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The Victor Popov Academy

30 November 2019

Online at <https://mpra.ub.uni-muenchen.de/97657/>

MPRA Paper No. 97657, posted 18 Dec 2019 12:28 UTC

# Observing the Evolution in Macroeconomic Theory

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**Abstract:** The principal purpose of the given work is to summarize certain observations on the evolution of thought in macroeconomic theory with the original (rather than conventional) notation where appropriate. The observations are organized by topic and supplied with respective references.

**JEL Classification Numbers:** B20, E12, E19, E21, E24, E31, E43, E49, E52

**Keywords:** Economic Thought, Economic History, Macroeconomic Models, Models Derivation, Notation, Expectations, Growth, Consumption, Unemployment, Inflation, Random Walk Hypothesis, Money, Natural Rate

## 1. Expectations

Rational behavior suggests that economic agents intent to choose actions that maximize their benefits (minimize costs) in prolonged periods of time. To be able to evaluate the optimality of decisions affecting their future in present, agents need information about relevant conditions and trajectory of change in such conditions in future. Expectations are beliefs used by agents as substitutes to factual information about said conditions. Accounting for expectations leads to more detailed and correct reflection of agents' behavior in economic models.

Consider, for example, the CBOE Volatility index (VIX) futures fair value model

$$FV=f(P_t, \text{var}_t[F_T])=100\text{sqrt}((365-30)(P_t-\text{var}_t[F_T]))$$

where FV is the fair value of the futures contract on VIX,  $P_t$  – forward price of de-annualized variance in the 30 days after the futures expiration, and  $\text{var}_t[F_T]$  - concavity adjustment (“Additional Features of VIX Futures”).

Since the settlement is conditioned on the “spot” value of VIX, it seems reasonable to expect actual futures prices to follow the model quite closely, as deviations will create arbitrage opportunity. In reality, however, this is not always the case (Rhoads 2015). Speculating on the nature of such deviations, one can observe that although VIX itself is influenced by expectations about the underlying market, there is no easy way to “spot” trade and directly affect its value.

Expectations about VIX, though not reflected in its value, can nevertheless be observed in its derivatives' prices. Therefore, CBOE futures model could be modified from  $FV=f(P_t, \text{var}_t[F_T])$  to  $FV=f(P_t, \text{var}_t[F_T], E_t)$ , where  $E_t$  stands for expectations over the “spot” value of VIX at time  $t$ . Augmented by expectations, such model is expected to be reflect the instrument more accurately.

The early use of expectations can be found in Cobweb modeling of agriculture markets, essentially adding the expected price at time  $t+1$  terms to conventional supply and demand functions (Kaldor 1934). Later, the adaptive expectations model was introduced by Philip Cagan and further developed by Milton Friedman to reflect permanent or expected income  $Y_p$  (Chow 2011):

$$Y_p - Y_p(t-1) = b(Y - Y_p(t-1)) = bY + (1-b)Y_p(t-1) = bY(t) + b(1-b)Y(t-1) + b(1-b)^2Y(t-2) + \dots$$

Dissatisfaction with the way the existed models reflected the dynamics, John Muth (1961) and later Robert Lucas (1976) put emphasis on the information circulation and the mechanics of decision making within the system. The resulting rational expectations approach was derived directly from the concept of mathematical expectation.

Consider the following form of the model (Nimark 2016):

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + k(y_t - \bar{y}_t) \\ y_t &= E_t y_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \\ i_t &= \phi \pi_t \\ \bar{y}_t &= p \bar{y}_{t-1} + u_t: u_t \sim N(0, \sigma_u^2),\end{aligned}$$

where  $\pi_t$  is inflation,  $y_t$  – output,  $\bar{y}_t$  – potential output and  $i_t$  – interest rate.

$$\begin{aligned}\bar{y}_t &= b \bar{y}_t & a \bar{y}_t &= \beta a p \bar{y}_t + k(b \bar{y}_t - \bar{y}_t) & a - \beta a p - k b &= -k \\ \pi_t &= a \bar{y}_t & \rightarrow & b \bar{y}_t = b p \bar{y}_t - \sigma(\phi a \bar{y}_t - a p \bar{y}_t) & \rightarrow & b - b p + \sigma \phi a - \sigma a p = 0 & \rightarrow \\ y_t &= b \bar{y}_t\end{aligned}$$

$$\rightarrow \begin{pmatrix} 1 - \beta p & k \\ \sigma \phi - \sigma p & 1 - p \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -k \\ 0 \end{pmatrix} \rightarrow \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 - \beta p & -k \\ \sigma \phi - \sigma p & 1 - p \end{pmatrix}^{-1} \begin{pmatrix} -k \\ 0 \end{pmatrix} =$$

$$= \begin{pmatrix} -k \frac{p-1}{p + \beta p - \beta p^2 - k \sigma \phi + k \sigma p - 1} \\ -k \frac{\sigma \phi - \sigma p}{p + \beta p - \beta p^2 - k \sigma \phi + k \sigma p - 1} \end{pmatrix}$$

#### References:

“Additional Features of VIX Futures.” *CBOE Futures Exchange*, CBOE, [cfe.cboe.com/cfe-education/cboe-volatility-index-vx-futures/vix-primer/vix-features](http://cfe.cboe.com/cfe-education/cboe-volatility-index-vx-futures/vix-primer/vix-features).  
Chow, Gregory. *Usefulness of Adaptive and Rational Expectations in Economics*. Working paper no. 221. Princeton University, Sept. 2011. Web. 16 Oct. 2017.

