Optimal Value-at-Risk Disclosure

Seixas, Mário and Barbosa, António

Banco de Portugal, ISCTE - Instituto Universitário de Lisboa

2019

Optimal Value-at-Risk Disclosure

Mário Seixas
Banco de Portugal

António Barbosa*
ISCTE-IUL - Instituto Universitário de Lisboa

This version: December 2019

Abstract

In 1995, the Basel Accords introduced an alternative method to compute the market risk charge through the use of a risk model developed internally by the financial institution. These internal models, based on the Value-at-Risk (VaR), follow certain rules that are defined under the Basel Accords. From this moment on, risk analysts and financial academics focused their attentions on how to accurately estimate the VaR in order to reduce the regulatory capital. However, considering the market risk framework defined in the Basel Accords, the best strategy to optimize the regulatory capital may not lie in truthfully disclosing an accurate VaR estimation. In this study, we propose to solve, through dynamic programming, for the optimal policy function for disclosing the reported VaR based on the estimated value that minimizes the daily capital charge. This policy function will provide the optimal percentage of the estimated 1-day VaR that should be disclosed, taking into account the impact that this disclosure decision will have in future capital charges, by managing the rules defined in the Basel Accords. Our goal is to prove that truthful disclosure of an accurately estimated VaR is suboptimal. The main results from our investigation show that using the optimal reporting strategy leads to an average daily reduction in the capital requirements of 4.32% in a simulated environment, compared with a normal strategy of always truthfully disclosing the estimated 1-day VaR, and leads to an average daily saving of 7.22% when applied to our S&P500 test portfolio.

JEL Classification: G32, G28, G11, G17

Keywords: Value-at-Risk, Regulatory Capital, Market Risk Charge, Optimal Disclosure, Dynamic Programming

*ISCTE Business School, Av. Forças Armadas, 1649-026 Lisbon, Portugal. Phone: +351 21 790 39 16. E-mail: antonio.barbosa@iscte.pt

1 Introduction

In 1995, the Basel Committee on Banking Supervision (BCBS) introduced an internal model to compute the market risk – Value-at-Risk (VaR) – and define the regulatory capital charge used to cover future losses related to this type of risk. This model was described as the estimate for the worst possible loss that a certain portfolio could suffer considering a predetermined statistical confidence (Basel Committee on Banking Supervison, 1995).

Since then, the Basel Accords, and the VaR method, were adopted by more than 100 countries (Alexander, 2008). Due to this worldwide use, there exists a big literature on this topic and, in particular, about the methods to estimate the VaR (Angelovska, 2013; Aussenegg and Miazhynskaia, 2006; Hull and White, 1998; Totić et al., 2011; and Ünal, 2011).

However, as far as we know, there exists few literature on possible strategies to optimize the daily capital charge based on VaR internal models. Some of these studies focus on the idea that the best method to optimize the daily capital charge is by accurately estimating the VaR (McAleer, 2009), while others explored the use of financial options to minimize VaR and, consequently, the capital charge (Ahn et al., 1999; and Deelstra et al., 2007).

Nevertheless, there is one study that stands out from the others that was introduced by McAleer et al. (2010). In it, the authors developed a function that minimizes the daily capital charge through the management of the number of exceedances (i.e. number of days where the actual loss exceeds the VaR estimate) that a financial institution (FI) is allowed to have, according to the Basel rules (Basel Committee on Banking Supervison, 1995). With this model, risk analysts obtain the optimal percentage of the forecasted VaR that should be reported taking into account two variables: the number of exceedances recorded since the beginning of the regulatory period; and the number of exceedances recorded in the last 25 days.

The authors reached the conclusion that using their function together with a VaR model could reduce the daily capital charges up to 14.3% when compared with the RiskMetrics Model.[1] However, to achieve the perfect model it is necessary to do a calibration test for the parameters, which is computational intensive and time consuming, and no out-of-sample tests were performed in order to verify if this would work in a real life context.

Considering this and the small number of studies in this field, we propose to create a model for the optimization of the regulatory capital charge by choosing the VaR disclosure based on the estimated VaR and other key variables. This optimal disclosure policy is solved using dynamic programming methods.

Dynamic programming is a method to solve for optimization problems that can be stated recursively. The solution that is obtained is a policy function that defines the optimal course of action in each possible state of nature, taking into consideration the effect of those choices in the future.

[1] Method to compute the VaR created by Reuters and JP Morgan that assumes that the portfolio returns are normally distributed.

In the case of our model, the policy function represents the percentage of the estimated 1-day VaR that should be disclosed, and this optimal decision will depend on three state variables which, together, fully describe the state of nature: time remaining for the regulator to do the backtesting and review the multiplier (TtoB); multiplier that is currently in use (K); and the number of exceedances that were recorded until the moment of the decision, during the current regulatory period (EC). For each combination of these state variables, there exists an optimal decision that optimizes today's capital charge taking into account the effect that this decision will have in the likelihood of future states of nature and, as a consequence, in future capital charges. The range of these variables was chosen according to the rules defined by the Basel Accords for the use of internal models (Basel Committee on Banking Supervison, 1995).

This problem will be solved as an infinite horizon Markov decision process with the use of the function iteration algorithm on MATLAB$^{\mathrm{TM}}$ software.

The advantage of our model is the elimination of any subjective choice or calibration need for the parameters, defining the optimal policy according to the maximization of a value function that represents the present value of a FI's capital charges (that has a negative sign), which are directly related to the FI's cost of capital.

In a general way, the results of the optimal policy point out to an aggressive strategy when exceedances are low, i.e. underreport the estimated VaR, and to a more conservative strategy when exceedances are high, i.e. overreport the estimated VaR. There are some deviations from this general strategy that will be analyzed later on in Section 4.

The results from this model will then be tested in a Monte Carlo simulation and afterwards they will be applied to a real portfolio in order to compare the performance of the optimal reporting strategy (i.e. reporting the 1-day VaR according to the optimal policy) with that of a strategy that truthfully reports the estimated VaR. The main results from these 2 analyses were very promising. In the Monte Carlo simulation, we concluded that our model performed better in, approximately, 78% of the cases, which translated into an average relative saving in the daily capital charge of 4.32%. In our S&P500 portfolio simulation test, the results were even better, pointing out to a better performance of the optimal strategy model in 82% of the cases and an average relative saving in the daily capital charge of 7.22%.

This investigation gives four main contributions for the financial literature: defines an optimal rule to minimize the regulatory capital charge that takes advantage of the Basel rules; demonstrates that the minimization of the capital requirements goes beyond the formulation of a good VaR estimation model even though, as we will see, the accurate estimation of the VaR plays an important role in the effectiveness of the optimal strategy; gives some insights related with the impact that the state variables may have in the choice of the disclosed 1-day VaR by risk analysts; and provides an incentive for the development of more complex studies and models to do this optimization.

The next sections are organized as follows: Section 2 reviews the necessary concepts for the development of the model; Section 3 analyses the most relevant literature for this work as well as the different uses of VaR in the literature; Section 4 introduces our model, the methodology used to achieve the optimal policy function and analyses the results; Section 5 tests the performance of the optimal policy in the context of a Monte Carlo simulation while Section 6 does this in a real life context by applying the optimal policy to a real portfolio; Section 7 concludes by summarizing the main findings and giving suggestions for future studies.

2 Theoretical Framework

In this section we will review the most important concepts related with this paper. First we will give a summary about the purpose of the Basel Committee on Banking and Supervision (BCBS) alongside with the Basel Accords. Afterwards the VaR model will be explained. Lastly, we will introduce the dynamic programming framework that will be used to attain the goal of this paper, i.e. the optimization of the capital charge.

2.1 Basel Accords

The Bank for International Settlements (BIS) was established in 1930 with the mission of promoting the cooperation between central banks, the financial and monetary stability and to act as a bank for central banks. The BIS has 60 member central banks, including Portugal's Central Bank and the European Central Bank (ECB).

In order to promote this cooperation, it hosts independent organizations and committees that have the purpose of improving financial stability. One of these committees is the Basel Committee on Banking Supervision (BCBS). The BCBS was created in 1974 by the central bank governors of the G10 countries as a forum to discuss regulation of banks with the goal of increasing the efficiency of risk management and banking supervision. From these forums a report arises with the name of Basel Accords that consists in a number of suggestions for the improvement of the current risk management framework.

The first Basel Accord of 1988 introduced minimum capital requirements to cover financial risks (in a standardized approach) and was the main driver for the importance of this issue in the future. Subsequent to this, in 1995 an amendment, known as Market Risk Amendment, was released in which it was discussed an internal model to address the market risk (Value at Risk), that was defined as an estimate of the worst possible loss that a bank could suffer according to a predetermined statistical confidence.

With the increase of the complexity of the financial products and the need to distinguish between the different types of financial risk, it was made a revision to the previous framework, which resulted in the Basel Accords II, launched in 2004. In this new framework there were defined three pillars: minimum capital requirements, supervisory reviews (recommendations for supervisors) and market discipline (disclosure requirements in order to increase transparency). And it was distinguished three types of risk: credit risk, operational risk and market risk. Since the focus of this paper is in the market risk, we will only analyze the impact of the Basel Accords for this type of risk.

Although Basel II has defined some important guidelines about the purpose of the BCBS and the different types of risk, it did not change the computation process for the market risk capital requirements – which is what is important for this study – therefore the following measures result from the analysis of the 1995 Market Risk Amendment.

According to the Basel rules (Basel Committee on Banking Supervision, 1995), the Market Risk Charge (MRC), i.e. the minimum capital requirement to cover the market risk, is divided in two types of risk: General Risk Charge (GRC) and Specific Risk Charge (SRC).

$$MRC = GRC + SRC \tag{1}$$

To compute these, two methods are described: the Standardized Method and Internal Models. The Standardized Method consists in predetermined rules that define the market risk according to the nature of the assets, e.g. bond or stock (for more information on this topic refer to Alexander, 2008). The internal model – the focus of this paper – is a model developed by a bank, which needs to be approved by the regulator, that can accurately predict the market risk. When an entity chooses the internal model, these models should be based on the Value-at-Risk (VaR). The value computed with this method is the GRC while the SRC will depend on the ability of the risk model to capture specific risk.

However, to use the internal model, some rules need to be followed: the model must estimate the 10-day VaR (2 weeks holding period) at a 1% significance level;[2] the VaR should be estimated on a daily basis; the historical sample used should be at least 1 year of data; and the data must be updated every 3 months or when a sharp change in prices occurs. Considering these rules, one would assume that the GRC would be equal to the 10-day VaR at a 1% significance level. However, there are some risks in setting the GRC as the 10-day VaR at 1% significance level that the BCBS have identified, e.g. past observations cannot reflect the future behavior, normality assumption may not be verified, and, in statistical terms, a confidence level of 99% would mean that the bank could go bankrupt once every 4 years.

Because of this, the Market Risk Amendment (Basel Committee on Banking Supervison, 1995) also stipulates that the 10-day VaR should be multiplied by a factor (k) that is related to the performance of the FI's model. This performance is evaluated by producing a backtesting that evaluates the model's ability to predict the 1-day VaR in the last 250 business days. This is done by comparing the VaR estimate with the profit/loss on that day, in order to determine whether an exceedance has occurred (i.e. if the loss was higher than the one predicted by the VaR).

The factor, commonly known as the multiplier, is then defined according to Table 1. This allows supervisors to have a higher confidence when approving models and provides an incentive for risk analysts to create good models in order to reduce the capital requirements

[2] If the portfolio is linear (i.e. does not have options) the 1-day VaR can be scaled by using the square root of time.

Exceedances	k factor
≤ 4	3
5	3.4
6	3.5
7	3.65
8	3.75
9	3.85
≥ 10	4

Table 1: Relation between the number of exceedances and the k factor.

(i.e. the capital charge) through lower multipliers.

Lastly, in order to avoid the need of sharp increases in the minimum required capital related with the high volatility of the market risk, especially when a sharp move in prices occurs, it is used the average of the last sixty days VaR in the calculation of the GRC.

In short, the previous rules can be translated into the following equation

$$GRC = \max\left(k \times \frac{1}{60}\sum_{i=0}^{59} VaR_{1,1\%,t-i} \times \sqrt{10}, VaR_{1,1\%,t} \times \sqrt{10}\right) \tag{2}$$

In terms of the SRC the BCBS defines that it should be zero when regulators approve that the VaR model is capturing all the specific risk, otherwise it should be estimated according to the standardized method.

An important note about the Basel accords is that none of these measures are mandatory, these are recommendations that are discussed by the committee, in order to improve risk management efficiency. The application of these measures depend on their adoption by the regulators in each country. Nonetheless, the first Basel accord and the 1995 Market Risk Amendment were adopted by more than 100 countries (Alexander, 2008) – including the Eurozone countries – which demonstrates the large influence of the BCBS.

Due to the effect of the 2007/2008 financial crisis the BCBS revised Basel II and published a new report in 2009 entitled “Revisions to the Basel II market risk framework”. The main reason for this was that the previous measures did not account for some key risks, e.g. default risk and migration risk.

In this report, some of the rules to use internal models were changed. FIs are now obliged to update their data once every month and they must compute, on a weekly basis, a stressed- VaR (sVaR) measure in order to verify how their model would react to a period of financial stress. This sVaR is also part of the GRC, and it is added to equation 2.

Regarding the SRC, for VaR models that have specific risk recognition it should now be added an IRC (Incremental Risk Charge) to the MRC, which is an estimate of the exposure to systematic and specific default, credit migration, credit spread and equity price risk in a period of 1 year and with a confidence level of 99.9%.

These changes will not be considered in this study since the computation of the sVaR is ambiguous and difficult to implement in an optimization model. Therefore it will only be considered the rules reported until the Basel Accords II, meaning that our model will optimize one part of the GRC (equation 2) and not the full amount as it is defined nowadays.

2.2 Value-at-Risk

As it was stated above, the Value-at-Risk represents the loss that will not be exceeded with a certain confidence level $(1-\alpha)$ over a certain period of time (h) (Alexander, 2008). From this it is easy to understand that the VaR is a statistic and it is related with a probability:

$$\mathbb{P}\left(X_h < -VaR_{h,\alpha}\right) = \alpha \tag{3}$$

The popular use of this risk measure to address market risk is mainly due to its comprehensibility and ability to be disaggregated and aggregated while taking into account the dependencies between the assets, allowing the determination of the origin of the risks in a portfolio. In order to estimate VaR it is necessary to define the values for the two parameters α and h, the distribution of h-day returns (X_h) and the method used to estimate the VaR.

The first two parameters have to be chosen according to the rules under the Basel II – α is equal to 1% and h is equal to 10 trading days. The return distribution will depend on the estimation method used. The choice of this method is the most important step in the development of the VaR model since it is one of the main drivers for its performance. In a study made by Beder (1995) the VaR was computed according to eight different methodologies for three portfolios and the results indicated eight significantly different values, varying in some cases by more than fourteen times in the same portfolio.

Alexander (2008) explores three main methods for VaR estimation: Parametric VaR, Historical Simulation and Monte Carlo Simulation. The Parametric VaR model is based on the assumption that the return distribution follows a certain parametric distribution, usually the Normal distribution. The historical simulation is a nonparametric method that uses the empirical distribution of the returns in a given sample. The Monte Carlo Simulation is a more advanced method that consists in using a statistic model to simulate the future returns, being at the same time extremely flexible but also more prone to risk model risk.

According to a survey made by Pérignon and Smith (2010), the most common method used by FIs is the historical simulation, choice that is not difficult to understand since it is the one that has the least assumptions and is easier to explain in case of failing.

In this paper the estimation method used is the Parametric Normal – assumes that the return distribution follows a normal distribution – due to its simplicity and easy computation. Therefore the focus, from now on, will be on this method.

To derive the formula for the VaR under the parametric normal it is necessary to assume that the return distribution follows a normal i.i.d. (independent and identically distributed) with a certain mean and standard-deviation (i.e. volatility):

$$X_h \sim N\left(\mu_h, \sigma_h\right) \tag{4}$$

Both parameters are the forecasts of the future expected return and volatility over the following h-days. From this assumption it is easy to derive the formula for the VaR.

$$VaR_h = \Phi^{-1}\left(1-\alpha\right) \times \sigma_h - \mu_h \tag{5}$$

where $\Phi^{-1}\left(\cdot\right)$ is the quantile function for the standard normal random variable.

Alexander (2008) demonstrates that using an expected value (i.e. mean) equal to zero makes the computation of VaR easier and only has a big effect in its value when the risk horizon (h) is higher than one month. Thus, to simplify this equation, we will assume that the mean is equal to zero.

$$VaR_h = \Phi^{-1}\left(1-\alpha\right) \times \sigma_h \tag{6}$$

Now that the formula is derived there is one last concept to understand: scalling. Basel rules define that the 1-day VaR can be scalled in order to obtain the 10-day VaR. The purpose of this strategy is to avoid the use of a big sample when a 10-day VaR is estimated, since this would lead to a forecast of the parameters based in a considerable amount of past data.

This scalling is done by multiplying the 1-day VaR by the square root of time (in our case, 10 days). However, it is necessary to do an extra assumption to be able to use this rule: the arithmetic returns are approximately equal to the geometric returns.

When this rule is used it means that we are computing a dynamic VaR. This assumes that the portfolio is rebalanced every day, i.e. its value has to remain the same in the end of each trading day over the h-day horizon. This is why the Basel Accords states that the VaR must be computed on a daily basis.

Lastly, it is necessary to forecast the volatility to compute the VaR. This is another of the main drivers that are directly related with the performance of the model. Some of the most common methods, in ascending order of complexity, are the equally weighted variance, the EWMA (Exponentially Weighted Moving Average) and GARCH (Generalized Autoregressive Conditional Heteroscedasticity) models.

The difference between the equally weighted variance and the others is that the first is sensitive to the sample size, while the others are not, because the weight of an observation in the volatility estimation decays exponentially with the age of that observation. Due to this, it becomes a better method to estimate future values. For these reasons, the method that will be used in the empirical tests is the EWMA since it is the simplest method that gives good forecasts for the future variance.

The recursive equation of this method is the following (Alexander, 2008):

$$\hat{\sigma}_t^2 = \left(1-\lambda\right) X_{1,t-1}^2 + \lambda \hat{\sigma}_{t-1}^2 \tag{7}$$

According to this equation, the variance is computed in a recursive form and this is the reason for the smaller impact of past observations on the variance, as time passes. This can be observed in the next equation.

$$\hat{\sigma}_t^2 = (1-\lambda)\sum_{i=1}^{\infty}\lambda^{i-1}X_{1,t-i}^2 \tag{8}$$

To apply this method it is necessary to give a value to the parameter λ, i.e. the smoothing constant, in order to define the different weights that the most recent and older observations have in the future variance. In JPMorgan (1996) a calibration for this parameteris performed using the RMSE (Root Mean Square Error). The values achieved were 0.94, when considering daily data, and 0.97, for monthly data.

2.3 Dynamic Programming

Dynamic programming (DP) is used to do optimizations when a multi-stage decision problem is addressed, i.e. the optimal decision depends on the current state.

The base for the DP model is Bellman's principle of optimality (Bellman, 1954) that states that an optimal policy should always define the optimal decision to take considering all the previous decisions in order to maintain the final result in its optimal level.

Dynamic models are categorized depending on whether time, state space (S) and action space (X) are discrete or continuous (Miranda and Fackler, 2002). As it will be demonstrated later, the proposed optimization problem has a finite discrete space of states and an infinite continuous space of actions. However, in order to reduce the complexity of the problem, we will discretize the action variable, meaning that the correct model to use is one that is discrete in time, state and actions. The reason for this choice will be explained in Section 4.

The discrete model that will be used is the discrete Markov Decision Model. Its structure is: in each period t it is observed the current state s_t and, according to the characteristics of that state, an action x_t is taken by an agent, resulting in a reward $f(x_t, s_t)$ that is directly related with the state and the action Miranda and Fackler, 2002). Given the decision, the agent will pass to the next state s_{t+1} . This transition is deterministic, if the next state can be determined just from knowing the current state of nature and action, otherwise it is stochastic. If it is stochastic, it is necessary to define a transition function $g(x_t, s_t)$ that contains the probabilities of transitioning from one state to another.

Taking all of this into account it is now possible to derive the Value Function according to Bellman's principle of optimality (Miranda and Fackler, 2002):

$$V_t(s) = \max_{x \in X(s)} f(x,s) + \delta \sum_{s' \in S} \mathbb{P}\left(s' \middle| x, s\right) \times V_{t+1}(s'),\ s \in S \tag{9}$$

In the previous equation we have the reward function $f(x,s)$, a discount factor δ, the probability of transitioning to the next state $\mathbb{P}(s'|x,s)$ (which represents $g(x_t, s_t)$) and the value function of the next period $V_{t+1}(s')$. This equation reflects the maximization problem that is: take an action x today in order to maximize today's reward and the discounted expected future rewards.

To achieve the optimal policy $x^*(s)$, it is necessary to guarantee that, in each state, the decision maximizes the present and future rewards. This is achieved by using a recursion method, starting from the last period T until today. This equation is named Bellman's recursion equation (Miranda and Fackler, 2002).

