Negative Network Externalities in Two-Sided Markets: A Competition Approach

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Abstract

Consider a firm advertising in a job matching agency with the aim of employing the most qualified workers. Its chances of success would be higher for a smaller number of competitor firms advertising in the same job matching agency, i.e. careerbuilder.com. How would the resulting competitive behavior among the firms which are advertising to this job matching agency affect the agency's optimal pricing behavior?

I analyze the optimal market structures and pricing strategies of a monopolist platform in a two-sided market setup in which the agents on each side prefer the platform to be less competitive on their side; that is, a market with negative intra-group network externalities. I find that the equilibrium market structure varies with the extent of negative externalities. If the market's negative network externalities are substantial, that is, if an agent's disutility given the size of the agent pool on his side is high (enough), then the profit-maximizing strategy for the matchmaker will be to match the highest types of one side with all of the agents on the other side, by charging a relatively high price from the former side and allowing free entrance for the agents of the latter side. However, if the network externalities on one side are not substantial, then the matchmaker will maximize profits by matching an equal number of agents from each side. This paper thus provides an explanation of the asymmetric pricing schedules in two-sided markets where the matchmaker uses a one-program pricing schedule.

JEL Codes: D42, L11, L12

1 Introduction

A two-sided market consists of two groups of agents who can meet each other through an intermediary market: Each agent is interested in interacting with the agents on the other side, and each is willing to pay for a network –or platform– to provide this mediation service. The celebrated examples of two-sided markets are newspapers and TV, where viewers and advertisers meet; operating systems, where application developers and clients meet; and internet informational intermediary services, such as dating or job search services.

Network externalities, by definition, are a feature of two-sided markets: The properties of a marketplace affect an agent's utility, as well as the utility of the agent with whom he is matched.[1] Market size is an important feature that affects an agent's utility. The effect of the size of the network on an agent's utility can be positive and/or negative: Positive externalities occur when a larger-sized market on one side increases the utility of agents on the other side, as a larger network provides a greater chance of finding a partner with whom to interact. This type of network externalities are usually called positive inter-group size externalities in the two-sided market literature. Negative intra-group network externalities, on the other hand, occur when a larger-sized market on one side decreases the utility of agents on that same side: A thick market on one's own side means higher number of competitors and a lower chance of finding a trading partner.[2] An example would be a group of men and women who are interested in meeting each other and who can meet each other through the only bar of the town. A man's utility from attending the bar would increase with the number of women attending this bar, as a higher number of women will imply a higher chance to be matched with a preferred woman, which in turn implies the existence of positive network externalities. On the other hand, a man's utility from attending the bar would decrease with the number of men attending this bar, as a higher number of men will imply higher competition among men and lower chances to be matched with a preferred woman, which in turn implies the existence of negative network externalities.

[1] An extensive definition of externalities in two sided markets can be found in Rochet and Tirole (2004).

[2] A negative effect can also occur on the other side, along with the positive effects: agents may prefer a network smaller on the other side, i.e., the viewers may prefer a newspaper with a lower number of advertisers. This aspect of networks is not investigated in this study.

1.1 Agent Heterogeneity and Negative Intra-Group Externalities

Two-sided markets with positive inter-group network externalities and heterogeneous agents is examined by Ambrus and Argenziano(2005). According to their research, heterogeneity among agents affects the quantity of optimal markets such that a large enough heterogeneity among agents is a necessary condition for the existence of multiple networks in equilibrium. In the case of monopoly, if there is enough heterogeneity among consumers then the monopolist might want to establish two networks rather than one. In the case of duopoly with Bertrand competition, heterogeneity of consumers is needed for firms to earn positive profits by matching the high types of one side of the market with the low types of the other side, where stealing consumers by undercutting rivals' prices is no longer profitable. Li and Damiano (2004) showed that, in the presence of network externalities due to sorting effects rather than size effects, that is, when the agents' utility depends on the type of agent on the other side of the market with whom he is matched, an equilibrium exists only if types are sufficiently diffused.

Jullien (2001) showed that, in the presence of both inter-group (between groups) and positive intra-group (within groups) network externalities with more than two subgroups of consumers, where the established firm has an advantage of reputation, the market can be shared between two firms with the strategy of divide-and-conquer. In the strategy of divide-and-conquer, firms form a large subsidized base group whom they sell to the group from which they extract the surpluses due to network effects, in particular bandwagon effects. As a consequence, no network can capture the surplus generated by the intergroup network externalities in this equilibrium. When intra group network effects are small, and perfect price discrimination is feasible, both firms are better off selling compatible goods with the strategy of divide-and-conquer. And thus, unless one firm can exploit large intra group network effects, cross-subsidization is less profitable. Thus intra-group network externalities can make cross-subsidization and asymmetric networks profitable in the equilibrium. In the case of two competing auction sites, as in Ellison, Fudenberg and Mobius (2003), the concentration of auction markets appears to exist due to scale effects or efficiency effects. That is because larger markets provide a greater expected surplus to participants. The reason for incomplete concentration is the market impact effect, which occurs when a seller switches to the market and takes into account the increase in the seller-buyer ratio there, thereby lowering the expected price. Thus an auction market can be dominated by a single auction site, but there are also equilibria in which sites of quite different sizes coexist. In a setup where negative intra-group and positive inter-group network externalities are present, and where the agents are homogeneous on each side, Bellaflamme and Toulemonde (2007) investigated if it is possible for a new platform to divert agents from an existing platform by using divide-and-conquer strategies. They found that the new platform can make positive profits either in the case where intra-group externalities are strong enough or in the case where they are weak enough, by using the divide-and-conquer strategies. For the values of intra-group network externalities that are in between, it is not possible for a newcomer to launch a platform by any strategy.

As in Ambrus and Argenziano (2005), in the presence of only positive network externalities, setting up multiple networks is the most profitable strategy for a monopolist when agents are of heterogeneous types. On the other hand, whenever the monopolist is constrained to form a single network, the profit-maximizing strategy is to form a single symmetric network where agents of equal size and type from each side join. Negative network externalities affect the structure of two-sided markets. Ellison, Fudenberg and Mobius (2003) showed that, in auction markets, agents can take into account the seller-buyer ratio while deciding whether to switch to another auction site, and thus are negatively affected by the size of the market on their side. This prevents the complete concentration of the auction market and allows multiple auction sites to coexist.