- Kaldor, Nicholas. "A Classificatory Note on the Determinateness of Equilibrium." *The Review of Economic Studies*, vol. 1, no. 2, Feb. 1934, pp. 122–136. JSTOR.
- Lucas, Robert E. "Econometric policy Evaluation: A Critique." *Carnegie-Rochester Conference Series on Public Policy*, vol. 1, 1976, pp. 19-46. ScienceDirect.
- Muth, John F. "Rational Expectations and the Theory of Price Movements." *Econometrica*, vol. 29, no. 3, 1961, pp. 315–335. JSTOR.
- Nimark, Kristoffer P. "Solving Linear Rational Expectations Models." *Kristoffer Nimark's Economics etc*, 2016,  
[www.kris-nimark.net/Applied\\_Macro\\_2016/Solving\\_RE\\_notes.pdf](http://www.kris-nimark.net/Applied_Macro_2016/Solving_RE_notes.pdf).
- Rhoads, Russell. "The History of VIX Contango and Backwardation." *CBOE Blogs*, CBOE, 30 Aug. 2015, [www.cboe.com/blogs/options-hub/2015/08/30/the-history-of-vix-contango-and-backwardation](http://www.cboe.com/blogs/options-hub/2015/08/30/the-history-of-vix-contango-and-backwardation).

## 2. Solow Growth Model

The demand for a new model of economic growth in the mid-1900s can be associated with criticism of once dominant Harrod-Domar model. In particular, this neo-classical model was criticized for its adjustment process depiction (Sato 1964). Solow model, at least in part, can be considered an answer to such demand.

Assuming single commodity and implying closed economy with no government intervention, Solow (1956) treats flow of capital over time as augmented function of output saved ( $s$ ), namely

$$dK/dt = sF(K, L_0 e^{(nt)})$$

where the exogenous growth of initial labor force  $L_0$  is represented by  $n$  (the model was further extended to include multiplier  $A(t)$ , so that  $Q = A(t)f(K, L)$ ). Introducing capital-labor ratio  $r = K/L$ ,  $K = rL = rL_0 e^{(nt)}$ , we are led to

$$dK/dt = L_0 e^{(nt)} (dr/dt) + nrL_0 e^{(nt)}$$

and, consequently, to

$$\left(\frac{dr}{dt} + nr\right)L_0 e^{(nt)} = sL_0 e^{(nt)} F\left(\frac{K}{L_0 e^{(nt)}}, 1\right)$$

for constant returns to scale. This leads to the final differential equation

$$\frac{dr}{dt} = sF(r, 1) - nr$$

where  $s$  is the exogenous savings rate.

Steady-state can be found at the intersection  $nr = sF(r, 1)$  where  $\frac{dr}{dt} = 0$  yields us  $r^*$ ; that is, the equilibrium ratio of capital to labor. Using constant return to scale that Solow (1957) considers “practically unavoidable”, such  $r^*$  will be stable, so that for other  $r \neq r^*$  there will be tendency to converge to  $r^*$  (unless the assumptions on the shape of productivity curve  $F(r, 1)$  will allow other equilibria as well).

The following Inada (1963) conditions are applied:  $f'(k) > 0$  (here and in other literature  $k$  is conveniently used for capital-labor ratio, that is,  $k = K/L$ ),  $f''(k) < 0$ ,  $\lim_{k \rightarrow 0} f(k) = 0$  (Inada originally used wage-rental ratio that can be skipped here for simplicity's sake),  $\lim_{k \rightarrow 0} f'(k) = \infty$  and  $\lim_{k \rightarrow \infty} f'(k) = 0$ .

With this, the equation can be rewritten as  $\frac{dk}{dt} = sf(k) - nk$  or, more extensively, as  $\frac{dk}{dt} = sAk(t) - (d+n)k(t)$ . Solving, we set  $sAk^* = (d+n)k^*$  we are led to  $k^* = (sA)/(d+n)$ ,  $s^* = (d+n)/A$ . Returning to Solow's original notation, one can define consumption as  $c = F(r, 1) - (n+d)r$ , where  $d$  represents exogenous capital depreciation rate. Maximizing in  $c$ , one is led to  $F'(r) = n+d$ ; hence the optimal “golden rule” savings rate  $s^* = ((n+d)r^*)/F(r^*)$ .

One important consideration of Solow model corresponds to the standard utility maximization logic. As additional output that is not consumed is impractical, we consider the set of conditions that maximizes consumption over time to be optimal. The model's optimality was explored by Phelps (1961) by elaborating on the example of the Golden Age (hence the name) in the fictional Kingdom of Solovia. Phelps defines consumption as  $c = (1-s)f(s)e^{(gt)}$ , where  $g$  stands for parametric “natural rate of output growth” (exogenous). The consumption function is further maximized to yield

$$-f(s)e^{gt}+(1-s)f'(s)e^{gt}=0 \quad \rightarrow \quad f(s)=(1-s)f'(s),$$

which is indeed maximum by the second derivative test. One advantage of this form is that it clearly shows the consequences of excessive savings (so  $1-s=0$ ).

The significance of the model was hard to overestimate at its time, as it attempted to answer questions about the disparities in economic growth between different countries and economic systems. The other significant features of the model are its non-linearity and dynamic nature. Later more sophisticated dynamic models of Rebelo, Romer and Lucas seem to be at least in part based on Solow's findings.

One major criticism of Solow model is its treatment of factors augmenting labor and capital as exogenous (the rate of technical progress). As the economy became less industrialized by the end of the century, the significance of ideas and knowledge demanded greater attention. Models of Endogenous growth theory attempted to solve this problem by expanding the understanding of such augmentation and inserting it inside the model (Romer 1986).

#### References:

- Inada, Ken-Ichi. "On a Two-Sector Model of Economic Growth: Comments and a Generalization." *The Review of Economic Studies*, vol. 30, no. 2, Jun. 1963, pp. 119-127. *JSTOR*.
- Phelps, Edmund. "The Golden Rule of Accumulation: A Fable for Growthmen." *The American Economic Review*, vol. 51, no. 4, Sept. 1961, pp. 638-643. *JSTOR*.
- Romer, Paul M. "Increasing Returns and Long-Run Growth." *Journal of Political Economy*, vol. 94, no. 5, Oct. 1986, pp. 1002-1037. *JSTOR*.
- Sato, Ryuzo. "The Harrod-Domar Model vs the Neo-Classical Growth Model." *The Economic Journal*, vol. 74, no. 294, June 1964, pp. 380-387. *JSTOR*.
- Solow, Robert M. "A Contribution to the Theory of Economic Growth." *The Quarterly Journal of Economics*, vol. 70, no. 1, Feb. 1956, pp. 65-94. *JSTOR*.
- Solow, Robert M. "Technical Change and the Aggregate Production Function." *The Review of Economics and Statistics*, vol. 39, no. 3, Aug. 1957, pp. 312-320. *JSTOR*.

