New Evidence on the Size and Drivers of the Shadow Economy in Spain: A Model Averaging Approach

Rios, Vicente

Public University of Navarre

July 2019

New Evidence on the Size and Drivers of the Shadow Economy in Spain *

Vicente Rios
Universidad Pública de Navarra
E-mail: vicente.rios@unavarra.es

Abstract

This study investigates the evolution of the shadow economy in Spain during the period 1986-2016 using the Currency Demand Approach by means of Bayesian Model Averaging econometric techniques. The results of the empirical analysis suggest that the average share of the underground economy as a percentage of the GDP during 1986-2016 was the 18.2% whereas in 2016, the estimated size was the 11.95%. The estimated figures for the period under consideration are in line with the averaged estimates of previous studies for the same time frame. Nevertheless, a remarkable difference between previous analysis and the estimated pattern stemming from this study is that the size of the shadow economy depicts an inverted U shape time profile, with a marked reduction in the years after the Great Recession. With the estimates of the shadow economy in hand, the importance of the drivers of the shadow economy in Spain is investigated by means of Bayesian Model Averaging methods. The main finding of this analysis is that the key factors driving variations in the size of the shadow economy are the taxes, the level of education and the distribution of employment across sectors.

Keywords: Shadow Economy, Spain, Currency Demand, Bayesian Model Averaging.

*I would like to thank Alberto Vaquero García for sharing the data on previous shadow economy estimates. Also, I would like to thank Pedro Pascual and Antonio Gómez Gómez-Plana for their suggestions and comments. I am grateful to Jose-María Rubio and Gonzalo Echevarría for their work in the compilation of the database used in the empirical analysis. This research has benefited from the financial support of the Spanish Ministry of Economy and Competitiveness (Project ECO2016-76681-R).

1 Introduction

The aim of this study is to obtain estimates of the size of the shadow economy in Spain for the period ranging from 1986 to 2016 using the Currency Demand Approach (CDA). To that end, this study employs Bayesian Model Averaging (BMA) econometric techniques.

The shadow economy is defined as the realization of legal or illegal economic activities, which the law requires to be reported in full to the tax administration, but which are hidden to the authorities in order to avoid paying taxes, quotes or to avoid certain regulations or administrative procedures, OECD (2017).[1]

The concern and interest raised by this phenomenon has increased after the Great Recession. According to the 2018 survey on public opinion and fiscal policy, 59.7% of the surveyed Spanish citizens consider that tax fraud is a widespread behavior among taxpayers (CIS, 2018). Undoubtedly, undeclared activities erode the ability to collect revenues through the tax system causing negative distorting effects on efficiency, equity, morality and trust in public institutions (Schneider and Enste, 2000). From a policy perspective, the topic is relevant given that the reduction of tax fraud is a common objective of the tax policy of most OECD countries and the European Union, because of an improvement in the degree of tax compliance could have a positive impact on economic activity (Caballé and Panadés, 1997).

However, despite the relevance of the problem, not much is known about the magnitude of the underground economy as different studies provide very different estimates. This is because of its measurement and analysis remains a challenging task (Schneider and Buehn, 2017). In the empirical literature, there are two main approaches to estimate the shadow economy: direct and indirect ones. *Direct approaches* refer to microeconomic approaches, that employ either well-designed surveys and samples based

[1]Traditionally the shadow economy has been defined as all unregistered economic activities that would contribute to the officially calculated (or observed) Gross National Product if observed. However, after the 2014 statistical and methodological review, some illegal activities such as prostitution, drug sales, tobacco smuggling and illegal gambling have been included in the National Accounts of Spain, in accordance with a new standard approved in the European Union, the so-called European System of National and Regional Accounts 2010 (ESA 2010).

on voluntary replies, or tax auditing and other compliance methods. While these approaches can provide detailed information they also present important drawbacks: they are expensive to carry out, the willingness to cooperate of the respondents is low and overall, they are likely to underestimate the shadow economy given that people are likely to under-declare what they hide from authorities (Schneider and Buehn, 2017). On the other hand, *indirect approaches* are mostly macroeconomic and use various economic indicators that contain information about the development of the shadow economy over time.

The most commonly employed indirect approaches to investigate the shadow economy are: (i) the Currency Demand Approach (CDA) (Tanzi, 1983; Ahumada *et al.*, 2007, 2008), (ii) the Physical Input (electricity consumption) method with its two variants: the Kauffmann and Kaliberda (1996) approach and the Lackó Approach (2000) and finally (iii) the Multiple Indicators Multiple Cuases (MIMIC) method (Giles, 1999; Dell'Anno *et al.*, 2007; Schneider and Dell'Anno, 2009).[2] In this study, the estimation of the shadow economy is based on the CDA given that is the most commonly employed framework in the empirical literature and allows to compare the BMA results obtained here with a larger amount of previous estimates.

A review of previous empirical studies using the CDA reveals that in the econometric analysis researchers usually face limited and short time-series, and as a consequence, they restrict the number of regressors to avoid multicollinearity problems and obtaining inefficient parametric estimates (see González-Fernández y González-Velasco, 2015; Pickhardt and Sardá, 2015). However, efficiency is obtained at the cost of a potential bias in the parameter estimates due to the omission of potentially relevant factors. From an econometric perspective, the omission of explanatory variables that could affect money demand patterns is of major importance given that estimates may be inefficient and/or biased. This is specially relevant in the CDA framework, as these estimates are used to estimate the demand of money for hidden transactions, a key metric that serves as the

[2]Although there are other methods such as the (i) the discrepancy between national expenditure and income statistics, the discrepancy between the official and actual labor force and the transactions approach, these are of more questionable reliability (see Schneider and Buehn (2017) for a critical review the various methods that have been used to estimate the size of the shadow economy and discuss their strengths and weaknesses).

basis to calculate hidden GDP. Second, by ignoring the uncertainty existing around the currency demand model, researchers assume there is just one true model to estimate the evolution of cash and treat the estimates as if they were from the true model, when in fact, there are many candidate models based on the inclusion of different explanatory factors which have a certain probability of being the true one (Moral-Benito, 2015; Steel, 2019). Since it is often not clear a priori which set of variables should be included in the "true" regression model, a naive approach that ignores specification and data uncertainty results in biased estimates , overconfident (too naroow) standard errors and misleading inference and predictions (Doppelhofer and Weeks, 2009).

In this regard, the key contribution of this study to the measurement of the shadow economy is methodological given that the BMA analysis performed here is based on a large set of regressors and allows to solve the aforementioned problems. This procedure allows to properly account for model uncertainty, reducing biases due to omitted variables and avoids the problems caused by multicollinearity, when there is a large number of potential factors driving currency demand and the sample size of the study is small. In a second step, with the estimated size of the shadow economy in hand, BMA is employed to produce a probabilistic ranking of the various determinants of the shadow economy in Spain.

The document is organized as follows. In Section (2) the theoretical framework underlying the CDA is presented. Section (3) reviews the literature and presents a collection of the estimates of the size of the shadow economy, based on ten previous studies using the CDA in Spain for the time-frame considered. Section (4) discusses the empirical strategy to obtain estimates and presents the data set employed in the analysis. Section (5) discusses the results of the Bayesian analysis of the demand of currency and presents the estimates on the size shadow economy. Section (6) analyzes the drivers of the shadow economy in Spain whereas Section (7) offers the main conclusions of this research.

2 Theoretical Framework

The CDA assumes that opaque transactions are carried out in the form of cash payments, with the objective of leaving no traces traceable to the authorities. The intuition behind this assumption is that while transactions made with cash are difficult to track, those made with other forms of money, recorded in financial institutions, can be easily inspected. Therefore, an increase in the shadow economy will tend to increase the demand for currency or cash. The second assumption is that if there were no taxes, the incentives to carry out opaque transactions would also disappear. Therefore, a key idea that underlies this approach is that, keeping everything else constant, high tax levels should translate into a larger size of the shadow economy. This point has been confirmed in other international studies (Dell'Anno *et al.*, 2007, Schneider, 2005).

As pointed out by Ahumada *et al.* (2008), currency demand can be expressed by means of a Cagan (1958) function as in Equation (1):

$$C_{Ot} = A\left(1+\theta_t\right)^{\alpha} Y_{Ot}^{\phi} e^{-\delta i_t} \tag{1}$$

where C_{Ot} denotes currency in t, θ_t is a fiscal variable that induces agents to operate in the underground sector, Y_{Ot} is a scale variable such as the observed GDP, i_t measures the opportunity cost of holding cash (i.e, the interest rate or inflation) and A, α, ϕ y δ are parameters that take positive values. Observable currency (C_{Ot}), is equal to the total amount of cash (C_{Tt}), which includes currency used for registered transactions (C_{Rt}) and for hidden ones (C_{St}):

$$C_{Ot} = C_{Tt} = C_{Rt} + C_{St} \tag{2}$$

Thus, this expression can be rewritten as:

$$C_{Tt} = AY_{Rt}^{\phi} e^{-\delta i_t} + AY_{St}^{\phi} e^{-\delta i_t} = AY_{Rt}^{\phi} e^{-\delta i_t} \left(1 + \frac{Y_{St}}{Y_{Rt}}\right)^{\phi} \tag{3}$$

Observed GDP (Y_{Ot}), is registered GDP (Y_{Rt}) and does not include shadow GDP (Y_{St}).

Therefore:

$$Y_{Tt} = Y_{Ot} + Y_{St} = Y_{Rt} + Y_{St} \tag{4}$$

For this reason, the size of the shadow economy in the GDP can be obtained by imposing $\theta_t = 0$ in Equation (1) to obtain an estimate of the demand of currency when there are no incentives to perform hidden transactions $\hat{C}_{Rt}$:

$$\hat{C}_{Rt} = \hat{A} Y_{Ot}^{\hat{\phi}} e^{-\hat{\delta} i_t} \tag{5}$$

Given that $\hat{C}_{Rt}$ is known by the previous equation and C_{Tt} is observable (i.e C_{Ot}), then the demand of cash for hidden transactions can be obtained as the difference:

$$\hat{C}_{St} = C_{Tt} - \hat{C}_{Rt} \tag{6}$$

The ratio between C_{Rt} and C_{St} is given by:

$$\frac{C_{Rt}}{C_{St}} = \frac{A Y_{Rt}^{\phi} e^{-\delta i_t}}{A Y_{St}^{\phi} e^{-\delta i_t}} = \left(\frac{Y_{Rt}}{Y_{St}} \right)^{\phi} \tag{7}$$

Once $\hat{C}_{St}$ is estimated, in order to determine the size of the shadow economy the quantitative Fisher Equation $M \times V = P \times T$ is employed. In the Fisher's equation, M is money, V is the speed of money, and the right hand side is the value of all the transactions of goods and services. Since $P \times T$ is unknown the practical implementation requires to assume that $P \times T$ is well approximated by nominal GDP. To estimate the size of the shadow economy, the literature has assumed that velocity in the hidden sector and the official sector are the same (i.e, $v_{Rt}/v_{St} = 1$), such that:

$$v_{Rt} = \frac{Y_{Rt}}{C_{Rt}} = \frac{Y_{St}}{C_{St}} \tag{8}$$

Hence, the size of the shadow economy (Y_{St}) can be obtained as:

$$\hat{Y}_{St} = \hat{v}_{Rt} \hat{C}_{St} \tag{9}$$

where $\hat{Y}_{St}$ is the estimate of the shadow economy obtained using $\hat{C}_{St}$ from Equation (6). However, as pointed out by Ahumada *et al.* (2008) the $\hat{Y}_{St}$ estimate based on Equation (9) will be biased if the money velocities in the underground and official sector are different from each other (i.e, $v_{Rt} \neq v_{St}$). This can be corroborated easily computing the ratio of the speeds which is given by:

$$\frac{v_{St}}{v_{Rt}} = \left(\frac{Y_{St}}{Y_{Rt}}\right)^{1-\phi} \tag{10}$$

Equation (10) shows that the assumption of equal speeds in both sectors is only correct if $\phi = 1$, or in the unlikely situation of $Y_{Rt} = Y_{St}$. In fact, the empirical evidence suggests that $\phi \neq 1$. Mark and Soul (2003) report a value of 1.2, Hamori and Hamori (2008) obtain a value of 4.13, whereas in González-Fernández and González-Velasco (2015) this parameter oscillates between [1.15-1.34]. Thus, ignoring this point can be troublesome to obtain reliable estimates. Recognizing that the value of v depends on the estimated value of $\hat{\phi}$, implies the correct expression to calculate the shadow economy with respect to the GDP is given by:

$$SE_t = \left(\frac{Y_{St}}{Y_{Rt}}\right) = \left(\frac{\hat{C}_{St}}{C_{Rt}}\right)^{\frac{1}{\hat{\phi}}} \tag{11}$$

Previous equation shows how to obtain the ratio of the shadow economy with respect the GDP given Y_{Rt}, C_{Rt}, $\hat{C}_{St}$ y $\hat{\phi}$. Since the work of Ahumada *et al.* (2007), this correction has been applied in different studies (see Macias and Cazzavillan, 2009; Pickhardt and Sardá, 2015; González-Fernández and González-Velasco, 2015; among others).