This paper investigates the markets formed by a monopolist matchmaker in the presence of negative intra-group network externalities and their implications for equilibrium prices and market coverage, while letting the extent of the negative network externality vary. The results reflect the effect of negative intra-group network externalities on the two-sided markets for each value of negative network externality parameter that varies within a wide range. The intuition behind negative network externality is that it reflects the importance of competitiveness among agents on each side. On each side, an agent's utility should decrease, ceteris paribus, with an increase in the number of competitors, who decrease his chances and choices to find a match. Another interpretation of negative network externalities is one where agents who can take into account the ratio of each side of the market –the ratio of the number of their potential partners to their competitors– which reflects their probability of being matched. An advertiser's utility will increase as fewer of his competitors advertise in the Boston Globe; a job seeker who sends his resume to monster.com will rightly think that his chances of finding a job on monster.com will decrease if there are many other job seekers who have done the same.

In the model presented, the two sides of the market are assumed to be symmetric, where each side consists of heterogeneous agents with a continuum of types. Firstly the model is presented where the monopolist is restricted to forming at most one search market on each side, that is, he is able to set up at most one network, by charging a single price of each side of that network. The setup of the model is a two-stage game where in the first stage the monopolist matchmaker creates search markets on each side by setting the network access price for each side. After observing the prices, agents make their decisions about joining the markets created. The model assumes the presence of positive network externalities; an agent is always better off if she or he joins a market that is thicker on the other side. By contrast, the extent of negative network externalities can vary from as much as equaling that of the positive network externalities to a minimum of total absence. Letting the negative network externalities vary so widely allows the analysis of the effects of different degrees of competition within a side. The variation in the competitiveness parameter reflects the variation in the extent of negative network externalities in two-sided markets: while job seekers on monster.com prefer less competition on monster.com, Visa card holders are not competing with each other for sellers. Alternatively, when the agent's utility is decreasing with an incease in the size of the market on his own side, this can be interpreted as the ratio of potential partners to competitors, where the number of potential partners per competitor reflects the probability of finding a match, and the weight of this ratio in an agent's utility is allowed to vary widely. The assumption of heterogeneous agents allows the monopolist matchmaker to sort agents into different marketplaces by charging different prices. Heterogeneity in the types of consumers reflects the differences in agents' valuations of the network good. Although agents are heterogeneous in their valuation of the network good, it is assumed that agents do not differ in their valuation of network externalities.

Previous studies, (i.e. Jullien (2001), Caillaud and Jullien (2003)) have shown that the strategy of divide-and-conquer, where a firm forms a large subsidized base group whom they match with the group from which they extract the surpluses due to network effects, can be an equilibrium in the case of duopoly. In this paper, it is conjectured that in the presence of substantial negative network externalities, the optimal strategy is similar to the strategy of divide-and-conquer even in the case of monopoly, where the monopolist matchmaker gains zero profits from the base side and extracts the revenues from the other side. This is supported by numerical analysis. In this case, the monopolist will match a relatively small number of highest types, from whom it extracts the revenues, with the entire population of the other side of the market, the base side. In this equilibrium, the number of high types who join the market decreases as the extent of negative network externalities increase. Thus prices charged in that setting will be very high for the former side and will be zero for the latter. In the absence of substantial negative network externalities, by contrast, this strategy will no longer be optimal. The monopolist will maximize his revenues by matching equal amounts of higher types through the formation of a symmetric network and the same prices will be charged to both sides.

The organization of the chapter is as follows. Introduction presents the related literature and the discussion. Then in section 2, the basic model is introduced. In section 3, the optimal network for one side given the participation from the other side is solved for and the two networks which will be called "symmetric network" and "asymmetric network", as candidates of revenue-maximizing networks are presented. Afterwards, by an analytical proof it is shown that in the presence of extensive negative network externalities, asymmetric network yields higher revenues compared to the symmetric, but on the other hand, if the negative network effects are not extensive, in that case the symmetric network yields higher revenues compared to the asymmetric. Lastly, the results of numerical analysis which points that the two candidate networks are the only revenue-maximizing networks are presented.

2 The Basic Model

The setup of the model is a two-stage game played between a monopolist matchmaker and the agents of a two-sided market. It is assumed that the matchmaker can create at most one search market for each side; in other words, the monopolist can set up one network through which agents can meet. For convenience, the two sides of the market are hereby said to be composed of men and women respectively, following the literature on matching markets. Agents of both sides have heterogeneous one-dimensional characteristics, called "types". The set of players in the game is (F, M, W), namely the matchmaker firm F, a unit mass of agents from the men's side M consisting of agents of type $t_m \in [0, 1]$, and a unit mass of agents from the women's side W consisting of agents of type $t_w \in [0, 1]$. Type distribution is uniform for each side with a support fo each side: $U[0, 1]$. The monopolist matchmaker's objective is to maximize his revenue, which is the sum of the network access fees from each agent who choose to participate in the network. The matchmaker is unable to observe the types of agents, and he uses entrance fees to create search markets. The game consists of two stages: in the first stage the matchmaker sets the prices for each side of the network, and in the second stage agents of each side simultaneously make their decisions about whether tp join the network. The two sides of the market are assumed to be symmetric. It is assumed that search markets are costless to organize.

The price for each side of the market will be called P_M and P_W, for the men's and women's side respectively. The equilibrium concept used will be subgame perfect Nash equilibrium, in which agents will choose to participate in the equilibrium if it is their interest after observing prices (entrance fees) P_M and P_W, set by the matchmaker. A man of type t_m will have a utility of

$$U(t_m) = t_m \frac{\text{\# of women who joined the market}}{(\text{\# of men who joined the market})^{1-y}} - P_M$$

if he joins the market and he will choose to participate if $U(t_m) \geq 0$. Symmetrically the utility for a type t_w woman who decides to join will be

$$U(t_w) = t_w \frac{\text{\# of men who joined the market}}{(\text{\# of women who joined the market})^{1-y}} - P_W$$

given the price vector (P_W, P_M). The positive size externalities are represented by this function as an increase in a male agent's utility with an increase in the number of women who join the market, and negative size externalities are represented as a decrease in a male agent's utility with an increase in the number of men who join the market. The intuition behind positive and negative network externalities in this utility function can be explained respectively as follows: a male agent thinks that his chances of finding a partner will increase with the size of the network on the women's side as the choices from which he can find a partner increase, whereas, with an increasing size of the network on his side, his chances of finding a partner will decrease as it will imply higher competition and fewer options to choose from. It is assumed that negative network externalities can at most be as important as positive network externalities, where, in that case, a male agent's utility would depend only on the ratio of the number of women to men. The symmetric case would apply for a woman agent. Thus the parameter $y \in [0, 1]$ shows the valuations of agents for negative size externalities as shown in the utility function below. The utility function for a type t_m man in the presence of negative network externalities can also be represented as:

$$U(t_m) = t_m \left(\text{\#women in the market}\right)^y \left(\frac{\text{\#women in the market}}{\text{\#men in the market}}\right)^{1-y} - P_M$$

