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Social Connectivity, Media Bias, and Correlation Neglect*

Philipp Denter[†], Martin Dumav[‡], and Boris Ginzburg[§]

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Abstract

We propose a model of political persuasion in which a biased newspaper aims to convince voters to vote for the government. Each voter receives the newspaper's report, as well as an independent private signal. Voters then exchange this information on social media and form posterior beliefs, neglecting correlation among signals. An increase in connectivity increases the newspaper's bias if voters are ex ante predisposed to vote against the government, and reduces the bias if they are predisposed in favour of the government. While more precise independent signals reduce the newspaper's optimal bias, the bias remains positive even when connectivity becomes large. Thus, even with a large number of social connections, the election produces an inefficient outcome with positive probability, implying a failure of the Condorcet jury theorem.

Keywords: social media; media bias; correlation neglect; Bayesian persuasion; voting; deliberation
JEL codes: D72, D82, D91

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1 Introduction

"*Repetition does not transform a lie into a truth.*"

Franklin Delano Roosevelt

"*Want to make a lie seem true? Say it again. And again. And again.*"

Emily Dreyfuss, *Wired*

Individuals receive a substantial amount of political information from their social media connections. Surveys indicate that 62% of US adults receive their news via social media,[1] and a 26-country study has shown that over half 50% of all web users use social media for news each week.[2] The effect of social media on voting outcomes, ideological polarisation, and collective action has been the focus of a number of recent studies.[3]

Our paper proposes a new channel through which social media can affect political outcomes. Namely, we explore the effect of social media connectivity on biased information providers. As voters exchange information with their peers on social media, they are exposed to the same news report from multiple sources. This would have no effect on voters' beliefs if they could fully realise that different messages they receive from their friends are partly based on a common news story, and are therefore positively correlated. Crucially, however, there is evidence that individuals exhibit correlation neglect: they treat the same information received from different sources as independent signals[4]. This causes them to overvalue the information content of these messages. A biased media outlet that supplies the news report that forms part of these messages can use this to manipulate the voters' beliefs. Its ability to do so, however, depends on the number times an individual is exposed to the same news story on social media – which in turn depends on the number of social media connections individuals have. Accordingly, the main objective of this paper is to analyse the effect of increased social media connectivity on media bias, as well as on information aggregation and voting outcomes.

To address these questions, we model a population of voters choosing whether to vote for the government, and a biased newspaper that aims to maximise the probability that the government wins the election. Each voter is connected to a number of other voters on social media. There is a binary state of the world. All voters prefer voting for the government in the high state, and voting against the government in the low state. The newspaper commits to an editorial policy that, conditional on the realisation of the state, sends a report to all voters. Each voter observes the newspaper's report, as well as an independent continuous signal about the state. Each voter then communicates this information to her social media friends. Hence, each voter observes a number of messages which are not identical (since

[1] Pew Research Center, May 2016, "*News use across social media platforms 2016.*"

[2] The Guardian, June 15, 2016, "*Facebook's rise as news source hits publishers' revenues.*"

[3] See Gentzkow and Shapiro (2011), Battaglini (2017), Giovanniello (2017), Enikolopov et al. (2018), Buechel and Mechtenberg (2019), Campbell et al. (2019), Pogorelskiy and Shum (2019).

[4] See Eyster and Weizsacker (2016), Enke and Zimmermann (2017) for experimental evidence. For a discussion of the role of correlation neglect in political settings, see Ortoleva and Snowberg (2015) and Levy and Razin (2015a; 2015b).

they contain the independent signal), but are correlated, as they are partly based on the newspaper's report. Voters, however, fail to realise that these messages are correlated when forming their posterior beliefs and choosing whether to vote for the government.

We analyse what happens if the distribution of the number of connections shifts in a way that increases the number of connections that voters tend to have. If voters did not neglect correlation, an increase in connectivity would make each voter exposed to more independent signals. Hence, voters would become more informed, and to retain persuasive power, the newspaper would have to send more informative reports. If connectivity became arbitrarily large, the newspaper's editorial policy would approach truthful revelation. In the limit, voters would become fully informed about the state, and would almost surely re-elect the government if and only if the state is high. Hence, a version of the Condorcet jury theorem would hold.

However, if voters neglect correlation, an increase in connectivity has two effects. On the one hand, each voter is exposed to more independent signals, making it harder for the newspaper to manipulate their beliefs. On the other hand, however, each voter receives more realisations of the newspaper's report, which she perceives as independent messages. Hence, the newspaper's report becomes more persuasive, counteracting the first effect.

Our first result shows that as connectivity increases, the optimal bias of the newspaper can both increase and decrease, depending on the prior and the voters' preferences. If the prior is such that without additional information voters are willing to vote for the government, then greater connectivity induces the newspaper to send more informative signals. On the other hand, if the prior is low enough that voters are ex ante predisposed to vote against the government, then an increase in connectivity leads the newspaper to send more biased signals. Thus, greater connectivity – for example, as a result of greater social media penetration – decreases the optimal bias of the newspaper if voters tend to be ex ante supportive of its position, and increases the optimal bias if voters tend to oppose it. In the former case, increasing social connectivity makes it less likely that voters reelect the government in the low state, whereas the opposite holds in the latter case, i.e., when voters tend to oppose the government ex ante.[5]

As connectivity becomes arbitrarily large, in the limit each voter observes infinitely many independent signals about the state. However, our second result shows that even in the limit, the newspaper continues to send biased reports. In the limit the bias is lower when independent signals are more informative; nevertheless, the probability that the newspaper lies remains positive. Hence, even when connectivity is arbitrarily large – for example, in the case of a large fully connected electorate – the government is re-elected with a positive probability in the low state, implying a failure of the Condorcet jury theorem.

The rest of this section discusses the related literature. Section 2 introduces the model. Section 3 characterises the newspaper's optimal editorial policy. Section 4 first analyses the

[5]These results have implications for contexts other than voting. Consider, for example, the question of the effect of social connectivity and communication technologies on the ability of citizens to mobilise for a protest against the government (Enikolopov et al., 2018; Manacorda and Tesei, forthcoming; González, 2019). Suppose that a government-run media outlet is attempting to ensure that protest participation does not exceed a certain threshold. Our results suggest that in this situation, increased social connectivity makes it harder for citizens to mobilise for a protest if the government is ex ante unpopular, while organising a protest against an ex ante popular government becomes easier.

effect of connectivity on the newspaper's bias in the benchmark case in which voters do not neglect correlation. It then derives the main results of the paper by showing how an increase in connectivity affects the optimal bias in presence of correlation neglect. Section 5 extends the basic model to consider partial correlation neglect, arbitrary voting rules, and an imperfectly informed newspaper. Finally, Section 6 concludes. The appendix contains the proofs.

Related literature. Our model is related to the large literature on Bayesian persuasion initiated by Kamenica and Gentzkow (2011). More specifically, we add to the literature that studies Bayesian persuasion in a voting setup. Alonso and Câmara (2016) examines a sender who designs a policy experiment to persuade a heterogeneous group of voters to vote in a particular way. Wang (2013), Bardhi and Guo (2018), and Chan et al. (2019) study persuasion of heterogeneous voters when the sender is able to design different experiments for different groups of voters. Heese and Lauermann (2019) study persuasion of voters who, as in our model, receive exogenous private signals in addition to the sender's signal[6]. The distinguishing feature of our paper relative to that literature is that we consider information exchange on a social network by receivers who neglect correlation.

Levy et al. (2018b) study Bayesian persuasion with correlation neglect. In their paper, a single sender controls a number of media outlets, which, conditional on a state of the world, send messages to a single reader with certain accuracy p. The sender can select the degree of correlation between these outlets' messages. The authors show that it is optimal for the sender to positively correlate bad news and negatively correlate good news. In our setup, the multiplicity of signals emerges from information exchange between voters on social media. Hence, the sender can choose neither the number of signals each voter observes—it is exogenously determined by the structure of the social network—nor the correlation between signals. Instead, we focus on the sender's choice of bias, that is, of the probability of sending a high report in the low state. In this constrained optimal framework, we show how an exogenous increase in the number of reports observed by voters affects the optimal bias. In addition, in our model observe independent signals in addition to the newspaper's reports. These signals counteract the newspaper's persuasive power, which implies that greater connectivity can both increase and reduce the optimal bias.

Levy et al. (2018a) construct a general model of information design in presence of correlation neglect, in which a sender designs an information structure to persuade a receiver that fully or partly neglects correlation between signals. They show that an increase in the number of signals increases the set of distributions over posteriors that the sender can induce. When the number of signals grows large, the sender can approach her first-best utility. In our model, receivers also observe independent signals. Because the multiplicity of signals appears as a result of information exchange on social media, an increase in the number of newspaper's reports received by each voter is always accompanied by an increase in the number of independent signals. Because of this, an increase in the number of reports as a result of greater connectivity can both increase and decrease the optimal bias. For the same reason, the newspaper's utility is below the first-best when the number of signals is large.

[6] More generally, a number of papers have studied persuasion of heterogeneous receivers in non-voting setups (Kolotilin et al., 2017; Kolotilin, 2018; Ginzburg, 2019).

Our paper also contributes to the literature studying information exchange in collective decision-making. Gerardi and Yariv (2007) extend the classic Condorcet jury theorem framework (see e.g. Austen-Smith and Banks, 1996, Feddersen and Pesendorfer, 1998) by allowing voters to exchange their private signals before taking a vote.[7] In a Bayesian framework, they show that all veto-free electoral rules yield the same equilibrium outcomes. In subsequent experimental work, Goeree and Yariv (2011) show that public communication increases efficiency of the vote, while Buechel and Mechtenberg (2019) find that private voter-to-voter communication can reduce efficiency. Perhaps the closest paper within this literature is Pogorelskiy and Shum (2019), which analyses individuals who receive private signals from media outlets with exogenous biases, and communicate them prior to voting. They find evidence of failure of Bayesian updating – individuals do not account for bias in the signals they share – which implies that social media may reduce efficiency of collective decisions. In our paper, individuals also deviate from Bayesian updating when exchanging information. However, rather than failing to recognise the bias in the signals they receive, they fail to realise that signals are correlated. We then endogenise the bias of the media outlet, and study the relation between connectivity and the optimal bias.[8]

Finally, our paper is related to the research on media bias or slant[9], specifically, to the literature that models biased media as committing to a reporting strategy ex ante (Gehlbach and Sonin, 2014; Gentzkow et al., 2015; Boleslavsky and Kim, 2018). We add to that research by analysing how media bias is affected by information exchange among receivers who neglect correlation.

