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**Abstract**

We build a standard overlapping generations model with endogenous fertility and involuntary unemployment. Being different from a log utility function, the capital income tax affects saving at the model of constant relative risk-averse utility function (CRRA function). In the parameter condition, to have the case of non-substitution between consumption in different periods, the capital income tax raises saving to compensate for consumption in the future. Then, results show that a capital income tax improves fertility and unemployment with no social security system.

**Keywords:** capital income tax, fertility, unemployment

**JEL Classifications:** J13 J60

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## 1. Introduction

Traditionally, a capital income tax is regarded as decreasing incentives for saving and as hindering economic growth if households exist infinitely, similarly to results reported by Chamly (1986). However, a capital income tax can contribute to macroeconomic performance within an overlapping generations model, as shown Uhlig and Yanagawa (1996). Kunze and Schuppert (2010) find that a capital income tax enhances not only economic growth but also employment. However, the impact of a capital income tax on fertility choice and unemployment under an overlapping generations model are considered only insufficiently in earlier studies.

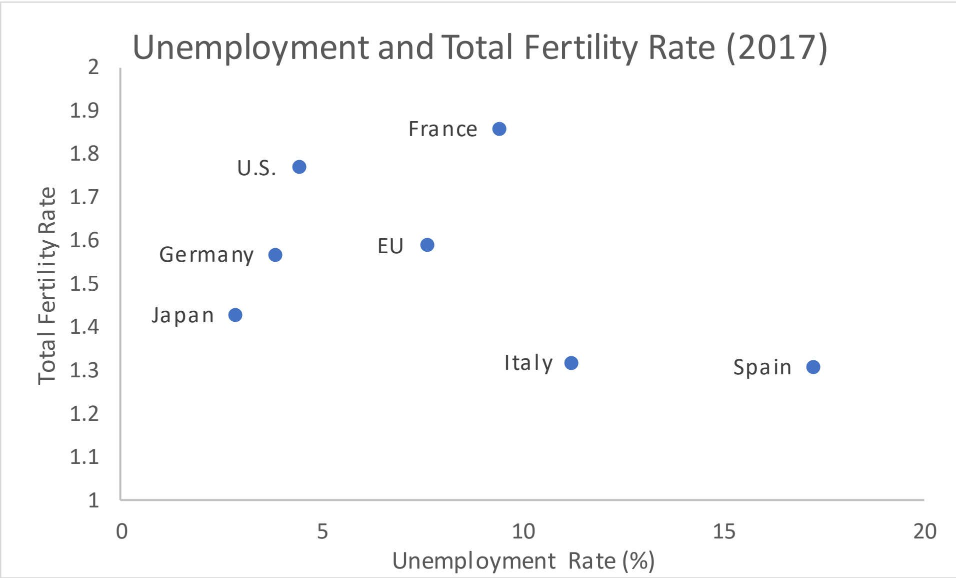


Fig. 1: Unemployment and Total Fertility Rate.

(Data: OECD Statistics. However, fertility within the EU is based on Eurostat.)

Following Fanti and Gori (2010), a decline of fertility and an increase in unemployment are crucially important issues in economically developed countries such as EU countries, as shown in Fig. 1. Therefore, a child tax is recommended as a remedy for the issues in their study. Wang (2015) states that pensions and child subsidies increase fertility and employment.

We extend Fanti and Gori (2010) and emphasize positive effects of a capital income tax. Results show that a capital income tax improves fertility and unemployment with no social security system such as one that provides a wage subsidy, child subsidy, and a pension. We assert the originality of the points in this paper as follows. For an endogenous fertility model, a logarithmic utility function is often assumed.[1] However, in the logarithmic utility function, saving is unaffected by the change of the interest rate: in fact, the capital income tax has no effect on saving. However, considering the

[1] Van Groezen, Leers and Meijdam (2003) and others assume a logarithmic utility function in the endogenous fertility model.

constant relative risk averse (CRRA) utility function, the capital income tax raises the saving if it is complementary between present consumption and future consumption. Then, by virtue of an increase in saving, the capital stock per capita rises.

The remainder of this article is constructed as follows. Section 2 builds our model. Section 3 derives the equilibrium of the model economy and examines the effect of a capital income tax on fertility and unemployment. The final section concludes our study.

## 2. The model

We construct a two-period overlapping generations (OLG) model with discrete time. The cohort born in period $t$ is regarded as generation $t$, with agents living for two periods: one for the young period and the other for the old period. This model economy has agents of three types: households, firms, and a government.

### 2.1. Firm

The production function in this model is assumed to have the following form:

$$Y_t = AK_t^{\alpha} L_t^{1-\alpha}, 0 < \alpha < 1, 0 < A, \tag{1}$$

where $Y_t$ represents total output. Here, $K_t$ stands for capital stock, $L_t$ denotes labor input, and $A$ expresses the technology level of the firm.