Here, $1 - y$ represents the importance of the ratio of the sizes of each side in an agent's utility function; the ratio of the network on the men's side to the women's side shows the number of men per women agents' and analogously for the women's side. The ratio of the sides represents the probability of being matched, assuming that each agent on the same side of the market who decides to join the matchmaker's network has an equal chance of being matched. $y = 1$ represents the extreme case where an agent's utility is not affected by the ratio of the sides and is only a function of the size of the network on the other side, i.e., the case of positive externalities. The other extreme case is when $y = 0$, where a male agent's utility is only a function of the ratio of women in the market to men of his type: this represents the case where an agent cares only about the probability of being matched, or where agents assign the same value to the market when it is thick on the other side as when it is thin on their side. For the values of $y \in (0, 1)$, the agent's utility is a weighted function of positive externalities and the probability of being matched in the platform, where a higher value of y implies a higher relative importance given to positive externalities rather than to the probability of being matched in the platform.

In the games of two-sided markets, multiple Nash equilibria exist due to network externalities.[3] The multiplicity of equilibria occurs as, given the price vector, multiple expectations about market participation can be supported. The celebrated example with positive network externalities is that non-participation in the network is always a Nash equilibrium; if agents on one side expect zero participation on the other side and will not participate, then agents of the other side will also choose not to participate. In this model, the uniqueness of the equilibrium is justified by intuitively refining the equilibria: consumers are assumed to be myopic and there are zero prices where the lower-type consumers drop off, as the matchmaker raises the prices until revenue-maximizing equilibrium prices are reached.

3 The Monopolist's Optimal Network

Given one-dimensional type space with infinitesimal players, for all $P_M > 0$ and $P_W > 0$, a sorting equilibrium realizes. Let $m \in [0, 1]$ be the cutoff type of man whose utility from joining the network is zero, that is, $U(m) - P_M = 0$; and let $w \in [0, 1]$ be the type of woman whose utility

[3]See Farell and Saloner (1985), Katz and Shapiro (1985, 1994).

from joining the network is zero, $U(w) - P_W = 0$; for a given price vector (P_M, P_W). Then as, $\frac{d(U(t_m)-P_M)}{dt_m} > 0$ for $t_m \in [0,1)$, the utility of joining increases with an increase in the type of agents, men of types $t_m \in [m,1]$ will join the network and men of $t_m \in [0,m)$ will choose not to join the network, with $(1-m)$ being the number of men who decided to join, and $(1-w)$ the number of women who decided to join. Then a type t_m man's utility gained by joining the market will be

$$U(t_m) = t_m \frac{(1-w)}{(1-m)^{1-y}} - P_M$$

and similarly it will be

$$U(t_w) = t_w \frac{(1-m)}{(1-w)^{1-y}} - P_W$$

for a type t_w woman. It is assumed that agents are risk neutral and care only about the difference between the expected match value and the entrance fee they pay. If an agent doesn't join any market his utility will be 0.

The matchmaker's revenue is the sum of revenues from the men's and women's sides of the market with the cutoff value $m \in [0,1]$ for the men's market and the cutoff value $w \in [0,1]$ for the women's market:

$$R = (1-m)P_M + (1-w)P_W$$

At the second stage of the game P_M and P_W will determine m and w such that at equilibrium the type m man's and type w woman's utility will be 0 and $U(m) - P_M = 0$ and $U(w) - P_W = 0$. The equilibrium price for the men's side can be written in terms of the cutoff values as

$$P_M = \frac{(1-w)}{(1-m)^{1-y}} m$$

similarly for the women's side the equilibrium price will be

$$P_W = \frac{(1-m)}{(1-w)^{1-y}} w$$

Then the revenue of the matchmaker can be written as:

$$R(m,w;y) = \frac{m(1-m)(1-w)}{(1-m)^{1-y}} + \frac{w(1-m)(1-w)}{(1-w)^{1-y}}$$

Thus, at the first stage of the game, the matchmaker can maximize his revenue with respect to cutoff values (m,w) and form the revenue-maximizing network by charging prices $P_M(m^*, w^*)$ and $P_W(m^*, w^*)$, where (m^*, w^*) are equilibrium cutoff values that maximize the matchmaker's revenue.

Lemma 1 *For a price vector (P_M, P_W), the revenue of the network is uniquely determined in subgame perfect Nash equilibrium by the corresponding cutoff values (m^*, w^*) and is equal to the maximum possible revenue among the Nash equilibria for the given price vector.*

Proof: First, the argument that the network is in the unique Nash equilibrium can be proved as follows. Assuming consumers are myopic, the matchmaker can attain the population-maximizing network by charging $(P_M, P_W) = (0, 0)$ at the beginning: all the consumers will join the network with cutoff values $[w, m] = [0, 0]$. Let (m', w') be the cutoff values of the man and woman respectively whose utility is 0 given (P_M, P_W); $U(m') = 0$ and $U(w') = 0$. Note that $\frac{dm'}{dP_M} > 0$ and $\frac{dw'}{dP_W} > 0$, and therefore (m', w') is uniquely determined by (P_M, P_W). As the matchmaker increases P_M and P_W gradually, agents of types $m \in [0, m')$ and $w \in [0, w')$ will have negative utility from joining the network and thus will drop out, whereas agents of types $m \in (m', 1]$ and $w \in (w', 1]$ will have positive utility from joining the network. The matchmaker would increase the price P_M and P_W until the revenue-maximizing price is reached.■

