathcal{F}_t^c\right)=\frac{1}{\frac{1}{\sigma_v^2}+\frac{1}{\alpha^2\sigma_\theta^2 z_t}} \tag{26}$$

and

$$K_t=\begin{bmatrix}\frac{V_t^i}{\alpha^2\sigma_\theta^2\sigma_{i,t}^2 w_t} & \frac{V_t^i}{\sigma_{i,t}^2}\\ \frac{1}{\alpha w_t}\left(\frac{V_t^i}{\alpha^2\sigma_\theta^2\sigma_{i,t}^2 w_t}-1\right) & \frac{V_t^i}{\alpha\sigma_{i,t}^2 w_t}\end{bmatrix},\ K_t^c=\begin{bmatrix}\frac{V_t^i}{\alpha^2\sigma_\theta^2\sigma_{i,t}^2 w_t}\\ \frac{1}{\alpha w_t}\left(\frac{V_t^i}{\alpha^2\sigma_\theta^2\sigma_{i,t}^2 w_t}-1\right)\end{bmatrix}. \tag{27}$$

From (26) and (24) we obtain the following 4 relations between conditional variances that will be extensively used throughout to simplify expressions:

$$\frac{1}{V_t^i}-\frac{1}{V_t^c}=\frac{1}{w_t},\ \frac{1}{V_t^i}-\frac{1}{V_{t-1}^i}=\frac{1}{\sigma_{i,t}^2}+\frac{1}{\alpha^2\sigma_\theta^2\sigma_{i,t}^4},\ \frac{1}{V_t^c}-\frac{1}{V_{t-1}^c}=\frac{1}{\alpha^2\sigma_\theta^2\sigma_{i,t}^4}. \tag{28}$$

Knowing K_t and K_t^c we can obtain the expressions for C_t, D_t, F_t, G_t and $\Sigma_{\Delta,t}$ as defined in Appendix A.2 of Barbosa (2011), with the necessary adaptations that the redefinition of ξ_t and ψ_t require.[10]. After simplification we obtain, for $t \leq T-1$

$$\begin{aligned}
C_t &= \begin{bmatrix} 0 & 1-(1-p_t)\frac{V_t^c}{V_{t-1}^c} & p_{\theta,t}\end{bmatrix} - \begin{bmatrix} 0 & p_{t-1} & p_{\theta,t-1}\end{bmatrix} = \hat{C}_t - \begin{bmatrix} 0 & p_{t-1} & p_{\theta,t-1}\end{bmatrix}\\
D_t &= \left[\frac{p_t w_t}{\sigma_{i,t}^2}+(1-p_t)\left(1-\frac{V_t^c}{V_{t-1}^c}\right)\right]\begin{bmatrix} 1 & 0 & -\alpha\sigma_{i,t}^2\end{bmatrix}\\
F_t &= \begin{bmatrix} 1 & 0 & 0\\ 0 & \frac{V_t^c}{V_{t-1}^c} & 0\\ 0 & 0 & 1\end{bmatrix}\\
G_t &= \begin{bmatrix} 0 & 0 & 0\\ \frac{V_t^c}{V_{t-1}^c}-\frac{V_t^i}{V_{t-1}^i} & \frac{V_t^i}{\sigma_{i,t}^2} & \frac{V_t^c-V_t^i}{\alpha\sigma_\theta^2\sigma_{i,t}^2}\\ -\frac{1}{\alpha V_t^c}\left(\frac{V_t^c}{V_{t-1}^c}-\frac{V_t^i}{V_{t-1}^i}\right) & \frac{V_t^i}{\alpha w_t\sigma_{i,t}^2} & 1-\frac{V_t^i}{\alpha^2\sigma_\theta^2\sigma_{i,t}^2 w_t}\end{bmatrix}\\
\Sigma_{\Delta,t} &= \begin{bmatrix} V_{t-1}^i & 0 & 0\\ 0 & \sigma_{i,t}^2 & 0\\ 0 & 0 & \sigma_\theta^2\end{bmatrix}.
\end{aligned}$$