3 What do we know about the size of the shadow economy in Spain?

This section draws from the literature review on the size of the shadow economy in Spain by Vaquero-García *et al.* (2018). In their review, they find ten different time series/cross-sectional data studies employing the CDA for Spain. The results of the existing evidence on the size of the shadow economy are shown in Figure (1)

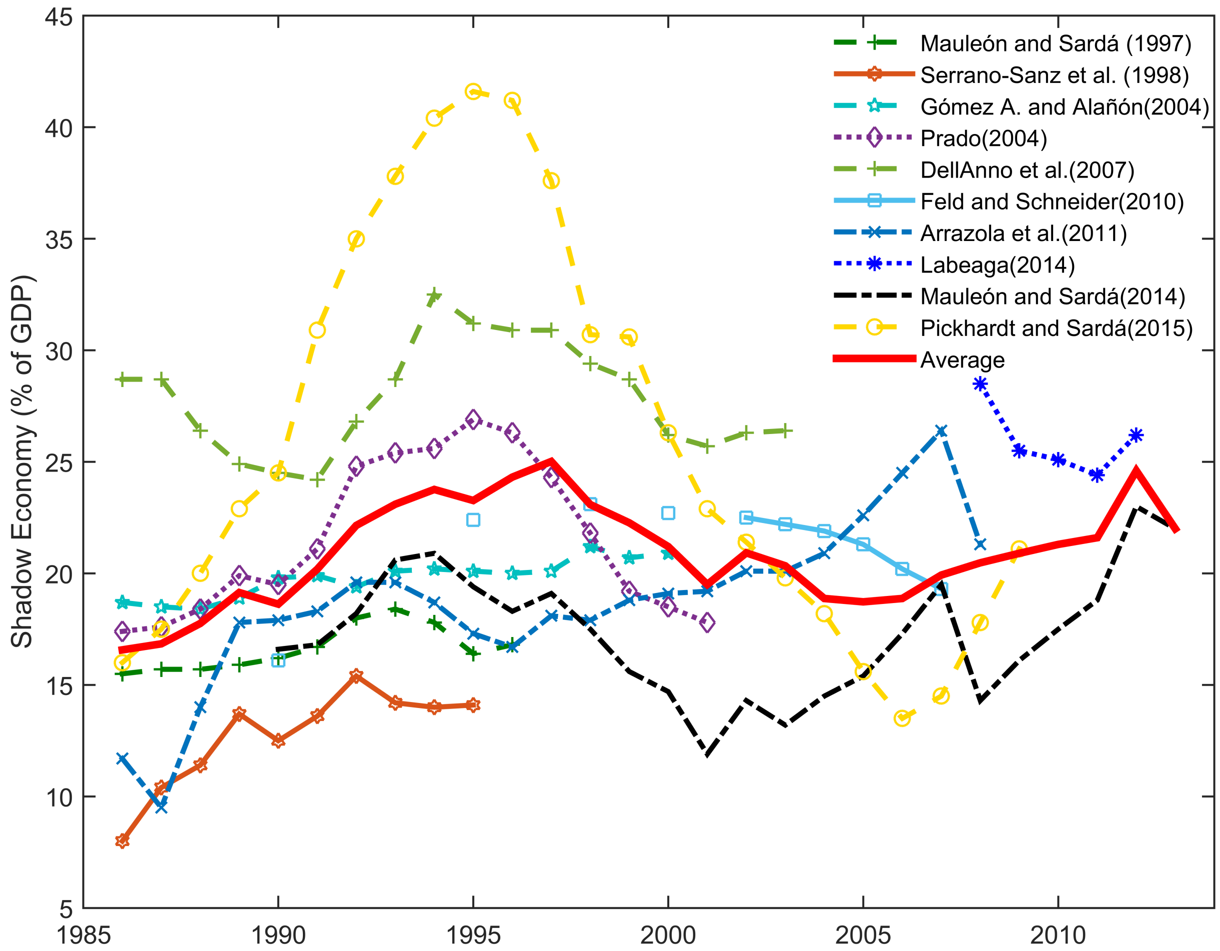
Gómez de Antonio and Alañón (2004), estimate that the underground economy in Spain during the 1980-2000 period oscillated between 18% and 21.2% of GDP. On the other hand, Serrano-Sanz *et al.* (1998) detected an increase in the shadow economy during the first decade of the sample period (8 % of GDP in 1986 to 14.1% in 1995). Mauleón and Sardá (1997) for the period 1986-1996 find that the shadow economy was on average the 16.6% of GDP. Prado (2004) quantifies that in 1986 the size was 17.4% it increased until 1995 with a value of the 26.9% and that later decreased to 17.8% in 2001. In Labeaga (2014) it is estimated that during the 2008-2012 period the size of the shadow economy was the 25.9%, moving from 28.5 % in 2008 to 26.2% in 2012.

The estimate based on the monetary method of Arrazola *et al.* (2011) points to an increase from the 9.5% in 1986 to the 26.4% in 2007, and a sharp fall with the outbreak of the crisis, rising later on to 21.3% in 2008. Finally, Pickhardt and Sardá (2015, use a monetary model to conclude that the underground economy during 1986-1996 was around 29.8%, decreased to 23.66% during 1997-2006 and that it bounced back between 2007 and 2009. These studies, which only use information from Spain, are complemented with the estimations of Dell'Anno *et al.* (2007) and Feld and Schneider (2010) that also use the monetary method to estimate the size of the informal sector in a panel of countries.

The average estimate of the shadow economy in Spain considering previous studies using the CDA for the period 1986-2016 is 20.9% of the GDP, with a standard deviation of the 2.28%. The observed pattern suggests an increase from the 16.57% in 1986 (which is the minimum value) to the 25.01% in 1997. A decrease until 2006 and then a rebound until the year 2011. While for the period ranging 1986-2007 there are numerous estimates, after the Great Recession to the year 2016, the number of estimates decreases. Thus, the U-shapped pattern observed from 1986-2007 is more reliable than the observed increase from 2008 to 2013.

In any case, given that the estimation of the evolution of the shadow economy in Spain is an empirical question, the rest of the study is devoted to obtain new estimates of this key variable.

Figure 1: Previous Shadow Economy Estimates 1986-2016



4 Empirical Strategy

4.1 The econometrics of the CDA

Most of CDA empirical studies draw from the pioneering analysis of Tanzi (1983).[3] The time-series model that is taken as a basis to estimate the underground economy in Spain is given by the Equation (12):

$$\ln C_t = \beta_0 + \beta_1 FP_t + \beta_2 \; lnY_t + \beta_3 R_t + \beta_4 \Pi_t + \gamma Z_t + \epsilon_t \tag{12}$$

where $\ln C_t$ is a $T \times 1$, vector of observations of the logarithm of currency in real terms, FP_t is a variable that measures fiscal pressure (expressed as a percentage of GDP), $\ln Y_t$ is the logarithm real GDP, R_t is the money interest rate, Π_t is the inflation rate calculated using 2010 as the base year. The term ϵ_t is a vector of random perturbations or shocks to currency demand, which is assumed to follow a normal distribution such that $\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$.

Once the Equation (12), is estimated, the demand of currency attributable to hidden activities $\hat{C}_{St}$ can be obtained using the condition $\hat{\beta}_1 = 0$ in order to obtain the counter-factual value $\hat{C}_t | \hat{\beta}_1 = 0, \vec{\hat{\beta}}_k$. Note that the predicted $\hat{C}_t | \beta_1 = 0, \vec{\hat{\beta}}_k$ is different to $\hat{C}_t | \vec{\hat{\beta}}_k$ if the effect of the fiscal indicator is different from zero (i.e, if in Equation (12) $\beta_1 \neq 0$). While the second term refers to currency demand predicted by the model using all the regressors and their respective parameters, the first fit is equivalent to performing a counter-factual asking: "What would have been the demand of currency if fiscal pressure had been the 0% or if $\Delta FP_t = 0$?". Thus, the demand of currency that can be attributed to shadow economy activities because of the existence of taxes is given by:[4]

$$\hat{C}_{St} = \exp\left[\left(\ln \hat{C}_t | \vec{\hat{\beta}}_k\right) - \left(\ln \hat{C}_t | \hat{\beta}_1 = 0, \vec{\hat{\beta}}_k\right)\right] \tag{13}$$

[3] A problem with the specification int he original work of Tanzi (1983) is that the dependent variable in that case is a ratio of monetary aggregates $\ln\left(\frac{C}{M_2}\right)_t$, which does not allow to capture the elasticity of money demand with respect income.

[4] This is the procedure by which one can obtain Equation (6) in Section (2).

With the base estimate of the demand of currency for hidden activities in hand ($\hat{C}_{St}$), the next step is to transform the data into a time series of hidden GDP (i.e, Y_{St}). To that end, it is necessary to use an estimate of money velocity v. As discussed in Section (2), usually, the following formula is employed:

$$\hat{Y}_{St} = \hat{v}\hat{C}_{St} \tag{14}$$

However, if $\hat{\beta}_2 \neq 1$ (the ϕ parameter in Section (2), one should correct for the elasticity of money with respect to income as follows:

$$\frac{Y_{St}}{Y_{Ot}} = \left[\frac{\hat{Y}_{St}}{Y_{Ot}}\right]^{\frac{1}{\hat{\beta}_2}} \tag{15}$$

where $\hat{Y}_{St}$ is obtained by means of Equation (14).

As observed, in the process of estimation of the shadow economy, there are to key parameters that come from the econometric estimation which deserve special attention to obtain reliable estimates. The first one is $\hat{\beta}_1$, which is the parameter associated to the fiscal indicator whose variation is assumed to influence shadow activities. An accurate estimation of this parameter is crucial to obtain the difference between ($\hat{C}_t$) and $\hat{C}_t|\beta_1 = 0$ .The second is $\hat{\beta}_2$, as it defines the magnitude of the adjustment of the initial ratio $\frac{\hat{Y}_{St}}{Y_{Ot}}$ in Equation (15).