According to the time horizon there are two possible types: finite and infinite horizons. If the horizon is finite it means that the last decision is taken at time T. On the other hand, an infinite horizon means that it is necessary to take a decision for an infinite number of time periods, therefore Bellman's equation does not depend on time.

$$V(s) = \max_{x \in X(s)} f(x, s) + \delta \sum_{s' \in S} \mathbb{P}\left(s' \middle| x, s\right) \times V_t\left(s'\right), \; s \in S \tag{10}$$

In this equation, $V_t(s')$ represents the value function considering the optimal results for $X(s)$ until that moment. The main difference between equations 9 and 10 is the time-dependence, i.e. while in equation 9 the purpose is to define an optimal decision for each point in time, in equation 10 the goal is to reach a single optimal policy that can be applied at every point in time.

The problem that will be presented later on will be solved as an infinite horizon problem therefore the next step is to describe the method that is commonly used to achieve the optimal policy in this type of problems.

Miranda and Fackler (2002) give some insight into the algorithms used to solve these problems using a computer software (e.g. MATLAB$^{\text{TM}}$), nevertheless it is first necessary to transform equation 10 into matrix notation.

$$V_1 = \max_x f(x) + \delta \times P(x) \times V_0 \tag{11}$$

The value functions $V(s)$ and $V(s')$ are represented, respectively, by the vector $V_1 \in \mathbb{R}^n$ and $V_0 \in \mathbb{R}^n$. The reward function is the vector $f(x) \in \mathbb{R}^n$ and the policy vector, that contains the optimal actions to be taken at each state, is denoted by $x \in X^n$. Lastly, $P(x) \in \mathbb{R}^{n \times n}$ represents the n-by-n probability transition matrix for a certain action. Finally, n represents the number of possible states of nature.

After this, one can apply the specific algorithm in order to achieve the optimal value and policy function. In the case of the infinite horizon Markov decision model, the algorithm used is the Function Iteration Algorithm (Miranda and Fackler, 2002):

1. Define the state variables S, action variable X, reward vector $f(x)$, discount factor δ, transition matrix $P(x)$, termination condition τ, and an initial value for V_0;

2. Compute $V_1 = \max_x f(x) + \delta \times P(x) \times V_0$;

3. Verify if the termination criterion is satisfied: $\|V_1 - V_0\| \leq \tau$;

4. If the termination criterion is satisfied: $x \to x^*$. Else, update V_0 to V_1 and return to step 2.

3 Literature Review

Since the introduction of Value-at-Risk, research has focused on Value-at-Risk potential uses and on how to accurately estimate it.

Studies have been produced where the VaR measure is used as a substitute for the variance in portfolio optimization problems. A recent study conducted by Deng et al. (2013) combines the VaR concept with the Sharpe Ratio – amount of expected return an investor gains for each aditional unit of risk that it takes – in order to optimize the value of the portfolio.

Other studies on this subject (e.g. Künzi-Bay and Mayer, 2006; Mansini et al., 2007; Lim et al., 2010; and Ogryczak and Sliwiński, 2011) used the CVaR (Conditional Value-at-Risk) – measures the risk of extreme losses calculating a weighted average of the expected losses that go beyond the VaR estimate – instead of the VaR to optimize the return on portfolios using Linear Programming models.

However, there are few studies related with the regulatory context of the VaR, namely optimization strategies for the market risk charge. Ahn et al. (1999) studied a method that could be used to find a put option that minimizes the VaR, considering a maximum hedging cost. This was performed by modeling a function that computes the optimal options' strike price taking into account the underlying asset's value, the mean and volatility of its return, the risk-free rate and the VaR hedging period. One of the main conclusions in this paper is that this optimal strategy can reduce the VaR by 45% comparing it with a normal strategy of using at-the-money options, in an equity portfolio, at the same time that it can reduce the cost of the strategy by up to 80%. Deelstra et al. (2007) studied the optimal risk management strategy for a portfolio of bonds and consisted in determining the optimal strike price of a put option that would minimize the VaR for a certain hedging cost.

Although these methods were proved to be good strategies to reduce VaR (hence reducing the capital charges), they focus in the use of other financial products and in portfolio management to achieve certain VaR objectives (i.e. its minimization).

What if there exists a method to optimize VaR without the need to incur in additional costs or in increasing the exposure to financial assets? McAleer (2009) addresses this question. In this paper, some guidelines are given about the improvements that can be made in VaR models in order to create better risk monitoring strategies and achieve superior forecasts for the VaR. The purpose is to manage the excessive risk taking, that is a characteristic of conservative financial institutions, and achieve an optimal VaR measure following the Basel rules.

These guidelines, like the title suggests, are ten and are related with, for example, the choice of the volatility estimation model (conditional, stochastic or realized volatility; using a symetric, asymetric or leverage model to estimate the volatility) and the method used to forecast VaR (parametric, semiparametric and nonparametric models).

Altough the presented methodology is of important use in the creation of a VaR model, it only gives an optimal strategy for risk monitoring and does not truly explores strategies to optimize the daily capital charges. Moreover, this study raises one question: Is disclosing a perfect estimation for the VaR the best strategy to optimize the capital charge?

The first answer to this question came in McAleer et al. (2010). This paper discussed the hypothesis that FIs could manage the number of exceedances that they are allowed to have according to the Basel Accords (10 exceedances), in order to optimize the daily capital charge. To achieve this purpose, they created a function named DYLES (Dynamic Learning Strategy) that was based in the trade- off between the expected number of exceedances and the expected capital requirements.

This function gives the percentage of the forecasted VaR that should be disclosed, taking into account the number of exceedances that the FI had since the beginning of the regulatory period and the number of exceedances recorded in the last 25 days. The foundations of this function are related with the premise that risk managers are conservative when the number of exceedances is high and aggressive when this number is small or zero. This method tries to optimize the capital charges by doing a policy that manages the Basel rules, taking advantage of them. The authors tested this theory and reached to the conclusion that using DYLES reduces the daily capital charges by up to 14.3% when compared with the RiskMetrics Policy.

While this seems to be a great tool to optimize the capital requirements taking into account the Basel rules, there is still some inconvenients with it. The function has three parameters for which the value is a subjective choice, meaning that the performance of the function has some dependence on those parameters' values.

The authors give some suggestions about the numerical intervals for the parameters, however to achieve the perfect model it is necessary to do a callibration that consists in computing the results for DYLES using all the possible parameter combinations and then choosing the one that has the lowest number of exceedances and average capital requirements. Thus, this process is time consuming and makes the model less user friendly.

The second incovenient is that it is an ad-hoc strategy, meaning that each strategy depends on the specific portfolio and does not have a general application. It is also important to point out that the authors only did an in-sample test, which means that the effectiveness of this model in a real life situation is unknown.

Following this thought, we propose to present an alternative method to do this optimization. Based in the same principle as DYLES, this paper consists in creating a model that optimizes the daily capital charge through the use of DP.

The objective is to achieve an optimal policy strategy that gives the percentage of the forecasted 1-day VaR that should be reported in every state of nature, taking into account the effects that each decision will have in the future capital charge. For this, we will consider three state variables: time that remains for the regulator to do the backtesting (TtoB); number of exceedances that were recorded until the moment of the decision, during the current regulatory period (EC); and multiplier that is currently in use (K). The focus is in the 1-day VaR because the exceedances are defined according to its value, and it is the only one that is in the control of the risk analyst (see equation 2).

The advantages of this model are: all the parameters are defined; in each state of nature there is an optimal decision; and the optimal policy strategy can be applied to any portfolio. The main contribute of this study for the financial literature is to demonstrate that, taking into account the market risk framework that is associated with the definition of the capital charge, the best strategy to optimize the regulatory capital may not be through the disclosure of a precise estimate for the 1-day VaR.

4 Optimization Model

The goal of this paper, as it was mentioned in the previous section, is to create a model that allows the optimization of the market risk charge when the VaR method is used. It consists of maximizing a value function, through the use of DP, in order to obtain an optimal policy function that provides the optimal decision that an agent should take regarding the reported VaR.

The policy function defines the percentage of the estimated VaR that should be reported in order to optimize the daily capital charge, taking into account all the future effects of today's decision, in particular the likelihood of future exceedances and the future value of the multiplier K.

The action space (X) corresponds to all percentages of the estimated VaR that can be reported, which can be any value from 0 to infinity, therefore making the action space continuous. However, we tested the possibility of solving the dynamic programming problem with a continuous action space and reached the conclusion that it was computationally intractable. Therefore we decided to discretize the action space, maintaining a wide range of possibilities, ranging from 0.001 to 3, in steps of 0.001.

$$X = \{0.001, 0.002, 0.003, \ldots, 3\} \tag{12}$$

This means that the lowest value that can be reported is 0.1% and the highest value is 300% of the estimated 1-day VaR.

The choice of the percentage to report will depend on three state variables: time remaining for the regulator to do the backtesting and review the multiplier ($TtoB$); multiplier that is currently in use (K) and the number of exceedances that were recorded until now (EC). All these state variables are discrete and the set of possible values are as follows:

$$TtoB = \{1, 2, 3, \ldots, 250\} \tag{13}$$

$$K = \{3, 3.4, 3.5, 3.65, 3.75, 3.85, 4, 100000\} \tag{14}$$

$$EC = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\} \tag{15}$$

K	EC
3	≤ 4
3.4	5
3.5	6
3.65	7
3.75	8
3.85	9
4	10
100000	11

Table 2: Relation between the number of exceedances at the end of the year and the multiplier.

The values of $TtoB$ are derived from the Basel Accords, where it is defined that, to do the backtesting analysis, the regulator must use data from the last 250 days to evaluate the performance of the risk model (Basel Committee on Banking Supervison, 1995). In terms of the EC, the Basel Accords state that, when it is higher than 10, the most probable consequence is the obligation to use the standardized method instead of using the internal model. Therefore we restricted the EC variable to a maximum value of 11 since we assume that the agent pretends, at all costs, to use the internal model for an infinite horizon. The K variable includes all the multipliers that are in the Basel accords plus an additional one (100000). The purpose of this multiplier will be explained later. Table 2 relates the multiplier with the EC at the end of each 250-day period.

For any combination of these three states, the optimal policy function gives the decision that should be taken in order to minimize the daily capital charge. This type of DP problem is known as infinite horizon Markov decision model and is solved with the function iteration algorithm (Miranda and Fackler, 2002) introduced in Subsection 2.3.

4.1 Methodology

In this subsection we are going to follow all the steps mentioned in the Subsection 2.3, in order to derive the Bellman's equation and apply the Function Iteration Algorithm.

First, it is necessary to change the notation of K and EC variables. For a better connection between this explanation and the program code, the state variable K and EC will be substituted by $Kindex$ and $ECindex$, respectively, which represent the index of each variable. For example, $Kindex = 2$ corresponds to the second smallest value of K, that is, 3.4, and $ECindex = 1$ corresponds to the smallest value of EC, that is, 0.

$$Kindex = \{1,2,3,4,5,6,7,8\} \tag{16}$$

$$ECindex = \{1,2,3,4,5,6,7,8,9,10,11,12\} \tag{17}$$

The Reward Function $f(x, TtoB, EC, K)$ represents the daily capital charge according to the Basel accords (Basel Committee on Banking Supervison, 1995). However, three simplifications were made to make the problem computationally tractable: the 60-day average will be ignored; it is assumed that the capital charges will always be the 10-day VaR multiplied by the multiplier K; and the capital charge is equal to the GRC, meaning that we assume that the risk model as specific risk recognition (SRC=0).

$$f(x, TtoB, EC, K) = -x \times K \times VaR_{1,1\%} \times \sqrt{10} \tag{18}$$

The Breakdown of this equation is the following: $VaR_{1,1\%} \times \sqrt{10}$ – represents the 10-day VaR that is in compliance with the Basel Accords; K is the multiplier for the current period; x is the percentage of the VaR to be reported (the action variable).

As one can see, this function has a negative sign because we are maximizing the FI's profit, meaning that the capital charge has a negative impact in the function.

Now that this was explained, it is possible to justify the additional multiplier: 100000. In the definition of the EC variable we limited the number of exceedances up to 11 because otherwise we would have 251 possible values for the $ECindex$ instead of 12. This would unnecessarily increase the dimensionality and complexity of the problem, making it intractable to reach a solution. To prevent this, we created a new multiplier (100000), associated with an $ECindex$ equal to 12, that represents the transition to a state with a very high cost (equation 19 below). With this mechanism we assure that the agent will do whatever it takes to prevent the transition to this state, which will probably result in disclosing the highest possible percentage of the 1-day VaR (i.e. 300%). Note that the value of this multiplier is only used to determine when the transition to this state occurs, meaning that it does not have an application in the general reward function (equation 18) as it can be seen in equation 19.

$$f(x, TtoB, EC, K) = \begin{cases} -x \times K \times VaR_{1,1\%} \times \sqrt{10} & \text{if } Kindex \leq 7 \\ -5000000000 & \text{if } Kindex = 8 \end{cases} \tag{19}$$

To be able to apply the Function Iteration algorithm it is essential to translate this function into matrix notation. This is done by creating a reward vector named RewFun with $250 \times 8 \times 12 = 24{,}000$ elements that represent the total number of possible state combinations (achieved by multiplying the total number of elements of each state variable).

Lastly, it is necessary to construct the Transition Function $g(x, TtoB, EC, K)$ that will define how the transitions occur between states of nature. In this problem, one can see that the state variables are not independent because the multiplier (K) depends on the EC when $TtoB$ is equal to 1. The reason for this is that the multiplier must be reviewed at the end of each 250-day period in accordance with the EC variable. Due to this, it is necessary to implement a transition function with all the possible state combinations instead of doing one function for each state variable. Next we discuss the main ideas behind these transitions.

The transition between $TtoB$ states is deterministic, decreasing from t to $t-1$ every period whenever $t > 1$. When $TtoB = 1$ the transition is to $TtoB = 250$, reflecting the fact that at this time the backtesting is performed and the number of periods for the next backtesting is reset to 250 days.

The transition between $Kindex$ states is, most of the times, deterministic. Whenever $TtoB > 1$ no backtesting is performed, which means that the multiplier is not revised. Hence, the state $Kindex$ remains unchanged. But, at $TtoB = 1$, on the eve of performing the next backtest process, the state $Kindex$ is updated, reflecting the results of the backtesting process and the Basel rules outlined in Table 2. In this scenario, the transition is stochastic, since it depends on the state $ECindex$ (how many exceedances were accumulated up to $TtoB = 1$) and whether there was an exceedance in the transition to the next period, which is a random event. The new state $Kindex$ is determined as follows: for $ECindex < 5$ (less than 4 exceedances up to the eve of the backtesting procedure) $Kindex$ transitions to 1 regardless of whether an exceedance is recorded in the transition to the next period; for $5 \leq ECindex < 12$, $Kindex$ transitions to the current value of $ECindex$ $-$ 4 if no exceedance is recorded and, otherwise, to the current value of $ECindex$ $-$ 3; and for $ECindex = 12$, $Kindex$ transitions to 8, the catastrophic scenario that the agent wants to avoid at all costs.

Finally, the transition between $ECindex$ states is, most of the times, stochastic. Whenever $TtoB > 1$, there is some probability that an exceedance is recorded (portfolio loss is higher than the reported VaR), in which case the $ECindex$ state transitions from i to $i+1$, and some probability that an exceedance is not recorded, in which case the EC state remains unchanged. At $TtoB = 1$ the transition is deterministic, since the exceedance count is reset when entering the new backtesting. Hence $ECindex$ transitions to 1.

On top of this, there is always a possibility of falling into bankruptcy if the portfolio loss is larger than the capital charge set based on the reported VaR. This possibility, and its associated high costs, effectively put a limit to extreme levels of VaR underreporting. In that case, and regardless of the current state of nature, the transition is made to a state of nature with $TtoB = 250$, $ECindex = 12$ and $Kindex = 8$ which, according to equation 19, is the state with a very large cost that reflect the bankruptcy cost. Table 3 summarizes the different transition scenarios and their probabilities.

To compute the probabilities associated with each scenario we assumed that returns follow a standard normal distribution (mean zero and standard deviation of one). These probabilities of transition between states are then represented by a 24,000 by 24,000 matrix named TrMat.

At this point we have all the essential tools to derive the value function that we want to maximize. This function is equation 11 that was introduced in subsection 2.3 (DP), with an adjustment for our notation.

$$V_1 = \max_x RewFun\,(x) + \delta \times TrMat\,(x) \times V_0$$

To initiate the algorithm it is still necessary to define the value for the parameter δ, which represents the discount factor and the importance of the future events into the

	State of nature	Probability
From	$TtoB = t > 1; ECindex = i; Kindex = k$	
To	$TtoB = t - 1; ECindex = i + 1; Kindex = k$	$\mathbb{P}(Z < VaR \times x)$
	$TtoB = t - 1; ECindex = i; Kindex = k$	$1 - \mathbb{P}(Z < VaR \times x) - \mathbb{P}(Z >$ Capital Charge$)$
	Bankruptcy state	$\mathbb{P}(Z >$ Capital Charge$)$
From	$TtoB = 1; ECindex = i < 5; Kindex = k$	
To	$TtoB = 250; ECindex = 1; Kindex = 1$	$\mathbb{P}(Z \leq$ Capital Charge$)$
	Bankruptcy state	$\mathbb{P}(Z >$ Capital Charge$)$
From	$TtoB = 1; ECindex = i; Kindex = k$	
To	$TtoB = 250; ECindex = i; Kindex = ECindex - 4$	$\mathbb{P}(Z < VaR \times x)$
	$TtoB = 250; ECindex = i; Kindex = ECindex - 3$	$1 - \mathbb{P}(Z < VaR \times x) - \mathbb{P}(Z >$ Capital Charge$)$
	Bankruptcy state	$\mathbb{P}(Z >$ Capital Charge$)$

Table 3: Transitions that occur in each scenario and the equations used to compute their associated probabilities (where Z represents a standard normal random variable and x represents the action variable).

current decision, the terminal condition τ, and an initial value for the value function, V_0.

We defined a yearly discount factor of 0.9, meaning that the daily discount factor is approximately 0.99957865.[3] Regarding the terminal condition, it is decided that the norm of the difference between V_0 and V_1 should be close to 0.001 in order to consider the policy function x as the optimal one (in the case of our model, this value was 0.0013).

Due to the complexity of this problem, it is essential to start with a reasonable guess for the initial value of V_0 in order to reduce the number of iterations necessary to achieve the terminal condition (and the optimal solution). Thus, the initial value for V_0 will be a 24,000 by 1 vector, that was obtained from the solution of a similar optimization problem, but with a smaller number of possible actions (300 instead of 3,000).

4.2 Model vs Reality

After modeling the problem, it is necessary to understand the differences between our model and the real life problem. These differences will be important, later on, in the identification of the limitations of this study.

One of the most important assumptions in the construction of the model is related with the distribution of returns. To simplify the computations, we assumed that the distribution and its parameters (expected return and volatility) are known with 100% certain.

As mentioned in Subsection 2.2, using a value of zero for the expected return is a good proxy and does not have a material impact in the results. The biggest problem lies in the volatility since, in reality, it is unlikely to predict the true value for the future volatility. Usually one works with estimates that can either under or overestimate the true value of the volatility. In our case, the scenario of underestimation of the true volatility is the one

[3] Assuming 250 trading days per year.

that represents a real danger. In that scenario, the underestimation of the volatility itself will be enough to generate too many exceedances. If, on top of that, the agent pursues the optimal VaR disclosing policy that is derived under the assumption that his volatility estimate is correct, he will tend to underreport and underestimate the VaR. That will inevitably lead to a fast accumulation of exceedances from which it is impossible to recover and that, next year, will result in a high multiplier being set by the regulator and thus high capital charges. The scenario of volatility overestimation is more benign. The agent will see too few exceedances being generated despite his VaR underreporting, but the only implication is that we will have more freedom to keep underreporting the VaR. These scenarios and their consequences for the application of the optimal VaR reporting policy will be tested later on in a Monte Carlo Simulation in order to investigate the extent to which the performance of the optimal policy is sensitive to the assumption that the true volatility is known.

The next assumption is related with the reward function. According to the 1995 Market Risk Ammendment (Basel Committee on Banking Supervison, 1995), the GRC is defined as the maximum between (a) the average of the 1-day VaR forecasted for the last sixty days multiplied by the square root of time and by K and (b) the last forecast for the 1-day VaR multiplied by the square root of time. However, in the reward function defined in equation 18 and 19, the maximization and the average are ignored, and it is assumed that the capital charge is always the reported 1-day VaR multiplied by the square root of time and by the multiplier K.

Ignoring the maximization does not make any effect in the results since, assuming a constant portfolio, it is very unlikely to have the last VaR estimate to exceed K times the average of the last sixty VaR's because K has a value between 3 and 4. Therefore the most important difference between the model and the real life problem is the averaging of the last sixty VaR's, which is omitted in the model.

This average is important to avoid a manipulation of the reported VaR, however it is hard to implement in an optimization problem and if implemented it dramatically increases its complexity, since we would have an additional 59 state variables (the VaR reported in the previous 59 days), making it intractable.