Since the sides are symmetric, it is assumed that $m \geq w$, to include the possibility of a non-symmetric solution where m is reformulated as $m \equiv w + a$, with $a \in [0, 1 - w]$ being the coefficient that describes the degree of asymmetry, showing the difference in cutoff values between the less populated men's side m and the more populated women's side w. Revenue with this new formulation can be written as:

$$R(w, a; y) = (w + a)(1 - w)(1 - w - a)^y + w(1 - w)^y(1 - w - a)$$

Claim 1 *The revenue function $R(w, a; y)$ is concave in the coefficient a, the coefficient of the degree of asymmetry, for all values of $y \in (0, 1]$.*

Concavity of revenue in the asymmetry coefficient a allows us to concentrate on values of a that optimize revenue with respect to a, in other words that satisfy

$$\frac{\partial R(w, a; y)}{\partial a} = R_a = 0$$

and allows us to use the envelope theorem to reach revenue-maximizing cutoff values. Thus for all local maxima of the revenue function $(w, a^*(w); y)$ for $a \in [0, 1 - w]$ we can conclude that $R_a = 0$ should hold. Thus we can find optimal $a^*(w)$ for all values of w, where $(w, a^*(w); y)$ satisfies $R_a = 0$.

The effects of the cutoff values on the revenue of the matchmaker can be decomposed into positive and negative size effects that affect the equilibrium price, the effects of the type and the quantity of agents joining the market.

The revenue obtained from the men's side can be written as:

$$R_M = (1-m)P_M = (1-m)\frac{m(1-w)}{(1-m)^{1-y}}$$

The marginal revenue from men's side when there is an increase in m is then

$$MR_M(m) = \frac{\partial R_M}{\partial m} = \frac{(1-w)}{(1-m)^{1-y}}[(1-m) - m + m(1-y)]$$

which can be decomposed into various effects:

$$MR_M(m) = MR_M^1(m) + MR_M^2(m) - MR_M^3(m)$$

where $MR_M^1(m)$, $MR_M^2(m)$ and $MR_M^3(m)$ represent the first second and third terms of the above, respectively. First, revenue increases by $MR_M^1(m)$ where

$$MR_M^1(m) \equiv \frac{(1-w)}{(1-m)^{1-y}}(1-m)$$

as the cutoff type increases due to the effect of a higher cutoff type who has a higher willingness to pay, thus resulting in a positive marginal revenue. Second, revenue increases by $MR_M^2(m)$ where

$$MR_M^2(m) \equiv \frac{(1-w)}{(1-m)^{1-y}}m(1-y)$$

as the cutoff type increases due to the effect of a higher cutoff type that corresponds to a smaller men's market which thus has a higher willingness to pay for a less competitive men's market resulting in a positive marginal revenue. Third, revenue decreases by $MR_M^3(m)$ where

$$MR_M^3(m) \equiv \frac{(1-w)}{(1-m)^{1-y}}m$$

as the cutoff type increases due to the effect of a higher cutoff type that corresponds to a smaller men's market and a decreased demand for the network service, thus resulting in a negative marginal revenue.

Correspondingly, the revenue from the women's side is:

$$R_W = (1-w)P_W = (1-w)\frac{w(1-m)}{(1-w)^{1-y}}$$

Regarding the marginal revenue from the women's side when there is an increase in m, the men's cutoff value affects women's revenue negatively, and this effect is due only to decreased positive network effects, i.e. the shrinking size of the market on the men's side:

$$MR_W(m) = \frac{\partial R_W}{\partial m} = -(1-w)\frac{w}{(1-w)^{1-y}}$$

This decomposition shows the four ways that an increase in the men's cutoff value m affects the matchmaker's revenue: First, it will have an increasing effect on R_M due to an increase in the price for men: P_M will increase as the cutoff type men's willingness to pay increases. Second, it will have a decreasing effect on R_M due to a decrease in the number of men joining and paying for the network. Third, it will have an increasing effect on R_M due to an increase in price for the men's side caused by decreased competition among men: it will decrease the negative network effects on the men's side. Fourth, it will have a decreasing effect on R_W due to a decreased price for the women's side caused by a smaller market on the men's side: it will decrease the positive network effects on the women's side.

Claim 2 *As the monopolist expands the network on the women's side by including lower types, it becomes relatively more profitable to increase the size of asymmetry, that is the difference between the sizes of each side. For the pairs* $(w, a^*(w))$*, the asymmetry coefficient* $a^*(w)$ *increases as the women's cutoff value* w *decreases, for all values of* $w \in [0, 1)$*.*

Claim 2 implies that as the monopolist expands the network on the women's side, he will always find it profitable also to expand the asymmetry of the network, i.e., the difference between the populations of the two sides who join the network. By setting up an asymmetric network that is larger on one side and smaller on the other, the monopolist will create a network relatively more valuable on the men's side; thus the larger the network on the women's side, the more profitable it will be to extract revenues from the men's side.

Claim 3 *As the monopolist expands the network on the women's side by including lower types, it becomes relatively more profitable to contract the network on the men's side. That is, for the pairs* $(w, a^*(w))$*, the men's cutoff value* $m^*(w)$ *increases as the women's cutoff value* w *decreases, for all values of* $w \in [0, 1)$*. This result also implies Claim 2.*

The intuition behind Claim 2 and Claim 3 can be explained by the network effects, the effects of the cutoff types and quantity of agents who join the network on the matchmaker's revenue. First, $MR_M(m) < 0$ for $m > \frac{1}{y+1}$, and as $MR_W(m) < 0$ for $m \in [0, 1]$, revenue from the men's side is strictly decreasing with m, for all $m \in [\frac{1}{y+1}, 1]$; thus for all values of w, the matchmaker can increase the revenues by decreasing m for $m \in [\frac{1}{y+1}, 1]$. So these values of w and m can be ruled out as they cannot be an equilibrium. Second, for $m < \frac{1}{y+1}$ the revenue from the men's side is increasing with m whereas the revenue from the women's side is decreasing with m. Also, $MR_M(m)$ is decreasing with m, and thus the revenue from the men's side is increasing at a decreasing rate where $MR_W(m)$ doesn't change with m, and thus revenue from the women's side is decreasing at a constant rate as the men's cutoff value m increases. From this we can conclude that to maximize revenue, the monopolist should form a network where the marginal revenue gained from the men's side by increasing m is equal to the marginal revenue lost from the women's side by increasing m.