### 3. Consumption

Keynes (1936) inserts consumption in macroeconomic theory by connecting it to net income. The attention is focused on rate of change of consumption with respect to such income or marginal propensity to consume. This derivative is treated as a function of income itself leading to further analysis of its response to income-targeting government policies.

The theory became subject to some critical scrutiny, namely with respect to its applicability in the long run (Kuznets 1946). Keynes' implications can be also questioned in the context of expectations in the way government may (or may not) affect the consumption by counter-cyclical measures.

Friedman (1957) introduced more detailed the concept of permanent income hypothesis. The model accounting for permanent (p) and transitory (t) components separately, as well as interest rate (i), general wealth to income ratio (w) and preferences (u):

$$C_p = k(i, w, u) Y_p$$

$$Y = Y_p + Y_t$$

$$C = C_p + C_t$$

Assuming no uncertainty (with regard to future income, interest rate etc.), linearity in form of  $C = \alpha + \beta Y$  and no-Ponzi condition, Friedman suggested conventional least square estimation of  $\beta$ , so that  $b = k P_y$ .

Several of the hypothesis' assumptions were questioned over time. The assumption of linearity implied non-decreasing marginal utility; Hall (1978), however, shows that quadratic function leads to the same conclusions. Relatively recent studies (Gross and Souleles 2001) brought up the significance of liquidity constraint through the difference between the average market interest rates and the actual rates available for consumption borrowing. The analysis of discriminated access to loanable funds in this manner, however, does not seem to cover the whole picture. Simply speaking, the borrowers paying 14-16% credit card interest are just as rational as the savers who keep positive balance on savings accounts yielding 0.01%. The analysis of would be borrowers discouraged by high borrowing cost case also does not seem reasonable without mirroring analysis of would be savers discouraged by low savings yields.



Adding life length ( $\tau$ ), time preference ( $\delta$ ) and constant real interest rate ( $r$ ), Hall extended the Friedman's approach by optimizing the long-run expected utility  $E_t \sum_{\tau=0}^{T-t} (1 + \delta)^{-\tau} u(c_{t+\tau})$  subject to life-long budget constraint  $A_t = \sum_{\tau=0}^{T-t} (1 + r)^{-\tau} u(c_{t+\tau} - w_{t-\tau})$  under uncertainty (so that the expectations are conditioned on information available at time  $t$ ):

$$V(A_t, Y_t) = \max(u(c_t) + \beta E_t(V(A_{t+1}, Y_{t+1}))) \text{ s.t. } A_{t+1} = Y_t + (1+r)A_t - c_t \text{ (where } A \text{ is assets)}$$

$$\text{in certainty: } V(A_t, Y_t) = \max(u(c_t) + \beta E_t(V(A_{t+1}, Y_{t+1}))) \text{ s.t. } \sum_{t=1}^T c_t \leq A_0 + \sum_{t=0}^T Y_t$$

Hall finds that once  $C_t$  is known, neither wealth nor income at periods  $t, t-1, \dots$  are relevant. In the empirical part of his paper, Hall rejects the absolute income hypothesis but not permanent income hypothesis, and describes the consumption as random walk with trend (i.e. consumption smoothing).

Later studies by Hall and Mishkin (1982) and Bernanke (1984) kept rejecting the pattern, although more sophisticated model accounting for two different kinds of consumers (PIH abiding and not):

$$\begin{aligned} \Delta c_t &= \alpha(\varphi \varepsilon_{t+1} + (1-\varphi)\varepsilon_t) + \mu\alpha(\varphi \Delta y_{t+1}^s + (1-\varphi)\Delta y_t^s) + (1-\mu)\alpha\beta(\varphi \eta_{t+1} + (1-\varphi)\eta_t) + \Delta y_t^s \\ \Delta y_t &= \varepsilon_t + \Delta y_t^s = \varepsilon_t + \eta_t - (1-\rho_1)\eta_{t-1} - (\rho_1 - \rho_2)\eta_{t-2} - \rho_2^2 \eta_{t-3}, \end{aligned}$$

where  $\varepsilon$  and  $\eta$  – unpredictable random innovations (stochastic income components) and  $\mu$  – income-tracking consumption (so that  $1-\mu$  – LC-PIH consumption).

Campbell and Mankiw (1989), however, reach different conclusion using instrumental variable approach. After highlighting the way to receive somewhat significant pro-Keynesian results, they combine  $\Delta c_y = \mu + \lambda \Delta y_t$  and  $\Delta c_y = \mu + \sigma r_t + \varepsilon_t$  into  $\Delta c_y = \mu + \lambda \Delta y_t + (1-\lambda)\sigma r_t + \varepsilon_t$ , thus formulating the consumption function of the form  $c_{t-y_t} = E_t \sum_{j=1}^{\infty} \rho^j (\Delta y_{t+j} - \sigma r_{t+j}) - \rho \mu / (1-\rho)$ .

This and similar studies led to additional considerations on consumption studying elements, as well as sophisticated further policy implications. One can question the assumption of perfect capital markets and borrowing accessibility, differentiate between discounting and the interest rate and inquire into non-optimality. Multitude of consumption factors also complicates the analysis through butterfly effect.

More recent studies took advantage of almost similar tax maneuvers of 2001 (Shapiro and Slemrod 2003) and 2009 (Sahm et al. 2010) to find out that one time fiscal stimuli have little or no effect on consumption behavior, that is consistent with permanent income hypothesis. Similar results could be observed overseas (Aisbett et al. 2016).