Considering this, a possible solution is to ignore the average and only consider the last estimated VaR. We concluded that this does not result in the most optimized strategy but it gives a quite good approximation for it. The following demonstration proves this. For simplification, assume that there is no uncertainty, so we can drop expectations. The dynamic programming problem we solved is equivalent to minimizing the present value of the capital charges.

$$PV\left(GRC\right) = \sum_{t=0}^{\infty} \delta^t GRC_t \tag{20}$$

The formula we consider for the GRC is

$$GRC_t^{mod} = k\sqrt{10} x_t VaR_t \tag{21}$$

In reality, the formula for GRC is (dropping the maximization):

$$GRC_t^{real} = k\sqrt{10}\frac{1}{60}\sum_{i=0}^{59} x_{t-i}VaR_{t-i} \tag{22}$$

If the impact of the choice variable, x_t, in $PV\left(GRC\right)$ is similar for both formulations of GRC_t (real and modeled), then the approximation has a small impact

$$\begin{aligned} \frac{\partial PV\left(GRC_t^{mod}\right)}{\partial x_t} &= \frac{\partial \sum_{t=0}^{\infty} \delta^t k\sqrt{10}x_t VaR_t}{\partial x_t} \\ &= \delta^t k\sqrt{10}VaR_t \end{aligned} \tag{23}$$

In turn,

$$\begin{aligned} \frac{\partial PV\left(GRC_t^{real}\right)}{\partial x_t} &= \frac{\partial \sum_{t=0}^{\infty} \delta^t k\sqrt{10}\frac{1}{60}\sum_{i=0}^{59} x_{t-i}VaR_{t-i}}{\partial x_t} \\ &= k\sqrt{10}\frac{1}{60}\sum_{i=0}^{59} VaR_t\delta^{t+i} \\ &= \delta^t k\sqrt{10}VaR_t\frac{1}{60}\sum_{i=0}^{59}\delta^i \\ &\approx \delta^t k\sqrt{10}VaR_t \end{aligned} \tag{24}$$

since

$$\frac{1}{60}\sum_{i=0}^{59}\delta^i \approx 1 \tag{25}$$

as long as δ is close to 1. In our case $\delta = 0.99957865$ and so[4]

$$\frac{1}{60}\sum_{i=0}^{59}\delta^i = 0.9877 \approx 1 \tag{26}$$

The last assumption is related to the Basel Accords. As it was stated in Subsection 2.1, our optimization model is in accordance with the rules introduced until Basel Accords II, and does not take into account the changes made by the revisions to the market risk framework published in 2009 (Basel Committee on Banking Supervison, 2009). One of the impacts that this has in the model is related with the probability of default, which is overestimated in the model. This happens because with the new standards the GRC would incorporate a new element – sVaR – meaning that the daily capital charge would be higher than the value considered by the model. On the other hand, not considering the sVaR means that our model only optimizes part of the capital charge, i.e. only the one related with the VaR.

Nevertheless, this does not invalidate the use of our model (to optimize the part related

[4]Note that this demonstration assumes that the multiplier k is constant over time.

with VaR) since, in the worst case, the effect of this change in the GRC would probably be a policy function where extreme underreporting is more likely to occur, especially when EC is low and in the last days of the 250-day cycle, due to lower probability of default.

4.3 Optimal Policy Function

In this subsection we are going to present the results of the optimization model. These represent the percentage of the estimated 1-day VaR that should be reported according to the three state variables: time remaining for backtesting ($TtoB$), number of exceedances so far (EC) and multiplier in use (K). This will be referred, from now on, as the *optimal policy*.

Figure 1 illustrates the percentage of the estimated 1-day VaR that should be reported for each point in time when the multiplier variable K is 3, which is divided in two panels according to the number of the EC variable. Thus, in the vertical axis one can find the action variable (that can range between 0.01% and 300%) and in the horizontal axis one of the state variables – $TtoB$ (ranges between 1 and 250). Each line represents the different decisions that should be taken according to the value of the exceedance count variable (EC) at that time, e.g. if the current number of exceedances is equal to 5, one should find the optimal decision in the lighter blue line in panel B. Figure 2 shows all the lines of the two panels of Figure 1 in one figure.

The careful reader may have noticed that in both Figures 1 and 2 it is not included the series for the EC equal to 10 and 11. These two will not be presented here because the optimal decision is always the same: report 300% when the EC is equal to 10 and report 0.1% when the EC is equal to 11. This happens for a simple reason: if in any day, during the 250- day period, a portfolio records the 10th exceedance, it means that an extra exceedance will result in a transition to a state that we considered in the formulation of the model as a state with big consequences (e.g. the obligation to use the standardized method instead of the internal models) – referred, from now on, as the worst case scenario. Assuming that the agent does not want to face these consequences, he will do whatever it takes to avoid the extra exceedance, and the only decision that has the highest probability of not having an exceedance is reporting the maximum possible percentage of the VaR, 300%.

In the opposite case, when the EC is equal to 11 it means that this extra exceedance already occurred, therefore there is nothing that the agent can do to avoid these consequences, and for this reason the percentage to report is the lowest possible (0.01%). In other words, the damage is done.

Now that this detail was explained, we go on with the analysis of Figure 1. The first aspect that can be noticed is that until the 3rd exceedance (panel A), in most cases, the percentage to report, in a given day, increases as the number of exceedances increases (each line is above the previous), and from the 5th to the 9th exceedance (panel B) the same happens until the last 50 days. In general, the reason for this different behavior in the last

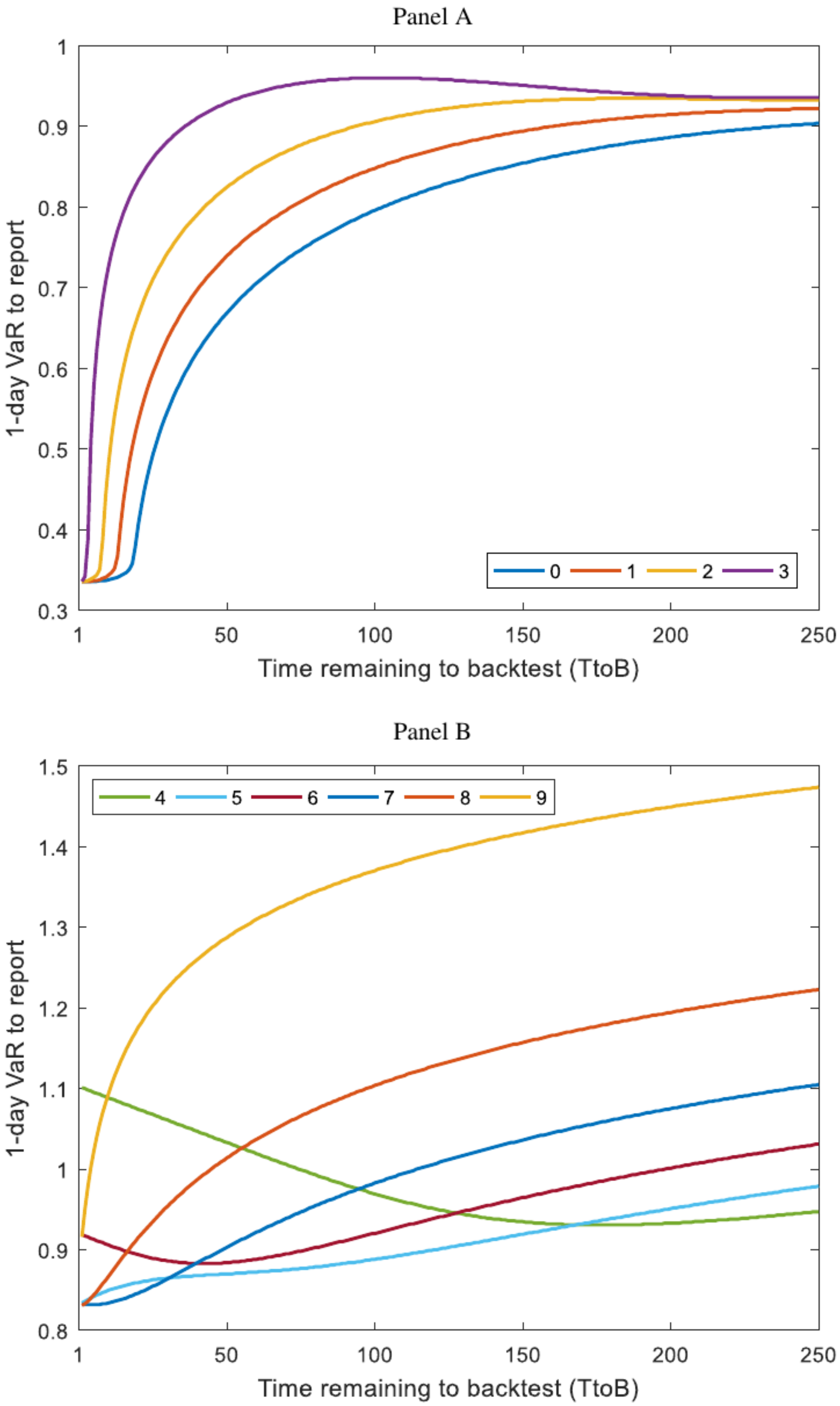

Figure 1: Optimal Policy Function for a number of exceedances of 0 to 3 (Panel A) and 4 to 9 (Panel B) when the multiplier K is equal to 3.

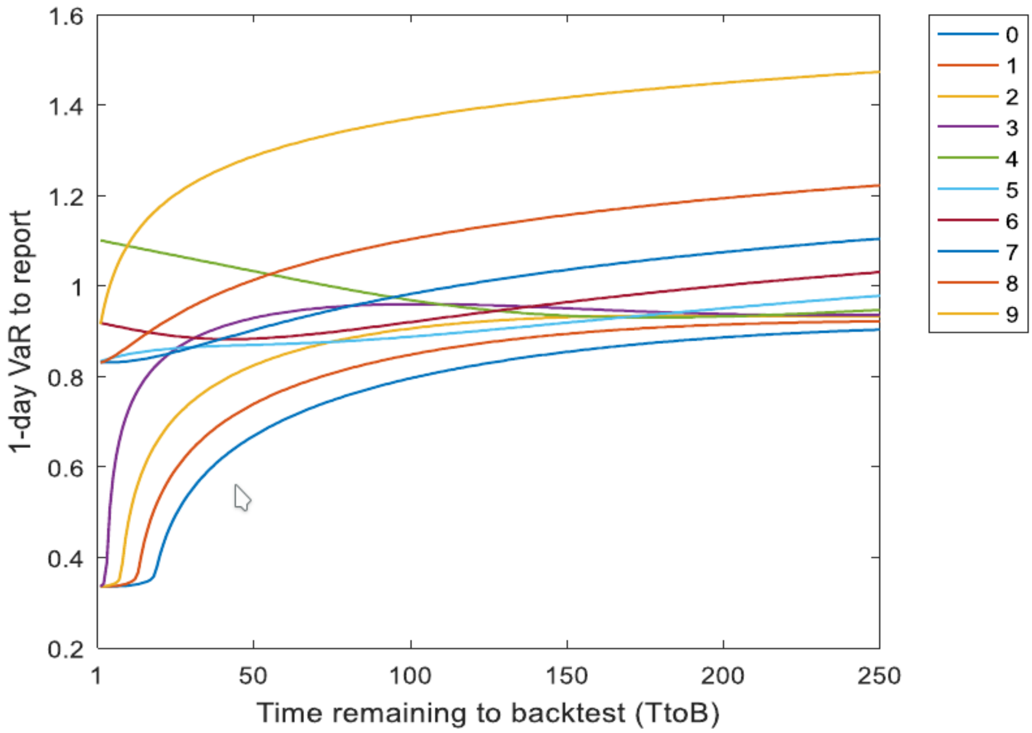

Figure 2: Optimal policy function for all the number of exceedances when the multiplier K is equal to 3.

EC	0	1	2	3	4	5	6	7	8	9
Mean	0.7655	0.8118	0.8634	0.9190	0.9771	0.9083	0.9476	0.9930	1.1029	1.3611
Median	0.8305	0.8765	0.9230	0.9410	0.9470	0.9035	0.9435	1.0125	1.1335	1.3965

Table 4: Statistical analysis of the optimal policy.

50 days of the last 5 series is that the agent has more factors to take into account when choosing the optimal decision to take, since the goal of the 4 exceedances was not attained. This difference will be analyzed later on.

In the series for the 4th exceedance, the behavior is different because this is the maximum number of exceedances that an agent can have in order to maintain the base multiplier. As one can see, the percentage to disclose increases as the time to backtesting decreases, being the only series that has an increasing trend, ranging from 94.7% to 110%. This effort is made in order to avoid another exceedance that would lead to a multiplier higher than the base value (3) in the future. It is also important to notice that, when the barrier of the 4 exceedances is exceeded, this effort decreases, mostly due to the consequence of having a multiplier higher than the base value in the future (which translates in higher capital charges and lower long- term benefits).

In Table 4, one can see the average and median of each series. Both statistics increase as the number of exceedances increase, except on the transition from the 4th to the 5th exceedance where the effort to avoid an extra exceedance has a slight decrease. This confirms the reasoning used on the previous paragraph related with the series for an EC equal to 4, by showing that the incentives to avoid an extra exceedance decrease after overcoming the barrier of 4 exceedances. From the 7th exceedance onwards, this effort is again high, reaching values higher than 100% when the worst case scenario is approaching.

Looking at the average of the whole series (0.9650), we conclude that, on average, it is better to disclose a lower 1-day VaR than the estimated one, i.e. to underreport. This is a natural conclusion because our model is designed in order to achieve 4 exceedances, objective that is attained through the underreporting of the 1-day VaR. If the estimated VaR is reported truthfully (i.e. no under nor overreporting), then on average there will be only 2.5 exceedances in a year. This leaves room to underreport and reap the short term benefit of lower capital charges without incurring in the cost of larger long term capital charges due to larger multipliers as consequence of accumulating more than 4 exceedances in a 250-day period.

Next we go into detail for each series. In the first three series (EC equal to 0, 1 and 2), there is a decreasing trend, i.e. the first percentage to report, when $TtoB$ is equal to 250, is, respectively, 90.4%, 92.2% and 93.3%, and after that, these values decrease until they both reach 33.6% when $TtoB$ is equal to 1. This behavior was expected since, in the beginning of the 250-day cycle, the agent is more risk averse due to the possibility of incurring in early exceedances that may accumulate to more than 4 by the end of the cycle, and, as time passes, this risk becomes smaller and the percentage to report decrease.

When the EC is equal to 3 (fourth series), the trend is also in general a downward one, although there is a slight increase between the 249 th and 92nd day to the next backtesting, going from 0.936 to 0.96. This is explained by the fact that the portfolio is 1 exceedance away from the 4th, i.e. the maximum value for the EC variable that guarantees the base multiplier, and it is related with the formulation of the optimization model. Since the goal is to optimize the present value of the capital charge, there are two factors that drive the optimal decision: report a lower percentage of the 1-day VaR to reduce the capital charge in the present (short-term benefits); or report a higher percentage to avoid an increase of the multiplier and benefit from this saving in the future (long-term benefits). To achieve an optimal model it is necessary to balance these two factors (referred from now on as "balancing factors").

Considering this, in the first 157 days the predominant factor is the long-term benefits of having the base multiplier in the next regulatory period (hence there is an increase in the percentage to report), whereas after this, since the end of the period is approaching and the likelihood of incurring in two more exceedances reduces, the short-term benefits seem to gain the lead as the predominant factor and the percentage to report decreases.

The next series (EC equal to 4) is the only one with an increasing trend, explained by the fact that it represents the maximum value that guarantees the base multiplier. Nevertheless, in the first 86 days the percentage to report marginally decreases from 94.7% to 93%. This is, again, a result of the balancing factors, explained previously, considering that in the first 86 days the predominant effect is the short-term benefits of having lower capital charges in the present and, following this, it seems to be better to report a higher percentage of the estimated VaR to benefit from a smaller multiplier in the future (in this case, the base multiplier).

Another possible explanation for this type of behavior (mix between decreasing and increasing trend) is related with the value of the EC variable in the early days. If, for example, an agent records 4 exceedances too quickly, it will be difficult to maintain this number until the end of the period, thus there is no incentive to be conservative and disclose an higher percentage of the estimated 1-day VaR (decreasing trend). However, as time passes, if this extra exceedance does not materialize, the agent will try to avoid it by increasing the percentage to report, in order to guarantee the smaller multiplier in the next period (increasing trend). This reasoning applies to all the series that show this type of behavior.

From the 4th exceedance on, every additional exceedance results in an increase of the multiplier up to the 11th exceedance, which represents the transition in the future to the worst case scenario. Hence, the balancing factors will also drive the trend of the next series.

For an EC equal to 5, 7, 8 and 9, one can notice that the curves have a decreasing trend and represent a higher risk aversion – the smallest percentage of the 1-day VaR to report is 83.1%. Also, for most of the time, the curves for the 7th, 8th and 9th exceedance are above 100%, which is due to the proximity to the worst case scenario (reaching 11 exceedances). Nevertheless, there is a curious observation when comparing the series for an EC equal to 5 and 7: in the last 30 days the values are higher in the former. The expectation for this would have been a persistence of higher values for an EC equal to 7. This different behavior can be explained with the effect of a future higher multiplier in the scenario where the EC is equal to 7 (3.65) compared with the scenario of an EC equal to 5 (3.4), meaning that the short-term benefits have an higher impact in the agent's decision – report a lower percentage to increase current savings, instead of a higher percentage to reduce future costs (the damage in the future multiplier is already done).

Considering the series for an EC equal to 6, one can observe a similar scenario to the series for an EC of 4: a decreasing trend until the last 48 days, and after that, an increasing trend. Like in that case, this change in the trend is due to the effect of the balancing factors because, following the increase of 0.4 in the base multiplier – EC equal to 5 – the second highest increase in the multiplier (0.15) occurs when the EC, in the end of the regulatory period, is equal to 7. Therefore in the first 202 days the predominant effect is the saving in the current capital charges (short-term benefits), while in the last 48 days it is the long-term benefits derived from the 0.15 saving in the future multiplier that leads to higher percentages to disclose.

An interesting feature from the analysis of Figure 1 and Figure 2 is that a higher number of exceedances does not always lead to a higher percentage to report, in a given day. Figure 3 represents the evolution of the optimal decision (vertical axis) depending on the number of exceedances (horizontal axis) for the same day.

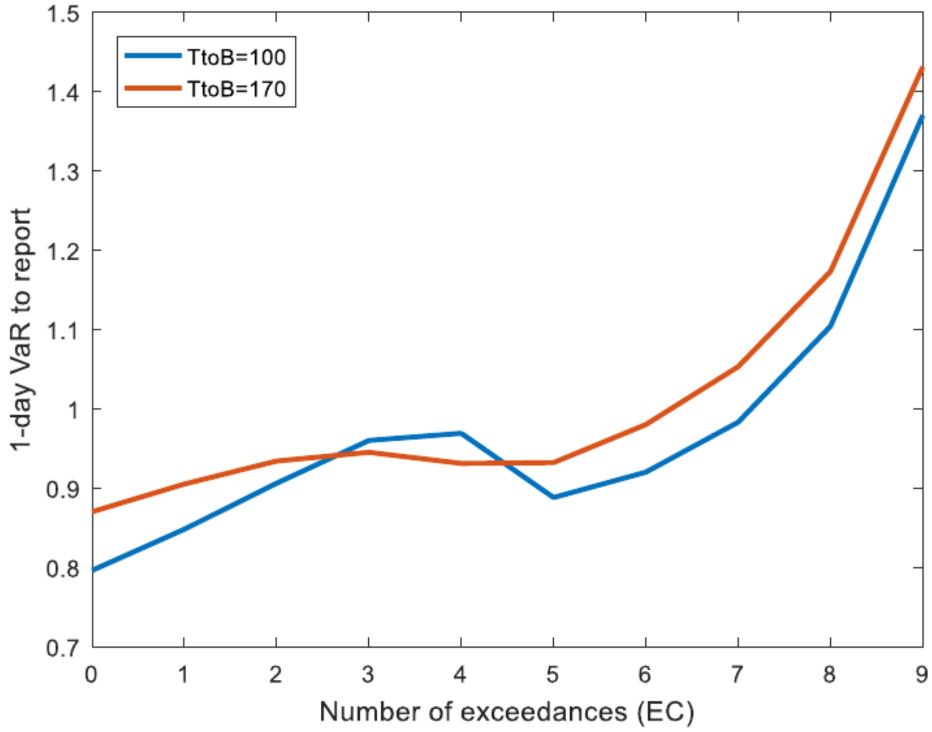

Figure 3: Evolution of the optimal decision as a function of the number of exceedances for the same day.

Following the blue line ($TtoB$ equal to 100), one can see that the percentage to report increases until the 4 th exceedance and then it falls from 0.969 to 0.888, followed by an increase until the highest possible number of exceedances. This decrease that occurs when the EC is equal to 5 is due to the increase of 0.4 in the base multiplier (the highest possible increase, compared with 0.1 and 0.15 for additional exceedances), which translates into lower future savings and consequently lower long-term benefits. Since this possible saving in the multiplier was not achieved, the incentives to avoid another exceedance decrease and the current savings (short-term benefits, benefit today from reporting a lower VaR) seem to have a higher impact in the optimization strategy. For the curve where $TtoB$ is 170 (red line), this fall occurs in the transition from the 3rd to the 4th exceedance, which is even more surprising since the change in the multiplier occurs only in the 5th exceedance. This seems to be an effect from the balance between having current savings in the capital charge (report smaller percentages of VaR) and benefit from these savings in the future (report higher percentages to assure a small multiplier in the future).