4.2 Bayesian Model Averaging

Denoting by $y_t = \ln C_t$, X_t the $T \times 5$ the matrix that collects the constant of the model, the macroeconomic variables and the fiscal aggregate, such that $X_t = [\iota_t, FP_t, \ln Y_t, R_t, \Pi_t]$ where ι_t is a vector of ones of size $T \times 1$, the previous model can be expressed in a compact form:

$$y_t = \beta X_t + \gamma Zt + \epsilon_t \tag{16}$$

In the Equation (16) the distinction between X and Z is made since, for consistency with the theoretical framework, the regressors included in X should always be part of the model that explains the time variation of currency. Nevertheless, Thomas (1999) points out that the miss-specification of the currency demand model may have important effects in the estimation of the shadow economy due to the biases implied by omitted variables. For this reason in the empirical specification Z_t includes a large variety of potential factors that could affect the demand for money and whose inclusion as part of the specification is uncertain (monetary events, temporary trends, socio-demographic characteristics, labor market and productive structure features, etc).

Let γ be of size $K_1 \times 1$ and Z of size $T \times K_1$. Note that there are many sub-models M_k of the model in Equation (16) given by the subsets of coefficients $\eta^k = \left(\beta, \gamma^k\right)$ and combinations of regressors $k \in \left[0, 2^{K_1}\right]$ where K_1 is the total number of non-fixed regressors in Z. A number of questions arise when there are many potential explanatory variables in Z. Which set of variables $Z_k \in Z$ should be then included in the model? And how important are they? Model averaging techniques solve these questions by estimating all the candidate models implied by the combinations of regressors in Z (or a relevant sample of them) and computing a weighted average of all the estimates of the corresponding parameter related to Z_h (the sub-index h denotes a single regressor and not a model or a combination of regressors k). By proceeding in this way, BMA estimates consider both the uncertainty associated to the parameter estimate conditional on a given model, but also the uncertainty of the parameter estimate across different models. In general, this approach leads to wider confidence intervals for the estimated effect of the exogenous regressors $\tilde{X} = [X, Z]$ on y.

By following the Bayesian logic, the posterior for the parameters η_k calculated using model M_k is written as:

$$g\left(\eta_k | y, \tilde{X} M_k\right) = \frac{f\left(y, \tilde{X} | \eta_k, M_k\right) g\left(\eta_k | M_k\right)}{f\left(y, \tilde{X} | M_k\right)} \tag{17}$$

where $g\left(\eta_k | y, \tilde{X} M_k\right)$ is the posterior, $f\left(y, \tilde{X} | \eta_k, M_k\right)$ is the likelihood and $g\left(\eta_k | M_k\right)$ is the prior. The key metrics in BMA analysis are the Posterior Mean (PM) of the distribution of η:

$$E\left(\eta|y,\tilde{X}\right)=\sum_{k=1}^{2^{K_1}}E\left(\eta_k|M_k,y,\tilde{X}\right)p\left(M_k|y,\tilde{X}\right) \tag{18}$$

and the Posterior Standard Deviation (PSD):

$$PSD=\sqrt{Var\left(\eta|y,\tilde{X}\right)} \tag{19}$$

where the $Var\left(\eta|y,\tilde{X}\right)$ is given by:

$$\begin{aligned}Var\left(\eta|y,X\right)= \quad & \textstyle\sum_{k=1}^{2^{K_1}}Var\left(\eta_k|M_k,y,\tilde{X}\right)p\left(M_k|y,\tilde{X}\right)+ \\ & \textstyle\sum_{k=1}^{2^{K_1}}\left(E\left(\eta_k|M_k,y,\tilde{X}\right)-E\left(\eta|y,\tilde{X}\right)\right)^2p\left(M_k|y,\tilde{X}\right)\end{aligned} \tag{20}$$

To derive these metrics, it is necessary to calculate the Posterior Model Probability $p\left(M_k|y,X\right)$ of each of the sub-models M_k. These can be obtained as:

$$p\left(M_k|y,\tilde{X}\right)=\frac{p\left(y,\tilde{X}|M_k\right)p\left(M_k\right)}{\sum_{k=1}^{2^{K_1}}p\left(y,\tilde{X}|M_k\right)p\left(M_k\right)} \tag{21}$$

where $p\left(y,\tilde{X}|M_k\right)$ is the marginal likelihood and $p\left(M_k\right)$ is the prior model probability. The marginal likelihood of a model k is calculated as:

$$p\left(y,\tilde{X}|M_k\right)=\int_0^{\infty}\int_{-\infty}^{\infty}p\left(y,\tilde{X}|\eta_k,\sigma^2,M_k\right)p\left(\eta_k,\sigma^2|g\right)d\eta d\sigma \tag{22}$$

where $p\left(y,\tilde{X}|\eta_k,\sigma^2,M_k\right)$ is the likelihood of model k and $p\left(\eta_k,\sigma^2|g\right)$ is the prior distribution of the parameters in model M_k conditional to g, the Zellgner's g-prior. In addition, the BMA framework can be extended to generate probabilistic on the relevance of the various regressors, using the Posterior Inclusion Probability (PIP) for a variable h:

$$p\left(\eta_h \neq 0|y,\tilde{X}\right)=\sum_{k=1}^{2^{K_1}} p\left(M_k|\eta_h \neq 0,y,\tilde{X}\right) \tag{23}$$

In addition, it is possible to compute the Conditional Posterior Positivity of h:

$$p\left(\eta_h \geq 0|y,\tilde{X}\right)=\sum_{k=1}^{2^{K_1}} p\left(\eta_{k,h}|M_k,y,\tilde{X}\right)p\left(M_k|y,\tilde{X}\right) \tag{24}$$

where values of conditional positivity close to 1 indicate that the parameter is positive in the vast majority of considered models and values close to 0 indicate the effect on the dependent variable is negative.

The calculation of previous metrics in the BMA approach requires to define priors on the model space and priors on the parameter space. As the baseline prior on the parameter space a Zellgner g-prior is implemented based on the Bayesian Risk Inflation Criterion (BRIC), whereas a Binomial prior on the model space is adjusted, such that every model has the same a priori probability.[5]

As regards, the numerical implementation of the BMA, a Markov Chain Monte Carlo Model Composition (MC^3) methodology proposed by Madigan and York (1995) based on the so called "reverse jump" algorithm is employed to explore the model space. The algorithm operates in the model space as follows. If we let M denote the current state of the chain, models are proposed using a neighborhood , $nbd(M)$ which consists on the model itself and models containing either one variable more (*birth step*) or one variable less (*death step*) than M. A transition matrix q, is defined by setting $q(M \rightarrow M')=0$ for all $M' \notin nbd(M)$ and $q(M \rightarrow M')$ constant for all $M' \in nbd(M)$. The proposed model

[5]In particular, the g-prior hyper-parameter takes the value of $g_k = max\left\{N, K_1^2\right\}$ such that $g(\eta_k) \sim N\left[0,\sigma^2 g\left(\tilde{X}_k'\tilde{X}_k\right)^{-1}\right]$. The Binomial prior on the model space, regulates prior model probabilities according to $p\left(M_k\right)=\phi^k\left(1-\phi\right)^{K_1-k}$, where each covariate k is included in the model with a probability of success ϕ. I set $\phi=1/2$ which assigns equal probability $p\left(M_k\right)=2^{-K_1}$ to all the models under consideration.

M', is compared with the current model state M using the acceptance probability:

$$P = min\left[1, \frac{p\left(M'|y, \tilde{X}\right)}{p\left(M|y, \tilde{X}\right)}\right] \tag{25}$$

In addition, to the sampling based on (*birth steps*) and (*death steps*) to the chain of models, the algorithm also implements a *move step* which consists on replacing randomly variables in $\tilde{X}$ with variables not included currently in the model. The key feature of this econometric procedure is that it eliminates the need to consider all possible models by constructing a sampler that explores relevant parts of the large model space.

Before continuining, a difference between the BMA-CDA analysis carried out in this study with respect traditional BMA exercises should be stressed. The BMA-CDA in this analysis does not rely exclusively on the model composition algorithm described above, given that in order to accept the model as a valid candidate to the estimate of the shadow economy, the following restrictions are imposed in the model space: (i) $0\% \geq \frac{Y_{S,0}}{Y_{O,0}} \leq 50\%$, (ii) $\frac{Y_{S,F}}{Y_{O,F}} \geq 2.5\%$ and (iii) $\hat{\beta}_1 \geq 0$. The restriction (i) is used to filter out absurd models with initial estimates for the initial period that due to numerical problems might give negative ratios or values above the 50% of the GDP, which is an extremely high magnitude far above all the reported estimates in the literature. The second restriction implies discarding models that point to a ratio of the shadow economy below the 2.5% in the final period, which much lower than the last available estimate in the literature. Finally, restriction (iii) discards models for which the theoretical core of the CDA does not hold (i.e, it excludes models in which the effect of increasing taxes decreases the demand of currency). Overall, this set of restrictions ensure shadow economy estimates to be produced by models conformable with the CDA theory and that the figures are not unreasonable due to bad model draws.

4.3 Data

This section describes the data set employed in the CDA-BMA analysis.

4.3.1 Dependent variable

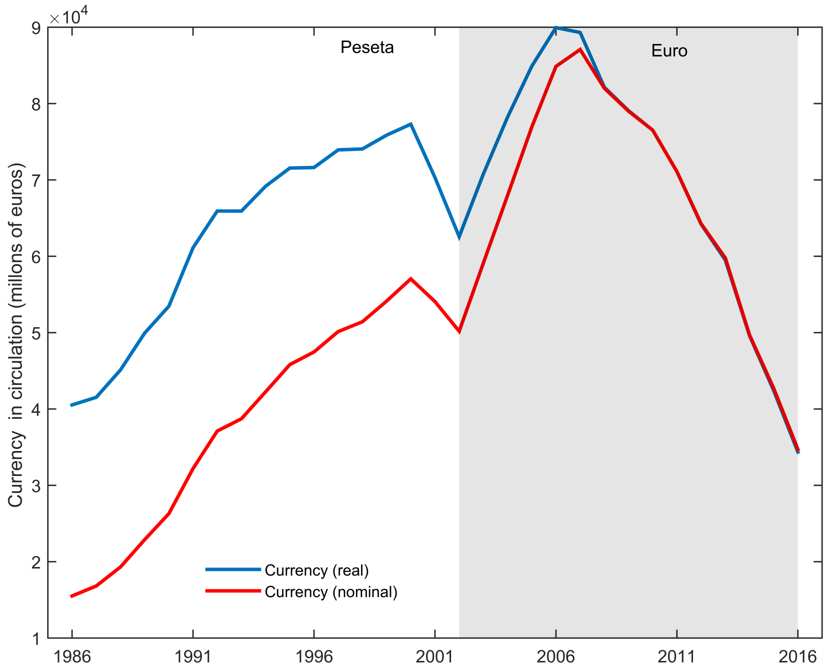
The dependent variable is the amount of cash held by the public (currency in circulation) in real terms based on data obtained from the Bank of Spain.[6] The price deflator used to obtain the amount of currency expressed in real terms, which is the variable of interest in the econometric model, comes from De la Fuente (2017a) and takes 2010 as base year, such that $P_{2010} = 1$. Figure (2) shows the evolution of the quantity of currency in Spain during the sample period. As it is observed, with the introduction of the euro, the ammount of currency decreased far below its previous trend. It grew until 2008 and since the outbreak of the Great Recession, it decreased sharply. Overall, the temporal pattern depicted in Figure (2) is that of an inverted U with a peak in 2008.

