Involuntary unemployment under indivisible labor supply: Perfect competition case

Tanaka, Yasuhito

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Yasuhito Tanaka

Faculty of Economics, Doshisha University,
Kamigyo-ku, Kyoto, 602-8580, Japan.

E-mail:yatanaka@mail.doshisha.ac.jp

Abstract

We show the existence of involuntary unemployment without assuming wage rigidity. A key point of our analysis is indivisibility of labor supply. We derive involuntary unemployment by considering utility maximization of consumers and marginal cost pricing behavior of firms in an overlapping generations model under perfect competition with indivisibility of labor supply and decreasing or constant returns to scale technology.

Key Words: involuntary unemployment, indivisible labor supply, perfect competition.

JEL Classification: E12, E24.

1 Introduction

Umada (1997) derived an upward-sloping labor demand curve from mark-up principle for firms, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firms' behavior is ad-hoc. In this paper we consider utility maximization of consumers and matginal cost pricing behavior of firms in an overlapping generations model under perfect competition according to Otaki (2019) and Otaki (2015), and show the existence of involuntary unemployment without assuming wage rigidity. A key point of our analysis is indivisibility of labor supply. As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if labor supply is divisible and it can be small, there exists no unemployment.

In the next section we analyze the relation between indivisibility of labor supply and the existence of involuntary unemployment under decreasing or constant returns to scale technology. We show that under decreasing returns to scale the real wage rate is decreasing in the employment and the reservation real wage rate for individuals is constant given the expected inflation rate. If the real wage rate is larger than the reservation real wage rate, an increase in the employment reduces the difference between them. However, it cannot be guaranteed that they are equalized. In Section 3 we present a general analysis of divisibility

[1] Lavoie (2001) presented a similar analysis.

and indivisibility of labor supply. Labor supply is determined by a parameter of utility function and productivity given the expected inflation rate. If labor supply of each consumer is not small, there may exist involuntary unemployment.

2 Indivisible labor supply and involuntary unemployment

We consider a two-period (young and old) overlapping generations model under perfect competition according to Otaki (2010 and 2015). There is one factor of production, labor, and there is one good which is produced under perfect competition. There is a continuum of firms. The volume of firms is one. Consumers are born at continuous density $[0,1] \times [0,1]$ in each period. They can supply only one unit of labor when they are young (the first period).

2.1 Consumers

We use the following notations.

c_i: consumption of the good at period $i,\ i = 1,2$.
p_i: the price of the good at period $i,\ i = 1,2$.
β: disutility of labor, $\beta > 0$.
$0 < \alpha < 1$.
W: nominal wage rate.
Π: profits of firms which are equally distributed to each consumer.
L: employment of each firm and the total employment.
L_f: population of labor or employment at the full-employment state.
$y(L)$: labor productivity, which is deceasing or constant with respect to the employment,
$y(L) \geq 1,\ y' \leq 0$.

δ is the definition function. If a consumer is employed, $\delta = 1$; if he is not employed, $\delta = 0$. The labor productivity is $y(L)$. It is decreasing or constant with respect to L. We define the employment elasticity of the labor productivity by

$$\zeta = \frac{y'}{\frac{y(L)}{L}}.$$

We assume that $-1 < \zeta \leq 0$, and ζ is constant. Decreasing (constant) returns to scale means $\zeta < 0$ ($\zeta = 0$). If the good is produced under constant returns to scale technology, $\Pi = 0$.

