A DSGE model for fiscal policy analysis in The Gambia

DJINKPO, Medard

27 December 2019

Online at https://mpra.ub.uni-muenchen.de/97874/
MPRA Paper No. 97874, posted 05 Jan 2020 05:13 UTC
A DSGE model for Fiscal Policy Analysis in The Gambia

Medard DJINKPO

December 2019

Abstract

The study investigates the effect of fiscal and monetary policies on domestic debt dynamics and provides fiscal rules useful to control domestic debt dynamics and maintain fiscal consolidation. Using a New-Keynesian model with the fiscal sector, this study analyses the contribution of government spending on aggregate demand measured by fiscal multipliers and the impact of tax adjustment on domestic debt dynamics. The findings indicate that while consumption and capital income tax have a stabilizing effect on domestic debt, labor income tax produces a weakly positive impact on domestic debt growth due to a higher fraction of Non-Ricardian households in the economy. The study provides a quantitative framework through a Bayesian estimation of steady-state tax rates as a benchmark to tax policy, aiming at mitigating fiscal distress without an adverse impact on output growth.

Keywords: New-Keynesian model, Fiscal multipliers effect, Non-Ricardian household, Fiscal and monetary policy, Bayesian estimation.

JEL Codes: C11; E62, E63

---

1 Department of Research and Statistics, West African Monetary Agency. Email address: djinkpomedard@yahoo.fr. The opinions expressed here are those of the author and do not involve the responsibility of any other person or Institution.
I. Introduction

The recent trend in the dynamic of public debt in the Gambia reveals a high debt-to-GDP ratio resulting from the expansionary fiscal policy. Fiscal authorities have increased the level of external debt (through borrowing on concessional terms in the international market) and domestic debt as a consequence of the ambition to scale up investment and promote economic growth. As a result, the ratio of domestic debt to GDP has increased in recent years. In particular, the Treasury bill accumulation has increased over the years. The expansion in government expenditure combined with the inadequate tax policy has contributed to the high budget deficit over the years and thus excessive government borrowing. Against this backdrop, the private sector faces a challenge of credit constraint and borrows at a higher cost because of the high T-bill rate offered by fiscal authorities. Besides, when monetary authorities adjust interest rates upward to control inflation, the private sector is constrained in the financial market as it faces a high-interest rate.

The main challenge faced by the government is how to coordinate fiscal policy with the monetary policy to curb down the dynamic of domestic debt and reduce its burden without undermining the efficiency of monetary policy. The challenge faced by monetary authorities in implementing effective monetary policy is that the lower interest rate provides more room for increasing public borrowing while crowding out private investment. For example, in the inflation-targeting monetary policy framework, the interest rate plays a crucial role in controlling inflation. Under inflation pressure, monetary authorities adjust the interest rate to control the money supply and reduce inflation. In this context, fiscal authorities face high borrowing costs as it was the case between 2002 and 2004 when the Treasury bill rate has skyrocketed to 27% on average, reaching a peak of 31% in 2003 while inflation has peaked up at an average of 13.3%. This fact is likely to cause financial stress in the case of government default, as 60% of domestic debt represents almost 50% of the short term assets in balance sheet of commercial banks. Also, the increasing interest payment on domestic debt (39% increase on average of interest payment on domestic debt from 2014 to 2017) constrains fiscal authorities from increasing capital spending to meet the Sustainable Development Goals as an essential proportion of government revenue goes to interest payment. Therefore, an increase in government spending implies new debt issuance to finance the deficit if the tax policy does not adjust subsequently.

The study seeks to analyze the main driving force of the dynamic of domestic debt in order to provide tax policy rules useful to fiscal authorities. The paper incorporates the fiscal
sector in a Dynamic Stochastic General Equilibrium (DSGE) model to account for domestic debt as a fiscal instrument and examine the extent to which the Government could coordinate fiscal and monetary policies to alleviate fiscal distress. Its purpose is to help answer the following questions: How can authorities coordinate fiscal and monetary policies to control domestic debt dynamics and maintain fiscal consolidation? To what extent tax adjustment restricts government borrowing by providing more revenue without an adverse effect on aggregate demand? To answer these questions, we incorporate three different taxes in our model and two categories of government spending to explain the contribution of effective tax policy to sound fiscal stance and the effect of government spending on aggregate demand.

The remainder of this paper is structured as follows: Section 2 provides an overview of relevant literature on the DSGE model with the fiscal sector. Section 3 deals with the background information on the Gambian domestic debt and the model specification, section 4 provides the equilibrium solution of the model, and section 5 addresses the issue of calibration, estimation and policy implications. Finally, section 6 concludes.

II. Literature review
Since Lucas’ critique (1976), macroeconomic models have gone through new developments with the introduction of a dynamic approach to account for agents’ optimization or expectation formation. This approach considers parameters of the model in their reduced form rather than the structural form where they remain invariant. This class of models has a common characteristic based on the preferences of economic agents and shocks (technological shocks, for instance) through the intertemporal maximization of consumers’ utility functions (subject to budget constraint) and the production function. There are several presentations based on the neoclassical growth theory initially developed by King, Plosser, and Rabelo (1988) as well as other applications of these models to the analysis of monetary policy that was initially developed by Christiano, Eichenbaum, and Evans (2005). For fiscal policy, a bunch of studies leveraged on DSGE models to analyze government fiscal policy (Cemi (2012), Smets and Wouters (2005), Yang and Traum (2011).

In recent years, the debt situation has become increasingly critical for both developed and developing countries. However, debt sustainability analysis tools developed by the International Monetary Fund (IMF) and the World Bank to guide policymakers in their debt policies, particularly developing countries, only focus on partial equilibrium analysis of debt sustainability without any interconnectedness between different sectors and different agents
in the economy. Therefore, recent studies focus on general equilibrium analysis of government debt to examine the extent to which government debt crowds out private investment (Yang and Traum 2011). Under different monetary policy regimes in coordination with fiscal policy, it is convenient to estimate time-varying parameters to account for monetary and fiscal policy switching between active and passive regime (Davig and Leeper, 2009). Using a New Keynesian model, they stipulated that government spending generates positive impact on consumption in some policy regimes.

Government spending effect on output is also explored using the DSGE model on US data to evaluate fiscal multiplier under different monetary regimes (Leeper et al. 2017). Recent empirical work in the context of Low-Income Countries (LIC) uses a New-Keynesian model to show analytically and through simulations how different sources of fiscal deficit financing play a key role in determining the effects of fiscal policy and related multipliers (Shen et al., 2015). This study is concerned with monetary and fiscal policy coordination under domestic debt stress, where the high-interest rate is likely to increase the debt burden and thus requires a discretionary fiscal policy to be implemented to alleviate government borrowing.

III. The Gambia domestic debt and model specification

This section presents the stylized fact about the domestic debt in the Gambia. It sets out the structural form of the New-Keynesian model that will serve to explain the dynamics of the domestic debt and its implication on the economy.

A closer look at data on public debt suggests that domestic debt-to-GDP ratio has increased significantly from 2010, while the external debt-to-GDP ratio has decreased. As a consequence, the interest on domestic debt as a ratio of total revenue has increased, reaching 22.5% on average. In contrast, the interest on external debt as a ratio of total revenue has reached 7.1% on average. As for the growth rate of both components of public debt, we can see that the domestic debt has increased over the period 2000-2017 at an average rate of 18.3% as opposed to external debt which has increased at an average rate of 4.3%; suggesting the prominence of domestic borrowing (figure 1). The growth rate of interest payment on domestic debt was moderate from 2000 to 2014, with a peak in 2015 similar to that on external debt with a peak in 2014.

However, before 2002, the expansionary fiscal policy was moderate before the situation deteriorated significantly during the period 2002-2004, leading to macroeconomic
imbalances (exchange rate depreciation, fiscal stress, and rising inflation to some extent). Thanks to a prudent monetary policy and sustained fiscal policy, the government had slowed down the domestic debt-to-GDP ratio from 2005 until the second half of 2010 before the situation worsened again with increasing borrowing.

Figure 1: Stylized facts on the public debt (interest and stock)

To better analyze the fiscal and monetary policy coordination, we consider a New-Keynesian model with price stickiness and monopolistic competition. Following Yang and Traum (2011), we adopt different fiscal instruments and shocks to allow for adjustment of fiscal policy to the economic situation. As for different agents in the economy, the paper draws on Yang and Traum (2011), Stähler and Thomas (2011) to specify the model. Following Smets and Wouters (2007), Yang and Traum (2011), we include a set of structural shocks such as productivity shock, three tax shocks (shock on consumption tax, capital tax, and tax on labor as the main components of tax revenues). Finally, we consider two shocks to occur on government spending: shock on current spending and shock on capital spending (as a way to increase investment and economic growth). Overall, the model accounts for seven (07) shocks.
Model set up

The model encompasses four agents in the economy: households, firms, the central bank, and fiscal sector. We consider a basic New-Keynesian model with price stickiness and wage rigidity in the sense that workers have no power in the labor market to set the wage. They face labor demand by firms, and the wholesale firms' maximization problem yields the equilibrium wage rate. Only prices are adjusted optimally in a monopolistic competition setting. The assumption that households cannot set the wage rate is supported by the feature of a small economy where workers have no power to sway the decision in the labor market. Therefore the optimal wage rate is derived from firms’ profit maximization and considered as given.

