The Optimal Mix of Monetary and Climate Policy

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Abstract

Given central banks' recent interest in "greening the financial system", this research theoretically investigates the relationship between monetary and climate policy and tries to find their “optimal mix”. We build an Environmental Dynamic Stochastic General Equilibrium (E-DSGE) model with the consideration of illegal emission which is pervasive in many countries. According to the model, we find: First, the dynamic of monetary policy is influenced by the selection of regimes of climate policy and the effectiveness of enforcement of environmental regulation. Second, the coefficients in the traditional Taylor rule of monetary policy can be better set to enhance welfare when a certain regime of climate policy is given in the economy. This helps find the constrained optimums of a policy mix. Third, if the mitigation of climate change is augmented into the target of monetary policy, the economy’s welfare can be enhanced. However, under certain circumstances, a dilemma in such monetary policy makes it incompatible with the traditional mandate of central bank.

Keywords: Optimal Mix, Monetary Policy, Environmental Policy, E-DSGE

1. Introduction

In 2015, a report published by the British central bank[3] proposed that climate change could become a risk that affect the financial stability and the economic

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[3] Bank of England’s Prudential Regulation Authority (2015). The impact of climate change on the UK insurance sector. https://www.bankofengland.co.uk/prudential-regulation/publication/2015/the-impact-of-climate-change-on-the-uk-insurance-sector

development. Since then and especially after the sign of Paris Agreement, climate change and the broader environmental issue have become a factor of consideration for many central banks. By forming the Network of Central Banks and Supervisors for Greening the Financial System (NGFS)[4] in 2017 and the International Platform on Sustainable Finance (IPSF) in 2019, they are starting to investigate ways to manage the risk from climate change and to support the green economy. In particular, China's central bank already started to use "green monetary policy" in 2018 as the pioneer[5].

In academia, the accelerated climate change and environmental problem has brought new waves of research in "environmental macroeconomics" (Hassler et. al, 2016). Since 2010, a group of theoretical frameworks are founded to explain how environmental risks and relevant policies could affect the macro-economy by the newly developed "Environmental Dynamic Stochastic General Equilibrium (E-DSGE)" models. Angelopoulos (2010), Fischer and Springborn (2011), Heutel (2012), Golosov et. al (2014), Doda (2014), Annicchiarico and DiDio (2015), Dissou and Karnizova (2016) investigated relationships between greenhouse gas (GHG)/pollutant emission and business cycle by setting GHG/pollutant as an externality in the economy and found how environmental policies can influence the fluctuation or the growth of economy. Some other researchers tried to find the role of weather in economic volatility. Chen (2014) built a model with weather shocks embedded finding its good explanatory power on China's business cycle. Gallic and Vermandel (2019) find weather shocks account for a very significant proportion of economic volatility in the long-run. Of the policy related researches, two regimes of environmental policy, cap-and-trade (permitting) and taxing, are mainly focused on. For example, Golosov et. al (2014) tried to find the optimal tax for fossil fuel. Dissou and Karnizova (2016) compared the different implications of reducing CO2 emissions with carbon permits and with carbon taxes.

Only until recently, has the monetary policy been linked with the environment issue

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[4] Please refer to their website https://www.ngfs.net/

[5] Please refer to http://www.greenfinance.org.cn/displaynews.php?id=2188

in macroeconomics. These two things are seemingly unrelated at first glance. However, it is not true. According to the above research, environment factors and policies are proven to influence the fluctuation or the growth of the economy which is just the thing that monetary policy cares about. Hence, some researchers have started to ask questions about the role of central bank and monetary policy in a changing climate. Early discussions including Haavio (2010), Campiglio (2016), Ma (2017) and McKibbin et. al (2017) qualitatively explained the linking mechanism between monetary policy and climate change. By E-DSGE model, Annicchiarico and DiDio (2017) studied the mix of monetary and climate policy at the first. They compared three specific mixes and showed that the optimal monetary policies should be slightly tightened when GHG emission is considered. Economides and Xepapadeas (2018) compared the monetary policy with and without the consideration of climate change in the model and found that the reaction of monetary policy to economic shocks will be affected by the climate change. Punzi (2019) introduced borrowing constraints and heterogeneous production sectors into the model to investigate the green financing activity, finding that only the differentiated capital requirement policy can sustain green financing. Huang and Punzi (2019) incorporated financial friction according to Bernanke et. al (1999) and found that environmental regulations can accelerate the risks that the financial system faces. There are

Together with the global combat for a sustainable future, central banks are eager to know more about monetary policy and the environment. This includes the macroeconomic and financial stability implications of climate change, the relationship between monetary policy and climate change as well as climate policy, the way to encourage green finance, the cost and benefit of "green monetary policy" and many others[6]. The existing researches, including aforementioned works by Annicchiarico and

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[6] Please refer to the Technical Supplement to the First Comprehensive Report of NGFS at https://www.banque-france.fr/sites/default/files/media/2019/08/19/ngfs-report-technical-supplement_final_v2.pdf and the research priorities listed by The International Network for Sustainable Financial Policy Insights, Research, and Exchange (INSPIRE) https://www.climateworks.org/inspire/

DiDio, Economides and Xepapadeas, Punzi and Huang, started to investigate these concerns. However, most concerns are still very new and have not been investigated deeply.

In this research, we aim to investigate the specific concern of “the relationship between monetary and climate policy”. This is a continuation of Annicchiarico and DiDio (2017) and we will answer three new questions which can better inform central bank’s policy making process facing climate change. These questions are: (1) If and how the monetary policy is influenced by the climate policy? (2) If and how can monetary policy be improved when the climate policy is considered in the framework of analysis? Whether there is an optimal monetary policy? (3) Should central bank use monetary policy to proactively take care for the environment? By answering these questions, we can understand the way for monetary policy to cooperate with climate policy and the mechanism of conduction of the policy mix.

Our method for research is an E-DSGE model based on Annicchiarico and DiDio (2017). Illegal emission and related regulation are additionally introduced. The basic DSGE model follows the standard New Keynesian framework. We introduce the “Environmental” features into the basic model by setting the greenhouse gas (GHG) emission from production, its negative externality on productivity and environmental policies that control emission. Considering the potential ineffectiveness of environmental regulation in many developing countries, we also introduce illegal (concealed) emission, potential penalty for it, and the effectiveness of enforcement of environmental regulation (EOEER) into the model. This is relaxing the hidden assumption of the perfect effectiveness of environmental regulation in most previous E-DSGE models. The relaxation of assumption is nontrivial since we find that EOEER is also a factor that influences the interaction between monetary and climate policy.

Based on the E-DSGE model, we first mix monetary policy with different types of climate policy and compare these different mixes to see if the climate policy can influence the monetary policy. For comparison, the impulse responses of major economic and environmental variables after shock, the conditional welfare and consumption equivalent under regimes with different mixes are calculated. The result shows that when monetary policy is mixed with different types of climate policy under different EOEER, its dynamic changes. Therefore, the design of monetary policy should consider these environmental factors.

We then want to improve and optimise the mix of monetary and climate policy. However, it is found that there is no unconstrained optimal mix of monetary and climate policy that is implementable in the real-world. So, we turn to find the constrained optimal mix by optimising policy coefficients under given regimes. It is found that the Taylor rule coefficients in monetary policy can be optimised when climate policy is given. This optimisation is both practicable and desirable.

We finally investigate a radically "climate-friendly" way to improve the policy mix, which can help find if it is good for central bank to use the monetary policy to proactively take care for the environment. This is to introduce a target for climate change mitigation into the Taylor rule of monetary policy. The results show that the economy's welfare can be enhanced when the new target is augmented into monetary policy and the coefficient of such target is optimised. However, under certain circumstances, such monetary policy has a dilemma which makes it incompatible with the traditional mandate of central bank. This means that proactively using monetary policy to protect the environment is not always desirable.

The paper proceeds as follow. Section 2 describes the modified E-DSGE model. Section 3 compares the mixes of monetary policy with different climate policy. Section 4 investigates the optimisation of policy mixes. Sections 5 concludes.

2. Model

We construct an Environmental Dynamic Stochastic General Equilibrium (E-DSGE) model based on the New Keynesian framework. GHG emission from production, its negative externality on productivity and environmental policies that control emission are introduced following Annicchiarico and DiDio (2017). Illegal (concealed) emission, potential penalty for it, and the effectiveness of enforcement of environmental regulation (EOEER) are innovatively set into the model.