Finally, it is important to remind that the analysis of the optimal policy function performed in this subsection was based on the scenario where the multiplier K is equal to 3. However, all the qualitative results obtained from this analysis stand for different values of K. In particular, the optimal policies for different values of K are very similar to each other. The only difference is that, for higher multipliers, the short-term benefit of underreporting the observed VaR obviously increases. Since the long-term benefit of reporting the VaR

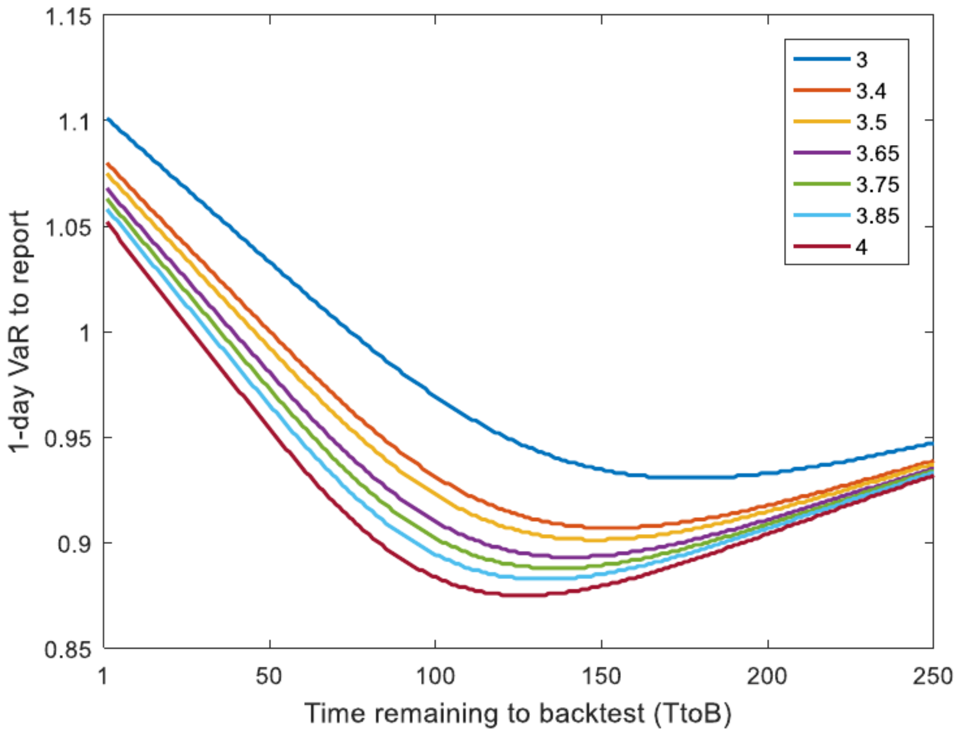

Figure 4: Optimal policy function for a number of exceedances (EC) of 4 considering different multipliers (K variable).

more conservatively is independent of the current multiplier (it is only a function of future multipliers), a higher multiplier will then increase the incentive to underreport the VaR. This can be clearly seen in Figure 4, which plots the optimal percentage of the observed VaR to report as a function of $TtoB$ for all different multipliers when EC is 4. For other values of EC, the conclusion is the same. Notice that the distance between each line is roughly proportional to the difference in the values of the corresponding multipliers, reflecting the fact that the more the multiplier increases, the more the incentive to underreport increases. In Appendix A one can find the figures for the optimal policy considering the other multipliers.

In general, these results seem to be, more or less, consistent with the principle of DYLES (McAleer et al., 2010): risk managers are conservative (report higher percentages) when the number of exceedances is high and aggressive (report lower percentages) when this number is small.

Since all the important aspects of the results were already explained and analyzed, the next step is to implement this strategy using simulated/historical data to test if it can be applied in a real market risk framework. First we will perform this evaluation withsimulated portfolio by doing a Monte Carlo Simulation, and after that we will apply the policy to a portfolio composed by the S&P500 index.

5 Monte Carlo Simulation

In this section, we perform a Monte Carlo simulation in order to obtain simulated portfolio returns and evaluate the performance of the optimal policy presented in the previous section.

To do this process, we use MATLAB$^{\text{TM}}$'s random number generator (RND) to generate returns from a normal distribution – using the inverse CDF of the normal – with an expected return of 0 and a certain standard deviation. Considering this, we will simulate the returns based on the following equation:

$$r = \Phi^{-1}\left(RND\right), \, r \sim N\left(0, \sigma\right) \tag{27}$$

The base value for the 1-day standard deviation (σ) is 1.7%, which we considered to be reasonable for a standard portfolio. To compute the VaR we use the same Parametric Normal Method that was considered when solving for the optimal strategy.

Two scenarios are going to be simulated. In the first one, labeled Normal Strategy scenario, it is always reported 100% of the estimated VaR. In the second one, labeled Optimal Strategy scenario, the percentage to report is defined according to the optimal policy. To limit the possible number of exceedances to 11 (like in the optimization model), when the EC is equal to 10 in both scenarios, it is reported 300% of the VaR (i.e. the same value defined by the optimal strategy). Another way of understanding this assumption is that, in both cases, going to the 11th exceedance means transitioning to the worst case scenario (mentioned in the previous section) which is something that the agent wants to avoid, regardless of the strategy in use. Due to this, the agent will always report a high value when the number of exceedances is equal to 10, in order to almost surely guarantee that the 11th exceedance is avoided.

It is also incorporated a mechanism for bankruptcy detection, i.e. if the loss in a certain day is higher than the daily capital charge it is assumed that the institution defaults, which brings that specific simulation to an end.

5.1 Normal Simulation

The first simulation that we will perform consists of 100,000 simulations of a period of 30 years, considering that each year has 250 business days. The value defined previously for the standard deviation will also be used to calculate the daily VaR according to equation 6. The multiplier used in the first year of each simulation is always the base value (3).

We start by analyzing the daily capital charge variable in order to verify if the optimal strategy delivers lower values than the normal strategy, and then we look closely at the other variables to confirm whether their behavior is in line with our expectations regarding the optimization model.

Figure 5 shows a histogram with the distribution of the average of the daily capital charge across simulations (i.e. in each 30-year period) for the two strategies, and two lines

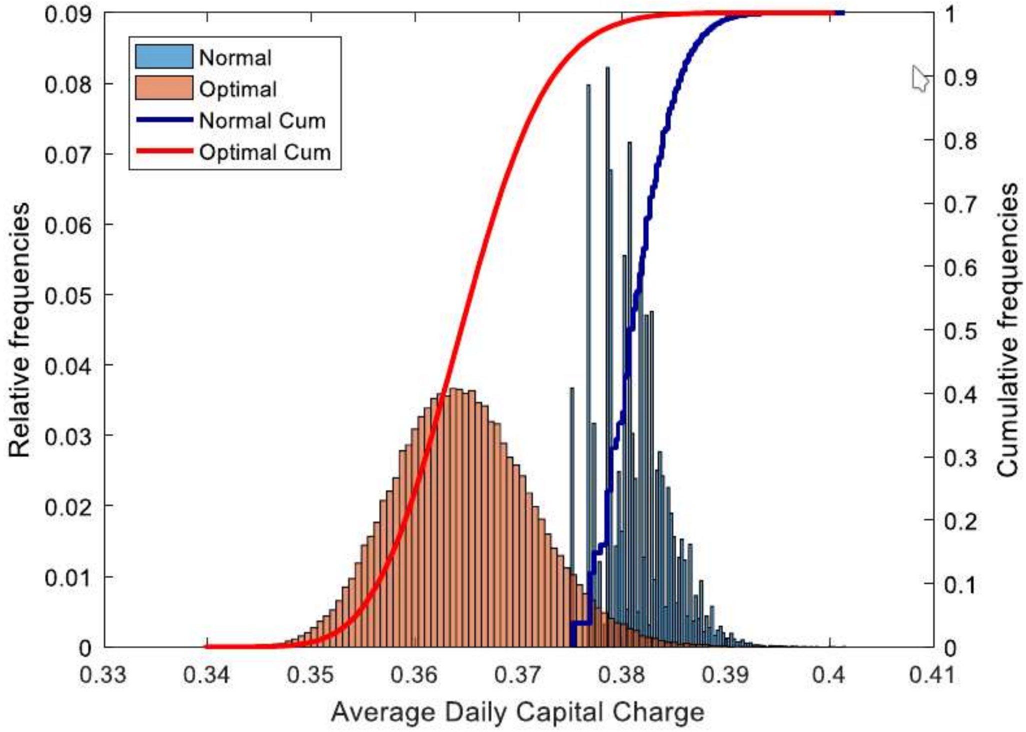

Figure 5: Distribution of the average daily capital charge across simulations, and the respective cumulative frequencies (right axis).

representing the cumulative frequencies.

From the analysis of Figure 5, one can notice that there is a clear separation between the optimal and the normal strategy. The values for the optimal strategy seem to be concentrated between 35% and 38% while, in the normal strategy, these are concentrated around 38% and 39%, clearly pointing out to a better performance of the optimal strategy. The line for the cumulative frequencies proves this, since with the optimal strategy the capital charge is below 38% in, approximately, 100% of the simulations, whereas, with the normal strategy, this only happens in, approximately, 40% of the simulations. Taking this into account, it is clear that the optimal strategy delivers lower values for the daily capital charge in almost all simulations.

Table 5 complements Figure 5 by analyzing the annual average of the daily capital charge with 5 important statistics computed in two different ways: (1) by averaging the statistics calculated in each simulation (i.e. in each 30-year period); and (2) by computing the statistic on all the data (i.e. the results of all simulations).

As expected from the analysis of Figure 5, Table 5 confirms the better performance of the optimal strategy in terms of the capital charge since, with the analysis of the mean, it is easy to see that this strategy offers a lower capital charge.

Continuing with Table 5, the average of the standard deviation for the optimal strategy is almost twice the value computed for the normal one. This higher value was expected due

	Capital Charge				
	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
Optimal Strategy					
Average of statistic	36.47%	35.45%	43.12%	32.51%	2.94%
Statistic of simulation	36.47%	35.34%	68.54%	31.20%	3.00%
Normal Strategy					
Average of statistic	38.12%	37.52%	44.03%	37.52%	1.72%
Statistic of simulation	38.12%	37.52%	70.24%	37.52%	1.82%

Table 5: Average of the statistics computed for the annual average of the daily capital charge in each simulation (30-year period) and the statistics for the global simulation.

	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
% Optimal better than Normal	77.81%	76.67%	100.00%	36.67%	8.55%

Table 6: Statistical analysis of the variable: percentage of times, in a 30-year period (each simulation), where the optimal strategy outperforms the normal strategy.

to the risk associated with the optimal strategy (i.e. the risk to report a higher value than the estimated 1-day VaR) that is translated in a higher standard deviation. This can be observed clearly in Figure 5, since the values for the optimal strategy are more dispersed. In the case of the maximum and minimum (average and global), the results confirm the outperformance of the optimal strategy as both statistics deliver lower values considering this strategy.

As the careful reader may have noticed, the value for the global maximum, in both strategies, is around 70%, which is a value that stands out from the others. The reason for such a high value is the trigger mechanism related with a number of exceedances of 10 – report 300% of the estimated 1-day VaR in both policies. From this we can conclude that, when the market conditions are adverse, the disclosure of the estimated 1- day VaR (normal strategy) does not necessarily lead to better results.

Table 6 shows the analysis of the percentage of times, in each simulation, where the optimal policy delivers lower values for the capital charge. This table shows that the optimal strategy outperforms, on average, the normal strategy in 78% of the times, which is a very good result taking into account the risks associated with this strategy.

Looking at the maximum and minimum, it can be noticed a big range between them. On one hand, we have simulations where the optimal policy is always better (100%) during the 30-year period and, on the other hand, simulations where this value goes as low as 36.67%. Once again, this behavior can be explained by the risk associated with the optimal strategy that is translated in the standard deviation of this variable (8.55%).

To increase the reliability of these results, it was created a confidence interval for the average of this percentage. Considering a confidence level of 99.99%, there is statistical evidence to believe that the true percentage of times where the optimal strategy outperforms the normal strategy is between 77.71% and 77.91%.

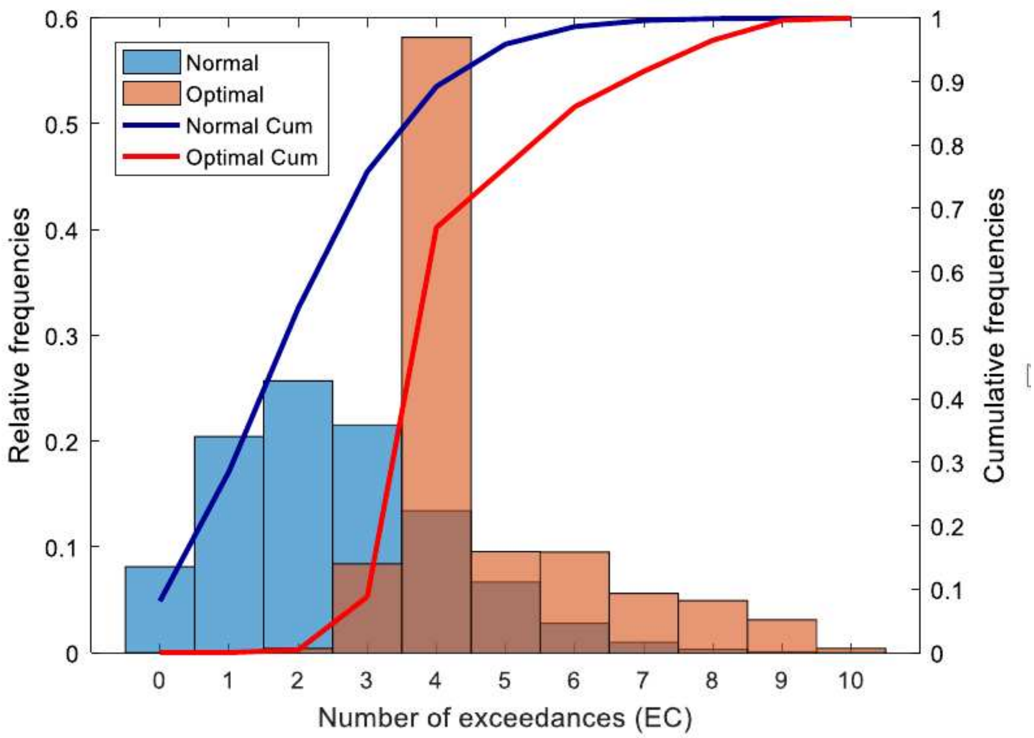

Figure 6: Distribution of the EC variable, in the end of the period, across the various simulations, and the respective cumulative frequencies (right axis).

Next, we move on to the analysis of the number of exceedances (EC) and multiplier (K) variables. In the formulation of the optimization problem, it was considered that the objective of the optimal policy would be to take advantage of the Basel rules by managing the number of exceedances, in order to maintain the base multiplier (maximum of 4 exceedances). The purpose of the next analysis is to show if this event is true when this policy is applied.

As one can see in Figure 6, the distribution of the EC variable is significantly different for each strategy. In the Normal Strategy, the variable seems to follow a lognormal distribution with a mode equal to 2, while in the Optimal Strategy the variable is clearly concentrated in the 4th exceedance – the maximum number of exceedances that does not result in the increase of the base multiplier. This proves that the optimal policy is taking advantage of the Basel rules, regarding the exceedances, because there is a big discrepancy between the numbers of years that end with 4 exceedances (58%) and also because the minimum relevant number of exceedances, in the optimal strategy, is 3 (vs 0 in the normal strategy).

Another way of seeing this is by analyzing the exceedance rate in the simulation. For the normal strategy this value is equal to the significance level used in the estimation of the 1-day VaR, i.e. 1.0%, while for the optimal strategy this rate is 1.9%, which is, as expected, higher than the significance level due to the criteria that the optimal strategy needs to follow (having 4 exceedances).

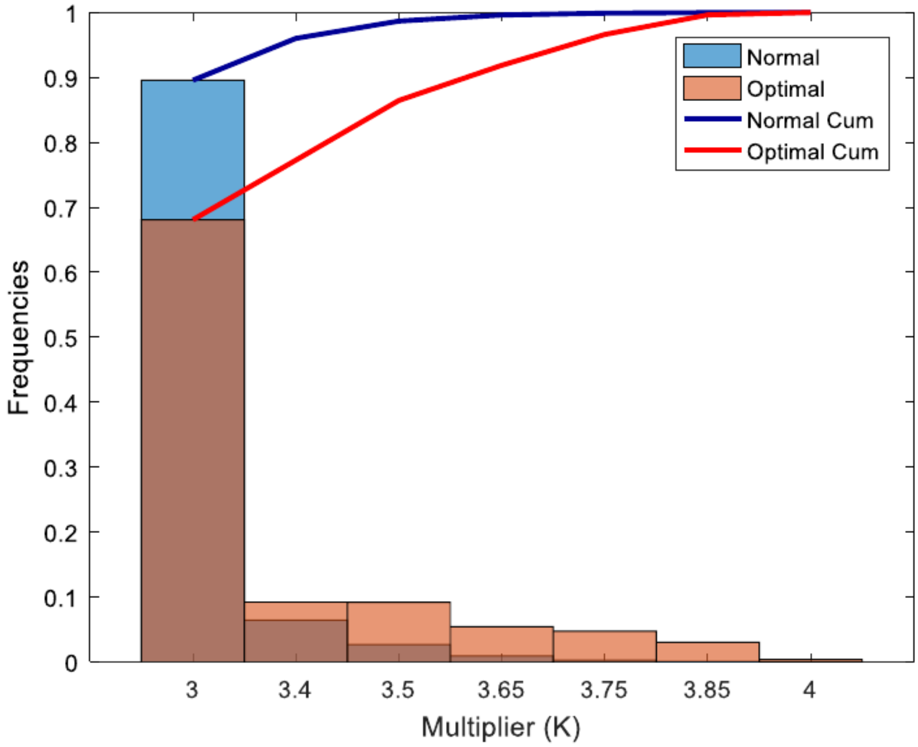

Figure 7: Distribution of the multiplier K variable across the various simulations and the respective cumulative frequencies.

An important detail that one can see in Figure 6 is that all the years that ended with an EC different than 4, in the optimal strategy, are unsuccessful tries to reach that value. Two factors justify these failed attempts: the bankruptcy risk/mechanism that prevents the agent from reporting sufficiently low values that result in an exceedance (EC smaller than 4), and the risk associated with the use of the optimal policy model (EC higher than 4). This risk is related with the disclosure of a smaller value than the estimated 1-day VaR and it can be observed in the lines for the cumulative probability because, in the optimal strategy, it is more likely to have an EC higher than 4 (33%) than in the normal strategy (11%).

In Figure 7, it is represented the relative frequencies of the multiplier variable. From its analysis, one can notice that, in both strategies, the base multiplier is used in more than 65% of the simulated years like it was expected with the analysis of the EC variable histogram. This result strengthens the hypothesis that the optimal strategy is better than the normal strategy, because it shows that the different data dispersion in the EC variable results in a similar multiplier histogram for both strategies.

Considering this, the major difference between the two strategies is that the optimal strategy optimizes the disclosed 1-day VaR in order to remain with the base multiplier (3). Sometimes this leads to higher multipliers due to the risk of the strategy (which can be observed in the lines for the cumulative frequencies since there is a higher probability associated with the higher multipliers in the case of the optimal strategy) but, overall, the

	EC				***K***			
	Mean	Mode	Median	Std. Dev.	Mean	Mode	Median	Std. Dev.
Optimal Strategy								
Average of statistic	4.74	4.00	4.05	1.46	3.18	3.00	3.01	0.28
Statistic of simulation	4.74	4.00	4.00	1.50	3.18	3.00	3.00	0.28
Normal Strategy								
Average of statistic	2.50	2.00	2.32	1.56	3.05	3.00	3.00	0.14
Statistic of simulation	2.50	2.00	2.00	1.57	3.05	3.00	3.00	0.15

Table 7: Average of the statistics computed for the EC and K variable in each simulation (30-year period) and the statistics for the global simulation.

	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
Relative saving	4.32%	4.39%	9.43%	-3.36%	1.45%
Relative annual gain	5.20%	5.25%	10.90%	-2.29%	1.51%

Table 8: Statistical analysis of the variables: average daily saving, in a 30 year period, that results from the use of the optimal strategy ("Relative saving"); and average of the annual relative return in each simulation ("Relative annual gain").

goal is successfully achieved. To complement Figures 5 and 6, Table 7 summarizes the statistical analysis of these two variables.[5]

With the analysis of Table 7 it is easy to understand that in the optimal strategy all figures point out to a number of exceedances around 4 and a multiplier around 3 (which is the base value), whereas in the normal strategy the results are around 2 exceedances and, also, a multiplier of 3. The only significant difference between the strategies is in the standard deviation that is higher in the optimal strategy for the K variable, as a result of the higher probability of having higher multipliers with this strategy, as it was mentioned previously.