From equation (42) of Barbosa (2011), we obtain

$$\hat{\Gamma}_t = \frac{V_t^i}{w_t}$$

which we can then use to obtain Q_t from the market clearing condition (13) of theorem 5 of Barbosa (2011),[11]

$$Q_t = \begin{bmatrix} 0 & \frac{1}{V_t^c} & \alpha\end{bmatrix}. \tag{29}$$

Using the recursive definition of Q_t, equation (34) of Barbosa (2011), and the above market clearing condition, we can determine p_t and $p_{\theta,t}$ recursively starting from $T-1$ as

$$\begin{bmatrix} p_{t-1} & p_{\theta,t-1}\end{bmatrix} = -D_t\Xi_t D_t'\begin{bmatrix} \frac{1}{V_{t-1}^c} & \alpha\end{bmatrix} + \left(\hat{C}_t - D_t\Xi_t G_t' H_t F_t\right)\begin{bmatrix} 0 & 0\\ 1 & 0\\ 0 & 1\end{bmatrix}. \tag{30}$$

Starting with the $T-1$ price coefficients, by definition we have $H_T = \mathbf{0}_{(3,3)}$. Therefore, from

[10]The modification in the state vector ψ_t implies that: the third column of C_t, the third row of G_t, the third column and row of F_t disappear; the second row of F_t and $G_t$become the difference between the old second and third rows. The modification in $\varepsilon_{\Delta,t}$ implies that: the first column of D_t and G_t and the first column and row of $\Sigma_{\Delta,t}$ disappear. And the modification of ξ_t implies that: $k^c_{\boldsymbol{s},\xi,t+1}$ and $k^c_{\theta,\xi,t+1}$ are divided by p_t wherever they show up in C_t, D_t, F_t and G_t.

[11]The modification in ψ_t implies that the third column disappears.

equation (33) of Barbosa (2011) we obtain $\Xi_T = \Sigma_{\Delta,T}$. Therefore,

$$\begin{bmatrix} p_{T-1} & p_{\theta,T-1} \end{bmatrix} = -D_T\Sigma_{\Delta,T}D_T'\begin{bmatrix} \frac{1}{V_{T-1}^c} & \alpha \end{bmatrix} + \hat{C}_T\begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{V_{T-1}^i}{w_{T-1}} & -\alpha V_{T-1}^i \end{bmatrix}.$$

To obtain the $T-2$ price coefficients we need first to determine H_{T-1} using equation (36) of Barbosa (2011). Once we get H_{T-1} we can then compute Ξ_{T-1}, $D_{T-1}\Xi_{T-1}D_{T-1}'$ and $D_{T-1}\Xi_{T-1}G_{T-1}'H_{T-1}F_{T-1}$, and finally obtain p_{T-2} and $p_{\theta,T-2}$. Starting with H_{T-1}, it can be written as

$$H_{T-1} = V_{T-1}^i\begin{bmatrix} 0 \\ \frac{1}{V_{T-1}^c} \\ \alpha \end{bmatrix}\begin{bmatrix} 0 & \frac{1}{V_{T-1}^c} & \alpha \end{bmatrix}.$$

After simplification, the matrix $G_{T-1}'H_{T-1}G_{T-1}$, necessary to compute Ξ_{T-1}, can be written as

$$G_{T-1}'H_{T-1}G_{T-1} = \begin{bmatrix} 0 \\ \frac{1}{\sigma_{i,T-1}^2} \\ \alpha \end{bmatrix}\begin{bmatrix} 0 & \frac{1}{\sigma_{i,T-1}^2} & \alpha \end{bmatrix}$$

and Ξ_{T-1} is then given by

$$\Xi_{T-1} = \Sigma_{\Delta,T-1} - \frac{V_{T-1}^i}{1 + V_{T-1}^i\left(\frac{1}{\sigma_{i,T-1}^2} + \alpha^2\sigma_\theta^2\right)}\begin{bmatrix} 0 \\ 1 \\ \alpha\sigma_\theta^2 \end{bmatrix}\begin{bmatrix} 0 & 1 & \alpha\sigma_\theta^2 \end{bmatrix}.$$

