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Abstract

In a storable good market, we investigate a firm's pricing policy and the welfare effects associated with the firm's ability to commit to future prices in the presence of time-varying production costs. We show that, if costs are expected to increase, the firm's lack of commitment leads to lower prices than full commitment when consumer storage costs are relatively small and demand is not too convex. This enhances consumer surplus and, under certain circumstances, total welfare. For intermediate consumer storage costs, the firm's full commitment generally benefits consumers and, a fortiori, the whole economy. Our analysis provides potentially significant empirical and policy implications, especially regarding the patterns of cost pass-through rates.

Keywords: commitment, consumer storage, cost variations, pass-through, storable goods.
JEL Classification: D21, D42, L12.

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1 Introduction

A critical issue for a firm that operates in a storable good market is to deal with the consumers' storage incentives. Systematic empirical evidence shows that consumers are willing to stockpile goods for later consumption when they anticipate higher future prices (e.g., Erdem et al. 2003; Hendel and Nevo 2004, 2006a, 2006b; Osborne 2018; Perrone 2017; Pesendorfer 2002; Pires 2016; Wang 2015). A natural reason that induces a firm to modify its prices over time is a variation in production costs. As reported in an article appeared on The New York Times in February 2018 (Horton 2018), Taiwanese consumers rushed to retail stores, including large-sized hypermarkets, and stockpiled significant quantities of toilet paper after they discovered that toilet paper suppliers would shortly increase product prices up to 30% as a result of rising pulp prices. An article appeared on the Financial Times in October 2018 (Abboud and Gray 2018) revealed that leading consumer goods companies, such as Procter & Gamble in the US and Unilever in Europe, notified their customers of higher future charges due to rising costs of raw materials.[1]

We consider a dynamic storable good market where a monopolistic firm exhibits production costs that evolve over time and faces a continuum of consumers that are willing to store in anticipation of higher future prices. In this framework, we characterize the firm's pricing policy and investigate the welfare effects associated with the firm's ability to commit to future prices. Under full commitment, the firm credibly announces a price for each period and complies with this pricing policy. Under limited commitment, the firm cannot refrain from revising the announced price in a sequentially optimal manner. The price comparisons between the two commitment regimes depend on a range of factors, such as the magnitude of consumer storage costs and the curvature of demand. When production costs are expected to increase, we show that, for *sufficiently small consumer storage costs*, a firm with limited commitment powers charges lower prices than under full commitment as long as demand is not too convex. Therefore, the pricing policy under limited commitment generates higher consumer surplus and, despite the firm's loss, under certain circumstances, it can even enhance total welfare.

As under limited commitment the firm cares about its continuation profits, one might believe that the firm should be more inclined to set higher prices than under full commitment in response to future cost increases. Indeed, we show that higher production costs over time can lead to lower prices under limited commitment than under full commitment. To understand the rationale for this result, it is helpful to start with the case of full commitment. When the increase in production costs exceeds the consumer storage cost, the firm prefers to stimulate consumer storage in order to avoid higher future production costs. In equilibrium, the firm commits to a price sequence that induces consumers to store the entire demand for future consumption. Under limited commitment, this outcome is no longer achievable, because the firm succumbs to the temptation to reduce the price below the full commitment level and to serve the future residual demand. Anticipating the firm's opportunistic behavior, consumers

[1] The costs of inputs and raw materials are likely to show increasing trends over a period of time. For instance, from December 2018 to April 2019 the Commodity Industrial Inputs Price Index increased by 11.1% and the Commodity Fuel (energy) Index increased by 15.9%. Data are available at https://www.indexmundi.com/commodities/ (last retrieved in December 2019).

are more reluctant to store. In order to discourage production and sales in the second period, the firm can manipulate the price in the first period. Notably, this affects the firm's problem in a non-trivial manner. On the one hand, a lower first period price stimulates consumer storage. Ceteris paribus, this reduces production and sales in the second period. On the other hand, a lower first period price leads to an increase in the second period demand gross of consumer storage, which is driven by a corresponding lower second period price. This is because the storability (no-arbitrage) constraint is binding in equilibrium in order to make consumers indifferent about storing, and therefore a change in the first period price translates into a change in the second period price in the same direction. The result of this trade-off is that, if the increase in consumer storage outweighs the increase in the second period gross demand, a lower first period price reduces the second period demand net of consumer storage. This occurs if and only if the second period residual demand is upward sloping with respect to the first period price. The decline in the second period production and sales stemming from a lower first period price mitigates the firm's loss from the lack of commitment. As shown in Section 6, the condition for upward sloping residual demand is that the demand function is not too convex. Given that the storability constraint is binding irrespective of the firm's commitment powers and therefore a lower price in the first period entails a lower price in the second period as well, the pricing policy under limited commitment definitely enhances consumer surplus. Remarkably, if the future residual demand is sufficiently small, the gain in consumer surplus more than compensates the firm's loss from the lack of commitment. Hence, the pricing policy under limited commitment can even increase total welfare.

For *intermediate values of consumer storage costs*, we find that the firm's full commitment tends to benefit consumers and, a fortiori, the whole economy. The lower capability to promote efficient consumer storage induces a firm with limited commitment powers to charge higher prices (at least in the first period) and to forgo consumer storage even when it is ex ante profitable. Alternatively, consumer storage cannot be prevented despite being ex ante suboptimal, which again translates into higher prices. If *consumer storage costs are large enough*, the static monopoly solution applies irrespective of the firm's commitment powers, and therefore the commitment problem is welfare inconsequential.

In the baseline model, we abstract from the possibility that the firm also engages in storage activities. This allows us to investigate the effects of consumer storage in a tractable and transparent manner. Our approach seems to be reasonable in various storable good markets, especially at the downstream level. Empirical and anecdotal evidence suggests that retailers prefer to induce consumers to stockpile some products rather than accumulate them in the form of inventories (e.g., Blattberg et al. 1981; Pesendorfer 2002).[2] Retail stores can have incentives to minimize the time period where the unsold products remain on their shelves by promoting sales that result in consumer storage. For instance, a number of goods, including food and dairy products, are delicate and require specific conditions to preserve their quality. Bulky items, such as paper products, often occupy valuable space. Despite these considera-

[2] As reported by Blattberg et al. (1981, p. 117), "[s]helf space is a major concern for food retailers. Products and suppliers vie vigorously for shelf space. On the other hand, for a number of consumers the cost of some additional storage space is extremely low. Another dozen boxes of tissue in the bathroom closet or an additional case of pickles in the fruit cellar is of almost no concern".

tions, it is plausible that, in some storable good markets where costs are expected to increase, firms benefit from holding inventories, which may coexist with consumer storage. In Section 8, we allow for inventory accumulation and find that, under fairly general circumstances, our qualitative results are unaffected.

The firm's commitment issue that we identify in a storable good market exhibits significant differences with respect to the classical Coase (1972) problem of a durable good monopolist, which succumbs to the temptation to charge lower future prices in order to capture the consumers with lower valuations. In Section 8, we show that our analysis reveals novel features in various aspects, such as the mechanics behind the results and the properties of the equilibrium price sequence. To appreciate even further the difference between storable and durable goods, it is worth noting that, contrary to the case of durable goods, the firm's commitment problem emerges with storable goods exactly when costs are expected to increase.

The predictions of our model are naturally pertinent to markets for storable goods with some degree of maturity, where demand tends to be stable but production costs vary over time. In developed countries, markets for various groceries and beverages, which can be generally stored for future consumption, are nowadays relatively mature and their demand tends to be flat over time.[3] As regards time-varying production costs, we focus on situations where input markets are in "contango". This means that the futures price is higher than the spot price, and therefore the price is expected to rise in the future.[4]

The model presented in our paper is robust and does not resort to any unduly restrictive assumptions on the functional forms. In Section 8, the analysis is extended to a number of directions, such as firm's inventories, convex storage costs, uncertainty about production costs, convex production costs, longer time horizon, and a more general discount factor. As extensively discussed in Section 9, our study sheds new light on the empirical evidence about the firms' propensity to pass their cost changes on to consumers. In various industries, it is possible to construct sufficiently accurate indicators to forecast cost fluctuations. We identify a novel channel that connects intertemporal cost variations, storability and demand curvature with the patterns of cost pass-through rates and firms' markups. Our study also provides potentially significant policy implications in different areas, including the welfare effects of commodity taxation and the antitrust scrutiny of the firms' instruments to improve their commitment power.

Related literature The economic literature on storable goods is fairly extensive. An early relevant contribution is Bénabou (1989), which characterizes the optimal pricing policy of a storable good monopolist operating in an inflationary environment vis-à-vis a continuum of speculators. In each period, the firm must decide whether to adjust its price to the rate of inflation by incurring a "menu cost", while the speculators engage in storage activities that are detrimental to the firm's profits. Differently from Bénabou (1989), in our setting the firm is

[3] For instance, the US consumer (real) expenditure on a number of food items has been quite stable over the last years. Details can be found at https://www.bls.gov/cex/2017/standard/multiyr.pdf (last retrieved in December 2019).

[4] As will be clear in the subsequent analysis, when input markets are in "backwardation" and therefore the price is expected to decline in the future, the firm's commitment powers are inconsequential.

able to costlessly change its price, which is not eroded by inflation, and may prefer to stimulate consumer storage in anticipation of higher future production costs. Moreover, we investigate the impact of the firm's commitment powers on the equilibrium pricing policy and the associated welfare effects. Jeuland and Narasimhan (1985) find that price discrimination among consumers with different demand functions provides an explanation for temporary discounts in storable good markets. In a model where a share of consumers can store the good for future consumption, Hong et al. (2002) show that consumer storage leads to equilibrium price dispersion. Our study is closely related to the seminal paper of Dudine et al. (2006), which considers a storable good market where demand varies deterministically over time and a monopolistic firm faces a continuum of consumers that have incentives to store in anticipation of higher future prices. In this framework, consumer storage unambiguously harms the firm's profits, because it reduces future sales occurring at higher prices. Hence, a firm with full commitment powers selects a price sequence that completely removes consumer storage. Under limited commitment, the firm succumbs to the temptation to increase the second period price to the static monopoly level in response to the absence of consumer storage. To mitigate wasteful storage driven by the consumers' anticipation of the firm's opportunistic behavior, the firm increases the price in the first period. As a result, the firm's lack of commitment reduces consumer surplus and the firm's profits, which is definitely welfare detrimental. In this setting, Antoniou and Fiocco (2019) show that a firm with limited commitment powers has strategic incentives to hold inventories when facing the possibility of buyer stockpiling. Inventory accumulation mitigates the firm's loss from the lack of commitment. Given that the costs of inventories are sunk once they have been incurred, the firm holds inventories to reduce future costs, which alleviates the firm's temptation to charge higher future prices and relaxes the consumers' storage incentives. In the current paper, we explore an alternative legitimate reason for time-varying prices, namely, intertemporal variations in the firm's production costs. As discussed in Section 9, our significantly different results provide a complementary picture to Dudine et al. (2006) and Antoniou and Fiocco (2019), which can contribute to the analysis of dynamic strategic interactions in storable good markets. In a model à la Dudine et al. (2006) with time-dependent buyer valuations, Berbeglia et al. (2019) characterize the optimal preannounced pricing policy and the optimal contingent pricing policy for a monopolistic retailer that sells indivisible items either to a finite number of buyers with unit demand or to a single buyer with arbitrary demand per period. Hendel et al. (2014) study non-linear pricing of storable goods and find cyclical patterns in prices and sales. Heterogeneity in consumers' ability to store makes larger bundles more likely to be on sale. Incorporating consumer storage into Su's (2007) analysis of a seller's optimal dynamic strategy vis-à-vis strategic buyers, Su (2010) shows that the seller may either charge a constant fixed price or offer periodic price promotions at predictable time intervals. Hendel and Nevo (2013) theoretically and empirically investigate the intertemporal price discrimination incentives of a firm that faces consumers with heterogeneous storage abilities. In equilibrium, the price pattern exhibits temporary reductions that allow the firm to discriminate among consumers.

The effects of competition in markets for storable goods have been studied as well. In a Cournot duopoly framework, Anton and Das Varma (2005) show that firms compete for consumer storage. The equilibrium price sequence is increasing and prices are higher with respect to the case where storage is unfeasible. In a differentiated good market with price competition, Guo and Villas-Boas (2007) find that preference heterogeneity leads to differential consumer storage propensity, which exacerbates future price competition and may remove consumer storage in equilibrium.[5] Nava and Schiraldi (2014) study the impact of consumer storage on the firms' incentives to promote periodic price reductions in order to sustain collusion.

Our paper can also contribute to the voluminous literature on durable goods. In Section 8, we contrast the Coase problem that emerges with durable goods and our mechanism that applies to storable goods. A recent relevant contribution by Ortner (2017) shows that in a durable good market stochastic costs introduce an option value of delaying trade, which restores the monopolist's power to extract some rents if the consumers' valuations are discrete. As in our setting an increase in production costs undermines the firm's commitment ability, our results tend to go in the opposite direction to Ortner (2017). This provides further corroboration for the different nature of the issue at hand. Analyzing the profit maximization problem of a durable good monopolist, Board (2008) explores the case where incoming demand evolves over time, and Garrett (2016) considers buyers arriving over time, whose valuations for the good vary stochastically. In a competitive dynamic market for durable goods with two incumbent sellers and potential entrants, Anton et al. (2014) investigate the equilibrium capacity choices and pricing strategies when capacities are chosen before competition takes place. Nava and Schiraldi (2019) show that selling multiple varieties of a durable good allows the monopolist to recoup some of its market power. The commitment issue of a durable good monopolist has also been addressed from a mechanism design perspective. In a setting where a seller of a durable good faces a privately informed buyer, Doval and Skreta (2019) characterize the revenue-maximizing equilibrium when the seller cannot commit to the mechanism offered to the buyer in the case of no trade.

Structure of the paper The rest of the paper unfolds as follows. Section 2 sets out the formal model. Section 3 considers the static solution to the firm's problem. Sections 4 and 5 characterize the firm's equilibrium pricing policy under full and limited commitment, respectively. Section 6 is devoted to price comparisons between the two commitment regimes. Section 7 conducts a welfare analysis. Section 8 discusses the robustness of the results and examines various possible extensions. Section 9 concludes and provides some empirical and policy implications of our results. All formal proofs are collected in the Appendix. Additional formal results and associated proofs are relegated to the Supplementary Appendix.

2 The model

Setting

Consumers We consider a two-period market for a storable good characterized by a (continuously differentiable) demand $D(p_\tau)$ in period $\tau \in \{1, 2\}$, which decreases with the price p_τ,

[5] When exploring competition among firms (Section 8), we discuss how our paper relates to Anton and Das Varma (2005) and Guo and Villas-Boas (2007).

i.e., $D'(p_\tau) < 0$. For the sake of simplicity, we assume no discounting on the second period. In Section 8, we allow for a more general discount factor. Consumers can store some units of the good in the first period for consumption in the second period at a unit cost $s_c \geq 0$. We refer to Section 8 for an extension to convex consumer storage costs. Competitive arbitrageurs can also engage in storage activities. Following Dudine et al. (2006), the consumer storage demand writes as

$$D_s(p_1) = \begin{cases} D(p_1 + s_c) & \text{if} \quad p_1 + s_c < p_2 \\ [0, D(p_1 + s_c)] & \text{if} \quad p_1 + s_c = p_2 \\ 0 & \text{if} \quad p_1 + s_c > p_2 \end{cases}. \tag{1}$$

For $p_1 + s_c < p_2$, the first period price augmented by the consumer storage cost is lower than the second period price, which implies that consumers prefer to store in the first period the entire quantity consumed in the second period. For $p_1 + s_c = p_2$, consumers are indifferent between storing the good and waiting until the second period to purchase it. Hence, they are willing to store any quantity between zero and consumption in the second period. For $p_1 + s_c > p_2$, consumers do not wish to store any quantity. As will be shown in the subsequent analysis, the last two cases are the only relevant outcomes in equilibrium. Throughout the paper, we refer to $p_1 + s_c \geq p_2$ as the *storability (no-arbitrage) constraint*, which becomes binding for $p_1 + s_c = p_2$.

Firm A monopolistic firm incurs a (constant) unit production cost c_τ in period $\tau \in \{1, 2\}$. The unit cost is c_1 in the first period and c_2 in the second period, where $\Delta c \equiv c_2 - c_1$ denotes the intertemporal cost variation. As will become clear in the sequel, we focus on the case where production costs rise over time, i.e., $\Delta c > 0$. In the baseline model, production costs vary deterministically. This assumption captures in a simple and tractable manner some features of production costs in retail storable good markets, where prices for primary commodities (which affect the retail costs) are strongly correlated over time (e.g., Deaton and Laroque 1996). In Section 8, we show the validity of our analysis in the presence of stochastic costs and discuss the implications of introducing cost uncertainty.

The firm's aggregate profits are $\Pi \equiv \Pi_1 + \Pi_2$, where

$$\Pi_1 = (p_1 - c_1)\,[D(p_1) + D_s(p_1)] \tag{2}$$

and

$$\Pi_2 = (p_2 - c_2)\,[D(p_2) - D_s(p_1)] \tag{3}$$

denote the profits in the first and second period, respectively. Consumer storage inflates the demand faced by the firm in the first period but depresses it in the second period, because consumers resort to the quantity stored in the first period.

The firm's profits Π_τ in period τ satisfy the following standard assumption.

Assumption 1 $\Pi_\tau''(p_\tau) < 0$, $\tau \in \{1, 2\}$.

Assumption 1 states that the firm's profits in each period must be concave in prices, which ensures that the second-order conditions for profit maximization are fulfilled.

Timing and equilibrium concept

Each period of the game includes the following two stages.
(I) The firm determines the price for the good.
(II) Consumers purchase a quantity of the good and consumption takes place.

Under full commitment, the firm is able to specify at the outset of the game the pricing policy that maximizes the ex ante aggregate profits. Under limited commitment, the price in each period is sequentially optimal and maximizes the firm's continuation profits, namely, it arises as the subgame perfect Nash equilibrium of the game.

3 Static solution

When consumer storage is not feasible, the firm's problem reduces to the static monopoly problem in each period $\tau \in \{1, 2\}$, which is given by

$$\max_{p_\tau} \left(p_\tau - c_\tau\right) D\left(p_\tau\right). \tag{4}$$

It is helpful for our analysis to consider the following auxiliary function

$$\phi_\tau(p_\tau) \equiv D\left(p_\tau\right) + \left(p_\tau - c_\tau\right) D'\left(p_\tau\right). \tag{5}$$

This represents the left-hand side of the first-order condition for the static monopoly problem in period τ. The equilibrium static monopoly price is $p_\tau^m = c_\tau - \frac{D(p_\tau^m)}{D'(p_\tau^m)}$. We define $\mu_\tau^m \equiv p_\tau^m - c_\tau = \frac{p_\tau^m}{\varepsilon_{p_\tau^m}}$ as the price-cost static monopoly markup in period τ, where $\varepsilon_{p_\tau^m} \equiv -\frac{D'(p_\tau^m)p_\tau^m}{D(p_\tau^m)}$ is the demand elasticity evaluated at p_τ^m. The difference in the static monopoly markups between the two periods is $\Delta\mu^m \equiv \mu_2^m - \mu_1^m$. Note that $\Delta\mu^m$ can be interpreted as a measure of cost pass-through, namely, the rate at which a cost change is passed on to consumers. The cost pass-through rate is lower (higher) than 1 if and only if $\Delta\mu^m < (>)\, 0$. The magnitude of the cost pass-through rate is related to the curvature of demand (e.g., Bulow and Pfleiderer 1983; Fabinger and Weyl 2012). Specifically, the cost pass-through rate is lower (higher) than 1 if and only if demand is log-concave (log-convex). Adopting the terminology of Rochet and Tirole (2011), a log-concave demand (e.g., linear) leads to "cost absorption", while a log-convex demand (e.g., iso-elastic) generates "cost amplification".[6] In Section 9, we discuss our results in the light of the empirical observations about the cost pass-through.