2.1 Household

A representative household maximise its expected lifetime utility which is determined by consumption C_t and labour L_t and has a form of

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\beta^t S_t\left(\ln C_t-\mu_L\frac{L_t^{1+\eta}}{1+\eta}\right)\right\} \tag{1}$$

where $0<\beta<1$ is the discount factor, $\eta\geq 0$ is the inverse of the elasticity of labour supply, $\mu_L>0$ is the coefficient of disutility of labour. S_t represents the stochastic shocks of time-preference which follows $\ln S_t=\rho_S\ln S_{t-1}+(1-\rho_S)\ln S+e_{S,t}$ to evolve, where $0<\rho_S<1$ and $e_{S,t}\sim i.i.d.N(0,\sigma_S^2)$.

The budget constraint of household is

$$P_tC_t+R_t^{-1}B_{t+1}=B_t+W_tL_t+D_t+P_tT_t \tag{2}$$

where P_t is the price of final good, B_t and B_{t+1} are the nominal quantity of riskless bond at period t and $t+1$, R_t is the riskless interest rate of the bonds which is determined by the central bank, W_t is the nominal wage of labour, D_t denotes nominal dividend derived from enterprises, T_t is the lump-sum transfer from government.

At the optimum we have the following first-order conditions

$$\beta R_t\mathbb{E}_\mathrm{t}\left[\frac{S_{t+1}}{S_t}\frac{C_t}{C_{t+1}}\frac{1}{\Pi_{\mathrm{t+1}}}\right]=1 \tag{3}$$

$$L_t^{\eta}=\frac{W_t}{\mu_L P_t C_t} \tag{4}$$

where $\Pi_{\mathrm{t+1}}=P_{t+1}/P_t$ is the inflation of period $\mathrm{t}+1$. Equation (1) is the Euler equation and (2) is the labour supply equation.

2.2 Enterprise and the Environment

Consistent with the standard New Keynesian framework, the enterprise sector is formed by final good and intermediate good producers. The final good Y_t is produced by competitive firms using the Constant Elasticity of Substitution (CES) technology

$$Y_t = \left[\int_0^1 Y_{j,t}^{\frac{\theta_t - 1}{\theta_t}} dj\right]^{\frac{\theta_t}{\theta_t - 1}} \tag{5}$$

where $Y_{j,t}$ denotes the intermediate goods produced by monopolistically competitive firms, the subscript $j \in [0,1]$ denotes the intermediate good firms of a continuum. $\theta_t > 1$ is the elasticity of substitution, and also a stochastic process that describe the cost-push shock (Smets and Wouters (2003)). It follows $\ln \theta_t = \rho_\theta \ln \theta_{t-1} + (1 - \rho_\theta) \ln \theta + e_{\theta,t}$ with $0 < \rho_\theta < 1$ and $e_{\theta,t} \sim i.i.d. N\left(0, \sigma_\theta^2\right)$.

Final good producers maximise their profit which is determined by

$$P_t Y_t - \int_0^1 Y_{j,t}^{\frac{\theta_t - 1}{\theta_t}} dj \tag{6}$$

So, we have the first-order condition which yields the demand function for intermediate goods

$$Y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\theta_t} Y_t \tag{7}$$

in which

$$P_t = \left[\int_0^1 P_{j,t}^{1-\theta_t} dj\right]^{\frac{1}{1-\theta_t}} \tag{8}$$

A typical intermediate good firm has a production function

$$Y_{j,t} = \Lambda_t A_t L_{j,t} \tag{9}$$

where A_t is the total factor productivity (TFP) factor or technology that follows a stochastic process $\ln A_t = \rho_A \ln A_{t-1} + (1 - \rho_A) \ln A + e_{A,t}$, in which $0 < \rho_A < 1$ and $e_{A,t} \sim i.i.d. N(0, \sigma_A^2)$. Following Golosov et al. (2014), Λ_t is a damage coefficient that describe the negative externality of GHG emission on productivity (via some channels including changing temperature). It is the pivot linking the economy and the environment. Λ_t is determined by the stock of emission following

$$\Lambda_t = \mathrm{e}^{-\chi(M_t - \widetilde{M})} \tag{10}$$

where M_t is the stock of emission of period t, $\widetilde{M}$ is the level before the industrial revolution, and χ>0 measures the intensity of negative externality.

According to Heutel (2012), GHG emission is a by-product of the production process. The original emission from production is $Z_{j,t}^{ori}$ which is proportional (measured by φ) to the volume of output of intermediate firms

$$Z_{j,t}^{ori} = \varphi Y_{j,t} \tag{11}$$

To dispose the original emission, a firm has three compatible choices: emission abating, legally emitting and illegally emitting. A firm can choose to abate a percentage of $U_{t,j}$ $(0 \leq U_{t,j} \leq 1)$ of the original emission which will bring a marginal increasing cost of $\phi_1 {U_{j,t}}^{\phi_2} Y_{j,t}$, where $\phi_1 = \phi_1' \varphi > 0$ and $\phi_2 > 1$ are cost coefficients. A firm can also choose to legally emit some original emission. This requires a firm to pay the carbon tax or buy the emission permit (depending on the climate regime) at a price $p_{Z,t}$ for every unit of GHG emission.

The novel features we introduce into the model are the illegal (concealed) emission and the potential penalty for it. This relaxes the hidden assumption of the perfect effectiveness of environmental regulation in most previous E-DSGE models. In many countries especially developing ones like China, the environmental regulation on emission is not very strict. The government or the environmental authority cannot accurately spot or strictly limit the real emission of firms. So, firms have some space to emit more than the legal level which is the level they paid tax or bought permit, so that to save some cost for emission abating or legal emitting. This results in the illegally emission or the concealed emission. Meanwhile, although a government may not able to fully prohibit the illegal emission, normally they still have environmental regulation at some degree. This means they will monitor the firms and penalise them (by fining) if illegal emission is spotted (with probability).

We assume that a firm faces an expected fine that equals to $\frac{\psi}{2} V_{t,j}^2 \varphi Y_{j,t}$, where $0 \leq V_{t,j} \leq 1$ is the proportion of illegal emission in the original emission. $\psi > 0$ is defined as the "effectiveness of enforcement of environmental regulation" ("EOEER" for short) which is proportional to the probability of government to spot illegal emission and the amount of fine for every unit of illegal emission. The amount of total fine is marginally increasing with regard to $v_{t,j}$ because the more a firm emit illegally, the easier it can be spotted. The introduction of illegal emission and EOEER is nontrivial since they will make the regimes of climate policy more similar or more different, and further influence the dynamics of financial and economic variables (see Sub-Section 3.3).

Now the three channels that firms can dispose their original emission, namely emission abating, legally emitting and illegally emitting, are all explained. This is helpful for illuminating the following variables. The real emission $Z_{j,t}^{real}$ is the amount of GHG that is really emitted to the atmosphere and can be monitored by the government. It equates the original emission $Z_{j,t}^{ori}$ minus the abated emission $Z_{j,t}^{abate} = U_{t,j}\varphi Y_{j,t}$. The claimed emission $Z_{j,t}^{claimed}$ is the amount of GHG emission that a firm reports to the government concealing its illegal emission $Z_{j,t}^{illegal}$. It determines the amount of legal emission $Z_{j,t}^{legal}$ and also the amount of tax or permit a firm needs to pay or buy ($p_{Z,t}Z_{j,t}^{legal}$). It equals to the real emission minus the illegal emission. Accordingly, we have

$$Z_{j,t}^{real} = Z_{j,t}^{ori} - Z_{j,t}^{abate} = \left(1 - U_{t,j}\right)\varphi Y_{j,t} = Z_{j,t}^{legal} + Z_{j,t}^{illegal} \tag{12}$$

$$Z_{j,t}^{claimed} = Z_{j,t}^{real} - Z_{j,t}^{illegal} = \left(1 - U_{t,j} - V_{t,j}\right)\varphi Y_{j,t} = Z_{j,t}^{legal} \tag{13}$$

The above relationship is illustrated in Figure 1.