The previous claims can be explained by $MR_M(m)$ and $MR_W(m)$ as follows: $MR_M(m)$ is decreasing with w; as the women's cutoff value increases, the number of women who join the network decreases, resulting in positive network effects for the men's side due to decreases in the thick women's market. So, as the women's market gets bigger and w gets smaller, the marginal revenue gained from the men's side by increasing m increases. On the other hand, $MR_W(m)$ is increasing with w, as, if w is higher, it will be more costly to increase m, because the loss of revenue from the women's side due to the smaller men's market will be higher; i.e.,because the value of women to the network and the revenue from the women's side will be higher. But if w is low, revenue from the women's side will be low, and thus, $MR_W(m)$, lost by increasing m, will be low. So, as the women's market get bigger and w gets smaller, the marginal revenue lost from the women's side by increasing m will be decreasing. Thus, a matchmaker who is maximizing the revenue with respect to the cutoff value m by equating the marginal revenues from the men's side to the marginal cost from the women's side, given the women's cutoff value w, will set a higher optimal m the lower the cutoff value w, as the marginal gain from increasing m, $MR_M(m)$, will be higher, and marginal cost from increasing m, $MR_W(m)$, will be lower.

The results of Claim 2 and Claim 3 can be summarized as follows: if the market is small containing only a small number of the top-type population of each side of the market then it is not so profitable to expand one side and contract the other and charge the smaller side more for the decreased negative network externality. First, revenues are decreasing by excluding the highest type agents regardless of the size of network on the other side, as the number of agents, who pay for the network becomes too low. For a smaller market including only the highest type women, marginal revenue gained by contracting the men's market is low as positive network effects to extract revenue are low on the women's side, and marginal costs are high as revenue extracted due to market size is high. This is because the positive size effects are not extensive when only a small amount of people are included in the network. On the other hand, if the network is large on the women's side, then the marginal revenue lost by the exclusion of lower-type women will be relatively low due to the low prices on their side. Moreover, marginal revenue gained from excluding low-type men will be higher in this case because the high-type man's value to the network is relatively high: negative network effects will be low if small numbers of men are included, and positive network effects will be high if high number's of women are included in the market. Therefore, positive and negative network effects working together create a highly valuable market for the less populated men's side. When the network is thick on the women's side, it becomes more profitable to extract revenues from high-type men by expanding the network on women's side, while contracting the network on the men's side, and thereby creating a more valuable network for them.

On the other hand, although an asymmetric network can be cost minimizing for a given cutoff value w, Claims 2 and 3 have no implications for weather an asymmetric network is revenue maximizing for any value of w. By Claim 4 it is shown that an asymmetric network can be revenue maximizing only if the network is large enough on the women's side.

Claim 4 *If both sides of the network are small enough, such that at most $\frac{y+1}{y+2}$ of the highest type agents with cutoff values of at least $w = m = \frac{1}{y+2}$ are included, then the only revenue-maximizing network is the symmetric network where $\frac{y+1}{y+2}$ of the highest type men and women join. In other words, for $w \in [\frac{1}{y+2}, 1]$, the optimal network is $(w, a^*(w)) = (\frac{1}{y+2}, 0)$ for all $y \in (0, 1]$. In this network, only the top $\frac{y+1}{y+2}$ type men and women join.*

Note that $\frac{2}{3} \geq \frac{y+1}{y+2} \geq \frac{1}{2}$ so this symmetric market will comprise at least $\frac{1}{2}$ and at most $\frac{2}{3}$ of the top types, and as $\frac{y+1}{y+2}$ increases with y, the optimum symmetric network will include an increased portion of top types as the negative network externalities become more important.

The reasons for Claim 4 are straightforward: First of all, for values $w \in [\frac{1}{y+2}, \frac{1}{y+1}]$ and $m \in [\frac{1}{y+2}, \frac{1}{y+1}]$, –and in general for values $w \in [0, 1]$ and $m \in [0, 1]$–, the marginal cost of increasing the cutoff value m in terms of the revenue lost from the women's side is (strictly) greater than zero, as

$$MC_W(m) = |MR_W(m)| = \left|\frac{\partial R_W}{\partial m}\right| = \left|-(1-w)\frac{w}{(1-w)^{1-y}}\right| > 0$$

and as $MR_M(m) = 0$ at $m = \frac{1}{y+1}$, and $MR_M(m) > 0$ for $m < \frac{1}{y+1}$, the critical m such that the marginal revenue from the men's side is equal to the marginal cost from the women's is $m < \frac{1}{y+1}$. The symmetric condition applies to the women's side. Note that the two conditions to maximize revenue by equating marginal revenues from one side to marginal costs from the other side, which thus sets the critical m and w, form twp symmetric equations with two unknowns. The solution to this pair of equations should give the revenue-maximizing symmetric cutoff values of m and w. As the proof of Claim 4 shows, for values of $w < \frac{1}{y+2}$, the optimal value of $m > \frac{1}{y+2}$. Thus the solution pair from Claim 4 shows the only symmetric solution where the matchmaker's revenue is maximized and the marginal cost of each cutoff value is equal to his marginal revenue.

The implication of Claim 4 for the optimal network is that, if the population is low enough and the willingness to pay of agents for the platform high enough, then an asymmetric market will never be optimal, and creating a large network on one side in order to extract revenue from the other side will never be the revenue-maximizing strategy. In that case, the value of the asymmetric market will be low due the low population in the market and therefore the impact of network externalities on agents will be low; but on the other hand, the revenue lost by excluding the lower types will be high as, the high-type agent's willingness to pay is high.

Claim 4 also determines the population of the women's side such that setting up an asymmetric network is revenue-maximizing, which is stated in Corollary 1:

Corollary 1 *An asymmetric network is revenue maximizing only if the cutoff value of the more populated side of the network is sufficiently small, i.e.* $w < \frac{1}{y+2}$*. This is when the women's side has a population larger than* $\frac{y+1}{y+2}$*.*

Lemma 2 *If the extent of negative network externalities is low, that is, for higher values of* y *such that* $y > y^*$ *where* y^* *is approximated as* $y^* \cong 0.561553$*, the symmetric solution, where an equal number of men and women join the network with* $(w, a^*(w)) = (\frac{1}{y+2}, 0)$*, is a local maximum of the revenue function* $R(w, a; y)$*. In other words, the symmetric network can be revenue maximizing only if negative network effects are not substantial.*