Considering all the previous points, it is possible to say that, as expected, the optimal strategy outperforms the normal strategy given that the different data dispersion in the exceedances histogram (Figure 6) leads to a similar multiplier histogram (Figure 7).

Table 8 answers the question: Are the savings in the capital charge significant to compensate the risk of using a policy that, in most cases, discloses a lower 1-day VaR than the estimated one? From the analysis of Table 8, one can see that the daily average savings in the capital charge (relative saving), from the use of the optimal strategy relative to the normal strategy, is 4.32%. This may seem a small saving, however in the case of a portfolio with a high market value it will make a significant difference, taking into account that it is a daily saving (we will get back to this below).

The maximum average relative saving achieved in a simulation was 9.34% while the minimum was -3.36%, reflecting the risk of using this policy. Nonetheless, the possible positive outcomes seem to outweigh the negative outcomes. To support this conclusion, a

[5]The method used to calculate the statistics is the same as in Table 5.

statistical test for the average of the saving was performed with the following hypothesis:

H_0 :The average daily saving in a 30-year period is higher than or equal to 4%

H_1 :The average daily saving in a 30-year period is lower than 4%

The test used was the z-test because the distribution of the average saving is approximately normal (central limit theorem). Following this, there is statistical evidence that confirms that, with a significance level of 5%, the average saving that results from the use of the optimal strategy is higher than 4%.

Lastly, it is important to understand the meaning of these savings. Considering that the agent can reduce the capital charge for a certain amount invested in the asset, this extra amount can be used to increase the exposure to the asset and increase the absolute accumulated return in the end of the period.

To show this, we are going to perform a simulation where the exposure to the asset is increased or decreased in order to always have a regulatory capital charge equal to the FI's defined capital charge limit, using the following assumptions: (1) the FI's capital charge limit is 100,000 monetary units (m.u.); (2) the price of the asset is fixed at 1 m.u./unit of the asset; (3) the annual return on the asset is fixed at 6%. The objective is to compute the accumulated return in each simulation and compare the performance of both strategies.

The result of this simulation and the associated statistics computed over the relative annual gain that an agent has with the use of the optimal strategy can be found in Table 8. By using this strategy, in this scenario, the agent can increase the average of the annual return up to 10.90% or reduce the gain by a maximum of 2.29%, when compared to the normal strategy. Looking at the mean, one can conclude that the annual gain is, on average, 5.20% in each simulation (30-year period), which is a considerably high reward taking into account that the return on the asset is 6%/year.

In a nutshell, all the results indicate that the behavior of the optimal strategy is in line with our expectations, namely: (1) it is clearly taking advantage of the Basel rules regarding the exceedances (EC and K variable analysis); (2) it has a better performance than the normal strategy in most of the times (mean of 77.81%); and (3) the relative saving from using the optimal strategy is, on average, 4.32%. These conclusions support our optimization model, meaning that reporting the 1-day VaR according to the optimal policy leads to positive outcomes when compared to reporting the estimated value.

5.2 Simulation with undervaluation of the volatility

Next it is important to understand the impact of using a poor VaR estimation model with the optimal strategy. As it was mentioned in Subsection 2.2 (VaR), the method used to estimate the volatility is one of the main drivers for a good VaR model due to its direct relation with the formula to compute the VaR (see equation 6).

If the estimated value for the volatility is undervalued, the forecast for the VaR will also be undervalued, resulting in a higher exceedance rate and, possibly, in the increase of the

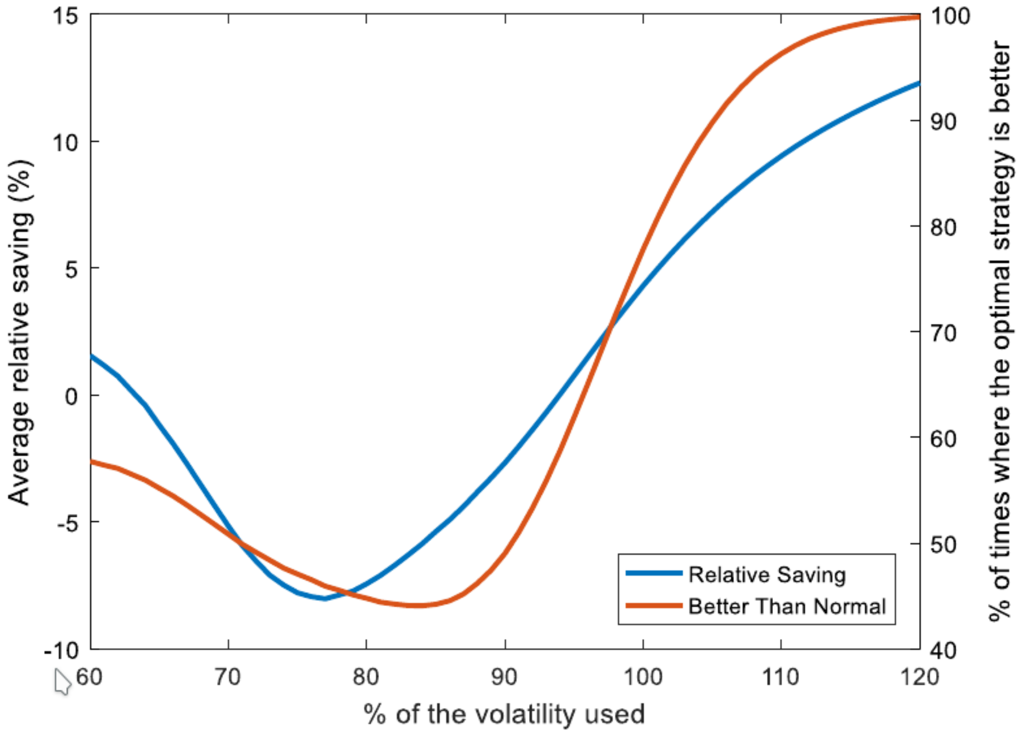

Figure 8: Average daily relative saving and percentage of times where the optimal strategy is better than the normal strategy considering different levels of under/overvaluation.

probability of ending a certain period in the critical scenario (having 11 exceedances). In addition, one of the assumptions of the optimization model is that the volatility is always estimated with precision, which is difficult to hold.

Due to these aspects it is necessary to test the impact of an undervaluation or overvaluation of the volatility in the results of the simulation, particularly in the capital charge. Since, in the simulation, we define the real value for the volatility of the return distribution, in the next simulation we will compute the value for the 1-day VaR considering different levels of under/overvaluation of the volatility used to generate the returns (1.7%). This level ranges from 60% to 120% of the base volatility, in steps of 1%. To evaluate the impact in the optimal strategy we created Figure 8 that indicates the relative saving (left axis) and the percentage of times where the optimal strategy outperforms the normal strategy (right axis) for each level of under and over-valuation.

Starting by analyzing the levels of overvaluation, as one can see in Figure 8, the performance of the optimal strategy increases as the level of overvaluation increases, reaching 100% of times where the optimal strategy is better than the normal strategy using 120% of the volatility. This is easy to understand because an overvaluation of the volatility means that the estimate for the 1-day VaR will be higher than necessary resulting in a lower likelihood of having an exceedance. Due to this, there is a higher safety margin to underreport the 1-day VaR without having an exceedance.

On the opposite case, an undervaluation of the volatility leads to a worst performance of the optimal strategy, however this only happens until a certain point. Before digging into this fact, we are going to analyze the downward part of the series.

Looking at the blue line, one can see that the average of the relative saving (i.e. the average saving in the simulation that results from the use of the optimal strategy compared with the normal strategy) decreases until -8%, which represents a level of 77% of the volatility. The explanation for this is exactly the opposite of the previous one used to explain the overvaluation: using a value for the volatility lower than the real one leads to a poor estimate for the 1-day VaR, i.e. a value lower than necessary, which results in a higher probability of having a negative return higher than the disclosed 1-day VaR (i.e. leading to an exceedance). Since the optimal strategy normally consists in disclosing a value for the 1- day VaR lower than the estimated one, in the end, the probability of having an exceedance would be even higher and the exceedances would occur too quickly without giving the model the necessary time to adjust to this scenario.

Nonetheless, Figure 8 shows that for small undervaluation levels, i.e. up to 6%, the optimal strategy continues to be the best one. At this level, the results are a slight relative saving of 0.14% and a better performance of the optimal strategy in 59% of the times. In the next subsection we will explore all the results associated with this level.

Despite of this, one can see that the two lines do not have a decreasing trend, as it would be expected. Instead, the trend of the lines changes when the level of undervaluation is 77% (blue line) and 84% (red line) of the volatility, with the relative saving reaching positive values again when this level is at 63% (relative saving of 0.12%). A possible explanation for this behavior is the ability of the optimal strategy to adapt to different scenarios (i.e. different values for the number of exceedances result in different percentages of the estimated 1-day VaR to disclose) while the normal strategy is always static.

In other words, since the normal strategy always reports the estimated VaR and a higher level of undervaluation leads to a lower estimated 1-day VaR, compared with the required value, from a certain level of undervaluation the exceedances start to occur quickly, resulting in a predominance of the critical scenario (having 11 exceedances) and the need to report substantially high values for the 1-day VaR (due to the trigger mechanism). In the case of the optimal strategy, its dynamic feature delays this scenario, for a little amount of time, which allows savings in the capital charge, relative to the normal strategy.

In short, Figure 8 demonstrates that: (1) a small level of undervaluation still results in a good performance of the optimal strategy (6%); (2) there is a positive correlation between the overvaluation of the volatility and the performance of the optimal strategy; and (3) there is a limit to the underperformance of the optimal strategy.

In the next 2 subsections we are going into more detail in two levels of undervaluation: 94% of the volatility (undervaluation of 6%) because it is the level where the optimal strategy continues to outperform the normal strategy; and 80% of the volatility (20% undervaluation), in order to do a statistical analysis of a catastrophic situation.

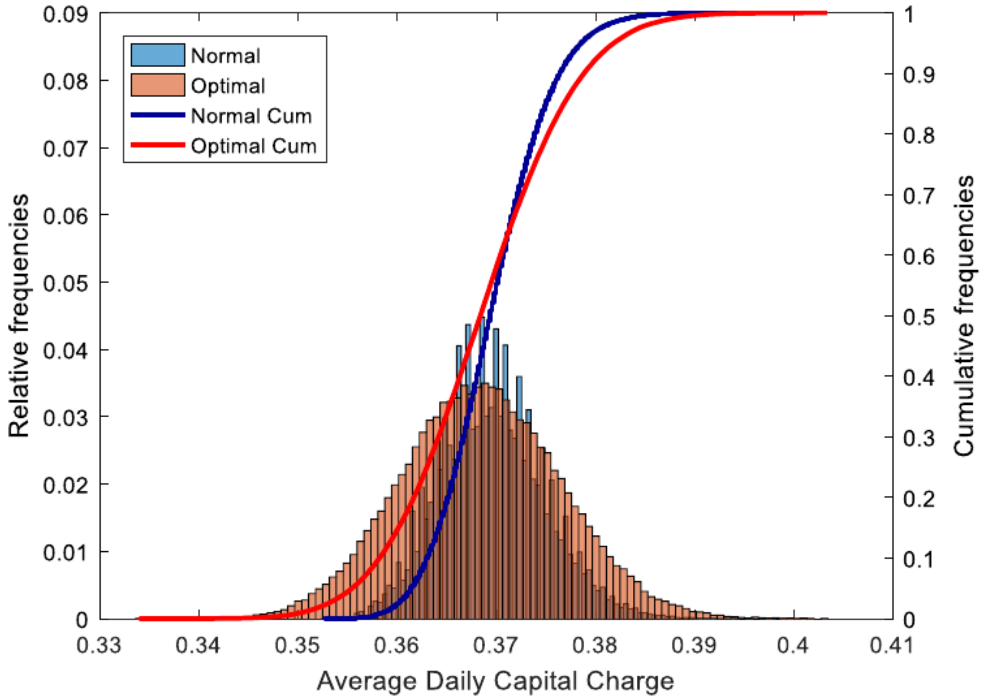

Figure 9: Distribution of the average daily capital charge across simulations, and the respective cumulative frequencies (right axis), for a 6% undervaluation level.

5.2.1 Undervaluation of 6%

In Figure 9 one can observe a completely different scenario compared with Figure 5. In the previous simulation the histogram of the capital charge showed a clear distinction between both policies, whereas in this simulation the histogram tells a different story, since the capital charge in both policies is more or less equivalent – the bars overlap. The most relevant differences between Figure 9 and Figure 5 are the higher dispersion and amplitude of the values for the normal strategy, which is a result of the undervaluation.

Nonetheless, when analyzing the cumulative frequencies, one can see that the optimal strategy offers a lower capital charge for more than 50% of the times. This can also be observed on Tables 9 and 10, since the average of the capital charge with the optimal strategy is lower than the same value for the normal strategy (36.85% vs 36.96%), and in 58.89% of the times the optimal strategy has a better performance. These points represent the main reasons that explain the better performance, on average, of the optimal strategy.

	Capital Charge				
	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
Optimal Strategy					
Average of statistic	36.85%	37.10%	43.47%	31.54%	3.47%
Statistic of simulation	36.85%	37.31%	93.50%	29.33%	3.53%
Normal Strategy					
Average of statistic	36.96%	35.30%	43.97%	35.27%	2.79%
Statistic of simulation	36.96%	35.27%	34.90%	35.27%	2.83%

Table 9: Average of the statistics computed for the annual average of the daily capital charge in each simulation (30-year period) and the statistics for the global simulation, for a 6% undervaluation level.

	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
% Optimal better than Normal	58.89%	60.00%	96.67%	20.00%	9.87%

Table 10: Statistical analysis of the variable: percentage of times, in a 30-year period (each simulation), where the optimal strategy outperforms the normal strategy, for a 6% undervaluation level.

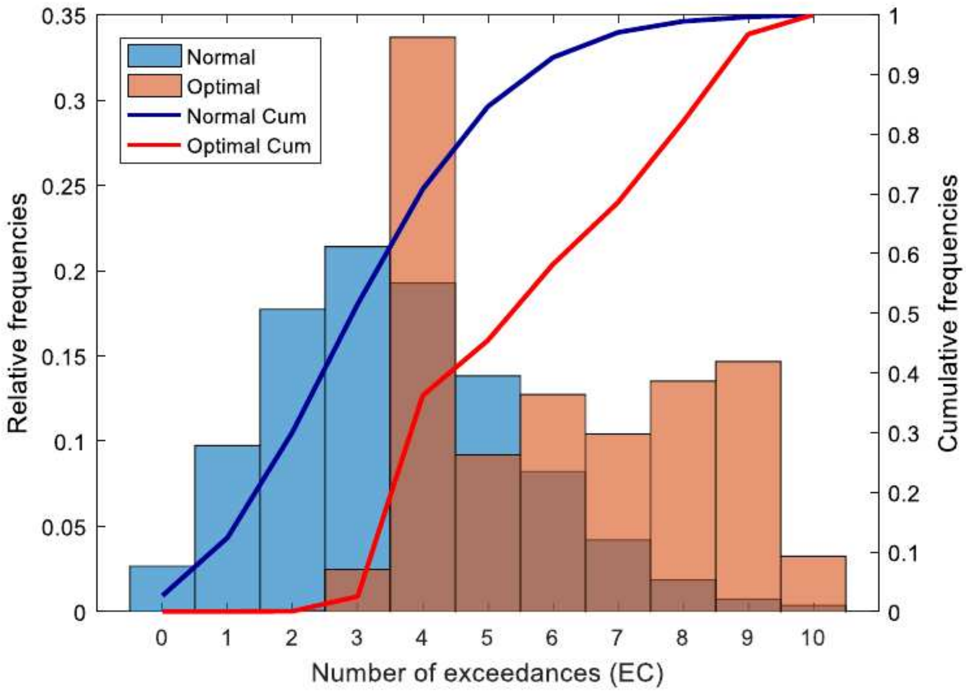

Figure 10: Distribution of the *EC* variable, in the end of the period, across the various simulations, and the respective cumulative frequencies (right axis), for a 6% undervaluation level.

In the case of the EC variable (Figure 10), in the optimal strategy there is still predominance of the value 4 and, in the normal strategy, the distribution continues to be similar to a lognormal distribution but now with a mode of 3 (in the normal simulation the mode

	EC				***K***			
	Mean	Mode	Median	Std. Dev.	Mean	Mode	Median	Std. Dev.
Optimal Strategy								
Average of statistic	6.10	4.50	5.83	2.01	3.41	3.05	3.44	0.35
Statistic of simulation	6.10	4.00	6.00	2.04	3.41	3.00	3.50	0.35
Normal Strategy								
Average of statistic	3.59	3.07	3.43	1.86	3.14	3.00	3.00	0.24
Statistic of simulation	3.59	3.00	3.00	1.88	3.14	3.00	3.00	0.24

Table 11: Average of the statistics computed for the EC and K variable in each simulation (30-year period) and the statistics for the global simulation, for a 6% undervaluation level.

	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
Relative saving	0.14%	0.18%	6.56%	-8.15%	1.72%
Relative annual gain	1.00%	1.02%	7.38%	-5.77%	1.65%

Table 12: Statistical analysis of the variable: average daily saving, in a 30 year period, that results from the use of the optimal strategy, for a 6% undervaluation level.

was 2).

The major difference between this simulation and the normal simulation is in the probabilities of each EC value for the optimal strategy. Compared with the normal simulation results, the probability of having an EC equal to 4 decreased to almost half (34% vs 58%) while the probability of having an EC higher than 4 increased from 33% to 62%. This means that the unsuccessful attempts to reach an EC equal to 4 increased, essentially due to the constant failure in estimating the real value of the volatility that is inherent to this simulation.

Looking at Table 11, and doing a comparison with Table 7, one can reach the same conclusion since the average of the EC variable increases from 4.74 to 6.10 and the median, considering all the simulations, increases from 4 to 6.

Figure 11 leads to the same conclusion as the probability of having the base multiplier, in the optimal policy, decreases from 68% to 38%, when compared with the normal simulation, and the probability of having the higher multipliers (3.75, 3.85 and 4) rises to 31% (vs 9% in the normal simulation). In the normal policy there is also a decrease in the probability of having the base multiplier (71% vs 89%), however it is not as significant as in the optimal strategy.

Even with these differences, we can see in Table 12 that the average relative saving that results from the use of the optimal strategy continues to be a positive number (0.14%), meaning that this strategy continues to have a slightly better performance even in a scenario of undervaluation. However, one can see that this saving results in a relatively small gain (1% return) when compared with the possible gains in the scenarios where the volatility has an inexistent undervaluation level (Table 8).

It is important to understand that the results of this simulation should be seen as a

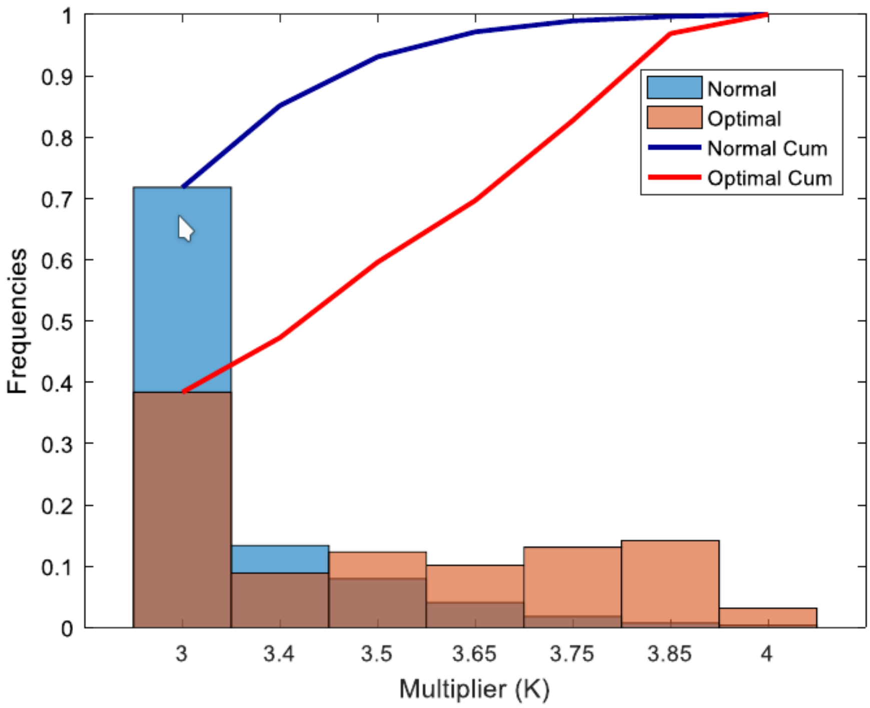

Figure 11: Distribution of the multiplier K variable across the various simulations and the respective cumulative frequencies, for a 6% undervaluation level.

pessimistic scenario because what normally happens in the estimation of the volatility is that sometimes the value is underestimated and other times it is overestimated.

In short, this simulation highlights one of the main drawbacks of the optimal strategy and demonstrates its effect in the capital charge and in the average relative saving. Two main conclusions are derived from these results: first, it is crucial to have a good estimation model to forecast the volatility with precision, as this will be critical to define the success of the optimal strategy; and second, the optimal strategy continues to have positive effects even when the volatility has a slight undervaluation (more or less up to 6%).