When storage is feasible, it follows from the consumer storage demand in (1) that the static monopoly solution is implementable if and only if $p_1^m + s_c \geq p_2^m$. This corresponds to the

[6]Standard computations show that (i) with linear demand $D\left(p_\tau\right) = \alpha - \beta p_\tau$ it holds $\Delta\mu^m = -\frac{\Delta c}{2} < 0$; (ii) with iso-elastic demand $D\left(p_\tau\right) = \gamma p_\tau^{-\eta}$ it holds $\Delta\mu^m = \frac{\Delta c}{\eta - 1} > 0$ ($\eta > 1$ due to the second-order condition for profit maximization); (iii) with exponential demand $D\left(p_\tau\right) = \lambda e^{-\sigma p_\tau}$ it holds $\Delta\mu^m = 0$. We refer to Fabinger and Weyl (2012) for an accurate taxonomy of demand functions according to the cost pass-through rates in a monopoly setting.

following static monopoly feasibility constraint

$$s_c \geq \Delta c + \Delta\mu^m, \tag{6}$$

which requires that the consumer storage cost s_c must be sufficiently large in order to remove storage at the static monopoly prices. Note that a smaller $\Delta\mu^m$ relaxes the feasibility constraint (6). When demand is log-concave ($\Delta\mu^m < 0$), the cost pass-through rate is lower than 1 and therefore a cost increase translates into a relatively smaller price increase, which makes the feasibility constraint (6) easier to be satisfied.

4 Full commitment

A firm equipped with full commitment powers can credibly announce a price for each period and adhere to this pricing policy. Formally, the firm sets a price sequence that maximizes the aggregate profits given by the sum of the first period profits in (2) and the second period profits in (3). In principle, there exist three pricing options that affect the consumer storage behavior. The first option for the firm is to set a price sequence such that the first period price augmented by the consumer storage cost is larger than the second period price, i.e., $p_1 + s_c > p_2$. The consumer storage demand in (1) vanishes, and in each period the firm's problem corresponds to the static monopoly problem described in Section 3. The static monopoly solution is implementable if and only if the feasibility constraint (6) is fulfilled. The second option for the firm is to implement a price sequence such that the first period price augmented by the consumer storage cost coincides with the second period price, i.e., $p_1 + s_c = p_2$. In this case, the storability (no-arbitrage) constraint is binding and consumers are indifferent between storing the good for future consumption and waiting until the second period to purchase it. When production costs increase in the second period ($\Delta c > 0$), the consumers' decision to store the entire demand for the second period can be profitable for the firm, because this allows the concentration of production in the first period and generates cost savings. The third option at the firm's disposal is to set a price sequence such that the first period price augmented by the consumer storage cost is lower than the second period price, i.e., $p_1 + s_c < p_2$. Consumers store in the first period the entire quantity that they are willing to consume in the second period. As this outcome can be replicated by setting $p_1 + s_c = p_2$, the third option for the firm is (at least weakly) dominated by the second option. Hence, we can restrict our attention to the first two pricing options.

The following proposition characterizes the consumer storage behavior and the price sequence in equilibrium when the firm can commit to future prices. The equilibrium outcome hinges on a number of factors, such as the magnitude of consumer storage costs, the curvature of demand, and the feasibility of the static monopoly solution.

Proposition 1 *Under full commitment,*

(i) if $s_c < \min\{\Delta c + \Delta\mu^m, \Delta c\}$, consumer storage is $D_s^{cs} = D\left(p_1^{cs} + s_c\right)$, and prices are $p_1^{cs} = c_1 - \frac{D\left(p_1^{cs}+s_c\right)+\phi_1\left(p_1^{cs}\right)}{D'\left(p_1^{cs}+s_c\right)}$ and $p_2^{cs} = p_1^{cs} + s_c$;

(iia) if $\Delta c + \Delta\mu^m \leq s_c \leq \Delta c$, there exists a threshold $\bar{s}_c^c \in (\Delta c + \Delta\mu^m, \Delta c)$ such that (1) for $s_c < \bar{s}_c^c$ the outcome in (i) applies, (2) for $s_c \geq \bar{s}_c^c$ consumer storage is $D_s^m = 0$, and prices are $p_1^m = c_1 + \mu_1^m$ and $p_2^m = c_2 + \mu_2^m$;

(iib) if, alternatively, $\Delta c \leq s_c < \Delta c + \Delta\mu^m$, consumer storage is $D_s^{cn} = 0$, and prices are $p_1^{cn} = c_1 - \frac{D(p_1^{cn}) + \phi_2(p_1^{cn} + s_c)}{D'(p_1^{cn})}$ and $p_2^{cn} = p_1^{cn} + s_c$;

(iii) if $s_c \geq \max\{\Delta c + \Delta\mu^m, \Delta c\}$, the outcome in (iia-2) applies.

To better appreciate the results in Proposition 1, we disentangle the analysis according to the sign of $\Delta\mu^m$, which measures the magnitude of the cost pass-through rate in a static monopoly setting (see Section 3). In Figure 1, panel (a) illustrates the case $\Delta\mu^m \leq 0$ formalized in Corollary 1, and panel (b) illustrates the case $\Delta\mu^m > 0$ formalized in Corollary 2.

We start with the case $\Delta\mu^m \leq 0$, which occurs if and only if demand is (weakly) log-concave (e.g., linear or exponential). The outcome in point (iia) of Proposition 1 is feasible instead of the outcome in point (iib).

Corollary 1 *Suppose $\Delta\mu^m \leq 0$. Then, under full commitment,*

(i) if $s_c < \bar{s}_c^c$, consumer storage is $D_s^{cs} = D(p_1^{cs} + s_c)$, and prices are p_1^{cs} and $p_2^{cs} = p_1^{cs} + s_c$;

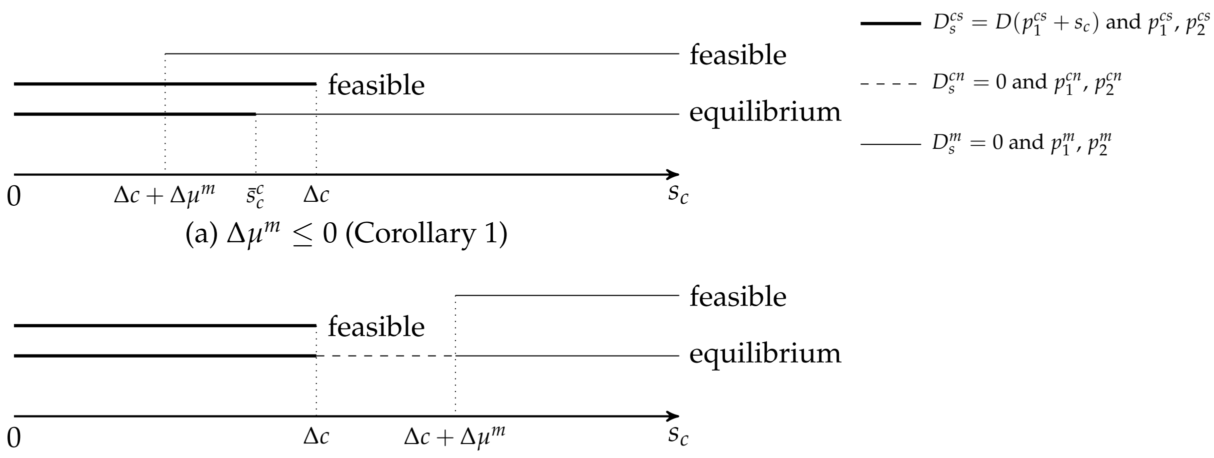
(ii) if $s_c \geq \bar{s}_c^c$, consumer storage is $D_s^m = 0$, and prices are p_1^m and p_2^m.

Prices exhibit the following features: (a) $\frac{\partial p_1^{cs}}{\partial s_c} < 0$ for $D''(\cdot) < \widehat{D}''$, (b) $\frac{\partial p_2^{cs}}{\partial s_c} > 0$. Moreover, it holds $p_1^m \geq p_1^{cs}$ for $D''(\cdot) < \widehat{D}''$, where the equality follows if and only if $s_c = 0$.

Point (i) of Corollary 1 indicates that, when the consumer storage cost is relatively small, i.e., $s_c < \bar{s}_c^c$, the firm finds it optimal to commit to a price sequence that induces consumers to store the entire quantity for the second period. Therefore, the firm shuts down in the second period. Given that the storability constraint is binding, i.e., $p_2^{cs} = p_1^{cs} + s_c$, consumers are indeed indifferent about storing. However, any outcome that departs from full storage is not sustainable in equilibrium, because the firm could slightly reduce the first period price and stimulate full storage, which yields a discontinuous increase in profits associated with cost savings. Note from panel (a) of Figure 1 that, for $\Delta c + \Delta\mu^m \leq s_c \leq \Delta c$, the firm can choose between allowing consumer storage and implementing the static monopoly solution (the feasibility constraint (6) is satisfied).[7] The firm's profits in the presence of consumer storage decrease with storage costs because consumers are more reluctant to store, but the static monopoly profits do not change. As formally shown in the proof of Proposition 1 in the Appendix, there exists a unique threshold $\bar{s}_c^c \in (\Delta c + \Delta\mu^m, \Delta c)$ such that the firm prefers to allow consumer storage if and only if $s_c < \bar{s}_c^c$. The threshold $\bar{s}_c^c$ increases with Δc, i.e., $\frac{\partial \bar{s}_c^c}{\partial \Delta c} > 0$ (for a given c_1). A larger cost increase leads to lower static monopoly profits but does not affect the profits in the presence of consumer storage. Hence, the firm is more likely to promote consumer storage in response to a larger cost increase that makes production more convenient in the first period. For $s_c \geq \bar{s}_c^c$, the firm sets the static monopoly prices and consumers abstain from storing, as point (ii) of Corollary 1 establishes.[8]

[7] For the sake of exposition, in Figure 1 the storing outcome is depicted as feasible only for $s_c \leq \Delta c$. When the additional price that consumers are willing to pay in the second period exceeds the additional production cost, i.e., $s_c > \Delta c$, consumer storage is clearly detrimental to the firm.

[8] When production costs decrease over time ($\Delta c < 0$), the static monopoly solution is implementable irrespective



(a) $\Delta\mu^m \leq 0$ (Corollary 1)

(b) $\Delta\mu^m > 0$ (Corollary 2)

Figure 1: Full commitment (Proposition 1)

Panel (a) of Figure 2 illustrates the equilibrium consumer storage and price sequence as a function of s_c, for the example of linear demand. Note that they exhibit a discontinuity at $\bar{s}_c^c$, where the firm is indifferent between allowing consumer storage and implementing the static monopoly solution. Clearly, consumer storage decreases with s_c, because consumers are more reluctant to store. When consumer storage is costless, i.e., $s_c = 0$, the equilibrium price is the same in the two periods, i.e., $p_1^{cs} = p_2^{cs}$, and coincides with the first period static monopoly price p_1^m. Consumers are so eager to store that the firm cannot discriminate between the two periods and faces twice the same demand in the first period. If s_c increases, the storing price p_1^{cs} declines in order to incentivize consumer storage. According to Corollary 1, this is the case when demand is not too convex.[9] The price pattern is non-monotonic with respect to s_c in the first period. This holds in the second period as well, especially when the second period cost is not too large.

Now, we turn to the case $\Delta\mu^m > 0$, which occurs if and only if demand is log-convex (e.g., iso-elastic). The results are formalized in the following corollary and illustrated in panel (b) of Figure 1. The outcome in point (iib) of Proposition 1 is feasible instead of the outcome in point (iia).

Corollary 2 *Suppose $\Delta\mu^m > 0$. Then, under full commitment,*

(i) if $s_c < \Delta c$, consumer storage is $D_s^{cs} = D\left(p_1^{cs} + s_c\right)$, and prices are p_1^{cs} and $p_2^{cs} = p_1^{cs} + s_c$;

(ii) if $\Delta c \leq s_c < \Delta c + \Delta\mu^m$, consumer storage is $D_s^{cn} = 0$, and prices are p_1^{cn} and $p_2^{cn} = p_1^{cn} + s_c$;

(iii) if $s_c \geq \Delta c + \Delta\mu^m$, consumer storage is $D_s^m = 0$, and prices are p_1^m and p_2^m.

Prices exhibit the following features: (a) $\frac{\partial p_1^{cs}}{\partial s_c} > 0$ for $D''(\cdot) > \widehat{D}''$, (b) $\frac{\partial p_1^{cn}}{\partial s_c} < 0$, (c) $\frac{\partial p_2^{cs}}{\partial s_c} > 0$, (d) $\frac{\partial p_2^{cn}}{\partial s_c} > 0$. Moreover, it holds (e) $p_1^{cs} \geq p_1^m$ for $D''(\cdot) > \widehat{D}''$, where the equality follows if and only if $s_c = 0$, (f) $p_1^{cn} > p_1^m$, (g) $p_2^m > p_2^{cn} > p_2^{cs}$.

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of the consumer storage costs ($\Delta c < 0$ implies $p_1^m > p_2^m$ and makes the feasibility constraint (6) satisfied). In this case, the static monopoly solution trivially applies, because the preferences of the firm and consumers are aligned against consumer storage.

[9] It follows from Section 3 that this is consistent with the case $\Delta\mu^m \leq 0$. To understand why, note that $\Delta\mu^m \leq 0$ if and only if $\frac{D(p_2^m)}{D(p_1^m)} \leq \frac{D'(p_2^m)}{D'(p_1^m)}$. As the left-hand side is lower than 1 ($\Delta c > 0$ implies $p_2^m > p_1^m$), a sufficient condition is that demand is (weakly) concave. By continuity, this holds as long as demand is not too convex.

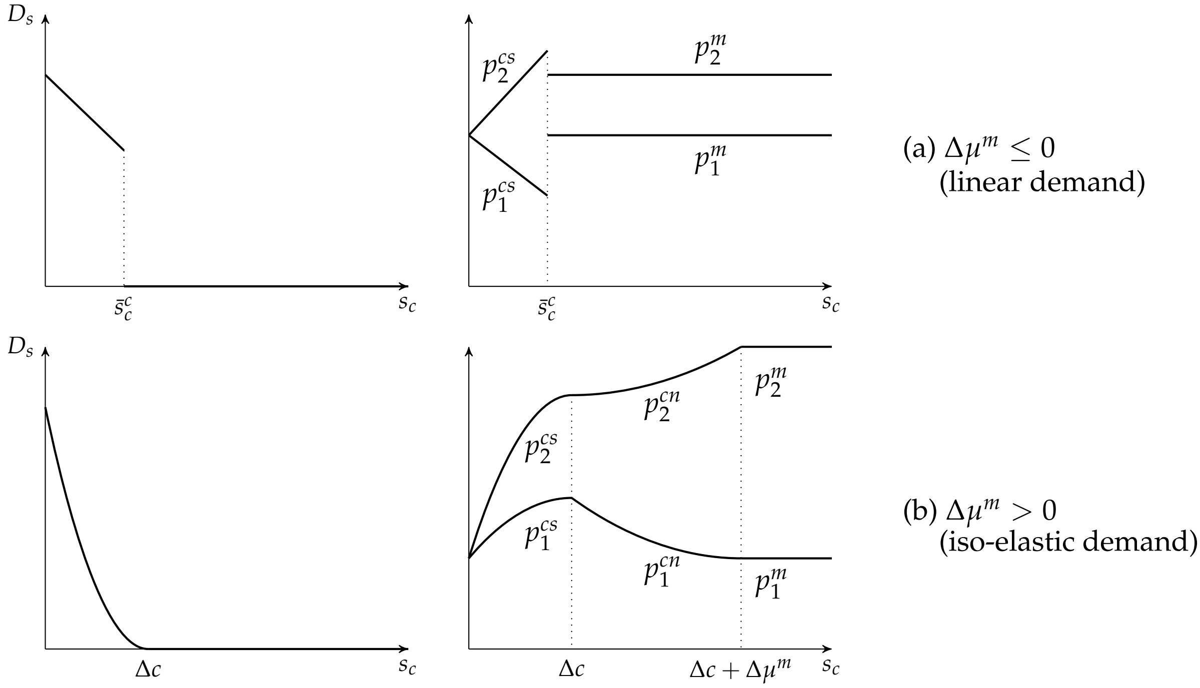


Figure 2: Equilibrium consumer storage and price patterns under full commitment

Point (i) of Corollary 2 shows that, as in point (i) of Corollary 1, when the consumer storage cost is small enough, i.e., $s_c < \Delta c$, the full storage outcome applies. A comparison between panels (a) and (b) in Figure 1 reveals that for $\Delta\mu^m > 0$ consumer storage is promoted as long as the additional price that consumers are willing to pay in the second period is lower than the additional production cost ($s_c < \Delta c$). As indicated in point (ii) of Corollary 2 and illustrated in panel (b) of Figure 1, there exists an interval for s_c, i.e., $\Delta c \leq s_c < \Delta c + \Delta\mu^m$, where consumer storage is profit detrimental ($\Delta c \leq s_c$) but the static monopoly solution is not implementable (the feasibility constraint (6) fails to hold). The firm must resort to prices distorted from the static monopoly level in order to remove consumer storage. Note from points (i) and (ii) that the storability constraint is binding and therefore consumers are indifferent about storing. Contrary to the outcome in point (i), consumer storage does not take place in the outcome in point (ii). As consumer storage is profit detrimental ($\Delta c \leq s_c$), the firm could slightly increase the first period price and fully remove consumer storage, which yields a discontinuous increase in profits. Clearly, the firm selects the static monopoly prices if and only if they are feasible, i.e., $s_c \geq \Delta c + \Delta\mu^m$, as point (iii) of Corollary 2 indicates.

Panel (b) of Figure 2 illustrates the equilibrium consumer storage and price sequence as a function of s_c, for the example of iso-elastic demand. Note that they are now continuous functions. An inspection of panels (a) and (b) of Figure 2 shows that for $\Delta\mu^m > 0$ the first period price p_1^{cs} is distorted above (rather than below) the static monopoly level and increases (rather than decreases) with s_c. According to Corollary 2, this holds when demand is sufficiently convex.[10] To appreciate the rationale for this result, it is important to realize that, with convex

[10]It follows from Section 3 that this is consistent with the case $\Delta\mu^m > 0$. To understand why, note that $\Delta\mu^m > 0$ if and only if $\frac{D(p_2^m)}{D(p_1^m)} > \frac{D'(p_2^m)}{D'(p_1^m)}$. As the left-hand side is lower than 1 ($\Delta c > 0$ implies $p_2^m > p_1^m$), this condition holds when demand is sufficiently convex.

demand, an increase in s_c generates two opposite effects. On the one hand, a higher s_c reduces the consumer storage demand, which calls for a price reduction in order to stimulate consumer storage (as with concave demand). On the other hand, a higher s_c mitigates the demand reduction associated with a price increase, because it makes the consumer storage demand flatter. This generates an incentive for a price increase. When demand is sufficiently convex, the latter effect dominates the former effect, and the first period price p_1^{cs} increases with s_c.

The no-storing prices p_1^{cn} and p_2^{cn} lie between the static monopoly prices p_1^m and p_2^m. To deter consumer storage, the firm distorts the price upward in the first period and downward in the second period compared to the static monopoly level. Contrary to p_1^{cs}, the no-storing price p_1^{cn} decreases with s_c. When storage becomes more costly for consumers, the firm can alleviate the price distortion from the static monopoly level to prevent consumer storage. The price sequence is non-monotonic with respect to s_c in the first period, but monotonically increases with s_c in the second period (due to the binding storability constraint) and achieves its maximum at the static monopoly price.

5 Limited commitment

We now investigate the situation where the firm is unable to commit to future prices. After the second period has commenced, the firm succumbs to the temptation to revise its price in a sequentially optimal manner. We know from point (i) of Proposition 1 that, when consumer storage costs are small enough, a firm with full commitment powers finds it optimal to announce a price sequence that induces consumers to store the entire future demand. Moreover, as point (iib) of Proposition 1 indicates, with log-convex demand and intermediate consumer storage costs, the firm prefers to commit to a price sequence such that the first period price is above while the second period price is below the static monopoly level, which fully removes consumer storage (see Corollary 2). These pricing policies cannot be implemented when the firm lacks the ability to commit to future prices. Specifically, in the first case, after consumers stored in the first period the entire second period demand at the announced prices, the firm has an incentive to decrease the second period price below the announced level in order to promote sales in the second period as well. In the second case, if consumers did not store in the first period, the firm's best response is to increase the second period price above the announced level up to the static monopoly price. Anticipating the firm's opportunistic behavior, consumers modify their storage strategies, and the full commitment solution is no longer achievable.