#### References:

- Aisbett, Emma, et al. "Fiscal Stimulus and Households' Non-Durable Consumption Expenditures: Evidence from the 2009 Australian Nation Building and Jobs Plan." *SSRN Electronic Journal*, 2013. SSRN.
- Bernanke, Ben S. "Permanent Income, Liquidity, and Expenditure on Automobiles: Evidence From Panel Data." *The Quarterly Journal of Economics*, vol. 99, no. 3, 1984, pp. 587–614. JSTOR.
- Campbell, John, and N. Gregory Mankiw. "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence." *NBER Macroeconomics Annual*, vol. 4, 1989. pp. 185-246. NBER. June 2001. Web. 30 Oct. 2017.
- Friedman, Milton. *A Theory of Consumption Function*. Princeton University Press, 1957.
- Gross, David, and Nicholas Souleles. *Do Liquidity Constraints and Interest Rates Matter for Consumer Behavior? Evidence from Credit Card Data*. Working paper no. 8314. NBER, June 2001. Web. 30 Oct. 2017.
- Hall, Robert E. "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence." *Journal of Political Economy*, vol. 86, no. 6, 1978, pp. 971–987.
- Hall, Robert, and Frederic Mishkin. "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households." *Econometrica*, vol. 50 no. 2, 1980, pp. 461-481.
- Keynes, John M. *The General Theory of Employment, Interest and Money*. The Macmillan Press, 1973.
- Kuznets, Simon. *National Income: s Summary of Findings*. National Bureau of Economic Research, 1946.
- Sahm, Claudia R., et al. "Check in the Mail or More in the Paycheck: Does the Effectiveness of

Fiscal Stimulus Depend on How It Is Delivered?” *American Economic Journal: Economic Policy*, vol. 4, no. 3, 2012, pp. 216–250. JSTOR.

Shapiro, Matthew, and Joel Slemrod. “Consumer Response to Tax Rebates.” *The American Economic Review*, vol. 93, no. 1, 2003. AEA.

#### 4. Unemployment

Criticism of conventional Walrasian equilibrium in the labor market dominated past century’s discourse in multiple ways. Loudly proclaimed by the chain of European revolution Marxian approach quite amusingly overlapped with later Keynes’ “General Theory”. Partly, due to political shocks of the time, partly due to ethical shifts, the attention to labor market inefficiencies grew stronger after WWII.

A conventional labor market description includes several key measures. Unemployment rate measures the percentage of active job seekers in the labor force. Labor force participation rate measures the percentage of employed and unemployed but employable adults in the population. Thus, students, retirees discouraged workers and certain other groups are excluded from the labor force (and, consequently, from unemployed). Unemployment, in turned, is divided into frictional (or temporary), cyclical and structural.

Not labor market equilibrium, but the process of reaching it became central in Shapiro and Stiglitz (1984) theory, known as the Efficiency Wage, aiming to explain structural unemployment. Observing the behavior of both sides of the market, they begin with modelling lifetime utility maximizing worker, who are unable to borrow and is neutral to risk:

$$W = \int_0^{\infty} u(w(t), e(t)) \exp(-rt) dt,$$

accounting for wage ( $w$ ), work effort ( $e$ ) and discount rate ( $r$ ). Value functions take form:

$$rV_E^S = w + (b+q)(V_u - V_E^S)$$

$$rV_E^N = w - e + b(V_u - V_E^S)$$

for shirking ( $V_E^S$ ) and non-shirking ( $V_E^N$ ) workers, where  $q$  and  $b$  are exogenous probabilities of losing job due to shirking (and being caught) or losing job for other reasons respectively,  $V_u$  – expected lifetime utility if unemployed. The model further solved to:

$$V_E^S = (w + (b+q)V_u) / (r+b+q)$$

$$V_E^N = ((w-e) + bV_u) / (r+b)$$

and, most famously, to no-shirking condition  $w \geq rV_u + (r+b+q)e/q$ . Accounting for the demand side  $rV_u = \bar{w} + a(V_E - V_u)$ , with unemployment benefit ( $\bar{w}$ ) and job acquisition rate ( $a$ ), it takes form:

$$rV_E = ((w-e)(a+r) + \bar{w}b) / (a+b+r)$$

$$rV_u = ((w-e) + \bar{w}(b+r)) / (a+b+r)$$

combined into  $w \geq e + \bar{w} + (e/q)(b/u+r)$  with unemployment ( $u$ ). Maximizing  $(w-e)L$  in  $w$  and  $L$  s.t.  $w \geq e + (e/q)((bN/(N-L)) + r)$  and  $wL \leq F(L)$  through Kuhn-Tucker conditions, we are led to  $\bar{w} = 0$  at optimum.

Unsurprisingly, this optimal result does not look satisfying after accounting for workers' risk awareness (and, consequently, their willingness to accept lower wages), separation from jobs for exogenous reasons (such as migration) and so on. The advantages of higher wages are found in Solow (1979), Weiss (1980), Akerlof and Yellen (1990) and can be ultimately tracked back to Keynes (1936). Higher wages lead to higher loyalty and, consequently, higher productivity. Resulting higher standard of living leads to spillovers into higher job satisfaction. Due to risk-adversity mentioned above, the amount of effort specifically to keep the job will be higher, as well as the overall quality of the job applicants' pool.

A different approach emerged from the emphasis on the process of job search and the cost of such search. McCall (1968) modeled it as  $c = \int_{\varepsilon}^{\infty} (x - \varepsilon)\varphi(x)dx$ , where  $x$  – job offer,  $\varphi(x)$  – PDF of  $x$ , and  $\varepsilon$  – expectation (such that  $\varepsilon = E(f(x))$ ). The implication of such approach allowed to differentiate between discouraged workers (dropouts) and frictionally unemployed.