5.2.2 Undervaluation of 20%

Like it was expected, all figures point out to a worst performance of the optimal strategy.

Starting by analyzing the capital charge, in Figure 12 one can see that the normal strategy seems to offer, on average, lower values than the optimal strategy. Moreover, the line for the normal strategy cumulative frequencies (blue line) is always one step behind the red line (optimal strategy cumulative frequencies), meaning that, in every point, it is more likely to have a lower capital charge when using the normal strategy.

An interesting comparison between the results of this simulation and the normal simulation is that in Figure 12 the normal strategy has values as low as 35% while in the normal simulation (Figure 5) this minimum value was around 37.5%. On the other hand, the maximum value in the case of this simulation increased compared with the normal

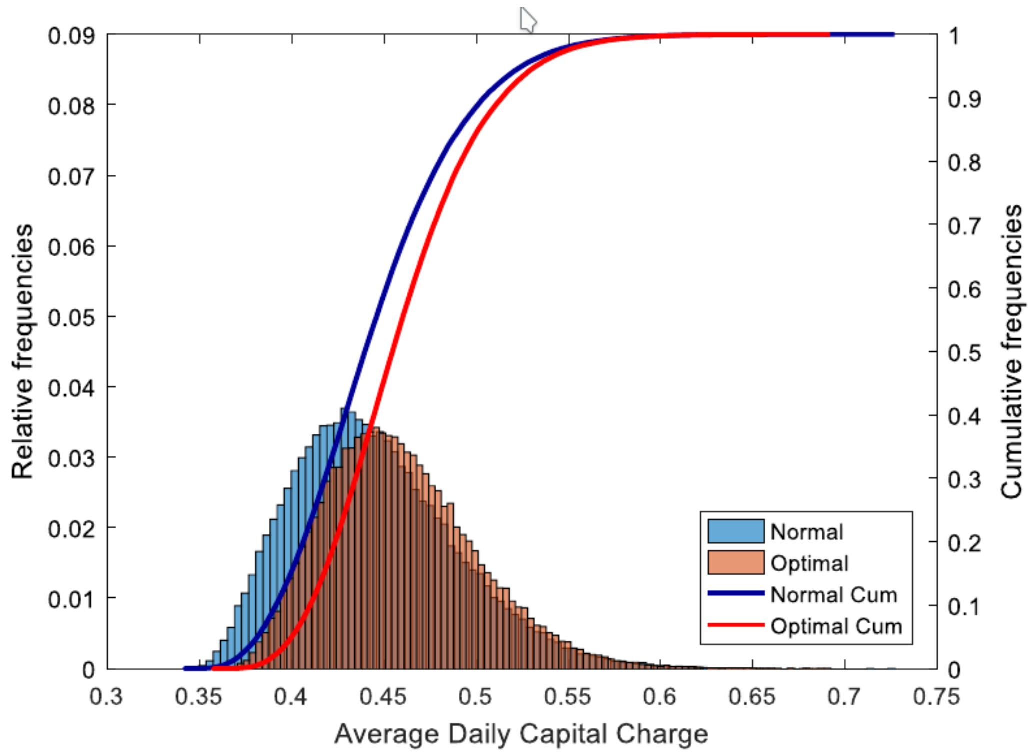

Figure 12: Distribution of the average daily capital charge across simulations, and the respective cumulative frequencies (right axis), for a 20% undervaluation level.

simulation (39.5% vs 62.5%). This is due to the constant undervaluation in the estimation of the 1-day VaR.

Like in the previous simulation (6% undervaluation), the capital charge for the normal strategy seems to follow more closely a normal distribution. From this we may conclude that as the level of undervaluation increases, the distribution of the normal strategy appears to be closer to the normal distribution.

Looking at Table 13, one can see that the average of the daily capital charge is in line with the histogram (Figure 12) because the normal strategy offers, on average, a lower value, meaning that it has a better performance when compared with the optimal strategy. This worst performance is confirmed in Table 14, as the normal strategy outperforms the optimal strategy in 55.73% of the times.

	Capital Charge				
	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
Optimal Strategy					
Average of statistic	45.89%	38.68%	122.75%	29.85%	20.30%
Statistic of simulation	45.89%	38.55%	289.40%	25.25%	21.78%
Normal Strategy					
Average of statistic	44.46%	38.15%	126.31%	30.09%	21.11%
Statistic of simulation	44.46%	37.74%	289.98%	30.01%	22.85%

Table 13: Average of the statistics computed for the annual average of the daily capital charge in each simulation (30-year period) and the statistics for the global simulation, for a 20% undervaluation level.

	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
% Optimal better than Normal	44.27%	43.33%	83.33%	10.00%	8.70%

Table 14: Statistical analysis of the variable: percentage of times, in a 30-year period (each simulation), where the optimal strategy outperforms the normal strategy, for a 20% undervaluation level.

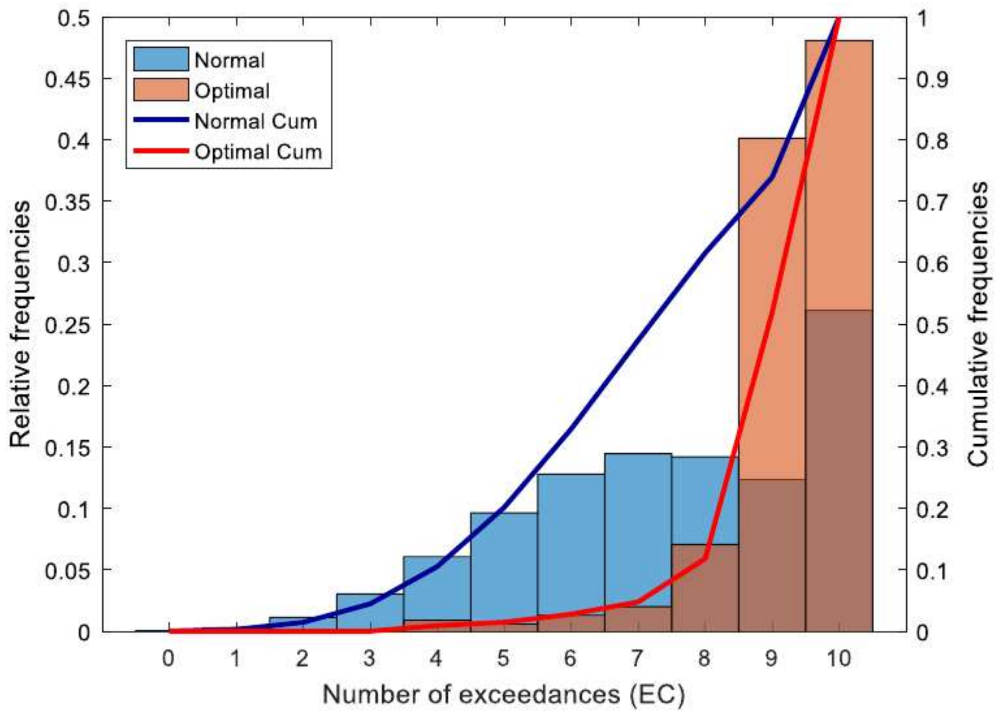

Figure 13: Distribution of the *EC* variable, in the end of the period, across the various simulations, and the respective cumulative frequencies (right axis), for a 20% undervaluation level.

Figure 13 shows that the objective of the optimal strategy is not being achieved, since the number of exceedances with the highest frequencies, in the end of each period, are 9

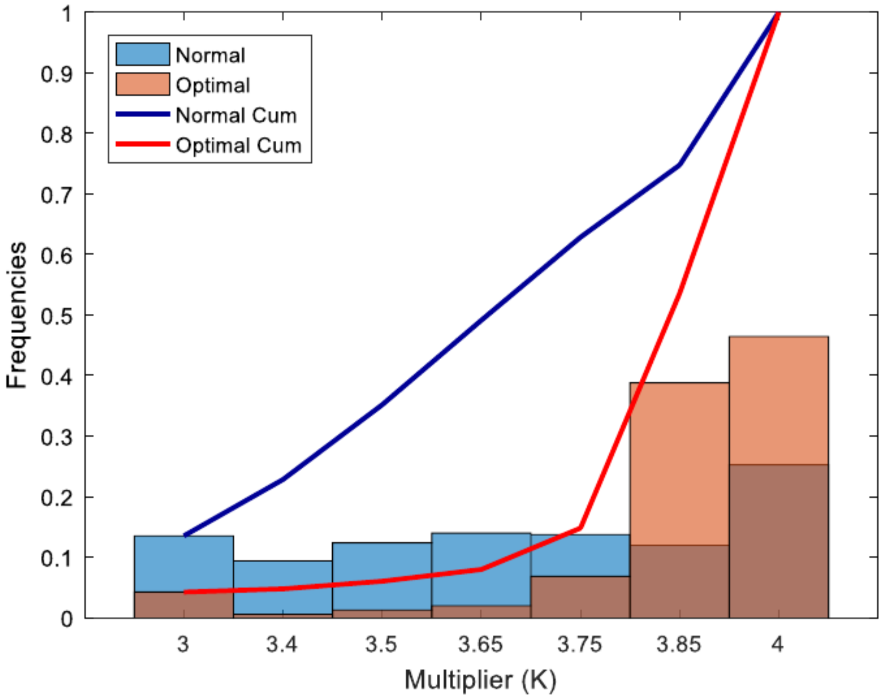

Figure 14: Distribution of the multiplier K variable across the various simulations and the respective cumulative frequencies, for a 20% undervaluation level.

and 10, representing almost 90% of the times. The explanation for this is simple: since a constant large undervaluation of the volatility (20%) leads to an undervaluation of the estimated 1- day VaR, the optimal strategy will be constantly failing the objective of ending each period with 4 exceedances and it will not be able to adapt to this scenario (by reporting higher values than the estimated one) in order to avoid more exceedances. For the normal strategy the scenario is quite similar, however in a smaller scale because it is always reported the estimated 1-day VaR (i.e. there is no underreporting).

Moreover, Figure 14 illustrates the real impact of having high values for the EC variable, as the multipliers for the optimal strategy are, in most of the times, the two highest ones: 3.85 and 4. The same happens to the normal strategy but the impact is a little bit less significant as it can be seen on Table 15 (average of 3.65 vs 3.87 in the optimal strategy).

This constant failure in reaching the 4th exceedance, and assuring the base multiplier, is the main reason for the failure of the optimal strategy in this undervaluation scenario.

All of this bad performance ends in a negative average relative saving of 11.90% in the capital charge and an effective average loss (relative annual gain) of 1.19% if the optimal strategy is used (Table 16).

In a nutshell, this simulation demonstrated the negative impact that a high undervaluation of the volatility has in the capital charge and in the effectiveness of the optimal strategy and, once again, strengthens the importance of having a good model to estimate the volatility.

	EC				***K***			
	Mean	Mode	Median	Std. Dev.	Mean	Mode	Median	Std. Dev.
Optimal Strategy								
Average of statistic	9.26	9.64	9.42	0.95	3.87	3.95	3.90	0.21
Statistic of simulation	9.26	10.00	9.00	1.00	3.87	4.00	3.85	0.21
Normal Strategy								
Average of statistic	7.47	8.91	7.69	2.16	3.65	3.80	3.71	0.32
Statistic of simulation	7.47	10.00	8.00	2.17	3.65	4.00	3.75	0.32

Table 15: Average of the statistics computed for the EC and K variable in each simulation (30-year period) and the statistics for the global simulation, for a 20% undervaluation level.

	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
Relative saving	–11.90%	-10.97%	6.97%	-57.53%	6.85%
Relative annual gain	-1.19%	-1.28%	11.04%	-8.61%	2.07%

Table 16: Statistical analysis of the variable: average daily saving, in a 30 year period, that results from the use of the optimal strategy, for a 20% undervaluation level.

5.3 Optimal Model Formulation Risk

In Subsection 2.1 we introduced the equation that we would use to compute the GRC (which is equal to the MRC assuming that the SRC is equal to 0). Equation 28 is a reproduction of that equation (equation 2).

$$GRC = \max\left(k \times \frac{1}{60}\sum_{i=0}^{59} VaR_{1,1\%,t-i} \times \sqrt{10}, VaR_{1,1\%,t} \times \sqrt{10}\right) \tag{28}$$

However, in Subsection 4.1, where the optimization problem was created, we simplified the problem by eliminating the maximization and the average of the last sixty 1-day VaRs, and came up with equation 29 (which is a reproduction of equation 18).

$$f\left(x, TtoB, EC, K\right) = -x \times K \times VaR_{1,1\%} \times \sqrt{10} \tag{29}$$

Regarding this simplification, in Subsection 4.2 we demonstrated that we were still able to achieve a quite good approximation of the real optimal policy.

In this subsection we are going to test how good this approximation is by performing a final simulation, referred from now on as the simplified model simulation, where we use equation 29 to compute the daily capital charge, i.e. we will ignore the average of the last sixty 1-day VaR in the definition of the daily capital charge.

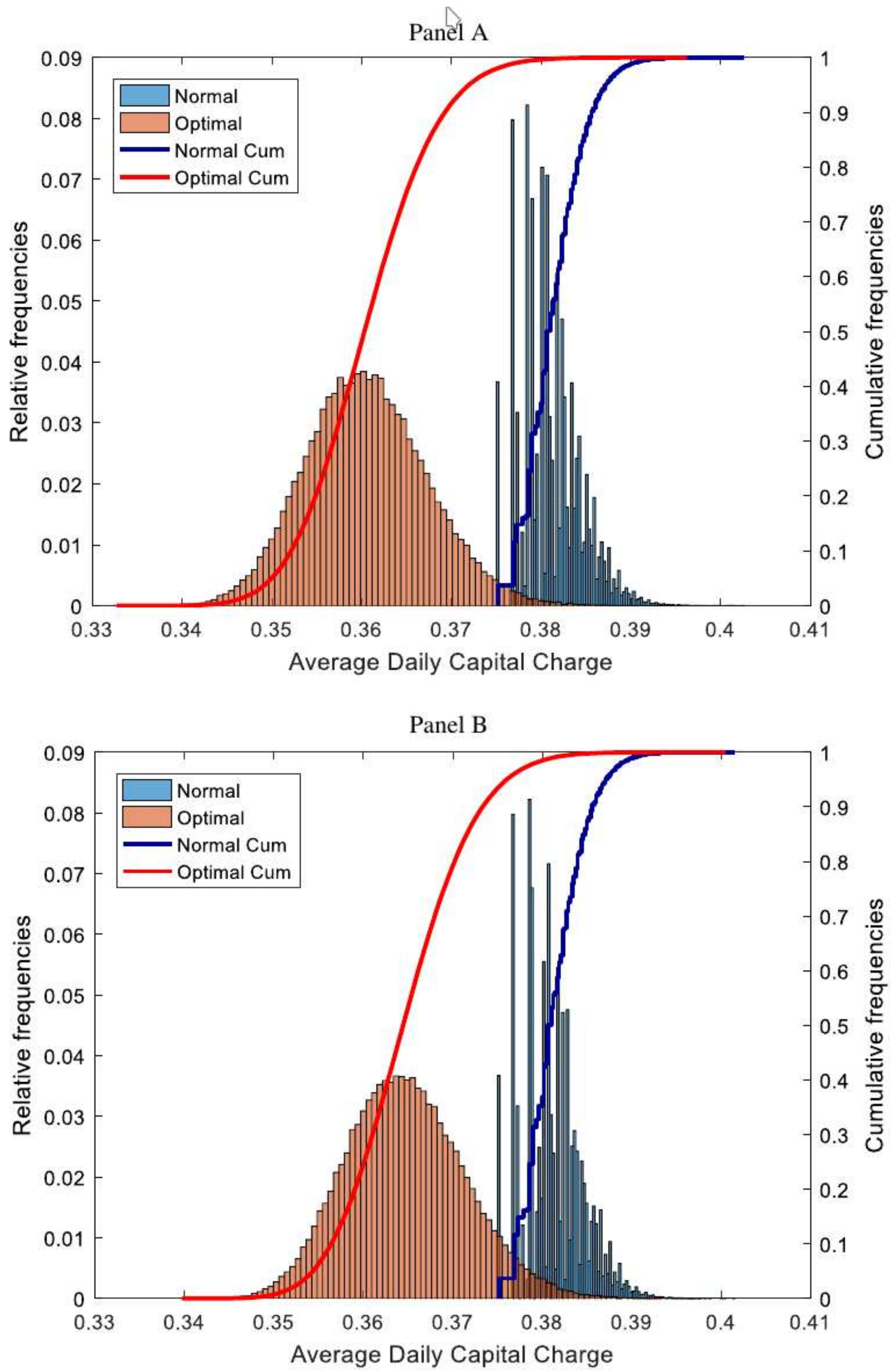

Figure 15: Distribution of the average daily capital charge in each simulation and the respective cumulative frequencies (right axis), for the simplified model simulation (Panel A) and the normal simulation (Panel B).

In Figure 15 one can see two histograms with the average of the daily capital charge

	Capital Charge				
	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
Optimal Strategy					
Average of statistic	36.05%	35.32%	43.00%	31.23%	3.05%
Statistic of simulation	36.05%	35.24%	78.87%	28.72%	3.11%
Normal Strategy					
Average of statistic	38.12%	37.52%	44.04%	37.52%	1.72%
Statistic of simulation	38.12%	37.52%	80.28%	37.52%	1.82%

Table 17: Average of the statistics computed for the annual average of the daily capital charge in each simulation (30-year period) and the statistics for the global simulation, considering the simplified model simulation.

	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
% Optimal better than Normal	80.00%	80.00%	100.00%	36.67%	8.24%

Table 18: Statistical analysis of the variable: percentage of times, in a 30-year period (each simulation), where the optimal strategy outperforms the normal strategy, considering the simplified model simulation.

for this simulation (panel A) and the normal simulation (panel B), which is a reproduction of Figure 5. Analyzing both histograms, one can see that there does not seem to be a significant difference in the normal strategy. However the histogram for the optimal strategy in panel A suffered a slight move to the left, compared with panel B, which means that the spread in the daily capital charge between the two strategies increased in the simplified model simulation.

Table 17 confirms the previous conclusion because, when compared with Table 5 (normal simulation) we can see that the statistics for the normal strategy are almost equal and the statistics for the optimal strategy are slightly smaller in this simulation (e.g. average of 36.05% vs 36.47% in the normal simulation), which explains the increase in the spread between the two strategies. This increase is also visible in Table 18, which shows a slightly higher percentage of times where the optimal strategy is better, when compared with Table 6 (80% vs 77.81% in the normal simulation).

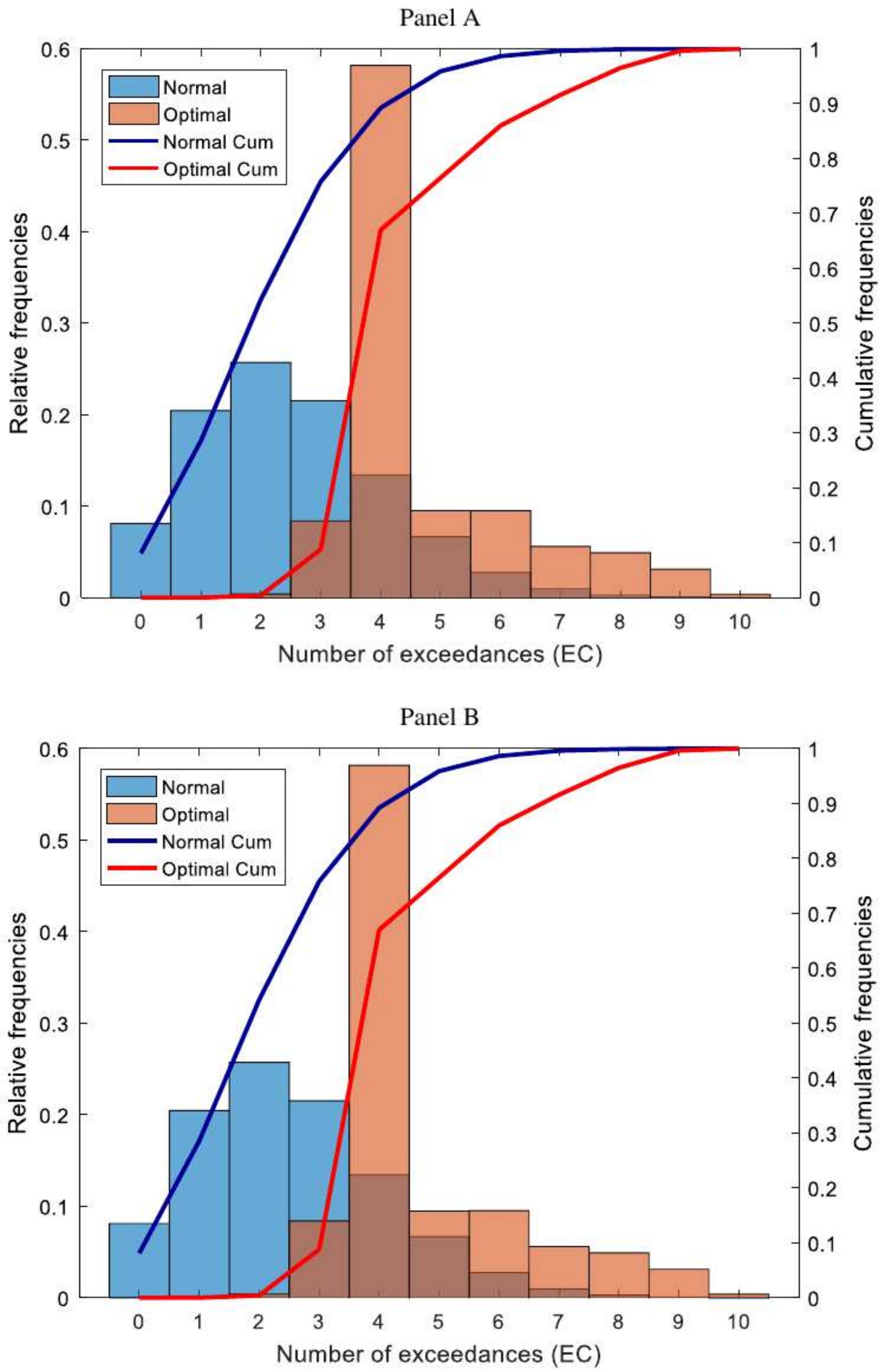

Figure 16: Distribution of the EC variable, in the end of the period, across the various simulations, and the respective cumulative frequencies (right axis), for the simplified model simulation (Panel A) and the normal simulation (Panel B).