2 Model

Players, network, and information. There are two groups of players: a continuum of voters whose mass is 1, and a newspaper. Each voter is connected to $n \in \{0, 1, ..., N\}$ other citizens on social media. Our model is agnostic about the actual structure of the network – the only object of interest is the distribution of the number of connections across voters. For all $n \in \{0, 1, ..., N\}$, let γ_n be the share of voters with n social media connections. Let $\boldsymbol{\gamma} \equiv (\gamma_0, ..., \gamma_N)$ be the distribution of the number of connections, that is, the degree distribution of the network.

There is an unknown binary state of the world $\theta \in \{0, 1\}$. The state reflects the government's competence, with $\theta = 1$ representing a more competent government. Let $q \equiv \Pr(\theta = 1)$ denote the common prior probability that the government is competent. The newspaper observes the state, and sends a binary *report* $r \in \{0, 1\}$.[10]

[7] In a setting without voting or strategic senders, Levy and Razin (2018) examine information exchange on a social network by agents who may neglect correlation, while Acemoglu et al. (2010) analyse exchange of information when some agents do not fully update their beliefs.

[8] In a related experimental paper, Kawamura and Vlaseros (2017) show that voters overweigh an unbiased public signal even in a setting without deliberation (and hence without observing multiple realisations of the same message). Our paper suggests that information exchange reinforces this effect, implying that a when the signal comes from a biased newspaper, the bias would increase when connectivity is larger.

[9] See, for example, Bernhardt et al., 2008, Gentzkow and Shapiro, 2010, Duggan and Martinelli (2011), Oliveros and Várdy, 2015, Piolatto and Schuett, 2015, and others.

[10] This is without loss of generality, as the report can always be interpreted as a recommendation. See, for

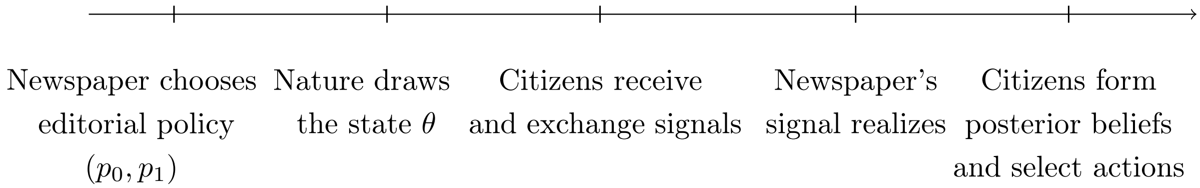

Figure 1: Sequence of events.

In addition, each voter i receives a private signal $s_i \in \mathbb{R}$. Conditional on the state, signals are independent across voters. In state 1, the s_i is drawn from a normal distribution with mean $\mu > 0$ and variance that is normalised to 1. In state 0 the signal is drawn from a normal distribution with mean $-\mu$ and variance 1. Note that these signal distributions satisfy monotone likelihood ratio property, and a higher realisation of the signal indicates that the state is more likely to equal 1. Higher value of μ corresponds to a more precise signal, while $\mu = 0$ corresponds to a signal that carries no information.

Preferences and actions. The newspaper commits ex ante to a strategy that specifies for each state $\theta \in \{0, 1\}$ the probability p_θ of sending report 1; report 0 is sent with the complementary probability. We will refer to the pair (p_0, p_1) as the newspaper's *editorial policy*. Without loss of generality, we will assume that $p_0 \leq p_1$ – thus, report 1 induces a weakly higher posterior belief that $\theta = 1$. After the exchange of information (described below), each voter i chooses an action $a_i \in \{0, 1\}$ where the action $a_i = 1$ ($a_i = 0$) corresponds to voting for (against) the government.

A voter who chooses the action 0 receives a payoff of 0. A voter who chooses the action 1 receives a payoff of $1 - \lambda$ if $\theta = 1$, and a payoff of $-\lambda$ if $\theta = 0$, where $\lambda \in (0, 1)$ is a parameter that represents voter's political preferences. Thus, a higher values of λ means voters are more inclined to vote against the government – one can interpret λ a a measure of government's popularity.[11] Since $\lambda \in (0, 1)$, voters prefer voting for the government in state 1, and voting against it in state 0.[12] We assume that each voter who is indifferent chooses action 1.[13]

The newspaper receives a payoff of 1 if at least half of the voters vote for the government, and zero otherwise. The newspaper thus aims to maximize the probability that the government wins the election.

Timing, information exchange, and belief formation. The sequence of events is depicted in Figure 1. At the beginning of the game, the newspaper chooses an editorial policy

example, Alonso and Câmara (2016).

[11]Note that the payoff of each voter only depends on her vote, and not on the outcome of the election. Thus, each voter has a preference for voting "the right way" (supporting the government if it is competent, and opposing to it otherwise), as in models of expressive voting (Brennan and Hamlin, 1998; Hillman, 2010; see also Tyran and Wagner, 2016, for experimental evidence of expressive voting). Note that since the model posists a continuum of voters, the probability of a given voter being pivotal is zero, and hence strategic motives related to pivotality are irrelevant.

[12]Section 5 considers the case in which some voters are partisans, that is, have a preferred choice that does not depend on the state.

[13]This ensures the existence of an optimal editorial policy.

(p_0, p_1).[14] Then, Nature draws the state θ according to the common prior q. For each voter i, Nature also draws her number of social media connections from distribution $\boldsymbol{\gamma}$, as well as her private signal. Upon observing θ, the newspaper sends report 1 with probability p_θ, and report 0 with the complementary probability $1 - p_\theta$. Each voter observes the newspaper's report r and her private signal s_i. She then forms a likelihood ratio $x_i = \frac{\Pr(\theta=0|r,s_i)}{\Pr(\theta=1|r,s_i)}$, and truthfully reveals x_i to her friends on social media.[15] Thus, a voter with n social media connections observes $n+1$ such messages (those of her n friends, plus her own). These x_i's are correlated, as they are based on identical realisations of the newspaper's report, in addition to independent signals. Voters, however, fail to realise this correlation. Specifically, we assume that when updating their beliefs, voters perceive n fully correlated signals as $k(n)$ independent signals. The main part of our analysis focuses on the case of full correlation neglect, in which $k(n) = n$. We will compare this to the benchmark case of Bayesian voters, for whom $k(n) = 1, \forall n$. Partial correlation neglect, in which $k(n)$ is an arbitrary function, will be considered in Section 5.

After updating her belief, each voter i chooses an action $a_i \in \{0, 1\}$. After this, payoffs are realised.

3 Optimal Editorial Policy

Consider the last stage of the game. If a voter has a posterior belief π, her expected utility from voting for the government equals $\pi(1-\lambda) - (1-\pi)\lambda = \pi - \lambda$. She thus votes for the government if and only if $\pi \geq \lambda$, that is, if and only if her posterior belief is sufficiently strong.

When choosing its editorial policy, the newspaper takes into account voters' preference parameter λ and prior q. Let $A \equiv \frac{\lambda}{1-\lambda}\frac{1-q}{q}$. Informally, a low (high) value of A indicates that the situation is favourable (unfavourable) for the newspaper. In particular, $A < 1$ if and only if $q > \lambda$, that is, if and only if voters are ex ante willing to vote for the government without additional information.

A possible strategy for the newspaper is to select an editorial policy $p_0 = p_1$ – that is, to send an uninformative report that does not affect the voters' behaviour. All such babbling strategies are payoff-equivalent for the newspaper. We will thus without loss of generality exclude editorial policies $(0, 0)$ and $(1, 1)$. Since we have assumed that $p_0 \leq p_1$, this means that we restrict attention without loss of generality to editorial policies in which $p_1 > 0$ and $p_0 < 1$.

Suppose the newspaper chooses an editorial policy (p_0, p_1). Consider a voter j with n social media connections. This voter observes $n+1$ messages $x_1, ..., x_{n+1}$. For the purposes of belief updating, this is equivalent to observing $n+1$ independent signals $s_1, ..., s_{n+1}$, as well as $n+1$ realisations of r which the voter perceives as $k(n+1)$ independent realisations.

[14] Thus, the newspaper commits to an editorial policy before the state is realised. In practice, commitment to an editorial policy can mean, for example, hiring staff with particular political views. Alternatively, an outcome similar to the equilibrium under commitment can emerge in a repeated interaction if the newspaper cares about its reputation, as Mathevet et al. (2019) show.

[15] Note that voters have no incentive to lie, since they share the same preferences. Furthermore, as voters have expressive preferences only, they have no interest in manipulating the beliefs of other voters.

If the newspaper sends report $r = 1$, this voter will form a posterior belief

$$\pi_1(p_0, p_1) = \frac{q p_1^{k(n+1)} \prod_{i=1}^{n+1} e^{-\frac{(s_i-\mu)^2}{2}}}{q p_1^{k(n+1)} \prod_{i=1}^{n+1} e^{-\frac{(s_i-\mu)^2}{2}} + (1-q)\, p_0^{k(n+1)} \prod_{i=1}^{n+1} e^{-\frac{(s_i+\mu)^2}{2}}} = \frac{p_1^{k(n+1)}}{p_1^{k(n+1)} + \frac{1-q}{q} p_0^{k(n+1)} e^{-2S\mu}}$$

where $S \equiv \sum_{i=1}^{n+1} s_i$. We will refer to S as *evidence*; it summarises all information that is available to the voter, other than the newspaper's report.