4.3.2 Fiscal indicators

The key explanatory variable that encourages the demand for cash for hidden transactions in the CDA is the fiscal pressure. The intuition is that some individuals in the society may decide that the burden of paying taxes is greater than the benefits obtained through the provision of public goods and services. Thus, high tax levels should translate into a larger size of the shadow economy given that incentives to move from the formal to the informal sector would be greater in this context. For the period 1986-2016 the fiscal data have been taken from the annual reports of the Spanish Tax Agency. In this analysis, the following ratios are taken into account as candidates to define the fiscal pressure: (i) the aggregate fiscal pressure is calculated as the ratio of the sum of social contributions and total tax collection to GDP (FPA_t), (ii) the total fiscal pressure excluding social contributions (FP_t), (iii) the direct fiscal pressure (DIR_t), (iv) indirect fiscal pressure ($INDIR_t$), (v) the ratio of Personal Income Tax to GDP (PIT_t), (vi)

[6]The INE identifier for the time series of Currency in circulation and deposits of resident banking institutions in the Bank of Spain is "FMI62 - Central Bank Accounts". This time series is initially calculated in monthly frequency and covers the period 2001-2016 On the other hand, the Bank of Spain on its website, provides data for the period 1962-2001, also in monthly frequency `https://www.bde.es/webbde/es/estadis/infoest/bolest7.html`. These data have been compared with those of the IMF to ensure consistency in the period considered with the other monetary macro magnitudes such as M_1, M_2 and M_3. This is issue is relevant as the proper measurement of money in circulation affects the estimation of the velocity $\hat{v}$ in Equation (14). Since the rest of the variables of the model are measured in annual frequency, the money data of each year are obtained as the mean over the 12 months of the year.

Figure 2: The Evolution of Currency in Spain



the ratio of the collection of the Value Added Tax with respect to the GDP (VAT_t) and (vii) the ratio of the Corporate Tax with respect to the GDP (CT_t).

4.3.3 Other explanatory factors

The set of variables included in the matrices X_t and Z_t considered in the analysis other than fiscal indicators whose omission could bias the estimates of the response of money to changes in taxes and the elasticity of money with respect income, refer to (i) macroeconomic factors, (ii) socio-demographic characteristics (iii) productive structure characteristics and (iv) monetary events. The descriptive statistics and the respective sources of all the variables used in the analysis are shown in the Table (A1) in the Appendix.

The econometric model proposed in Equation (16) explains the variations in demand for money as a result of changes in (i) the logarithm of real GDP ($RGDP_t$), (ii) the nominal interest rate (R_t), (iii) the inflation rate (Π_t), (iv) the unemployment rate (U_t), (v) the share of wages in the total income (WS_t) and (vi) the logarithm of the number of hours worked per capita ($HOURS_t$). The historical data of real GDP and the share of wages in total income have been obtained from De la Fuente (2017a, b), the interest rate is taken from the Bank of Spain, inflation has been calculated as the annual growth rate of the nominal GDP price deflator calculated with the linked series of De la Fuente (2017a) and the unemployment rate was obtained from the series of unemployed and active population of De la Fuente (2017b). Additionally, demographic factors that could be related to the demand for cash haven been considered, such as the (vii) net migratory balance (MIG_t) and (viii) the average number of years of education of the population ($EDUC_t$). The data comes from De la Fuente (2017c).

The productive structure and the industry mix might also be correlated with money in circulation, tax pressure and income. According to Schneider (2013), the construction sector has historically been very susceptible to underground activities. The estimate for Spain is that approximately the 31% of activity in this sector is shadow economy. The wholesale sector is the second in size, with a 20%. For other sectors such as agriculture or manufacturing, the estimates presented are around the 15%. Since the productive structure of the Spanish economy has changed significantly during the study period and different sectors can have different levels of hidden activity, the shares in employment of the various sectors with respect to the total employment are included. In particular, (ix) agricultural ($AGRI_t$), (x) construction ($CONS_t$), (x) industrial (IND_t), (xi) non-market services (NMS_t), (xii) financial services (FIN_t) and (xiii) wholesale services, retail, transportation, lodging and food, information and communication (Other services, OS_t). The data comes from the database *Cambridge Econometrics* 2017.

Moreover, during the period under consideration, there are two drastic monetary and financial events that may have had an effect on the demand for currency: (xiv) the entry into the euro zone (EUR_t) and (xv) the outbreak of the financial crisis ($CRISIS_t$). As observed in Figure (2), in the year of entry into the euro zone, there was a significant fall in cash due to the uncertainty associated with the new currency. The reaction of citizens was to convert cash into other monetary instruments such as deposits and long-run deposits. A second event is linked to the outbreak of the financial crisis and the burst of the housing bubble. To control for these events, two dummy variables are created. The dummy variable of the Euro takes value 1 from 2002 onwards while the dummy variable of the financial crisis takes the value of 1 in the 2008-2013 interval and 0 otherwise.

Finally, with the aim of improving the fit to the data, a linear and a quadratic trend are included to capture the non-linear evolution of currency depicted in Figure (2).

5 Estimates of the Shadow Economy in Spain

Table (1) reports the results obtained from the BMA analysis when the dependent variable is natural logarithm of real currency in circulation. However, before continuing with the discussion of the results, it is worth mentioning the problems that the methodology applied here is able to solve and those problems that may persist, affecting the quality of the estimates. The strong point of the BMA methodology employed here is that it accounts for the uncertainty of the parameter estimates across different models while controlling for omitted variable bias (Moral-Benito, 2015; Steel, 2019). However, it does not correct for the potential negative effect of endogeneity generated by reverse causal relationships or measurement errors. Thus, the results should be interpreted with caution. In fact, how to tackle the issue of endogeneity in a model averaging framework is an important line of open research.[7]

The PIPs of the different variables are scaled in intervals to classify evidence of robustness of currency drivers into two categories so that regressors with PIP $\in [0-50\%]$ are considered as weak determinants and variables with PIP $\in [50-100\%]$ as significant determinants. This metric is reported in Column (1). Besides the factors included in X which are always part of the model, the only important drivers in Z are the quadratic trend (96.9%), the net migration (71.2%) and the hours worked (54.5%). Finally, weak currency demand drivers include a myriad of factors related to the composition of the productive structure (e.g. agriculture, industry, financial services, non-market services),

[7] This is because in the context of endogenous regressors the model posterior probabilities are based on pseudo-likelihoods that are not fully comparable across models.

financial and monetary events or the distribution of income.

Table 1: Baseline Results (fiscal pressure including social contributions)

Variable	PIP (1)	Cond. Post. Mean (2)	Cond Post . Std (3)	Cond. Post Sign (4)	Cond Post. Mean & Restrict(5)	Cond Post Std & Restrict (6)
Fiscal pressure$^{(a)}$	1	1.045	1.498	0.805	1.761	1.559
Real GDP	1	3.41	1.555	0.979	3.103	2.566
Inflation rate	1	2.463	1.941	0.963	1.236	2.660
Interest rate	1	1.173	1.617	0.798	0.126	1.617
Quadratic trend	0.969	-0.003	0.001	0	-0.002	0.001
Net migration	0.712	-0.098	0.036	0	-0.086	0.045
Hours Worked	0.545	-2.968	1.174	0.001	-2.761	2.144
Education	0.483	1.088	0.692	0.999	1.437	0.966
Other Servs.	0.378	9.996	4.967	0.998	11.474	6.672
Unemployment rate	0.361	3.733	1.875	0.969	3.147	0.346
Linear trend	0.28	-0.169	0.152	0.162	-0.249	0.169
Non Market Servs.	0.168	-2.305	4.438	0.169	0.131	2.078
Euro	0.123	-0.111	0.102	0.058	-0.061	0.055
Financial Servs.	0.107	4.452	4.108	0.981	1.578	2.083
Industry	0.09	-2.799	4.06	0.172	-4.193	4.786
Crisis	0.088	0.077	0.089	0.922	0.112	0.073
Agriculture	0.077	-0.556	6.856	0.546	-13.258	8.455
Wage Share	0.07	0.141	1.684	0.561	0.782	0.794
Construction	0.069	1.426	2.437	0.868	3.421	1.842

Notes: The dependent variable in all regressions the logarithm of real currency. Prior mean model size is 11.5 whereas posterior mean model size is 8.5. Variables are ranked by Column (1), the posterior inclusion probability. Columns (2) and (3) reflect the posterior mean and standard deviations for the linear marginal effect of the variable conditional on inclusion in the model, respectively. Column (4) denotes the sign certainty probability, a measure of the posterior confidence in the sign of the coefficient. The results reported in Columns (2) to (4) correspond to the estimation of the top 10,000 models from the 524288 million possible regressions including any combination of the 19 variables whereas the the results reported in Columns (5) and (6) correspond to the estimation of the models that satisfy the restrictions in Section (4.2). (a) The fiscal pressure indicator employed in these regressions is the FPA_t including social contributions.

Columns (2) and (3) report the mean and the standard deviation of the posterior parameters distributions, conditional on the variable being included in the model.[8] To complement these statistics, Column (4) presents the results of the posterior sign certainty, which measures the posterior probability of a positive coefficient expected value, conditional on inclusion. For the factors included in X, which are always part of the model, it is found that the tax pressure (80.5%), real GDP (97.9%), inflation rate (96.3%) and interest rates (79.8%) present a high sign certainty, exerting positive effects on the amount of cash. Note that while the positive impacts of the fiscal indicator and income

[8]The key difference with respect to unconditional posterior estimates of Equations (18) and (19) is that conditional posterior estimates for a particular variable are obtained as the weighted average over the models where the variable is included. On the contrary, the unconditional posterior estimate is the averaged coefficient over all models, including those in which the variable does not appear, hence having a zero coefficient. Thus, the unconditional posterior mean can be computed by multiplying the conditional mean in Column (3) times the PIP in Column (1)

are in line with the CDA, the positive effect of inflation and interest rate are not, since these factors in the CDA are expected to capture the opportunity cost of holding cash. Columns (6) and (7) report the conditional mean and the standard deviation of the posterior parameter distributions for the sampled models that satisfy conditions outlined in Section (4.2). These are the parameter estimates used to obtain $(\hat{C}_t)$ and $\hat{C}_t|\beta_1 = 0$. As observed, the posterior of mean of fiscal pressure is 1.76 whereas the elasticity of money with respect income is 3.1, which is in the range of previous studies.

Using the probabilistic weighted average of the natural logarithm of the real cash for the set of models sampled by the algorithm $MC3$ that also satisfy the restrictions, the adjustment to the demand of currency $(\ln \hat{C}_t)$ is obtained and shown in the Figure (3).[9]

The red dashed line in Figure (3a) represents the model's fit using all information $\left[\ln \hat{C}_t|\vec{\hat{\beta}}_k\right]$, while the black solid line represents the historical trajectory. As can be seen, except for the year 2002, which corresponds to the entry in the euro, the adjustment of the model to the data is quite satisfactory given that it replicates quite accurately the movements in time of the currency. On the other hand, the dash-dot blue line is the counter-factual of the hypothetical evolution of the logarithm of cash money "shutting down" the effect of changes in taxes (i.e, the term $\left[\ln \hat{C}_t|\hat{\beta}_1 = 0, \vec{\hat{\beta}}_k\right]$ of the Equation (13)). The difference between the two time series gives us the amount of cash which can be attributed to hidden or underground transactions $(\ln \hat{C}_{St})$, whose evolution is plotted in Figure (3b). As observed, the evolution of the currency held by the public for hidden activities has its maximum in the year 2007 which coincides with the peak of the housing bubble. Specifically, in the year 2007, the estimated amount of currency for hidden activities was 19,414 millions of euros.