Our paper presents consideration of unemployment with a minimum wage, as examined by Fanti and Gori (2010) and by Wang (2015). Particularly,Wang (2015) explain how a pension premium affects income per capita in the long run through capital markets. If the pension premium decreases, then the effect for capital accumulation is mitigated. This effect raises income per capita. An increase in income per capita can raise the pension benefit under the constant premium rate for pensions.

In the unemployment model, labor input can be presented as

$$L_t = (1 - u_t)N_t, \tag{2}$$

where $u_t$ signifies the unemployment rate and $N_t$ is the population size. We define the fertility rate as $n_t$. Actually, $n_t$ can be regarded as the population growth rate and $N_{t+1} = n_t N_t$ can be reduced. The minimum wage is assumed in this model. We consider a model economy in which the minimum wage is higher than the competitive wage. We also assume that the capital stock is fully depreciated. We can obtain the following equation to maximize the firm profit as

$$\overline{w} = (1 - \alpha)Ak_t^{\alpha}(1 - u_t)^{-\alpha}, \tag{3}$$

$$1 + r_t = \alpha Ak_t^{\alpha-1}(1 - u_t)^{1-\alpha}. \tag{4}$$

Therein, $\overline{w}(> 0)$ represents the minimum wage, $k_t$ denotes the capital stock per capita $\left(= \frac{K_t}{N_t}\right)$, and $r_t$ is the interest rate. Because of (3), employment rate $1 - u_t$ can be represented as

$$1 - u_t = \left(\frac{(1-\alpha)A}{\overline{w}}\right)^{\frac{1}{\alpha}} k_t. \tag{5}$$

By substitution of (5) into (4), the interest rate can be given as a constant level over time as

$$r_t = r = \alpha A^{\frac{1}{\alpha}} \left(\frac{1-\alpha}{\overline{w}}\right)^{\frac{1-\alpha}{\alpha}} - 1. \tag{6}$$

We assume that $r > 0$.

### 2.2. Household

Households care for the consumption in both young and old period and the number of children (fertility). We assume the following Constant Relative Risk Averse (CRRA) utility function as

$$v_t = \frac{c_{y,t}^{1-\theta} - 1}{1-\theta} + \beta \frac{c_{o,t+1}^{1-\theta} - 1}{1-\theta} + \frac{n_t^{1-\theta} - 1}{1-\theta}, 0 < \beta < 1, 0 < \theta, \tag{7}$$

where $c_{y,t}$ and $c_{o,t+1}$ respectively denote consumption in the young and old period. With $\theta = 1$, a utility function (7) becomes a logarithmic utility function. Our paper presents consideration of equation (7), which is more of a general type than a logarithmic type.

In the young period, the household obtain wage income $(1-u_t)\overline{w}$.[2] Here, $\varepsilon(>0)$ and $s_t$ respectively denote the childcare cost and saving. Then, the budget constraint in the young period can be shown as

$$c_{y,t} + \varepsilon n_t + s_t = (1-u_t)\overline{w}. \tag{8}$$

Denoting $\tau$ as the capital income tax rate $(0 < \tau < 1)$, the budget constraint in the old period can be presented as

$$c_{o,t+1} = [1 + (1-\tau) r_{t+1}] s_t. \tag{9}$$

Subject to budget constraints (8) and (9), we derive the following allocations to maximize utility (7).

$$n_t = \frac{(1-u_t)\overline{w}}{\varepsilon + \varepsilon^{\frac{1}{\theta}} + (\varepsilon\beta)^{\frac{1}{\theta}} [1 + (1-\tau) r_{t+1}]^{\frac{1-\theta}{\theta}}}, \tag{10}$$

$$\frac{s_t}{n_t} = (\varepsilon\beta)^{\frac{1}{\theta}} [1 + (1-\tau) r_{t+1}]^{\frac{1-\theta}{\theta}}, \tag{11}$$

### 2.3. Government

If one assumes the following government budget constraint with a balanced budget

$$\tau r_t s_{t-1} N_{t-1} = G_t, \tag{12}$$

where $G_t$ denotes non-productive government expenditure; then $G_t$ does not contribute to the productivity of firms or marginal utility at all.[3] Non-productive expenditure $G_t$ is assumed because

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[2] We consider that the share of $u_t$ remains unemployed in the young period. Then, the employment period is given as $1 - u_t$.

[3] This assumption is considered by Chamley (1986) and by others. It examines optimal taxation.

we can check the direct effect of the tax on the employment rate and on the fertility rate.