The utility of consumers of one generation over two periods is

$$U(c_1, c_2, \delta, \beta) = c_1^{\alpha} c_2^{1-\alpha} - \delta\beta.$$

The budget constraint is

$$p_1 c_1 + p_2 c_2 = \delta W + \Pi.$$

p_2 is the expectation of the price of the good at period 2. The Lagrange function is

$$\mathcal{L} = c_1^{\alpha} c_2^{1-\alpha} - \delta\beta - \lambda(p_1 c_1 + p_2 c_2 - (\delta W + \Pi)).$$

λ is the Lagrange multiplier. The first order conditions are

$$\alpha c_1^{\alpha-1} c_2^{1-\alpha} = \lambda p_1,$$

and

$$(1-\alpha)c_1^{\alpha}c_2^{-\alpha} = \lambda p_2.$$

They are rewritten as

$$\alpha c_1^{\alpha}c_2^{1-\alpha} = \lambda p_1 c_1,$$

$$(1-\alpha)c_1^{\alpha}c_2^{1-\alpha} = \lambda p_2 c_2.$$

Then, we obtain

$$c_1^{\alpha}c_2^{1-\alpha} = \lambda(p_1c_1 + p_2c_2) = \lambda(\delta W + \Pi),$$

$$p_1c_1 = \alpha(\delta W + \Pi),\ p_2c_2 = (1-\alpha)(\delta W + \Pi).$$

The indirect utility of consumers is written as follows

$$V = \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p_1^{\alpha}p_2^{1-\alpha}}(\delta W + \Pi) - \delta\beta, \tag{1}$$

with

$$\lambda = \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p_1^{\alpha}p_2^{1-\alpha}}.$$

The reservation nominal wage rate W^R is a solution of the following equation.

$$\frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p_1^{\alpha}p_2^{1-\alpha}}(W^R + \Pi) - \beta = \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p_1^{\alpha}p_2^{1-\alpha}}\Pi.$$

From this

$$W^R = \frac{p_1^{\alpha}p_2^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\beta.$$

The labor supply is indivisible. If $W > W^R$, the total labor supply is L_f. If $W < W^R$, it is zero. If $W = W^R$, employment and unemployment are indifferent for consumers, and there exists no involuntary unemployment even if $L < L_f$. Indivisibility of labor supply may be due to the fact that there exists minimum standard of living even in the advanced economy (please see Otaki (2015)).

Let $\rho = \frac{p_2}{p_1}$. This is the expected inflation rate (plus one). The reservation real wage rate is

$$\omega^R = \frac{W^R}{p_1} = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}\rho^{1-\alpha}\beta.$$

If the value of ρ is given, ω^R is constant. Otaki (2015) assumes that the wage rate is equal to the reservation wage rate at the equilibrium. However, there exists no mechanism to equalize them.

2.2 Firms

The demand for the good of a consumer of younger generation is

$$c_1 = \frac{\alpha(\delta W + \Pi)}{p_1}.$$

Similarly, his demand for the good in the second period is

$$c_2 = \frac{(1-\alpha)(\delta W + \Pi)}{p_2}.$$

Let M be the income of an individual of older generation including the saving carried over from his first period. Then, his demand for the good is $\frac{M}{p_1}$. The government expenditure constitutes the national income as well as consumptions of younger and older generations. The total demand for the good is

$$c = \frac{Y}{p_1}.$$

Y is the effective demand defined by

$$Y = \alpha(WL + \Pi) + G + M.$$

G is the government expenditure (about this demand function please see Otaki (2007), Otaki (2015)).

Let x and z be the output and the employment of a firm. We have $x = y(z)z$. Thus, we have

$$\frac{dz}{dx} = \frac{1}{y(z) + y'z}.$$

The profit of the firm is

$$\pi = p_1 x - \frac{x}{y(z)} W.$$

The condition for profit maximization under perfect competition is

$$p_1 - \frac{y(z) - xy'\frac{dz}{dx}}{y(z)^2} W = p_1 - \frac{1 - y'z\frac{dz}{dx}}{y(z)} W = p_1 - \frac{1}{y(z) + y'z} W = p_1 - \frac{1}{(1+\zeta)y(z)} W = 0.$$

Therefore,

$$p_1 = \frac{1}{(1+\zeta)y(z)} W.$$

This means the marginal cost pricing. Since at the equilibrium $x = c$ and $z = L$, we obtain

$$p_1 = \frac{1}{(1+\zeta)y(L)} W.$$

With decreasing (constant) returns to scale $-1 < \zeta < 0$ ($\zeta = 0$).