The voter will then vote for the government if and only if $\pi_1(p_0, p_1) \geq \lambda$. This is always true if $p_0 = 0$. If $p_0 > 0$, the voter votes for the government if and only if her evidence is sufficiently convincing, that is, if and only if $e^{2S\mu} \geq \frac{\lambda}{1-\lambda} \frac{1-q}{q} \left(\frac{p_0}{p_1}\right)^{k(n+1)}$. After taking logs of both sides, this is equivalent to

$$S \geq S_n(p_0, p_1) \tag{1}$$

where

$$S_n(p_0, p_1) \equiv \frac{1}{2\mu} \ln A + \frac{1}{2\mu} k(n+1) \ln\left(\frac{p_0}{p_1}\right)$$

At the same time, consider the case when the newspaper sends report $r = 0$. Then a voter with n connections who observed independent signals $s_1, ..., s_{n+1}$ will form a posterior belief

$$\pi_0(p_0, p_1) = \frac{q (1-p_1)^{k(n+1)} \prod_{i=1}^{n+1} e^{-\frac{(s_i-\mu)^2}{2}}}{q (1-p_1)^{k(n+1)} \prod_{i=1}^{n+1} e^{-\frac{(s_i-\mu)^2}{2}} + (1-q)(1-p_0)^{k(n+1)} \prod_{i=1}^{n+1} e^{-\frac{(s_i+\mu)^2}{2}}} = \frac{(1-p_1)^{k(n+1)}}{(1-p_1)^{k(n+1)} + \frac{1-q}{q}(1-p_0)^{k(n+1)} e^{-2S\mu}}$$

The voter votes for the government if and only if $\pi_0(p_0, p_1) \geq \lambda$. When $p_1 = 1$, this condition is never satisfied. Otherwise, the voter votes for the government if and only if $e^{2S\mu} \geq \frac{\lambda}{1-\lambda} \frac{1-q}{q} \left(\frac{1-p_0}{1-p_1}\right)^{k(n+1)}$, or, equivalently, if and only if

$$S \geq S_n(1-p_0, 1-p_1) \tag{2}$$

Let $V(\theta, r)$ be the share of voters that vote for the government in state θ when the newspaper sends report r. Conditions (1) and (2) imply that conditional on the report, a given voter with n connections will vote for the government if she observes independent signals whose sum S is sufficiently large. Since signals are informative, this implies that $V(1, r) > V(0, r), \forall r \in \{0, 1\}$, that is, the government receives a larger share of votes when $\theta = 1$ then when $\theta = 0$.

In addition, because $p_1 \geq p_0$, it is true that $\ln\left(\frac{p_0}{p_1}\right) \leq \ln\left(\frac{1-p_0}{1-p_1}\right)$, and thus $S_n(p_0, p_1) \leq S_n(1-p_0, 1-p_1)$. Then (1) and (2) imply that in a given state, a voter is more likely to vote for the government when the newspaper sends report 1 then when it sends report

0. Thus, $r = 1$ gives the government a weakly larger vote share than $r = 0$, that is, $V(\theta, 1) \geq V(\theta, 0), \forall \theta \in \{0, 1\}$.

In each state $\theta \in \{0, 1\}$ after each report $r \in \{0, 1\}$, the newspaper receives a payoff of 1 if $V(\theta, r) \geq \frac{1}{2}$, and a payoff of 0 otherwise. One possible strategy for the newspaper is to select $p_0 = 0$ and $p_1 = 1$ – that is, to reveal the state truthfully. In that case, all voters vote for the government in state 1, and no voters vote for the government in state 0. The newspaper's payoff then equals q. We can show, however, that the newspaper can always guarantee itself a higher payoff. Specifically, the following result shows that truthfully revealing the state is never optimal for the newspaper:

Lemma 1. *Editorial policy* $(p_0, p_1) = (0, 1)$ *is never optimal.*

The opposite strategy for the newspaper is to send an uninformative signal, that is, to set $p_0 = p_1$. Consider the following assumption, where Φ is the c.d.f. of the standard normal distribution:

Assumption 1. $\sum_{n=0}^{N} \gamma_n \Phi \left[\frac{\frac{1}{2\mu} \ln A + (n+1)\mu}{\sqrt{n+1}} \right] > \frac{1}{2}$

In the sequel we assume that the assumption holds unless stated differently. The assumption is important because from the condition stated in it both a necessary and sufficient condition for such a babbling strategy to be optimal follow:

Lemma 2. *Editorial policy* (p_0, p_1) *such that* $p_0 = p_1$ *is optimal if and only if Assumption 1 is violated.*

In plain words, babbling is an optimal strategy when Assumption 1 fails. This happens when A is small (that is, voters' preferences favour the government), μ is small (independent signals are imprecise), and $\boldsymbol{\gamma}$ is such that n tends to be small (so voters tend to have few social media connections). When Assumption 1 is satisfied, babbling is not an equilibrium strategy.

Intuitively, suppose that $q > \lambda$ (so that $A < 1$), and that $\mu = 0$, so independent signals are completely uninformative. Then without additional information all voters want to vote for the government. Then by babbling the newspaper achieves its maximum payoff. More generally, if q is much larger than λ, μ is small, and voters tend to have few connections (and hence few voters observe sufficient evidence to push their posterior beliefs below λ), then in each state sufficiently many voters vote for the government when the newspaper babbles.

On the other hand, if A is large, μ is large (that is, independent signals are informative), and voters tend to have many connections, then by babbling the newspaper can convince $\frac{1}{2}$ of all voters to vote for the government in, at most, state 1 only. In this case its expected payoff is then at most q, which equals its payoff from truthful revelation. Since we have already shown that truthful revelation is suboptimal, this implies that babbling is also not optimal under these conditions.

If Assumption 1 is violated, then babbling is an optimal policy, and small changes in connectivity do not affect it. For most of the analysis, we will consider the more interesting case in which Assumption 1 is satisfied. In particular, the following simple condition is sufficient for Assumption 1 to hold (and hence for babbling not to be optimal) for any $\boldsymbol{\gamma}$:

Lemma 3. *A sufficient condition for Assumption 1 to hold is* $\ln A \geq -2\mu^2$.

Intuitively, it is optimal for the newspaper to reveal some information under any $\boldsymbol{\gamma}$ if the government is sufficiently unpopular and voters are sufficiently well-informed.

When Assumption 1 holds, the newspaper selects an editorial policy such that $p_0 < p_1$. Then the government receives strictly more votes after report $r = 1$ then after report $r = 0$. Together with the fact that for each r more voters vote for the government in state 1, this implies that

$$V(1,1) > \max\{V(1,0), V(0,1)\} > \min\{V(1,0), V(0,1)\} > V(0,0)$$

The newspaper receives a payoff of 1 if and only if $V(\theta, r) \geq \frac{1}{2}$. What are the (θ, r) pairs under which this happens? We can show that the optimal editorial policy has the following property:

Lemma 4. *Under an optimal editorial policy* (p_0, p_1)*, for all* $\theta \in \{0, 1\}$*, we have* $V(1,1) > V(0,1) \geq \frac{1}{2}$*, and* $V(0,0) < V(1,0) < \frac{1}{2}$.

In words, at the equilibrium the newspaper selects an editorial policy under which in every state it receives a payoff of 1 after sending report $r = 1$, and a payoff of 0 after sending report $r = 0$. Its expected utility then equals $qp_1 + (1-q)p_0$.

We can now characterise the optimal editorial policy as follows:

Proposition 1. *There exists a unique optimal editorial policy* (p_0, p_1) *such that* $p_1 = 1$ *and* $p_0 \in (0, 1)$ *that is given by*

$$\sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left[\frac{S_n(p_0, 1) + (n+1)\mu}{\sqrt{n+1}}\right]\right) = \frac{1}{2}$$

Thus, the newspaper always sends report 1 in state 1. In state 0, it sends report 1 with some probability p_0 that is distinct from zero and from one. That probability is chosen in such a way as to ensure that $V(0,1) = \frac{1}{2}$, that is, in state 0 after report 1 exactly $\frac{1}{2}$ of all voters vote for the government. On the other hand, $p_1 = 1$ implies that upon receiving report 0, all voters know that $\theta = 0$ with certainty.

Consequently, the government will always receive $\frac{1}{2}$ of all votes in state 1. In state 0 the government receives no votes with probability $1 - p_0$, and with probability p_0 it receives $\frac{1}{2}$ of all votes. Hence, a competent government always wins the election, while an incompetent government wins the election with probability p_0. Hence, the probability that the election produces the correct decision is

$$\Psi \equiv 1 - (1-q)p_0$$

The newspaper's equilibrium payoff is $q + (1-q)p_0$.

We can think of p_0 as a measure of bias: the higher it is, the more likely it is that the newspaper misreports the state. Our analysis will focus on how a change in the distribution of links $\boldsymbol{\gamma}$ towards more connectivity affects p_0.

4 The Effect of Connectivity

In this section we will look the effect of an increase in connectivity on the equilibrium. Specifically, consider a shift from connectivity distribution $\boldsymbol{\gamma}$ to another distribution $\tilde{\boldsymbol{\gamma}}$ that first order stochastically dominates $\boldsymbol{\gamma}$.[16] How will such a shift affect the optimal bias p_0 and the probability of the correct decision Ψ? We will start with a benchmark case of no correlation neglect, before analysing the effect of an increase in connectivity when voters neglect correlation.

4.1 A Benchmark: Persuasion Without Correlation Neglect

If voters do not neglect correlation, then $k(n) = 1, \forall n$. Thus, $S_n(p_0, 1) = \frac{1}{2\mu}\ln A + \frac{1}{2\mu}\ln p_0$. Rearranging the expression in Proposition 1 using the fact that $\sum_{n=0}^{N}\gamma_n = 1$, we can then write the expression that defines optimal bias as

$$\sum_{n=0}^{N}\gamma_n\Phi\left[\frac{\frac{1}{2\mu}\ln A + \frac{1}{2\mu}\ln p_0 + (n+1)\mu}{\sqrt{n+1}}\right] = \frac{1}{2} \tag{3}$$

Then we have the following result

Proposition 2. *If voters do not neglect correlation, then a shift from* $\boldsymbol{\gamma}$ *to* $\tilde{\boldsymbol{\gamma}}$ *reduces the optimal* p_0.

Intuitively, when the number of connections tends to increase, each voter observes more independent signals, and thus her information becomes more precise. In particular, in state 0 voters become more reluctant to vote for the government. Then to ensure that report $r = 1$ induces sufficiently many voters to vote for the government when $\theta = 0$, the newspaper needs to make this report a stronger signal. Hence, it reduces the bias.

We can also look at what happens in the limit as connectivity becomes arbitrarily large. Formally, suppose that $\gamma_N = 1$, and let $N \to \infty$, i.e., the number of social media connections of each voter approaches infinity. The next result shows that in the limit the optimal strategy approaches truthful revelation as the number of connections becomes arbitrarily large:

Proposition 3. *Suppose* $\gamma_N = 1$. *If voters do not neglect correlation, then* $\lim_{N\to\infty} p_0 = 0$.

Propositions 2 and 3 imply the following result about the effect of connectivity on the probability of the correct decision:

Corollary 1. *Suppose voters do not neglect correlation. Then a shift from* $\boldsymbol{\gamma}$ *to* $\tilde{\boldsymbol{\gamma}}$ *increases* Ψ. *Furthermore, if* $\gamma_N = 1$, *then* $\lim_{N\to\infty}\Psi = 1$.