Relying, at least in part, on this, as well as on the Efficiency Wages, Matching Theory attempts to give equal treatment to both sides of labor market. Summarizing earlier findings of

Diamond, Mortensen as well as his own, Pissarides (1990) starts with somewhat familiar system for workers

$$\begin{aligned} rU &= z + \theta q(\theta)(E - U) \\ rE &= w + s(U - E) \end{aligned}$$

for present-discounted value of the expected income stream for unemployed (U) and employed (E) workers with some real return (z), probability of obtaining employment ( $\theta q(\theta)$ ) and rate of becoming unemployed (s). Neither skills nor work effort are in the model and workers are assumed to be heterogeneous. The system is further developed to

$$\begin{aligned} rU &= ((r+s)z + \theta q(\theta)w) / (r+s + \theta q(\theta)) \\ rE &= (sz + (r + \theta q(\theta))w) / (r+s + \theta q(\theta)). \end{aligned}$$

On the firm's side we have

$$rV = -\gamma_0 + q(\theta)(J - V) \text{ leading to } f(k) - (r + \delta)k - w - ((r+s)\gamma_0) / (q(\theta)) = 0.$$

for vacant job cost ( $\gamma_0$ ) and present-discounted value of expected profit from an occupied (J) and a vacant (V) job, so that net return J-V follows Poisson process with rate  $q(\theta)$ . Other variables conventionally represent capital (k) and discounting ( $\delta$ ). Assuming Cobb-Douglas of the form  $(E-U)^\beta (J-V)^{1-\beta}$ , no cost of job search and no cost of vacancy filling, equilibrium is achieved at  $u = s / (s + \theta q(\theta))$  and  $w = (1-\beta)z + \beta(f(k) - (r + \delta)k + \theta \gamma_0)$  with  $f'(k) = (r + \delta)$  and "bargaining strength" parameter  $0 \leq \beta \leq 1$ .

Since the parameters' values for each individual worker are different (workers are heterogeneous), the equilibrium does not clear the market. This is one of the major difference from the Efficiency Wage, where the equilibrium wage, although undesirable, still clears the market. Thus, while the Efficiency Wage tend to focus of structural unemployment, while Matching Theory attempts to model frictional dynamics. Being more expandable, matching model, therefore, is believed to be more representative and is used to describe processes outside

the labor market, such as investment banking (matching potential investors with potential entrepreneurs) and even online dating (Hitsch et al. 2010).

#### References:

- Akerlof, George, and Janet Yellen. "The Fair Wage-Effort Hypothesis and Unemployment." *The Quarterly Journal of Economics*, vol. 105, no. 2, May 1990, pp. 255-283. JSTOR.
- Hitsch, Gunther, et al. "Matching and Sorting in Online Dating." *American Economic Review*, vol. 100, no. 1, Mar. 2010, pp. 130-163. AEA.
- Keynes, John M. *The General Theory of Employment, Interest and Money*. The Macmillan Press, 1973.
- McCall, J. "Economics of Information and Job Search." *The Quarterly Journal of Economics*, vol. 84 no. 1. Feb. 1970, pp. 113-126. JSTOR.
- Pissarides, Christopher. *Equilibrium Unemployment Theory*. Basil Blackwell, 1990.
- Shapiro, Carl, and Joseph Stiglitz. "Equilibrium Unemployment as a Worker Discipline Device." *The American Economic Review*, vol. 74, no. 3, Jun. 1984., pp. 433-444. JSTOR.
- Solow, Robert. "Alternative Approaches to Macroeconomic Theory: A Partial View." *The Canadian Journal of Economics*, vol. 12, no. 3, Aug. 1979, pp. 339-354. JSTOR.
- Weiss, Andrew. "Job Queues and Layoffs in Labor Markets with Flexible Wages." *Journal of Political Economy*, vol. 88, no. 3, Jun. 1980, pp. 526-538. JSTOR.

#### 5. Inflation

Phillips (1958) famously plotted 1861-1957 unemployment percentage (x) against percentage change in wages (y) in the form

$$y+a=bx^c \text{ or } \log(y+a)=\log b+c\log x$$

with estimation  $y+.9=9.683x^{-1.394}=\log(y+.9)=.984-1.394\log x$ . Quite amusingly, credits for the discovery were later claimed by Irving Fisher in the very title of his 1973 paper (although Fisher's version accounted for prices instead of wages). Therefore, Phillips curve in its most

generic form signifies the balance between unemployment and inflation and implies correlation between inflation and the distance from full employment output (i.e. output gap).

The relation was further popularized by Samuelson and Solow (1960) and, with certain reservations, by Friedman (1968), who noted the original model's nominal units and short time horizon. The idea was further expanded by Phelps (1967) to form

$$\dot{p}/p = f(y) - x$$

with time rate of change  $\dot{p}$  (so that  $\dot{p}/p$  is inflation) and expected inflation ( $x$ ). This version of the curve relies on adaptive expectations as well as the existence of natural rate of unemployment.

Later approach of post-Lucas macroeconomics was, at least in part, built on micro elements. Using Rotemberg's (1982) and Calvo's (1983) optimal pricing for the firm's side, Roberts (1995) formulates the New Keynesian Phillips Curve (NKPC):

$$\Delta p_t - E_t \Delta p_{t+1} = c_0 + \gamma y_t + \varepsilon_t,$$

where the difference between price  $p$  and its expectation in future is related to menu cost ( $c > 0$  due to stickiness) and output gap ( $y$ ). Clarida et al. (2000) further expanded the model to account for information ( $\Omega$ ) (and, consequently, to be based on rational expectations), so that at the equilibrium (here  $y$  conventionally represents actual output,  $z$  – “natural” output and  $x$  - output gap):

$$\begin{aligned} \pi_t &= \delta E(\pi_{t+1} | \Omega_t) + \lambda(y_t - z_t) \\ y_t &= E(y_{t+1} | \Omega_t) - (1/\sigma)(r_t - E(\pi_{t+1} | \Omega_t)) + g_t \\ r_t^* &= \beta E(\pi_{t+1} | \Omega_t) + \gamma x_t \\ r_t &= \rho r_{t-1} + (1-\rho)r_t^* \end{aligned}$$

One can see that the model takes into consideration the interest rate and the way it affects (once being set) the prices (as  $z_t$  stands for instruments known after setting the rate). The other implication that the increase in  $y$  in response to announced decrease in money supply (and,

consequently, decrease in expected inflation) does not seem to hold empirically. However, the model still builds a bridge between Calvo's staggered contracts, Rotemberg's quadratic cost adjustment and expectations of Friedman and Phelps.