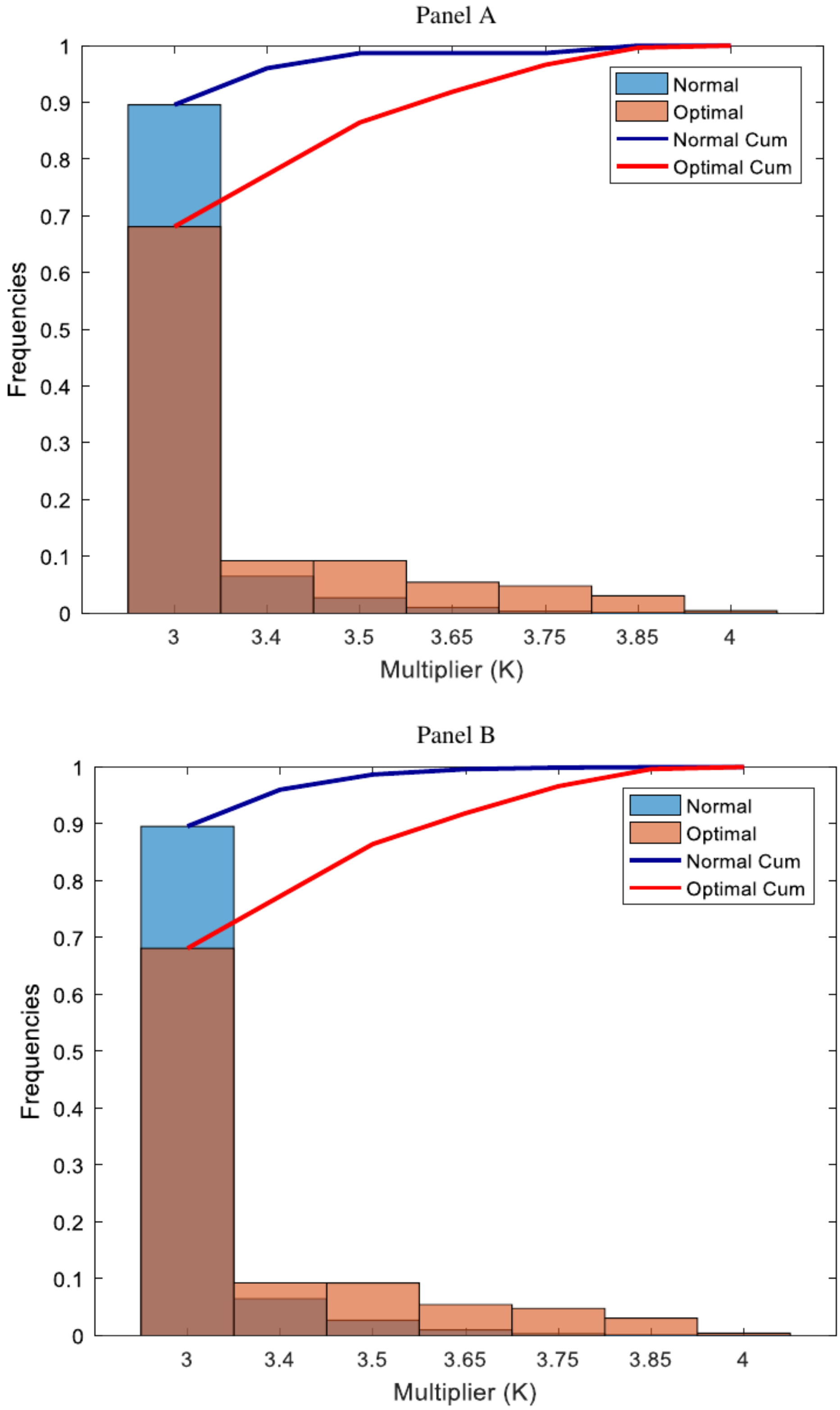

Figure 17: Distribution of the multiplier K variable across the various simulations and the respective cumulative frequencies, for the simplified model simulation (Panel A) and the normal simulation (Panel B).

Figure 16 compares the histogram of the EC variable in the end of each period for

	EC				***K***			
	Mean	Mode	Median	Std. Dev.	Mean	Mode	Median	Std. Dev.
Optimal Strategy								
Average of statistic	4.74	4.00	4.05	1.46	3.18	3.00	3.01	0.28
Statistic of simulation	4.74	4.00	4.00	1.50	3.18	3.00	3.00	0.28
Normal Strategy								
Average of statistic	2.50	2.01	2.32	1.56	3.05	3.00	3.00	0.14
Statistic of simulation	2.50	2.00	2.00	1.57	3.05	3.00	3.00	0.15

Table 19: Average of the statistics computed for the EC and K variable in each simulation (30-year period), considering the simplified model simulation.

	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
Relative saving	5.41%	5.46%	11.29%	-2.26%	1.48%
Relative annual gain	6.98%	7.00%	14.51%	-1.13%	1.69%

Table 20: Statistical analysis of the variables: average daily saving, in a 30 year period, that results from the use of the optimal strategy ("Relative saving"); and average of the annual relative return in each simulation ("Relative annual gain", considering the simplified model simulation.

the normal simulation (Panel B) and this simulation (Panel A). At a naked eye, there is no difference between the two histograms, which means that the two simulations lead to the same results in terms of number of exceedances. The same happens in Figure 17 (histograms for the multiplier variable), as the relative and cumulative frequencies are very similar in the two panels.

In order to do a more detailed analysis, we can compare the values for the statistics, computed over the two variables (EC and K), that are in Table 19, with the ones in Table 7 (normal simulation). From this analysis one can see that both tables have the exact same values meaning that both simulations offer the same performance in terms of these variables.

The higher spread in the capital charge of both strategies that we mentioned earlier is now reflected in a higher average relative saving of 5.41% presented in Table 20, compared with 4.32% (Table 8). This superior saving explains the higher relative annual gain presented also in Table 20, compared with the one obtained in the normal simulation (Table 8).

In brief, these results point out to a slightly better performance of the optimal strategy in the case where the sixty days average is ignored in the equation for the capital charge, which ends in an approximately 25% higher average daily relative saving. This means that our optimization model is a good approximation for the final solution, however there is some room to improve this model in order to reflect the real equation and have slightly better results.

	Capital Charge				
	Mean	**Median**	**Max.**	**Min.**	**Std. Dev.**
Optimal Strategy	24.99%	20.59%	136.16%	9.42%	14.93%
Normal Strategy	26.61%	21.74%	133.77%	10.81%	15.31%
Relative Strategy	7.22%	6.90%	57.13%	-19.15%	8.86%

Table 21: Statistics computed over the daily capital charge, for the portfolio simulation.

6 Portfolio Simulation

Until now we have tested the performance of the optimal strategy in a simplified and controlled world where returns follow a normal distribution with a pre-determined mean and volatility. From this analysis we concluded that reporting the 1-day VaR according to the optimal policy can provide savings in the capital charge when compared with a policy of reporting the estimated 1-day VaR. We also demonstrated that these savings lead to a relative annual gain when considering different levels of exposure to the asset in order to achieve always the same value for the market risk charge. However, in a real world, these assumptions do not hold, meaning that this strategy may have a different performance.

In this section we are going to test the performance of the optimal policy when applied to a diversified portfolio. This portfolio will be represented by the S&P 500 index.

To perform a more relevant simulation, we will use the Historical VaR model, which is the most worldwide used method. This model consists in using the empirical return distribution to compute the VaR instead of imposing a parametric distribution to the data. However, instead of using the plain-vanilla method that would only reflect the previous years' market conditions, we will follow Hull and White (1998) and do a refinement that consists in adjusting all the returns in order to reflect the current market conditions (i.e. the future volatility). This refinement is known as volatility adjustment.

To do this adjustment we will use equation 35. What this equation does is to compute the adjusted return by considering the estimate for the future 1-day volatility ($\hat{\sigma}_T$).

$$\hat{r}_t = \frac{\hat{\sigma}_T}{\hat{\sigma}_t} \times r_t \tag{30}$$

To estimate the volatility, we will use the EWMA method (explained in Subsection 2.2), with a parameter lambda of 0.94, since we are considering daily data.

The capital charge will be computed according to equation 2 (using the same methodology as previously). We will use the data from 22-04-1986 to 20-01-2016, representing 30 periods of 250 days (the same number of periods that were simulated in the previous section).

We will start this analysis by observing the time series for the daily capital charge. In Panel A of Figure 18 one can observe that the optimal strategy (red line) is generally below the line for the normal strategy (blue line). Panel B of the same figure confirms this since the relative saving of using the optimal strategy is almost always above 0.

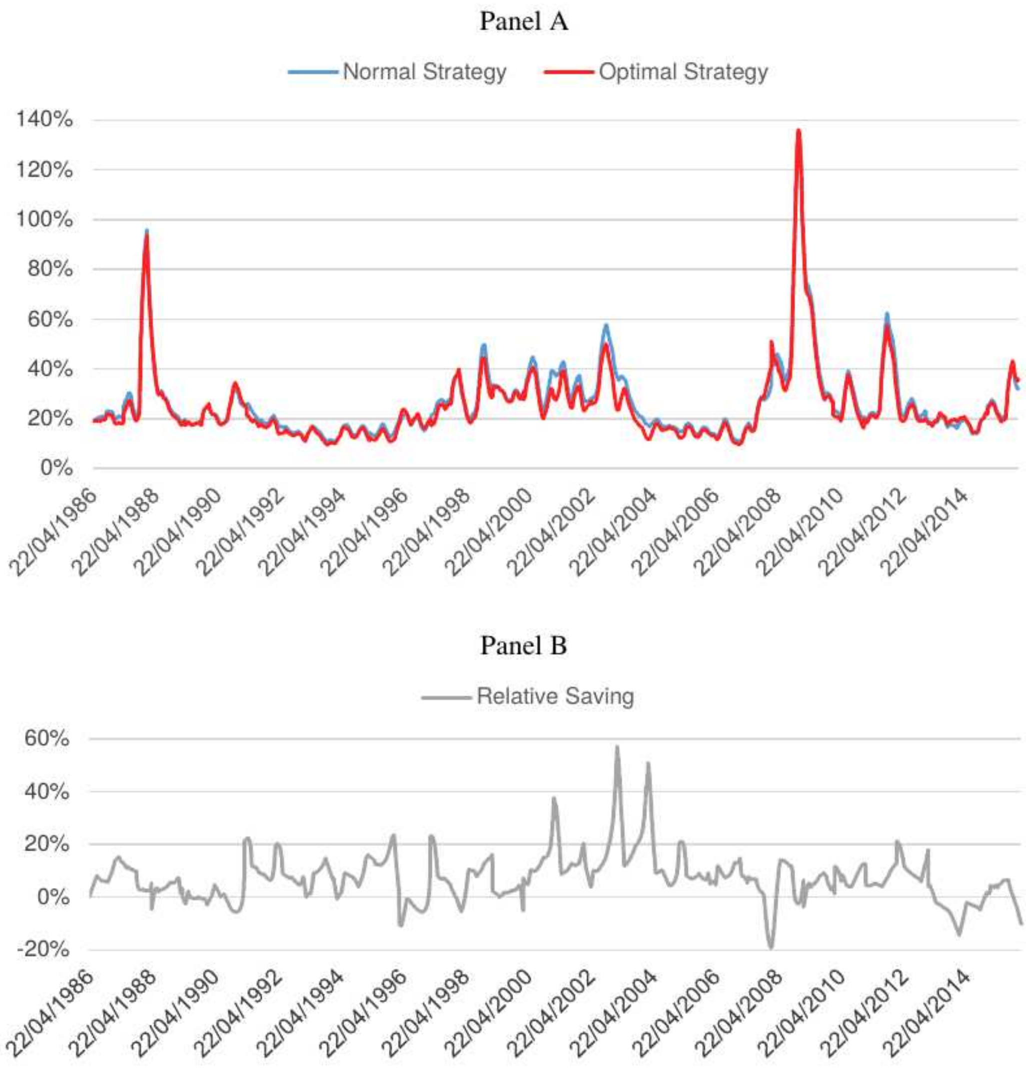

Figure 18: Time series for the daily capital charge for each strategy (Panel A) and for the daily saving from using the optimal strategy (Panel B), for the portfolio simulation.

To complement the analysis of the figure we have in Table 21 the statistics computed for the daily capital charge. As one can see, there is a clear difference between both strategies that lead to a better performance of the optimal strategy, namely a smaller mean, median and minimum value. A curious fact is that even the standard deviation of the capital charge is smaller in the case of the optimal strategy, which is something that did not happen in the Monte Carlo simulations.

Moreover, in Table 21 we can also see that the optimal strategy offers an average relative saving of 7.22%, which is a consequence of the better performance of this strategy in 82.29% of the days (relative to the normal strategy). This result is even higher than the one achieved in the normal simulation (77.81%). This higher relative saving may be a result of an overestimation of the volatility by the employed model, which would consequently mean an overestimation of the VaR. In that case, and under the assumptions of our model,

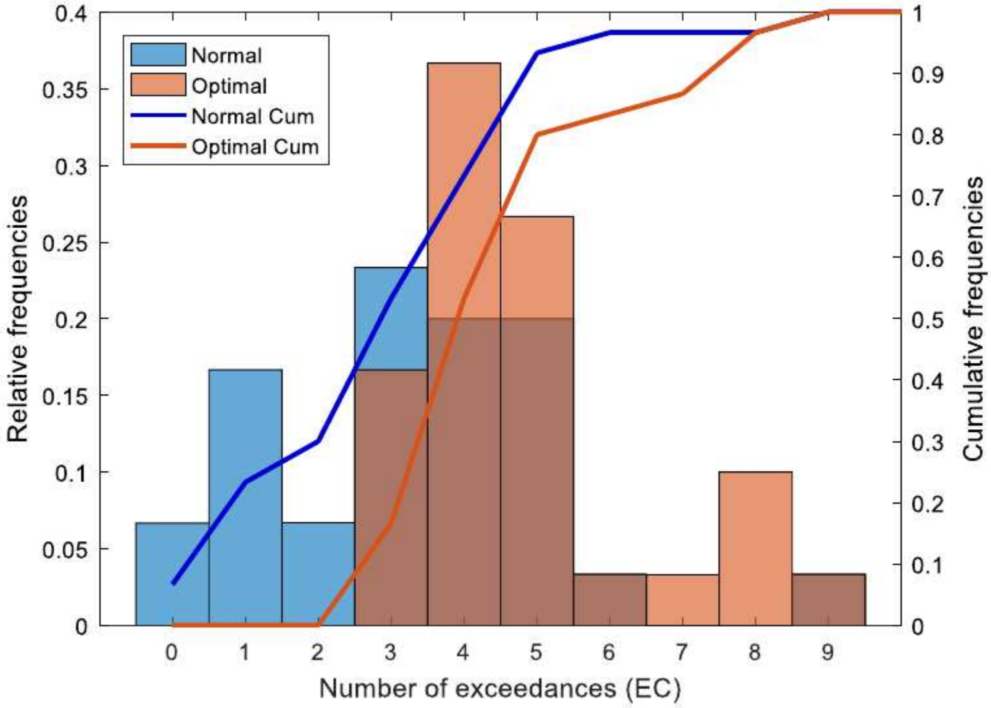

Figure 19: Distribution of the EC variable, for the portfolio simulation.

	EC				***K***			
	Mean	Mode	Median	Std. Dev.	Mean	Mode	Median	Std. Dev.
Optimal Strategy	4.83	4.00	4.00	1.61	3.25	3.00	3.00	0.29
Normal Strategy	3.30	3.00	3.00	1.93	3.30	3.00	3.00	0.22

Table 22: Statistics computed over the number of exceedances (EC) and multiplier (K), for the portfolio simulation.

the level of overestimation would be around 8%, according to Figure 8.

Figure 19 shows the distribution of the number of exceedances in the end of each period. These exceedances, in the optimal strategy, are concentrated around 4 and include some failed attempts to reach this number (3 and 5 exceedances). These failed attempts do not represent a big impact in the performance as it can be seen in Table 22, since all the statistics for the optimal strategy are near 4. Concerning the Normal Strategy, the distribution is more conservative, because there are years where the number of exceedances is below 3 (something that does not happen in the optimal strategy), which translates in a value around 3 for the mean, mode and median.

Finally, Figure 20 demonstrate, as expected, that the different data dispersion in the EC histogram (Figure 19) leads to a similar histogram for the multiplier in both strategies.

All in all, this section provided the necessary information to complete the evaluation of the optimal strategy. In the end we conclude that all figures point out to a better performance of the optimal strategy, relative to the normal strategy, going from the capital charge, which is smaller in the optimal strategy, to the exceedances and multiplier man-

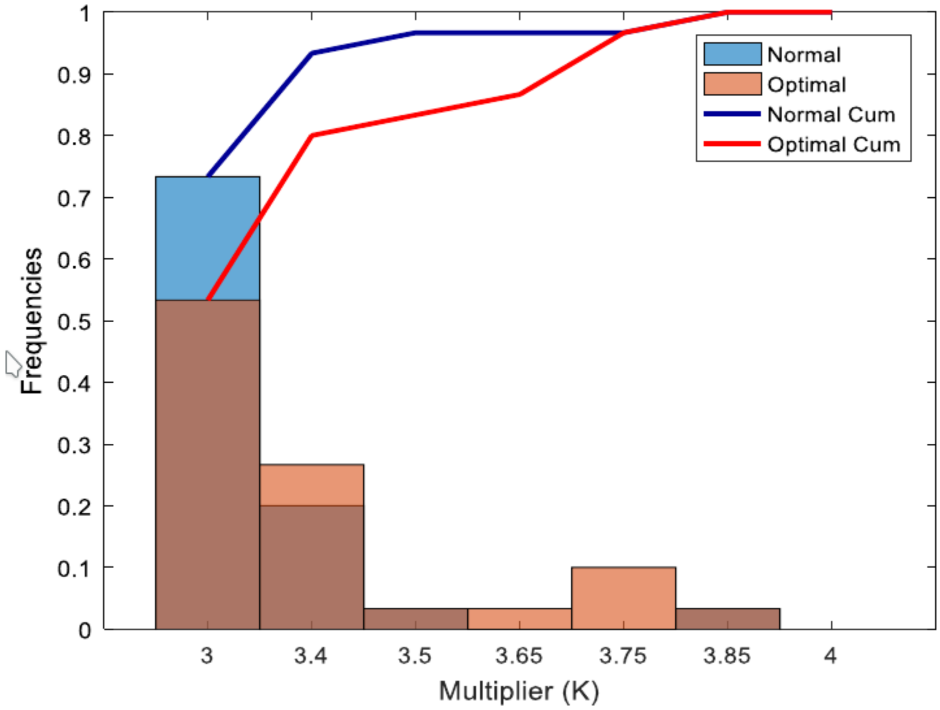

Figure 20: Distribution of the EC variable, for the portfolio simulation.

agement, meaning that this strategy can deliver positive outcomes when applied to a real portfolio.

7 Conclusion

In this study, we used dynamic programming to solve a model that would allow a FI to optimize its capital charges related with the market risk regulation (that is defined in the Basel Accords), by giving the optimal percentage of the 1-day VaR that should be disclosed in every state of nature, also named the optimal policy. In this model, we identified the three state variables that are relevant to choose the VaR disclosure: the time remaining for the regulator to do the backtesting and review the multiplier ($TtoB$); the multiplier for the current regulatory period (K); and the number of exceedances recorded during the current regulatory period until the moment of the decision (EC).

In the analysis of the results of the optimal policy, we concluded that two combined factors (the balancing factors) were driving the results: use an aggressive strategy, i.e. disclose a lower 1-day VaR than estimated in order to save in the current capital charges and have short-term benefits; or use a conservative strategy, i.e. disclose a higher 1-day VaR, to avoid more exceedances that would lead to a multiplier higher than the base value (3) in the future regulatory period, which would reduce the capital charges in the future (long-term benefits). Moreover, we found that, on average, the model led to a strategy of underreporting the estimated 1-day VaR.