Corollary 1 says that when voters do not neglect correlation, a version of the Condorcet jury theorem holds in this setting: as each voter observes more independent signals from

[16] Many standard network models have one or more parameters such that changing this parameter changes the degree distribution in exactly the same way, i.e., such that we can rank connectivity using first order stochastic dominance. Two examples are the random network models by Erdös and Rényi (1959) or by Jackson and Rogers (2007).

her friends on social media, she becomes increasingly more informed. Hence, it becomes increasingly more likely that the outcome of the vote corresponds to voters' preferences. The presence of the biased newspaper does not change this, since the newspaper optimally reveals more information as voters become more informed.

4.2 Persuasion with Correlation Neglect

We will now derive the main results of the paper: comparative statics for the case when voters neglect correlation. In that case, $k(n) = n$. Thus, $S_n(p_0, 1) = \frac{1}{2\mu}\ln A + \frac{1}{2\mu}(n+1)\ln p_0$. Rearranging the expression in Proposition 1, we can then write the expression that defines optimal bias as

$$\sum_{n=0}^{N} \gamma_n \Phi\left[\frac{\frac{1}{2\mu}\ln A + \frac{1}{2\mu}(n+1)\ln p_0 + (n+1)\mu}{\sqrt{n+1}}\right] = \frac{1}{2} \tag{4}$$

Then we have the following result

Proposition 4. *If voters neglect correlation, then a shift from* $\boldsymbol{\gamma}$ *to* $\tilde{\boldsymbol{\gamma}}$ *reduces the optimal* p_0 *if* $A < 1$*, increases the optimal* p_0 *if* $A > 1$*, and leaves the optimal* p_0 *unchanged if* $A = 1$*.*

Hence, when $A < 1$ – that is, when voters are ex ante willing to vote for the government – an increase in connectivity reduces the optimal bias, as in the case without correlation neglect. However, when voters are ex ante predisposed against the government, higher connectivity leads the newspaper to increase the bias.

We can also look at the optimal bias in the limit, as connectivity becomes arbitrarily large. As before, suppose that $\gamma_N = 1$, and let $N \to \infty$. The next result shows that in the limit the optimal strategy approaches truthful revelation as the number of connections becomes arbitrarily large:

Proposition 5. *Suppose* $\gamma_N = 1$*. If voters neglect correlation, then* $\lim_{N\to\infty} p_0 = e^{-2\mu^2}$.

Hence, when correlation neglect is present, the newspaper continues to send biased reports even when connectivity—and hence the overall precision of independent signals—is arbitrarily large.

The following intuition underlies these results. When connectivity becomes very large, voters receive a large number of independent signals. In itself, this should make the voters almost perfectly informed. On the other hand, voters also observe a large number of realisations of the newspaper's report, which they treat as independent messages. Hence, the newspaper's report becomes increasingly more convincing. Thus, in the limit, the bias is bounded away from 0.

As connectivity increases, the newspaper adjusts its bias to ensure that in state 0 report $r = 1$ continues to induce exactly half of all voters to vote for the government. When independent signals are more informative – that is, when μ is larger – the newspaper, at each $\boldsymbol{\gamma}$, needs to send a more persuasive message to convince enough voters to vote for the government in state 0. Thus, its optimal bias is lower – therefore, the limit of p_0 as $N \to \infty$ is decreasing in μ. This implies the result described in Proposition 5.

An the same time, note that when connectivity is very large, the posterior belief in the limit does not depend on the prior, because voters observe infinitely many independent signals and an infinitely strong report from the newspaper. These messages overcome any prior belief. Hence, the limit of p_0 as $N \to \infty$ does not depend on q. However, when connectivity is low, the prior has a large effect on the posterior belief. When $A < 1$, and hence $q < \lambda$, voters are ex ante unwilling to vote for the government. To induce voters to do so, the newspaper needs to send a very credible report when connectivity is low – that is, to select a low bias. Hence, when the prior is unfavourable to the government, as connectivity increases the bias converges to $e^{-2\mu^2}$ from below. On the other hand, when $A > 1$, the newspaper can select a high bias when connectivity is low – thus, as connectivity increases, the bias converges to $e^{-2\mu^2}$ from above. This underlies the result in Proposition 4.

Note that if $\mu = 0$, then p_0 converges to 1 as as $N \to \infty$. As the expected payoff of the newspaper equals $q + (1-q)\,p_0$, the newspaper can achieve its first-best utility in the limit if independent signals are uninformative. Furthermore, if $\mu = 0$, then Assumption 1 only holds when $A > 1$ – thus, either babbling is an equilibrium for all connectivity levels, or an increase in connectivity always increases the optimal bias and the newspaper's expected payoff.[17] In general, however, the existence of independent signals puts a bound on the maximum utility that the newspaper can attain. It also ensures that p_0 (and hence the newspaper's utility) is decreasing in connectivity when $A < 1$.

Propositions 4 and 5 imply the following result about the effect of connectivity on the probability of the correct decision:

Corollary 2. *Suppose voters fully neglect correlation. Then a shift from $\boldsymbol{\gamma}$ to $\tilde{\boldsymbol{\gamma}}$ increases Ψ if $A < 1$, reduces Ψ if $A > 1$, and leaves Ψ unchanged if $A = 1$. Furthermore, if $\gamma_N = 1$, then* $\lim_{N \to \infty} \Psi = 1 - (1-q)\,e^{-2\mu^2}$.

Corollary 2 implies that Condorcet jury theorem fails when voters neglect correlation, as the probability of an incorrect decision remains positive even when each voter observes a large number of signals.

5 Extensions

5.1 Partial Correlation Neglect

In our model of correlation neglect we have assumed that $k(n) = n$ – that is, that voters perceive any number of perfectly correlated signals as an equivalent number of independent signals. One can argue that this may not always be the case. For example, the strength of correlation neglect may decrease when the number of signals becomes larger.

As connectivity becomes large, the optimal bias in the limit depends on the behaviour of $k(n)$ when n is large. If k becomes flat when n is large – that is, if voters cease to perceive additional reports as independent when they observe many of them – then for large n a further increase in connectivity only increases the number of independent signals that voters receive, without increasing the newspaper's persuasive power. Then a similar characterisation to the

[17] Similar results emerge in Levy et al. (2018a), where independent signals do not exist, and the sender can approach her first-best when the number of messages she sends approaches infinity.

case with no correlation neglect emerges: an increase in connectivity reduces the optimal bias, which converges to zero as connectivity becomes arbitrarily large.

On the other hand, if k remains an increasing function when n is large, then an increase in connectivity continues to increase the newspaper's ability to manipulate beliefs. Hence, the optimal bias remains positive, and results from Section 4.2 continue to qualitatively hold.

The following proposition captures this intuition:

Proposition 6. *Suppose $\gamma_N = 1$. If $\lim_{N\to\infty} \frac{k(N)}{N} = L$ for some $L > 0$, then $\lim_{N\to\infty} p_0 = e^{-\frac{2\mu^2}{L}}$. If $\lim_{N\to\infty} \frac{k(N)}{N} = 0$, then $\lim_{N\to\infty} p_0 = 0$.*

Note that the bias in the limit is increasing in L, because the greater is correlation neglect in the limit, the more scope the newspaper has to send a biased report.

5.2 Generic Voting Rule

So far we have assumed that the newspaper aims to maximise the probability that at least $\frac{1}{2}$ of voters vote for the government. Suppose instead that the newspaper receives a payoff of 1 if and only if the share of voters who vote for the government is at least $\tau \in (0, 1)$.[18] For example, some voters may be partisans, who are willing to support or oppose the government regardless of the state. The newspaper then needs to persuade a sufficient fraction of the remaining voters to vote for the government. Alternatively, the newspaper may be trying to ensure that the fraction of voters who join an anti-government protest is below a certain threshold.

It is straightforward to show that Lemma 1 holds in this more general setting. At the same time, the condition that rules out babbling as an optimal editorial policy is different but similar to Assumption 1, as the following result shows:

Lemma 5. *An editorial policy (p_0, p_1) such that $p_0 = p_1$ is optimal if and only if*

$$\sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left[\frac{\frac{1}{2\mu}\ln A + (n+1)\mu}{\sqrt{n+1}}\right]\right) \geq \tau.$$

When babbling is not an optimal policy, the same approach as the one used to prove Lemma 4 and Proposition 1 then implies that the optimal editorial policy ensures that the government receives at least τ votes if and only if $r = 1$. Hence, at the optimum, the newspaper selects $p_1 = 1$ and p_0 such that

$$\sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left[\frac{\frac{1}{2\mu}\ln A + \frac{1}{2\mu}(n+1)\ln p_0 + (n+1)\mu}{\sqrt{n+1}}\right]\right) = \tau \tag{5}$$

It is straightforward to see that when τ increases, p_0 needs to decrease for (5) to continue to hold. Hence, a more restrictive voting rule reduces the optimal bias. Since the probability

[18] It is straightforward to show that if $\tau = 0$, any strategy that does not perfectly reveal the state guarantees the newspaper a payoff of 1, as a positive mass of voters will always receive sufficiently high signals. If $\tau = 1$, the only way for the newspaper to attain a payoff of 1 with positive probability is to reveal the true state when it equals 1 – hence, truthful revelation is an optimal strategy.

of the correct decision is decreasing in p_0, it follows that a more restrictive voting rule makes the correct decision more likely[19].

How does connectivity affect the optimal bias? We can show that the following result holds:

Proposition 7. *Suppose babbling is not optimal. If voters neglect correlation, then a shift from $\boldsymbol{\gamma}$ to $\tilde{\boldsymbol{\gamma}}$ reduces the optimal p_0 if $A < 1$ and $\tau \leq \frac{1}{2}$, and increases the optimal p_0 if $A > 1$ and $\tau \geq \frac{1}{2}$.*

This result parallels the result derived in Proposition 4. The latter showed that an increase in connectivity increases the optimal bias if preferences are ex ante unfavourable to the government, and vice versa. Proposition 7 shows that an increase in connectivity reduces the optimal bias if preferences are unfavourable, and, in addition, the newspaper has to overcome an unfavourable voting rule. Similarly, an increase in connectivity increases the optimal bias if preferences and the voting rule favour the government.