Using (2) and (3), the firm's maximization problem can be written as

$$\max_{p_1} \left(p_1 - c_1\right)\left[D\left(p_1\right) + D_s\left(p_1\right)\right] + \left(p_2 - c_2\right)\left[D\left(p_2\right) - D_s\left(p_1\right)\right] \tag{7}$$

subject to the following constraint of sequential optimality

$$p_2\left(D_s\left(p_1\right)\right) \equiv \arg\max_{\widetilde{p}_2} \left(\widetilde{p}_2 - c_2\right)\left[D\left(\widetilde{p}_2\right) - D_s\left(p_1\right)\right]. \tag{8}$$

As under full commitment, the firm can resort to three pricing options. First, the firm may select a pricing policy such that consumer storage does not occur, i.e., $p_1 + s_c > p_2$. This leads to the static monopoly prices, provided that the feasibility constraint (6) is satisfied. The second pricing option is to make consumers indifferent between storing in the first period and purchasing in the second period, i.e., $p_1 + s_c = p_2$. Differently from full commitment, the firm cannot freely manipulate the equilibrium storage level, which is dictated by the constraint of sequential optimality (8). The third pricing option for the firm is $p_1 + s_c < p_2$, which induces full consumer storage. Yet, this pricing policy is not implementable because the firm succumbs to the temptation to reduce the price in the second period in order to stimulate its sales.[11]

Intuitively, the firm faces the following trade-off. A lower price in the first period encourages consumer storage, which improves the firm's cost efficiency in the presence of cost increases over time. However, the firm's profit margin deteriorates. Despite this basic trade-off, things are far from being trivial. As under full commitment, the equilibrium outcome hinges on a number of factors, such as the magnitude of consumer storage costs, the curvature of demand, and the feasibility of the static monopoly solution. Sequential optimality imposes an additional relevant constraint. The following proposition characterizes the consumer storage behavior and the price sequence in equilibrium when the firm cannot commit to future prices.

Proposition 2 *Under limited commitment,*

(i) if $s_c < \min\{\Delta c + \Delta\mu^m, \widetilde{s}_c^*\}$, *consumer storage is* $D_s^{*s} = \phi_2(p_1^{*s} + s_c)$, *and prices are* $p_1^{*s} = c_1 - \frac{D(p_1^{*s}) + \phi_2(p_1^{*s}+s_c) + (\Delta c - s_c)\phi_2'(p_1^{*s}+s_c)}{D'(p_1^{*s})}$ *and* $p_2^{*s} = p_1^{*s} + s_c$;

(iia) if $\Delta c + \Delta\mu^m \leq s_c \leq \widetilde{s}_c^*$, *there exists a threshold* $\bar{s}_c^* \in (\Delta c + \Delta\mu^m, \widetilde{s}_c^*)$ *such that (1) for* $s_c < \bar{s}_c^*$ *the outcome in (i) applies, (2) for* $s_c \geq \bar{s}_c^*$ *consumer storage is* $D_s^m = 0$, *and prices are* $p_1^m = c_1 + \mu_1^m$ *and* $p_2^m = c_2 + \mu_2^m$;

(iib) if, alternatively, $\widetilde{s}_c^* \leq s_c < \Delta c + \Delta\mu^m$, *consumer storage is* $D_s^{*n} = 0$, *and prices are* $p_1^{*n} = p_2^{*n} - s_c$ *and* $p_2^{*n} = p_2^m$;

(iii) if $s_c \geq \max\{\Delta c + \Delta\mu^m, \widetilde{s}_c^*\}$, *the outcome in (iia-2) applies.*

In line with the analysis of full commitment in Section 4, we identify two main cases. In Figure 3, panel (a) illustrates the case $\Delta c + \Delta\mu^m \leq \widetilde{s}_c^*$ formalized in Corollary 3, and panel (b) illustrates the case $\Delta c + \Delta\mu^m > \widetilde{s}_c^*$ formalized in Corollary 4. As shown in the proof of Proposition 2 in the Appendix, the threshold $\widetilde{s}_c^*$ represents the highest value for s_c such that consumer storage is feasible.

We start with the case $\Delta c + \Delta\mu^m \leq \widetilde{s}_c^*$, which implies that the outcome in point (iia) of Proposition 2 is feasible instead of the outcome in point (iib). A necessary condition is that $\Delta\mu^m \leq 0$, namely, demand is (weakly) log-concave.[12] For instance, this case applies with linear demand.

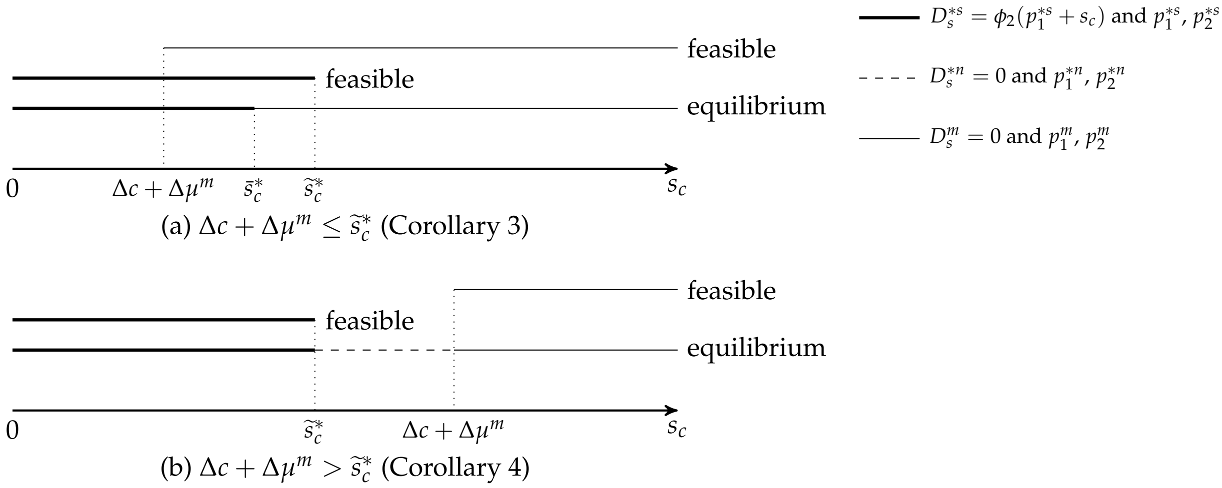
Corollary 3 *Suppose* $\Delta c + \Delta\mu^m \leq \widetilde{s}_c^*$. *Then, under limited commitment,*

(i) if $s_c < \bar{s}_c^*$, *consumer storage is* $D_s^{*s} = \phi_2(p_1^{*s} + s_c)$, *and prices are* p_1^{*s} *and* $p_2^{*s} = p_1^{*s} + s_c$;

(ii) if $s_c \geq \bar{s}_c^*$, *consumer storage is* $D_s^m = 0$, *and prices are* p_1^m *and* p_2^m.

[11] We focus on the plausible situation where the cost increase is not so pronounced as to make production ex post unprofitable in the second period. When the full commitment price with consumer storage is below the costs in the second period, the full commitment storing outcome can be trivially replicated under limited commitment.

[12] We refer to the proof of Proposition 2 in the Appendix for technical details.



(a) $\Delta c + \Delta\mu^m \leq \widetilde{s}_c^*$ (Corollary 3)

(b) $\Delta c + \Delta\mu^m > \widetilde{s}_c^*$ (Corollary 4)

Figure 3: Limited commitment (Proposition 2)

Prices exhibit the following features: (a) $\frac{\partial p_1^{*s}}{\partial s_c} = 0$ *for* $D''(\cdot) = 0$, *(b)* $\frac{\partial p_2^{*s}}{\partial s_c} > 0$. *Moreover, it holds (c)* $p_1^m > p_1^{*s}$ *for* $D''(\cdot) = 0$, *(d)* $p_2^m > p_2^{*s}$.

Point (i) of Corollary 3 shows that, if the consumer storage cost is sufficiently small, i.e., $s_c < \bar{s}_c^*$, consumers partially store in the first period the quantity demanded in the second period. As the storability constraint is binding, i.e., $p_2^{*s} = p_1^{*s} + s_c$, consumers are indeed indifferent about storing. Contrary to the case of full commitment, the equilibrium storage level is now established by the sequential optimality constraint. Under limited commitment, the firm can only resort to the first period price to promote consumer storage in anticipation of higher future costs. Consumers realize that, after storing in the first period the entire quantity that they are willing to consume in the second period at the announced prices, the firm will invariably succumb to the temptation to decrease the price in the second period below the announced level in order to stimulate its sales. This mitigates the consumers' storage incentives, and the firm can only induce partial storing of future demand. As illustrated in panel (a) of Figure 3, for $\Delta c + \Delta\mu^m \leq s_c \leq \widetilde{s}_c^*$, the firm can choose between allowing consumer storage and implementing the static monopoly solution (the feasibility constraint (6) is satisfied). The firm's profits in the presence of consumer storage decrease with storage costs while the static monopoly profits are unaffected. As formally shown in the proof of Proposition 2 in the Appendix, there exists a unique threshold $\bar{s}_c^* \in (\Delta c + \Delta\mu^m, \widetilde{s}_c^*)$ such that for $s_c < \bar{s}_c^*$ the storing option is profit superior. For $s_c \geq \bar{s}_c^*$, the firm sets the static monopoly prices and consumers abstain from storing, as point (ii) of Corollary 3 indicates.

Panel (a) of Figure 4 illustrates the equilibrium consumer storage and price sequence as a function of s_c, for the example of linear demand. Similarly to the case of full commitment, they are discontinuous at $\bar{s}_c^*$. The first period price p_1^{*s} is independent of s_c, which is, however, an artifact of the linear demand specification. To attract consumer storage, p_1^{*s} is distorted below p_1^m. A more general result is that the second period price p_2^{*s} is now unambiguously lower than p_2^m. The firm's lack of commitment removes the possibility of a second period price above the static monopoly level, because the firm would have an incentive to reduce this price irrespective of the magnitude of consumer storage.

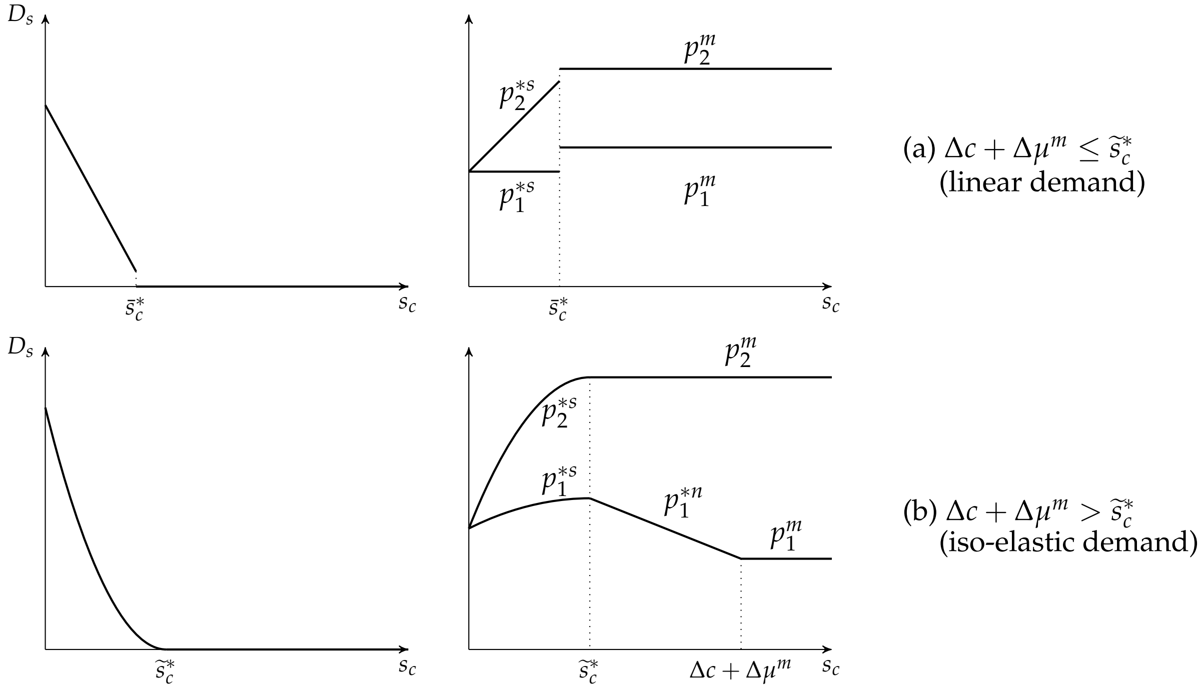


Figure 4: Equilibrium consumer storage and price patterns under limited commitment

Now, we consider the case $\Delta c + \Delta\mu^m > \widetilde{s}_c^*$, which is formalized in the following corollary and illustrated in panel (b) of Figure 3. The outcome in point (iib) of Proposition 2 is feasible instead of the outcome in point (iia). This case applies if $\Delta\mu^m > 0$, namely, demand is log-convex (e.g., iso-elastic).

Corollary 4 *Suppose $\Delta c + \Delta\mu^m > \widetilde{s}_c^*$. Then, under limited commitment,*

(i) if $s_c < \widetilde{s}_c^*$, consumer storage is $D_s^{*s} = \phi_2\left(p_1^{*s} + s_c\right)$, and prices are p_1^{*s} and $p_2^{*s} = p_1^{*s} + s_c$;

(ii) if $\widetilde{s}_c^* \leq s_c < \Delta c + \Delta\mu^m$, consumer storage is $D_s^{*n} = 0$, and prices are $p_1^{*n} = p_2^{*n} - s_c$ and $p_2^{*n} = p_2^m$;

(iii) if $s_c \geq \Delta c + \Delta\mu^m$, consumer storage is $D_s^m = 0$, and prices are p_1^m and p_2^m.

Prices exhibit the following features: (a) $\frac{\partial p_1^{*n}}{\partial s_c} < 0$, (b) $\frac{\partial p_2^{*s}}{\partial s_c} > 0$. Moreover, it holds (c) $p_1^{*n} > p_1^m$, (d) $p_2^m > p_2^{*s}$.

Point (i) of Corollary 4 indicates that, as in point (i) of Corollary 3, consumer storage occurs in equilibrium for sufficiently small consumer storage costs. A comparison between panels (a) and (b) of Figure 3 shows that the firm now prefers to induce consumer storage as long as it is feasible, i.e., $s_c < \widetilde{s}_c^*$. As point (ii) of Corollary 4 reveals, for intermediate consumer storage costs, i.e., $\widetilde{s}_c^* \leq s_c < \Delta c + \Delta\mu^m$, consumer storage is unfeasible but the firm cannot implement the static monopoly solution (the feasibility constraint (6) is violated). Although consumers are indeed indifferent about storing (the storability constraint is binding), we find that no storage takes place in equilibrium. This resembles the full commitment outcome in point (ii) of Corollary 2 illustrated in panel (b) of Figure 1. However, under limited commitment, the first period no-storing price p_1^{*n} is distorted above the static monopoly level, whereas second period no-storing price p_2^{*n} coincides with the static monopoly level, which constitutes the firm's best response to the absence of consumer storage. As point (iii) of Corollary 4 indicates, the static monopoly solution is implemented if and only if it is available, i.e., $s_c \geq \Delta c + \Delta\mu^m$.

Panel (b) of Figure 4 illustrates the equilibrium consumer storage and price sequence as a function of s_c, for the example of iso-elastic demand. As under full commitment in panel (b) of Figure 2, they are continuous functions. The depicted pattern of first period price p_1^{*s} does not hold generally, because p_1^{*s} varies with s_c according to the demand curvature. The first period no-storing price $p_1^{*n} = p_2^m - s_c$ lies above the static monopoly level and decreases linearly with s_c (due to the binding storability constraint). Hence, the first period pricing policy is typically non-monotonic. The second period pricing policy monotonically increases with s_c and coincides with the static monopoly level when consumer storage is no longer feasible.

6 Price comparisons

Equipped with the results of the previous sections, we are now in a position to compare the equilibrium prices under the two commitment regimes. For the sake of convenience, we define $D_2^N(p_1) \equiv D(p_1 + s_c) - D_s(p_1)$ as the second period demand net of consumer storage.

Proposition 3 *Suppose $s_c < s_c^l$, where s_c^l is defined by (A14) in the Appendix. Then, in each period the price under limited commitment is lower than the price under full commitment, i.e., $p_\tau^{*s} < p_\tau^{cs}$, $\tau \in \{1,2\}$, if and only if $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} > 0$.*

Proposition 3 shows that, under certain circumstances, the firm's lack of commitment leads to lower prices in each period. Given that production costs rise in the second period and a firm with limited commitment powers only cares about its continuation profits after the second period starts, one might be tempted to believe that the firm ends up charging excessively high prices, at least in the second period. Indeed, we show that an increase in production costs can translate into lower prices in each period under limited commitment. To appreciate the rationale for this result as substantiated in the introduction, recall from Proposition 1 that the full commitment price sequence in the presence of consumer storage is such that consumers store the entire second period demand, and therefore the firm shuts down in the second period. However, as shown in Section 5, this pricing policy is not sequentially optimal, because the firm succumbs to the temptation to charge a lower price than under full commitment in order to serve the market in the second period. As stated in Proposition 3, suppose that consumer storage costs are sufficiently small, i.e., $s_c < s_c^l$, where s_c^l is the threshold for s_c below which consumer storage occurs irrespective of the firm's commitment powers, which implies that the storability constraint is binding. Given that $s_c^l \leq \Delta c$, the additional cost Δc of producing in the second period exceeds the additional price s_c that consumers are willing to pay in the second period. Hence, the firm has an ex ante incentive to discourage purchases in the second period. To this aim, the only instrument to which the firm can resort under limited commitment is the price in the first period. A manipulation of the first period price generates two opposite effects. A lower p_1 stimulates consumer storage $D_s(p_1)$. Ceteris paribus, this reduces production and sales in the second period, and allows the firm to enjoy cost savings. However, given the binding storability constraint, a lower p_1 translates into a lower $p_2 = p_1 + s_c$, which inflates the second period gross demand $D(p_1 + s_c)$ and hinders the firm's cost efficiency. When the second period demand net of consumer storage is upward sloping, i.e., $\frac{\partial D_2^N}{\partial p_1} > 0$, the increase in consumer storage associated with a lower first period price more than compensates the increase in the second period gross demand. A reduction in the first period price leads to lower production and sales in the second period, which alleviates the firm's loss from the lack of commitment. As formally shown in the proof of Proposition 3 in the Appendix, the sign of the slope of the second period residual demand crucially depends on the curvature of demand. It turns out that the second period residual demand is upward sloping as long as the demand function is not too convex. Widely-used demand specifications that satisfy this condition are linear, exponential and, under some circumstances, iso-elastic demand functions.[13] Given that the second period price declines as well due to the binding storability constraint, a firm with limited commitment powers charges lower prices than under full commitment in each period, i.e., $p_\tau^{*s} < p_\tau^{cs}$, $\tau \in \{1, 2\}$. For illustrative purposes, in the Supplementary Appendix (Section 7) we characterize the equilibrium price sequence and consumer storage under the two commitment regimes as well as the associated welfare properties in a linear demand framework.

If the second period net demand is downward sloping, i.e., $\frac{\partial D_2^N}{\partial p_1} < 0$, the firm must increase the first period price in order to reduce the second period net demand. This occurs when the demand function is significantly convex.[14] The reduction in the second period gross demand arising from a higher first period price exceeds the reduction in consumer storage, which implies that the second period residual demand declines. The firm charges higher prices than under full commitment in order to reduce production and sales in the second period, which mitigates the firm's loss from the lack of commitment. Despite higher prices, consumer storage still occurs in equilibrium. As the storability constraint is binding, consumers are indeed indifferent about storing, and the equilibrium consumer storage is dictated by the sequential optimality constraint.