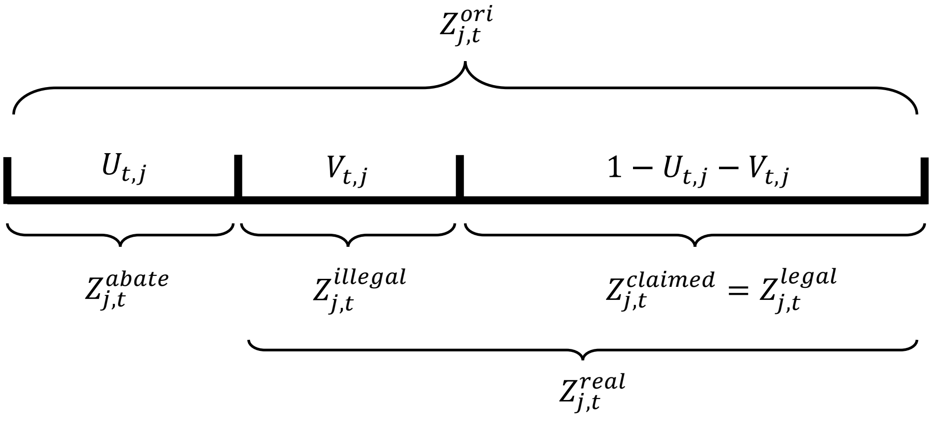

Figure 1: The relationship among emission variables

Considering the cost of disposing emission via the three channels and the sticky pricing assumption in the standard New Keynesian framework (Rotemberg, 1982), the objective of an intermediate firm is to maximise

$$\mathbb{E}_0\left\{\sum_{t=0}^{\infty}\Omega_{0,t}\left[\frac{P_{j,t}}{P_t}Y_{j,t}-TC_{j,t}-\frac{\gamma}{2}\left(\frac{P_{j,t}}{P_{j,t-1}}-1\right)^2 Y_t\right]\right\} \tag{14}$$

which is subject to

$$TC_{j,t}=\frac{W_t}{P_t}L_{j,t}+\phi_1 U_{j,t}^{\phi_2}Y_{j,t}+p_{Z,t}\left(1-U_{j,t}-V_{j,t}\right)\varphi Y_{j,t}+\frac{\psi}{2}V_{j,t}^2\varphi Y_{j,t} \tag{15}$$

where $\Omega_{0,t}=\beta^t\frac{C_0}{C_t}$ is the stochastic discount factor.

The above settings and assumptions yield the following first-order conditions (see Appendix)

$$(1-\theta_t)-\gamma(\Pi_t-1)\Pi_{\mathrm{t}}+\beta\gamma\mathbb{E}_{\mathrm{t}}\left[\frac{C_t}{C_{t+1}}(\Pi_{t+1}-1)\Pi_{t+1}\frac{Y_{t+1}}{Y_t}\right]+\theta_t MC_t=0 \tag{16}$$

$$MC_t=\frac{W_t}{\Lambda_t A_t P_t}+\phi_1 U_t^{\phi_2}+p_{Z,t}(1-U_t-V_t)\varphi+\frac{\psi}{2}V_t^2\varphi \tag{17}$$

$$p_{Z,t}=\frac{1}{\varphi}\phi_1\phi_2 U_t^{\phi_2-1} \tag{18}$$

$$V_t=\frac{1}{\psi\varphi}\phi_1\phi_2 U_t^{\phi_2-1}=\frac{p_{Z,t}}{\psi} \tag{19}$$

where MC_t is the marginal cost of production, $\gamma>0$ is the price adjusting cost coefficient, $\Pi_t=\frac{P_t}{P_{t-1}}$ denotes inflation. Equation (16) is the New Keynesian Phillips Curve.

2.3 Monetary and Environmental Authorities

The monetary policy authority (central bank) decides nominal interest rate following a traditional Taylor rule

$$\frac{R_t}{R}=\left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi}\left(\frac{Y_t}{Y_t^{na}}\right)^{\rho_Y} \tag{20}$$

where Y_t^{na} is the natural output without price stickiness, R and Π are the steady state of nominal interest rate and inflation, ρ_Π and ρ_Y are the intensity coefficients for targeting on inflation and output gap.

The environmental authority decides the regime of climate policy. In this research we consider two major regimes: cap-and-trade (“CA” for short) and carbon tax (“TX” for short). Under the CA regime, the environmental authority sets an emission cap Z_t^{cap} and sell emission permit to the market on a price decided by the market competition. In equilibrium, the total legal emission Z_t^{legal} equates Z_t^{cap}. Under the TX regime, the authority set a fixed carbon tax level for every unit of legal emission. The authority does not set a ceiling for total legal emission. As stated earlier, the environmental authority also monitors the firms and fine them if illegal emission is spotted, however, their effectiveness of enforcement (EOEER) is exogenously determined by the governance capacity of the country. The earnings of the authority including the income from selling emission permit or levying carbon tax and from the fine are transferred to households directly.

2.4 Market Clearing and Aggregation

In equilibrium, we have the market clearing condition

$$Y_t = C_t + \phi_1 U_t^{\phi_2} Y_t + \frac{\gamma}{2}(\Pi_t - 1)^2 Y_t \tag{21}$$

We assume all the firms are symmetrical following Rotemberg (1982). So, the gross variables share the same form of expressions with individual variables. The total production function is

$$Y_t = \Lambda_t A_t L_t \tag{22}$$

The totalities of emission are

$$Z_t^{legal} = \int_0^1 Z_{j,t}^{legal} dj = (1 - U_t - v_t)\varphi Y_t \tag{23}$$

$$Z_t^{real} = \int_0^1 Z_{j,t}^{real} dj = (1 - U_t)\varphi Y_t \tag{24}$$

The total transfer is

$$T_t = p_{Z,t} Z_t^{legal} + \frac{\psi}{2} v_t^2 \varphi Y_t \tag{25}$$

The total stock of emission is

$$M_t = (1 - \delta_M) M_{t-1} + Z_t^{real} + \tilde{Z} \tag{26}$$

where $\tilde{Z}$ is the emission from the nature without human influence, $0 < \delta_M < 1$ is the natural decay rate of GHG stock.

2.5 Calibration

We calibrate the parameters as follow and list them in Table 1. Following Gali (2015), discount factor β is set as 0.99, elasticity of substitution in steady state θ is set as 6, inverse of the Frisch elasticity η is set as 1. The adjusting cost coefficient γ which measures price stickiness is set as 58.25 so that the stickiness has a duration of three quarters when it is converted into Calvo pricing. The disutility coefficient of labour μ_L is set as 24.9983 so that the steady-state of labour is 0.2 without monopoly. Following tradition, the persistent coefficients of shocks (including TFP shock, preference shock and cost-push shock) are set as 0.9, the Taylor-rule elasticities (coefficients) of monetary policy ρ_Π and ρ_Y are set as 1.5 and 0.5 respectively in Section 3. Following Annicchiarico and DiDio (2017), the scale coefficient of abatement cost ϕ_1 is set as 0.185 and the elasticity ϕ_2 is set as 2.8. The parameter determining the damage caused by emission on output χ is set as 0.000457. Following Heutel (2012), the decay rate of emission stock δ_M is set as 0.0021. Following Xu et. al (2016), the coefficient measuring the original emission per unit of output φ is set as 0.601. As to the effectiveness of enforcement of environmental regulation (EOEER) ψ, according to the proportion of environmental punishment cost in total GDP in China which is around 0.01%[7], ψ should be around 0.45. This is within the magnitude of 0.1 to 1. For comparison propose, we need to set a large ψ and a small ψ. Considering the magnitude, the benchmark of ψ (in Sub-Section 3.1 and 3.2) is set as 1 which is the upper bound of the magnitude, and the value describing a relative ineffective regulation is set as 0.1 (in Section 3.3) which is the lower bound.

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[7] The State Council of China http://www.gov.cn/xinwen/2019-02/26/content_5368758.htm

Table 1: Calibrated values of the parameters

Parameter		Value	Target
β	Discount factor	0.99	Risk free rate
η	Inverse of the Frisch elasticity,	1	Literature
μ_L	Disutility coefficient of labour	24.9983	Steady labour time is 0.2 under fully competition market
θ	Elasticity of substitution in steady state	6	Literature
γ	Adjusting cost coefficient of sticky price	58.25	Literature
ρ_A	Persistent coefficient of TFP shocks.	0.9	Commonly used value
ρ_S	Persistent coefficient of preference shocks.	0.9	Commonly used value
ρ_θ	Persistent coefficient of cost-push shocks.	0.9	Commonly used value
ϕ_1	Scale coefficient of abatement cost	0.185	Literature
ϕ_2	Elasticity of abatement cost	2.8	Literature
χ	Intensity of negative externality	0.000457	Literature
φ	Emissions per unit of output in the absence of abatement	0.601	Literature
ψ	EOEER	0.1, 1	Proportion of environmental punishment cost in GDP
δ_M	Decay rate of GHG stock	0.0021	Literature
A	TFP in steady state	5.1151	Steady output is 1 under fully competition market
S	Preference in steady state	1	No influence at steady state
ρ_Π	Policy Response to Inflation	0.5	Literature
ρ_Y	Policy Response to Output Gap	1.5	Literature

3. The Mixes of Monetary Policy with Different Climate Policy

In this section, we mix the monetary policy with four different types of climate policies: cap-and-trade, carbon tax, no control (with climate policy absent), and Ramsey optimal which constitute four regimes, and compare the mixes in terms of differences in fluctuation and welfare. We will also consider the differences brought by the (in)effectiveness of enforcement of environmental regulation. The comparison in this section will show if and how the monetary policy will differ when the type of climate policy and the effectiveness of environmental regulation is different.