Lemma 3 *If the extent of negative network externalities is high, that is, for lower level values of* y *such that* $y < \tilde{y}$ *where* $\tilde{y}$ *is approximated as* $\tilde{y} \cong 0.5628$*, the asymmetric solution, where a smaller-sized portion of top-type men will meet all the women with* $(w, a^*(w)) = (0, \frac{1}{y+1})$*, is a local maximum. In other words, an asymmetric network can be revenue maximizing only if negative network effects are substantial.*

Lemma 3 shows the case where the matchmaker maximizes the revenue from only one side of the network, i.e., he is using the strategy of divide and conquer, using the women's side as a base by setting $w = 0$. Although he does not get any revenue from the women's side, in this case he compensates for the losses from the other side of the network, where now he is able to extract higher revenues. The reason the revenues are higher in that case is the matchmaker's ability to extract the value of network externalities, as positive and negative network externalities working together create a more valuable network on the men's side. Recall that the divide-and-conquer strategy involves sacrificing all the revenues from one side, or even subsidizing that side, to make the most valuable network that can be attained for the other side. As subsidization is not possible here, the only way for the matchmaker to increase the value of the network on the men's side is by increasing the network's size on the women's side, as the only effect of the women's side on the men's side is that of positive network effects or effects, of the thick women's market. In this case the monopolist is able to attain this configuration by including all the women in the network by setting the price at that side to be 0 and the revenues from the women's market thus will be 0, and giving up the cutoff value of the women's side w as a policy variable. Then, given $w = 0$, the matchmaker maximizes the revenues from the men's side, again by equating the marginal revenue for increasing m to the marginal cost for the women's side. Note that this time both the revenue and the marginal revenue from the women's market are 0 and the critical m should equate the marginal revenue from the men's side by increasing m to zero. Thus, the critical value for equilibrium is $m = \frac{1}{y+1}$, as shown before.

By Lemma 2 and Lemma 3, it is shown that the network $(w, a^*(w); y) = (\frac{1}{y+2}, 0; y)$ is a local maximum for $y \in (0.561553, 1]$ and the network $(w, a^*(w); y) = (0, \frac{1}{y+1}, y)$ is a local maximum for $y \in [0, 0.5628)$. Thus it can be conjectured that the matchmaker's revenue is either maximized by forming a symmetric network that matches the $\frac{y+1}{y+2}$ amount of top-type men and women, or by forming a network that matches the $\frac{y}{y+1}$ amount of top types of one side (men's) with all the agents of the other side (women's).

Proposition 1 *There exists a threshold value for the extent of negative network externalities, $y^* \in [0,1]$, such that:*

If $y < y^*$, that is, if substantial negative network externalities are present, then $R(0, \frac{1}{y+1}) > R(\frac{1}{y+2}, 0)$ $\forall y \in [0,1]$, that is, the asymmetric network with cutoff values $(0, \frac{1}{y+1})$ for (w, m) respectively yields higher revenues than the symmetric network.

If $y > y^*$, that is, if negative network externalities in the market are not substantial, then $R(\frac{1}{y+2}, 0) > R(0, \frac{1}{y+1})$ $\forall y \in [0,1]$, that is, the symmetric network with cutoff values $(\frac{1}{y+2}, \frac{1}{y+2})$ for (w, m) respectively yields higher revenues than the asymmetric network.

If $y = y^*$, then $R(\frac{1}{y+2}, 0) = R(0, \frac{1}{y+1})$ that is, the symmetric network and the asymmetric network yield equal revenues to the matchmaker.

Note that the asymmetric network matches the top $\frac{y}{y+1}$ agents from the men's side to with all the agents of the women's side, whereas the symmetric network matches the top $\frac{y}{y+2}$ agents from both the men's and women's side. For $y = y^*$, the symmetric network and the asymmetric network yield the same revenues to the matchmaker.

Numerical Result: The local maxima, that is, the symmetric network at $\left[\frac{1}{y+2}, 0; y\right]$ and the asymmetric network at $\left[0, \frac{1}{y+1}; y\right]$ are the only candidates for the revenue-maximizing global maximum for $y \in [0,1]$.

A graphical presentation of the numerical result is given in section 1.6.2. The graphs show the values of the revenue function maximized with respect to the asymmetry coefficient a, for values of $w \in [0,1]$. I graphed the values of $R^*(w, a^*(w))$, for increasing values of y, shown as respective graphs with dot values. The relationship shown in smooth graphs is attained by connecting the single values shown as dots.

The analysis of the numerical result is as follows. By Lemma 1 and Lemma 2 it can be shown that for $y \in [0, 0.561335)$, the symmetric network at $(w, a^*(w)) = (\frac{1}{y+2}, 0)$ is a local minimum, and the asymmetric network at $(w, a^*(w)) = (0, \frac{1}{y+1})$ is a local maximum. In addition, numerical analysis at various values of y shows that for $y \in [0, 0.561335)$, the revenue function evaluated at $a^*(w)$, namely $R(w, a^*(w))$, is decreasing for $w \in [0, \frac{1}{y+2}]$. Thus it can be conjectured that when negative network externalities are substantial, that is, for lower values of y such that $y < 0.561335$, the only revenue-maximizing strategy for the matchmaker is to form an asymmetric network with cutoff values $(w, a^*(w)) = (0, \frac{1}{y+1})$.

On the other hand, by Lemma 1 and Lemma 2 it is also shown that for $y \in [0.5628, 1]$ the asymmetric network at $(w, a^*(w)) = (0, \frac{1}{y+1})$ is a local minimum and asymmetric network at $(w, a^*(w)) = (\frac{1}{y+2}, 0)$ is a local maximum. In addition to that, numerical analysis at various values of y shown below points out that for $y \in [0.5628, 1]$ the revenue function evaluated at $a^*(w)$, namely $R(w, a^*(w))$ is increasing for $w \in [0, \frac{1}{y+2}]$. Thus it can be conjectured that when negative network externalities are not substantial, that is for higher values of y such that $y > 0.5628$, the only revenue maximizing strategy for matchmaker is to form a symmetric network with cutoff values $(w, a^*(w)) = (\frac{1}{y+2}, 0)$.