The condition about the slope of the second period residual demand in Proposition 3 can be formulated in terms of a relationship between the convexity of demand and the firm's relative markup in the second period. Following Mrázová and Neary (2017), the convexity of demand is defined as the elasticity of the slope of demand, which corresponds to $r_2^{*s} \equiv -\frac{d \log D'(p_2)}{d \log p_2}\Big|_{p_2 = p_2^{*s}} = -\frac{p_2^{*s} D''(p_2^{*s})}{D'\left(p_2^{*s}\right)}$ if evaluated at the second period equilibrium limited commitment price p_2^{*s}. The relative markup, or Lerner index, in the second period is equal to the ratio between the profit margin and the price in equilibrium, i.e., $m_2^{*s} \equiv \frac{p_2^{*s} - c_2}{p_2^{*s}}$. We find the following result.

Corollary 5 *Suppose $s_c < s_c^l$, where s_c^l is defined by (A14) in the Appendix. Then, in each period the price under limited commitment is lower than the price under full commitment, i.e., $p_\tau^{*s} < p_\tau^{cs}$, $\tau \in \{1, 2\}$, if and only if $r_2^{*s} < \frac{1}{m_2^{*s}}$.*

Corollary 5 provides an alternative condition for the result in Proposition 3, according to which limited commitment leads to lower prices if and only if the convexity of demand is

[13]For sufficiently small consumer storage costs, the degree of elasticity of the iso-elastic demand must be high enough.

[14]An example is the iso-elastic demand with a sufficiently low degree of elasticity.

lower than the inverse of the Lerner index in the second period, i.e., $r_2^{*s} < \frac{1}{m_2^{*s}}$.[15] As discussed in Section 9, the appealing feature of this condition is that it is suitable for an empirical estimation.

The following proposition completes the analysis of the price comparisons between the two commitment regimes.

Proposition 4 *A. Suppose $s_c^l \leq s_c < s_c^h$, where s_c^l and s_c^h are defined by (A14) and (A15) in the Appendix. Then, in the first period the price under limited commitment is higher than the price under full commitment. If $\Delta\mu^m > 0$, the price under limited commitment is also higher in the second period.*

B. Suppose $s_c \geq s_c^h$. Then, in each period the price under limited commitment coincides with the price under full commitment and corresponds to the static monopoly price.

Proposition 4 delivers results that substantially differ from Proposition 3. Specifically, Proposition 4A indicates that, for intermediate consumer storage costs, i.e., $s_c^l \leq s_c < s_c^h$, the price in the first period is higher under limited commitment irrespective of the demand curvature. Although the equilibrium price varies under each commitment regime according to the parameter constellations, a common rationale for this result can be identified. It follows from the discussion after Proposition 3 that for $s_c \geq s_c^l$ consumer storage disappears at least under one commitment regime. In particular, it may occur that under limited commitment consumer storage is removed but it is profitable under full commitment. Alternatively, under limited commitment consumer storage is either allowed or removed but it is unprofitable under full commitment. The firm's lower capability to promote efficient consumer storage under limited commitment implies that the firm charges a first period price higher than under full commitment at which consumer storage disappears even when it is ex ante profitable. If consumer storage cannot be prevented despite being ex ante unprofitable, we find that limited commitment leads to higher prices in both periods.[16] As Proposition 4A indicates, with log-convex demand ($\Delta\mu^m > 0$), the second period price is also unambiguously higher under limited commitment, because the storability constraint is binding under the two commitment regimes. However, this result may not hold if demand is (weakly) log-concave ($\Delta\mu^m \leq 0$). The comparison between the second period prices becomes problematic when the static monopoly prices p_1^m and p_2^m are set under limited commitment while the prices p_1^{cs} and p_2^{cs} with consumer storage are chosen under full commitment. We know from the feasibility constraint (6) that $p_1^m + s_c \geq p_2^m$ and from Proposition 1 that $p_1^{cs} + s_c = p_2^{cs}$. Hence, a higher first period price under limited commitment does not necessarily imply a higher price in the second period as well. Given that p_2^{cs} increases with s_c (see Corollaries 1 and 2) but p_2^m is unaffected, there may exist a threshold for s_c above which the second period price is indeed lower under limited commitment.

[15] The proof of Corollary 5 directly follows from Proposition 3 and therefore it is omitted. Note from Proposition 2 that, in the presence of consumer storage $D_s^{*s} = \phi_2\left(p_1^{*s} + s_c\right)$ (where $\phi(\cdot)$ is defined by (5)), the Lerner index is lower than the inverse of the demand elasticity, i.e., $m_2^{*s} < \frac{1}{\varepsilon_{p_2^{*s}}}$, differently from the static monopoly.

[16] This case occurs for $s_c > \Delta c$ and the binding storability constraint under the two commitment regimes. As the additional price that consumers are willing to pay in the second period outweighs the additional production cost, i.e., $s_c > \Delta c$, consumer storage is ex ante unprofitable. Hence, a firm with limited commitment powers resorts to a higher first period price in order to mitigate consumer storage. This translates into a higher second period price due to the binding storability constraint. We refer to the proof of Proposition 4 in the Appendix for technical details.

As Proposition 4B reveals, the firm opts for the static monopoly prices irrespective of its commitment powers when consumer storage costs are sufficiently large, i.e., $s_c \geq s_c^h$, where $s_c^h \geq \Delta c + \Delta\mu^m$. Consumer storage is so costly that it renders the static monopoly solution not only implementable (the feasibility constraint (6) holds) but also optimal under the two commitment regimes.

It is worth exploring the impact of the cost increase Δc (for a given c_1) on the equilibrium prices in the presence of consumer storage under the two commitment regimes. This is formalized in the following remark.

Remark 1 *For a given c_1, it holds*
(i) $\frac{\partial p_\tau^{cs}}{\partial \Delta c} = 0$, $\tau \in \{1, 2\}$;
(ii) $\frac{\partial p_\tau^{*s}}{\partial \Delta c} < 0$ *if and only if* $\frac{\partial}{\partial \Delta c}\left[(\Delta c - s_c)\frac{\partial D_2^N(p_1^{*s})}{\partial p_1}\right] > 0$, $\tau \in \{1, 2\}$.

As Remark 1 indicates, the full commitment price p_τ^{cs} is independent of the cost increase Δc, because production only takes place in the first period. The relation between the cost increase Δc and the limited commitment price p_τ^{*s} is more sophisticated. It reflects the impact of Δc on the firm's loss $(\Delta c - s_c)$ from serving the market in the second period weighted by the slope of the second period net demand function $\frac{\partial D_2^N}{\partial p_1}$.[17] This is the outcome of the trade-off between two effects. To gain some intuition, note that $\frac{\partial}{\partial \Delta c}\left[(\Delta c - s_c)\frac{\partial D_2^N}{\partial p_1}\right] = \frac{\partial D_2^N}{\partial p_1} + (\Delta c - s_c)\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c}$. The first term captures the direct effect of Δc, which is equal to the price impact on the second period net demand, i.e., $\frac{\partial D_2^N}{\partial p_1}$. The second term measures the indirect effect of Δc through the price channel, which corresponds to the responsiveness of $\frac{\partial D_2^N}{\partial p_1}$ to Δc, i.e., $\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c}$, weighted by the firm's loss $(\Delta c - s_c)$. As $\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c} = D''(p_1 + s_c)$, the trade-off between the two effects crucially depends on the curvature of demand. First, consider a concave demand, i.e., $D''(\cdot) < 0$. We know from Proposition 3 and the associated Corollary 5 that the second period net demand is upward sloping, i.e., $\frac{\partial D_2^N}{\partial p_1} > 0$. Hence, the first effect is positive and pushes toward a price reduction in order to dampen the second period net demand. A higher Δc induces the firm to implement a more aggressive pricing policy, which allows saving production costs in the second period. However, the second effect is negative, i.e., $\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c} = D''(p_1 + s_c) < 0$. The reason is that a higher Δc mitigates the negative slope of the consumer storage function, which becomes flatter ($\frac{\partial^2 D_s}{\partial p_1 \partial \Delta c} = -\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c} > 0$). A more rigid consumer storage demand tempers the reduction in consumer storage associated with a higher price. This creates an incentive for a price increase. When Δc is relatively small, the first effect dominates the second effect due to the small size of the firm's loss $(\Delta c - s_c)$, and therefore the limited commitment price decreases with Δc. For relatively large values of Δc, the second effect can prevail, and the limited commitment price increases with Δc.

Consider now a (weakly) convex demand, i.e., $D''(\cdot) \geq 0$. We know from Proposition 3 and Corollary 5 that, if demand is not too convex, the first effect is still positive, i.e., $\frac{\partial D_2^N}{\partial p_1} > 0$. The second effect is now (weakly) positive as well, i.e., $\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c} = D''(p_1 + s_c) \geq 0$. The idea is

[17] For the sake of convenience, we focus on the most relevant case where $s_c < \Delta c$. However, with log-convex demand ($\Delta\mu^m > 0$), consumer storage may occur under limited commitment even for $s_c > \Delta c$ (see the proofs of Propositions 3 and 4 in the Appendix). The result in Remark 1 is unaffected but the explanation should be qualified accordingly.

that a higher Δc steepens the consumer storage demand function ($\frac{\partial^2 D_s}{\partial p_1 \partial \Delta c} = -\frac{\partial^2 D_2^N}{\partial p_1 \partial \Delta c} \leq 0$). A more elastic consumer storage demand exacerbates the reduction in consumer storage associated with a higher price. This strengthens the incentive for a price reduction. Consequently, the limited commitment price unambiguously decreases with Δc. Notably, this result is evocative of Edgeworth's paradox that a tax on a monopolist may lead to lower prices (Hotelling 1932). However, the rationale for Edgeworth's paradox, derived in a static framework with substitutable goods, hinges on merely analytical conditions (which are not satisfied with linear demand and cost functions) and significantly differs from the mechanism behind our result.

When demand is sufficiently convex, it follows from Proposition 3 and Corollary 5 that the first effect becomes negative, i.e., $\frac{\partial D_2^N}{\partial p_1} < 0$. Given that the second effect is still positive, the trade-off between the two effects generates opposite results to those with concave demand. When Δc is small enough, the limited commitment price increases with Δc but the converse can occur for sufficiently large values of Δc.

7 Welfare analysis

We now investigate consumer surplus and total welfare associated with the firm's ability to commit to future prices. Total welfare is computed as the (unweighted) sum of consumer surplus and the firm's profits. In the following proposition, we consider the case where consumer storage costs are relatively small, as in Proposition 3.

Proposition 5 *Suppose $s_c < s_c^l$, where s_c^l is defined by (A14) in the Appendix. Then, for $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} > 0$,*

(i) consumer surplus is higher under limited commitment than under full commitment;

(ii) total welfare is higher under limited commitment than under full commitment if the second period net demand $D_2^N(p_1^{*s})$ is small enough.

For $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} \leq 0$, consumer surplus and total welfare are lower under limited commitment than under full commitment.

Proposition 5 characterizes the welfare comparison between the two commitment regimes when consumer storage costs are sufficiently small so that storage takes place irrespective of the firm's commitment powers, i.e., $s_c < s_c^l$. Note that the quantity bought at the unit price p_1 and stored by consumers in the first period is actually consumed in the second period at the additional unit cost s_c. This generates the same consumer surplus as if that quantity had been bought in the second period at the unit price $p_1 + s_c$. Given that for $s_c < s_c^l$ consumers are indifferent about storing under full and limited commitment, i.e., $p_2 = p_1 + s_c$, consumer surplus is higher under the commitment regime that generates lower prices, irrespective of the level of consumer storage. It follows from Proposition 3 that limited commitment increases consumer surplus if and only if the second period net demand is upward sloping, i.e., $\frac{\partial D_2^N}{\partial p_1} > 0$, which occurs as long as the demand function is not too convex.

The comparison in terms of total welfare between the two commitment regimes differs from the standard static case of a price change. Intuitively, a lower price under limited commitment raises total welfare because it mitigates the deadweight loss from monopoly power.

However, limited commitment harms per se the firm's profits, in addition to the mere price reduction. We find that, despite the firm's loss, limited commitment enhances total welfare if the second period demand net of consumer storage is small enough. Recall from the discussion in Section 6 that for $s_c < s_c^l$ the additional price s_c that consumers are willing to pay is lower than the additional cost Δc of producing in the second period ($s_c^l \leq \Delta c$). This implies that production in the second period is socially inefficient. Given that a firm with limited commitment powers cannot refrain from serving the market in the second period, limited commitment is total welfare superior if the second period sales are low enough. As formally shown in the Supplementary Appendix (Section 7), in a linear demand framework, there exists a threshold for the cost increase Δc above which limited commitment increases total welfare. The rationale for this result can be grasped in the light of our analysis so far. It follows from Remark 1 that, with linear demand, under limited commitment the firm lowers its prices in response to a higher Δc, which stimulates consumer storage and reduces the second period residual demand. If Δc is above a certain threshold, the gain in consumer surplus more than compensates the firm's loss, and therefore limited commitment enhances total welfare.[18] Notably, we find that the threshold for Δc declines with the slope of demand. This relaxes the condition for the total welfare superiority of limited commitment. A more elastic demand leads to lower prices and higher consumer storage, which reduces the socially inefficient production and sales in the second period. We know from Proposition 3 that, when the second period residual demand is downward sloping, prices are higher under limited commitment. As Proposition 5 indicates, this reduces consumer surplus and, a fortiori, total welfare.

For the sake of completeness, the following remark formalizes the welfare results for larger values of consumer storage costs, as in Proposition 4.

Remark 2 *A. Suppose $s_c^l \leq s_c < s_c^h$, where s_c^l and s_c^h are defined by (A14) and (A15) in the Appendix. Then, if $\Delta\mu^m > 0$, consumer surplus and total welfare are lower under limited commitment than under full commitment.*

B. Suppose $s_c \geq s_c^h$. Then, consumer surplus and total welfare are the same under the two commitment regimes and coincide with the static monopoly level.

The results in Remark 2 are a direct consequence of Proposition 4.[19] For intermediate consumer storage costs, i.e., $s_c^l \leq s_c < s_c^h$, prices are unambiguously higher under limited commitment if demand is log-convex ($\Delta\mu^m > 0$), which leads to lower consumer surplus. As the firm's profits are also lower, limited commitment is definitely welfare detrimental. When demand is (weakly) log-concave ($\Delta\mu^m \leq 0$), the first period price is higher under limited commitment, but no clear-cut result can be derived in the second period. A higher first period price and lower profits suggest that limited commitment is still welfare detrimental, but a more rigorous analysis can only be conducted in a more specific setting. As formally shown in the Supplementary Appendix (Section 7), with linear demand, limited commitment reduces consumer surplus and, a fortiori, total welfare. For sufficiently large consumer storage costs, i.e., $s_c \geq s_c^h$,

[18]The cost increase Δc cannot be too large in order to guarantee a positive profit margin in the second period. As discussed in Section 5, throughout the paper we focus on cost increases whose magnitude is not so significant that the firm's commitment problem trivially disappears.

[19]The proof of Remark 2 is therefore omitted.

the firm implements the static monopoly solution irrespective of its commitment powers, and therefore the firm's commitment problem is welfare inconsequential.

8 Robustness and extensions

Firm's inventories and convex storage costs

Along with consumers, the firm can conduct storage activities by accumulating inventories for future sales. Intuitively, the amount of the firm's inventories and consumer storage shall depend on the relative magnitude of their storage costs, which varies with the industry at hand. As discussed in the introduction, there exists empirical and anecdotal evidence according to which in different markets retailers are more inefficient at storing than consumers and therefore prefer to delegate storage activities to them (e.g., Blattberg et al. 1981; Pesendorfer 2002).[20] For instance, Walmart has over 11,000 stores worldwide that generally face physical constraints on the storage capacities and prefer to induce consumer storage rather than return the unsold products to their warehouses. The existence of price differences over time can spur storage at least by consumers with relatively low opportunity costs. Competitive arbitrageurs can also engage in storing for speculative purposes, especially in the presence of price volatility (e.g., Mitraille and Thille 2009, 2014).[21] However, in other markets, mainly at the upstream level, a large firm can incur lower storage costs than its customers. Inventory accumulation reduces future production costs and therefore can be the firm's preferred option. In addition, as shown by Antoniou and Fiocco (2019) in the context of a growing market, a firm with limited commitment powers can exhibit strategic incentives to hold inventories in order to credibly affect its future price. In our setting, consumer storage can still emerge in equilibrium, particularly when convex production costs or capacity constraints prevent the firm from covering the entire future demand. We refer to the subsequent analysis in Section 8 for extensions of our model in these directions.

In practice, the firm's inventories and consumer storage are likely to coexist for reasons mainly related to limited storage capacities. In line with some relevant literature (e.g., Dudine et al. 2006), we consider linear storage costs in the baseline model. Yet, consumers may find it more costly to store an additional unit of the good when their storage is higher. Along these lines, Hendel et al. (2014) assume that consumers are able to store for free but face a storage capacity. Alternatively, consumers can be heterogeneous in their unit storage costs. The firm can also face capacity constraints or increasing marginal storage costs. In the Supplementary Appendix (Section 2), we extend our analysis to accommodate these features by introducing convex storage costs. Then, the firm accumulates inventories and promotes consumer storage for production smoothing purposes. In case of homogeneous consumers, the firm can benefit from the relatively small consumer storage costs up to some storage level. When consumers differ in their storage abilities, the firm resorts to the storage activities by the most efficient

[20] Inventories affect the firms' taxable profits so that higher inventory accumulation translates into a higher tax burden. Direct inventory taxation is imposed in some US states (e.g., Kentucky, Louisiana, Texas, and West Virginia).

[21] Recent technological advances facilitate these arbitrage operations. For instance, Fulfillment by Amazon allows small sellers to store their products in Amazon's fulfillment centers.

consumers. In a general framework with convex storage costs, we show that, following the rationale behind the results in the baseline model, a firm with limited commitment powers has an incentive to charge lower prices than under full commitment when consumers are more reluctant to store in anticipation of the firm's opportunistic behavior in the second period. In a more specific setting with linear demand and quadratic storage costs, we explicitly derive the equilibrium prices under the two commitment regimes. We find that limited commitment leads to lower prices as long as the increase in marginal production costs is not significantly pronounced. The firm succumbs to the temptation to reduce the second period price below the full commitment level in order to stimulate its sales, and therefore the firm's commitment problem persists.

Uncertainty about production costs

Cost expectations can be formed in a number of relevant industries with some degree of accuracy. For instance, systematic empirical evidence about the oil market indicates that crude oil prices evolve according to a mean reversion pattern (e.g., Anderson et al. 2018; Bessembinder et al. 1995; Deltas 2008). Following a negative (positive) cost shock, future costs are expected to be higher (lower) than current costs.[22] Nowadays, several central banks around the world, including the Federal Reserve, the Bank of England and the European Central Bank, significantly resort to "forward guidance", via which they inform the public about the intended future path of monetary policy. Their purpose is to influence the operators' expectations about the future cost of capital (e.g., Dell'Ariccia et al. 2018). More generally, predictions regarding future input costs (at least in the short run) are available in sectors where commodities can be traded in the stock markets (e.g., coffee, cereals, corn, oil, and chemicals). In a similar vein, in international markets the exporters' forecasts about their future production costs depend on the expected variations in exchange rates.

Our model can be extended to allow for uncertainty about future production costs. To preserve the relevance of our analysis, we focus on the case where the second period expected marginal cost is higher than the first period marginal cost. Formal details are provided in the Supplementary Appendix (Section 3). Under full commitment, for sufficiently small storage costs, the introduction of cost uncertainty does not crucially affect the firm's pricing policy, because the firm induces full storage in the first period and shuts down in the second period. Under limited commitment, the firm cannot refrain from serving the market in the second period, and the associated price will depend on the realization of the cost shock. A stochastic cost process may yield significant welfare effects. As formally shown in the Supplementary Appendix (Section 3), the firm's (second period) profits are convex with respect to the stochastic term that positively affects the cost realization. This suggests that uncertainty about future costs mitigates the firm's loss from the lack of commitment. In the light of Waugh's (1944) classical result that consumer surplus is convex in prices, we may expect that the variability in the second period price can benefit consumers as well. In the Supplementary Appendix (Section 3), we characterize the condition under which consumer surplus is convex with respect to the stochastic term. This is satisfied under widely-used demand functions, such as linear,

[22] This stochastic process resembles the one adopted by Antoniou et al. (2017).

exponential, and iso-elastic. Hence, cost uncertainty creates a shift in consumer preferences in favor of limited commitment for a relevant number of cases.