When preferences and the voting rule affect the newspaper in the opposite ways – that is, if $A < 1$ and $\tau > \frac{1}{2}$, or if $A > 1$ and $\tau < \frac{1}{2}$ – the effect of connectivity on the bias is, in general, non-monotone. Nevertheless, we can show that for any voting rule, as connectivity becomes arbitrarily large, the optimal bias converges to the same limit:

Proposition 8. *Suppose $\gamma_N = 1$. If voters neglect correlation, then $\lim_{N\to\infty} p_0 = e^{-2\mu^2}$.*

Hence, the result of Proposition 5 holds regardless of the voting rule. In other words, the bias converges to the same limit for all voting rules $\tau \in (0, 1)$.

5.3 Newspaper Receives Imprecise Signals

In our analysis so far the newspaper perfectly observed the true state θ. This enabled it to choose an editorial policy with arbitrary precision, and hence it was always able to persuade voters. But the assumption of a newspaper perfectly observing the state is strict. In this section we study the implication of a newspaper that receives a signal σ with precision $\alpha = Pr[\sigma = \theta] \in (\frac{1}{2}, 1)$.

Receiving an imprecise signal constrains the newspaper's editorial policy's maximum precision, and hence may also impede the newspaper's ability to persuade a majority of voters. Intuitively, when α is small, even truthfully revealing σ may not be sufficient to persuade voters, if voters are ex ante biased against the government ($q < \lambda$). If μ is small this may be true even independent of the state. For larger μ, however, the newspaper might be able to persuade the electorate in state 1, while in state 0 this remains impossible. The reason is that the voters' independent signals shift the distribution of beliefs by more when μ is larger. Finally, when α is large, persuasion is possible, i.e., persuasion is possible in both states as before. In this section we focus on the case where persuasion is *generally possible*, meaning that independent of the true state and the network structure the newspaper is able

[19] A similar result emerges in Alonso and Câmara (2016), in a setting without private information but with heterogeneous voters.

to persuade a majority of voters by choosing an appropriate editorial policy.[20] The following lemma establishes under which conditions this is the case:

Lemma 6. *For a given $\boldsymbol{\gamma}$ persuasion is possible in both states if and only if*

$$\sum_{n=0}^{N} \gamma_n \Phi\left(\frac{\frac{1}{2\mu}\ln A + \frac{1}{2\mu}(n+1)\ln\frac{1-\alpha}{\alpha} + (n+1)\mu}{\sqrt{n+1}}\right) < \frac{1}{2}.$$

A sufficient and necessary condition for persuasion to be generally possible, i.e., in both states and for any $\boldsymbol{\gamma}$, is

$$\max\{1, A\}\frac{1-\alpha}{\alpha} \leq e^{-2\mu^2}. \tag{6}$$

Intuitively, when α is close to 1 persuasion is generally possible, while it is generally impossible when $\alpha = \frac{1}{2}$. Moreover, as the precision of voters' independent signals increases, the newspaper's signal's precision may need to increase as well for the newspaper to remain able to persuade a majority of voters.

Note that the condition under which babbling is optimal is independent of the newspaper's signal's quality α, because when the newspaper babbles it does not transmit any information. Therefore, when Assumption 1 holds babbling cannot be optimal (see Lemma 2).

The next proposition shows that all of our earlier results qualitatively remain valid when Assumption 1 holds and when persuasion is generally possible.

Proposition 9. *Assume persuasion is generally possible for any $\boldsymbol{\gamma}$, i.e., that* (6) *holds. If voters neglect correlation, then a shift from $\boldsymbol{\gamma}$ to $\tilde{\boldsymbol{\gamma}}$ reduces the optimal p_0 if $A < 1$, increases the optimal p_0 if $A > 1$, and leaves the optimal p_0 unchanged if $A = 1$. If $\gamma_N = 1$, then $\lim_{N\to\infty} p_0 = \frac{e^{2\mu^2}(1-\alpha)-\alpha}{(1-\alpha)-e^{2\mu^2}\alpha} \in (0,1)$.*

When persuasion is generally possible, the bias is increasing (decreasing) in social connectivity if $A > 1$ ($A < 1$), and in the limit the optimal bias remains strictly positive.

Note that the bias in the limit is increasing in the newspaper's signals precision. That is, as the newspaper receives a more precise signal, it sends a less precise signal itself.

Proposition 9 further implies that the probability that voters collectively take the correct decision depends on social connectivity in the same way as before. Furthermore, even in the limit the probability that the collective decision is incorrect remains strictly positive:

Corollary 3. *Suppose voters neglect correlation. Then a shift from $\boldsymbol{\gamma}$ to $\tilde{\boldsymbol{\gamma}}$ increases Ψ if $A < 1$, reduces Ψ if $A > 1$, and leaves Ψ unchanged if $A = 1$. Furthermore, if $\gamma_N = 1$, then*

$$\lim_{N\to\infty} \Psi = 1 - (1-q)\frac{e^{2\mu^2}(1-\alpha)-\alpha}{(1-\alpha)-e^{2\mu^2}\alpha} \in (0,1).$$

[20] If persuasion is not generally possible, two distinct case can emerge. First, persuasion is generally not feasible. This is the case when α is very small, where small depends on the other parameters of the game such as q, μ, and λ. In this case any editorial policy yields the same outcome of zero utility to the newspaper and hence also any editorial policy is optimal. Second, it is possible that the newspaper can only persuade a majority in state $\theta = 1$, while it is impossible when $\theta = 0$. The reason is that when $\mu > 0$ and $\theta = 0$ the newspaper needs to be more persuasive than when $\theta = 1$, but it cannot send any signal with precision greater than $(1-\alpha)/\alpha > 0$. In this case the newspaper is indifferent between all sufficiently informative editorial policies, i.e., those policies that help the government to win if $\theta = 1$. One of these optimal editorial policies is truthfully revealing its signal σ, $p_1 = 1$ and $p_0 = 0$.

6 Conclusions

This paper developed a model of political persuasion in which voters observe a report about a state from a biased newspaper, as well as independent signals, and communicate this information to other voters on social media. The messages that each voter receives are thus correlated, because they are partly based on the newspaper's report. Voters ignore this correlation and form posterior beliefs about the state, before voting based on these beliefs. As connectivity increases, the newspaper optimally sends more biased or less biased reports, depending on the prior belief of the voters. At the same time, even when connectivity is large, the bias does not disappear in the limit, and voters make incorrect collective decision with positive probability.

Future research can extend this analysis in a number of ways. One potential extension could be to endogenise the decision of whether to read the newspaper. Another way to extend the model would be to consider a dynamic setting in which voters observe messages from their social media friends before choosing which social media connections to maintain in the next stage. This would enable the model to account for endogenous formation of social networks based on common political views.

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Appendix

Proof of Lemma 1. Take the editorial policy $(p_0, p_1) = (0, 1)$. Then $\pi_0 (p_0, p_1) = 0$ and $\pi_1 (p_0, p_1) = 1$. Therefore, all voters vote for the government if $\theta = 1$, and no voters vote for the government if $\theta = 0$. The newspaper's expected payoff then equals q. Consider a deviation to an editorial policy $(p_0, p_1) = (\varepsilon, 1)$ for some small $\varepsilon > 0$. Then with a positive probability, the newspaper sends message 1 in state 0. Note that

$$\lim_{\varepsilon \to 0} S_n (\varepsilon, 1) \equiv \lim_{\varepsilon \to 0} \left[\frac{1}{2\mu} \ln A + \frac{1}{2\mu} k (n+1) \ln (\varepsilon) \right] = -\infty$$

Hence, when $\varepsilon \to 0$, (1) implies that under editorial policy $(\varepsilon, 1)$ almost all voters vote for the government when $r = 1$. By continuity, there exists a sufficiently small $\hat{\varepsilon} > 0$ such that under editorial policy $(\hat{\varepsilon}, 1)$ at least half of voters vote for the government when $r = 1$. Then the newspaper's expected payoff under editorial policy $(\hat{\varepsilon}, 1)$ equals $q + (1-q)\hat{\varepsilon} > q$. Hence, editorial policy $(0, 1)$ is not optimal. □

Proof of Lemma 2. Suppose that $p_0 = p_1$. Then, $S_n (p_0, p_1) = S_n (1 - p_0, 1 - p_1) = \frac{1}{2\mu} \ln A$. Note that S is distributed normally with variance $n+1$ and mean $(n+1)\mu$ if $\theta = 1$, and mean $-(n+1)\mu$ if $\theta = 0$. Hence, the probability that a voter with n connections votes for the government in state θ is

$$1 - \Phi \left[\frac{\frac{1}{2\mu} \ln A - \theta (n+1) \mu + (1-\theta)(n+1)\mu}{\sqrt{n+1}} \right]$$

Thus, for each $\theta \in \{0, 1\}$ we have

$$V(\theta, 0) = V(\theta, 1) = \sum_{n=0}^{N} \gamma_n \left(1 - \Phi \left[\frac{\frac{1}{2\mu} \ln A - \theta (n+1) \mu + (1-\theta)(n+1)\mu}{\sqrt{n+1}} \right] \right)$$

This implies that $V(0,r) < V(1,r)$ for each $r \in \{0,1\}$. Furthermore, for each $r \in \{0,1\}$, we have

$$V(0,r) = \sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left[\frac{\frac{1}{2\mu}\ln A + (n+1)\mu}{\sqrt{n+1}}\right]\right)$$

If Assumption 1 holds, then $V(0,r) < \frac{1}{2}$. Thus, the share of voters that vote for the government can only reach $\frac{1}{2}$ in state 1. Hence, the payoff of the newspaper cannot exceed q. But q is also the payoff of the newspaper under editorial policy $(0,1)$. We have shown in Lemma 1 that the newspaper can guarantee itself a payoff strictly higher than q. Hence, babbling is not optimal. This proves the first statement.