We can now compute the $D_{T-1}\Xi_{T-1}D_{T-1}'$, obtaining

$$D_{T-1}\Xi_{T-1}D_{T-1}' = \frac{V_{T-2}^i\left(\frac{1}{V_{T-1}^i} - \frac{1}{V_{T-2}^i}\right)^2}{\frac{1}{V_{T-1}^i}\left(\frac{1}{V_{T-1}^c} + \frac{1}{V_{T-2}^c}\right) + \frac{1}{\sigma_{i,T-1}^2}\left(\frac{1}{V_{T-1}^i} - \frac{1}{V_{T-2}^i}\right)}.$$

I turn, $G_{T-1}'H_{T-1}F_{T-1}$ can be written as

$$G_{T-1}'H_{T-1}F_{T-1} = V_{T-1}^i\begin{bmatrix} 0 \\ \frac{1}{\sigma_{i,T-1}^2} \\ \alpha \end{bmatrix}\begin{bmatrix} 0 & \frac{1}{V_{T-2}^c} & \alpha \end{bmatrix}$$

from which we obtain

$$D_{T-1}\Xi_{T-1}G_{T-1}'H_{T-1}F_{T-1} = -V_{T-1}^i\frac{\frac{1}{\sigma_{i,T-1}^2}\left(\frac{1}{V_{T-1}^i} - \frac{1}{V_{T-2}^i}\right)}{\frac{1}{V_{T-1}^i}\left(\frac{1}{V_{T-1}^c} + \frac{1}{V_{T-2}^c}\right) + \frac{1}{\sigma_{i,T-1}^2}\left(\frac{1}{V_{T-1}^i} - \frac{1}{V_{T-2}^i}\right)}.$$

Bringing everything together, from equation (30) we obtain, after a great deal of simplification that

$$\begin{bmatrix} p_{T-2} & p_{\theta,T-2} \end{bmatrix} = \begin{bmatrix} \frac{V_{T-2}^i}{w_{T-2}} & -\alpha V_{T-2}^i \end{bmatrix}.$$

Since $\hat{C}_t$, D_t, F_t, G_t and $\Sigma_{\Delta,t}$ have the same form at all dates $t \leq T-1$, if the same is true for H_t, which would imply that Ξ_t also retains the same form, then we would obtain

$$\begin{bmatrix} p_t & p_{\theta,t} \end{bmatrix} = \begin{bmatrix} \frac{V_t^i}{w_t} & -\alpha V_t^i \end{bmatrix}. \tag{31}$$

So the next and final step is to derive H_{T-2} and verify that it has the same form of H_{T-1}, which would imply that H_t have the same form at all dates $t \leq T-1$. Once again, using equation (36) of Barbosa (2011), we obtain

$$H_{T-2} = V_{T-2}^i \begin{bmatrix} 0 \\ \frac{1}{V_{T-2}^c} \\ \alpha \end{bmatrix} \begin{bmatrix} 0 & \frac{1}{V_{T-2}^c} & \alpha \end{bmatrix} + \begin{bmatrix} \ln\left|\frac{\Sigma_{\Delta,T-1}}{\Xi_{T-1}}\right| & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

For the purpose of computing the price coefficients the second term can be ignored, since the first row of G_t is full of zeros. This means that dropping the second term has no impact on $G_t' H_t G_t$ and thus on Ξ_t; and it also has no impact on $D_t \Xi_t G_t' H_t F_t$. Therefore, we can confirm that the price function coefficients are given by equation (29) for all t.

Next, I use the solution obtained above to compute the expected price change, showing that on average the AI does not expect a price change at any future dates, and to compute the demand function, showing that it is the same that arises in a static model or, equivalently, in the base-case model where there is no additional release of exogenous information nor mean-reversion of net supply.