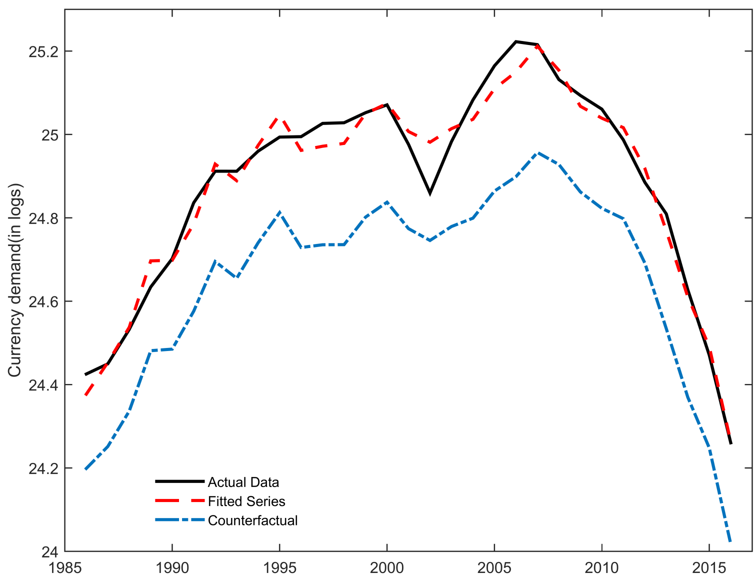
With the estimate of the currency for hidden transactions, velocity of monetary circulation is applied using the corresponding monetary aggregate and Fisher's equation.

[9]The estimates of $\hat{C}_t$ and $\hat{C}_{St}$ in line with the BMA framework are obtained as the probabilistic weighted average of all the specific $\hat{C}_t$ and $\hat{C}_{St}$ predicted by each model M_k such that:

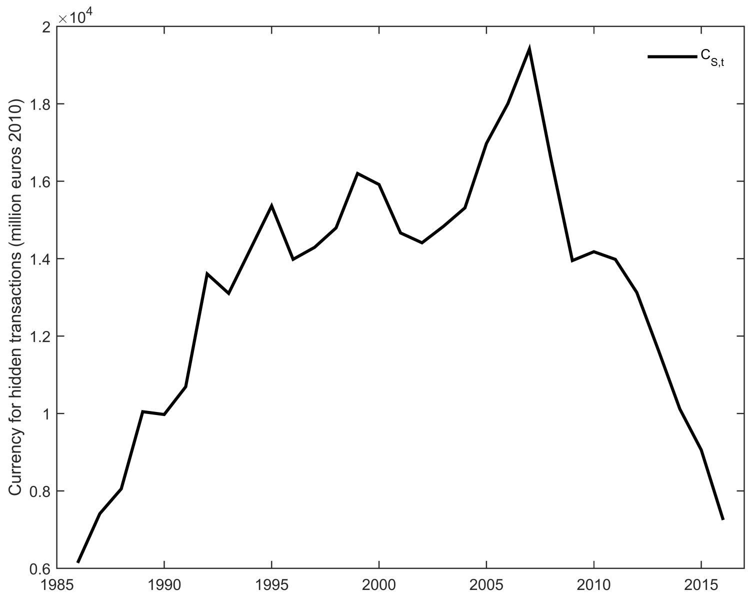
$$\hat{C}_{St} = e^{\sum_{k=1}^{2^K}\left(\ln \hat{C}_{St}|M_k, C, \tilde{X}\right)}p\left(M_k|C, \tilde{X}\right)$$

$$\hat{C}_t = e^{\sum_{k=1}^{2^K}\left(\ln \hat{C}_t|M_k, C, \tilde{X}\right)}p\left(M_k|C, \tilde{X}\right)$$

Figure 3: Currency demand estimates



(a) Total currency demand



(b) Currency demand for hidden transactions

The velocity of monetary circulation is obtained by solving Fisher's equation using data from a monetary aggregate and nominal GDP. As discussed by Gadea and Serrano-Sanz (2002) and González-Fernández y González-Velasco (2015) the selection of the monetary aggregate is a relevant issue in the computation of the shadow economy. The use of smaller monetary aggregates implies a greater speed of circulation in the underground economy than in the rest of the economy while, on the contrary, the use of broader monetary aggregates goes with the assumption of a slower speed of circulation in the informal sector. The speed of circulation of hidden money could be faster due to the productive structure of the informal sector, or slower due to a higher level of accumulation. In this context, the researcher should use a broad monetary indicator that reflects savings decisions, but not in the same proportion as in the official economy, since economic agents will prefer to save with legal money where there are higher returns and use cash for current transactions. According to Gadea and Serrano-Sanz (2002), because of hidden activities can not benefit from the returns provided by other liquid assets available as forms of payment, the use of M_1 could lead to an overvaluation of the size of the informal sector, since its speed has increased in recent years due to financial innovation. Due to these issues, most of the empirical studies use the velocities implied by the monetary aggregates M_2 and M_3. In this analysis, the average velocity implied by M_2 and M_3 is taken as the estimate of $\hat{v}$ in Equation (14).[10]

Using $\hat{\beta}_2$, the first estimate of $\hat{Y}_{St}$ is corrected using Equation (15). Figure (4) displays the results of this procedure and reports the model averaged estimates of $\frac{\hat{Y}_{S,t}}{Y_{O,t}}$:

$$\hat{SE}_t = \frac{\hat{Y}_{S,t}}{Y_{O,t}} = \left(\frac{\hat{Y}_{S,t}|M_k}{Y_{O,t}}\right) p\left(M_k\right) \tag{26}$$

In addition, Figure (4) reports the results of the BMA-CDA and the average estimate of the size of the shadow economy implied by the previous literature using the CDA framework. The distribution of the estimates of $\frac{\hat{Y}_{S,t}}{Y_{O,t}}$ for $t = 1986, \ldots, 2016$ is simulated by means of a Monte Carlo simulation of the distribution $N\left(\mu_t, \sigma_t\right)$, where μ_t is the mean size of the shadow economy at time t and the standard deviation at each period

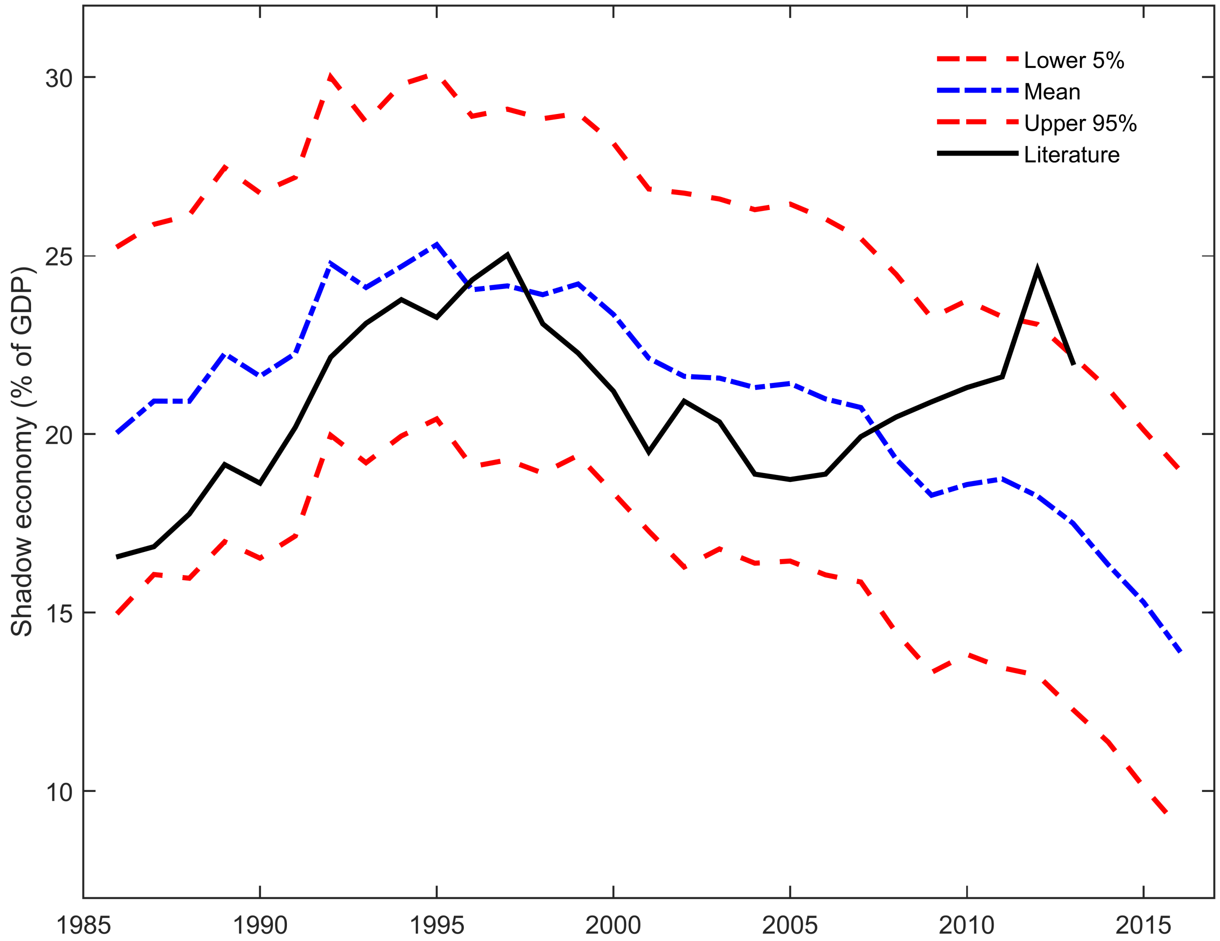
[10]Figure (A1) in the Appendix depicts the evolution of the velocity of the different monetary aggregates considered. As observed the money velocity of M_2M_3 has decreased over time and lies within the speeds implied by M_2 and M_3

σ_t, reflects the variability of the shadow economy estimate across models and within models.[11] This distribution is used to construct confidence intervals. Previous literature estimates are in line with the mean estimate of the BMA and fall within the upper 95% and lower 5% confidence bands for most of the period under consideration. In fact, the average size of the shadow economy for the study period implied by the BMA is the 21.05%, whereas the average in the literature is the 20.9% of the GDP. The similarity in the estimated pattern is specially remarkable in the first twenty-five years, where both predict an increase in the size of the shadow economy and then a decrease until the year 2007. However, the maximum size in the BMA using the FPA indicator (a 25.3% of the GDP) is obtained in year 1995, whereas the peak in the literature occurs in 1997 (with a 25.01% of the GDP). After achieving its maximum in the 90's, both predict a decrease until 2007. However, while previous literature suggests a rebound and a raising ratio after the Great Recession, BMA estimates suggest a continuous decrease of the ratio of the underground economy with respect the GDP. In fact, the figure of shadow economy estimated by the BMA in the year 2016 is the 13.95% of the GDP.