These results are in line with the premises that were used by McAleer et al. (2010): risk managers are conservative (report higher percentages) when the number of exceedances is high and aggressive (report lower percentages) when this number is small. Nevertheless, in our study we were able to reduce some of the incovenients of DYLES by creating a model that does not have unknown parameters, neither is an ad-hoc model.

The optimal policy was then tested in a controled environment (Monte Carlo simulation), where the true value for the volatility was known. Here, we compared the optimal strategy, i.e. a strategy were the optimal policy was applied, with the normal strategy, i.e. a stratagy were it is always truthfully reported the estimated 1-day VaR. The main conclusions were that the optimal strategy outperformed the normal strategy in about 78% of the times, which translated into an average saving in the daily capital charge of 4.32% (relative to the normal strategy). The major explanation for this is that the optimal strategy is clearly taking advantage of the Basel rules, because in about 66% of the times, the number of exceedances in the end of the regulatory period is smaller than or equal to 4 (meaning that the multiplier is the base value 3). This extra saving can be translated into a higher annual gain of 5.20%, due to the possible increase in the exposure to the asset (considering a fixed value for the maximum market risk charge of a certain portfolio and a fixed annual return).

Nonetheless, there are three important limitations regarding our investigation. (1) In the derivation of the optimal policy we assumed that the distribution of returns and its parameters (mean and volatility) were known with 100% certainty; (2) in the reward function, which represents the value for the daily capital requirements in the optimization model, we ignored the average of the last sixty 1-day VaR; (3) and we only considered the regulatory framework until the Basel accords II.

Concerning the distribution assumption (1), we tested the impact of under/overestimating the true volatility by performing a simulation where the volatility used to compute the 1-day VaR was always affected by a level of under/overvaluation. From this, we concluded that the optimal strategy continues to outperform the normal strategy when there is a slight undervaluation of the volatility (up to 6%). Moreover, there is a limit to the underperformance of the optimal strategy, attained when the level of undervaluation is at 37% (mainly due to the dynamic aspect of the optimal policy which sometimes report values higher than 100% of the estimated 1-day VaR). In turn, an overvaluation always leads to an even better performance of the optimal strategy.

In terms of the sixty days average assumption (2), in Subsection 4.3 we demonstrated that even though our optimization model was not optimizing the real value for the capital charge, it was a good proxy for the final solutions. This was also tested by performing a simulation where the daily capital charge was computed with the formula used for the reward function, i.e. ignoring this average, and the results were only slightly better compared to our first simulation (the optimal strategy was better in 80% instead of 78% of the times and the daily average relative saving was 5.41% instead of 4.32%). This means that there is some room to improve this model by considering the real equation defined in the Basel accords to compute the daily capital charge, however from these results one can conclude that our solution is a good approximation for the final result.

The last assumption (3) is related with the complexity of the regulation that was defined after the Basel accords II, namely the concept of sVaR (stressed VaR). In this study, we considered that the capital charge is calculated by multiplying the average of the last sixty 1- day VaR by the multiplier factor and the square root of 10 (considering 10 trading days), without considering sVaR that was added to this formula later on in the revisions that were made to this regulatory framework. Therefore, in our model we only optimize one part of the market risk charge and not the full amount.

Considering all of these limitations, we ended this study by analyzing the performance of our model in a real life context by applying the optimal strategy to the S&P 500 index (using data from the last 30 years) and compare it to the normal strategy. This analysis supported the results of the simulation because the optimal strategy outperformed the normal strategy in 82% of the times and the average daily saving in the capital charge was around 7.22%, which represent an even better performance than the one achieved in the simulation. This result may be just luck of the draw, or it may be a consequence of an overestimation of the VaR by the model employed (which according to Figure 8 would be consistent with a level of overvaluation of 8%).

All in all, in this study we demonstrated that, contrary to common sense, the best solution that optimizes the daily capital charge may not be the disclosure of a good estimate for the 1-day VaR but the management of the rules defined in the market risk framework in order to benefit from them in the best possible way.

For new studies on this subject, it would be interesting to: compare the performance of our model against DYLES (McAleer et al., 2010); apply our optimal strategy to a large amount of financial products and understand if there is a significant difference in the results between different kinds of portfolios (stock, bond and mix); develop an optimization model without the limitations that our model have, namely using the real equation for the capital charge and optimizing the capital charge according to the risk framework defined after the Basel accords II.

References

Ahn, D.-H., Boudoukh, J., Richardson, M., and Whitelaw, R. F. (1999). Optimal risk management using options. *The Journal of Finance*, 54:359–375.

Alexander, C. (2008). *Market Risk Analysis IV: Value-at-Risk Models*. John Wiley & Sons.

Angelovska, J. (2013). Managing market risk with var (value at risk). *Journal of Contemporary Management Issues*, 18(2):81–96.

Aussenegg, W. and Miazhynskaia, T. (2006). Uncertainty in value-at-risk estimates under parametric and non-parametric modeling. *Financial Markets and Portfolio Management*, 20(3):243–264.

Basel Committee on Banking Supervison (1995). An internal model-based approach to market risk capital requirements. Bank for International Settlements.

Basel Committee on Banking Supervison (2009). Revisions to the basel ii market risk framework. Bank for International Settlements.

Beder, T. (1995). Var: Seductive but dangerous. *Financial Analysts Journal*, 51(5):12–24.

Bellman, R. (1954). The theory of dynamic programming. *Santa Monica, CA: RAND Corporation.*

Deelstra, G., Ezzine, A., Heyman, D., and Vanmaele, M. (2007). Managing value-at-risk for a bond using bond put options. *Computational Economics*, 29(2):133–149.

Deng, G., Dulaney, T., McCann, C. J., and Wang, O. Y.-H. (2013). Robust portfolio optimization with value-at-risk-adjusted sharpe ratios. *Journal of Asset Management*, 14:293–305.

Hull, J. and White, A. (1998). Value at risk when daily changes in market variables are not normally distributed. *Journal of Derivatives*, 5:9–19.

JPMorgan (1996). Riskmetrics technical document. New York: JPMorgan.

Künzi-Bay, A. and Mayer, J. (2006). Computational aspects of minimizing conditional value-at-risk. *Computational Management Science*, 3(1):3–27.

Lim, C., Sherali, H. D., and Uryasev, S. (2010). Portfolio optimization by minimizing conditional value-at-risk via nondifferentiable optimization. *Computational Optimization and Applications*, 46(3):391–415.

Mansini, R., Ogryczak, W., and Speranza, M. G. (2007). Conditional value at risk and related linear programming models for portfolio optimization. *Annals of Operations Research*, 152(1):227–256.

McAleer, M. (2009). The ten commandments for optimizing value-at-risk and daily capital charges. *Journal of Economic Surveys*, 23(5):831–849.

McAleer, M., Jimenez-Martin, J.-A., and Pérez-Amaral, T. (2010). A decision rule to minimize daily capital charges in forecasting value-at-risk. *Journal of Forecasting*, 29(7):617–634.

Miranda, M. J. and Fackler, P. L. (2002). *Applied Computational Economics and Finance*. The MIT Press.

Ogryczak, W. and Sliwiński, T. (2011). On solving the dual for portfolio selection by optimizing conditional value at risk. *Computational Optimization and Applications*, 50(3):591–595.

Pérignon, C. and Smith, D. R. (2010). The level and quality of value-at-risk disclosure by commercial banks. *Journal of Banking & Finance*, 34:362–377.

Totić, S., Bulajić, M., and Vlastelica, T. (2011). Empirical comparison of conventional methods and extreme value theory approach in value-at-risk assessment. *African Journal of Business Management*, 5(33):12810–12818.

Ünal, G. (2011). Value-at-risk forecasts: A comparison analysis of extreme-value versus classical approaches. *Journal of Risk Model Validation*, 5(3):59–76.

A Optimal policy function for different values of the multiplier K

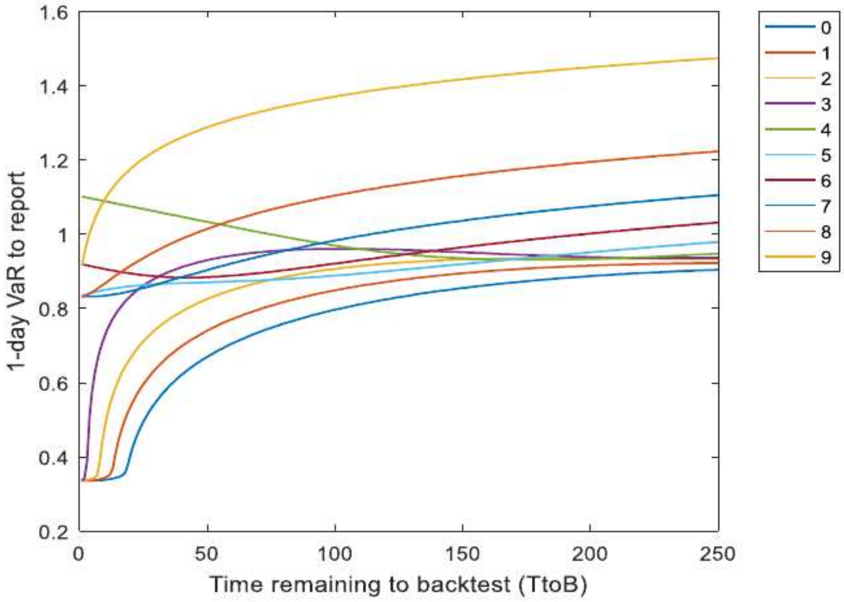

Figure 21: Optimal policy function for all the number of exceedances when the multiplier K is equal to 3.

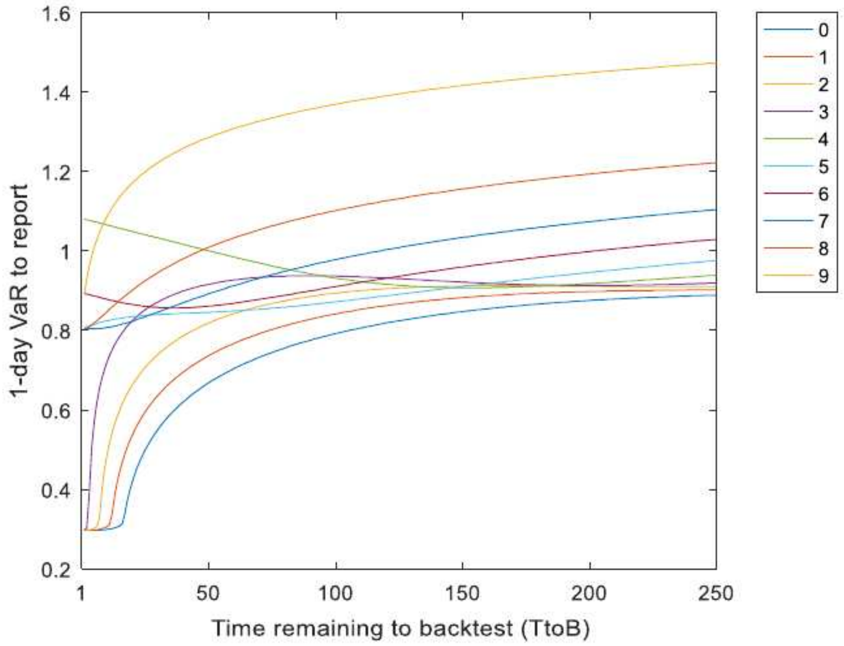

Figure 22: Optimal policy function for all the number of exceedances when the multiplier K is equal to 3.4.

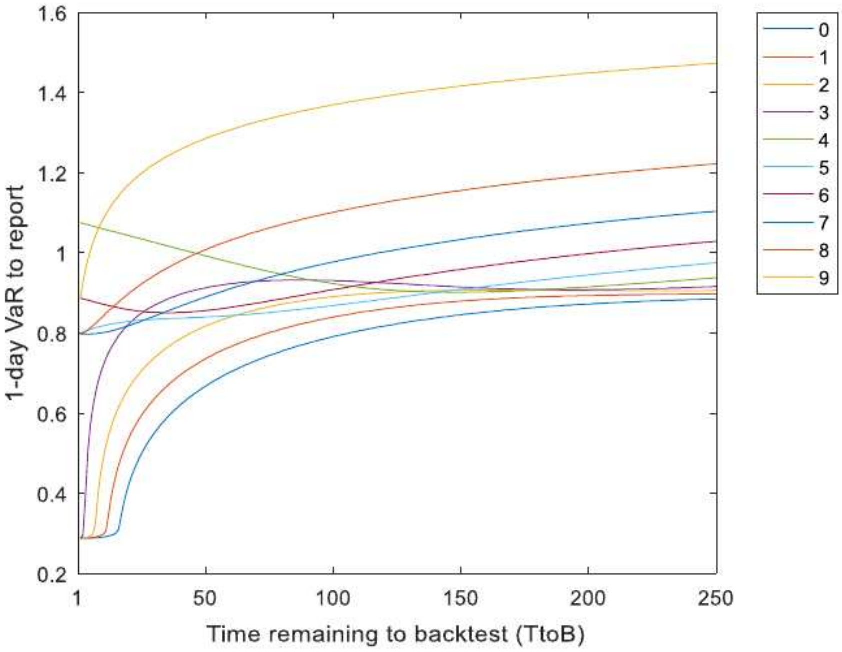

Figure 23: Optimal policy function for all the number of exceedances when the multiplier K is equal to 3.5.

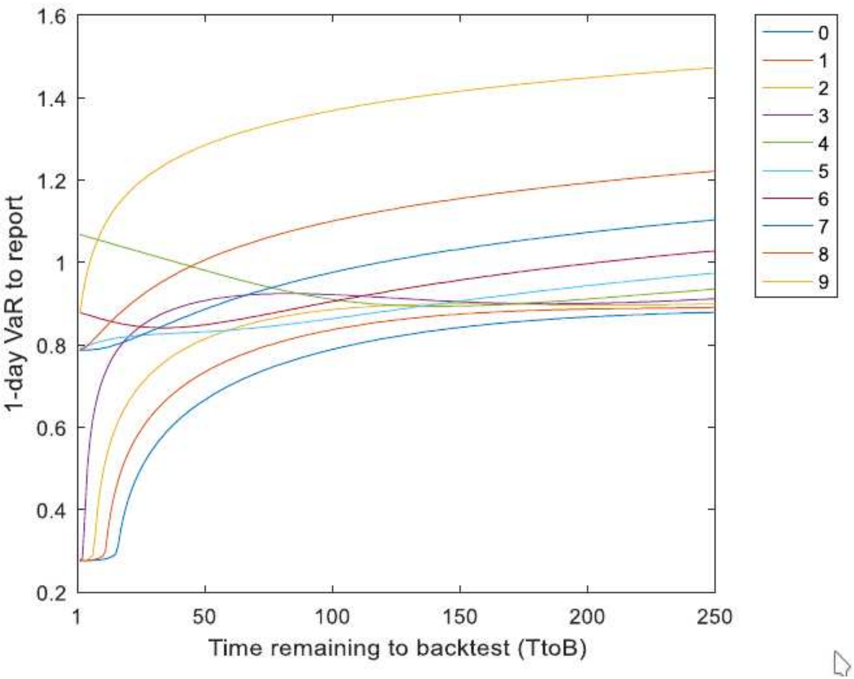

Figure 24: Optimal policy function for all the number of exceedances when the multiplier K is equal to 3.65.

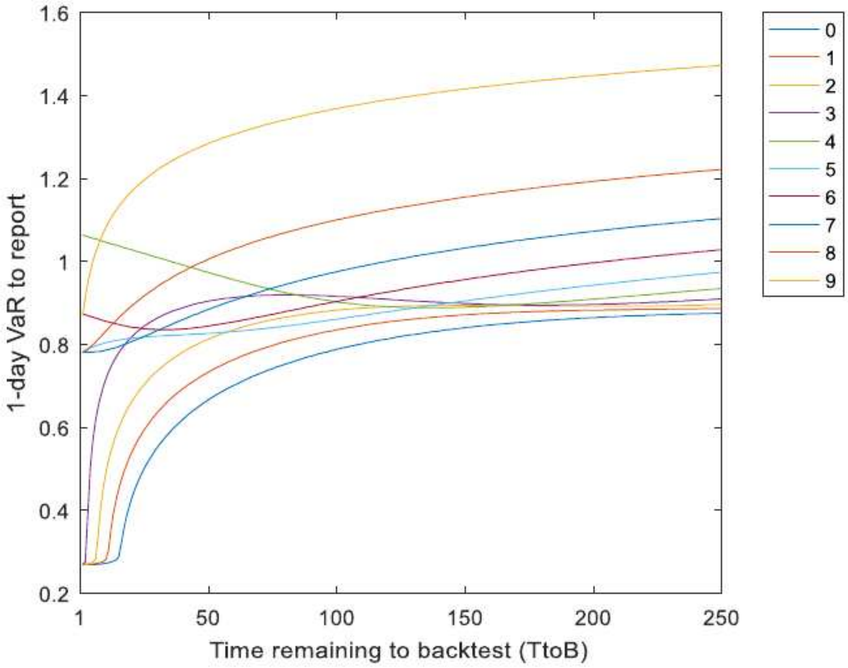

Figure 25: Optimal policy function for all the number of exceedances when the multiplier K is equal to 3.75.

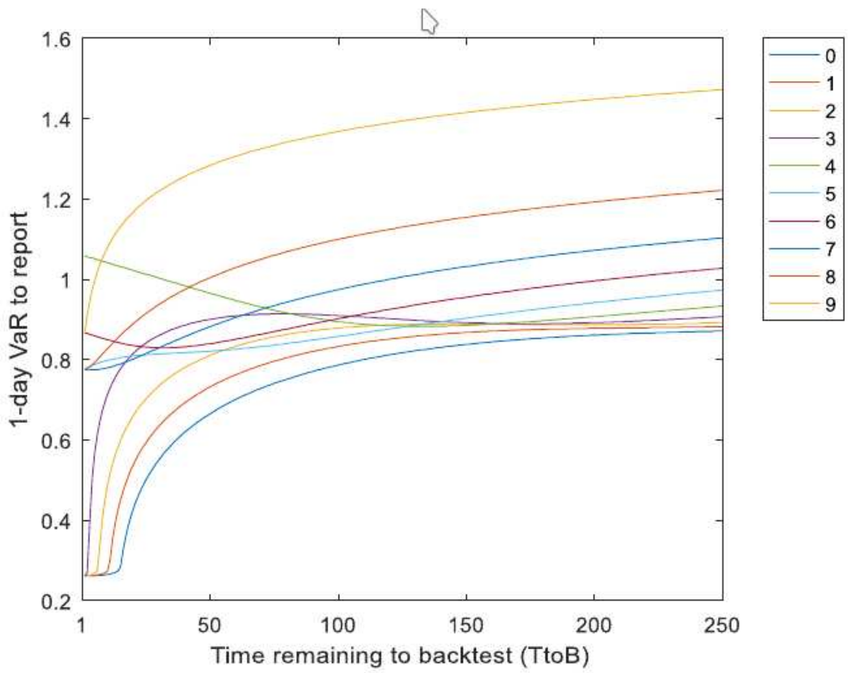

Figure 26: Optimal policy function for all the number of exceedances when the multiplier K is equal to 3.85.

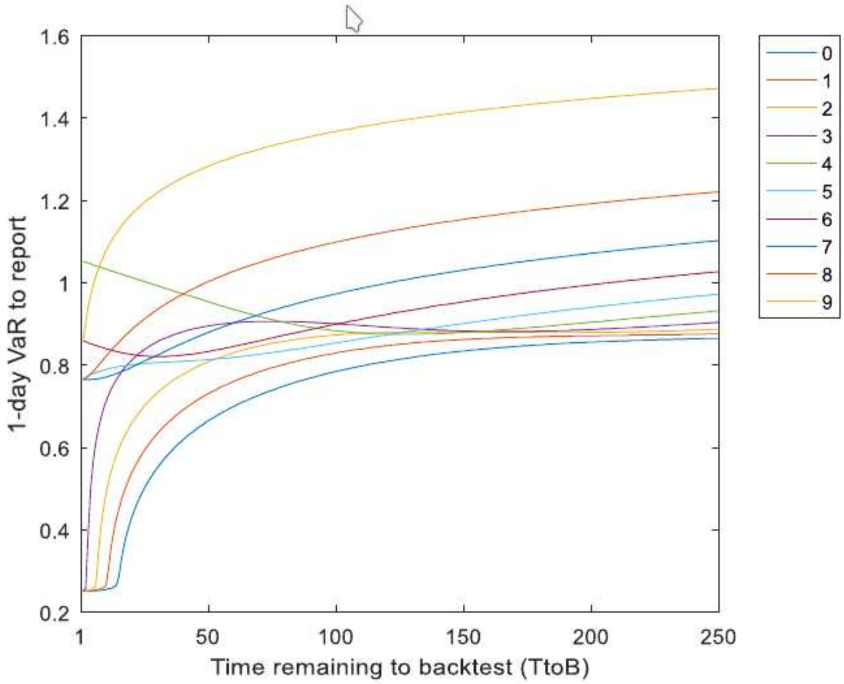

Figure 27: Optimal policy function for all the number of exceedances when the multiplier K is equal to 4.