B.1 Expected Price Change

Expected price changes can be computed from the price change and state vector processes, equations (9) and (10) of theorem 3 of Barbosa (2011), as

$$\begin{aligned} \mathbb{E}\left(\Delta P_{t+\tau} | \mathcal{F}_t^i\right) &= \mathbb{E}\left(C_{t+\tau} \boldsymbol{\psi}_{t+\tau-1} + D_{t+\tau} \boldsymbol{\varepsilon}_{\Delta,t+\tau} | \mathcal{F}_t^i\right) \\ &= C_{t+\tau} \mathbb{E}\left(\boldsymbol{\psi}_{t+\tau-1} | \mathcal{F}_t^i\right) \\ &= C_{t+\tau} \mathbb{E}\left(F_{t+\tau-1} \boldsymbol{\psi}_{t+\tau-2} + G_{t+\tau-1} \boldsymbol{\varepsilon}_{\Delta,t+\tau-1} | \mathcal{F}_t^i\right) \\ &= C_{t+\tau} F_{t+\tau-1} \mathbb{E}\left(\boldsymbol{\psi}_{t+\tau-2} | \mathcal{F}_t^i\right) \\ &= C_{t+\tau} \prod_{j=1}^{\tau-1} F_{t+\tau-j} \boldsymbol{\psi}_t \end{aligned}$$

The product is easily to determined as

$$\prod_{j=1}^{\tau-1} F_{t+\tau-j} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{V_{t+\tau-1}^c}{V_t^c} & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

The expression for C_t can be simplified by substituting for the p_t and $p_{\theta,t}$ using equation (29), obtaining

$$C_t = \left(V_{t-1}^i - V_t^i\right) \begin{bmatrix} 0 & \frac{1}{V_{t-1}^c} & \alpha \end{bmatrix}.$$

It the follows that

$$\begin{aligned}
\mathbb{E}\left(\Delta P_{t+\tau}|\mathcal{F}_t^i\right) &= \left(V_{t+\tau-1}^i - V_{t+\tau}^i\right)\left[\begin{array}{ccc} 0 & \frac{1}{V_t^c} & \alpha \end{array}\right]\psi_t \\
&= \left(V_{t+\tau-1}^i - V_{t+\tau}^i\right) Q_t\psi_t \\
&= \alpha\left(V_{t+\tau-1}^i - V_{t+\tau}^i\right) X_t^i
\end{aligned}$$

where the second equality follows from equation (29) and the last equality from equation (6). We can see that the expected price change between dates $t+\tau-1$ and $t+\tau$ is the product of the expected reduction in uncertainty from $t+\tau-1$ to $t+\tau$ with the current demand and the coefficient of risk-aversion. Therefore, prices are expected to change only in the periods where exogenous information is released.

Averaging over net supply and investors, it follows that

$$\mathbb{E}_{\theta,i}\left[\mathbb{E}\left(\Delta P_{t+\tau}|\mathcal{F}_t^i\right)\right] = \alpha\left(V_{t+\tau-1}^i - V_{t+\tau}^i\right)\mathbb{E}_{\theta,i}\left(X_t^i\right) = 0$$

which means that, on average, the AI does not expect a price change at any future date, regardless of whether exogenous information is released or not.

B.2 Demand Function

The demand is given by equation (6). Simplification yields

$$\begin{aligned}
X_t^i &= \frac{1}{\alpha}Q_t\psi_t \\
&= \frac{1}{\alpha}\left[\begin{array}{ccc} 0 & \frac{1}{V_t^c} & \alpha \end{array}\right]\left[\begin{array}{ccc} 1 & \mathbb{E}\left(s|\mathcal{F}_t^i\right)-\mathbb{E}\left(s|\mathcal{F}_t^c\right) & \mathbb{E}\left(\theta_t|\mathcal{F}_t^i\right) \end{array}\right]' \\
&= \frac{\mathbb{E}\left(s|\mathcal{F}_t^i\right)-\mathbb{E}\left(s|\mathcal{F}_t^c\right)}{\alpha V_t^c} + \mathbb{E}\left(\theta_t|\mathcal{F}_t^i\right) \\
&= \frac{\mathbb{E}\left(s|\mathcal{F}_t^i\right)-\mathbb{E}\left(s|\mathcal{F}_t^c\right)}{\alpha V_t^i}\frac{V_t^i}{V_t^c} + \mathbb{E}\left(\theta_t|\mathcal{F}_t^i\right) \\
&= \frac{\mathbb{E}\left(s|\mathcal{F}_t^i\right) - p_t\mathbb{E}\left(s|\mathcal{F}_t^i\right) - (1-p_t)\,\mathbb{E}\left(s|\mathcal{F}_t^c\right) - p_{\theta,t}\mathbb{E}\left(\theta_t|\mathcal{F}_t^i\right)}{\alpha V_t^i} \\
&= \frac{\mathbb{E}\left(s|\mathcal{F}_t^i\right) - \left[p_t s + (1-p_t)\,\mathbb{E}\left(s|\mathcal{F}_t^c\right) - p_{\theta,t}\theta_t\right]}{\alpha V_t^i} \\
&= \frac{\mathbb{E}\left(s|\mathcal{F}_t^i\right) - P_t}{\alpha V_t^i},
\end{aligned}$$