[11]The uncertainty on the size of the shadow economy is computed as $\sigma_t = \sqrt{Var\left(\frac{\hat{Y}_{S,t}}{Y_{O,t}}\right)}$ which is given by:

$$Var\left(\frac{\hat{Y}_{S,t}}{Y_{O,t}}\right) = \sum_{k=1}^{2^{K_1}} p\left(M_k\right) Var\left[\frac{\hat{Y}_{S,t}}{Y_{O,t}}|M_k\right] + \sum_{k=1}^{2^{K_1}} p\left(M_k\right)\left[\frac{\hat{Y}_{S,t}}{Y_{O,t}} - E\left(\frac{\hat{Y}_{S,t}}{Y_{O,t}}\right)\right]^2$$

Figure 4: Shadow Economy Estimates: 1986-2016

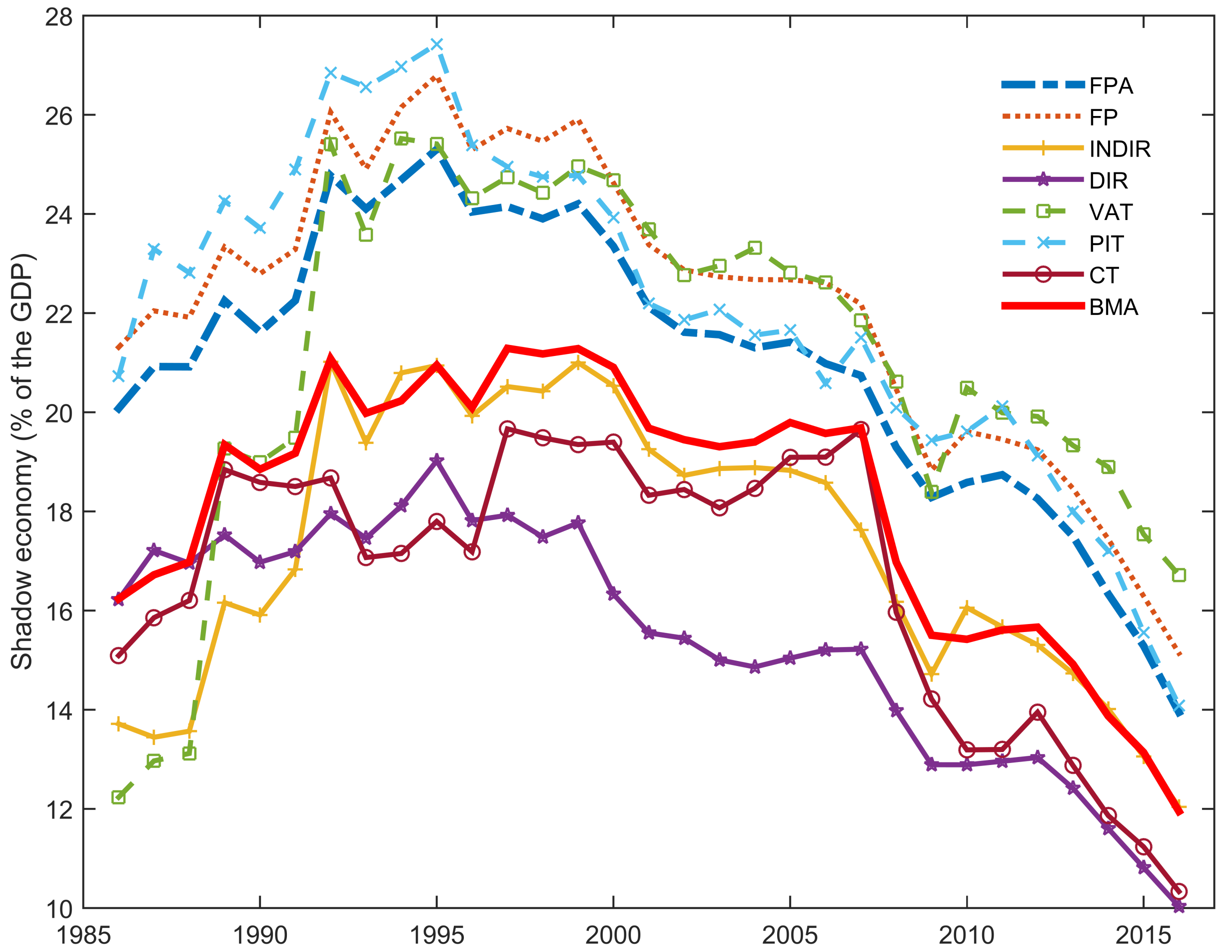


As regards the estimation of the shadow economy, it is important to recall that different fiscal indicators may produce different results. For this reason, researchers usually report estimates of the impacts of different fiscal variables such as direct taxes, indirect taxes or social contributions as a robustness checks (see Arrazola *et al.*, 2011; González-Fernández and González-Velasco, 2015). However, the question of how much one should trust the estimate implied by a specific tax variable is difficult to solve in a frequentist framework. Nevertheless, this issue can be easily solved within a BMA framework by calculating the marginal likelihoods of all the models of each indicator, integrating over them to derive a new set of model probabilities $p\left(M_k\right)'$ and weighting the underground economy estimates by their corresponding probability. The results of the BMA using all the information of all the sampled models for each indicator are reported in Figure (5).

The Figure (5) shows that models that generated higher shadow economy size predictions based on the PIT, the VAT or the FPA indicators are unlikely to be the true data generating process driving currency demand and therefore, the higher shadow economy ratios implied by these indicators might not be good descriptions of reality.[12] In fact the final result of applying the BMA for all the indicators implies a lower size than that of the baseline FPA indicator used as a baseline, as it yields a historical average for the size of the hidden sector of 18.2% which is lower than the 21.05%. Notice that although the temporal U-shaped pattern is very similar, the decrease after 2007 in the BMA is much more pronounced. This estimate suggests that by the year 2016, the shadow economy ratio in Spain was the 11.95%.

[12]The estimated weights to produce the Bayesian Model Averaged estimate across all models for all the indicators are: $DIR_t = 0.06$, $INDIR_t = 0.23$, $FP_t = 0.18$, $FPA_t = 0.07$, $VAT_t = 0.02$, $PIT = 0.00$, $CT_t = 0.43$.

Figure 5: Shadow Economy Estimates: Alternative Fiscal Indicators



6 The Drivers of the Shadow economy in Spain

6.1 Baseline results

In this section, the drivers of the evolution of the shadow economy in Spain are investigated by means of the BMA econometric procedure. At the international level there is a consensus that some country characteristics such as (i) the tax burden, (ii) the quality of the institutions, (iii) the regulatory intensity, (iv) the fiscal morale, or (iv) deterrence help to explain differentials in the size of the underground economy (Johnson *et al.* 1998, Friedman *et al.*, 2000; Feld and Schneider, 2010; Schneider and Williams, 2013). However, these are mostly slow-moving variables for which the are no available time-series covering the period under consideration. For this reason, this study focuses is on the alternative set of (i) macroeconomic factors, (ii) socio-demographic characteristics (iii) productive structure characteristics and (iv) monetary events, that have been employed before as controls in the CDA.

Again, the PIPs of the variables and the previous classification in two levels: [0-50%] and [50%-100%] are used to analyze the importance of the regressors. The most significant determinants of the evolution of the shadow economy in Spain are (i) the average years of education (92.6 %), (ii) the ratio of the VAT revenues to GDP (88.2 %), (iii) the share of the agricultural sector in total employment (88 %), (iv) the crisis dummy (85.9%), (v) the ratio of the CT revenues to GDP (85.7 %), (vi) the share of financial services in the total employment (83.7%) and (vii) the share of the construction sector in the employment (66.6%). The unemployment rate has a PIP of the 48.9%, which in this context is slightly below the cut-off the 50% which is the prior probability of inclusion. Nevertheless, since the posterior mean size is of 8 regressors, it seems reasonable to consider it as a significant determinant.

As observed, the certainty of the impacts across models for these top variables is quite high. The factors that exert a positive effect on the size of the shadow economy are the VAT revenues as a percentage of the GDP, the crisis dummy, the CT revenues as a percentage of the GDP, the share of employment in the construction sector and the unemployment rate. On the other hand, the average years of education and the share of employment in the agriculture and financial sector are negatively related to the size of the shadow economy.

Table 2: The drivers of the shadow economy

Variable	PIP (1)	Cond. Post. Mean (2)	Cond Post . Std (3)	Cond. Post Sign (4)
Education	0.926	-0.030	0.019	0.000
VAT (% of GDP)	0.882	0.791	0.189	0.995
Agriculture	0.880	-0.742	0.250	0.000
Crisis	0.859	0.015	0.005	1.000
Corporate Tax (% of GDP)	0.857	1.122	0.305	1.000
Financial Servs.	0.837	-0.753	0.203	0.000
Construction	0.660	0.317	0.137	1.000
Unemployment rate	0.489	0.320	0.325	0.986
Net migration	0.349	-0.005	0.002	0.006
Hours worked	0.316	0.185	0.300	0.502
Euro	0.197	-0.013	0.009	0.097
Fiscal pressure (% of GDP) $^{(a)}$	0.192	0.528	0.460	0.798
Non Market Servs.	0.169	-0.296	0.218	0.044
Social contributions (% of GDP)	0.138	-0.497	0.436	0.032
Personal Income Tax (% of GDP)	0.124	-0.022	0.472	0.347
Inflation rate	0.104	-0.140	0.159	0.106
Interest rate	0.101	-0.057	0.095	0.145
Industry	0.100	-0.121	0.299	0.244
Wage Share	0.082	-0.057	0.121	0.215
Other Servs.	0.072	-0.071	0.295	0.308

Notes: The dependent variable in all regressions is the ratio of the shadow economy to GDP obtained by means of the CDA-BMA using all the indicators of tax pressure. The results reported here correspond to the estimation of the top 10,000 models from the 1 million possible regressions including any combination of the 20 variables. Prior mean model size is 10 whereas posterior mean model size is 8.33. Variables are ranked by Column (1), the posterior inclusion probability. Columns (2) and (3) reflect the posterior mean and standard deviations for the linear marginal effect of the variable conditional on inclusion in the model, respectively. Column (4) denotes the sign certainty probability, a measure of our posterior confidence in the sign of the coefficient. (a) The fiscal pressure indicator employed in these regressions is the FP_t excluding social contributions.

Note that these findings are in line with previous international evidence and corroborate the intuition that taxes are among the key drivers of variations in the underground economy. Moreover, the fact that variations in the tax pressure of the VAT and CT are among the top determinants (i.e, with PIPs above the 50%) whereas the PIT, the Social Contributions or the overall fiscal pressure are not significant, suggests that fraud in these taxes might be of major importance. This result is partially in line with previous studies of Gadea and Serrano-Sanz (2002) and González-Fernández and González-Velasco (2015), who find evidence supporting the claim that changes in direct taxes (which include both the PIT and the CT) are those that have greater impact on the size of the shadow economy.

A novel result that stems from this analysis is the role of education. The fact that education appears as the most important determinant exerting a negative impact on hidden activities has important policy implications and contradicts the negative view emerging from the studies of Goneaga (2018) or De Neve *et al.* (2019) who suggest that educative actions and informative policy measures aiming at curving the shadow economy are not effective and enjoy a low receptiveness.

On the other hand, the positive effect observed in the crisis dummy and the unemployment rate suggests that the shadow economy reacts to falls in output and to the deterioration of the labor market as suggested by Lackó (2000) or Dell'Anno *et al.* (2007). In these contexts, it seems likely that economic agents activate survival strategies (such as household production or production and service for sale without registration), which increase the size of the shadow economy.

Finally, it is observed that industry mix matters driving the shadow economy. Whereas the share of construction increases underground economy, there is a negative link between the share of financial services and agriculture and the shadow economy. The observed negative link between the shadow economy and the share of employment in the financial sector is in line with Schenider (2013) where he finds that in European countries, the growing use of electronic payment systems decreases the room for opaque transactions. This, in turn, requires a developed financial sector with a significant share of employment devoted to financial activities. In addition, the positive effect of construction is in line with the finding that the greatest bags of undeclared work in European economies are located in the construction sector (see the review of Schneider and Williams (2013) pp 66-73 on the micro studies of Denmark and Germany). The only variable that displays a somewhat unexpected effect is the share of employment in agriculture given that most of the empirical international evidence suggest that a higher share in the agricultural sector increases the size of the shadow economy, because of local governments in rural areas enjoy a more limited ability to control the economy (Vuletin, 2008). Nevertheless, while this argument may hold to explain cross-country differentials, it might not apply in a developed country such as Spain, where the regulation and protection of the agricultural sector is high. In fact, the result obtained here could be explained because of these regulations.

6.2 Robustness checks

The results presented so far rely on Bayesian econometric modeling. An implication of Bayesian econometrics is that inferences drawn on the relevance of different regressors depend on prior distributions assigned to the model parameters and to the models. Often, Bayesian analysis try to avoid situations where the conclusions depend heavily on subjective prior information. For this reason, some robustness checks with respect the role of the priors are performed.