3.1 Fluctuation Comparison

Annicchiarico and DiDio (2017) investigated the mixes of monetary policy and climate policy by giving one policy as Ramsey type and the other as varying types. They showed that key macroeconomic variables including labour, emission, interest rate, inflation have different response to productivity shock when policy type differs.

Their analysis can be extended in three aspects. First, at least one policy was assumed as Ramsey type in any regime they studied. This kind of policy mix is the ideal optimisation and difficult to carry out directly in the reality. The practically realizable "optimal mix" is not studied. So, more real-world practicable policy mixes can be investigated. Second, the potential ineffectiveness of environmental regulation that will cause illegal emission and change the dynamics of the economy can be considered. This relaxes the hidden assumption on the perfect effectiveness of environmental regulation. Third, the regime with "no climate policy" and "Ramsey climate policy" can be introduced into the comparison to serve as benchmarks.

We extend Annicchiarico and DiDio (2017)'s work by comparing the mixes of Taylor rule type monetary policy with four different types of climate policy (therefore constituting four regimes) with the consideration of the effectiveness of enforcement of environmental regulation. The four types of climate policy include cap-and-trade, carbon tax, no control (with climate policy absent) and Ramsey optimal (see Appendix for equations). The first three and the Taylor rule monetary policy are all practicable in the real-world. In this sub-section, we compare the fluctuation of economy in different regimes by conducting impulse response analysis. To be specific, we give 1% positive TFP shock and find the dynamics of economic variables afterwards. The EOEER ψ is set as 1 here as a benchmark. The values of tax level and emission target are set so that all regimes (except for the No Control regime ("NO" for short)[8]) share the same steady state with the case of Ramsey.

The results of impulse response analysis (absolute deviation from steady states) are show in Figure 2. It can be found that the responses of endogenous variables to the shock have different paths under the four different regimes. For economic and monetary variables, output under the cap-and-trade regime (CA) decrease more than the Ramsey optimal regime (RM), while under the carbon tax regime (TX) decrease less than RM. TFP damage coefficient (Λ_t), inflation together and the resulting interest rate under CA drop less than under RM, while under TX the negative changes are larger than the case under RM. The differences in output, inflation and interest rate among regimes are not big. For environmental related variables, abatement, illegal emission and emission price under CA rise more than under RM, while under TX they change less than under RM or even do not change. Legal emission and real emission under TX increase more than under RM, while under CA real emission rise less than under RM and legal emission does not change. The differences among regimes in environmental variables are more significant than in economic and monetary variables.

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[8] The No Control regime is equivalent to a carbon tax regime with a tax level a 0. This makes the steady state different.

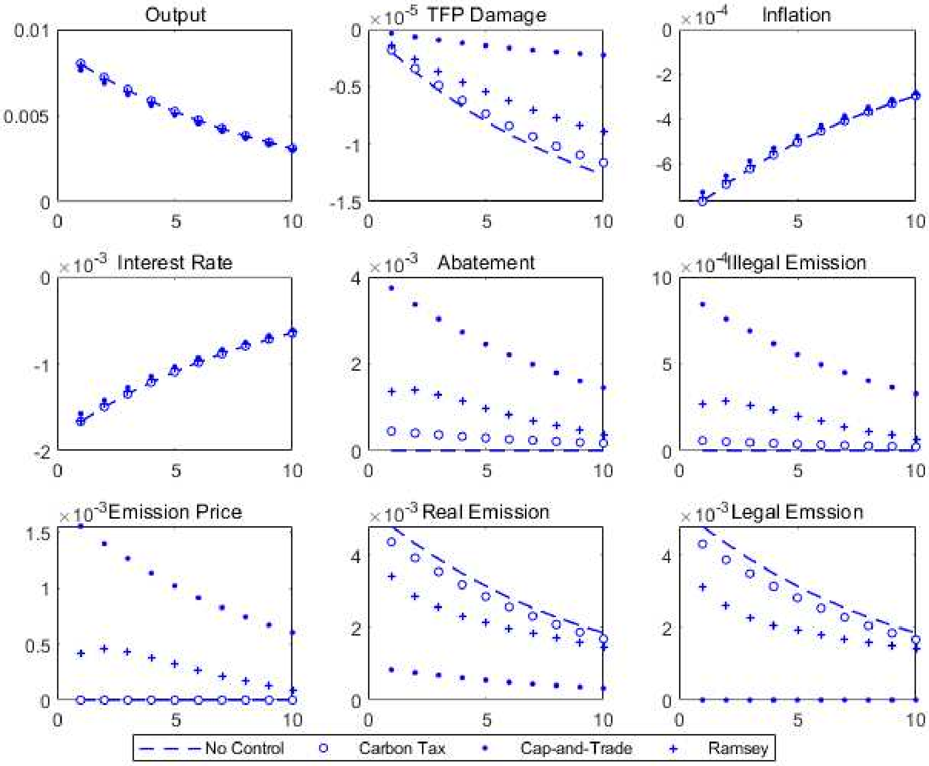

Figure 2: The dynamic of endogenous variables after 1% positive TFP shock under different regimes (EOEER=1)

To understand the mechanism behind the differences of the changes, we first need to understand that after a positive TFP shock, **emission price** and **real emission** under RM will **rise**. When the shock happens, every unit of output will have a lower cost. This decreases the price level and increases the demand. An increased demand causes an increased supply or output. When the level of output increases, the original emission from production also increase. This can cause a higher marginal damage to TFP so the Ramsey optimization requires a higher rate of abatement U_t. According to equation (18), the **emission price $p_{Z,t}$ also needs to be higher simultaneously** under RM. To dispose the extra original emission from production under RM, firms will be arranged to use all three channels, namely abating, legally emitting and illegally emitting, since all the channels have an increasing marginal cost for the society. Hence, abatement, legal emission, and illegal emission will all rise. As a result, **real emission, which**

equates the sum of legal and illegal emission, will also rise under RM.

Then the differences between CA and TX can be explained. Under TX (and NO), the **emission price** (for legal emission) is fixed at the carbon tax level (or 0) no matter how much firms emit. After shock, this is lower than the Ramsey optimal (increased) emission price. The relative lower emission price has three implications. (1) On **output**. The emission price is fixed hence its marginal level is also fixed and equates to the tax level. At optimum, the costs of all three channels for disposing the original emission from production share this same marginal level. The costs for disposing every unit of emission via illegal emitting and abatement are marginal increasing, hence the average cost of these two channels are lower than the tax level. Given the tax level is lower than the Ramsey optimal emission price, the average cost for disposing every unit of emission via all the three channels is less than the case under RM. When the unit emission cost is lower, the price level decreases which causes a higher demand for production output. So, it is higher than the case under RM. (2) On **real emission** and **TFP damage coefficient**. The relative lower disposing costs of legal and illegal emitting allow real emission which is the sum of legal and illegal emission rises more than the case under RM. Real emission accumulates into emission stock and directly decreases TFP damage coefficient (N.B. it is negative). Therefore, TFP damage coefficient drops more than it under RM. (3) On **legal emission, abatement and illegal emission**. With a lower emission price, the legal emission increases more than it under RM. When relatively more original emission from production is disposed by the legal emitting channel, less of it needs to be disposed by the other two channels, namely abating and illegal emitting. This causes the abatement and illegal emission to increase less than the case under RM. (4) On **inflation and interest rate**. A lower than RM emission price causes a lower marginal cost of production, then lower inflation and interest rate in succession. Hence, both the change in inflation and interest rate is lower than the case under RM.

Under CA, the mechanism of change is the antithesis of TX. The legal emission volume is fixed at emission target, so it is lower than the new Ramsey optimal (increased) legal emission level. After the shock and the rise of original emission, the illegal emitting and abatement channels need to dispose more emission than the case under RM. This brings a higher marginal disposing costs of these two channels. At optimum, the costs of all three channels for disposing the original emission share a same marginal level, hence emission price (for legal emission) rises higher than the case under RM. The higher than RM emission price (which is opposite to the lower than RM price under TX) has implications on endogenous variable that is exactly antithetical to those in the TX. Therefore, we can see the differences of change between CA and TX. Meanwhile, we can say that there exists a "cost-offsetting" effect in the CA regime which can better stabilize the economy when shock happens. This is because the fixed legal emission volume causes a higher/lower price for disposing emission and offsets the lowering/heightening price level (and also attenuates monetary policy). Under the TX regime, the fixed carbon price does not have such function.