which corresponds to the demand function in a static 2-period model (i.e. one trading date and one liquidation date). The fifth equality follows from equation (31) and (28); the sixth follows from the fact that $\xi_t = s + \frac{p_{\theta,t}}{p_t}\theta_t$ is observationally equivalent to the price, and so $\mathbb{E}\left(\xi_t|\mathcal{F}_t^i\right) = \xi_t$; and the last equality follows from equation (5) and the fact that $\hat{K} = 1$ since $v = s$ and, from equation (28) of Barbosa (2011), $\mathbb{E}\left[v|s\right] = \hat{K}s$.

C Discussion on Correlation of Price Changes

Here I discuss how mean-reversion of net supply and risk reduction affect the correlation between price changes.

To have a base case on which to build the intuition, I start by considering the case where there is no mean-reversion of net supply nor risk reduction through endogenous or exogenous release of information. This corresponds to the base case discussed in Section 4.1. The absence of risk reduction implies that $p_{\theta,t} = p_\theta$. Therefore, a supply shock at date t has the same impact on P_t as it has (on average) on all subsequent prices except P_T. This exception follows from the fact that the asset is liquidated at date T and so $P_T = v$ regardless of the net supply level. As a result, a negative shock at t, which increases all prices from dates t until $T-1$, implies that: ΔP_t increases; $\Delta P_{t+\tau} = 0$, $\tau = \{1, 2, ..., T-t-1\}$ remains unchanged; and ΔP_T decreases. Thus, ΔP_t is uncorrelated with all subsequent price changes, except ΔP_T with which it is negatively correlated. In turn a supply shock at date $t-1$ has no impact on ΔP_t and all subsequent price changes, except ΔP_T. This means that, following a negative supply shock at date $t-1$, ΔP_t is uncorrelated with all subsequent price changes, including ΔP_T.

Therefore, when there is no mean-reversion of net supply nor risk reduction, ΔP_t is uncorrelated with all subsequent price changes, except ΔP_T with which it is negatively correlated. In this case, we have

$$\chi^Q_{\tau,t} = \chi^Q_{\tau+1,t} > 0 \, \forall \tau \in \{1, 2, ..., T-t-1\}.$$

The hedging demand will be only a function of the expected price change from date $t+1$ to T, that is $\sum_{\tau=2}^{T-t} \mathbb{E}\left[\Delta P_{t+\tau} | \mathcal{F}^i_t\right]$, and will have the same sign of the expected price change.

Next, consider the effect of mean-reversion in net supply ($0 < \rho < 1$). Now the impact of a supply shock on future prices decreases as time passes. Therefore, a supply shock at date t makes ΔP_t negatively correlated with *all* subsequent price changes. In turn, a supply shock at date $t-1$ makes ΔP_t positively correlated with all subsequent price changes. But overall, the impact of supply shocks at date t dominates, and price changes are negatively correlated. This is the case because ΔP_t increases only if the net supply level decreased from $t-1$ to t which, on average, means that the net supply at date t is negative. The expected mean-reversion of this negative net supply in the periods that follow generates negative price differences, which in turn generates negative correlation between ΔP_t and all subsequent price changes. In this case we have

$$\chi^Q_{\tau,t} > \chi^Q_{\tau+1,t} > 0 \, \forall \tau \in \{1, 2, ..., T-t-1\},$$

that is, hedging demand has the same sign of expected future price changes, but less weight is given to price changes more distant into the future.