In our model, storage activities can be conducted either by competitive arbitrageurs or directly by final consumers. Consider a continuum of risk-neutral, profit-maximizing, competitive arbitrageurs (or speculators) that purchase the good at the price p_1 from the firm in the first period and, after incurring the storage cost s_c, resell it to final consumers in the second period. The presence of arbitrageurs implies that the second period expected price reflects the first period price augmented by the storage cost, i.e., $\mathbb{E}[p_2] = p_1 + s_c$. Now, suppose that arbitrageurs do not operate in the market, and final consumers can directly engage in storing activities. It follows from the convexity of consumer surplus with respect to the stochastic term that consumers prefer ex ante to buy a unit of the good in the second period at a random price p_2 rather than at a deterministic price equal to $\mathbb{E}[p_2]$. Given that storing involves a unit deterministic price $p_1 + s_c$, the condition under which consumers are indifferent about storing is such that $p_1 + s_c < \mathbb{E}[p_2]$. In this case, the firm must reduce the first period price to a larger extent in order to stimulate consumer storage, which tends to increase consumer surplus. The welfare superiority of limited commitment derived in Section 7 can be (ex ante) even more pronounced in the presence of cost uncertainty.

Convex production costs

Our results can be extended to nonlinear technologies, such as convex production costs. Technical details are available in the Supplementary Appendix (Section 4). It is well-known that the firm has an incentive for production smoothing in order to achieve cost efficiency when production costs differ across periods. A natural extension of our framework is that marginal costs (at given quantities) rise in the second period. When the magnitude of the cost increase is significant compared to the consumer storage cost, a firm with full commitment powers prefers to induce consumer storage in order to smooth production over time. Differently from the setting with linear costs, consumer storage generally covers only a part of the second period demand. Under limited commitment, consumers are more reluctant to store because they anticipate the firm's temptation to reduce the second period price below the full commitment level in order to stimulate its sales. As formally shown in the Supplementary Appendix (Section 4), when consumer storage is profitable for the firm, limited commitment leads to lower prices than full commitment, provided that the second period net demand is positively sloped. Notably, this holds true despite the fact that lower prices imply higher marginal costs due to the increase in production. In line with the baseline model, the firm resorts to lower prices in order to stimulate consumer storage and dampen the second period production and sales, which mitigates the firm's loss from the lack of commitment. Hence, the rationale behind our main results carries over to the presence of convex production costs.

Longer time horizon

Our analysis can be generalized to a time horizon with more than two periods. There are various reasons that make a two-period model suitable for our purposes, in addition to its analytical tractability. Forecasts about the evolution of costs tend to be accurate only in the short run. Furthermore, storable goods are subject to depreciation over time and can be generally accumulated only for a limited amount of time. Despite these considerations, the study of a longer time horizon warrants some attention. To fix ideas, consider a setting with $T \geq 2$ periods, where production costs increase in each period, i.e., $\Delta c_\tau \equiv c_\tau - c_{\tau-1} > 0$, $\tau \in \{2, ..., T\}$, and consumer storage costs are sufficiently small so that storage is profitable in each period irrespective of the firm's commitment powers, which requires $s_c < \Delta c_\tau$.[23] It follows from our analysis that the storability constraint is binding in each period, i.e., $p_\tau = p_{\tau-1} + s_c$, $\tau \in \{2, ..., T\}$. Formally, under full commitment, the firm's maximization problem is given by

$$\max_{p_1} \; (p_1 - c_1) \sum\nolimits_{\tau=1}^{T} D(p_\tau) \;\; s.t. \;\; p_\tau = p_{\tau-1} + s_c.$$

In line with the baseline model, a firm that can fully commit to a price sequence induces consumers to store in the first period the entire quantity consumed in the following periods. This is because the cost increase Δc_τ from period $\tau - 1$ to period τ exceeds the additional price s_c that consumers are willing to pay.

Under limited commitment, the firm cannot refrain from reducing its prices below the full commitment level to serve the residual demand. Anticipating the firm's opportunistic behavior, consumers are less inclined to store. Formally, under limited commitment, the firm's maximization problem can be written as

$$\max_{p_1} \; (p_1 - c_1) \sum\nolimits_{\tau=1}^{T} D(p_\tau) - \sum\nolimits_{\tau=2}^{T} D_\tau^N(p_\tau) \left[\sum\nolimits_{t=2}^{\tau} \Delta c_t - (\tau - 1) s_c \right] \;\; s.t. \;\; p_\tau = p_{\tau-1} + s_c,$$

where $D_\tau^N(p_\tau) \equiv D(p_\tau) - D_s(p_{\tau-1}) + D_s(p_\tau)$ constitutes the net demand in period $\tau \in \{2, ..., T\}$, namely, the demand for consumption $D(p_\tau)$ in period τ, reduced by the consumer storage $D_s(p_{\tau-1})$ in period $\tau - 1$ and inflated by the consumer storage $D_s(p_\tau)$ in period τ (where $D_s(p_\tau) = 0$ for $\tau = T$ because no storage takes place in the final period). The amount of consumer storage in each period is determined in equilibrium by the condition of sequential optimality and the binding storability constraint. Each unit of the net demand $D_\tau^N(p_\tau)$ in any future period τ involves a loss equal to $\sum_{t=2}^{\tau} \Delta c_t - (\tau - 1) s_c$, which corresponds to the excess of the aggregate cost increase from the initial period over the aggregate additional price that consumers are willing to pay. In line with the baseline model, when the future residual demand is upward sloping, a firm with limited commitment powers sets lower prices in order to stimulate consumer storage, which reduces future production and sales that occur at higher costs.[24] Notably, a longer sequence of periods with increasing costs aggravates the firm's loss from serving the future demand, which magnifies the firm's incentive for a price reduction.

[23] A finite horizon setting adequately captures the main features of our framework. In an infinite horizon setting, even abstracting from the fact that forecasts about future costs are likely to be accurate only in the short run, the existence of a period where costs are expected to decrease identifies a terminal period, analogously to a finite horizon setting. We refer to Hendel et al. (2014) and Mitraille and Thille (2016) for insightful investigations in an infinite horizon model.

[24] For instance, in a three-period setting with a linear demand of the form $D(p_\tau) = \alpha - \beta p_\tau$, $\tau \in \{1, 2, 3\}$, the residual demand increases with the price in the second and third period, i.e., $\frac{\partial D_2^N}{\partial p_1} = D'(p_2) - D_s'(p_1) + D_s'(p_2) = 3\beta > 0$ and $\frac{\partial D_3^N}{\partial p_1} = D'(p_3) - D_s'(p_2) = \beta > 0$.

Therefore, under limited commitment, prices can be lower to a further extent compared to full commitment as the time horizon becomes longer.

Durable goods

It has been well-established in the literature on durable goods since the seminal contribution of Coase (1972) that, if a monopolist cannot refrain from price-discriminating over time among consumers with different valuations of the good, consumers have an incentive to postpone their purchases in expectation of future better deals. Consistently with the consumers' beliefs, the firm charges lower prices than under full commitment and loses (at least partially) its monopoly power. With storable goods, the nature of the consumers' intertemporal incentives is significantly different. Consumers are willing to store in anticipation of higher future prices, which implies that the consumers' behavior is driven by demand anticipation rather than by demand postponement. The equilibrium price sequence also exhibits relevant differences. It follows from our analysis that the price comparisons between full and limited commitment in a storable good market crucially depend on a number of factors, such as the magnitude of consumer storage costs and the curvature of demand. Conversely, prices for durable goods are unambiguously lower under limited commitment. Moreover, with higher future costs of production and costly consumer storage, the price pattern for storable goods increases over time. Yet, under limited commitment, prices for durable goods decline across periods. For illustrative purposes, in the Supplementary Appendix (Section 5) we characterize the price sequence for a durable good monopolist in a stylized two-period framework with linear demand.

In contrast with the case of durable goods, the firm's commitment problem with storable goods originates from an (expected) increase in future production costs. To better appreciate the different implications of the storability vis-à-vis the durability of a good, it is helpful to explore the impact of a cost variation on the price behavior of a durable good monopolist. As formally shown in the Supplementary Appendix (Section 5), the second period price rises in response to a more pronounced cost increase. The reduction in the price drop from the full commitment level leads to lower sales in the second period and therefore mitigates the firm's commitment problem at the cost of more expensive production. This might be misperceived as a softer constraint for the firm, which could charge a higher price in the first period in order to recoup some monopoly power. Indeed, the firm decreases the first period price even further below the full commitment level to stimulate current purchases and save future costs of production. Hence, a larger cost increase moves prices in opposite directions. The price rise in the second period more than compensates the price reduction in the first period, which can lead to lower consumer surplus. In a market for storable goods, the firm's price response to a larger cost increase and the associated welfare effects are substantially different. As shown in Remark 1, with linear demand, a larger cost increase translates into lower prices in each period, which definitely enhances consumer surplus but aggravates the firm's commitment problem.

Discount factor

In the baseline model, we consider no discounting on the second period. This assumption is imposed for the sake of simplicity and our qualitative results carry over to a more general discount factor $\delta \in (0, 1]$. Technical details are provided in the Supplementary Appendix (Section 6). Naturally, when consumer storage costs are sufficiently small so that storage occurs in equilibrium, the storability constraint is binding, with implies that the first period price augmented by the consumer storage cost equals the discounted second period price, i.e., $p_1 + s_c = \delta p_2$. In line with the result in Proposition 3, we find that, for sufficiently small consumer storage costs, limited commitment leads to lower prices than full commitment if and only if the second period residual demand is upward sloping, namely, the demand function is not too convex. The impact of the discount factor δ on equilibrium prices exhibits features of some interest. As shown in the Supplementary Appendix (Section 6), in a linear demand framework, equilibrium prices increase with δ under the two commitment regimes. A higher δ makes consumers more eager to store for future consumption, which allows the firm to charge higher prices. Remarkably, the price gap between full and limited commitment increases with δ. A firm with limited commitment powers increases its prices in response to a higher δ less significantly than under full commitment in order to spur consumer storage and mitigate the more valuable loss from serving the market in the second period.

Competition

Throughout the analysis, we focus our attention on a single firm in the market. This captures in a simple and tractable manner the presence of market power. As documented by Besanko et al. (2005), for many grocery products (including storable goods), retailers can have high market power in their pricing decisions. Nonetheless, it is worth discussing the impact of competition among firms on our results in the light of the existing literature. In a two-period Cournot duopoly setting where consumers engage in storage activities, Anton and Das Varma (2005) show that firms behave more aggressively to attract consumer storage. In a two-period differentiated good Bertrand framework, Guo and Villas-Boas (2007) find that the opportunity of consumer storage exacerbates price competition. When production costs are expected to increase over time, the firms' incentives to compete for consumer storage are magnified, which creates further pressure to cut prices. Therefore, the forces described in our paper complement those identified by Anton and Das Varma (2005) and Guo and Villas-Boas (2007). Our analysis suggests that, when consumer storage costs are small enough, the benefits for consumers can be larger under limited commitment, at least in sufficiently concentrated markets.

9 Concluding remarks: empirical and policy implications

The dynamic interactions between firms and consumers are a relevant issue in many settings, such as markets for storable goods. In this paper, we characterize a firm's pricing policy and the welfare effects associated with the firm's ability to commit to future prices in a dynamic storable good market where consumers are willing to store in anticipation of higher future prices and production costs evolve over time. When production costs are expected to increase, we find that, for sufficiently small consumer storage costs, the firm's lack of commitment generates lower prices if and only if the future residual demand is upward sloping, namely, the demand function is not too convex. The firm resorts to lower prices in order to reduce future production and sales, which mitigates the firm's loss from the lack of commitment. Despite the firm's loss, under certain circumstances, limited commitment can be even total welfare superior. For intermediate values of consumer storage costs, the firm's inefficient behavior toward consumer storage under limited commitment generally leads to higher prices. This reduces consumer surplus and, a fortiori, total welfare.

Our analysis sheds new light on some empirical regularities about the firms' pricing behavior, especially regarding the patterns of cost pass-through rates. The empirical evidence indicates that typically cost changes are not fully passed through to prices at the firm's level. However, in a static monopoly setting, the log-linear and log-convex demand specifications usually adopted in the empirical literature (such as exponential and iso-elastic demands) lead to a cost pass-through rate equal and higher than 1, respectively. We find that, for widely-used demand functions (including exponential and iso-elastic demands), the cost pass-through rate is indeed lower than 1, consistently with the empirical evidence.[25] Hence, our study provides theoretical support for the commonly observed incomplete cost pass-through in a setting with empirically relevant demand specifications, even when the market is relatively concentrated. Remarkably, our results can also explain the puzzling phenomenon of "perverse" pass-through rates documented by Froot and Klemperer (1989), according to which a cost increase leads to lower prices.[26] Using a data set with 78 products across 11 categories of storable goods sold by a major US supermarket chain, Besanko et al. (2005) find that 5.6% of the estimated pass-through rates are "perverse" (i.e., negative), and this percentage becomes substantially higher for some items (e.g., more than 30% for toothpaste). In our model, "perverse" pass-through rates emerge with moderately convex demand, provided that consumer storage costs are small enough (see Remark 1 in Section 6). As emphasized by Ravn et al. (2010), the main theoretical gap in the existing empirical literature on cost pass-through is the pervasive use of static demand systems. Our study advocates that the dynamic interactions between firms and consumers and the identification of anticipated and unanticipated future cost shocks should be incorporated into the econometric estimations of pass-through rates. The rationale for the dynamic patterns of pass-through rates provided in our paper is different from — but potentially complementary to — the idea of Ravn et al. (2010) based on good-specific habit formation, and lies in the intertemporal incentives associated with storable goods.

Cost variations may also stem from changes in commodity taxation. Miravete et al. (2018) empirically characterize the tax pass-through rates for alcoholic beverages in Pennsylvania and show that market power crucially affects government tax revenues. Our analysis suggests that a dynamic econometric model tends to generate lower estimations of tax pass-through rates

[25]This follows from the inspection of (A11) in the Appendix for exponential and iso-elastic demands, provided that consumer storage costs are sufficiently small.

[26]In the international trade framework of Froot and Klemperer (1989), a foreign firm increases its dollar prices on exports to the US in response to the dollar appreciation that leads to a reduction in the foreign firm's costs expressed in dollars.

with respect to a static model. This can significantly affect the relationship between commodity tax rates and government tax revenues, i.e., the Laffer curve, and the corresponding design of the optimal tax policy. The predictions of our model about the impact of storability on pass-through rates and firms' markups lend themselves to an empirical validation.

Our analysis is particularly suitable for industries where cost expectations can be formed with some degree of accuracy, as discussed in Section 8. A prominent example is provided by tradable pollution allowances in the European Union. To achieve the overall greenhouse gas emissions reduction target for 2030, the sectors covered by the EU Emissions Trading System must decrease during Phase IV (2021-2030) their emissions by 43% compared to the levels in 2005.[27] As the total number of emission allowances will decline at an annual rate of 2.2% from 2021 onwards, the carbon price is expected to increase over time and therefore energy-intensive firms will incur higher costs. Our analysis suggests that firms operating in markets for storable goods where demand is moderately convex will reduce their prices over time. To the extent that inflation affects relatively more input costs, our study establishes new micro-foundations for the empirical evidence that the rate of inflation can be negatively correlated with the average markups (e.g., Bénabou 1992; Banerjee et al. 2001; Banerjee and Russell 2001; Head et al. 2010). For sectors where costs evolve according to a mean reversion pattern (such as the oil market), an empirical test of our model is to estimate the relation between cost mean reversion and price patterns.

Our findings can substantiate the stance of regulators and antitrust authorities on the firms' adoption of instruments that improve their commitment powers in storable good markets. A well-known contractual policy that a firm can implement to restore (or approach) the full commitment outcome is a money-back guarantee — sometimes called "most-favored nation" clause — which commits the firm to reimburse its customers if the future price falls below the preannounced level. In markets for durable goods where full commitment leads to higher prices, these price protection policies harm consumers, and therefore they should be prohibited. However, as shown by Dudine et al. (2006), in markets for storable goods where demand increases over time, the firm's lack of commitment is unambiguously welfare detrimental, which induces a positive evaluation of such contracts that improve the firm's commitment ability. Our analysis indicates that a more sophisticated assessment is warranted in relatively mature markets for storable goods, where demand tends to remain stable but production costs vary over time. Specifically, when consumer storage costs are relatively small and demand is not too convex, contractual clauses that enhance the firm's commitment powers should be banned, because they reduce consumer surplus and, possibly, total welfare. Otherwise, antitrust authorities should approve these policies, which tend to benefit consumers and the whole economy. For relatively small consumer storage costs, limited commitment leads to lower prices if and only if the convexity of demand is lower than the inverse of the Lerner index (see Corollary 5 in Section 6). This condition can be empirically identified in a parsimonious manner by resorting to the concept of "demand manifold" proposed by Mrázová and Neary (2017), which relates the curvature and the elasticity of demand. Estimates of the de-

[27] Further details can be found at https://ec.europa.eu/clima/policies/ets/revision_en (last retrieved in December 2019).

mand manifold and the Lerner index are instrumental to potentially fruitful applications of our results.

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Appendix

Proof of Proposition 1. The firm faces the following three pricing options: (I) $p_1 + s_c > p_2$; (II) $p_1 + s_c = p_2$; (III) $p_1 + s_c < p_2$.

Option (I) $p_1 + s_c > p_2$. It follows from (1) that $D_s(p_1) = 0$. The firm's maximization problem is

$$\max_{p_1, p_2} (p_1 - c_1) D(p_1) + (p_2 - c_2) D(p_2).$$

The first-order condition for p_τ, $\tau \in \{1, 2\}$, is given by

$$D(p_\tau) + (p_\tau - c_\tau) D'(p_\tau) = 0, \tag{A1}$$

which yields the equilibrium static monopoly prices

$$p_1^m = c_1 + \mu_1^m \text{ and } p_2^m = c_2 + \mu_2^m, \tag{A2}$$

where $\mu_\tau^m \equiv p_\tau^m - c_\tau$, $\tau \in \{1, 2\}$ (see Section 3). Equilibrium consumer storage is $D_s^m = 0$. This solution is implementable if and only if the feasibility constraint (6) holds.

Option (II) $p_1 + s_c = p_2$. It follows from (1) that $D_s(p_1) \in [0, D(p_1 + s_c)]$. The firm's maximization problem is

$$\max_{p_1} (p_1 - c_1)[D(p_1) + D_s(p_1)] + (p_1 + s_c - c_2)[D(p_1 + s_c) - D_s(p_1)]. \tag{A3}$$

The following two cases emerge:
(IIa) $s_c \leq \Delta c$. As the firm's profits in the maximand of (A3) increase with $D_s(p_1)$, the firm prefers to induce full consumer storage, i.e., $D_s(p_1) = D(p_1 + s_c)$. The firm's maximization

problem reduces to

$$\max_{p_1} \ (p_1 - c_1)\left[D(p_1) + D(p_1 + s_c)\right]. \tag{A4}$$

The first-order condition for p_1 is

$$D(p_1) + D(p_1 + s_c) + (p_1 - c_1)\left[D'(p_1) + D'(p_1 + s_c)\right] = 0. \tag{A5}$$

We obtain the equilibrium full commitment storing prices

$$p_1^{cs} = c_1 - \frac{D\left(p_1^{cs} + s_c\right) + \phi_1\left(p_1^{cs}\right)}{D'\left(p_1^{cs} + s_c\right)} \text{ and } p_2^{cs} = p_1^{cs} + s_c, \tag{A6}$$

where $\phi_\tau(\cdot)$ is defined by (5).