For $0.561335 < y < 0.5628$, Lemma 1 and Lemma 2 show that both the asymmetric network at $(w, a^*(w)) = (0, \frac{1}{y+1})$ and the symmetric network at $(w, a^*(w)) = (\frac{1}{y+2}, 0)$ are local maxima. Additionally, numerical analysis at various values of y shows that for $y \in (0.561335, 0.5628)$ the revenue function evaluated at $a^*(w)$, namely $R(w, a^*(w))$, is first decreasing and reaches a global minimum for some value $w \in (0, \frac{1}{y+2})$, and is then increasing until the local maximum at $(w, a^*(w)) = (\frac{1}{y+2}, 0)$. Thus it can be conjectured that for the interval $0.561335 < y < 0.5628$ there are two global maxima, and in this interval, the asymmetric network at $(w, a^*(w)) = (0, \frac{1}{y+1})$ is a global maximum for relatively lower values of y, that is, for $y \in (0.561335, 0.5621)$, and the symmetric network at $(w, a^*(w)) = (\frac{1}{y+2}, 0)$ is a global maximum for relatively higher values of y that is, for $y \in (0.5621, 0.5628)$.

The intuition for the numerical result is such that with the revenue can be maximized by equating the marginal revenue with the marginal cost of increasing each critical value, w and m, which yields two equations with two unknowns, w and m, and which are symmetric. Thus one solution to these equations should be symmetric, yielding the same value for cutoff values $w^* = m^*$. The alternative strategy for maximizing the revenue of the matchmaker is to decrease the policy variable to one of the cutoff values, and to maximize the revenue from one particular side –the men's side– sacrificing the revenue from the other side –the women's side– to obtain the maximum possible revenue from the men's side, that is, to follow the divide-and-conquer strategy. This is the asymmetric solution which is unique: revenue from the men's side R_M is strictly decreasing in the cutoff value w, the only equilibrium candidate for w is $w = 0$, and given $w = 0$, the unique value such that $MR_M(m) = 0$ is $m = \frac{1}{y+1}$, which yields the candidate asymmetric solution shown to be a local maximum.

As y increases competition within agents of one side becomes less important and the impact of negative network externalities decreases. If competition within agents of one side is not significant, that is, if $y > y^*$ then the only impact of negative network externalities is for the matchmaker to contract sides, to decrease competition within sides as well, charging the agents for decreased competition by increasing the price for that side. As the model assumes symmetry of the two sides, the impact of negative network externalities is not extensive enough for one side to be less populated than the other, and therefore the revenue-maximizing optimal network is symmetric. Note that this symmetric network expands as negative network externalities become even less substantial, as competition decreases.

On the other hand, if $y < y^*$, then the negative network externalities are large enough for matchmaker to set up a completely different optimal network, where on the women's side all women join but on the men's side only the highest types join. Competition between agents in this case is more important and agents will value a network higher which is less competitive on their side, and will be willing to pay a higher amount for a less competitive market. Thus the matchmaker includes only a small portion of the highest type men in the network by charging a high price for men. As the men's side is small, positive network effects for the women's side decrease; thus agents on the women's side will value the network less, and their willingness to pay will fall. Thus it becomes optimal for the matchmaker to construct a very valuable network for men where the women are included, and thus that network will have the highest positive network effects. As a relatively small number of men are included, the competition and negative network effects are low; consequently men's willingness to pay will be substantially high, so that for the matchmaker extracting all the revenues from the highest type men will become the revenue-maximizing strategy.

Corollary 2 *When negative network effects are substantial, at the optimum, the network will be free for women but highly priced for men. (more highly priced than the symmetric network). On the other hand, in the absence of substantial negative network effects, the network is moderately priced (lower priced than for the men of asymmetric network) and the price is the same for men and* *women. That is, if $y < y^*$, then $P_W = 0$ and $P_M > 0$, and if $y > y^*$, then $P_W = P_M > 0$, where, for $y \in (0, 1]$, $P_W(y > y^*) = P_M(y > y^*) < P_M(y < y^*)$.*

Corollary 3 *For $y \in [0, 1]$, the total number of people who decide to join the market decreases as negative network externalities become more important.*

As the extent of negative network externalities increases, the increase in an agent's utility corresponding with a decrease in the population of the network on their side will be higher. Thus the network shrinks as the negative network externalities become more important.

The implication of Proposition 1 is that, although the two sides of the market are symmetric, in the presence of substantial negative network externalities, the optimal network will be asymmetric, one that is expensive and small on one side and free and covers all agents on the other side. This network picks one side randomly and matches the top types of this side will all of the population of the other side.

4 Conclusion

This chapter investigated the negative effects of one side of a network on the participants of this network from this particular side of a monopolist platform's optimal markets and pricing strategy. It is shown that if negative effects are high enough, then an asymmetric network with a divide-and-conquer strategy, where the revenues from one side are sacrificed, yields higher revenues than a symmetric network that matches equal numbers of agents from each side. Although it has not been proved, the numerical analysis points out that the symmetric and the asymmetric networks are the only possible revenue-maximizing networks, for all values of $y \in [0, 1]$.

5 Appendix

5.1 Proofs

Proof of Claim 1: $R(w, a; y)$ is concave with respect to a as

$$\frac{\partial^2 R(w, a; y)}{\partial a^2} < 0$$

And thus there is an optimal $a^*(w)$ for all values of w, where $(w, a^*(w); y)$ satisfies $\frac{\partial R(w,a;y)}{\partial a} = 0$. Note that all values of $a^*(w)$ and w which satisfy $\frac{\partial R(w,a;y)}{\partial a} = 0$ construct a line on (a, w) space.■

Proof of Claim 2: If $\frac{\partial R(w,a;y)}{\partial a} = 0$, then

$$(1 - w - a)^{y-1}(1 - (y + 1)(a + w)) = (1 - w)^{y-1}w$$

and then by taking the total derivative of both sides, the below relationship can be found. Defining A and B respectively as:

$$A \equiv y(2 - (y + 1)(a + w)) \geq 0$$

as $y + 1 \leq 2$ and $a + w = m \leq 1$, and

$$B \equiv ((1 - y)w + (1 - w)) \geq 0$$

. The relationship follows as

$$\frac{da^*(w)}{dw} = -\frac{\big(A(1 - w - a)^{y-2} + (1 - w)^{y-2}B\big)}{A(1 - w - a)^{y-2}} < 0$$

■

Proof of Claim 3: Remember that we assumed $m = w + a$ where $a \geq 0$, as the two sides are symmetric, which implies $m \geq w$. Thus $m^*(w) = a^*(w) + w$. Then we can calculate:

$$\frac{d(a^*(w) + w)}{dw} = \frac{d(m^*(w))}{dw} = -\frac{B(1 - w)^{y-2}}{A(1 - w - a)^{y-2}} < 0$$

■

Proof of Claim 4: First, if $w = \frac{1}{y+2}$ then $a^*(w) = 0$, that is,

$$\frac{\partial R(w, a; y)}{\partial a}(\frac{1}{y + 2}, 0; y) = 0$$

As $R(w, a; y)$ is concave in a, we know that for $w = \frac{1}{y+2}$ the only $a^*(\frac{1}{y+2})$is $a = 0$ as $R(\frac{1}{y+2}, 0; y) > R(\frac{1}{y+2}, a; y)$ for $a \in \{[-w, 1 - w] - 0\}$.