Finally, consider the effect of risk reduction. Risk reduction can be achieved either through endogenously produced information or through the release of exogenous information. If the asset becomes less risky as time passes, then a supply shock at date t has a decreasing effect on all prices from date t up to date $T-1$. This means that, following a negative shock at date t, ΔP_t increases whereas all subsequent price changes decrease, inducing negative correlation between ΔP_t and all subsequent price changes. However, a negative supply shock at any date before date t will also decrease ΔP_t, which contributes to positive correlation. Whether, overall, ΔP_t is positively or negatively correlated with subsequent price changes will depend on how fast the risk decreases. For example, an increase in ΔP_t can be driven by: a decrease in the net supply from dates $t-1$ to t, which on average means that $\theta_t < 0$; or by a decrease in risk reduction from date $t-1$ to date t, which means that $\theta_t > 0$. In the first case subsequent risk reductions will decrease subsequent price changes and result in negative correlation between ΔP_t and those price changes. Exactly the opposite occurs in the second case. Obviously, risk reduction is more likely to be driving changes in ΔP_t if it is large. Therefore, unless risk reduces sharply from $t-1$ to t, ΔP_t is negatively correlated with all subsequent price changes.

As I mention in the main text, mean-reversion of net supply generates endogenous information. Thus, if the risk reduction generated by endogenous information is very sharp, it may cause ΔP_t to be positively correlated with future price changes, even though mean-reversion of net supply by itself induces negative correlation. However, extensive numerical simulations suggests that this is never the case. The amount of risk reduction originating from endogenous information is very gradual and spread out through time. Therefore, even after accounting for endogenous information, mean-reversion of net supply leads to

$$\chi^Q_{\tau,t} > \chi^Q_{\tau+1,t} > 0 \, \forall \tau \in \{1, 2, ..., T-t-1\},$$

and hedging demand has the same sign of future expected price changes.

When risk reduction originates from exogenous information, however, risk from date $t-1$ to t can decrease enough over one period for ΔP_t to become positively correlated with future price changes.[12] In that case we have

$$\chi^Q_{\tau,t} < 0 \, \forall \tau \in \{1, 2, ..., T-t-1\}$$

and hedging demand has the opposite sign of future expected price changes.

D Discussion of Assumption 5

The lack of a closed form solution for the general version of the model makes it impossible to prove the validity of assumption 5. And even in the special case of no residual uncertainty, the complexity of the solution makes this task infeasible for more than 3 periods.

I will start by considering a 2-period model ($T = 3$) with an exogenous return process (i.e., without the learning component). This exercise is fruitful because it shows that assumption 5 does not hold for all return processes, and indicates the conditions that would lead to the violation of this assumption. As we will see, these conditions are unlikely to arise in the model considered in this paper. I support this conjecture by proving that assumption 5 holds in a 3-period and no residual uncertainty version of the model.

D.1 Exogenous Return Process

I will consider that the expected return follows the AR(1) process

$$\Delta P_t = \mu_t + \rho_t (\Delta P_{t-1} - \mu_{t-1}) + \sigma_t \varepsilon_t, \, t = 1, 2$$

[12]For this we also need residual uncertainty.

with $\varepsilon_t \sim N(0,1)$. This process can be rewritten as equations (9) and (10) of theorem 5 of Barbosa (2011) by defining

$$\begin{aligned} \psi_t &= \begin{bmatrix} 1 \\ \Delta P_t - \mu_t \end{bmatrix}, \boldsymbol{\varepsilon}_{\Delta,t} = \varepsilon_t,\ \Sigma_{\Delta,t} = 1, \\ C_t &= \begin{bmatrix} \mu_t & \rho_t \end{bmatrix}, D_t = \sigma_t,\ F_t = \begin{bmatrix} 1 & 0 \\ 0 & \rho_t \end{bmatrix}, G_t = \begin{bmatrix} 0 \\ \sigma_t \end{bmatrix}. \end{aligned}$$

The objective is to determine the expressions for $\chi^Q_{1,1}$ and $\chi^Q_{2,1}$, from equations (19) and (20), and verify the conditions for which $\chi^Q_{1,1} - \chi^Q_{2,1} > 0$, that is for which assumption 5 holds. To that end, we need to determine Ξ_2 and Q_2, for which we use equations (34) and (30) of Barbosa (2011). To determine Ξ_2 and Q_2 we need to work backward from date 3 and we need H_2, which is given by equation (36) of Barbosa (2011).