To prove the second statement, note that if Assumption 1 is violated, then $\frac{1}{2} \leq V(0,r) \leq V(1,r)$. Hence, under babbling the newspaper receives a payoff of 1 with certainty. This is the highest payoff the newspaper can receive, so babbling is an optimal strategy. □

Proof of Lemma 3. If $\ln A \geq -2\mu^2$, then $\ln A \geq -2(n+1)\mu^2$, and hence $\frac{1}{2\mu}\ln A + (n+1)\mu \geq 0$ for all n, with strict inequality for $n > 0$. Hence, $\frac{\frac{1}{2\mu}\ln A+(n+1)\mu}{\sqrt{n+1}} \geq 0$, implying that $\Phi\left[\frac{\frac{1}{2\mu}\ln A+(n+1)\mu}{\sqrt{n+1}}\right] \geq \frac{1}{2}$ for all n, with strict inequality for $n > 0$. Thus,

$$\sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left[\frac{\frac{1}{2\mu}\ln A + (n+1)\mu}{\sqrt{n+1}}\right]\right) < \sum_{n=0}^{N} \gamma_n \frac{1}{2} = \frac{1}{2}$$

□

Proof of Lemma 4. Note that S is distributed normally with variance $n+1$ and mean $(n+1)\mu$ if $\theta = 1$, and mean $-(n+1)\mu$ if $\theta = 0$. Hence, the probability that (1) holds for a voter with n connections is

$$1 - \Phi\left[\frac{S_n(p_0,p_1) - \theta(n+1)\mu + (1-\theta)(n+1)\mu}{\sqrt{n+1}}\right]$$

Hence, if $p_0 = 0$, then $V(\theta,1) = 1, \forall\theta \in \{0,1\}$. If $p_0 > 0$, we have

$$V(\theta,1) = \begin{cases} \sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left[\frac{S_n(p_0,p_1)+(n+1)\mu}{\sqrt{n+1}}\right]\right) & \text{if } \theta = 0 \\ \sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left[\frac{S_n(p_0,p_1)-(n+1)\mu}{\sqrt{n+1}}\right]\right) & \text{if } \theta = 1 \end{cases} \tag{7}$$

Similarly, the probability that (2) holds for a voter with n connections is

$$1 - \Phi\left[\frac{S_n(1-p_0,1-p_1) - \theta(n+1)\mu + (1-\theta)(n+1)\mu}{\sqrt{n+1}}\right]$$

Hence, if $p_1 = 1$, then $V(\theta,0) = 0, \forall\theta \in \{0,1\}$. If $p_1 < 1$, we have

$$V(\theta,0) = \begin{cases} \sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left[\frac{S_n(1-p_0,1-p_1)+(n+1)\mu}{\sqrt{n+1}}\right]\right) & \text{if } \theta = 0 \\ \sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left[\frac{S_n(1-p_0,1-p_1)-(n+1)\mu}{\sqrt{n+1}}\right]\right) & \text{if } \theta = 1 \end{cases} \tag{8}$$

Comparing $V(\theta,0)$ and $V(\theta,1)$, we obtain

$$V(1,1) > \max\{V(1,0), V(0,1)\} > \min\{V(1,0), V(0,1)\} > V(0,0)$$

We will now show that a profitable deviation exists from any editorial policy that does not satisfy the condition in the lemma.

If $V(1,1) < \frac{1}{2}$, then the newspaper's payoff is 0, so the newspaper can gain by deviating to editorial policy $(0,1)$, which ensures it a payoff of q.

If $V(1,1) \geq \frac{1}{2}$ and $\max\{V(1,0), V(0,1)\} < \frac{1}{2}$, then the newspaper receives a payoff of 1 if and only if $\theta = 1$ and $r = 1$. Its expected utility is then $qp_1 \leq q$. But we have shown in Lemma 1 that the newspaper can guarantee itself a payoff strictly larger than q.

If $\min\{V(1,0), V(0,1)\} \geq \frac{1}{2}$ and $V(0,0) < \frac{1}{2}$, then the newspaper receives a payoff of 1 unless $\theta = 0$ and $r = 0$. The newspaper's payoff then equals $q + (1-q)p_0$. Furthermore, $V(1,0) \geq \frac{1}{2}$ implies that $p_1 < 1$, because otherwise $V(1,0) = 0$. Now consider a deviation to an editorial policy $(p_0', p_1') = \left(\frac{p_0}{p_1}, 1\right)$. This deviation does not affect the value of $S_n(p_0, p_1)$, and hence $V(1,1)$ and $V(0,1)$ remain unchanged. Furthermore, under this deviation in state 1 the newspaper always sends report 1. Hence, the newspaper always receives a payoff of 1 when $\theta = 1$, and when $\theta = 0$ it receives a payoff of 1 with probability $p_0' = \frac{p_0}{p_1}$. Its expected utility is thus $q + (1-q)\frac{p_0}{p_1}$, which is higher than the original utility since $p_1 < 1$.

Finally, if $V(0,0) \geq \frac{1}{2}$, we have

$$\frac{1}{2} \leq V(0,0) \leq \sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left[\frac{\frac{1}{2\mu}\ln A + (n+1)\mu}{\sqrt{n+1}}\right]\right)$$

which violates Assumption 1.

Hence, at the equilibrium we must have $V(1,1) > \max\{V(1,0), V(0,1)\} \geq \frac{1}{2} > \min\{V(1,0), V(0,1)\} > V(0,0)$. If $V(1,0) \geq \frac{1}{2} > V(0,1)$, then the newspaper receives a payoff of 1 if and only if $\theta = 1$. Then its payoff is equal to a payoff from truthfully revealing the state. But Lemma 1 shows that this payoff is not optimal. Hence, at the equilibrium, $V(1,1) > V(0,1) \geq \frac{1}{2}$, and $V(0,0) < V(1,0) < \frac{1}{2}$. □

Proof of Proposition 1. Consider any editorial policy (p_0, p_1) that satisfies Lemma 4. Suppose that $p_1 < 1$. Then the expected utility of the newspaper is $qp_1 + (1-q)p_0$. Consider a deviation to an editorial policy $(p_0', p_1') = \left(\frac{p_0}{p_1}, 1\right)$ (note that we have ruled out the case when $p_1 = 0$). This deviation leaves $S_n(p_0, p_1)$ unchanged, as it only depends on the ratio of p_0 and p_1. Hence $V(1,1)$ and $V(0,1)$, given by (7) in the proof of Lemma 4, do not change. Thus, (p_0', p_1') satisfies Lemma 4. The newspaper's expected utility under (p_0', p_1') then equals $qp_1' + (1-q)p_0' = q + (1-q)\frac{p_0}{p_1}$, which is higher than the utility under (p_0, p_1), as $p_1 < 1$. Hence, this deviation is profitable, so at the optimum we must have $p_1 = 1$.

The equilibrium expected utility of the newspaper then equals $q + (1-q)p_0$. The newspaper chooses p_0 to maximise it, subject to satisfying the condition given in Lemma 4. The maximum is attained when $V(0,1) = \frac{1}{2}$, which, given, $p_1 = 1$ is equivalent to the expression in the proposition.

Uniqueness follows from the fact that the left-hand side of that expression is monotone in p_0. Existence is guaranteed because the left-hand side of the expression approaches 1 as $p_0 \to 0$, and is smaller than $\frac{1}{2}$ when $p_0 = 1$ as long as Assumption 1 holds. □

Proof of Proposition 2. We can rewrite (3) as

$$\sum_{n=0}^{N} \gamma_n \Phi\left[\hat{h}(n)\right] = \frac{1}{2}$$

where

$$\hat{h}(n) \equiv \frac{1}{2\mu}\left[\ln A + \ln p_0\right](n+1)^{-\frac{1}{2}} + (n+1)^{\frac{1}{2}}\mu$$

Note that $\hat{h}(n)$ is increasing in p_0. If $\hat{h}(n)$ is increasing in n, then a shift from $\boldsymbol{\gamma}$ to $\tilde{\boldsymbol{\gamma}}$ increases the left-hand side of (3), so p_0 has to decrease to restore equality. Hence, to prove the result, it is sufficient to show that $\hat{h}(n)$ is increasing in n. Replacing n with a continuous variable and differentiating yields

$$\frac{d\hat{h}(n)}{dn} = -\frac{1}{4\mu}\left[\ln A + \ln p_0\right](n+1)^{-\frac{3}{2}} + \frac{1}{2}(n+1)^{-\frac{1}{2}}\mu$$

for this to be positive for all n, it is sufficient to have $\ln A + \ln p_0 \le 0$. Suppose the opposite is the case, i.e. $\ln A + \ln p_0 > 0$. Then $\hat{h}(n) > 0$ for all n. Hence,

$$\sum_{n=0}^{N} \gamma_n \Phi\left[\hat{h}(n)\right] > \sum_{n=0}^{N} \gamma_n \Phi(0) = \frac{1}{2}\sum_{n=0}^{N} \gamma_n = \frac{1}{2}$$

which violates (3). Hence, $\ln A + \ln p_0 \le 0$, implying that $\frac{d\hat{h}(n)}{dn} > 0$ for all n, and hence $\hat{h}(n)$ is increasing in n. □

Proof of Proposition 3. When $\gamma_N = 1$, Assumption 1 is equivalent to

$$\Phi\left[\frac{\frac{1}{2\mu}\ln A + (N+1)\mu}{\sqrt{N+1}}\right] > \frac{1}{2}$$

Since $\lim_{N\to\infty} \Phi\left[\frac{\frac{1}{2\mu}\ln A + (N+1)\mu}{\sqrt{N+1}}\right] = \lim_{N\to\infty} \Phi\left[\sqrt{N+1}\mu\right] = 1$, Assumption 1 holds when N is sufficiently large. Proposition 1 then implies that for sufficiently large N, p_0 is given by (3) as

$$\Phi\left[\frac{\frac{1}{2\mu}\ln A + \frac{1}{2\mu}\ln p_0 + (N+1)\mu}{\sqrt{N+1}}\right] = \frac{1}{2}$$

This is equivalent to

$$\frac{\frac{1}{2\mu}\ln A + \frac{1}{2\mu}\ln p_0 + (N+1)\mu}{\sqrt{N+1}} = 0$$

and hence

$$p_0 = \frac{1}{A}e^{-2(N+1)\mu^2}$$

which converges to 0 as $N \to \infty$. □

Proof of Corollary 1. The first statement follows from from Proposition 2 and the fact that Ψ is decreasing in p_0. The second statement follows from Lemma 3, which implies that $\lim_{N\to\infty} \Psi = \lim_{N\to\infty} [1 - (1-q)\, p_0] = 1$. □

Proof of Proposition 4. We can write (4) as

$$\sum_{n=0}^{N} \gamma_n \Phi \left[h\left(n\right)\right] = \frac{1}{2}$$

where

$$h\left(n\right) \equiv \frac{1}{2\mu} \left(n+1\right)^{-\frac{1}{2}} \ln A + \left(\frac{1}{2\mu} \ln p_0 + \mu\right) \left(n+1\right)^{\frac{1}{2}}$$

Note that $h\left(n\right)$ is increasing in p_0. If $h\left(n\right)$ is increasing in n, then a shift from $\boldsymbol{\gamma}$ to $\tilde{\boldsymbol{\gamma}}$ increases the left-hand side of (3), so p_0 has to decrease to restore equality. If $h\left(n\right)$ is decreasing in n, then a shift from $\boldsymbol{\gamma}$ to $\tilde{\boldsymbol{\gamma}}$ decreases the left-hand side of (3), so p_0 has to increase to restore equality. If $h\left(n\right)$ is constant in n, then a shift from $\boldsymbol{\gamma}$ to $\tilde{\boldsymbol{\gamma}}$ leaves the left-hand side of (3) unchanged, so p_0 remains unchanged.