2.3 Involuntary unemployment

The real wage rate is

$$\omega = \frac{W}{p_1} = (1 + \zeta)y(L).$$

Under decreasing (constant) returns to scale, since ζ is constant, ω is decreasing (constant) with respect to L.

The aggregate supply of the good is equal to

$$WL + \Pi = p_1 L y(L).$$

The aggregate demand is

$$\alpha(WL + \Pi) + G + M = \alpha p_1 L y(L) + G + M.$$

Since they are equal,

$$p_1 L y(L) = \alpha p_1 L y(L) + G + M,$$

or

$$p_1 L y(L) = \frac{G + M}{1 - \alpha}.$$

In real terms[2]

$$Ly(L) = \frac{1}{1-\alpha}(g + m), \tag{2}$$

where

$$g = \frac{G}{p_1},\ m = \frac{M}{p_1}.$$

(2) means that the employment L is determined by $g + m$. It can not be larger than L_f. However, it may be strictly smaller than L_f ($L < L_f$). Then, there exists *involuntary umemployment*. Under decreasing returns to scale the real wage rate $\omega = (1 + \zeta)y(L)$ is decreasing with respect to L. Since the reservation real wage rate ω^R is constant, if $\omega > \omega^R$ an increase in the employment reduces the difference between them. However, it cannot be guaranteed that they are equalized.

If we consider the following budget constraint for the government with a lump-sum tax T on the younger generation consumers,

$$G = T,$$

the aggregate demand is

$$\alpha(WL + \Pi - G) + G + M = \alpha(p_1 Ly(L) - G) + G + M.$$

Then, we get[3]

$$Ly(L) = \frac{1}{1-\alpha}[(1-\alpha)g + m],$$

Summary of discussions

The real aggregate demand and employment are determined by the value of $g + m$. The employment may be smaller than the population of labor, then there exists involuntary unemployment.

Under decreasing returns to scale the real wage rate is decreasing with respect to the employment and the reservation real wage rate is constant. When the real wage rate is larger than the reservation real wage rate, an increase in the employment reduces the difference between them. However, it cannot be guaranteed that they are equalized.

Comment on the nominal wage rate

In the model of this section no mechanism determines the nominal wage rate. When the nominal value of $G + M$ increases, the nominal aggregate demand and supply increase. If the nominal wage rate rises, the price also rises. If the rate of an increase in the nominal wage rate is smaller than the rate of an increase in $G + M$, the real aggregate supply and the employment increases. Partition of the effects by an increase in $G + M$ into a rise in the nominal wage rate (and the price) and an increase in the employment may be determined by bargaining between labor and firm[4].

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[2] $\frac{1}{1-\alpha}$ is a multiplier.

[3] This equation means that the balanced budget multiplier is 1.

[4] Otaki (2009) has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow (1981). The arguments of this paper, however, do not depend on

Full-employment case

If $L = L_f$, full-employment is realized. Then, (2) is written as

$$L_f y(L) = \frac{1}{1-\alpha}(g+m). \tag{3}$$

Since L_f is constant, this is an identity not an equation. On the other hand, (2) is an equation not an identity. (3) should be written as

$$L_f y(L) \equiv \frac{1}{1-\alpha}(g+m).$$

From this we have

$$p_1 = \frac{1}{(1-\alpha)L_f y(L)}(G+M),$$

where

$$g = \frac{G}{p_1},\ m = \frac{M}{p_1}.$$

Therefore, the price level p_1 is determined by $G+M$, which is the sum of nominal values of government expenditure and consumption of the older generation. Also the nominal wage rate is determined as

$$W = (1+\zeta)y(L)p_1.$$

3 Divisibility and indivisibility of labor supply

The utility of the representative consumer is

$$c_1^{\alpha} c_2^{1-\alpha} - G(l),$$

with the budget constraint

$$p_1 c_1 + p_2 c_2 = Wl.$$

l is labor supply of an individual ($0 < l \leq 1$), and $G(l)$ is a function of disutility of labor which is continuous, strictly increasing, differentiable and strictly convex. Similarly to (1), we obtain the following indirect utility given l for en employed worker,

$$V = \frac{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}{p_1^{\alpha} p_2^{1-\alpha}} Wl - G(l).$$