At the terminal date, date 3, we have

$$H_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \Rightarrow \Xi_3 = \Sigma_{\Delta,3} = 1.$$

We can then determine Q_2 as

$$Q_2 = \left(D_3 D_3'\right)^{-1} C_3 = \frac{1}{\sigma_3^2} \begin{bmatrix} \mu_3 & \rho_3 \end{bmatrix}.$$

Next we obtain H_2 as

$$H_2 = Q_2' D_3 D_3' Q_2 = \frac{1}{\sigma_3^2} \begin{bmatrix} \mu_3^2 & \mu_3\rho_3 \\ \mu_3\rho_3 & \rho_3^2 \end{bmatrix},$$

from which we can then finally obtain Ξ_2 as

$$\Xi_2 = \left(\Sigma_{\Delta,2} + G_2' H_2 G_2\right)^{-1} = \frac{\sigma_3^2}{\sigma_3^2 + \sigma_2^2\rho_3^2}.$$

We can now derive the expressions for $\chi^Q_{1,1}$ and $\chi^Q_{2,1}$:

$$\begin{aligned} \chi^Q_{1,1} &= \left(D_2 \Xi_2 D_2'\right)^{-1} = \frac{\sigma_3^2 + \sigma_2^2\rho_3^2}{\sigma_2^2\sigma_3^2} = \frac{Var\left(\Delta P_3 | \mathcal{F}_1\right)}{\alpha Var\left(\Delta P_2 | \mathcal{F}_1\right) Var\left(\Delta P_3 | \mathcal{F}_2\right)} \\ \chi^Q_{2,1} &= -\left(D_2 \Xi_2 D_2'\right)^{-1} D_2 \Xi_2 G_2' Q_2' = -\frac{\rho_3}{\sigma_3^2} = -\frac{Cov\left(\Delta P_2, \Delta P_3 | \mathcal{F}_1\right)}{\alpha Var\left(\Delta P_2 | \mathcal{F}_1\right) Var\left(\Delta P_3 | \mathcal{F}_2\right)}. \end{aligned}$$

Assumption 5 holds if and only if

$$\chi^Q_{1,1} - \chi^Q_{2,1} > 0 \Leftrightarrow \sigma_3^2 + \sigma_2^2\rho_3\left(1+\rho_3\right) > 0 \Leftrightarrow Var\left(\Delta P_3 | \mathcal{F}_1\right) + Cov\left(\Delta P_2, \Delta P_3 | \mathcal{F}_1\right) > 0.$$

Fixing the variances, the left hand side is minimized when $\rho_3 = -\frac{1}{2}$. In the worst case scenario, assumption 5 holds if and only if $\sigma_3^2 > \frac{\sigma_2^2}{4}$. This means that for the assumption 5 to be false, we need a very large reduction of uncertainty *and* a relatively large negative correlation between price changes. However, as discussed in Appendix C, in the general model with learning, the more uncertainty is resolved, the more prices tend to be positively correlated. Therefore, it is highly unlikely that such a large reduction in uncertainty can coexist with a large negative correlation in price changes.

D.2 Endogenous Return Process With No Residual Uncertainty

It is feasible to show that assumption 5 holds for the model with learning considered when there is no residual uncertainty, and $T = 3$. In that case, we simply need to show that

$$\chi_{1,1}^{Q} - \chi_{2,1}^{Q} = \chi_{1,1}^{Q} \left(1 + D_2 \Xi_2 G_2' Q_2'\right) > 0$$

which was already accomplished in Appendix A.4. Therefore, assumption 5 holds in this simplified version of the model.

I was able also to verify that the same holds true in the case of $T = 4$ with two announcements, one at $t = 1$ and the other at $t = 3$ (the expressions are too long to report here, though). But the expressions become intractable for models with more periods or announcement dates.