Thus, to prove the result, it is sufficient to show that for all n, $h\left(n\right)$ is (i) strictly increasing in n if $A < 1$; (ii) strictly decreasing in n if $A > 1$; and (iii) constant in N if $A = 1$. Replacing n with a continuous variable and differentiating yields

$$\frac{dh\left(n\right)}{dn} = -\frac{1}{4\mu} \left(n+1\right)^{-\frac{3}{2}} \ln A + \frac{1}{2} \left(\frac{1}{2\mu} \ln p_0 + \mu\right) \left(n+1\right)^{-\frac{1}{2}}$$

Note that $h\left(n\right)$ is increasing (decreasing) in n if and only if the derivative is positive (negative) for all n.

To show (i), suppose that $A < 1$. Then $\ln A < 0$. To show that $\frac{dh(n)}{dn} > 0$ for all n, it is then sufficient to show that $\frac{1}{2\mu} \ln p_0 + \mu \geq 0$. Suppose the opposite is the case, i.e. $\frac{1}{2\mu} \ln p_0 + \mu < 0$. Then $h\left(n\right) < 0$ for all n. Hence,

$$\sum_{n=0}^{N} \gamma_n \Phi \left[h\left(n\right)\right] < \sum_{n=0}^{N} \gamma_n \Phi\left(0\right) = \frac{1}{2} \sum_{n=0}^{N} \gamma_n = \frac{1}{2}$$

which violates (4). Hence, $\frac{1}{2\mu} \ln p_0 + \mu \geq 0$, implying that $\frac{dh(n)}{dn} > 0$ for all n, and hence $h\left(n\right)$ is strictly increasing in n.

To show (ii), suppose that $A > 1$. Then $\ln A > 0$. To show that $\frac{dh(n)}{dn} < 0$ for all n, it is then sufficient to show that $\frac{1}{2\mu} \ln p_0 + \mu \leq 0$. Suppose the opposite is the case, i.e. $\frac{1}{2\mu} \ln p_0 + \mu > 0$. Then $h\left(n\right) > 0$ for all n. Hence,

$$\sum_{n=0}^{N} \gamma_n \Phi \left[h\left(n\right)\right] > \sum_{n=0}^{N} \gamma_n \Phi\left(0\right) = \frac{1}{2} \sum_{n=0}^{N} \gamma_n = \frac{1}{2}$$

which violates (4). Hence, $\frac{1}{2\mu} \ln p_0 + \mu \leq 0$, implying that $\frac{dh(n)}{dn} < 0$ for all n, and hence $h\left(n\right)$ is strictly decreasing in n.

To show (iii), suppose that $A = 1$. Then $\ln A = 0$. To show that $\frac{dh(n)}{dn} = 0$ for all n, it is then sufficient to show that $\frac{1}{2\mu} \ln p_0 + \mu = 0$. Suppose the opposite is the case, i.e. $\frac{1}{2\mu} \ln p_0 + \mu = B$ for some $B \neq 0$. Then $h(n) = B(n+1)^{\frac{1}{2}} \neq 0$, and

$$\sum_{n=0}^{N} \gamma_n \Phi[h(n)] > \sum_{n=0}^{N} \gamma_n \Phi\left[B(n+1)^{\frac{1}{2}}\right] = \Phi\left[B(n+1)^{\frac{1}{2}}\right] \neq \frac{1}{2}$$

which violates (4). Hence, $\frac{1}{2\mu} \ln p_0 + \mu = 0$, implying that $\frac{dh(n)}{dn} = 0$ for all n, and hence $h(n)$ is constant in n. □

Proof of Proposition 5. We have shown in the proof of Lemma 3 that when $\gamma_N = 1$, Assumption 1 holds for sufficiently large N. Proposition 1 then implies that for sufficiently large N, p_0 is given by (4) as

$$\Phi\left[\frac{\frac{1}{2\mu} \ln A + \frac{1}{2\mu}(N+1) \ln p_0 + (N+1)\mu}{\sqrt{N+1}}\right] = \frac{1}{2}$$

This is equivalent to

$$\frac{\frac{1}{2\mu} \ln A + \frac{1}{2\mu}(N+1) \ln p_0 + (N+1)\mu}{\sqrt{N+1}} = 0$$

Solving for p_0 yields

$$p_0 = A^{-\frac{1}{N+1}} e^{-2\mu^2}$$

which converges to $e^{-2\mu^2}$ as $N \to \infty$. □

Proof of Corollary 2. The first statement follows from Proposition 4 and the fact that Ψ is decreasing in p_0. The second statement follows from Lemma 3 and the fact that $\Psi = 1 - (1-q) p_0$. □

Proof of Proposition 6. We have shown in the proof of Lemma 3 that when $\gamma_N = 1$, Assumption 1 holds for sufficiently large N. Then, given Proposition 1 and our expression for $S_n(p_0, p_1)$, the optimal value of p_0 is given by

$$\Phi\left[\frac{\frac{1}{2\mu} \ln A + \frac{1}{2\mu} k(n+1) \ln p_0 + (n+1)\mu}{\sqrt{n+1}}\right] = \frac{1}{2}$$

Equivalently,

$$\frac{\frac{1}{2\mu} \ln A + \frac{1}{2\mu} k(n+1) \ln p_0 + (n+1)\mu}{\sqrt{n+1}} = 0$$

and hence

$$\ln p_0 = \frac{-\ln A - (N+1) 2\mu^2}{k(N+1)}$$

Thus,

$$\lim_{N\to\infty} \ln p_0 = \lim_{N\to\infty} -2\mu^2 \frac{N}{k(N)}$$

If $\lim_{N\to\infty} \frac{k(N)}{N} = L$, then $\lim_{N\to\infty} \ln p_0 = -\frac{2\mu^2}{L}$, so $\lim_{N\to\infty} p_0 = e^{-\frac{2\mu^2}{L}}$. If $\lim_{N\to\infty} \frac{k(N)}{N} = 0$, then $\lim_{N\to\infty} \ln p_0 = -\infty$, so $\lim_{N\to\infty} p_0 = 0$. □

Proof of Lemma 5. Analogous to the proof of Lemma 2, with τ replacing $\frac{1}{2}$. □

Proof of Proposition 7. We can write (5) as

$$\sum_{n=0}^{N} \gamma_n \Phi\left[h(n)\right] = 1 - \tau$$

where $h(n)$ is defined as in the proof of Proposition 4. Following the logic of that proof, it is sufficient to show that for all n, $h(n)$ is (i) strictly increasing in n if $A < 1$ and $\tau \leq \frac{1}{2}$; and (ii) strictly decreasing in n if $A > 1$ and $\tau \geq \frac{1}{2}$. Replacing n with a continuous variable and differentiating yields

$$\frac{dh(n)}{dn} = -\frac{1}{4\mu}(n+1)^{-\frac{3}{2}} \ln A + \frac{1}{2}\left(\frac{1}{2\mu}\ln p_0 + \mu\right)(n+1)^{-\frac{1}{2}}$$

Note that $h(n)$ is increasing (decreasing) in n if and only if the derivative is positive (negative) for all n.

To show (i), suppose that $A < 1$ and $\tau \leq \frac{1}{2}$. Then $\ln A < 0$. To show that $\frac{dh(n)}{dn} > 0$ for all n, it is then sufficient to show that $\frac{1}{2\mu}\ln p_0 + \mu \geq 0$. Suppose the opposite is the case, i.e. $\frac{1}{2\mu}\ln p_0 + \mu < 0$. Then $h(n) < 0$ for all n. Hence,

$$\sum_{n=0}^{N} \gamma_n \Phi\left[h(n)\right] < \sum_{n=0}^{N} \gamma_n \Phi(0) = \frac{1}{2}\sum_{n=0}^{N} \gamma_n = \frac{1}{2} \leq 1 - \tau$$

which violates (4). Hence, $\frac{1}{2\mu}\ln p_0 + \mu \geq 0$, implying that $\frac{dh(n)}{dn} > 0$ for all n, and hence $h(n)$ is strictly increasing in n.

To show (ii), suppose that $A > 1$ and $\tau \leq \frac{1}{2}$. Then $\ln A > 0$. To show that $\frac{dh(n)}{dn} < 0$ for all n, it is then sufficient to show that $\frac{1}{2\mu}\ln p_0 + \mu \leq 0$. Suppose the opposite is the case, i.e. $\frac{1}{2\mu}\ln p_0 + \mu > 0$. Then $h(n) > 0$ for all n. Hence,

$$\sum_{n=0}^{N} \gamma_n \Phi\left[h(n)\right] > \sum_{n=0}^{N} \gamma_n \Phi(0) = \frac{1}{2}\sum_{n=0}^{N} \gamma_n = \frac{1}{2} \geq 1 - \tau$$

which violates (4). Hence, $\frac{1}{2\mu}\ln p_0 + \mu \leq 0$, implying that $\frac{dh(n)}{dn} < 0$ for all n, and hence $h(n)$ is strictly decreasing in n. □

Proof of Proposition 8. When $\gamma_N = 1$, the condition in Lemma 5 is equivalent to

$$\Phi\left[\frac{\frac{1}{2\mu}\ln A + (N+1)\mu}{\sqrt{N+1}}\right] > 1 - \tau$$

Since $\lim_{N\to\infty}\Phi\left[\frac{\frac{1}{2\mu}\ln A+(N+1)\mu}{\sqrt{N+1}}\right] = \lim_{N\to\infty}\Phi\left[\sqrt{N+1}\mu\right] = 1$, thus condition holds when N is sufficiently large. Thus, babbling is not optimal, and the optimal bias is defined by (5) as

$$\Phi\left[\frac{\frac{1}{2\mu}\ln A + \frac{1}{2\mu}(N+1)\ln p_0 + (N+1)\mu}{\sqrt{N+1}}\right] = 1 - \tau$$

which is equivalent to

$$\frac{\frac{1}{2\mu}\ln A + \frac{1}{2\mu}(N+1)\ln p_0 + (N+1)\mu}{\sqrt{N+1}} = \Phi^{-1}(1-\tau)$$

and hence

$$\ln p_0 = -\frac{1}{N+1}\ln A - 2\mu^2 + 2\mu^2\frac{1}{\sqrt{N+1}}\Phi^{-1}(1-\tau)$$

and hence

$$p_0 = A^{-\frac{1}{N+1}}e^{-2\mu^2+2\mu^2\frac{1}{\sqrt{N+1}}\Phi^{-1}(1-\tau)}$$

which converges to $e^{-2\mu^2}$ as $N \to \infty$. $\square$

Proof of Lemma 6. Note that for any $\mu > 0$ and a given report sent by the newspaper r, the share of voters voting for the government is greater if $\theta = 1$ than if $\theta = 0$. Hence, if it is possible to convince a majority of voters in state $\theta = 0$ if is also possible to do so in state $\theta = 0$. Hence, to prove the lemma it suffices to provide a condition such that persuasion is possible if $\theta = 0$.