Households’ problem:

There are two categories of households known as savers and non-savers in the economy. The savers (also known as Ricardian households) represent a fraction $\omega$ of the households in the economy. They have access to financial market; save part of their income for future consumption. They lend capital to firms at a rental rate $r^*_f$; buy government bonds in a form of financial asset at a return rate $r^*_t$. This type of household follows the life-cycle theory where consumers save for future consumption when push comes to shove. Conversely, non-savers, a fraction $(1 - \omega)$, have no access to credit and cannot buy financial instruments such as government bonds for future yields. This type of household known as “Rule-of-thumb” consumers lives only on the income from labor. In the specification, both households have the same utility function $U_s(C_s, L_s) = \frac{(C_s)^{1-\sigma}}{1-\sigma} - \frac{(L_s)^{1+\gamma}}{1+\gamma}$ where $\sigma > 0$ is the risk aversion parameter and $\gamma > 1$ is the substitution parameter between labor and leisure. The households pay tax on consumption, labor income, and capital income (only savers pay capital income tax). We assume that government’s transfer to households is a form of government investment in social services and does not appear in the consumer budget constraint.

As such, the utility maximization problem for these two categories of agents is as follows:

Savers’ problem: They maximize the utility function $U_s(C_s^*, L_s^*)$ subject to budget constraint
\begin{align*}
(1 + \tau_r^c) P_t C_t^r + I_t^p + B_{t+1}^r &= (1 - \tau_r^k) R_t^p K_t^r + (1 - \tau_r^w) W_t L_t^r + r_{t-1} B_t, \quad \text{(1)} \\
(1 + \tau_r^c) P_t C_t^r \text{ is after-tax consumption spending in the period } t, \\
I_t^p \text{ is the households spending on durable goods in period } t \\
B_{t+1} \text{ Stands for bonds purchased by households in period } t \text{ and carried over to period } t+1, \\
(1 - \tau_r^k) R_t^p K_t^r \text{ is the income from capital saved in the previous period,} \\
(1 - \tau_r^w) W_t L_t^r \text{ represents the labor income in the household budget} \\
r_{t-1} B_t \text{ is the interest payment on bonds purchased in the previous period with maturity in the period } t. \\
The law of motion of private capital for this set of households is defined as follows:} \\
K_{t+1}^p = I_t^p + (1 - \delta_r) K_t^p \text{ where } \delta_r \text{ is the depreciation rate of the stock of private capital.} \\
The budget constraint boils down to:} \\
(1 + \tau_r^c) P_t C_t^r + K_{t+1}^p + B_{t+1} = (1 - \tau_r^k) R_t + (1 - \delta_r) K_t^p + (1 - \tau_r^l) W_t L_t^r + r_{t-1} B_t \\
\text{The solution to the households’ problem provides the following equations:} \\
L_t^r = \frac{(1 - \tau_r^l)(C_t^r)^{-\sigma} W_t}{(1 + \tau_r^c) P_t} \\
\beta \left[ 1 - \delta_r + E_t (1 - \tau_{t+1}^k) R_{t+1} \right] = E_t \left[ \frac{(C_t^r)^{-\sigma} (1 + \tau_{t+1}^c) \pi_{t+1}^r}{(1 + \tau_r^c)(C_{t+1}^r)^{-\sigma}} \right] \quad \text{(2)} \\
E_t \left[ \frac{(C_{t+1}^r)^{-\sigma}}{(1 + \tau_{t+1}^c) \pi_{t+1}^r} \right] r_t = \frac{1}{\beta} \left( C_t^p \right)^{-\sigma} \quad \text{(3)} \\
\text{Non-Savers problem: Non-savers maximize the utility function } U_t \left( C_t^{ns}, L_t^{ns} \right) \text{ subject to budget constraint:} \\
(1 + \tau_r^c) P_t C_t^{ns} = (1 - \tau_r^{ns}) W_t L_t^{ns} \quad \text{(4)} \\
\text{1 The upper scripts } s \text{ and } ns \text{ on variable } C \text{ stand for saver and non-saver.}
The first-order condition with respect to consumption gives \( (C_i^{ns})^{-\sigma} = \lambda_i(1+\tau^e_i)P_t \) or 
\[ \lambda_i^{ns} = \frac{(C_i^{ns})^{-\sigma}}{(1+\tau^e_i)P_t} \] where \( \lambda_i^{ns} \) is the Lagrangian multiplier for non-savers.

The aggregate labor in the economy is 
\[ L_t = (1-\omega)L_t^{ns} + \omega L_t^s \] with \( \omega \) the fraction of savers in the economy. Similarly, the aggregate consumption of households equals 
\[ C_i = (1-\omega)C_i^{ns} + \omega C_i^s \]

**Firms’ problem:**

There are two categories of firms producing two categories of goods: intermediate goods produced by wholesale firms and sold to retail firms. The final aggregate product of retail firms has a functional form known as Dixit-Stiglitz aggregator represented by
\[ Y_t = \left( \int_0^1 Y_{j,t} \right)^{\psi} \] where \( Y_{j,t} \) for \( j \in [0,1] \) is the wholesale good \( j \) and \( \psi \) is the elasticity of substitution between wholesale goods. The general price of retail goods is \( P_t \).

As commonly set in the New-Keynesian model, the representative retail firm maximizes its profit subject to \( \int_0^1 Y_{j,t} \) by considering its price \( P_{j,t} \) as given.

\[ \max_{P_{j,t}} \left( P_tY_t - \int_0^1 P_{j,t}Y_{j,t}d_j \right) = \max_{P_{j,t}} \left( P_t\left( \int_0^1 Y_{j,t} \right)^{\psi} \right) - \int_0^1 P_{j,t}Y_{j,t}d_j \right). \]

The first-order condition of this problem yields 
\[ Y_{j,t} = Y_t \left( \frac{P_t}{P_{j,t}} \right)^{\psi} \] and \( P_t = \left[ \int_0^1 P_{j,t}d_j \right]^{\frac{1}{\psi}} \).

To a way consistent with Weitzman (1970), Barro (1990) and, more recently, Yang and Traum (2011), Stähler and Thomas (2011), the production of intermediate goods by wholesale firms follows Cobb Douglas technology with a slight modification to include public capital as input to the production. This feature is essential to the setting of a DSGE model for a small economy because the public sector is vital in the formation of output. Most of the private sector activities depend on the performance of the public sector, which is the availability of public infrastructure (roads, energy, etc.) necessary for private sector activities. The function is specified as follows:

\[ \text{(1)} \text{ See Costa J. C. Jose. “Understanding DSGE models: Theory and Applications” for mathematical details.} \]
\[ F \left( K_i^p, K_i^G, L_i, Z_i \right) = Y_i = Z_i \left( K_i^p \right)^{\theta_1} \left( K_i^G \right)^{\theta_2} \left( L_i \right)^{\theta_3}. \] Where \( Z_i \) is the productivity factor reflecting the growth of technology. It follows an AR (1) process specified as 
\[ \log(Z_i) = \rho \log(Z_{i-1}) + \varepsilon_i \quad \text{and} \quad \varepsilon_i \sim N(0, \sigma^2). \] In line with Weitzman (1970), Yang and Traum (2011), Stähler and Thomas (2011), we assume constant return to scale in labor and private capital \((\theta_1 + \theta_2 = 1)\) because private and public capital are specific to the role for which they have been created and cannot be shifted (Weitzman 1970). Therefore, it is reasonable to assume that the private sector can shift factors by substituting labor for capital according to the return of each factor. The law of motion of public capital is set as follows:
\[ K_i^G = I_i^G + (1 - \delta_t) K_{i-1}^G. \]

These firms first minimize the cost of producing given the factor capital and labor cost and return on capital then maximize their profit by setting the price optimally. So the first problem consists of minimizing the cost subject to the output

Thus the problem is set as follows:
\[
\min_{K_i^p, L_i} W_i L_i - R_i K_i^p \quad s.t. Y_i = Z_i \left( K_i^p \right)^{\theta_1} \left( K_i^G \right)^{\theta_2} \left( L_i \right)^{\theta_3}
\]

The solution to the Lagrangian problem provides the following equations:

So
\[ L_i = \mu_i Y_i = \theta_i MC_i \frac{Y_i}{W_i} \quad (5) \]

and
\[ K_i^p = \theta_i \mu_i \frac{Y_i}{R_i} = \theta_i MC_i \frac{Y_i}{R_i} \quad (6) \]

\[ K_i^p = \theta_i MC_i \frac{Y_i}{R_i} \Rightarrow MC_i = \frac{R_i K_i^p}{\theta_i Y_i} \]

Note that the derivative of the Lagrangian with respect to \( Y_i \) is equal to \( \mu_i \). Hence the marginal cost \( \mu_i = MC_i \), after simplification equals
\[ MC_i = \frac{1}{Z_i \left( K_i^G \right)^{\theta_2} \left( L_i \right)^{\theta_3}} \left( \frac{W_i}{\theta_3} \right)^{\theta_2} \left( \frac{R_i}{\theta_1} \right)^{\theta_3} \quad (7) \]

---

1 See Costa Junior, Celso Jose. “Understanding DSGE models: Theory and Applications” for details on algebra
The second step consists of setting the optimal price by maximizing the profit. We assume a certain degree of price stickiness in the model since only a fraction \( \alpha \) of wholesale firms can set the price optimally at \( P_{jt}^* \) and \( (1 - \alpha) \) keep their price unchanged at \( P_{j,t-1} \).