In general, the above analysis shows that when monetary policy is mixed with different climate policy, the monetary policy itself (interest rate) and policies' effect on the economy (other endogenous variables) will differ facing TPF shock. Under TX the monetary policy (interest rate) is strengthened compared with RM; while the TX type climate policy is looser than the RM type (real emission too more and abatement too less). On the opposite, under CA the monetary policy is weakened; the CA type climate policy is tighter than the RM type.

The above analysis conveys two key messages: (1) The cap-and-trade regime of climate policy could be an attenuator for monetary policy. (2) The design of monetary policy should consider the existing regime of climate policy since the dynamic of monetary policy is influenced by the selection of climate policy.

3.2 Welfare Comparison

To further investigate the above policy mixes, we compare the welfare of the four regimes in addition to the above fluctuation analysis. This will help us find which mix among the four is better and which is worse.

In the comparison, we maintain all parameters including the coefficients in the Taylor rule and the EOEER fixed. We set the steady states of CA and TX equalling to RM's. The steady state of NO comes from the $p_{Z,t} = 0$ case of TX. So, the differences in welfare among CA, TX and RM is only caused by the difference in regime. We follow Mendicino and Pescatori (2007)'s welfare criterion and calculate the conditional welfare of individual. The expression is

$$W_j = \mathbb{E}_t \sum_{m=0}^{\infty} \beta^m \left(\ln C_{j,t+m} - \mu_L \frac{L_{j,t+m}^{1+\eta}}{1+\eta} \right) \tag{27}$$

where W_j is the conditional welfare, $j = \{\mathrm{NO}, \mathrm{TX}, \mathrm{CA}, \mathrm{RM}\}$ means the four regimes of climate policy: no control, carbon tax, cap-and-trade and Ramsey optimal.

In order to show more intuitive results, we also calculate the consumption equivalents (CE) of each case. CE is the additional fraction of consumption that households under no policy can obtain if a certain policy is introduced for them. Let

$$W_{j'} = \mathbb{E}_t \textstyle\sum_{m=0}^{\infty} \beta^m \left[\ln\left(1 + CE_{j'}\right) C_{NO,t+m} - \mu_L \frac{L_{NO,t+m}^{1+\eta}}{1+\eta} \right] \tag{28}$$

we have

$$CE_{j'} = \exp\{(1-\beta)\left(W_{j'} - W_{NO}\right)\} - 1 \tag{29}$$

where $j' = \{\mathrm{TX}, \mathrm{CA}, \mathrm{RM}\}$ represents a certain regime of climate policy.

The welfares of all four regimes and the corresponding CEs is shown in Table 2.

Table 2: Welfare and Consumption Equivalent of the four regimes

	Welfare	CE
NO	-59.469	0
TX	-58.583	0.0088972
CA	-58.585	0.0088727
RM	-58.566	0.0090715

We can find

$$W_{RM} > W_{TX} > W_{CA} > W_{NO} \tag{30}$$

and

$$CE_{RM} > CE_{TX} > CE_{CA} > CE_{NO} \tag{31}$$

In specific, (1) Any regime with climate policy has a better welfare than NO since any climate policy can somehow reduce emission and so do its externality. (2) RM has the highest welfare and CE among all regimes. This is the nature of Ramsey policy. (3) CA is a little better than TX in terms of welfare and CE, however, the differences between them are not big.

TX tends to be a better choice among the three real-world achievable regimes (CA, TX and NO) when TFP shock happens in terms of the welfare standard. However, according to sensitivity analysis, it is not always the best choice. We find that when the parameter EOEER is small enough or the shock is changed to demand-type, the result $W_{TX} > W_{CA}$ and $CE_{TX} > CE_{CA}$ will reverse to $W_{TX} < W_{CA}$ and $CE_{TX} < CE_{CA}$. Hence, no mix of policy is always dominant to others regardless of parameters and shocks. This means that there is no absolute or “unconstrained” optimal mix of monetary and climate policy that is implementable in the real-world in terms of welfare standard. When parameters or shock changes, the optimal mix could change to another form. In Sub-Section 4.1, we will try to find “constrained” optimal mixes by optimising policy coefficients in given regimes of policy mix.

3.3 The Role of Environment Regulatory Effectiveness

This section tries to find that if the effectiveness of enforcement of environmental regulation, in addition to the choice of climate policy type, will also bring differences to the economy and the monetary policy.

To do this, we set a lower effectiveness parameter ψ equalling to 0.1. This is a much smaller value than the benchmark case in Sub-Section 3.1 where $\psi = 1$. The small value means the environmental regulation is less effective. As in Figure 3, we show the fluctuation of economy following the same way in Sub-Section 3.1. It needs to be noted that the units of some vertical-axis in Figure 2 and Figure 3 are different. Then, we compare the results in Figure 2 ($\psi = 1$) and in Figure 3 ($\psi = 0.1$) to find any differences brought by the effectiveness of enforcement of environmental regulation.

It can be found that when the effectiveness is lower, for variables excepting legal and illegal emission, the differences of fluctuation between CA and TX becomes smaller – mainly because that the variables' paths under CA move more approximate to the paths under TX. For legal emission, under TX it changes more than the case when environmental regulation is more effective. For illegal emission, under CA it changes more.

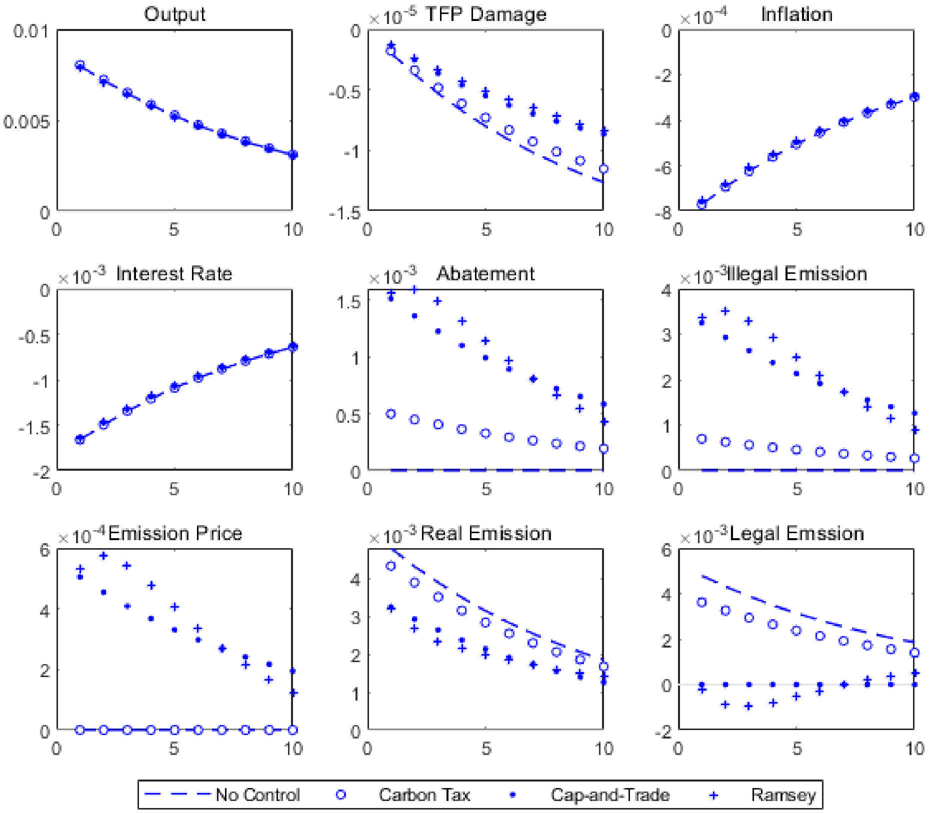

Figure 3: The dynamic of endogenous variables after 1% positive TFP shock under different regimes (EOEER=0.1)

The pivotal reason of the changed differences is that the less effective of enforcement of environmental regulation gives firms more space to dispose their emission via the illegal emitting channel. When ψ is lower, the unit cost for illegal emission and the total cost for disposing every unit of original emission will decrease. This allows the steady state share of illegal emission in original emission (i.e. V_t) and original emission to increase. After TFP shock under TX, illegal emission rises more than the case with higher ψ because of the increased steady state V_t. The path of abatement is almost not changed because the extra original emission after shock does not change significantly, and the share of abatement for disposing every unit of original emission (i.e. U_t) is not changed according to equation (18) which does not include ψ. The path of real emission whose share is $1 - U_t$ neither changed significantly for the same reason. The legal emission rises less since its share in disposing every unit of original emission $1 - U_t - V_t$ is decreased due to an increased V_t. The paths of inflation and interest rate are almost not changed due to a fixed $p_{Z,t}$ under TX.