Second if $w > \frac{1}{y+2}$ revenue is maximized at $a = 0$.

We can show this: We know $a^*(\frac{1}{y+2}) = 0$. Also from Claim 2 we know that $\frac{da^*(w)}{dw} < 0$ for $w[0, 1)$. Thus we can conclude that for $w(\frac{1}{y+2}, 1]$, $a^*(w) < 0$. But we have assumed that $a \geq 0$.

As $R(w, a; y)$ is concave in a, we know that if $a[0, 1-w]$ then $R(w, 0; y) > R(w, a; y)$ for $w(\frac{1}{y+2}, 1]$. Thus for $w > \frac{1}{y+2}$ revenue is maximized at $a = 0$.

Third, revenue $R(w, 0; y)$ is decreasing with w for $w(\frac{1}{y+2}, 1]$. This result follows from $\frac{\partial R(w,a;y)}{\partial w} < 0$ as $w > \frac{1}{y+2}$.

Then as $R(w, 0; y) > R(w, a; y)$ when $a \neq 0$ and $R(\frac{1}{y+2}, 0; y) > R(w, 0; y)$ for $w(\frac{1}{y+2}, 1]$ we can conclude that $R(\frac{1}{y+2}, 0; y) > R(w, a; y)$ for $w(\frac{1}{y+2}, 1]$. We also showed that $R(\frac{1}{y+2}, 0; y) > R(\frac{1}{y+2}, a; y)$ for $a \in \{[-w, 1-w] - 0\}$. From here Claim 4 follows, as for values of $w[\frac{1}{y+2}, 1]$, revenue is maximized at cutoff values $(w, a^*(w)) = (0, \frac{1}{y+2})$ for all $y(0, 1]$.■

Proof of Corollary 1: This follows from Claim 4 as, if $w = \frac{1}{y+2}$ then $a^*(w) = 0$, and from Claim 2 it is known that $\frac{da^*(w)}{dw} < 0$, and thus if $w < \frac{1}{y+2}$ then $a^*(w) > 0$.■

Proof of Lemma 2: At $(w(a), a) = (\frac{1}{y+2}, 0)$ both first-order conditions are satisfied: $\frac{\partial R(w,a;y)}{\partial a}(\frac{1}{y+2}, 0) = 0$. The second-order condition is satisfied as $\frac{\partial^2 R(w,a;y)}{\partial a^2}(\frac{1}{y+2}, 0) < 0$. And for higher values of y such that $y > 0.561553$, the revenue function is concave with a positive determinant of its Hessian matrix:

$$|H| = (\frac{\partial^2 R(w,a;y)}{\partial w^2}\frac{\partial^2 R(w,a;y)}{\partial a^2} - (\frac{\partial^2 R(w,a;y)}{\partial a \partial w})^2)(0, \frac{1}{y+2}) > 0$$

■

Proof of Lemma 3: From the envelope theorem, for optimal a

$$\frac{dR(w,a;y)}{dw}(0, \frac{1}{y+1}) = \frac{y}{y+1} - \frac{y^y}{(y+1)^{y+1}}$$

which is a strictly increasing function for $y \cong 0.5628$.

This result can also be seen by Taylor expansion of the revenue function at $(w*(a), a) = (0, \frac{1}{y+1})$:

$$L(a, w) = R(0, \frac{1}{y+1}) + R_w(0, \frac{1}{y+1})(w - 0) + R_a(0, \frac{1}{y+1})(a - \frac{1}{y+1})$$

$$L(a, w) = \frac{y^y}{(y+1)^{y+1}} + (\frac{y}{y+1} - \frac{y^y}{(y+1)^{y+1}})w$$

which is decreasing with w for $y < \tilde{y}$. ■

Proof of Proposition 1: We know that the two local maxima $\left[\frac{1}{y+2}, 0\right]$ for $y > 0.561335$ and $\left[0, \frac{1}{y+1}\right]$ for $y < 0.5628$ are the candidates for the profit-maximizing equilibrium network. To determine the network that attains higher profits among them we should compare the value of $R(w, a; y)$ evaluated at these points.

It is enough to show

- if y has the minimum value of $y = 0$ then $R(0, \frac{1}{y+1}) = 1 > R(\frac{1}{y+2}, 0) = \frac{1}{2}$
- if y has the maximum value of $y = 1$ then $R(0, \frac{1}{y+1}) = \frac{1}{4} > R(\frac{1}{y+2}, 0; y) = 0.2963$
- $\frac{d\frac{R(\frac{1}{y+2},0)}{R(0,\frac{1}{y+1})}}{dy} > 0$

Note that $R(\frac{1}{y+2}, 0; y)(y = 0) = R(0, \frac{1}{y+1}; y)(y = 0)$ is satisfied for $y \cong 0.562183$ where both candidates are local maxima.■

Proof of Corollary 3: If $y \in [y^*, 1]$, then the total number of people that join the market increases with y as $\frac{\partial 2(1-\frac{1}{y+2})}{\partial y} > 0$; also if $y \in [0; y^*]$ then the total number of people that join the market increases with y as $\frac{\partial(1+1-\frac{1}{y+1})}{\partial y} > 0$, and we note that the total number of people included in the market is larger at $y \in [y^*, 1]$ than the equilibrium of $y \in [0, y^*]$, as $2(1 - \frac{1}{y+2}) = 1 - \frac{2}{y+2} > 1 + 1 - \frac{1}{y+1} = 1 - \frac{1}{y+1}$. And thus we can conclude that the total number of people included in the market increases with y.■

5.2 Numerical Results

The following graphs show the values of the revenue function when it is optimized with respect to the asymmetry coefficient a. The maximum value of the revenue function depends on the extent of network externality: y.

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