Subsequently, the resulting profit maximization problem boils down to:

\[
\max_{P_{jt}} \sum_{i=0}^{\infty} (\beta \alpha)^i \left( P_{jt}^* Y_{j,t+i} - CT_{j,t+i} \right)
\]

\[
\max_{P_{jt}} \sum_{i=0}^{\infty} (\beta \alpha)^i \left( P_{jt}^* Y_{j,t+i} \left( \frac{P_{jt}^*}{P_{jt}} \right)^\psi - Y_{j,t+i} \left( \frac{P_{jt}^*}{P_{jt}} \right)^\psi \right) MC_{j,t+i}
\]

The first-order condition implies that:

\[
E_t \sum_{i=0}^{\infty} (\alpha \beta)^i \left( (1 - \psi) Y_{j,t+i} + \psi Y_{j,t+i} MC_{j,t+i} \right) = 0. \quad \text{The solution to the problem yields:}
\]

\[
P_{jt}^* = \frac{\psi}{(\psi - 1)} E_t \sum_{i=0}^{\infty} (\alpha \beta)^i MC_{j,t+i}.
\]

As in the new Keynesian specification, we assume that all firms resetting their price have the same marginal cost as follows:

\[
MC_{j,t+i} = MC_i = \frac{1}{Z_i(G_i^{\theta_1})(\theta_3^{\theta_2})} \left( R_i^{\theta_1} \right)^{\theta_1}.
\]

The aggregate price level boils down to \( P_t = \left[ (1 - \alpha) P_{t-1}^\psi + \alpha P_t^\psi \right]^{1-\psi} \). This expression implies that when all firms reset their prices at \( P_t^* \) (\( \alpha = 1 \)), the aggregate price level \( P_t = P_t^* \). Therefore, the new price level depends on the fraction of firms with the ability to reset the price in the economy.

**Fiscal policy**

Government levies taxes on three different goods at different rates: tax on capital income (\( \tau_k^t \)), labor income tax (\( \tau_w^t \)), and tax on consumption (\( \tau_c^t \)). Following Stähler and Thomas (2011), we assume that public spending has two components, which are public investment spending and public consumption spending. The latter is the sum of current expenditure (purchase of goods and services by the public sector), interest payment, and payroll.

---

Government budget constraint implies that the sum of revenue from different taxes and the bonds issued in each period equals the expenditure (current expenditure, capital expenditure and interest payment on domestic debt stock). In each period \( t \), government domestic debt stock accumulation emerges from the primary deficit, which is the gap between government revenue and its total expenditure. The primary deficit financing implies the issuance of new debt \( d_{t+1} \) with maturity in the period \( t+1 \). Thus the government budget constraint is equivalent to:

\[
\tau^c_t P C_t + \tau^K_t R_t K^p_t + \tau^W_t W_t + d_{t+1} = P G^c_t + P G^i_t + r_{t-1} d_t,
\]  
(8)

Following Yang and Traum (2011), we assume that tax rates are endogenous variables which depend on a lag (an AR(1) term as feedback effects), the output gap and debt-to-GDP ratio to reflect the response of tax rates to domestic debt-to-GDP ratio\(^1\). This specification illustrates the adjustment of tax rates by fiscal authorities following an explosive debt. The specification of the tax rates is as follows\(^2\):

\[
\tilde{\tau}^c_t = \rho^c \tilde{\tau}_{t-1}^c + (1 - \rho^c) \left( \varphi^c \tilde{Y}_t + \kappa^c \tilde{S}_{t-1} \right) + \epsilon_t^c
\]  
(9)

\[
\tilde{\tau}^w_t = \rho^w \tilde{\tau}_{t-1}^w + (1 - \rho^w) \left( \varphi^w \tilde{Y}_t + \kappa^w \tilde{S}_{t-1} \right) + \epsilon_t^w
\]  
(10)

\[
\tilde{\tau}^k_t = \rho^k \tilde{\tau}_{t-1}^k + (1 - \rho^k) \left( \varphi^k \tilde{Y}_t + \kappa^k \tilde{S}_{t-1} \right) + \epsilon_t^k
\]  
(11)

\[
S_{t-1} = \frac{d_{t-1}}{y_{t-1}}
\]  
(12)

Where \( S_t \) is the domestic debt-to-GDP ratio.

On the government spending side, both expenditures (capital and consumption expenditures) react to the debt-to-GDP ratio. Following Traum and Yang, we set these fiscal rules as follows:

\[
\tilde{G}_t^c = \rho_{c^c} \tilde{G}_{t-1}^c + (1 - \rho_{c^c}) \varphi_{c^c} \tilde{S}_t + \epsilon_t^{c^c} \]  
(13)

\[
\tilde{G}_t^i = \rho_{i^c} \tilde{G}_{t-1}^i + (1 - \rho_{i^c}) \varphi_{i^c} \tilde{S}_t + \epsilon_t^{i^c} = I_t^G
\]  
(13)

The shock components are exogenous shocks reflecting the innovations in government fiscal policy and are causes of the explosiveness of domestic debt. They are represented here by

---

\(^1\)The term of output gap reflects the macroeconomic conditions on the tax rate. We assume that in period of boom, fiscal authorities increase tax rate and in recession, they reduce tax rate.

\(^2\)Variable with tilde represents the deviation from the steady-state value.
the random term \( iid \) process \( \varepsilon_i' \sim N\left(0, \sigma_{\varepsilon_i'}\right) \). The introduction of contemporaneous term of debt-to-GDP ratio implies that any innovation in government fiscal policy has a direct impact on the domestic debt.

**Monetary policy rule**

The monetary policy of the central bank has two objectives: the inflation targeting and the reserve requirement. Thus, in the conduct of monetary policy, the central bank has two main instruments amid others, which are the interest rate and open market. The central bank applies the Taylor rule gradually adjusting the interest rate in response to the inflation target and the economic growth target. The monetary policy interest rate is as follows:

\[
   r_t = \rho r_{t-1} + (1 - \rho) \left( \phi_y \bar{Y}_t + \phi_\pi \bar{\pi}_t \right) + \varepsilon_t' \tag{14}
\]

where \( \varepsilon_t' \sim N\left(0, \sigma_{\varepsilon_t'}\right) \) and \( \pi_t = \frac{P_t}{P_{t-1}} \).

The random term in the equation illustrates monetary policy shock enabling monetary authorities to adjust interest rates to meet the inflation target and output growth. The parameter \( \rho \) is the smoothing parameter of interest rates reflecting the feed-back effect. The parameters \( \phi_y \) and \( \phi_\pi \) represent the response of inflation and output gap when monetary authorities adjust interest rates to achieve inflation target or output growth. These parameters reflect the importance that the central bank attaches to inflation targeting and economic growth.

**Markets clearing condition in the economy:**

The labor market clearing implies that labor supply by households equals labor demand by firms \( N_t = L_t \). Furthermore, capital market clearing implies that government borrowing equals household bonds. The government budget constraint equals:

\[
   \tau^c C^c_t + \tau^K R^K_t + \tau_W W_t L_t + d_t = G^c_i + G^i_t + r_{t-1} d_{t-1} \tag{15}
\]

Introducing the bonds market-clearing condition \( d_t = B_t \) and adding household budget and simplifying we get the aggregate resource constraint:

\[
   Y_t = G^c_t + G^i_t + C_t + I^P_t \tag{16}
\]
IV. The dynamic equilibrium of the economy

4.1 Equilibrium equations

The equilibrium conditions provide the expressions of consumption, labor supply, capital, and bonds held by households $\{C_t, L_t, K_{i,t+1}, B_{i,t+1}\}$.

The equilibrium equations are summarized as follows:

**Savers equilibrium equations**

$$\left( C_t^s \right)^\sigma \left( L_t^s \right)^\psi = \frac{(1 - \tau_t^w)W_t}{(1 + \tau_t^c)P_t}$$

$$\beta \left[ 1 - \delta_p + E_t (1 - \tau_{t+1}^c) R_{t+1} \right] = E_t \left( \left( C_t^s \right)^{-\sigma} \left( 1 + \tau_t^c \right) \pi_{t+1} \right) \left( C_{t+1}^s \right)^{\sigma} \left( 1 + \tau_t^c \right)$$

$$r_t = \frac{1}{\beta} E_t \left( \left( C_t^s \right)^{-\sigma} \left( 1 + \tau_t^c \right) \pi_{t+1} \right)$$

**Non-savers equilibrium equations**

$$\left( 1 + \tau_t^c \right) P_t C_{i,t}^n = (1 - \tau_t^c) W_t L_t^s$$

$$\left( C_{i,t}^n \right)^{-\sigma} = \lambda_i \left( 1 + \tau_t^c \right) P_t$$

**Aggregate labor and consumption equations**

$L_t = (1 - \omega) L_t^n + \omega L_t^s$ and $C_t = (1 - \omega) C_{i,t}^n + \omega C_t^s$

**Labor and capital equation (firms’ equations)**

$L_t = \theta_3 MC_t \frac{Y_t}{W_t}$ and $K_{i,t}^p = \theta_1 MC_t \frac{Y_t}{R_t}$

$$MC_t = \frac{1}{Z_t \left( K_t^G \right)^{\theta_3}} \left( \frac{W_t}{\theta_3} \right)^{\theta_1} \left( \frac{R_t}{\theta_1} \right)^{\theta_3}$$

$$P_t = \left[ (1 - \alpha) P_{t-1}^{-\psi} + \alpha \left( P_t^* \right)^{-\psi} \right]^{-\frac{1}{1-\psi}}$$

$$P_{j,t} = \frac{\psi}{1-\psi} E \sum_{i=0}^{\infty} \left( \alpha \beta \right)^i MC_{j,t+i}$$
Aggregate resource constraint

\[ C_t + I_t^p + G_t^c + G_t^i = Y_t \]