## 3. Equilibrium

The equilibrium condition in capital markets is represented as $k_{t+1} = \frac{s_t}{n_t}$. Equations (6) and (11) can be derived as

$$k_{t+1} = (\varepsilon\beta)^{\frac{1}{\theta}}[1 + (1-\tau)r_{t+1}]^{\frac{1-\theta}{\theta}} = (\varepsilon\beta)^{\frac{1}{\theta}}\left\{1 + (1-\tau)\left[\alpha A^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\overline{w}}\right)^{\frac{1-\alpha}{\alpha}} - 1\right]\right\}^{\frac{1-\theta}{\theta}} \tag{13}$$

$$= k.$$

As shown by (13), the capital stock per capita is constant over time. If $\theta$ is larger than $1(\theta > 1)$, then the capital stock accumulation can be facilitated if the capital income tax rate rises because of a decrease in the interest rate. With $\theta = 1$ as the logarithmic utility function, the interest rate and the capital income tax rate do not affect the capital stock.

Equilibrium in our model economy can be given as equilibrium in the capital stock market. In this section, we show that a decrease in capital income brought about by an increase in the capital income tax rate induces households to increase saving. Equations (5), (6) and (13) give the long-run employment rate as

$$1 - u = \left(\frac{(1-\alpha)A}{\overline{w}}\right)^{\frac{1}{\alpha}} k$$

$$= \left(\frac{(1-\alpha)A}{\overline{w}}\right)^{\frac{1}{\alpha}} (\varepsilon\beta)^{\frac{1}{\theta}}\left\{1 + (1-\tau)\left[\alpha A^{\frac{1}{\alpha}}\left(\frac{1-\alpha}{\overline{w}}\right)^{\frac{1-\alpha}{\alpha}} - 1\right]\right\}^{\frac{1-\theta}{\theta}}. \tag{14}$$

In general, the minimum wage raises unemployment because of a decrease in labor demand. Therefore, equation (14) shows $\frac{du}{d\overline{w}} > 0 \ for\ any \ \ \theta > 0$. In addition, the following expression can be obtained because of equations (13) and (14) as

$$\frac{dk}{d\tau} > 0 \ and \ \frac{d1-u}{d\tau} > 0 \ \ if \ \theta > 1\,. \tag{15}$$

The output per capita can be shown as $= k^{\alpha}(1-u)^{1-\alpha}$ .

Then, the following proposition can be established.

**Proposition 1**

With $\theta > 1$, a capital income tax improves long-term employment and output per capita.

Kunze and Schuppert (2010) show that an increase in the capital income tax facilitates capital stock accumulation and employment. Finally, the effect of capital income tax affects the fertility rate in the long run because the household income rises. Considering equations (10), (13), and (14), fertility can be derived as

$$n = \frac{(1-u)\overline{w}}{\varepsilon + \varepsilon^{\frac{1}{\theta}} + \underbrace{(\varepsilon\beta)^{\frac{1}{\theta}}[1+(1-\tau)r]^{\frac{1-\theta}{\theta}}}_{=k}} = \frac{\left(\frac{(1-\alpha)A}{\overline{w}}\right)^{\frac{1}{\alpha}} k\overline{w}}{\varepsilon + \varepsilon^{\frac{1}{\theta}} + k}. \tag{16}$$

Because of (13) and (16), one can obtain the following and the following proposition:

$$\frac{dn}{d\tau} = \frac{dn}{dk}\frac{dk}{d\tau} > 0 \; if \; \theta > 1 . \tag{17}$$

**Proposition 2**

An increase in the capital income tax rate raises the fertility rate in the long-run if $\theta > 1$.

Fanti and Gori (2010) show that a child tax raises the fertility rate. However, our manuscript shows that a tax only increases the fertility rate in the long-run, even if the child care cost does not change.[4]

The case of $\theta > 1$ represents a complementary case or a non-substitution case. Then, an increase in capital income tax reduces consumption during the old period. Then, to avoid reduction of consumption in the old period, households raise saving to increase consumption in the old period. Then, the capital stock per capita increases. Income per capita increases. By virtue of an increase in income per capita, the household income rises and fertility increases. Moreover, marginal labor productivity rises and the labor demand increases. Consequently, the unemployment rate decreases.

**4. Conclusion**

Economically developed countries among the EU membership are confronting a common problem: a high unemployment rate and a low fertility rate. As described herein, we set an endogenous fertility model with a minimum wage and derive that the capital income tax can raise not only fertility but also income per capita. Based on results of our study, as future work we can assess aging population effects by considering the survival rate of elderly people.

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[4] Fanti and Gori (2010) consider taxation for child care. This taxation increases the child care cost. However, the child care cost used for our analyses does not change. Capital accumulation is facilitated by a capital income tax. Then income per capita raises fertility. In the setting by Fanti and Gori (2010), a child care tax reduces the fertility directly. Then, the capital stock accumulation per capita increases and income per capita rises. Finally, household income rises; also, the fertility rate rises indirectly.

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