The g-prior specification

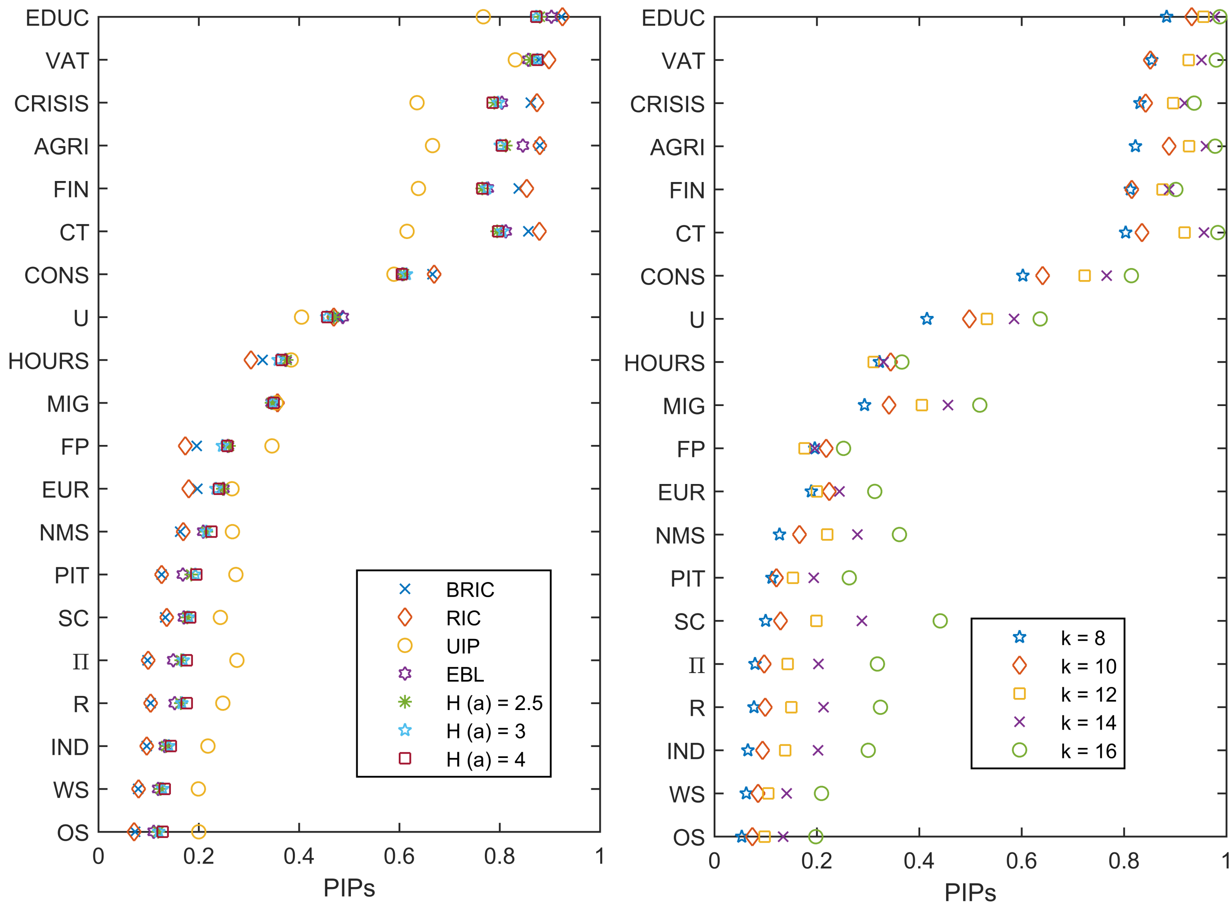
First, I consider fixed g-priors following Fernández *et al.* (2001) as it is the case of the baseline g-prior, the BRIC which sets $g = max\left(N; K^2\right)$. In this group of priors I also consider the (i) Unit information prior (UIP) which sets $g = N$; the (ii) Risk information criteria prior (RIC) where $g = K^2$ and (iii) the Hannan-Quinn (HQ) g-prior setting $g = log(N)^3$. However, we also consider the (iv) Empirical Bayes prior (EBL) of Liang *et al.* (2008) which is a model k specific g-prior estimated via maximum likelihood. In this case $g = max\left(0, F_k\right)$ where $F_k = \frac{R_k^2(N-1-k)}{\left(1-R_k^2\right)}$. Finally, I consider the Hyper-g prior of Liang *et al.* (2008) who suggest a Beta prior on the shrinkage factor of the form $\frac{g}{1+g} \sim Beta(1, \frac{a}{2} - 1)$ where in this specific case, a is set to 2.5, 3 and 4. Figure (6) shows the PIPs for the different regressors when using different g-prior configurations. As observed, the ranking of regressors and their relevance does not change significantly due to changes in g for the set of significant regressors whereas only minor changes arise for determinants of low importance.

Priors on the model space

I also check the sensitivity of the results to the Binomial prior in the model space. Here I depart from the baseline specification of $\phi = K/2$ and the parameter controlling model size ϕ is set to produce models of prior size of 8, 10, 12, 14 and 16 regressors respectively.

As observed in Figure (6), the effect of increasing the prior model size has a stronger impact on the PIPs than the g-prior given that the employment of priors favoring a large model size, increases slightly the PIPs of most of the determinants. However, for most of the regressors, the use of large model size priors does not generate a change in their classification. Only factors in close to the threshold of significance such as the unemployment rate or the net migration increase substantially their PIPs up to the level where they could be considered significant. Thus, the results obtained in the baseline scenario appear to be robust and are not driven by the implementation of priors beliefs.

Figure 6: The role of priors



7 Conclusions

This study investigates the evolution of the shadow economy in Spain during the period 1986-2016 using the Currency Demand Approach by means of Bayesian Model Averaging econometric techniques which have the advantage of accounting for model uncertainty and reducing the biases implied by omitted variables.

The results of the empirical analysis suggest that the use of a unique aggregate fiscal pressure metric such as the overall fiscal pressure including social contributions, may over-estimate the average size of the shadow economy. By integrating over the log-marginal likelihoods of the MC^3 sampling algorithm for a variety of tax indicators, a new set of model weights is obtained. This allows to produce an estimate of the shadow economy that accounts for the uncertainty with respect to the taxes driving the demand of currency for hidden transactions. The average share of the underground economy as a percentage of the GDP during 1986-2016 was the 18.2% whereas in 2016, the estimated size was the 11.95%. The estimated figures of the shadow economy for the period under consideration are in line with the averaged estimates of previous studies for the same time frame. Nevertheless, a remarkable difference between previous analysis and the estimated pattern stemming from this study is that the size of the shadow economy depicts an inverted U shape time profile, with a marked reduction in the years after the Great Recession.

With the estimates of the shadow economy in hand, the importance of the drivers of the shadow economy in Spain is investigated by means of Bayesian Model Averaging methods, and in particular by the inspection of the PIPs of the different variables. The main finding of this exercise is that the key factors driving variations in the size of the shadow economy are the taxes, the level of education and the distribution of employment across sectors. The results of this analysis highlight the importance of the VAT and CT as the key taxes driving the shadow economy, which suggests that policy-makers aiming at decreasing the tax-fraud should focus compliance efforts on these taxes. As regards the key role played by the education of the population, it seems desirable to continue investing in this public good as it may have important economic returns in the future by lowering the size of the shadow economy. Finally, even if the share of employment in the construction has decreased considerably with respect the values in the peak of the housing bubble, the high importance of the construction sector driving the underground economy from the historical point of view and its positive link with the size of the shadow economy, suggests that labor inspections should be focused in this branch of activity.

8 References

Ahumada, H., Alvaredo, F. and Canavese, A. (2007): The Monetary Method and the Size of the Shadow Economy: A Critical Assessment. *Review of Income and Wealth*, 53, 2, 363-371.

Ahumada, H., Alvaredo, F. and Canavese, A. (2008): The Monetary Method to Measure the Shadow Economy: The Forgotten problem of the initial Conditions. *Economics Letters*, 101, 2, 97-99.

Arrazola, M., Hevia, J., Mauleón, I. and Sánchez, R. (2011): Estimación del volumen de economía sumergida en Espaa. *Cuadernos de Información Económica*, 220, 81-87.

Caballé, J. and Panadés, J. (1997): Tax evasion and economic growth. *Public Finance/Finances Publiques*, 52 (2), 318-340

Cagan, P. (1958): The demand for Currency relative to the Total Money Supply. *Journal of Political Economy*, 66, 4, 303-328.

CIS (2018): Opinión pública y política fiscal (XXXV), Estudio n 3221, julio 2018.

De la Fuente, A. (2017a): Series enlazadas de algunos agregados económicos regionales, 1955-2014. Parte I: Metodología, VAB, PIB y puestos de trabajo. RegData 55-14, Versión 5.0-parte I. *Documentos de trabajo FEDEA* .

De la Fuente, A. (2017b): Series enlazadas de algunos agregados económicos regionales, 1955-2014. Parte II: Otras variables de empleo, rentas del trabajo y paro

RegData 55-14, Versión 5.0-parte II. *Documentos de trabajo FEDEA.*

De la Fuente, A. (2017c): Series largas de algunos agregados económicos y demográficos regionales: Actualización de RegData hasta 2016. *Documentos de trabajo FEDEA.*

Dell'Anno, R. (2003): Estimating the shadow economy in Italy: A structural equation approach. Economics WorkingPapers, (2003-7). *School of Economics and Management, University of Aarhus.*

Dell'Anno, R., Gómez-Antonio, M., and Alañón-Pardo, A. (2007): The shadow economy in three Mediterranean countries: France Spain and Greece. A MIMIC approach. *Empirical Economics*, 33 (1), 51-84.

Dell'Anno, R. and Schneider, F. (2009): A Complex Approach to Estimate Shadow Economy: The Structural Equation Modelling, 111-130. M. Faggini and T. Lux, eds. (2009) *Coping with the Complexity of Economics.* Springer, Milan.

De Neve, J.E., Imbert, C., Spinnewijn, J., Tsankova, T. and Luts, M. (2019): *How to improve tax compliance? Evidence form Population-wide Experiments in Belgium.* CEPR Discussion paper 13733. Centre for Economic Policy Research, Londres.

Doppelhofer, G. and Weeks, M. (2009): Jointness of Growth Determinants. *Journal of Applied Econometrics*, 24, 209-244.

Feld, L. P., and Schneider, F. (2010): Survey on the shadow economy and undeclared earnings in OECD countries. *German Economic Review*, 11 (2), 109-149.

Fernandez, C., Ley, E., and Steel, M. F. (2001): Benchmark priors for Bayesian model averaging. *Journal of Econometrics*, 100, 2, 381-427.

Gadea, M. D., and Serrano-Sanz, J. M. (2002): The hidden economy in Spain - A monetary estimation, 1964-1998. *Empirical Economics*, 27, 499-527.

Giles, D.E.A. (1999): Measuring the Hidden Economy: Implications for Econometric

Modelling. *Economic Journal*, 109/3, 370-380.

Goenaga, M. (2018): Qué hacer para combatir la economía sumergida y el fraude fiscal en España?: Una perspectiva social. En *Economía sumergida y fraude fiscal en España: Qué sabemos?Qué podemos hacer?,* Lago, S. (dir.), FUNCAS, Madrid.

Gómez de Antonio, M. and Alañón-Pardo, A. (2004): Evaluación y análisis espacial del grado de incumplimiento fiscal para las provincias españolas (1980-2000). *Hacienda Pública Española*, 171, 9-32.

González-Fernández, M. and González-Velasco, C. (2015): Analysis of the shadow economy in the Spanish regions. *Journal of Policy Modeling*, 37(6), 1049-1064.