### 4.2 Steady-state

In steady-state, all variables in equilibrium equations are constant, and we drop the subscript \( t \) and solve for steady-state values.

\[
\left( C_{ss}^{\sigma} \right)^\gamma \left( L_{ss}^\rho \right)^\beta = \frac{\left(1 - \tau_{ss}^c \right) W_{ss}}{\left(1 + \tau_{ss}^c \right) P_{ss}}
\]

\[
\left(1 - \delta_P^i \right) + \left(1 - \tau_{ss}^c \right) R_{ss} = \frac{1}{\beta}, \quad r_{ss} = \frac{1}{\beta} \pi_{ss}
\]

\[
\left(1 + \tau_{ss}^c \right) P_{ss} C_{ss}^{\alpha s} = \left(1 - \tau_{ss}^w \right) W_{ss} L_{ss}^{\alpha s} \text{ and } \left( C_{ss}^{\alpha s} \right)^{-\sigma} = \lambda_{ss} \left(1 + \tau_{ss}^c \right) P_{ss}
\]

Aggregate labor and consumption equations

\[ L_{ss} = (1 - \omega) L_{ss}^{\alpha s} + \omega L_{ss}^i \text{ and } C_{ss} = (1 - \omega) C_{ss}^{\alpha s} + \omega C_{ss}^i \]

Labor and capital equation (firms’ equations)

\[
L_{ss} = \theta_3 MC_{ss} Y_{ss} W_{ss}, \quad K_{ss}^p = \theta_4 MC_{ss} Y_{ss} R_{ss}, \quad MC_{ss} = \frac{1}{Z_{ss}} \left( K_{ss}^G \right)^{\theta_1} \left( R_{ss} \right)^{\theta_1} \frac{1}{\left(1 - \alpha \right)^{\rho_1}}
\]

\[
P_{ss} = \left[ \left(1 - \alpha \right) P_{ss}^{\rho_1} + \alpha \left( P_{ss}^* \right)^{\rho_1} \right]^{1/(1-\rho_1)}, \quad P^* = \frac{\psi}{(1-\psi)} \frac{1}{\left(1 - \alpha \beta \right)} MC_{ss}
\]

\[
Y_{ss} = Z_{ss} \left( K_{ss}^G \right)^{\theta_1} \left( K_{ss}^G \right)^{\theta_1} \left( L_{ss} \right)^{\theta_1} \text{ and } \delta_{ss} K_{ss}^G = I_{ss}^G, \quad \delta_{ss} K_{ss}^p = I_{ss}^P
\]

\[
\tau_{ss}^c P_{ss} C_{ss} + \tau_{ss}^c R_{ss} K_{ss}^p + \tau_{ss}^c W_{ss} L_{ss} + d_{ss} = P_{ss} G_{ss}^c + P_{ss} G_{ss}^i + r_{ss} d_{ss}
\]

\[
S_{ss} = \frac{d_{ss}}{Y_{ss}} \Rightarrow d_{ss} = S_{ss} Y_{ss} \text{ and the aggregate resource constraint yields: } C_{ss} + I_{ss}^p + G_{ss}^c + G_{ss}^i = Y_{ss}
\]

To solve the steady-state equations, we made some assumptions on the steady-state values:

\[ L_{ss} = L_{ss}^{\alpha s} = L_{ss}^i, \quad C_{ss} = C_{ss}^{\alpha s} = C_{ss}^i, \quad Z_{ss} = P_{ss} = \pi_{ss} = 1 \text{ and. Solving these equations recursively, we get the steady-state value for the remaining endogenous variables.} \]
4.3 Linearized form model

To log linearize, we use the following properties (Uhlig 1999) as follows:

\[
\hat{X}_t = \log (X_t) - \log (X_{ss}) \Rightarrow X_t = X_{ss} \exp (\hat{X}_t) = X_{ss} e^{\hat{X}_t}, \quad \text{and } e^{\hat{X}_t} \approx 1 + \hat{X}_t \text{ in the neighborhood of } \hat{X}_t = 0, \quad e^{\hat{X}_t + aY_t} \approx 1 + \hat{X}_t + a\tilde{Y}_t \quad \text{with } \hat{X}_t, \tilde{Y}_t \approx 0 \quad \text{E}_t \left[ ae^{\hat{X}_{i,t}} \right] \approx a + aE_t \left[ a\hat{X}_{i,t} \right].
\]

Each variable \( X_t \) is replaced by \( X_{ss} e^{\hat{X}_t} \). The transformation provides the following linear equations.

The log-linearized form of Ricardian optimal solution equations are as follows:

\[
\sigma \tilde{C}_t^i + \gamma \tilde{L}_t^i + \tilde{T}_t^i + \tilde{P}_t = \tilde{T}_t^i + \tilde{W}_t
\]

where \( \tilde{T}_t^i = -\frac{\tau^i t}{1 - \tau^i t} \), \( \tilde{T}_t^C = \frac{\tau^C_t \tau^C_t}{1 - \tau^C_t t} \) and \( \tilde{T}_t^K = -\frac{\tau^K_t \tau^K_t}{1 - \tau^K_t t} \).

By the same way,

\[
\beta E_t \left[ \left( \frac{C_{i+1}^t}{(1 + \tau^c_{i+1})} \right)^{-\sigma} \right] \left[ (1 - \delta_p) + E_t \left( 1 - \tau^K_{i+1} \right) R_{i+1} \right] = \left( \frac{C_t^i}{(1 + \tau^c_t)} \right)^{-\sigma} E_t \pi_{i+1} \text{ turns to }
\]

\[
\frac{\sigma}{\beta} \left( \tilde{C}^i_{i+1} - \tilde{C}^i_t \right) + \frac{1}{\beta} E_t \left( \tilde{T}^C_{i+1} - \tilde{T}^C_t + \tilde{\pi}_{i+1} \right) = T^C_{i+1} R_{i+1} E_t \left( \tilde{R}_{i+1} + \tilde{T}^K_{i+1} \right)
\]

\[
r_t = \frac{1}{\beta} E_t \left( \frac{C_t^i}{(1 + \tau^c_t)} \right)^{-\sigma} \pi_{i+1} \text{ becomes } \frac{\sigma}{\beta} \left( \tilde{C}^i_{i+1} - \tilde{C}^i_t \right) + \frac{1}{\beta} E_t \left( \tilde{\pi}_{i+1} + \tilde{T}^C_{i+1} - \tilde{T}^C_t \right) = r_t
\]

Non-savers

\[
(1 + \tau^c_t) P_t C^w_t = (1 - \tau^w_t) W_t L^w_t \text{ is equivalent to } \tilde{T}^C_t + \tilde{C}^w_t + \tilde{P}_t = \tilde{T}^w_t + \tilde{W}_t + \tilde{L}^w_t
\]

\[
\left( C^w_t \right)^{-\sigma} = \lambda_t \left( 1 + \tau^c_t \right) P_t \text{ is equivalent to } -\sigma \tilde{C}^w_t = \lambda_t + \tilde{T}^C_t + \tilde{P}_t
\]

Aggregate labor and consumption equations

\[
L_t = (1 - \omega) L^w_t + \omega L^s_t \text{ equivalent to } \tilde{L}_t = (1 - \omega) \tilde{L}^w_t + \omega \tilde{L}^s_t
\]

\[
C_t = (1 - \omega) C^w_t + \omega C^s_t \text{ equivalent to } \tilde{C}_t = (1 - \omega) \tilde{C}^w_t + \omega \tilde{C}^s_t
\]

Labor and capital equation (firms’ equations)
\[ L_t = \theta_3 M_{C_t} \frac{Y_t}{W_t} \text{ equivalent to } \tilde{L}_t = mc_t + \tilde{Y}_t - \tilde{W}_t \text{ where } mc_t = MC_t - MC_{ss} \]

\[ K_t^P = \theta_4 M_{C_t} \frac{Y_t}{R_t} \text{ equivalent to } \tilde{K}_t^P = mc_t + \tilde{Y}_t - \tilde{R}_t \]

\[ MC_t = \frac{1}{Z_t \left( K_t^G \right)^{\theta_t}} \left( \frac{W_t}{\theta_t} \right)^{\theta_t} \left( \frac{R_t}{\theta_t} \right)^{\theta_t} \text{ equivalent to } mc_t = \theta_5 \tilde{W}_t + \theta_6 \tilde{R}_t - \theta_7 \tilde{Z}_t - \theta_8 \tilde{K}_t^G \]

Some algebra on price equations for \( P_t \) and \( P_t^* \) provides the New-Keynesian Philips equation for inflation.

\[ \tilde{\pi}_t = \beta E \tilde{\pi}_{t+1} + \left[ \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \right] \left( mc_t - \tilde{P}_t \right) \]

\[ \tilde{\pi}_{t+1} = \tilde{P}_{t+1} - \tilde{P}_t \]