More sophistication was added to form the hybrid version of NKPC due to empirical exercises (Gali and Gertler 1999)

$$\pi_t = \delta x_t + (1 - \phi) E_t(\pi_{t+1}) + \phi \pi_{t-1},$$

so that inflation depends on convex combination of expected future and actual past (lagged) inflation.

Although Gali and Gertler show their version of the model to be descriptive on 1960-1997 US data, overall empirical studies of NKPC are controversial. Estimating the model of the following form:

$$\pi_t = \alpha_0 + \phi E_{t-1} \pi_{t+1} + \alpha_1 E_{t-1} z_{p,t} + \alpha_{11} E_{t-1} (y_t - y_t^*) + \alpha_{12} E_{t-1} (p_t^w - p_t^*) + \alpha_1 E_{t-1} s_{L,t} + \alpha_2 \phi E_{t-1} \Delta n_{t+1} + \alpha_2 E_{t-1} \Delta n_t + u_t$$

on UK data, Batini et al. (2005) find that the inflation can be explained both by changes in employment and by changes in real import prices. Bihan and Jondeau (2004) find that different versions of the Phillips curve (including the hybrid version) can be fitted for different countries to a varying degree. Bardsen et al. (2004), as well as Rudd and Whelan (2007) do not find the model useful.

#### References:

- Bardsen, Gunnar, et al. "Econometric Evaluation of the New Keynesian Phillips Curve." *Oxford Bulletin of Economics and Statistics*, vo. 66, no. 1, Sep. 2004, pp. 671-686. Wiley.
- Batini, Nicoletta, et al. "An open-economy new Keynesian Phillips curve for the U.K." *Journal of Monetary Economics*, vol. 52, no. 6, Sep. 2005, pp. 1061-1071. ScienceDirect.
- Bihan, Herve, and Eric Jondeau. "Testing for the New Keynesian Phillips Curve. Additional



- international evidence.” *Economic Modelling*, vol. 22, no. 3, May 2005, pp. 521-550. ScienceDirect.
- Calvo, Guillermo. “Staggered Prices in a Utility Maximizing Framework.” *Journal of Monetary Economics*, vol. 12, no. 3, Sep. 1983, pp. 383–398.
- Clarida, Richard, et al. “Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory.” *The Quarterly Journal of Economics*, vol. 115, no. 1, Feb. 2000, pp. 147–180. Oxford Academic.
- Gali, Jordi, and Mark Gertler. “Inflation dynamics: A structural econometric analysis.” *Journal of Monetary Economics*, vol. 44, no. 2, 1999, pp. 195-222. Elsevier.
- Fisher, Irving. “I Discovered the Phillips Curve: "A Statistical Relation between Unemployment and Price Changes." *Journal of Political Economy*, vol. 81, no. 2, p. 1, Mar. 1973, pp. 496-502. JSTOR.
- Friedman, Milton. “The Role of Monetary Policy.” *The American Economic Review*, vol. 58, no. 1, Mar. 1968, pp. 1-17. JSTOR.
- Phelps, Edmund. “Phillips Curves, Expectations of Inflation and Optimal Unemployment over Time.” *Economica*, vol. 34, no. 135, Aug. 1967, pp. 254-281. JSTOR.
- Phillips, A.W. "The Relationship between Unemployment and the Rate of Change of Money Wages in the United Kingdom, 1861-1957." *Economica*, vol. 25, no. 100, Nov. 1958, pp. 283-299. Wiley.
- Roberts, John. “New Keynesian Economics and the Phillips Curve.” *Journal of Money, Credit and Banking*, vol. 27, no. 4, Nov. 1995, pp. 975-984. JSTOR.
- Rotemberg, Julio. “Sticky Prices in the United States.” *Journal of Political Economy*, vol. 90, no. 6, Dec. 1982, pp. 1187-1211. JSTOR.
- Rudd, Jeremy, and Karl Whelan. “Modeling Inflation Dynamics: A Critical Review of Recent Research.” *Journal of Money, Credit and Banking*, vol. 39, no. 1, Feb. 2007, pp. 155-170. Wiley.
- Samuelson, Paul, and Robert Solow. “Analytical Aspects of Anti-Inflation Policy.” *The American Economic Review*, vol. 50, no. 2, May 1960, pp. 177-194. JSTOR.

## 6. Monetary Policy

Putting aside speculations on normative nature of idealism, a desirable monetary policy would achieve the objectives set in the most efficient way possible. Being the US monetary authority, the Federal Reserve System is required by law to “promote” maximum employment, stable prices and moderate long-term interest rates. In addition, the Federal Reserve Reform Act of 1977 focuses on long run growth of money supply (MS) corresponding to “long run potential to increase production”. One strict interpretation, therefore, does not suggest that the Fed is supposed to do anything to actually achieve this potential growth, but rather rise the MS at the rate corresponding to such potential. In other words, it is not the Fed’s job to keep the economy at the maximum growth rate of output, but rather to keep MS from preventing it.

One may also focus on the goals the Fed is supposed to “promote” with regard to employment, prices and interest rates. If “maximum employment” is understood as maximum attainable value of Employment-Population Ratio (EPR), it can be achieved by either minimizing unemployment rate ( $u$ ), or maximizing Labor Force Participation Rate (LFPR) or some combination of both. For a number of rather complex reasons (such as ethical intolerance of child labor, for example), the actual Fed policies tend to focus on minimizing  $u$ .

Stable prices intuitively should mean prices that do not change, or, more formally, with minimized inflation (i.e.  $\Delta p$  representing either  $\Delta \text{CPI}$  or  $\Delta(\text{GDP deflator})$ ). Due to modern mainstream macroeconomics’ awareness of the problems associated with “too low” inflation (known as inflation bias), however, it is believed that there exists some optimal degree of price instability  $(\Delta p)^*$  (sometimes, though not always, assumed to be constant). Another way of looking at this is to remember that the optimal MS is required to be kept in accordance with maximum or full employment output. Since in practice, however, full employment is never achieved, the MS the Fed is required to keep is greater than the optimal MS for the actual economic conditions, hence the positive inflation. In reality, therefore, the goal of “stable prices” turns into minimizing  $(\Delta p - (\Delta p)^*)^2$ .