The newspaper is maximally persuasive when it reports it's signal truthfully, $p_1 = 1$ and $p_0 = 0$. However, in this case it never sends report $r = 1$ in state $\theta = 0$. Hence, consider $p_0 = \epsilon$. When $\epsilon \to 0$ implies that a majority of voters votes for the government in state $\theta = 0$, persuasion is possible in both states.

When $p_1 = 1$ and $p_0 = 0$, the belief after receiving report $r = 1$ and evidence S is

$$\pi = \frac{q}{q + (1-q)\left(\frac{1-\alpha}{\alpha}\right)^{n+1}e^{-2S\mu}}.$$

Thus, a voter with n connections needs to receive at least at evidence $S_n(1-\alpha,\alpha)$, where S_n is defined as above. This implies that the share of voters voting for the government is

$$\sum_{n=0}^{N}\gamma_n\left(1-\Phi\left(\frac{S_n(1-\alpha,\alpha)+(n+1)\mu}{\sqrt{n+1}}\right)\right).$$

Using the definition of $S_n(1-\alpha,\alpha)$, this policy of reporting the own signal truthfully achieves the newspaper a majority if and only if

$$\begin{aligned} \sum_{n=0}^{N} \gamma_n \left(1 - \Phi\left(\tfrac{S_n(1-\alpha,\alpha)+(n+1)\mu}{\sqrt{n+1}}\right)\right) &> \tfrac{1}{2} \\ \Leftrightarrow \sum_{n=0}^{N} \gamma_n \Phi\left(\tfrac{S_n(1-\alpha,\alpha)+(n+1)\mu}{\sqrt{n+1}}\right) &< \tfrac{1}{2} \\ \Leftrightarrow \sum_{n=0}^{N} \gamma_n \Phi\left(\tfrac{\frac{1}{2\mu}\ln A+\frac{1}{2\mu}(n+1)\ln\frac{1-\alpha}{\alpha}+(n+1)\mu}{\sqrt{n+1}}\right) &< \tfrac{1}{2} \end{aligned} \tag{9}$$

The strict inequality follows from the fact that $p_0 = \epsilon > 0$, as otherwise the probability of $r = 1$ in state $\theta = 0$ is zero. If the inequality would be weak there might be no $p_0 > 0$ such that at least half of the voters vote for the government.

The lemma states that a necessary and sufficient condition for (9) to hold for *any* $\boldsymbol{\gamma}$ is $\max\{1, A\}\frac{1-\alpha}{\alpha} \leq e^{-2\mu^2}$. We prove both parts separately.

Sufficiency: To see that this is sufficient observe that Φ is a strictly increasing function. If no voter is connected to another voter, $\gamma_0 = 1$, (9) simplifies to

$$\begin{aligned} \Phi\left(\tfrac{1}{2\mu}\ln A + \tfrac{1}{2\mu}\ln\tfrac{1-\alpha}{\alpha} + \mu\right) < \tfrac{1}{2} \Leftrightarrow \tfrac{1}{2\mu}\ln A + \tfrac{1}{2\mu}\ln\tfrac{1-\alpha}{\alpha} + \mu < 0 \\ \Leftrightarrow \ln A + \ln\tfrac{1-\alpha}{\alpha} < -2\mu^2 \Leftrightarrow A\tfrac{1-\alpha}{\alpha} < e^{-2\mu^2} \end{aligned}$$

This is the condition stated in the lemma if $\max\{1, A\} = A$. If $\kappa := \frac{1}{2\mu}\ln A + \frac{1}{2\mu}(n+1)\ln\frac{1-\alpha}{\alpha} + (n+1)\mu$ weakly decreases in n, truthfully revealing its signal persuades a majority of voters for *any* number of connections n, and hence persuasion is generally possible. Approximate n by a continuous variable and take the derivative of κ with respect to n:

$$\frac{\partial\kappa}{\partial n} = \frac{1}{2\mu}\ln\frac{1-\alpha}{\alpha} + \mu.$$

For this to be weakly negative it has to hold that $e^{-2\mu^2} \geq \frac{1-\alpha}{\alpha}$, which is the stated condition if $\max\{A, 1\} = 1$. Thus, if $\max\{1, A\}\frac{1-\alpha}{\alpha} < e^{-2\mu^2}$, persuasion is feasible for any $\boldsymbol{\gamma}$, and hence it is generally feasible. Note that when $\alpha = 1$, the inequality always holds as $e^{-2\mu^2} > 0$.

Necessity: Necessity follows immediately. Assume the condition was violated and that $\max\{1, A\}\frac{1-\alpha}{\alpha} > e^{-2\mu^2}$. If $\max\{1, A\} = 1$, κ linearly increases in n and converges to positive infinity. This means that for $\gamma_N = 1$ and sufficiently large N persuasion is impossible. To the contrary, if $\max\{1, A\} = A$ and $\max\{1, A\}\frac{1-\alpha}{\alpha} > e^{-2\mu^2}$, then for $\gamma_N = 1$ and $N = 0$ persuasion is impossible. Finally, note that when $\max\{1, A\}\frac{1-\alpha}{\alpha} = e^{-2\mu^2}$ truthfully revealing natures signal might persuade a majority of voters for any $\boldsymbol{\gamma}$, but since $p_0 = 0$ the newspaper would never send this signal, and hence persuasion would be possible only in state $\theta = 1$. Hence, $\max\{1, A\}\frac{1-\alpha}{\alpha} < e^{-2\mu^2}$ is necessary for persuasion to be possible. □

Proof of Proposition 9. Given that the state is $\theta = 1$, the newspaper sends report $r = 1$ with probability $z_1 = \alpha p_1 + (1-\alpha)p_0$. In state $\theta = 0$ this probability is $z_0 = \alpha p_0 + (1-\alpha)p_1$. Thus, the belief of a voter with n connections and independent evidence S after report r are

$$\pi_1 = \frac{q}{q + (1-q)\left(\frac{z_0}{z_1}\right)^{n+1} e^{-2\mu S}}$$

and

$$\pi_0 = \frac{q}{q + (1-q)\left(\frac{1-z_0}{1-z_1}\right)^{n+1} e^{-2\mu S}}.$$

These beliefs are structurally identical to before except that now $p_i = z_i$. Thus, the newspaper now chooses z_i instead of p_i. Recall that we assume without loss of generality that $p_1 \geq p_0$, which implies that $z_0 \leq z_1$. When $\alpha = 1$, the newspaper can choose any $\frac{z_0}{z_1} \in [0,1]$. However, when $\alpha \in (\frac{1}{2}, 1)$ this changes, as

$$\frac{z_0}{z_1} = \frac{\alpha p_0 + (1-\alpha)p_1}{\alpha p_1 + (1-\alpha)p_0} = \frac{\alpha\rho + (1-\alpha)}{\alpha + (1-\alpha)\rho} \in \left[\frac{1-\alpha}{\alpha}, 1\right],$$

where $\rho \equiv p_0/p_1 \in [0,1]$. Thus, as we have noted before, the newspaper now has to choose a likelihood ratio z_0/z_1 from a constrained set. This impacts the newspaper because it might not be able to persuade a majority of voters in general. However, under the conditions of Lemma 6 persuasion is generally possible.

To prove that a unique optimal editorial policy exists when persuasion is generally possible we follow similar steps as those leading to Proposition 1. First note that truth telling cannot be optimal, because, as before, this would imply a probability that the government wins of q. A very small deviation from this to $p_1 = 1$ and $p_0 = \epsilon$ yields a greater probability for the government to win, and hence truthfully revealing σ cannot be optimal. Moreover, also as before, when a policy of babbling is not optimal ($p_0/p_1 = z_0/z_1 = 1$), the newspaper can only persuade a majority after sending report $r = 1$ (see Lemma 4). Hence, the optimal editorial policy leads to vote shares $V(1,1) > V(0,1) \geq \frac{1}{2}$ and $\frac{1}{2} > V(1,0) > V(0,0)$. We can thus also proceed analogous to Proposition 1 to prove that the optimal editorial policy must have $p_1 = 1$ and $p_0 \in (0,1)$ such that $V(0,1) = \frac{1}{2}$.

Note that this also implies that Proposition 4 remains valid if persuasion is generally possible. In this case we can go through its proof step by step while replacing p_i with z_i, as general persuadability assures that the optimal ratio z_0/z_1 is feasible.

Finally, if $\gamma_N = 1$, the optimal editorial policy follows from

$$1 - \Phi\left(\frac{S_N(z_0,1) + \mu(n+1)}{\sqrt{n+1}}\right) = \frac{1}{2} \Leftrightarrow S_N(z_0,1) = -\mu(n+1).$$

Solving for p_0 we find

$$p_0 = \frac{\alpha A^{\frac{1}{N+1}} - e^{2\mu^2} + \alpha e^{2\mu^2}}{\alpha A^{\frac{1}{N+1}} - A^{\frac{1}{N+1}} + \alpha e^{2\mu^2}}$$

In the limit, as $N \to \infty$, we get

$$\lim_{N\to\infty} p_0 = \frac{\alpha - e^{2\mu^2} + \alpha e^{2\mu^2}}{\alpha - 1 + \alpha e^{2\mu^2}}$$

This is strictly between 0 and 1 if $\frac{1-\alpha}{\alpha} < e^{-2\mu^2}$, which follows from the condition in Lemma 6. □