**Production sector**

\[ Y_t = Z_t \left( K_t^P \right)^{\theta_t} \left( K_t^G \right)^{\theta_t} \left( L_t \right)^{\theta_t} \text{ equals } \tilde{Y}_t = \tilde{Z}_t + \theta_9 \tilde{K}_t^P + \theta_2 \tilde{K}_t^G + \theta_3 \tilde{L}_t \]

\[ \log(Z_t) = \rho \log(Z_{t-1}) + \epsilon_t \text{ equals } \tilde{Z}_t = \rho \tilde{Z}_{t-1} + \epsilon_t \]

\[ \tilde{K}_{t+1}^G = \delta^G \tilde{I}_t^G + (1-\delta^G) \tilde{K}_t^G \text{ and } \tilde{K}_{t+1}^P = \delta_p \tilde{I}_t^P + (1-\delta_p) \tilde{K}_t^P \]

**Government budget constraint** boils down to:

\[ \tau_{ss} c_{ss} \left( \tilde{C}_t + \tilde{P}_t + \tilde{\tau}_t^c \right) + \tau_{ss} K_{ss} \left( \tilde{K}_t^P + \tilde{\tau}_t^K + \tilde{R}_t \right) + \tau_{ss} W_{ss} \left( \tilde{\tau}_t^w + \tilde{W}_t + \tilde{L}_t \right) \]

\[ = P_{ss} G_{ss}^C \left( \tilde{G}_t + \tilde{P}_t \right) + P_{ss} G_{ss}^G \left( \tilde{P}_t + \tilde{G}_t \right) + r_{ss} d_{ss} \left( \tilde{r}_{t-1} + \tilde{d}_t \right) - d_{ss} \tilde{d}_{t+1} \]

And the aggregate resource constraint turns to:

\[ C_{ss} C_t + I_{ss}^P I_t^P + G_{ss}^C G_t^C + G_{ss}^G G_t^G = Y_{ss} Y_t \]

**5 calibration and estimation**

**5.1 Data and Calibration**

We combined different data sources to achieve the calibration as well as the estimation of the model parameters. First, the national sources, mainly government finance statistics, provide data on government consumption expenditure, capital expenditure, tax revenues, and stock of domestic debt. We also retrieve a series of Treasury bill rates from central bank statistics. These data span from 1999 to 2017 on an annual basis. The World Bank
Development Indicator database (WDI) provides data on real GDP, consumer price index inflation, household consumption, stock of capital (gross fixed capital formation and gross capital formation), and labor force participation.

We refer to Cooley and Prescott (1995), Uribé and Schmitt-Grohé (2017), to perform the calibration. As described in Uribé and Schmitt-Grohé, we combined two ways to accomplish the calibration: econometric estimation and calibration based on parameters’ values matching moments of data that the model aims to explain. Following this approach, we calibrate the autocorrelation parameters with the regression method (OLS approach), such as persistence parameters \( (\rho_c, \rho_w, \rho_k, \rho_g, \rho_g, \rho_j) \) and other parameters in the linear equations \( (\varphi_c, \varphi_w, \varphi_k, \varphi_g, \varphi_g, \varphi_g, \varphi_y) \).\(^1\)

Following Cooley and Prescott (1995), we calibrate the capital depreciation rate \( \delta_p \) and \( \delta_G \). Starting from the law of motion of capital \( K_{t+1}^G = I_t^G + (1-\delta_G) K_t^G \), and using some algebra, we arrive at the following identity:\[ \frac{Y_{t+1}}{Y_t} \frac{K_{t+1}^G}{K_t^G} = \frac{I_t^G}{Y_t} + (1-\delta_G) \frac{K_t^G}{Y_t} \] Assuming that \( Y_{t+1} = g Y_t \) equals gross GDP growth rate and arranging the expression, we get \( g - 1 + \delta_G = \frac{I_t^G}{Y_t} / \frac{K_{t+1}^G}{Y_{t+1}} \). By using the average ratios in the expression over the period 1990-2017, it follows that \( \delta_G = 0.035 \) and \( \delta_p = 0.060 \), implying that 6% of private capital depreciates each period while 3.5% of the public capital depreciates each period. For parameters in the production function, we set the share of private capital and labor to their conventional value according to the literature \( (\theta_1 = 0.34) \) and \( (\theta_2 = 0.66) \). The parameter \( \theta_2 \) of the public capital is calibrated to the average ratio of gross fixed public capital formation (% of GDP). The persistence term of the total factor productivity is calibrated to \( \rho = 0.80 \) to avoid the explosive solution.

For the set of parameters calibrated to match the moments of data (first and second moments), we use average ratios for data collected on key macroeconomic variables. The steady-state values or deterministic equilibrium relationships allow us to assign values to these parameters. For instance, the static equilibrium derived from the Euler equation

---

\(^1\) Using the OLS approach, we get the confidence interval for these parameters in which we choose the convenient values to avoid indeterminacy or explosive solution. In cases where the OLS does not provide an accurate value for the parameter, the average ratio of the relevant variable over GDP is used instead of the OLS estimates.
implies that the discount factor equals the inverse of the gross interest rate \( \beta = \frac{1}{r} \).

Therefore, we calibrate the parameter \( \beta \) to match the average T-bill rate \( r \). The fraction of savers \( \omega \) is calibrated to 0.20 (20% of households are savers), and the price stickiness parameter \( \alpha \) is calibrated to 0.70 equivalents to average price duration of three quarters. The risk aversion parameter \( \sigma \) is set to 1.5 according to the literature as well as the labor substitution parameter \( \gamma \), and the degree of substitution between intermediate goods \( \psi \) equals 2. For fiscal variables, we use the steady-state values equal to the mean value of each variable ratio to GDP. These are \( \tau_{ss}^c = 0.192 \), \( \tau_{ss}^w = 0.019 \), \( \tau_{ss}^k = 0.043 \) and \( S_{ss} = 0.37 \). Table (4) in the appendix provides the calibrated values of parameters in the model.

### 5.2 Estimation

We estimate the model using seven observable variables on an annual basis: real GDP \( Y_t \), household consumption \( C_t \), government consumption \( G^C_t \), consumer price index inflation \( \pi_t \), revenue from consumption tax \( \tau^C_t \), revenue from the capital income tax \( \tau^k_t \), and T-bill rate \( r_t \). The choice of these variables is motivated by the issue of identification of deep parameters in the model and the problem of singularity arising from the choice of linearly dependent observable variables\(^1\). Since the model is log-linearized around its steady-state, we applied the same transformation to observable variables (the first difference to the log of each variable), which is equivalent to the closed-form expression of the growth rate of each variable (figure 2 in appendix).

We perform the estimation using the Bayesian approach, which requires the prior distribution as well as the support of the distribution of each parameter. As such, we refer to Herbst and Schorfheide (2016) to choose the prior distribution of the parameters to be estimated. In literature, the prior of the variances of exogenous shocks follows an inverse gamma distribution with support \((0, \infty)\). The autoregressive parameters \((\rho_c, \rho_w, \rho_k, \rho_g, \rho_k, \rho_r)\), the discount factor \( \beta \), the Calvo price stickiness parameter \( \alpha \), the capital depreciation rates \((\delta_c, \delta_p)\), the steady-state tax rates \((\tau_{ss}^c, \tau_{ss}^k, \tau_{ss}^w)\) and the production function parameters (capital and labor share \( \theta_1, \theta_2, \theta_3 \)) have prior distribution Beta

\(^1\) See Iskrev (2010b) for details on the identification of parameters.
because they are bounded on $[0,1]$. The risk aversion and labor substitution parameters follow Gamma distribution. Finally, the parameters relating GDP and debt ratio to fiscal variables have normal distribution. For parameters relating GDP and debt to fiscal variables such as consumption tax, capital income tax, and labor income tax, their priors are set to normal distribution because of ambiguity underlying the sign of these parameters. We then perform the identification test on the parameters and find that they are all identified except the fraction of savers ($\omega$) due to the choice of observables available for the estimation. Therefore it is not estimated. The outcome of the identification test indicates that 99.4% of the prior support gives a unique saddle-path solution to the model. The sensitivity analysis shows the importance of parameters on the model in its linear expectation representation and confirms the identifiability of the parameters (figure 14). We conclude that the model does not feature any identification problems inherent to the structure of the DSGE model. The estimation provides the results summarized in table 1. The posterior mean values, the variances as well as the Highest Probability Density Interval (HPDI) for each parameter are computed from 10,000 draws from the prior support using Metropolis-Hastings (M-H) algorithm.

The general picture emerging from the outcomes is that the prior distributions match the posterior accurately in most cases, suggesting that the information in the data used to estimate the parameter are consistent with our prior beliefs (figure 16). In addition, figure 15 suggests that the calibrated values of the parameters provide non-explosive solutions to the model. It also suggests that Blanchard-Kahn conditions are satisfied because the estimated mode is at the maximum of the posterior likelihood for almost all the parameters.

The most exciting aspect of these estimates is the steady-state tax rates and the 90% HPDI derived from the posterior density. It stands out that contrary to the claim that the capital should not be taxed at steady-state (Chamley (1986), Chari et al. (1991) and others), our findings reveal a steady-state value of 0.0676 (6.76%) and a range of [0.0430, 0.0962] within which the fiscal authorities could set the capital income tax. This finding corroborates the view of Piketty (2015) who argues that capital should be taxed because its return is always higher than the economic growth throughout history and the gap between the return

---

1 See Herbst and Schorfheide (2016) for details on Bayesian estimation of DSGE. The choice of prior is guided by the restriction on the domain of some parameters and the uncertainty about the sign.
2 See Iskrev (2010b) and Ratto (2011) for details on the Identification test and sensitivity analysis.
3 See Herbst and Schorfheide (2016) for details on the methodology.
4 The red dots in some graphs indicate that for parameter values in this range, the model cannot be solved due to violations of the Blanchard-Kahn conditions (indeterminacy or no bounded solution).
to capital and GDP growth is the leading cause of inequality\textsuperscript{1}. Unquestionably, the steady-state tax on capital income cannot be around zero in a developing economy even though the capital stock is essential to economic growth and job opportunities. As far as consumption tax is concerned, our estimates provide a reasonable rate compared to that applicable in similar economies where the value-added tax (VAT) is 18%. As for the labor income steady-state tax rate, its value is close to the rate of capital income, and the lower value almost equal to the posterior mean of the capital income tax.