Moderate long-term interest rates seems to be even more confusing, due to normative nature of moderation. Oxford Dictionary, for example, defines “modest” as “relatively moderate, limited, or small”, “small” - as “of a size that is less than normal”, and “normal” - “conforming to a standard; usual, typical, or expected.” Targeting less than expected rates, however, does not

seem to make sense, since even the most primitive form of expectations leads to  $i_t^e = a \Rightarrow i_t = a - b$  followed by  $i_{t+1}^e = a - b \Rightarrow i_{t+1} = a - b - b_1$  and so on all the way into negative infinity for nominal rates. Perhaps, the key term here is “long-term” as opposed to short term; this made sense in the historical, economic, and political context of 1977, but still leaves room for ambiguity in understanding what “moderate” is.

For these and other reasons, the majority of approaches to design an ideal monetary policy tend to focus on unemployment and inflation. As important as these are, the goals of employment, prices and interest rate are only to be “promoted”. What the Act is specific about is the adequacy of MS in terms of potential output growth. Such an accent, again, does not sound surprising given its historical context. Nevertheless, the actual monetary policy models attempt to address some or all of these elements.

For further discussion, however, the following should be noted. Besides being a monetary policy maker, Fed acts also as banking regulator. It seems reasonable to believe that the significant share of its employees are or were associated with the industry. No surprise here, as who can possibly know banking industry better than the bankers themselves? This, however, creates the potential for conflict of interests. With this in mind, some of the Fed’s decision (such as to start paying interest on reserves) might raise suspicions with regard to its true intentions (Ozdemir 2015).

More importantly, it makes harder to differentiate between monetary and regulatory nature of instruments at Fed’s disposal. Even more, since the Fed is not independent completely, it’s decisions may have a degree of political influence. If the Treasury will, for whatever reasoning, alter the amount of borrowing and, therefore, the amount of treasuries in the market, the Fed will more likely to react accordingly. If, however, the Fed, will decide to make a monetary move, the specific response likely will be at Congress’ and the Treasury’s discretion.

Some of the instruments at Fed’s discretion can have asymmetric effects as well. For example, adjusting FFR through the market operations with treasuries have direct universal effect (due to universal openness of the market). However, adjusting the FFR through the interests on reserves direct affects banks only. While the reaction to such move will affect other market participants as well, the mechanics of these policies are different, thus creating additional concerns.

One key issue in designing such policy is demand for money. Expressed by Fisher (1911) as  $MV = \sum pQ$  with velocity, prices and quantity, the equation can be solved for money demand:  $M_D = pQ/V$ . Baumol (1952) presents cash (c) demand as  $c = \sqrt{(2bT)/i}$  for “transaction stream” (T), fee (b) and interest (i). Sidrauski (1967), understanding money in classical sense and using utility  $u = u(c, m)$  develops “money-in-utility” function model of the form

$$M_D/(PK) = L(y'(k) - \mu + \pi) = L(r)$$

for prices (P), capital (K), labor (L), capital-ratio (k), depreciation ( $\mu$ ), expected inflation ( $\pi$ ) and “expected opportunity cost of holding cash” (r). Finally, Gali (2007) presents the log-linear version:

$$m_t - p_t = y_t - \eta_t$$

As one can see, the variety of opinions on the nature of single monetary policy element (other important elements, such as inflation, unemployment, consumption and so on are described in respective previous reviews and omitted here for brevity’s sake) ultimately makes monetary policy one of the most complicated (and, consequently, one of the most interesting) topics in economics. While designing a policy, the Fed has to jointly track its objectives as well as the instruments used. Though the theory of such approach can be traced at least back to Tinbergen (1952), in more practical terms it was brought up by Poole (1970). Using IS-LM model, he suggest complimentary use of different policy instruments to make up for shocks and advocates for active monetary policy based on empirical evidences. The resulting optimal interest rate is supposed to be set as

$$r = r_0 = a_1^{-1}(Y_t(1 - S_1 - S_2)a_0),$$

coming from the lagged income equation of the form

$$Y_t = a_0 + a_1 r_t + S_1 Y_{t-1} + S_2 Y_{t-2} + u_t.$$

The variety of opinions about the elements inevitably led to a multiplicative variety of actual monetary policy approaches. The common foundations may include the IS-LM-model, the Solow Growth Theory, the Ramsey-Cass-Koopmans model and others. Later sophistications were influenced by to a varying degree by the emergence of game theory, monetarism and Lucas critique together with the evolution (easily trackable from Friedman through Kydland and Prescott to Taylor and beyond) of straightforward targeting into rule-based approach.

Svensson (1997) starts with the following (in logs):

$$\begin{aligned}\pi_{t+1} &= \pi_t + \alpha_1 y_t + \alpha_2 x_t + \varepsilon_{t+1} \\ y_{t+1} &= \beta_1 y_t - \beta_2 (i_t - \pi_t) + \beta_3 x_t + \eta_{t+1} \\ x_t &= \gamma x_t + \theta_{t+1}\end{aligned}$$

for output gap ( $y$ ), some exogenous variable ( $x$ ), some monetary policy instrument ( $i$ ) and shocks ( $\varepsilon$ ,  $\eta$  and  $\theta$ ). Targeting optimal inflation  $\pi^*$  will require to minimize  $E_t \sum_{\tau=t}^{\infty} \delta^{(\tau-t)} (1/2(\pi_{\tau} - \pi^*)^2)$  with discount factor ( $0 < \delta < 1$ ) (it is interesting to note that Svensson focuses on repo as a monetary instrument example). Focusing on 2 years period (because of the lagged effect from change in  $i$ ), Svensson conventionally optimized it through the first order conditions to

$$\begin{aligned}i_t &= \pi_t + (1/(\alpha_1 \beta_2))(\pi_t - \pi^*) + ((1 + \beta_1)/\beta_2)y_t + ((\alpha_1 \beta_3 + \alpha_2(1 + \gamma))/(\alpha_1 \beta_2))x_t \\ \pi_{t+2} &= \pi^* + \varepsilon_{t+1} + \alpha_1 \eta_{t+1} + \alpha_2 \theta_{t+1} + \varepsilon_{t+2} \\ y_{t+1} &= -\alpha_2/\alpha_1 \gamma x_t - 1/\alpha_1 \varepsilon_t - \eta_t - \alpha_2/\alpha_1 \theta_t + \eta_{t+1}\end{aligned}$$

Assuming unit income velocity, Svensson further considers money demand of the form (also in logs)  $m_{t+1} - p_{t+1} = y_{t+1} - \kappa_t + v_{t+1}$  and solves it for the money growth rate. Therefore, his approach can be defined as inflation-targeting and backward-looking.