Hamori, S. and Hamori, N. (2008): Demand for money in the Euro area. *Economic Systems*, 32, 274-284.

Kaufmann, D., and Kaliberda, A. (1996): Integrating the unofficial economy into the dynamics of post socialist economies: a framework of analyses and evidence. *The World Bank, Policy Research Working Paper*, 1691.

Lackó, M. (2000): Hidden Economy - an Unknown Quantity? Comparative Analysis of Hidden Economies in Transition Economies, 1989-95. *Economics of Transition*, 8/1, 117-149.

Liang, F., Paulo, R., Molina, G., Clyde, M. A., and Berger, J. O. (2008). Mixtures of g priors for Bayesian variable selection. *Journal of the American Statistical Association*, 103, 481, 410-423.

Macias, J.B. and Cazzavillan, G. (2009): The dynamics of parallel economies. Measuring the informal sector in Mexico. *Research in Economics*, 63 (3), 189-199.

Madigan, D. and York, J. (1995): Bayesian graphical models for discrete data. *International Statistical Review*, 63 215-232.

Mark, N.C. and Soul, D. (2003): Cointegration vector estimation by panel DOLS and long-run money demand. *Oxford Bulletin of Economic and Statistics*, 65, 665-680.

Mauleón, I., and Sardá. J. (1997): Estimación cuantitativa de la economía sumergida en España. *Ekonomiaz*, 39, 125-135.

Mauleón, I., and Sardá. J. (2014): La Economía Sumergida en Navarra. *Informe al Parlamento de Navarra.*

Moral-Benito, E. (2015): Model averaging in economics: An overview. *Journal of Economic Surveys, 29*(1), 46-75.

Labeaga, J.M. (2014): Estimación del volumen de economía sumergida a travs del mtodo monetario. *Mimeo.*

OECD (2017): *Shining Light on the Shadow Economy: Opportunities and Threats.* OECD Publishing, Paris.

Pickhardt, M., y Sardá, J. (2015): Size and causes of the underground economy in Spain: a correction of the record and new evidence from the MCDR approach. *European Journal of Law and Economics*, 39, 2, 403-429.

Prado-Domínguez, A.J. (2004): Una estimación de la economía informal en España según un enfoque monetario, 1964-2001. *El Trimestre Económico*, 71 (82), 417-452.

Schneider, F. (2005): Shadow economies around the world: What do we really know?. *European Journal of Political Economy*, 21 (3), 598-642.

Schneider, F. (2013): The shadow economy in Europe, 2013. Available in `http://www.atkearney.com/documents/10192/1743816/The+Shadow+Economy+in+Europe+2013.pdf/42062924-fac2-4c2c-ad8b-0c02e117e428`.

Schneider, F. and Buehn, A. (2017): Estimating a Shadow Economy: Results, Methods, Problems, and Open Questions. *Open Economics*, (1), 1-29.

Serrano-Sanz, J.M., Bandrés, E., Gadea, M.D. and Sanau, J. (1998): Desigualdades territoriales en la economía sumergida, Instituto Aragonés de Desarrollo.

Steel, M. (2019): Model Averaging and its Use in Economics. forthcoming in *Journal of Economic Literature*. `https://arxiv.org/abs/1709.08221v3`.

Tanzi, V. (1983): The Underground Economy in the United States: Annual Estimates, 1930-80. *IMF Staff Papers*, 30, 2, 799-811.

Thomas, J. (1999): Quantifying the Black Economy: Measurement without Theory, yet again?. *Economic Journal*, 109, 381-389.

Vaquero-García, A., Lago-Peñas, S., Martínez-Vázquez, J. and Fernández-Leiceaga, X. (2018): Economía sumergida y fraude fiscal en España: qué sabemos? qué podemos hacer? Una panorámica de la literatura. *Estudios de la Fundación de Cajas de Ahorro.*

Appendix: Figures and Tables

Figure A1: The Evolution of Money Velocity in Spain

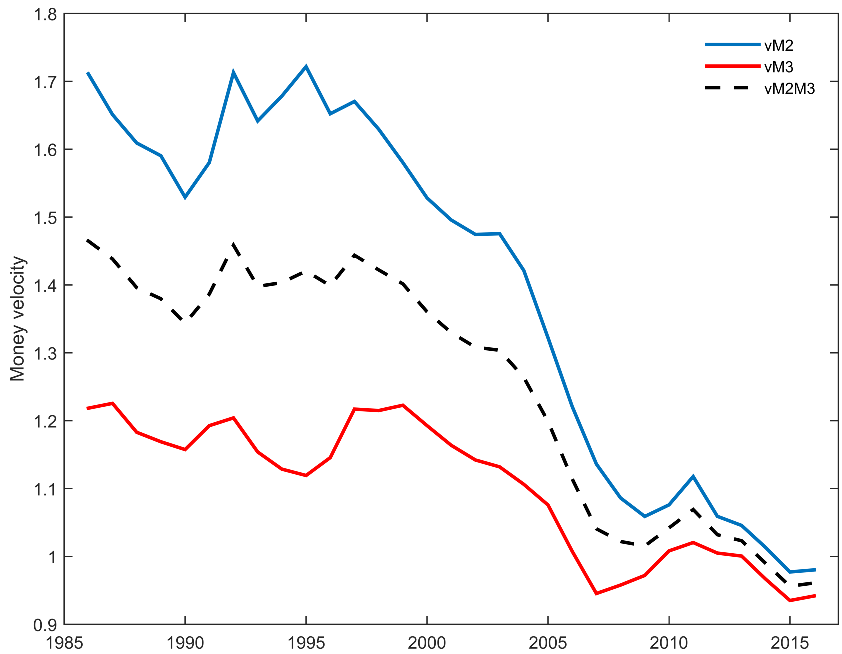


Table A1: Data and descriptive statistics

Variables	Mean	Standard deviation	Min	Max	Units / Definition	Source
Currency	6.587E+10	1.490E+10	3.436E+10	8.994E+10	(constant euros 2010)	BS, IMF
Monetary aggregate M1	2.977E+11	2.007E+11	8.397E+10	7.339E+11	(constant euros 2010)	BS, IMF
Monetary aggregate M2	6.442E+11	2.679E+11	3.117E+11	1.092E+12	(constant euros 2010)	BS, IMF
Monetary aggregate M3	7.869E+11	2.448E+11	4.367E+11	1.171E+12	(constant euros 2010)	BS, IMF
Prices	0.7645	0.2139	0.38265	1.009	GDP deflator $P_{t=2010} = 1$	De la Fuente (2017a)
Total FP (including social contribs)	0.305	0.023	0.237	0.339	% of the GDP	STA, SS
Total FP	0.199	0.019	0.154	0.232	% of the GDP	STA
Indirect FP	0.086	0.021	0.036	0.111	% of the GDP	STA
Direct FP	0.112	0.011	0.092	0.133	% of the GDP	STA
Value Added Tax FP	0.051	0.018	0.007	0.070	% of the GDP	STA
Personal Income Tax FP	0.072	0.006	0.053	0.081	% of the GDP	STA
Corporate Tax FP	0.025	0.007	0.017	0.045	% of the GDP	STA
ln Real GDP	27.466	0.231	27.009	27.744	log of real GDP	De la Fuente (2017a)
Unemployment rate	0.151	0.047	0.079	0.253	% of the active population	De la Fuente (2017b)
Inflation rate	0.035	0.0270	-0.002	0.1106	Annual Inflation rate (%)	De la Fuente (2017b)
Interest rate	0.055	0.051	0	0.158	3-months nominal interest rate (%)	BS, ECB
Linear trend	16	9.0921	1	31	Linear numeric progression [1:1:31]	
Quadratic trend	336	299.87	1	961	Quadratic numeric progression [1:1:961]	
Euro	0.48	0.51	0	1	Dummy: takes a value of 1 in 2002-2016 and 0 otherwise	
Crisis	0.19	0.4	0	1	Dummy: takes a value of 1 in 2008-2013 and 0 otherwise	
Education	8.73	1.19	6.75	10.51	Average years of education	De la Fuente (2017c)
Net migration	0.66	0.74	-0.61	2.39	% of total population	De la Fuente (2017c)
Agriculture employment	0.06	0.02	0.04	0.11	% of total employment	CE
Industrial employment	0.28	0.04	0.23	0.34	% of total employment	CE
Construction employment	0.08	0.02	0.04	0.11	% of total employment	CE
Non market services employment	0.25	0.03	0.18	0.31	% of total employment	CE
Financial services employment	0.24	0.01	0.22	0.25	% of total employment	CE
Other services employment	0.09	0.01	0.07	0.11	% of total employment	CE
Wage share	0.56	0.02	0.51	0.59	% of total employment	De la Fuente (2017b)

Notas: BS denotes Bank of Spain, ECB, European Central Bank, IMF International Monetary Fund , STA Spanish Tax Agency, SS Social Security, CE Cambridge Econometrics.

Table A2: Estimates of the Shadow Economy of Figure (5)

	Alternative Fiscal Indicators							Model
Year	FPA	FP	Indirect	Direct	VAT	CT	PIT	Averaged
1986	20.07	21.31	13.72	16.22	12.23	20.73	15.09	16.24
1987	20.92	22.05	13.44	17.21	12.97	23.29	15.86	16.72
1988	20.92	21.91	13.57	16.96	13.12	22.81	16.20	16.98
1989	22.25	23.33	16.16	17.52	19.27	24.26	18.84	19.34
1990	21.62	22.79	15.91	16.97	18.99	23.72	18.59	18.85
1991	22.25	23.28	16.83	17.19	19.49	24.89	18.50	19.17
1992	24.77	26.05	21.02	17.95	25.41	26.85	18.67	21.08
1993	24.10	24.91	19.38	17.46	23.58	26.56	17.07	19.98
1994	24.69	26.15	20.79	18.11	25.52	26.97	17.15	20.23
1995	25.30	26.79	20.94	19.02	25.41	27.42	17.80	20.95
1996	24.04	25.30	19.93	17.82	24.32	25.39	17.19	20.09
1997	24.15	25.72	20.52	17.92	24.74	24.95	19.66	21.29
1998	23.90	25.47	20.43	17.48	24.43	24.75	19.48	21.18
1999	24.20	25.91	21.00	17.77	24.96	24.78	19.35	21.28
2000	23.35	24.63	20.53	16.33	24.68	23.92	19.40	20.91
2001	22.12	23.38	19.25	15.55	23.69	22.20	18.33	19.68
2002	21.62	22.87	18.73	15.44	22.77	21.86	18.44	19.45
2003	21.57	22.73	18.87	15.01	22.96	22.07	18.07	19.30
2004	21.30	22.68	18.88	14.86	23.32	21.56	18.46	19.40
2005	21.41	22.67	18.83	15.04	22.82	21.66	19.09	19.79
2006	20.98	22.61	18.58	15.20	22.62	20.58	19.10	19.57
2007	20.74	22.19	17.63	15.22	21.85	21.51	19.65	19.68
2008	19.29	20.47	16.18	13.98	20.62	20.09	15.96	16.98
2009	18.28	18.83	14.72	12.89	18.40	19.43	14.22	15.50
2010	18.58	19.61	16.06	12.89	20.49	19.61	13.19	15.42
2011	18.74	19.45	15.67	12.96	19.99	20.12	13.20	15.61
2012	18.26	19.24	15.31	13.04	19.92	19.12	13.95	15.67
2013	17.50	18.46	14.73	12.42	19.33	18.00	12.88	14.92
2014	16.34	17.44	14.02	11.60	18.90	17.20	11.87	13.87
2015	15.29	16.29	13.06	10.82	17.54	15.56	11.24	13.14
2016	13.95	15.14	12.04	10.03	16.72	14.09	10.33	11.95