Further analysis of the estimated parameters of the fiscal rules provides evidence that tax rates respond positively to explosive domestic debt-to-GDP ratio meaning that fiscal policy adjusts to any increase of debt-to GDP ratio during the previous period. Additionally, the tax rates respond positively to the economic situation as the coefficients of the output gap in the tax rate equations are positive in the three instrument equations. The positive values of the posterior mean, for the prior normal distribution, illustrate the stabilization role of tax instruments. However, the 90\% interval encompasses zero for the parameters of output gap and domestic debt in tax equations. This suggests that fiscal authorities at some time over the period of the estimation do not consider neither the output gap nor the deviation of domestic debt from its steady-state to adjust tax rate. This illustrates the time inconsistency of fiscal policy that could lead to higher than expected domestic debt level\textsuperscript{2}.

Turning to the government spending side, we observe that the estimates of debt-to-GDP coefficients have the expected sign. Although their prior distributions are normal, their lower bounds of the 90\% HPDI are both positive. This finding supports the idea that government spending (both consumption and investment) is the main driving force of domestic debt dynamics.

Equally important is the analysis of the estimated parameters in the Taylor rule, especially the inflation and output parameters, which reflect the importance of the output gap and inflation in monetary policy decision by central bank authorities. The prior means for these two parameters are set to their conventional calibrated values, giving more weight to inflation and a relatively smaller weight to output, bearing in mind the problem of indeterminacy. As provided by the estimate, both coefficients are very close to their prior

\textsuperscript{1}Although our model is not meant to define the optimal taxation for this economy, zero tax on capital income is positively discriminatory to Ricardian households.

\textsuperscript{2}The 90\% interval of the coefficient of domestic debt in the labor income tax equation does not encompasses zero.
mean values, suggesting that monetary authorities react timely to inflation in a way consistent with the monetary policy framework.

An important finding which deserves comments is the production function parameters, especially private capital and labor inputs share. We observe that the estimated values of these two parameters imply a decreasing return to scale in these two factors. Looking closely at the 90% interval, we can infer that neither combination of the two parameters yields a constant or increasing return to scale in labor and private capital meaning that for any two values from the 90% HPDI of the parameters, the sum is always less than a unit. This feature is mainly due to the share of private capital, which is lower than the calibrated value.

The estimation provides the magnitude of different shocks in the model as an illustration of the significance of innovations in the model. The results suggest that the shock to labor income tax was the most prominent during the period of estimation and contributes to the fluctuation in all fiscal variables (government spending, debt, and tax rates) as well as output growth, consumption, and inflation. The productivity shock is another sizeable shock with a significant contribution to the fluctuation in output growth. The government capital expenditure shocks have also been sizeable in magnitude. From the historical shock decomposition, it appears that the fluctuation in the domestic debt stems from the following shocks: shocks to government capital expenditure, interest rate, labor income tax, and productivity. The most important contribution to the positive movement of domestic debt (deviation above its steady-state) arises from positive disturbances in government capital spending while government consumption spending has no contribution to fluctuation during the period of estimation.
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Prior distribution</th>
<th>Prior mean</th>
<th>Posterior mean</th>
<th>90% HPDI</th>
<th>Post deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>Gamma 1.5</td>
<td>1.5296</td>
<td>[1.3856, 1.6514]</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Gamma 1.5</td>
<td>1.6946</td>
<td>[1.5435, 1.8299]</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Beta 0.88</td>
<td>0.8840</td>
<td>[0.8705, 0.8988]</td>
<td>0.0100</td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>Beta 0.330</td>
<td>0.1214</td>
<td>[0.0651, 0.1683]</td>
<td>0.0500</td>
<td></td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>Beta 0.070</td>
<td>0.0360</td>
<td>[0.0014, 0.0712]</td>
<td>0.0500</td>
<td></td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>Beta 0.660</td>
<td>0.6587</td>
<td>[0.6441, 0.6744]</td>
<td>0.0100</td>
<td></td>
</tr>
<tr>
<td>$\rho$</td>
<td>Beta 0.800</td>
<td>0.8007</td>
<td>[0.7978, 0.8040]</td>
<td>0.0020</td>
<td></td>
</tr>
<tr>
<td>$\delta_g$</td>
<td>Beta 0.035</td>
<td>0.0293</td>
<td>[0.0194, 0.0401]</td>
<td>0.0100</td>
<td></td>
</tr>
<tr>
<td>$\delta_p$</td>
<td>Beta 0.060</td>
<td>0.0616</td>
<td>[0.0494, 0.0724]</td>
<td>0.0100</td>
<td></td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Beta 0.900</td>
<td>0.8974</td>
<td>[0.8824, 0.9121]</td>
<td>0.0100</td>
<td></td>
</tr>
<tr>
<td>$\varphi_c$</td>
<td>Normal 0.042</td>
<td>0.1053</td>
<td>[-0.1363, 0.3335]</td>
<td>0.1500</td>
<td></td>
</tr>
<tr>
<td>$\kappa_c$</td>
<td>Normal 0.037</td>
<td>0.0848</td>
<td>[-0.0285, 0.1910]</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>Beta 0.600</td>
<td>0.6003</td>
<td>[0.5989, 0.6016]</td>
<td>0.0010</td>
<td></td>
</tr>
<tr>
<td>$\varphi_w$</td>
<td>Normal 0.097</td>
<td>0.1338</td>
<td>[-0.1132, 0.3915]</td>
<td>0.1500</td>
<td></td>
</tr>
<tr>
<td>$\kappa_w$</td>
<td>Normal 0.239</td>
<td>0.2149</td>
<td>[0.0709, 0.3672]</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>$\rho_k$</td>
<td>Beta 0.600</td>
<td>0.6003</td>
<td>[0.5993, 0.6017]</td>
<td>0.0010</td>
<td></td>
</tr>
<tr>
<td>$\varphi_k$</td>
<td>Normal 0.140</td>
<td>0.1640</td>
<td>[-0.2623, 0.5676]</td>
<td>0.5000</td>
<td></td>
</tr>
<tr>
<td>$\kappa_k$</td>
<td>Normal 0.064</td>
<td>0.0934</td>
<td>[-0.0581, 0.2592]</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>$\rho_{gc}$</td>
<td>Normal 0.800</td>
<td>0.8008</td>
<td>[0.7632, 0.8300]</td>
<td>0.0200</td>
<td></td>
</tr>
<tr>
<td>$\varphi_{gc}$</td>
<td>Normal 0.300</td>
<td>0.2023</td>
<td>[0.0918, 0.3134]</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>$\rho_{si}$</td>
<td>Beta 0.600</td>
<td>0.5882</td>
<td>[0.5625, 0.6156]</td>
<td>0.0200</td>
<td></td>
</tr>
<tr>
<td>$\varphi_{si}$</td>
<td>Normal 0.30</td>
<td>0.2636</td>
<td>[0.0972, 0.4144]</td>
<td>0.1000</td>
<td></td>
</tr>
<tr>
<td>$\rho_j$</td>
<td>Beta 0.590</td>
<td>0.6012</td>
<td>[0.5727, 0.6297]</td>
<td>0.0200</td>
<td></td>
</tr>
<tr>
<td>$\varphi_z$</td>
<td>Gamma 1.500</td>
<td>1.4960</td>
<td>[1.4828, 1.5082]</td>
<td>0.0100</td>
<td></td>
</tr>
<tr>
<td>$\varphi_y$</td>
<td>Gamma 2.00</td>
<td>1.9950</td>
<td>[1.9806, 2.0089]</td>
<td>0.0100</td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Beta 0.70</td>
<td>0.7009</td>
<td>[0.6994, 0.7024]</td>
<td>0.0010</td>
<td></td>
</tr>
<tr>
<td>$\tau_{ss}$</td>
<td>Beta 0.192</td>
<td>0.1882</td>
<td>[0.1774, 0.1973]</td>
<td>0.0100</td>
<td></td>
</tr>
<tr>
<td>$\tau_{k}$</td>
<td>Beta 0.043</td>
<td>0.0676</td>
<td>[0.0430, 0.0962]</td>
<td>0.0100</td>
<td></td>
</tr>
<tr>
<td>$\tau_{\psi}$</td>
<td>Beta 0.019</td>
<td>0.0771</td>
<td>[0.0642, 0.0899]</td>
<td>0.0100</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{z}$</td>
<td>inv_gamma 0.001</td>
<td>3.2301</td>
<td>[2.6247, 3.7693]</td>
<td>Inf</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{i}$</td>
<td>inv_gamma 0.044</td>
<td>0.1193</td>
<td>[0.0881, 0.1499]</td>
<td>Inf</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{g}$</td>
<td>inv_gamma 0.031</td>
<td>0.1394</td>
<td>[0.1012, 0.1711]</td>
<td>Inf</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{w}$</td>
<td>inv_gamma 0.070</td>
<td>30.9523</td>
<td>[24.533, 35.691]</td>
<td>Inf</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{gc}$</td>
<td>inv_gamma 0.038</td>
<td>0.0753</td>
<td>[0.0537, 0.0944]</td>
<td>Inf</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{si}$</td>
<td>inv_gamma 0.164</td>
<td>1.7749</td>
<td>[0.9090, 2.6136]</td>
<td>Inf</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{i}$</td>
<td>inv_gamma 0.010</td>
<td>0.0492</td>
<td>[0.0347, 0.0620]</td>
<td>Inf</td>
<td></td>
</tr>
</tbody>
</table>
5.3 The impulse response of endogenous variables to shocks

Based on the parameter values, the time paths of endogenous variables of the model are simulated following different shocks. We first consider the shocks on fiscal variables (government expenditure and distortionary tax rates) to assess the impact on fiscal variables and the domestic debt path. Therefore, the analysis focuses on the effects of fiscal shocks on the debt path and economic growth and the crowding-out effect on the private sector.