Maximization of V with respect to l implies

$$W = \frac{p_1^{\alpha} p_2^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} G'(l). \tag{4}$$

Let $\rho = \frac{p_2}{p_1}$. (4) is rewritten as

$$\omega = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \rho^{1-\alpha} G'(l), \tag{5}$$

where $\omega = \frac{W}{p_1}$ is the real wage rate. If the value of ρ is given, l is obtained from (5) as a function of ω. l is increasing in ω because $G'' > 0$. In our model, however, assuming $L = L_f$, $\omega = (1+\zeta)y(L_f l)$. Thus, we have

$$(1+\zeta)y(L_f l) = \frac{1}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} \rho^{1-\alpha} G'(l). \tag{6}$$

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bargaining.

The value of l is obtained from (6). It does not depend on W given ρ. The total labor supply is $L_f l$. It is constant. L_f is the population of labor. If $L_f l$ is not strictly larger than the labor demand in (2), there exists no unemployment, that is, full-employment is realized. Then, we obtain

$$L_f l y(L_f l) \equiv \frac{1}{1-\alpha}(g+m),$$

and

$$p_1 = \frac{1}{(1-\alpha)L_f l y(L_f l)}(G+M).$$

When $L_f l$ is strictly larger than the labor demand in (2), there exists involuntary unemployment similarly to the arguments in the previous section. The real wage rate in this case is

$$\omega = (1+\zeta)y(Ll).$$

(6) is written as

$$(1+\zeta)y(Ll) = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\rho^{1-\alpha}G'(l).$$

From this l is obtained as a function of L. Denote it by $l(L)$. We assume that $Ll(L)$ is increasing in L.

$$\varphi G''(l) - (1+\zeta)y'L > 0$$

guarantees that $l(L)$ is decreasing and $Ll(L)$ is increasing in L because $y' < 0$, $G'' > 0$,

$$\frac{dl(L)}{dL} = \frac{(1+\zeta)y'l(L)}{\varphi G''(l) - (1+\zeta)y'L} < 0,$$

and

$$\frac{dLl(L)}{dL} = L\frac{dl(L)}{dL} + l(L) = \frac{\varphi G''(l)l(L)}{\varphi G''(l) - (1+\zeta)y'L} > 0,$$

where

$$\varphi = \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\rho^{1-\alpha}.$$

Then, the real wage rate is decreasing in L (because $y' < 0$). Similarly to (2) we get

$$Ll(L) = \frac{1}{(1-\alpha)y(Ll(L))}(g+m).$$

This is the employment. If $Ll(L) < L_f l(L_f)$, there exists involuntary unemployment. If labor supply of each consumer is not small, there may exist involuntary unemployment. If it is small, however, it is likely that there exists no unemployment.

If

$$(1+\zeta)y(Ll) \geq \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\rho^{1-\alpha}G'(l) \text{ for any } 0 < l \leq 1, \text{given } L,$$

consumers choose $l = 1$, and then the labor supply is indivisible.

On the other hand, if

$$\lim_{Ll \to 0}\left(1 - \frac{1}{\eta}\right)(1+\zeta)y(Ll) < \frac{1}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\rho^{1-\alpha}G'(0),$$

we have $l = 0$. However, if $G'(0)$ is sufficiently small, $l > 0$.

4 Concluding Remark

In this paper we have examined the existence of involuntary umemployment using a perfect competition model with decreasing or constant returns to scale. We have derived involuntary unemployment from indivisibility of labor supply. We think that although the labor supply must not be infinitely divisible, it need not be infinitely indivisible. In the future research we want to analyze the problem of involuntary unemployment in a monopolistic competition model with constant or increasing returns to scale technology.

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