After TFP shock under CA, $p_{Z,t}$ increases less than the case when ψ is higher since the cost for illegal emission rises less[9]. This brings more similar changes in the path of inflation and interest rate. Illegal emission rises more than the case with higher ψ for the same reason under TX. Abatement increases less since more original emission is disposed by the illegal emitting channel. Real emission rises more since the illegal emission increased more and the legal emission is fixed under CA.

Besides the fluctuation analysis, we also calculate and compare the welfare of each regime after the EOEER is changed to 0.1. We find that the order of welfare and consumption equivalent comparison will change to $W_{ET} > W_{TX}$ and $CE_{CA} > CE_{TX}$. The reason is that consumption, as one of the determinants of welfare, is increased more under CA than under TX. A lower ψ brings a lower cost for illegal emission. Under CA this also brings a lower $p_{Z,t}$. Then price level decreases; demand, production output and consumption increase. However, under TX, $p_{Z,t}$ is fixed, hence price level decreases not as much as the case under CA. Then consumption does not rise so much.[10] The output under CA rises more than it under TX after shock, which makes the output

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[9] There is a marginal increasing cost for illegal emission $\frac{\psi}{2} v_{t,j}^2 \varphi Y_{j,t}$. When ψ is lower, the steady state cost for illegal emission is lower. Hence the cost for illegal emission rises less here. Meanwhile, at optimal, the three channels for disposing original pollution have a same marginal cost, hence $p_{Z,t}$ equals to the cost for illegal emission.

[10] The fluctuation of price also influences welfare according to Rotemberg (1982). However, the result here means the influence of consumption on welfare is stronger.

gap under CA relatively smaller and welfare larger.

The above analysis shows that the ineffectiveness of enforcement of environmental regulation will make climate policy less effective and different regimes become more similar by giving more space for illegal emission. This implies that the economy and the monetary policy will fluctuate differently when the EOEER is different. The extent of the attenuation effect of climate policy on monetary policy will be changed by the strength of EOEER. Therefore, in addition to the type of climate policy, the effectiveness of enforcement of environmental regulation also needs to be considered when designing monetary policy. Otherwise, the dynamic of monetary policy and its effect on the economy will be somewhat different (too strong or too weak) from what is originally envisaged. This should be particularly noticed by the authorities of developing countries like China.

4. The Optimisation of Policy Mixes

It is found from Sub-Section 3.2 that there is no realistic and “unconstrained” optimal mix of monetary and climate policy in terms of welfare standard. In this section, we try to find the “constrained” optimal mixes by optimising policy coefficients in the traditional Taylor rule of monetary policy under given regimes. We also investigate a radically “climate-friendly” way to improve the policy mix. This is to introduce the emission control (i.e. climate change mitigation) target into the Taylor rule of monetary policy. We will see if it can become a useful practice and try to find the optimal coefficient for the new target.

4.1 Optimisation in the Traditional Monetary Policy

The Ramsey optimal monetary and climate policy (RM) is the ideally optimal policy mix. However, it is difficult for policy makers to carry out in the reality since the RM assumes that all endogenous variables in the economy can be controlled and adjusted by the authority.

The realistic way to improve or to optimise the policy mix is to choose between the CA and TX (and NO) regime or (and) to adjust the policy coefficients in them. It was found from Sub-Section 3.2 that there is no “unconstrained” optimal mix (regime) of monetary and climate policy in terms of welfare standard. So, we can only adjust the parameters in a given regime to improve itself. This is to find the “constrained” optimal mix. To do this, we have three potential options. The first is to give a fixed strength of climate policy (carbon price level or emission cap fixed) and optimise the coefficients in the Taylor rule of monetary policy (ρ_Y and ρ_Π). The second is to fix the monetary policy coefficients and optimise the climate policy strength. The third is to optimise the climate strength and the monetary coefficients simultaneously. We choose the first method since it is the financial regulator who recently and prominently wants to know what monetary policy needs to do facing the climate problem. The second method is on the angle of environmental regulator. The third is more comprehensive however more complex and difficult for policy makers to coordinate and carry out.

We combine different values of monetary policy coefficients with different types of climate policy (CA or TX) under different EOEER and shocks to form the candidates of regime with “constrained” optimal policy mix. Shocks include TFP, cost-push and preference shock since these three can cover both supply and demand side shocks. Then, we calculate the welfare and CE of every candidate of regime. If there is a maximum of welfare and CE under certain climate policy, EOEER and shock, the corresponding policy coefficients ρ_π and ρ_Y are the (constrained) optimal values of that regime. For simplicity, we only consider the regimes that can solve the model with a unique solution.

We find that under cost-push shock (a positive θ_t shock), there exist optimal monetary policy coefficients for every climate policy and EOEER, as shown in Table 3. This means that if the cost-push shock is dominant in the economy, the central bank has a best choice of coefficients in the Taylor rule of monetary policy, when climate policy and EOEER is given.

Table 3: Optimal policy coefficients in the Taylor rule of monetary policy under different climate policy and EOEER (cost-push shock)

φ	Cap-and-Trade		Carbon Tax	
	ρ_{π}	ρ_Y	ρ_{π}	ρ_Y
0.1	3.2335	0.4573	3.4792	0.4591
0.5	2.8024	0.4573	3.4948	0.4593
1	2.6819	0.4589	3.4969	0.4593
10	2.5549	0.4619	3.4984	0.4593
100	2.5418	0.4624	3.4985	0.4591

We can find from Table 3 that ρ_Y does not vary significantly across climate policy regimes, however, ρ_{Π} is always larger under TX than under CA. This is because the emission price in the CA regime changes when shock happens. When cost-push shock (a positive θ_t shock) happens, the price level becomes lower which causes a higher demand, production output and emission. The higher emission then causes a higher price for disposing emission under CA (see Sub-Section 3.1 for detail). Hence the price level under TX (which is fixed) is relatively lower than the case under CA. To suppress deflation, a stronger ρ_{Π} is needed. This again shows the “cost-offsetting” effect in the CA and the basic mechanism that differs the two climate regimes.

Under TFP and preference shocks, we find that the welfare and CE becomes higher when ρ_{π} and ρ_Y become larger. This is a common result of the New-Keynesian model. However, this means there is no optimal values of ρ_{π} and ρ_Y if the ranges of the coefficients are not limited and TFP (or preference) shock is dominant in the economy.

To summarise, it is found that when climate policy is considered, the monetary policy can be improved by adjusting Taylor rule coefficients. If the cost-push shock is dominant in the economy, there exists optimal coefficients. Both the regime of climate policy and the EOEER can affect the value of the optimal coefficients. Till this section, we can also summarise that when the existing climate policy is brought into the framework of central bank's policy making, at least three things can be considered to improve the monetary policy: the type (regime) of climate policy, the effectiveness of enforcement of environmental regulation and the coefficient in the Taylor rule of monetary policy.

4.2 Monetary Policy Pegging on Emission

In this section, we turn to a radical way to optimise the traditional policy mix. This is to change the form of the Taylor rule of monetary policy by incorporating the emission control target into it. Considering central banks' recent interest in helping solve the climate change problem, this will help answer their question of "if it is good for central bank to use the monetary policy to proactively take care for the environment".

Our method is to add the emission gap as a third target into the traditional inflation and output gap targeting Taylor rule. The new form of the Taylor rule is

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\rho_\Pi} \left(\frac{Y_t}{Y_t^{na}}\right)^{\rho_Y} \left(\frac{Z_{t-1}}{Z}\right)^{\rho_Z} \tag{32}$$

where Y_t^{na} is the natural output without nominal price stickiness, and R, Π, Z are the steady states of nominal interest rate, inflation rate and real emission respectively. The Z_{t-1} is the real emission of one period earlier. The real emission target is not a replication of the output target since it is not the original emission that is proportional to output. Meanwhile, the real emission has direct effect on the environment so it can reflect the environmental objective. We assume the authority uses Z_{t-1} not Z_t to form the emission target since the real emission includes the illegal emission which is often concealed and cannot be detected at the period of policy making. This new form of Taylor rule makes the monetary policy proactively take care for the environment.