**Government spending shocks:** The first part of the analysis examines the effect of expansionary fiscal policy on the dynamic of domestic debt. Figure 3 presents the model implied impulse response of key variables following a shock to government consumption spending. As can be seen from the figure, the dynamic of domestic debt has the expected impulse response to shock on government consumption spending. It appears that the increase of public consumption spending implies a high growth rate of domestic debt, contraction of public and private investments. What stands out here is that the increase of government consumption expenditure implies an increase of domestic debt which crowds out private investment meaning that any increase in government borrowing leads to a reduction of access to credit by private sector. The increase of government consumption spending also leads to the contraction of public investment growth rate. The two effects are further exemplified by the fact that government consumption shock is not followed by a convenient tax adjustment policy to provide more revenue. As the impulse response shows, we observe a little adjustment of different tax rates; yielding a gap between government revenue and expenditure thus higher domestic debt to finance budget deficit resulting from the increase in government consumption. Turning to the impact on real variables, it can be noted that expansionary government spending has a positive impact on output and households’ consumption as it appears on the impulse response figure 3. This result was also reported by Galí et al (2007) in a New Keynesian model with rule-of-thumb households. According to the literature, the reason for the positive effect of government spending shock on output and household consumption is the existence of an important fraction of non-Ricardian households; financially constrained and consume their income fully in each period as opposed to Ricardian households. Another factor adding to the positive response of household consumption to government spending shock is the price stickiness featured in the New-Keynesian model. The estimated value of the Calvo price stickiness parameter suggests an average price duration of three quarters. This provides evidence of a certain degree of price stickiness in the goods market as a necessary condition.
to a positive response of consumption following a shock to government spending (Galí et al 2007).  

In much the same way, a shock to government investment expenditure creates the same impact on fiscal variables but with different magnitudes as can be seen in figure 4. As it is noted above, the increase in government capital expenditure leads to a restriction of government consumption. The same impact on domestic debt occurs as a result of unchanged tax policy to respond to a higher increase in capital spending. As government borrowing increases, private sector access to financing shrinks and private investment growth reduces. As a way of comparison, this type of government spending has a similar effect on output and household consumption as does the government expenditure. 

**Tax policy shocks:** On the revenue side, a shock to consumption tax and capital income tax leads to a decrease in domestic debt growth. First, figure 5 shows the stabilizing effect of a shock to consumption tax on the domestic debt through an increase of government revenue; leading to a reduction of government borrowing. Conversely, households’ consumption responds negatively especially the consumption of Non-Ricardian households as those households do not have any saving in previous period to smooth their consumption. As a result, they react to the shock on consumption tax by supplying more labor to increase their after tax income; bringing the wage rate below the steady-state level. By the same token, a shock to capital income tax leads to a contraction of domestic debt growth (figure 6). A higher capital tax causes debt to fall below the steady-state as a result of less borrowing but causes a decrease in the return to capital. Firms respond to this decrease of return to capital by increasing demand for labor, leading to an increase of output growth above the steady-state growth rate. As we can see, a shock on each of the two tax rates triggers a decrease in domestic debt with no adverse effect on output growth and household consumption. This effect seems preferable as it enables fiscal authorities to control domestic debt through an increase in tax rate and less borrowing.

Counter to the previous findings, the results of labor income tax are somewhat counterintuitive. It appears that a shock to the labor income tax rate causes an increase in domestic debt growth while output and households’ consumption decline as a result of a higher nominal wage rate and lower demand for labor by firms (figure 7). This result is illustrative of Non-Ricardian behavior depicted by a decline in their consumption because

---

1. The average duration of the price is computed as \( \frac{1}{1 - \alpha} \) where \( \alpha \) is the Calvo price stickiness parameter.
2. The magnitudes are presented as fiscal multipliers below.
they face lower demand for labor by firms. Since a higher fraction of households is Non-Ricardian, the contraction of their consumption drives down the aggregate consumption which is the larger component of aggregate demand; thus a contraction in output growth. Conversely, Ricardian agents increase consumption by substitution of private capital leading to a contraction of private investment and capital stock with a negative impact on output. As consumption, private capital and labor demand growth decline, government revenue growth decline and the gap between government total expenditure and revenue increases. Consequently, the government resort to the issuance of a new bond to finance the budget deficit.

**Monetary policy effect on government debt:** When monetary authorities increase the nominal interest rates to control inflation, we observe that domestic debt explodes as interest payment on new bond increases (figure 8). The fiscal authorities respond by tax adjustment in order to increase government revenue and less borrowing because government consumption and capital expenditure does not decrease following the shock to the nominal interest rate. Therefore, the debt returns to the equilibrium path. The increase of interest rates also leads to the contraction of private investment (crowding-out effect) and consequently the contraction of output growth. Although the tight monetary policy reduces inflation, the households’ consumption growth responds negatively as a result of the contraction of Ricardian households ‘consumption due to the substitution between bond and consumption. The contraction in private investment and household consumption leads to that of the output growth rate.

**Fiscal multiplier effect:**
In the previous section, we explain the driving forces of domestic debt and the impact of fiscal policy on output. To provide a quantitative measure of fiscal policy effect on output and debt dynamics over time, the analysis of the fiscal multipliers is carried out. We define cumulative multiplier as the expected cumulative change in output given one unit cumulative change in government spending as follows: \[ \sum_{t=0}^{T} dY_t / \sum_{t=0}^{T} dG_t \]. The impact multiplier is the first-period measure following the shock that is \( \frac{dY}{dG} \). The measures are computed from the equilibrium solution and the impulse response function for different time horizons: short, medium and long term (table 2). These results imply that one unit increase in any of government spending increases output by the corresponding multiplier (in unit). What emerges from the results is that government spending multiplier for consumption
expenditure is lower than that of capital expenditure at any time horizon; revealing the driving force of government investment in the economy. These findings are consistent with those of Shen et al (2015) who examined the fiscal multipliers under different sources of government budget deficit financing. Their findings reveal that under domestic debt financing, the fiscal multiplier for government consumption is 0.4 on impact and -0.4 in five years while fiscal multipliers for public investment is 0.3 on impact and -0.5 in five years period\(^1\). The small size of the fiscal multiplier (less than one) could be explained by the crowding-out effect of private investment created by fiscal expansion.

Another relevant issue emerging from the finding is the analysis of fiscal multipliers of tax policy on debt path. As it was noted, both consumption tax and capital tax play the role of the stabilizer of domestic debt. As such, it is important to figure out the impact in the form of the multiplier effect. To perform this analysis, we proceed to the computation of the fiscal multipliers, replacing output by debt and government spending by tax variable in the computation above. The results are reported in table 3. As shown in the table, a significant difference in the impact effect is observed. Consumption tax shock provides a more stabilizing effect on the dynamics of domestic debt as the multiplier is larger at any time horizon than the capital income tax multiplier. Additional revenue generated by a shock of 1% of the standard deviation of consumption tax rate reduces the domestic debt growth by 0.35 percentage point on impact (in the first period) and the cumulative effect over five years period is 0.59.

| Table 3: Fiscal multipliers of the tax rate on domestic debt\(^2\) |
|---------------------------------|-----------------|---------------|
|                               | Consumption tax | Capital income tax |
| On impact                      | -0.3568         | -0.0057        |
| Five years                     | -0.5857         | -0.0141        |
| Cumulative                     | -0.2068         | -0.0491        |

\(^1\) Shen and al (2015) Developed a new-Keynesian small open economy model in the context of low-Income countries to show that some features (different types of financing including aid, the marginal efficiency of public investment, and the degree of home bias) play a key role in determining the effects of fiscal policy and related multipliers.

\(^2\) The multiplier of labor income tax is not reported as its effect is not negative and not important.
Scenario analysis: To provide a clear picture of how these two tax rates shocks affect debt, we proceed to scenario analysis by changing the steady-state tax rate within the estimated interval (90% HPDI) reported in table 1. The impulse response of stock of domestic debt over five years periods reveals that the negative effect of consumption tax on debt is low on impact for any the steady-state tax rate in the interval and increases gradually over time (figure 9). The debt-to-GDP ratio, on the other hand, drops significantly on impact and this effect decays over time (from -10 to -4 over 5 years). The possible explanation of this result could be the rapid decay of the positive response of the output to shock on consumption tax.