(IIb) $s_c > \Delta c$. As the firm's profits in the maximand of (A3) decrease with $D_s(p_1)$, the firm prefers to deter consumer storage, i.e., $D_s(p_1) = 0$. The firm's maximization problem becomes

$$\max_{p_1} \ (p_1 - c_1) D(p_1) + (p_1 + s_c - c_2) D(p_1 + s_c).$$

The first-order condition for p_1 is

$$D(p_1) + D(p_1 + s_c) + (p_1 - c_1) D'(p_1) + (p_1 + s_c - c_2) D'(p_1 + s_c) = 0. \tag{A7}$$

Using (5), we obtain the equilibrium full commitment no-storing prices

$$p_1^{cn} = c_1 - \frac{D\left(p_1^{cn}\right) + \phi_2\left(p_1^{cn} + s_c\right)}{D'\left(p_1^{cn}\right)} \text{ and } p_2^{cn} = p_1^{cn} + s_c. \tag{A8}$$

Option (III) $p_1 + s_c < p_2$. It follows from (1) that $D_s(p_1) = D(p_1 + s_c)$. This yields the same profits (and effective prices paid by consumers) as in (IIa), and therefore is irrelevant.

We obtain the following results.

(i) Suppose $s_c < \min\{\Delta c + \Delta\mu^m, \Delta c\}$. The only relevant option is (IIa), and the equilibrium prices are described by (A6).

(iia) Suppose $\Delta c + \Delta\mu^m \leq s_c \leq \Delta c$. This interval is non-empty if and only if $\Delta\mu^m \leq 0$. The relevant options are (I) and (IIa), whose associated profits are Π^m and Π^{cs}. It follows from the feasibility constraint (6) that at the lower bound $s_c = \Delta c + \Delta\mu^m$ it holds $p_2^m = p_1^m + s_c$. Substituting p_1^m and $p_2^m = p_1^m + s_c$ into the maximand of (A3) yields $\Pi^{cs}(p_1^m) = \Pi^m(p_1^m)$, where the equality follows because $D_s(p_1^m) = 0$. Then, the profit outcome in (I) can be replicated by (IIa). As $\Pi^{cs}(\cdot)$ is maximized at p_1^{cs}, which differs from p_1^m, we find that $\Pi^{cs} > \Pi^m$ at the lower bound $s_c = \Delta c + \Delta\mu^m$. Now, consider the upper bound $s_c = \Delta c$. Note that $D_s(\cdot)$ disappears in the maximand of (A3). Then, it holds $\Pi^m > \Pi^{cs}$, because Π^m is the solution to an unconstrained maximization problem. Taking the derivative of Π^{cs} with respect to s_c and using (A5) yields $\frac{\partial\Pi^{cs}}{\partial s_c} = (p_1^{cs} - c_1) D'(p_1^{cs} + s_c) < 0$. As Π^m is independent of s_c, there exists a unique threshold $\bar{s}_c^c \in (\Delta c + \Delta\mu^m, \Delta c)$ such that for $s_c < \bar{s}_c^c$ it holds $\Pi^{cs} > \Pi^m$ and the equilibrium prices are described by (A6), while for $s_c \geq \bar{s}_c^c$ it holds $\Pi^m \geq \Pi^{cs}$ and the equilibrium prices are described by (A2), where $\Pi^{cs} = \Pi^m$ if and only if $s_c = \bar{s}_c^c$.

(iib) Suppose, alternatively, $\Delta c \leq s_c < \Delta c + \Delta\mu^m$. This interval is non-empty if and only if $\Delta\mu^m > 0$. The only relevant option is (IIb), and the equilibrium prices are described by (A8).[28]

(iii) Suppose $s_c \geq \max\{\Delta c + \Delta\mu^m, \Delta c\}$. The relevant options are (I) and (IIb), whose associated profits are Π^m and Π^{cn}. Given that consumer storage is absent under both options and Π^m is the solution to an unconstrained maximization problem, it holds $\Pi^m > \Pi^{cn}$ and the equilibrium prices are described by (A2). ■

Proof of Corollary 1. As $\Delta\mu^m \leq 0$, the equilibrium consumer storage and prices in points (i) and (ii) of the corollary are a direct consequence of the outcomes in points (i), (iia) and (iii) of Proposition 1. Taking the derivative of the left-hand side of the first-order condition for p_1^{cs} in (A5) with respect to s_c yields $D'(p_1^{cs} + s_c) + (p_1^{cs} - c_1) D''(p_1^{cs} + s_c) < 0$, where the inequality holds if and only if $D''(p_1^{cs} + s_c) < \widehat{D}'' \equiv -\frac{D'(p_1^{cs}+s_c)}{p_1^{cs}-c_1}$, with $\widehat{D}'' > 0$. It follows from the implicit function theorem that $\frac{\partial p_1^{cs}}{\partial s_c} < 0$ for $D''(\cdot) < \widehat{D}''$. Moreover, the derivative of the left-hand side of the first-order condition for p_2^{cs} — obtained by replacing p_1 with $p_2 - s_c$ in (A5) — with respect to s_c is $-2D'(p_2^{cs} - s_c) - (p_2^{cs} - s_c - c_1) D''(p_2^{cs} - s_c) - D'(p_2^{cs}) > 0$, where the inequality follows from Assumption 1 and $D'(\cdot) < 0$. By the implicit function theorem, we obtain $\frac{\partial p_2^{cs}}{\partial s_c} > 0$. Now, we turn to the price comparisons. Substituting the first-order condition for p_1^m in (A1) into the left-hand side of the first-order condition for p_1^{cs} in (A5) yields $D(p_1^m + s_c) + (p_1^m - c_1) D'(p_1^m + s_c)$. This expression vanishes if and only if $s_c = 0$, which implies $p_1^m = p_1^{cs}$. As $\frac{\partial p_1^{cs}}{\partial s_c} < 0$ for $D''(\cdot) < \widehat{D}''$ but p_1^m is independent of s_c, for $s_c > 0$ we obtain $p_1^m > p_1^{cs}$ when $D''(\cdot) < \widehat{D}''$. ■

Proof of Corollary 2. As $\Delta\mu^m > 0$, the equilibrium consumer storage and prices in points (i), (ii) and (iii) of the corollary are a direct consequence of the outcomes in points (i), (iib) and (iii) of Proposition 1. It follows from the proof of Corollary 1 that $\frac{\partial p_1^{cs}}{\partial s_c} > 0$ for $D''(\cdot) > \widehat{D}''$ and that $\frac{\partial p_2^{cs}}{\partial s_c} > 0$. Taking the derivative of the left-hand side of the first-order condition for p_1^{cn} in (A7) with respect to s_c yields $2D'(p_1^{cn} + s_c) + (p_1^{cn} + s_c - c_2) D''(p_2^{cn} + s_c) < 0$, where the inequality follows from Assumption 1. Using the implicit function theorem, we obtain $\frac{\partial p_1^{cn}}{\partial s_c} < 0$. Moreover, the derivative of the left-hand side of the first-order condition for p_2^{cn} — obtained by replacing p_1 with $p_2 - s_c$ in (A7) — with respect to s_c is $-2D'(p_2^{cn} - s_c) - (p_2^{cn} - s_c - c_1) D''(p_2^{cn} - s_c) > 0$, where the inequality follows from Assumption 1. We find from the implicit function theorem that $\frac{\partial p_2^{cn}}{\partial s_c} > 0$. Now, we turn to the price comparisons. Recall from the proof of Corollary 1 that $p_1^{cs} = p_1^m$ if and only if $s_c = 0$. For $s_c > 0$, we have $p_1^{cs} > p_1^m$ when $D''(\cdot) > \widehat{D}''$, because $\frac{\partial p_1^{cs}}{\partial s_c} > 0$ but p_1^m is independent of s_c. Moreover, given $\frac{\partial p_1^{cn}}{\partial s_c} < 0$ and the continuity of the price strategy, we obtain $p_1^{cn} > p_1^m$. Finally, it follows from $\frac{\partial p_2^{cs}}{\partial s_c} > 0$, $\frac{\partial p_2^{cn}}{\partial s_c} > 0$, $\frac{\partial p_2^m}{\partial s_c} = 0$ and the continuity of the price strategy that $p_2^m > p_2^{cn} > p_2^{cs}$. ■

Proof of Proposition 2. Proceeding backwards, we find from (8) that the second period price is given by

$$p_2 = c_2 - \frac{D(p_2) - D_s(p_1)}{D'(p_2)}. \tag{A9}$$

Moving to the first period, the firm faces the following three pricing options: (I) $p_1 + s_c > p_2$;

[28]Clearly, for $s_c = \Delta c$ cases (IIa) and (IIb) coincide.

(II) $p_1 + s_c = p_2$; (III) $p_1 + s_c < p_2$.

Option (I) $p_1 + s_c > p_2$. It follows from (1) that $D_s(p_1) = 0$. As under full commitment, the equilibrium prices are set at the static monopoly level in (A2). This solution is implementable if and only if the feasibility constraint (6) is fulfilled.

Option (II) $p_1 + s_c = p_2$. It follows from (1) that $D_s(p_1) \in [0, D(p_1 + s_c)]$. The firm's first period maximization problem is given by (7), subject to (A9). We find from (A9) and $p_1 + s_c = p_2$ that $D_s(p_1) = \max\{0, D(p_1 + s_c) + (p_1 + s_c - c_1 - \Delta c) D'(p_1 + s_c)\}$. The following two cases emerge.

(IIa) Let $D_s(p_1) > 0$. The firm's first period maximization problem becomes

$$\max_{p_1} \; (p_1 - c_1)[D(p_1) + D(p_1 + s_c)] + (\Delta c - s_c)(p_1 + s_c - c_1 - \Delta c) D'(p_1 + s_c). \quad \text{(A10)}$$

The first-order condition for p_1 is

$$\begin{aligned} & D(p_1) + D(p_1 + s_c) + (p_1 - c_1)\left[D'(p_1) + D'(p_1 + s_c)\right] \\ & + (\Delta c - s_c)\left[D'(p_1 + s_c) + (p_1 + s_c - c_1 - \Delta c) D''(p_1 + s_c)\right] = 0. \end{aligned} \quad \text{(A11)}$$

Using (5), we obtain the equilibrium limited commitment storing prices

$$p_1^{*s} = c_1 - \frac{D(p_1^{*s}) + \phi_2(p_1^{*s} + s_c) + (\Delta c - s_c)\phi_2'(p_1^{*s} + s_c)}{D'(p_1^{*s})} \text{ and } p_2^{*s} = p_1^{*s} + s_c. \quad \text{(A12)}$$

Consumer storage is $D_s^{*s} = D(p_1^{*s} + s_c) + (p_1^{*s} + s_c - c_1 - \Delta c) D'(p_1^{*s} + s_c) = \phi_2(p_1^{*s} + s_c)$. To derive the condition under which this solution is feasible, i.e., $D_s^{*s} > 0$, we compute the derivative of D_s^{*s} with respect to s_c. This yields

$$\frac{\partial D_s^{*s}}{\partial s_c} = \frac{\partial p_2^{*s}}{\partial s_c}\left[2D'(p_2^{*s}) + (p_2^{*s} - c_1 - \Delta c) D''(p_2^{*s})\right] < 0,$$

where the inequality follows from $\frac{\partial p_2^{*s}}{\partial s_c} > 0$ (see Corollaries 3 and 4) and the negative sign of the expression in square brackets (by Assumption 1). Now, we prove that $D_s^{*s} > 0$ at $s_c = 0$. Note that this is the case if and only if $D(p_1^{*s}) + (p_1^{*s} - c_1 - \Delta c) D'(p_1^{*s}) > 0$. This means from the first-order condition for p_2^m in (A1) that $p_1^{*s} = p_2^{*s} < p_2^m$. Substituting (A1) into the left-hand side of the first-order condition for p_1^{*s} in (A11) evaluated at $s_c = 0$ yields after some manipulation $\Delta c\,[2D'(p_2^m) + (p_2^m - c_2) D''(p_2^m)] + \Delta c D'(p_2^m) < 0$, where the inequality holds because the expression in square brackets is negative (by Assumption 1) and $D'(\cdot) < 0$. This implies that $p_1^{*s} = p_2^{*s} < p_2^m$ and therefore $D_s^{*s} > 0$ at $s_c = 0$. By continuity, we have $D_s^{*s} > 0$ for s_c small enough. As $D_s^{*s} < 0$ for s_c arbitrarily large, we can conclude that there exists a unique threshold $\widetilde{s}_c^* > 0$ such that $D_s^{*s} > 0$ if and only if $s_c < \widetilde{s}_c^*$.

(IIb) For $s_c \geq \widetilde{s}_c^*$, consumers do not store, i.e., $D_s^{*n} = 0$. We find from (A9) that the equilibrium limited commitment no-storing prices are

$$p_1^{*n} = p_2^{*n} - s_c \text{ and } p_2^{*n} = p_2^m = c_2 + \mu_2^m. \quad \text{(A13)}$$

As the firm's maximization problem in (A10) allows for any $D_s(\cdot)$, option (IIa) dominates option (IIb) as long as it is feasible, i.e., $D_s^{*s} > 0$.

Option (III) $p_1 + s_c < p_2$. It follows from (1) that $D_s(p_1) = D(p_1 + s_c)$. As discussed in Section 5, this option is not implementable, because the firm has an incentive to reduce p_2 below $p_1 + s_c$ and serve the market in the second period.

We obtain the following results.

(i) Suppose $s_c < \min\{\Delta c + \Delta\mu^m, \tilde{s}_c^*\}$. The only relevant option is (IIa), and the equilibrium prices are described by (A12).

(iia) Suppose $\Delta c + \Delta\mu^m \leq s_c \leq \tilde{s}_c^*$. The relevant options are (I) and (IIa).[29] We know from the feasibility constraint (6) that at the lower bound $s_c = \Delta c + \Delta\mu^m$ it holds $p_2^m = p_1^m + s_c$. Substituting p_1^m and $p_2^m = p_1^m + s_c$ into the maximand of (7) yields $\Pi^{*s}(p_1^m) = \Pi^m(p_1^m)$, where the equality follows because $D_s(p_1^m) = 0$. Hence, the profit outcome in (I) can be replicated by (IIa). As $\Pi^{*s}(\cdot)$ is maximized at p_1^{*s}, which differs from p_1^m, we find that $\Pi^{*s} > \Pi^m$ at the lower bound $s_c = \Delta c + \Delta\mu^m$. At the upper bound $s_c = \tilde{s}_c^*$, there is no storing in (IIa). This implies that $\Pi^m > \Pi^{*s}$, because Π^m is the solution to an unconstrained maximization problem. Taking the derivative of Π^{*s} with respect to s_c and using (A11) yields after some manipulation $\frac{\partial\Pi^{*s}}{\partial s_c} = (\Delta c - s_c)\left[2D'(p_1^{*s} + s_c) + (p_1^{*s} + s_c - c_2) D''(p_1^{*s} + s_c)\right]$, where the expression in square brackets is negative (by Assumption 1). In principle, the following three cases emerge: (1) if $\tilde{s}_c^* < \Delta c$, then $\frac{\partial\Pi^{*s}}{\partial s_c} < 0$; (2) if $\Delta c + \Delta\mu^m \leq \Delta c \leq \tilde{s}_c^*$, then $\frac{\partial\Pi^{*s}}{\partial s_c} < 0$ for $s_c < \Delta c$ and $\frac{\partial\Pi^{*s}}{\partial s_c} > 0$ for $s_c > \Delta c$ (with a minimum at $s_c = \Delta c$); (3) if $\Delta c < \Delta c + \Delta\mu^m$, i.e., $\Delta\mu^m > 0$, then $\frac{\partial\Pi^{*s}}{\partial s_c} > 0$. Note that case (3) is impossible, because it contradicts the previous result that $\Pi^{*s} > \Pi^m$ at the lower bound $s_c = \Delta c + \Delta\mu^m$ and $\Pi^m > \Pi^{*s}$ at the upper bound $s_c = \tilde{s}_c^*$, where Π^m is independent of s_c. First, consider case (1). As $\frac{\partial\Pi^{*s}}{\partial s_c} < 0$, there exists a unique point of equalization between Π^{*s} and Π^m. Now, consider case (2). Given that the impossibility of case (3) implies $\Delta\mu^m \leq 0$, we know from point (ii) of Corollary 1 that for $s_c \geq \bar{s}_c^c$ the static monopoly solution is implemented under full commitment. Being sequentially optimal, the static monopoly solution must be implemented under limited commitment as well. As $\Pi^{*s} > \Pi^m$ at the lower bound $s_c = \Delta c + \Delta\mu^m$ and $\Pi^m > \Pi^{*s}$ for $s_c \geq \bar{s}_c^c$, where $\bar{s}_c^c < \Delta c$, the point of equalization between Π^{*s} and Π^m must lie in the region where $s_c < \Delta c$, namely, in the declining part of Π^{*s}. This implies that the point of equalization is again unique. Summarizing, either in case (1) or case (2), we find that $\frac{\partial\Pi^{*s}}{\partial s_c} < 0$ in the relevant range for s_c. Hence, there exists a unique threshold $\bar{s}_c^* \in (\Delta c + \Delta\mu^m, \tilde{s}_c^*)$ such that for $s_c < \bar{s}_c^*$ it holds $\Pi^{*s} > \Pi^m$ and the equilibrium prices are described by (A12), while for $s_c \geq \bar{s}_c^*$ it holds $\Pi^m \geq \Pi^{*s}$ and the equilibrium prices are described by (A2), where $\Pi^m = \Pi^{*s}$ if and only if $s_c = \bar{s}_c^*$.

(iib) Suppose, alternatively, $\tilde{s}_c^* \leq s_c < \Delta c + \Delta\mu^m$. The only relevant option is (IIb), and the equilibrium prices are described by (A13).

(iii) Suppose $s_c \geq \max\{\Delta c + \Delta\mu^m, \tilde{s}_c^*\}$. The relevant options are (I) and (IIb). As consumer storage is absent under both options and Π^m is the solution to an unconstrained maximization problem, it holds $\Pi^m > \Pi^{*n}$ and the equilibrium prices are described by (A2). ■

Proof of Corollary 3. As $\Delta c + \Delta\mu^m \leq \tilde{s}_c^*$, the equilibrium consumer storage and prices in points (i) and (ii) of the corollary are a direct consequence of the outcomes in points (i), (iia) and (iii) of Proposition 2. The derivative of the left-hand side of the first-order condition for

[29]Clearly, for $s_c = \tilde{s}_c^*$ cases (IIa) and (IIb) coincide.

p_1^{*s} in (A11) with respect to s_c vanishes for $D''(\cdot) = 0$. Using the implicit function theorem, this implies that $\frac{\partial p_1^{*s}}{\partial s_c} = 0$. Furthermore, taking the derivative of the left-hand side of the first-order condition for p_2^{*s} — obtained by replacing p_1 with $p_2 - s_c$ in (A11) — with respect to s_c yields after some manipulation $-\left[2D'\left(p_2^{*s} - s_c\right) + \left(p_2^{*s} - s_c - c_1\right) D''\left(p_2^{*s} - s_c\right)\right] - \left[2D'\left(p_2^{*s}\right) + \left(p_2^{*s} - c_2\right) D''\left(p_2^{*s}\right)\right] > 0$, where the inequality holds because each expression in square brackets is negative (by Assumption 1). We find from the implicit function theorem that $\frac{\partial p_2^{*s}}{\partial s_c} > 0$. Now, we now turn to the price comparisons. Substituting the first-order condition for p_1^m in (A1) into the left-hand side of the first-order condition for p_1^{*s} in (A11) with $D'' = 0$ yields $D\left(p_1^m + s_c\right) + \left(p_1^m - c_1\right) D' + \left(\Delta c - s_c\right) D'$. As for $s_c = 0$ this expression reduces to $\Delta c D' < 0$ and both p_1^{*s} (with $D'' = 0$) and p_1^m do not depend on s_c, we obtain from Assumption 1 that $p_1^m > p_1^{*s}$ for $D'' = 0$. It follows from $D_s^{*s} = D\left(p_2^{*s}\right) + \left(p_2^{*s} - c_2\right) D'\left(p_2^{*s}\right) > 0$ that $p_2^m > p_2^{*s}$. ■

Proof of Corollary 4. As $\Delta c + \Delta\mu^m > \tilde{s}_c^*$, the equilibrium consumer storage and prices in points (i), (ii) and (iii) of the corollary are a direct consequence of the outcomes in points (i), (iib) and (iii) of Proposition 2. It is straightforward to see that $\frac{\partial p_1^{*n}}{\partial s_c} < 0$. Moreover, we know from the proof of Corollary 3 that $\frac{\partial p_2^{*s}}{\partial s_c} > 0$. Now, we turn to the price comparisons. Given $\frac{\partial p_1^{*n}}{\partial s_c} < 0$ and the continuity of the price strategy, we obtain $p_1^{*n} > p_1^m$. It follows from $\frac{\partial p_2^{*s}}{\partial s_c} > 0$, $\frac{\partial p_2^m}{\partial s_c} = 0$ and the continuity of the price strategy that $p_2^m > p_2^{*s}$. ■

Proof of Proposition 3. The proof proceeds through the following four cases: (I) $\Delta\mu^m \leq 0$ and $\Delta c + \Delta\mu^m \leq \tilde{s}_c^*$; (II) $\Delta\mu^m \leq 0$ and $\Delta c + \Delta\mu^m > \tilde{s}_c^*$; (III) $\Delta\mu^m > 0$ and $\Delta c + \Delta\mu^m > \tilde{s}_c^*$; (IV) $\Delta\mu^m > 0$ and $\Delta c + \Delta\mu^m \leq \tilde{s}_c^*$. Note that the thresholds $\bar{s}_c^c \in (\Delta c + \Delta\mu^m, \Delta c)$ and $\bar{s}_c^* \in (\Delta c + \Delta\mu^m, \tilde{s}_c^*)$ defined in Propositions 1 and 2 are such that $\bar{s}_c^* < \bar{s}_c^c$. The rationale is the following. The limited commitment profits are strictly lower than the full commitment profits (as long as they differ), and the profits under the two commitment regimes decrease with s_c in the presence of consumer storage (see the proofs of Propositions 1 and 2). Therefore, the point of equalization between the static monopoly profits and the limited commitment storing profits, which identifies $\bar{s}_c^*$, must be strictly lower than the corresponding point under full commitment, which identifies $\bar{s}_c^c$.