Then, we set the inflation and the output coefficient as fixed: $\rho_Y = 0.5$ and $\rho_\Pi = 1.5$; Calculate welfare values of the economy with different ρ_Z and different shocks. ρ_Z is taken ergodic of the its interval that can produce a unique solution for the equilibrium. Common shocks (TFP, cost-push and preference) that cover both supply and demand side shocks are introduced respectively. Under a same shock, if the welfare with a ρ_Z is higher than the welfare with $\rho_Z = 0$, a ρ_Z that can improve the policy mix is found. As ρ_Y and ρ_Π are fixed and allowing ρ_Z to change is introducing a new dimension for optimisation, there must be some ρ_Z that can improve the welfare by serving as a supplement of the potential over-strong or over-weak ρ_Y and ρ_Π.

By the above method, we find the intervals of ρ_Z that can improve the welfare, as well as the values of ρ_Z that can enhance the welfare at the greatest extent (i.e. the optimized value of ρ_Z) under different regimes with different shocks (as shown in Table 4). When the TFP or cost-push shock is dominant, the optimal ρ_Z is negative in the both climate regimes. When the preference shock is dominant, the optimal ρ_Z lie in the right boundary of possible values which means the higher ρ_Z is, the higher welfare will be.

Table 4: The interval of ρ_Z that can improve welfare and optimal ρ_Z under different climate policy and shock (original price stickiness)

Shock	Cap-and-Trade		Carbon Tax	
	Interval	Optimal	Interval	Optimal
TFP shock	(-0.866, 0)	-0.453	(-0.174, 0)	-0.091
Cost-push shock	(-0.509, 0)	-0.261	(-0.12, 0)	-0.062
Preference shock	The high the better			

Except with preference shock, the optimal ρ_Z can also be positive with other shocks if parameter changes. We find that if the price stickiness parameter γ is large enough (e.g. 10 times larger which is roughly in line with Gertler (2019)), the optimal ρ_Z becomes positive under both regimes with cost-push shock, as shown in Table 5.

Table 5: The interval of ρ_Z that can improve welfare and optimal ρ_Z under different climate policy and shock (price stickiness 10 times larger)

Shock	Cap-and-Trade		Carbon Tax	
	Interval	Optimal	Interval	Optimal
TFP shock	(-0.934, 0)	-0.508	(-0.16, 0)	-0.087
Cost-push shock	(0, 1.342)	0.602	(0, 0.184)	0.085
Preference shock	The high the better			

We must point out that when the optimal ρ_Z is negative, there is a dilemma between the welfare objective and the environmental objective. With a positive TFP or cost-push shock, the emission gap is positive due to the lower price level, higher output and emission. A negative ρ_Z will derive a lower interest rate which encourages demand and production and causes a higher emission. The higher emission is on the contrary of the environmental protection objective. Under this circumstance, if we change the ρ_Z to a positive value to realise the environmental objective (emission control), the welfare enhancing objective cannot be achieved. Failing to enhance welfare is incompatible with the traditional mandate of a central bank. So, it is doubtful to adopt the emission control target into the Taylor rule of monetary policy when the optimal ρ_Z is negative.

Only when the optimal ρ_Z is positive, improving the welfare by including the emission control target can simultaneously reduce emission. The welfare objective can be consistent with the environmental objective. On this occasion, the adoption of the emission control target into the Taylor rule of monetary policy is worth considering by the central bank for both the welfare and the environmental reason.

This section shows that changing the form of the Taylor rule of monetary policy by incorporating the emission control target into it can improve the policy mix in terms of welfare standard. The optimal value of the coefficient for targeting is found under different situations. However, under certain circumstances, this radically “climate-friendly” monetary policy will bring a dilemma between the welfare and the environmental objective, which makes it incompatible with the tradition mandate of central bank.

The analysis implies an answer to the question "if it is good for central bank to use the monetary policy to proactively take care for the environment": In terms of both welfare and environmental objective and by using a Taylor rule with emission control target, the answer is "yes", under certain shocks (e.g. cost-push shock) and parameters (e.g. a relatively high price stickiness); the answer will be "maybe not", under other shocks (e.g. TFP shock) and parameters (e.g. a relatively low price stickiness).

In the real world, one certain shock cannot always be dominant in the economy and it is difficult to change the form of monetary policy rule frequently. If we do not want to bring more dilemmas and difficulties to the central bank, it is better not to choose the radically "climate-friendly" rule of monetary policy.

5. Conclusion

In this paper, we have studied the relationship between monetary and climate policy and tried to find their optimal mix in an E-DSGE model with illegal emission and related regulation augmented. Using this model, we have compared the mixes of monetary policy with different climate policy to find whether and how climate policy will influence monetary policy; optimised the coefficients in the monetary policy rule under certain climate policy; given a climate-proactive monetary policy and investigated if and when it can be a good choice for the central bank.

Main findings include three parts. First, the dynamic of monetary policy is influenced by the selection of regimes of climate policy and the effectiveness of enforcement of environmental regulation. The pivotal reason of the difference between regimes is that the cap-and-trade regime can offset the fluctuation of price after shocks, while the carbon tax regime cannot. The effectiveness of environmental regulation also plays a role since it can make climate policy less effective by giving more space for illegal emission.

Second, there is no unconstrained optimal mix of monetary and climate policy that is implementable in the real-world, however, the coefficients in the traditional Taylor rule of monetary policy can be better set to enhance welfare when a certain regime of climate policy is given in the economy. If the cost-push shock is dominant in the economy, there exists optimal coefficients. Both the regime of climate policy and the effectiveness of environmental regulation can affect the value of the optimal coefficients. We can summarise from the above that, under the framework with climate factors, at least three things can be considered to improve the monetary policy: the type (regime) of climate policy, the effectiveness of enforcement of environmental regulation and the coefficient in the Taylor rule of monetary policy.

Third, if the mitigation of climate change is augmented into the target of monetary policy, the economy's welfare can be enhanced. The optimal value of the coefficient for targeting is found under different situations. However, under some circumstances, this radically "climate-friendly" monetary policy will bring a dilemma between the welfare and the environmental objective, which makes it incompatible with the tradition mandate of central bank. If we do not want to bring more difficulties to the central bank, it is better not to choose the "climate-friendly" rule of monetary policy.

The overall conclusion is that the design of monetary policy should consider the existing climate policy, otherwise, the dynamic of monetary policy and its effect on the economy will be different from what is originally envisaged. After this consideration, a given regime of policy mix can be improved by adjusting the coefficients in the monetary policy rule. Adding the climate target into the monetary policy rule may not be desirable.

Although the "climate-friendly" monetary policy is found to be controversial in this research, it does not mean this kind of monetary policy is useless from other angles of view. The DSGE model is mainly used for short-term analysis so the conclusions are mainly based on short-terms standards. Climate change could be a long-term problem for mankind. Considering that the "climate-friendly" monetary policy can support a green economic transition and reduce future climate risk, it could be a preferable choice in the long-run. From this angle, it is not conflict with the mandate of central bank.

This research can be extended in several aspects. For example: (1) Set EOEER as a shock to study the "transition risk" brought by climate change and the tightening process of environmental regulation (e.g. China's environmental inspection). (2) Set dynamic rule (e.g. Taylor rule) for climate policy. (3) Change the emission gap target in monetary policy to other forms. (4) Introduce more types of shocks in the study of economic fluctuation. (5) Introduced more financial fractions and features to describe the role of monetary policy more precise. (6) Besides the monetary policy, introduce and study more policy tools and measures that central banks can use to prevent climate risk and support the green economic transition (e.g. identifying green financing and differentiating reserve rate requirement, re-lending and collateral requirement (Pan, 2019), asset purchase and credit guidance)

Appendix

Derivation of the New Keynesian Phillips Curve

The maximization problem of firm j is

$$\begin{cases} V_0 = \max \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \Omega_{0,t} \left[\frac{P_{j,t}}{P_t} Y_{j,t} - TC_{j,t} - \frac{\gamma}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t \right] \right\} \\ s.t. \begin{cases} TC_{j,t} = \frac{W_t}{P_t} L_{j,t} + \phi_1 U_{j,t}^{\phi_2} Y_{j,t} + p_{Z,t} \left(1 - U_{j,t} - V_{j,t} \right) \varphi Y_{j,t} + \frac{\psi}{2} V_{j,t}^2 \varphi Y_{j,t} \\ Y_{j,t} = \Lambda_t A_t L_{j,t} \\ Y_{j,t} = \left(\frac{P_{j,t}}{P_t} \right)^{-\theta_t} Y_t \end{cases} \end{cases}$$