On the question of whether the steady-state capital income tax should be near zero, the scenario analysis provides an interesting result. The most obvious finding emerging from the analysis is that the negative response of debt following a shock to capital income tax decays as the steady-state tax increases (figure10). In the neighborhood of zero, the negative response is much stronger and remains steady over time but shifting the steady-state tax rate upward, the negative response becomes weaker and gets near zero at the horizon of five years. The debt ratio also follows the same pattern but with a stronger decay of the negative response. As the graph shows, on impact, the debt ratio responds strongly but the response decays and turns positive at the horizon of five years with the increase of the steady-state tax; suggesting that the convenient tax rate should remain within the estimated range.

Failure of Ricardian equivalence: Not only does government spending affect private investment but it also affects households' demand depending on the way the deficit is financed. This financing sources cause the Ricardian equivalence to fail because of the existence of a larger fraction of Non-Ricardian agents in the economy. The Ricardian Equivalence states that it does not matter whether a government finances its spending with debt or a tax increase because the effect on the total level of household demand in the economy is the same. However, from the findings it emerges that household consumption rises weakly on impact; showing a humped shape impulse response following a shock to government consumption and investment. Meanwhile, there is a jump in the response of households’ consumption on impact for a very short period followed by a rapid return to the steady-state even below the steady-state level following a shock to consumption tax and capital income tax (figure 11). The response is unequivocally negative for the shock to labor income tax. These patterns are similar to that of Non-Ricardian consumption’s response to the same shocks (figure 12). By contrast, the IRF differs significantly for Ricardian households where consumption responds negatively to government consumption and investment shocks and positively following shocks to tax on consumption. The negative
response of Ricardian consumption to expansionary fiscal policy is explained by the rise of government debt to which Ricardian households respond by substitution of the bond to consumption (figure 13). These findings are consistent with the literature of the New Keynesian model incorporating Non-Ricardian households and exemplify the finding of Mankiw et al (2009) about the important role of this category of agents in the economy.

VI. Concluding remarks

The present study set out to investigate the effect of fiscal and monetary policies on domestic debt dynamics. The findings indicate that growing domestic debt results from innovations in government capital spending more than government consumption spending. The research has also shown that government borrowing crowds out private sector investment and thus making monetary policy less effective and lower interest rate provides more room for public borrowing rather than an increase of private sector investment.

The second major finding was that consumption tax and capital income tax rates have a stabilizing effect on domestic debt whereas labor income tax produces a contraction of output growth and weakly positive impact on domestic debt due to a larger fraction of Non-Ricardian households in the economy. Furthermore, the study has provided a quantitative framework for tax policy to alleviate fiscal stress without adverse impact on output growth. The estimated steady-state tax rates confirm the claim of Piketty (2015) that capital should be taxed and the results are a useful benchmark to tax policy.

The research has provided additional evidence concerning the fiscal multiplier effect of government spending. It follows that government spending multiplier for consumption expenditure is lower than that of capital expenditure; revealing the driving force of government investment expenditure in the economy. This multiplier effect is also examined for the implementation of tax policy and the findings suggest that consumption tax shock provides a more stabilizing effect on the dynamics of domestic debt as the multiplier is larger than the capital income tax multiplier. Overall, the findings will be of interest to policymakers in conducting consistent fiscal and monetary policy for fiscal consolidation and output growth.
References


Hoelle, M. (2017) “Macroeconomics Course Manuscript”, *Department of Economics, Purdue University, Spring2017.*


Kara, E. & Sin, J. (2012). "Fiscal Multiplier in a Credit-Constrained New Keynesian Economy" *Bristol Economics Discussion Papers N° 12/634, Department of Economics, University of Bristol, UK.*


Appendix 1: Parameters’ definition and calibrated values

Table 4: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households Preferences</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.5</td>
<td>Risk aversion parameter</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.90</td>
<td>Discount factor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.5</td>
<td>Labor disutility factor</td>
</tr>
<tr>
<td>$\omega$</td>
<td>0.20</td>
<td>Fraction of savers</td>
</tr>
<tr>
<td><strong>Firms production</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>0.14</td>
<td>Private capital share in output</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>0.107</td>
<td>Public capital share in output</td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>0.107</td>
<td>Labor share in output</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.90</td>
<td>TFP autoregressive factor</td>
</tr>
<tr>
<td>$\sigma^e$</td>
<td>0.0327</td>
<td>Stochastic component of TFP</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.70</td>
<td>Calvo price stickiness parameter</td>
</tr>
<tr>
<td>$\psi$</td>
<td>2.0</td>
<td>The elasticity of substitution between goods</td>
</tr>
<tr>
<td>$\delta_G$</td>
<td>0.035</td>
<td>The public capital depreciation rate</td>
</tr>
<tr>
<td>$\delta_p$</td>
<td>0.060</td>
<td>Private capital depreciation rate</td>
</tr>
<tr>
<td><strong>Monetary policy rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>0.590</td>
<td>Feedback effect of nominal interest rate</td>
</tr>
<tr>
<td>$\phi'_i$</td>
<td>1.14</td>
<td>Inflation factor in Taylor rule</td>
</tr>
<tr>
<td>$\phi'_y$</td>
<td>0.125</td>
<td>Output gap factor in Taylor rule</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>0.2603</td>
<td>The stochastic component in Taylor rule</td>
</tr>
<tr>
<td><strong>Fiscal policy rule</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{ec}$</td>
<td>0.973</td>
<td>Feedback effect of Government current expenditure</td>
</tr>
<tr>
<td>$\phi_{ec}$</td>
<td>0.300</td>
<td>Current expenditures reaction to the debt-to-GDP ratio</td>
</tr>
<tr>
<td>$\sigma_{ec}$</td>
<td>0.1032</td>
<td>Stochastic component of GC</td>
</tr>
<tr>
<td>$\rho_{ei}$</td>
<td>0.913</td>
<td>Feedback effect of Government capital expenditure</td>
</tr>
<tr>
<td>$\phi_{ei}$</td>
<td>0.300</td>
<td>Capital expenditures reaction to the debt-to-GDP ratio</td>
</tr>
<tr>
<td>$\sigma_{ei}$</td>
<td>0.3558</td>
<td>Stochastic component of GI</td>
</tr>
<tr>
<td>$\rho_c$</td>
<td>0.945</td>
<td>Feedback effect of consumption tax</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>0.0415</td>
<td>Consumption tax reaction to output</td>
</tr>
<tr>
<td>$\kappa_c$</td>
<td>0.0375</td>
<td>Consumption tax reaction to debt-to-GDP ratio</td>
</tr>
<tr>
<td>$\sigma_{tc}$</td>
<td>0.1882</td>
<td>Stochastic component of consumption tax</td>
</tr>
<tr>
<td>$\rho_k$</td>
<td>0.764</td>
<td>Feedback effect of capital tax</td>
</tr>
<tr>
<td>$\phi_k$</td>
<td>0.1403</td>
<td>Capital tax reaction to output</td>
</tr>
<tr>
<td>$\kappa_k$</td>
<td>0.0643</td>
<td>Capital tax reaction to the debt-to-GDP ratio</td>
</tr>
<tr>
<td>$\sigma_{tk}$</td>
<td>0.1434</td>
<td>Stochastic component of capital tax</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>0.982</td>
<td>Feedback effect of labor tax</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>0.0967</td>
<td>Labor tax reaction to output</td>
</tr>
<tr>
<td>$\kappa_w$</td>
<td>0.239</td>
<td>Labor tax reaction to the debt-to-GDP ratio</td>
</tr>
<tr>
<td>$\sigma_{tw}$</td>
<td>0.2425</td>
<td>Stochastic component of labor tax</td>
</tr>
<tr>
<td><strong>Steady-state parameters of Fiscal variables</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_{ct}$</td>
<td>0.192</td>
<td>Steady-state value of consumption tax rate</td>
</tr>
<tr>
<td>$\tau_{ct}$</td>
<td>0.043</td>
<td>Steady-state value of capital income tax rate</td>
</tr>
<tr>
<td>$\tau_{ct}$</td>
<td>0.019</td>
<td>Steady-state value of labor income tax rate</td>
</tr>
</tbody>
</table>
Appendix 2: *Observed variables*

Figure 2: *Filtered variables used for estimation*

**Notes:** The observed variables are real data used to estimate the deep parameters in the model. To match the theoretical variables to the true data, we demeaned the first difference in the log transformation of each variable. Since the model is log linearized about its steady state, the transformed variables have mean zero like the theoretical variables simulated by the model.
Appendix 3: Impulse response to shocks

Figure 3: Shock to government consumption ($\varepsilon^{gc}$)

Figure 4: Shock to government investment ($\varepsilon^{gi}$)
**Figure 5:** Shock to consumption tax ($\varepsilon^c_t$)

**Figure 6:** Shock to capital income tax ($\varepsilon^k_t$)
Figure 7: Shock to labor income tax ($\varepsilon^w_t$)

Figure 8: Shock to nominal interest rate ($\varepsilon^r_t$)
Figure 9: Impulse response over steady-state tax on consumption

Figure 10: Impulse response over steady-state tax on capital income

Figure 11: Impulse response of household consumption

---

38
Figure 12: Impulse response of Non-Ricardian households’ consumption

Figure 13: Impulse response of Ricardian household consumption
Appendix 4: Some properties of the estimation

Figure 14: Identification and sensitivity graphs
Figure 15: Mode check plots
Figure 16: Prior and posterior distribution plots