Case (I) $\Delta\mu^m \leq 0$ and $\Delta c + \Delta\mu^m \leq \tilde{s}_c^*$. It follows from Corollaries 1 and 3 that the following three subcases emerge.

(Ia) If $s_c < \bar{s}_c^*$, the full commitment prices are p_1^{cs} and p_2^{cs} and the limited commitment prices are p_1^{*s} and p_2^{*s}. After substituting the first-order condition for p_1^{*s} in (A11) into the left-hand side of the first-order condition for p_1^{cs} in (A5) and using $D_2^N\left(p_1\right) \equiv D\left(p_1 + s_c\right) - D_s\left(p_1\right)$, we find that

$$-\left(\Delta c - s_c\right)\left[D'\left(p_1^{*s} + s_c\right) + \left(p_1^{*s} + s_c - c_2\right) D''\left(p_1^{*s} + s_c\right)\right] = \left(\Delta c - s_c\right)\frac{\partial D_2^N\left(p_1^{*s}\right)}{\partial p_1} > 0,$$

where the inequality holds if and only if $D''\left(p_1^{*s} + s_c\right) < -\frac{D'\left(p_1^{*s}+s_c\right)}{p_1^{*s}+s_c-c_2}$ or, equivalently, $\frac{\partial D_2^N\left(p_1^{*s}\right)}{\partial p_1} > 0$ (recall $s_c < \bar{s}_c^* < \bar{s}_c^c < \Delta c$). As $p_2^{cs} = p_1^{cs} + s_c$ and $p_2^{*s} = p_1^{*s} + s_c$, it follows from Assumption 1 that $p_\tau^{*s} < p_\tau^{cs}$, $\tau \in \{1,2\}$, if and only if $\frac{\partial D_2^N\left(p_1^{*s}\right)}{\partial p_1} > 0$.

(Ib) If $\bar{s}_c^* \leq s_c < \bar{s}_c^c$, the full commitment prices are p_1^{cs} and p_2^{cs} and the limited commitment prices are p_1^m and p_2^m. Substituting the first-order condition for p_1^m in (A1) into the left-hand side of the first-order condition for p_1^{cs} in (A5) yields

$$D\left(p_1^m + s_c\right) + \left(p_1^m - c_1\right) D'\left(p_1^m + s_c\right) < 0,$$

where the inequality holds because $p_1^m + s_c > p_2^m$ (which follows from $s_c \geq \bar{s}_c^* > \Delta c + \Delta\mu^m$ and (6)) implies $D\left(p_1^m + s_c\right) + \left(p_1^m - c_1\right) D'\left(p_1^m + s_c\right) < \left(\Delta c - s_c\right) D'\left(p_1^m + s_c\right) < 0$ (recall $s_c < \bar{s}_c^c < \Delta c$). By Assumption 1, we obtain $p_1^m > p_1^{cs}$. Substituting the first-order condition for p_2^m in (A1) into the left-hand side of the first-order condition for p_2^{cs} — obtained by replacing p_1 with $p_2 - s_c$ in (A5) — yields

$$D\left(p_2^m - s_c\right) + \left(p_2^m - s_c - c_1\right) D'\left(p_2^m - s_c\right) + \left(\Delta c - s_c\right) D'\left(p_2^m\right),$$

whose sign is ambiguous. Computing this expression at $s_c = \Delta c + \Delta\mu^m < \bar{s}_c^*$ and at $s_c = \Delta c > \bar{s}_c^c$ we find that $-\Delta\mu^m D'\left(p_2^m\right) \leq 0$ (where the inequality holds for $\Delta\mu^m < 0$) and $D\left(p_2^m - \Delta c\right) + \left(p_2^m - \Delta c - c_1\right) D'\left(p_2^m - \Delta c\right) > 0$ (as $p_1^m + \Delta c > p_2^m$), respectively. As $\frac{\partial p_2^{cs}}{\partial s_c} > 0$ (see Corollaries 1 and 2) but $\frac{\partial p_2^m}{\partial s_c} = 0$, there exists a unique threshold for s_c that lies between $\Delta c + \Delta\mu^m$ and Δc such that it holds $p_2^m > p_2^{cs}$ if and only if s_c is below this threshold. However, it cannot be generally established whether the threshold lies within the relevant interval for s_c.

(Ic) If $s_c \geq \bar{s}_c^c$, the full commitment prices coincide with the limited commitment prices and correspond to the static monopoly prices p_1^m and p_2^m.

Case (II) $\Delta\mu^m \leq 0$ and $\Delta c + \Delta\mu^m > \widetilde{s}_c^*$. It follows from Corollaries 1 and 4 that the following four subcases emerge.

(IIa) If $s_c < \widetilde{s}_c^*$, case (Ia) applies.

(IIb) If $\widetilde{s}_c^* \leq s_c < \Delta c + \Delta\mu^m$, the full commitment prices are p_1^{cs} and p_2^{cs} and the limited commitment prices are p_1^{*n} and p_2^{*n}. Substituting the first-order condition for p_2^{*n} in (A1) into the left-hand side of the first-order condition for p_2^{cs} — obtained by replacing p_1 with $p_2 - s_c$ in (A5) — yields

$$D\left(p_2^m - s_c\right) + \left(p_2^m - s_c - c_1\right) D'\left(p_2^m - s_c\right) + \left(\Delta c - s_c\right) D'\left(p_2^m\right) < 0,$$

where the inequality follows from $p_1^m + s_c < p_2^m$ (recall $s_c < \Delta c + \Delta\mu^m$ and (6)) and from $D'(\cdot) < 0$ (recall $s_c < \Delta c$). As $p_1^{cs} = p_2^{cs} - s_c$ and $p_1^{*n} = p_2^{*n} - s_c$, we find from Assumption 1 that $p_\tau^{*n} > p_\tau^{cs}$, $\tau \in \{1, 2\}$.

(IIc) If $\Delta c + \Delta\mu^m \leq s_c < \bar{s}_c^c$, case (Ib) applies.

(IId) If $s_c \geq \bar{s}_c^c$, case (Ic) applies.

Case (III) $\Delta\mu^m > 0$ and $\Delta c + \Delta\mu^m > \widetilde{s}_c^*$. It follows from Corollaries 2 and 4 that the following five subcases emerge.

(IIIa) If $s_c < \min\{\Delta c, \widetilde{s}_c^*\}$, case (Ia) applies.

(IIIb) If $\Delta c \leq s_c < \widetilde{s}_c^*$, the full commitment prices are p_1^{cn} and p_2^{cn} and the limited commitment prices are p_1^{*s} and p_2^{*s}. Substituting the first-order condition for p_1^{*s} in (A11) into the left-hand side of the first-order condition for p_1^{cn} in (A7) yields

$$-\left(\Delta c - s_c\right)\left[2D'\left(p_1^{*s} + s_c\right) + \left(p_1^{*s} + s_c - c_2\right) D''\left(p_1^{*s} + s_c\right)\right] \leq 0,$$

where the expression in square brackets is negative by Assumption 1 and the equality holds if and only if $s_c = \Delta c$. As $p_2^{cn} = p_1^{cn} + s_c$ and $p_2^{*s} = p_1^{*s} + s_c$, it follows from Assumption 1 that $p_\tau^{*s} \geq p_\tau^{cn}$, $\tau \in \{1, 2\}$, where the equality holds if and only if $s_c = \Delta c$.
(IIIc) If, alternatively, $\widetilde{s}_c^* \leq s_c < \Delta c$, case (IIb) applies.
(IIId) If $\max\{\Delta c, \widetilde{s}_c^*\} \leq s_c < \Delta c + \Delta\mu^m$, the full commitment prices are p_1^{cn} and p_2^{cn} and the limited commitment prices are p_1^{*n} and p_2^{*n}. Substituting the first-order condition for p_2^{*n} in (A1) into the left-hand side of the first-order condition for p_2^{cn} — obtained by replacing p_1 with $p_2 - s_c$ in (A7) — yields

$$D\left(p_2^m - s_c\right) + \left(p_2^m - s_c - c_1\right) D'\left(p_2^m - s_c\right) < 0,$$

where the inequality follows from $p_1^m + s_c < p_2^m$ (recall $s_c < \Delta c + \Delta\mu^m$ and (6)). As $p_1^{cn} = p_2^{cn} - s_c$ and $p_1^{*n} = p_2^{*n} - s_c$, it follows from Assumption 1 that $p_\tau^{*n} > p_\tau^{cn}$, $\tau \in \{1, 2\}$.
(IIIe) If $s_c \geq \Delta c + \Delta\mu^m$, case (Ic) applies.

Case (IV) $\Delta\mu^m > 0$ and $\Delta c + \Delta\mu^m \leq \widetilde{s}_c^*$. We show by contradiction that this case is impossible. It follows from Corollaries 2 and 3 that for $s_c \in (\Delta c + \Delta\mu^m, \overline{s}_c^*)$ the full commitment prices are p_1^m and p_2^m and the limited commitment prices are p_1^{*s} and p_2^{*s}. As the static monopoly solution is sequentially optimal, this solution should also be implemented under limited commitment. Hence, case (IV) is impossible.[30]

After defining

$$s_c^l \equiv \begin{cases} \overline{s}_c^* & \text{if} \quad \text{case (I) applies} \\ \widetilde{s}_c^* & \text{if} \quad \text{case (II) applies} \\ \min\{\Delta c, \widetilde{s}_c^*\} & \text{if} \quad \text{case (III) applies} \end{cases}, \tag{A14}$$

we find from cases (Ia), (IIa) and (IIIa) that for $s_c < s_c^l$ in each period the price under limited commitment is lower than the price under full commitment, i.e., $p_\tau^{*s} < p_\tau^{cs}$, $\tau \in \{1, 2\}$, if and only if $\frac{\partial D_2^N\left(p_1^{*s}\right)}{\partial p_1} > 0$. ■

Proof of Proposition 4. Using the results in the proof of Proposition 3, we define s_c^h as

$$s_c^h \equiv \begin{cases} \overline{s}_c^c & \text{if} \quad \text{case (I) or (II) applies} \\ \Delta c + \Delta\mu^m & \text{if} \quad \text{case (III) applies} \end{cases}. \tag{A15}$$

A. Suppose $s_c^l \leq s_c < s_c^h$. It follows from cases (Ib), (IIb), (IIc), (IIIb), (IIIc) and (IIId) in the proof of Proposition 3 that in the first period the price under limited commitment is higher than the price under full commitment. If $\Delta\mu^m > 0$, it follows from cases (IIIb), (IIIc) and (IIId) in the proof of Proposition 3 that the price under limited commitment is also higher in the second period.

B. Suppose $s_c \geq s_c^h$. It follows from cases (Ic), (IId) and (IIIe) in the proof of Proposition 3 that the prices under full and limited commitment coincide with the static monopoly prices in each period. ■

Proof of Remark 1. To see the result in point (i) of the corollary, note from (A5) that p_1^{cs} does

[30] Alternatively, it follows from point (iia) of the proof of Proposition 2 that $\Delta\mu^m > 0$ is impossible when $\Delta c + \Delta\mu^m \leq \widetilde{s}_c^*$.

not depend on Δc (for a given c_1). As $p_2^{cs} = p_1^{cs} + s_c$, it holds $\frac{\partial p_\tau^{cs}}{\partial \Delta c} = 0$, $\tau \in \{1,2\}$. To see the result in point (ii), note from the implicit function theorem that the sign of $\frac{\partial p_1^{*s}}{\partial \Delta c}$ is equal to the sign of the derivative of the left-hand side of the first-order condition for p_1^{*s} in (A11) with respect to Δc, which is given by

$$D'\left(p_1^{*s} + s_c\right) + \left(p_1^{*s} + 2s_c - c_1 - 2\Delta c\right) D''\left(p_1^{*s} + s_c\right).$$

To establish the sign of this expression, consider the second period net demand $D_2^N\left(p_1\right) \equiv D\left(p_1 + s_c\right) - D_s\left(p_1\right) = -\left(p_1 + s_c - c_1 - \Delta c\right) D'\left(p_1 + s_c\right)$. Then,

$$\frac{\partial}{\partial \Delta c}\left[\left(\Delta c - s_c\right) \frac{\partial D_2^N\left(p_1^{*s}\right)}{\partial p_1}\right] = -D'\left(p_1^{*s} + s_c\right) - \left(p_1^{*s} + 2s_c - c_1 - 2\Delta c\right) D''\left(p_1^{*s} + s_c\right).$$

As $p_2^{*s} = p_1^{*s} + s_c$, we find $\frac{\partial p_\tau^{*s}}{\partial \Delta c} < 0$, $\tau \in \{1,2\}$, if and only if $\frac{\partial}{\partial \Delta c}\left[\left(\Delta c - s_c\right) \frac{\partial D_2^N\left(p_1^{*s}\right)}{\partial p_1}\right] > 0$. ■

Proof of Proposition 5. Consumer surplus under full and limited commitment is respectively

$$\Psi^{cs} = \int_{p_1^{cs}} D\left(p\right) dp + \int_{p_1^{cs}+s_c} D\left(p\right) dp \;\text{ and }\; \Psi^{*s} = \int_{p_1^{*s}} D\left(p\right) dp + \int_{p_1^{*s}+s_c} D\left(p\right) dp.$$

Taking the difference between Ψ^{*s} and Ψ^{cs} yields

$$\Delta\Psi \equiv \Psi^{*s} - \Psi^{cs} = \int_{p_1^{*s}}^{p_1^{cs}} D\left(p\right) dp + \int_{p_1^{*s}+s_c}^{p_1^{cs}+s_c} D\left(p\right) dp. \tag{A16}$$

We know from Proposition 3 that $p_\tau^{*s} < p_\tau^{cs}$, $\tau \in \{1,2\}$, if and only if $\frac{\partial D_2^N\left(p_1^{*s}\right)}{\partial p_1} > 0$. This implies $\Delta\Psi > 0$ if and only if $\frac{\partial D_2^N\left(p_1^{*s}\right)}{\partial p_1} > 0$.

We now turn to total welfare $W \equiv \Psi + \Pi$. The firm's full commitment profits Π^{cs} are given by the maximand of (A4). The firm's limited commitment profits Π^{*s} are given by the maximand of (A10) and can be rewritten as

$$\begin{aligned}\Pi^{*s} = {} & \left(p_1^{*s} - c_1\right)\left[D\left(p_1^{cs}\right) + D\left(p_1^{cs} + s_c\right)\right] + \left(p_1^{*s} - c_1\right)\left[D\left(p_1^{*s}\right) - D\left(p_1^{cs}\right)\right. \\ & \left. + D\left(p_1^{*s} + s_c\right) - D\left(p_1^{cs} + s_c\right)\right] + \left(\Delta c - s_c\right)\left(p_1^{*s} + s_c - c_1 - \Delta c\right) D'\left(p_1^{*s} + s_c\right).\end{aligned}$$

Taking the difference between Π^{*s} and Π^{cs}, we obtain

$$\begin{aligned}\Delta\Pi \equiv \Pi^{*s} - \Pi^{cs} = {} & -\left(p_1^{cs} - p_1^{*s}\right)\left[D\left(p_1^{cs}\right) + D\left(p_1^{cs} + s_c\right)\right] + \left(p_1^{*s} - c_1\right)\left[D\left(p_1^{*s}\right) - D\left(p_1^{cs}\right)\right. \\ & \left. + D\left(p_1^{*s} + s_c\right) - D\left(p_1^{cs} + s_c\right)\right] + \left(\Delta c - s_c\right)\left(p_1^{*s} + s_c - c_1 - \Delta c\right) D'\left(p_1^{*s} + s_c\right).\end{aligned} \tag{A17}$$

Summing (A16) and (A17) yields

$$\begin{aligned}\Delta W \equiv \Delta\Psi + \Delta\Pi = {} & \int_{p_1^{*s}}^{p_1^{cs}} D\left(p\right) dp + \int_{p_1^{*s}+s_c}^{p_1^{cs}+s_c} D\left(p\right) dp - \left(p_1^{cs} - p_1^{*s}\right)\left[D\left(p_1^{cs}\right) + D\left(p_1^{cs} + s_c\right)\right] \\ & + \left(p_1^{*s} - c_1\right)\left[D\left(p_1^{*s}\right) - D\left(p_1^{cs}\right) + D\left(p_1^{*s} + s_c\right) - D\left(p_1^{cs} + s_c\right)\right] \\ & + \left(\Delta c - s_c\right)\left(p_1^{*s} + s_c - c_1 - \Delta c\right) D'\left(p_1^{*s} + s_c\right).\end{aligned} \tag{A18}$$

Suppose $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} > 0$, which implies from Proposition 3 that $p_\tau^{*s} < p_\tau^{cs}$, $\tau \in \{1,2\}$. The aggregate expression in the first line of (A18) is positive. To see this, note that this expression can be rewritten as

$$\int_{p_1^{*s}}^{p_1^{cs}} D(p)\, dp - \int_{p_1^{*s}}^{p_1^{cs}} D(p_1^{cs})\, dp + \int_{p_1^{*s}+s_c}^{p_1^{cs}+s_c} D(p)\, dp - \int_{p_1^{*s}+s_c}^{p_1^{cs}+s_c} D(p_1^{cs}+s_c)\, dp > 0,$$

where the inequality follows from $D'(\cdot) < 0$. The expression in the second line of (A18) is positive as well. The expression in the third line of (A18) corresponds to $-(\Delta c - s_c)\, D_2^N(p_1^{*s})$, where $D_2^N(p_1) \equiv D(p_1 + s_c) - D_s(p_1)$. As $s_c < s_c^l$ (by supposition in the proposition) and $s_c^l \leq \Delta c$ (see the proof of Proposition 3), we find $\Delta W > 0$ if $D_2^N(\cdot)$ is small enough. Now, suppose $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} \leq 0$. This implies $\Delta\Psi \leq 0$, where the equality holds if and only if $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} = 0$. Given that $\Delta\Pi < 0$ (the limited commitment profits are lower than the full commitment profits), we obtain $\Delta W \equiv \Delta\Psi + \Delta\Pi < 0$. ■

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Storable good market with intertemporal cost variations Supplementary Appendix

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1 Introduction

This Supplementary Appendix complements the paper and proceeds as follows. Section 2 extends our analysis to the firm's inventories and convex storage costs. Section 3 investigates uncertainty about production costs. Section 4 considers convex production costs. Section 5 explores the case of durable goods. Section 6 incorporates a more general discount factor into our model. Section 7 provides a full characterization of the results in a framework with linear demand.