We can rewrite the objective function by Bellman Equation as

$$V_t = \max \left\{ \frac{P_{j,t}}{P_t} Y_{j,t} - TC_{j,t} - \frac{\gamma}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t + \mathbb{E}_t \mathit{\Omega}_{t,t+1} V_{t+1} \right\}$$

Which yields the Lagrangian function as

$$\mathcal{L}_t = \frac{P_{j,t}}{P_t} Y_{j,t} - \left[\frac{W_t}{P_t} \frac{Y_{j,t}}{\Lambda_t A_t} + \phi_1 U_{j,t}^{\phi_2} Y_{j,t} + p_{Z,t} \left(1 - U_{j,t} - V_{j,t}\right) \varphi Y_{j,t} + \frac{\psi}{2} V_{j,t}^2 \varphi Y_{j,t} \right]$$

$$- \frac{\gamma}{2} \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right)^2 Y_t + \mathbb{E}_t \left[\Omega_{t,t+1} V_{t+1} \right] + \lambda_{j,t} \left[\left(\frac{P_{j,t}}{P_t} \right)^{-\theta_t} Y_t - Y_{j,t} \right]$$

where $\Omega_{t,t+1} = \beta \frac{C_t}{C_{t+1}}$ is the stochastic discount factor. So, we can obtain the FOC for $U_{j,t}$ and $V_{j,t}$

$$p_{Z,t} = \frac{\phi_1 \phi_2}{\varphi} U_{j,t}^{\phi_2 - 1}$$

$$V_{j,t} = \frac{p_{Z,t}}{\psi}$$

and derives

$$MC_{j,t} = \frac{W_t}{P_t} \frac{1}{\Lambda_t A_t} + \phi_1 U_{j,t}^{\phi_2} + p_{Z,t} \left(1 - U_{j,t} - V_{j,t}\right) \varphi + \frac{\psi}{2} V_{j,t}^2 \varphi$$

The FOCs for $P_{j,t}$ and $Y_{j,t}$ derive

$$1 - \theta_t - \gamma \left(\frac{P_{j,t}}{P_{j,t-1}} - 1 \right) \frac{P_{j,t}}{P_{j,t-1}} + \beta \gamma \mathbb{E}_t \left[\left(\frac{P_{j,t+1}}{P_{j,t}} - 1 \right) \frac{P_{j,t+1}}{P_{j,t}} \frac{C_t}{C_{t+1}} \frac{Y_{t+1}}{Y_t} \right] + \theta_t MC_{j,t} = 0$$

Equation Systems of First Order Conditions

Taylor Rule Monetary Policy Mix Cap-and-Trade Climate Policy

$$
\left\{
\begin{array}{c}
\beta R_t \mathbb{E}_{\mathrm{t}}\left[\frac{C_t}{C_{t+1}}\frac{1}{\Pi_{\mathrm{t+1}}}\right] = 1 \\
(1-\theta_t) - \gamma(\mathit{\Pi}_t - 1)\mathit{\Pi}_t + \beta\gamma\mathbb{E}_{\mathrm{t}}\left[\frac{C_t}{C_{t+1}}(\mathit{\Pi}_{t+1} - 1)\mathit{\Pi}_{t+1}\frac{Y_{t+1}}{Y_t}\right] + \theta_t MC_t = 0 \\
MC_t = \phi_1 \widetilde{U}_t^{\phi_2} + p_{Z,t}(1 - U_t - v_t)\varphi + \frac{W_t}{\mathit{\Lambda}_t A_t P_t} + \frac{\psi}{2}v_t^2\varphi \\
L_t^{\eta} = \frac{W_t}{\mu_L P_t C_t} \\
Y_t = C_t + \phi_1 U_t^{\phi_2} Y_t + \frac{\gamma}{2}(\mathit{\Pi}_t - 1)^2 Y_t \\
Z = (1 - U_t - v_t)\varphi Y_t + \tilde{Z} \\
M_t = (1 - \delta_M)M_{t-1} + (1 - U_t)\varphi Y_t + \tilde{Z} \\
Y_t = e^{-\chi(M_t - \widetilde{M})} A_t L_t \\
p_{Z,t} = \frac{1}{\varphi}\phi_1\phi_2 U_t^{\phi_2 - 1} \\
v_t = \frac{1}{\psi\varphi}\phi_1\phi_2 U_t^{\phi_2 - 1} = \frac{p_{Z,t}}{\psi} \\
\frac{R_t}{R} = \left(\frac{\mathit{\Pi}_t}{\mathit{\Pi}}\right)^{\rho_{\mathit{\Pi}}} \left(\frac{Y_t}{Y_t^{na}}\right)^{\rho_Y}
\end{array}
\right.
$$

Taylor Rule Monetary Policy Mix Carbon Tax Climate Policy

$$
\left\{
\begin{array}{c}
\beta R_t \mathbb{E}_{\mathrm{t}}\left[\frac{C_t}{C_{t+1}}\frac{1}{\Pi_{\mathrm{t+1}}}\right] = 1 \\
(1-\theta_t) - \gamma(\mathit{\Pi}_t - 1)\mathit{\Pi}_t + \beta\gamma\mathbb{E}_{\mathrm{t}}\left[\frac{C_t}{C_{t+1}}(\mathit{\Pi}_{t+1} - 1)\mathit{\Pi}_{t+1}\frac{Y_{t+1}}{Y_t}\right] + \theta_t MC_t = 0 \\
MC_t = \phi_1 U^{\phi_2} + p_{Z,t}(1 - U - v)\varphi + \frac{W_t}{\mathit{\Lambda}_t A_t P_t} + \frac{\psi}{2}v^2\varphi \\
L_t^{\eta} = \frac{W_t}{\mu_L P_t C_t} \\
Y_t = C_t + \phi_1 U_t^{\phi_2} Y_t + \frac{\gamma}{2}(\mathit{\Pi}_t - 1)^2 Y_t \\
M_t = (1 - \delta_M)M_{t-1} + (1 - U)\varphi Y_t + \tilde{Z} \\
Y_t = e^{-\chi(M_t - \widetilde{M})} A_t L_t \\
p_Z = \frac{1}{\varphi}\phi_1\phi_2 U^{\phi_2 - 1} \\
v = \frac{1}{\psi\varphi}\phi_1\phi_2 U^{\phi_2 - 1} = \frac{p_Z}{\psi} \\
\frac{R_t}{R} = \left(\frac{\mathit{\Pi}_t}{\mathit{\Pi}}\right)^{\rho_{\mathit{\Pi}}} \left(\frac{Y_t}{Y_t^{na}}\right)^{\rho_Y}
\end{array}
\right.
$$

Taylor Rule Monetary Policy Mix No Control Climate Policy

No control policy is a special case of the carbon tax policy with $p_Z = 0$. The equation system is all the same with the "Taylor Rule Monetary Policy Mix Carbon Tax Climate Policy" except p_Z is set as 0.

Taylor Rule Monetary Policy Mix Ramsey Optimal Climate Policy

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\ln C_t - \mu_L \frac{L_t^{1+\eta}}{1+\eta} \right)$$

$$s.t. \begin{cases} \beta R_t \mathbb{E}_\mathrm{t} \left[\frac{C_t}{C_{t+1}} \frac{1}{\Pi_{\mathrm{t+1}}} \right] = 1 \\ (1-\theta_t) - \gamma(\Pi_t - 1)\Pi_t + \beta\gamma \mathbb{E}_\mathrm{t} \left[\frac{C_t}{C_{t+1}} (\Pi_{t+1} - 1)\Pi_{t+1} \frac{Y_{t+1}}{Y_t} \right] + \theta_t MC_t = 0 \\ MC_t = \phi_1 \widetilde{U}_t^{\phi_2} + p_{Z,t}(1 - U_t - v_t)\varphi + \frac{W_t}{\Lambda_t A_t P_t} + \frac{\psi}{2} v_t^2 \varphi \\ L_t^{\eta} = \frac{W_t}{\mu_L P_t C_t} \\ Y_t = C_t + \phi_1 U_t^{\phi_2} Y_t + \frac{\gamma}{2} (\Pi_t - 1)^2 Y_t \\ M_t = (1 - \delta_M) M_{t-1} + (1 - U_t)\varphi Y_t + \tilde{Z} \\ Y_t = e^{-\chi(M_t - \widetilde{M})} A_t L_t \\ \frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi} \right)^{\rho_\Pi} \left(\frac{Y_t}{Y_t^{na}} \right)^{\rho_Y} \end{cases}$$

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