Clarida et al. (2000), basing their work on earlier studies of Bernanke and Goodfriend, employ forward-looking approach instead:

$$\begin{aligned}\pi_t &= \delta E(\pi_{t+1} | \Omega_t) + \lambda(y_t - z_t) \\ y_t &= \delta E\{y_{t+1} | \Omega_t\} + 1(1/\sigma)(r_t - E(\pi_{t+1} | \Omega_t)) + g_t \\ r_t^* &= \beta(E(\pi_{t+1} | \Omega_t) - \pi^*) + \gamma x_t\end{aligned}$$

$$r_t = \rho r_{t-1} + (1-\rho)r_t^*$$

for information ( $\Omega$ ), policy instrument ( $z$ ), exogenous demand factor ( $g$ ) and output gap ( $x$ ). Their account for information and, consequently, for expectations is closely connected to understanding the changes associated with Volker and Greenspan. This study has interesting empirical results reported separately for the periods associated with different Fed chairmen, as opposed to Taylor.

Taylor (1993) himself quite straightforwardly proposed setting Federal Funds Rate (FFR) as  $r = p + .5y + .5(p-2) + 2$ , or, more generally,

$$r = p + a(y - y^*) + b(p - p^*) + r^*$$

accounting for inflation ( $p$ ) and output gap ( $y$ ).

Additional concerns emerged after the era of stagflation effectively ruining the Phillips curve-dominated approach and bringing in the issue of time consistency. Elaborating on Barro and Gordon's (1983), Ireland (1999) presents the minimization problem

$$\min. E_{t-1} \left( \frac{1}{2} \left( (1-k)U_t^n - \alpha(\pi_t^p + \eta_t - \pi_t^e) \right)^2 + \frac{b}{2} (\pi_t^p + \eta_t)^2 \right)$$

for natural unemployment ( $U^n$ ) and planned inflation ( $\pi^p$ ) that is solved to

$$\Delta U = \lambda \Delta U_{t-1} + \varepsilon_t - \alpha \eta_t + \alpha(1+\lambda)\eta_{t-1} - \alpha\lambda\eta_{t-2}.$$

The model relies heavily on perfect information assumption (that is,  $\pi^p = \pi^e$ ). Lack of instruments at the Fed's discretion in the solution shows that that the Fed does not control the unemployment.

More extensive concerns are highlighted in recent empirical studies. Using Barnett's Divisia index instead of conventional MS measures, Belongia (1994, also Belongia and Ireland 2014) show that the effect of monetary policies can be switched in the analysis with different measures. Cochrane and Piazzesi (2002) show that the Fed follows rather than directs the market while setting target rates.

## References:

- Barro, Robert, and David Gordon. "Rules, discretion and reputation in a model of monetary policy." *Journal of Monetary Economics*, vol. 12, no. 1, 1983, pp. 101-121. Elsevier.
- Baumol, William. "The Transactions Demand for Cash: An Inventory Theoretic Approach." *Quarterly Journal of Economics*, vol. 66 no. 4, Nov. 1952, pp. 545–556. JSTOR.
- Belongia, Michael. "Measurement Matters: Recent Results from Monetary Economics Reexamined." *Journal of Political Economy*, vol. 104, no. 5, Oct. 1996, pp. 1065-1083. JSTOR.
- Belongia, Michael, and Peter Ireland. "The Barnett Critique After Three Decades: A New Keynesian Analysis." *Journal of Econometrics*, vol. 183, no. 1, Nov. 2014, pp. 5-21. Elsevier.
- Clarida, Richard, et al. "Monetary Policy Rules and Macroeconomic Stability: Evidence and Some Theory." *The Quarterly Journal of Economics*, vol. 115, no. 1, Feb. 2000, pp. 147–180. Oxford Academic.
- Cochrane, John, and Monika Piazzesi. "The Fed and Interest Rates - A High-Frequency Identification." *American Economic Review*, vol. 92, no 2, 2002, pp. 90-95.
- Fisher, Irving. *The Purchasing Power of Money*. 1922. Library of Economics and Liberty. 2 December 2017.
- "H.R. 9710 — 95th Congress: Federal Reserve Reform Act." GovTrack 1977. *Oxford Dictionary*, 2017. 2 December 2017.
- Ireland, Peter. "Does the time-consistency problem explain the behavior of inflation in the United States?" *Journal of Monetary Economics*, vol. 44, 1999, pp. 279-291. Elsevier.
- Ozdemir, Nilufer. "Money and Banking." Lecture given at the University of Mississippi, Fall 2015.
- Poole, William. "Optimal Choice of Monetary Policy Instruments in a Simple Stochastic Macro Model." *Quarterly Journal of Economics*, vol. 84, no. 2, May 1970, pp. 197-216. JSTOR.
- Sidrauski, Miguel. "Inflation and Economic Growth." *Journal of Political Economy*, vol. 75, no. 6, Dec. 1967, pp. 796-810. JSTOR.
- Svensson, Lars. "Inflation forecast targeting: Implementing and monitoring inflation targets." *European Economic Review*, vol. 41, 1997, pp. 1111-1146. Elsevier.

Taylor, John. "Discretion versus policy rules in practice." *Carnegie-Rochester Conference Series on Public Policy*, vol. 39, 1993, pp. 195-214.

Tinbergen, Jan. *On the Theory of Economic Policy*. 1952. North-Holland.