2 Firm's inventories and convex storage costs

We denote by $s_c(D_s)$ the storage costs associated with consumer storage D_s, where $s_c'(\cdot) \geq 0$ (with $s_c'(\cdot) > 0$ for $D_s > 0$) and $s_c''(\cdot) > 0$. The firm can also store a quantity I in the form of inventories intended for future sales at a cost $s_f(I)$, where $s_f'(\cdot) \geq 0$ (with $s_f'(\cdot) > 0$ for $I > 0$) and $s_f''(\cdot) > 0$. Consistently with the baseline model, we consider a framework where, under full commitment, the firm does not produce in the second period. Therefore, the aggregate quantity that the firm and consumers store in the first period suffices to cover the entire demand in the second period. Under limited commitment, the firm's inventories and consumer storage may still be sufficiently large so that no production takes place in the second period. Alternatively, the firm prefers to produce in the second period as well. In the following remark, we focus on the case where production only occurs in the first period. Moreover, we consider the plausible situation where consumers store to a lower extent under limited commitment for given prices. This captures the idea that consumers are more reluctant to store in anticipation of the firm's opportunistic behavior. Evaluating consumer storage at the equilibrium full commitment prices, we have $D_s^{*ci} < D_s^{ci}$. As shown in the linear-quadratic example presented in the sequel, this outcome emerges endogenously irrespective of whether the firm produces in the second period as well.

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Remark 3 *Suppose* $0 < D_s^{*ci} < D_s^{ci}$. *Then, in each period the price under limited commitment is lower than the price under full commitment, i.e.,* $p_\tau^{*i} < p_\tau^{ci}$, $\tau \in \{1, 2\}$.

Proof of Remark 3. Following the same rationale as in the baseline model, the storability constraint is binding in equilibrium, i.e., $p_2 = p_1 + s_c'(D_s(\cdot))$. The firm's aggregate profits are

$$\Pi^i = (p_1 - c_1)[D(p_1) + D_s(\cdot)] + [p_1 + s_c'(D_s(\cdot)) - c_1][D(p_1 + s_c'(D_s(\cdot))) - D_s(\cdot)] \\ - s_f(D(p_1 + s_c'(D_s(\cdot))) - D_s(\cdot)). \tag{S1}$$

Under full commitment, differentiating (S1) with respect to p_1 and D_s yields the following first-order conditions

$$\frac{d\Pi^{ci}}{dp_1} = \frac{\partial\Pi^i(p_1)}{\partial p_1} = 0 \tag{S2}$$

$$\frac{d\Pi^{ci}}{dD_s} = \frac{\partial\Pi^i(D_s)}{\partial D_s} = 0.$$

This gives the equilibrium first period price p_1^{ci} and consumer storage D_s^{ci} under full commitment. The corresponding equilibrium second period price follows from the binding storability constraint, i.e., $p_2^{ci} = p_1^{ci} + s_c'(D_s^{ci})$.

Under limited commitment, proceeding backwards, we first consider the firm's problem of maximizing $\Pi_2^i = (p_2 - c_2)[D(p_2) - D_s(p_1)]$ with respect to p_2.[1] Using the binding storability constraint $p_2 = p_1 + s_c'(D_s(p_1))$ yields the following first-order condition

$$D(p_1 + s_c'(D_s(p_1))) - D_s(p_1) + [p_1 + s_c'(D_s(p_1)) - c_2] D'(p_1 + s_c'(D_s(p_1))) = 0. \tag{S3}$$

Applying the implicit function theorem, we find that

$$\frac{\partial D_s}{\partial p_1} = -\frac{\partial^2\Pi_2^i/\partial p_1^2}{\partial^2\Pi_2^i/\partial p_1 \partial D_s} = \frac{\partial^2\Pi_2^i/\partial p_1^2}{1 - (\partial^2\Pi_2^i/\partial p_1^2) s_c''(D_s)} < 0, \tag{S4}$$

where the inequality follows from $\frac{\partial^2\Pi_2^i}{\partial p_1^2} < 0$ (by Assumption 1) and $s_c''(\cdot) > 0$. In the first period, differentiating (S1) with respect to p_1 yields

$$\frac{d\Pi^{*i}}{dp_1} = \frac{\partial\Pi^i(p_1)}{\partial p_1} + \frac{\partial\Pi^i(D_s)}{\partial D_s}\frac{\partial D_s}{\partial p_1} = 0. \tag{S5}$$

This gives the equilibrium first period price p_1^{*i} under limited commitment. Equilibrium consumer storage D_s^{*i} satisfies (S3). The corresponding equilibrium second period price follows from the binding storability constraint, i.e., $p_2^{*i} = p_1^{*i} + s_c'(D_s^{*i})$.

Comparing (S2) and (S5) evaluated at p_1^{ci}, we obtain that

$$\frac{d\Pi^{*i}(p_1^{ci})}{dp_1} - \frac{d\Pi^{ci}}{dp_1} = \frac{\partial\Pi^i(D_s^{*ci})}{\partial D_s}\frac{\partial D_s}{\partial p_1} < 0,$$

[1] Although the firm does not produce in the second period, the incorporation of the second period cost into the firm's maximization problem ensures sequential optimality in the light of the firm's temptation to reduce the price and produce in the second period. This solution generates the highest consumer storage.

where the inequality follows from $\frac{\partial \Pi^i\left(D_s^{*ci}\right)}{\partial D_s} > 0$ (as $D_s^{*ci} < D_s^{ci}$) and $\frac{\partial D_s}{\partial p_1} < 0$ (see (S4)). Hence, it holds $p_1^{*i} < p_1^{ci}$. Using the storability constraint and equation (S4), we find that $\frac{dp_2^{*i}}{dp_1} = 1 + s_c''(D_s)\frac{\partial D_s}{\partial p_1} > 0$. Then, we have $p_2^{*i} = p_1^{*i} + s_c'\left(D_s^{*i}\right) < p_1^{ci} + s_c'\left(D_s^{*ci}\right) < p_1^{ci} + s_c'\left(D_s^{ci}\right) = p_2^{ci}$ (recall $s_c''(\cdot) > 0$ and $D_s^{*ci} < D_s^{ci}$). This implies $p_2^{*i} < p_2^{ci}$. ∎

Linear-quadratic framework We now derive explicit results in a framework characterized by a linear demand function of the form $D(p_\tau) = \alpha - \beta p_\tau$, $\tau \in \{1,2\}$, with $\alpha > 0$ and $\beta > 0$. The unit production cost is c_1 in the first period and c_2 in the second period, where $\Delta c \equiv c_2 - c_1 > 0$ and $c_2 < \frac{\alpha}{\beta}$. Consumer storage costs are $s_c(D_s) = \frac{1}{2}k_c D_s^2$, with $k_c > 0$, and the firm's storage costs are $s_f(I) = \frac{1}{2}k_f I^2$, with $k_f > 0$. Given the binding storability constraint, i.e., $p_2 = p_1 + s_c'(D_s(\cdot))$, the firm's inventories are defined as $I \equiv D\left(p_1 + s_c'(D_s(\cdot))\right) - D_s(\cdot) - \mathbf{1}_{Q_2} \cdot Q_2$, where the indicator function $\mathbf{1}_{Q_2} \in \{0,1\}$ assumes a value of zero if no production takes place in the second period and a value of one otherwise. As previously discussed, we focus on the case where there is no production in the second period under full commitment.[2] The analysis is split into two separate cases according to whether or not production occurs in the second period under limited commitment. In Case A there is no production in the second period and the indicator function takes a value of zero, whereas in Case B production occurs in the second period and the indicator function takes a value of one.

Case A Given that no production takes place in the second period, it follows from the binding storability constraint $p_2 = p_1 + k_c D_s(\cdot)$ that the firm's aggregate profits are

$$\begin{aligned}\Pi^i &= (p_1 - c_1)\left[\alpha - \beta p_1 + D_s(\cdot)\right] + \left[p_1 + k_c D_s(\cdot) - c_1\right]\left[\alpha - \beta\left(p_1 + k_c D_s(\cdot)\right) - D_s(\cdot)\right] \\ &-\frac{1}{2}k_f\left[\alpha - \beta\left(p_1 + k_c D_s(\cdot)\right) - D_s(\cdot)\right]^2. \qquad \text{(S6)}\end{aligned}$$

Under full commitment, differentiating (S6) with respect to p_1 and D_s yields $p_1^{ci} = p_1^m = \frac{\alpha+\beta c_1}{2\beta}$ and $D_s^{ci} = \frac{k_f(\alpha-\beta c_1)}{2\left[k_f + k_c(2+\beta k_f)\right]}$. It follows from the binding storability constraint that $p_2^{ci} = p_1^{ci} + k_c D_s^{ci}$.

Under limited commitment, the firm's second period maximization problem gives $p_2 = \frac{\alpha+\beta c_2 - D_s}{2\beta}$. It follows from the binding storability constraint that $D_s(p_1) = \frac{\alpha+\beta c_2 - 2\beta p_1}{1+2\beta k_c}$. Substituting this expression into (S6) and differentiating with respect to p_1, we find after some manipulation $p_1^{*i} = \frac{2\alpha+\beta\left[2c_1(1+\beta k_c)(1+2\beta k_c)+\beta c_2\left(k_f+k_c\left(2+\beta k_f\right)\right)+k_c\left(8+4\beta k_c-\beta k_f\right)\alpha\right]}{\beta\left[4+\beta\left(8k_c(2+\beta k_c)+k_f\right)\right]}$ and $p_2^{*i} = p_1^{*i} + \frac{\beta k_c\left[k_f(\alpha-\beta c_2)+4\Delta c(1+\beta k_c)\right]}{4+\beta\left[8k_c(2+\beta k_c)+k_f\right]}$. To preserve the essence of the firm's commitment problem, consumer storage under limited commitment must be lower than under full commitment at given prices. Evaluating consumer storage at the equilibrium full commitment prices, we have $D_s^{*ci} < D_s^{ci}$, as in Remark 3. There exists a threshold for c_2, given by $c_2^{\max} \equiv \frac{\alpha(1+2\beta k_c)k_f+\beta c_1\left(4k_c+k_f\right)}{2\beta\left[k_f+k_c(2+\beta k_f)\right]}$, such that $D_s^{*ci} < D_s^{ci}$ if and only if $c_2 < c_2^{\max}$. For $c_2 = c_2^{\max} - \varepsilon$, where $\varepsilon > 0$, we find that

$$p_1^{*i} - p_1^{ci} = -\varepsilon\frac{\beta\left[k_f + k_c\left(2+\beta k_f\right)\right]}{4+\beta\left[8k_c\left(2+\beta k_c\right)+k_f\right]} < 0$$

[2] Our results do not qualitatively change if production takes place in the second period as well.

and

$$p_2^{*i} - p_2^{ci} = -\varepsilon \frac{\beta\left(6k_c + 4\beta k_c^2 + k_f\right)}{4 + \beta\left[8k_c\left(2+\beta k_c\right) + k_f\right]} < 0.$$

Finally, we derive the range of values for ε under which the firm prefers to accumulate inventories in order to cover the entire demand in the second period. This is the case if and only if the first period unit cost of production inflated by the marginal storage cost is (weakly) lower than the second period unit cost of production, i.e.,

$$c_1 + k_f\left[\alpha - \beta\left(p_1^{*i} + k_c D_s\left(p_1^{*i}, c_2^{\max} - \varepsilon\right)\right) - D_s\left(p_1^{*i}, c_2^{\max} - \varepsilon\right)\right] - c_2 \leq 0 \iff \varepsilon \leq \left(\alpha - \beta c_1\right)\omega,$$

where $\omega \equiv \frac{k_f\left[4+\beta\left(8k_c(2+\beta k_c)+k_f\right)\right]}{2\beta\left[4+8\beta k_c(2+\beta k_c)+\beta(5+2\beta k_c(5+2\beta k_c))k_f\right]\left[k_f+k_c\left(2+\beta k_f\right)\right]} > 0$. For $\varepsilon > \left(\alpha - \beta c_1\right)\omega$, Case B applies.

Case B Under full commitment, the solution is the same as in Case A. Under limited commitment, the firm now produces in the second period as well. Given the second period maximization problem and the binding storability constraint $p_2 = p_1 + k_c D_s\left(p_1\right)$, we find that $D_s\left(p_1\right) = \frac{\alpha+\beta c_2 - 2\beta p_1}{1+2\beta k_c}$. This implies that the firm's inventories are given by $I = \alpha - \beta\left(p_1 + k_c D_s\left(p_1\right)\right) - D_s\left(p_1\right) - Q_2 = \beta\frac{p_1 - c_2(1+\beta k_c)+k_c\alpha}{1+2\beta k_c} - Q_2$. The firm produces the quantity Q_τ in period $\tau \in \{1,2\}$ in order to solve the following cost minimization problem

$$\min_{Q_1,Q_2} \frac{1}{2}k_f I^2 + \Delta c Q_2 \;\; s.t. \;\; I + Q_2 = \beta\frac{p_1 - c_2\left(1+\beta k_c\right) + k_c\alpha}{1+2\beta k_c},$$

which reduces to

$$\min_{Q_2} \frac{1}{2}k_f\left[\beta\frac{p_1 - c_2\left(1+\beta k_c\right) + k_c\alpha}{1+2\beta k_c} - Q_2\right]^2 + \Delta c Q_2.$$

The first-order condition for Q_2 is given by

$$k_f\left[\beta\frac{p_1 - c_2\left(1+\beta k_c\right) + k_c\alpha}{1+2\beta k_c} - Q_2\right] - \Delta c = 0.$$

This yields $Q_2 = \beta\frac{p_1 - c_2(1+\beta k_c)+k_c\alpha}{(1+2\beta k_c)} - \frac{\Delta c}{k_f}$ and $I = \frac{\Delta c}{k_f}$. The firm's first period maximization problem is

$$\begin{aligned}&\max_{p_1} \left(p_1 - c_1\right)\left[\alpha - \beta p_1 + D_s\left(p_1\right)\right] + \left[p_1 + k_c D_s\left(p_1\right) - c_1\right] \\ &\times\left[\alpha - \beta\left(p_1 + k_c D_s\left(p_1\right)\right) - D_s\left(p_1\right)\right] - \frac{1}{2}k_f I^2 - \Delta c Q_2,\end{aligned}$$

where $D_s\left(p_1\right) = \frac{\alpha+\beta c_2-2\beta p_1}{1+2\beta k_c}$, $I = \frac{\Delta c}{k_f}$ and $Q_2 = \beta\frac{p_1 - c_2(1+\beta k_c)+k_c\alpha}{(1+2\beta k_c)} - \frac{\Delta c}{k_f}$. We find that $p_1^{*i} = p_1^{ci} - \frac{1}{4}\frac{\Delta c}{1+2\beta k_c(2+\beta k_c)}$ and $p_2^{*i} = \frac{p_1^{*i}+(\alpha+\beta c_2)k_c}{1+2\beta k_c}$. To compare prices under the two commitment regimes, we consider the minimum value for c_2 under full commitment (where no production takes place in the second period), i.e., $c_2 = c_2^{\min} \equiv \frac{(\alpha k_c + c_1)k_f + 2k_c c_1}{(1+\beta k_c)k_f + 2k_c}$. This yields

$$p_1^{*i} - p_1^{ci} = -\frac{\alpha - \beta c_1}{4\left[1 + 2\beta k_c\left(2 + \beta k_c\right)\right]\left[k_f + k_c\left(2 + \beta k_f\right)\right]} < 0$$

and

$$p_2^{*i} - p_2^{ci} = \left(3 + 2\beta k_c\right)\left(p_1^{*i} - p_1^{ci}\right) < 0.$$

Replacing the equilibrium limited commitment prices for $c_2 = c_2^{\min}$ into $D_s\left(p_1\right)$ and Q_2, we obtain an interior solution, i.e., $D_s^{*i} > 0$ and $Q_2^{*i} > 0$. Therefore, there exists a range for $c_2 \geq c_2^{\min}$ such that $p_\tau^{*i} < p_\tau^{ci}$, $\tau \in \{1, 2\}$.

3 Uncertainty about production costs

To introduce cost uncertainty, we consider a stochastic term θ, which positively affects c_2 such that $\mathbb{E}\left[c_2\right] > c_1$. For the sake of simplicity, we assume that $c_2 = c_1 + \Delta c + \theta$, where $\Delta c > 0$ is a constant and the stochastic term θ has zero mean. In the following remark, we characterize the convexity of the profit function $\Pi^{*s}\left(\cdot\right)$ and the convexity of the consumer surplus function $\Psi^{*s}\left(\cdot\right)$ with respect to θ under limited commitment in the presence of consumer storage.

Remark 4 *It holds*

(i) $\frac{\partial^2 \Pi^{*s}}{\partial \theta^2} > 0$;

(ii) $\frac{\partial^2 \Psi^{*s}}{\partial \theta^2} > 0$ *if and only if* $\left(\frac{\partial p_2^{*s}}{\partial \theta}\right)^2 D'\left(p_2^{*s}\right) + \frac{\partial^2 p_2^{*s}}{\partial \theta^2} D\left(p_2^{*s}\right) < 0$.

Proof of Remark 4. As θ does not affect p_1, we can restrict our attention to the second period profits

$$\Pi_2 = \left(p_2 - c_1 - \Delta c - \theta\right)\left[D\left(p_2\right) - D_s\left(p_1\right)\right]. \tag{S7}$$

Maximizing $\Pi_2\left(\cdot\right)$ with respect to p_2, we obtain the following first-order condition

$$D\left(p_2\right) - D_s\left(p_1\right) + \left(p_2 - c_1 - \Delta c - \theta\right) D'\left(p_2\right) = 0, \tag{S8}$$

which gives the equilibrium second period price p_2^{*s}. We find from the implicit function theorem that

$$\frac{\partial p_2^{*s}}{\partial \theta} = \frac{D'\left(p_2^{*s}\right)}{2D'\left(p_2^{*s}\right) + \left(p_2^{*s} - c_1 - \Delta c - \theta\right) D''\left(p_2^{*s}\right)} > 0, \tag{S9}$$

where the inequality follows from Assumption 1 and $D'\left(\cdot\right) < 0$. Using (S7) and (S8) yields

$$\begin{aligned}\frac{\partial \Pi^{*s}}{\partial \theta} = \frac{\partial \Pi_2^{*s}}{\partial \theta} &= \left(\frac{\partial p_2^{*s}}{\partial \theta} - 1\right)\left[D\left(p_2^{*s}\right) - D_s\left(p_1^{*s}\right)\right] + \left(p_2^{*s} - c_1 - \Delta c - \theta\right) D'\left(p_2^{*s}\right) \frac{\partial p_2^{*s}}{\partial \theta} \\ &= -\left[D\left(p_2^{*s}\right) - D_s\left(p_1^{*s}\right)\right].\end{aligned}$$

As $D_2^N(\cdot) \equiv D(p_2) - D_s(p_1)$, we find that

$$\frac{\partial^2 \Pi^{*s}}{\partial \theta^2} = -\frac{\partial D_2^N(p_2^{*s})}{\partial \theta} = -D'(p_2^{*s})\frac{\partial p_2^{*s}}{\partial \theta} > 0,$$

where the inequality follows from (S9).

In the second period, consumers buy at $p_2^{*s}(\theta)$ from the firm and competitive, risk-neutral arbitrageurs.[3] The equilibrium consumer surplus can be written as

$$\Psi^{*s}(\theta) = \int_{p_1^{*s}} D(p)\,dp + \int_{p_2^{*s}(\theta)} D(p)\,dp.$$

Applying Leibniz's rule yields

$$\frac{\partial \Psi^{*s}}{\partial \theta} = -\frac{\partial p_2^{*s}}{\partial \theta} D(p_2^{*s}).$$

Then, we find that

$$\frac{\partial^2 \Psi^{*s}}{\partial \theta^2} = -\left(\frac{\partial p_2^{*s}}{\partial \theta}\right)^2 D'(p_2^{*s}) - \frac{\partial^2 p_2^{*s}}{\partial \theta^2} D(p_2^{*s}).$$

A sufficient (albeit not necessary) condition for $\frac{\partial^2 \Psi^{*s}}{\partial \theta^2} > 0$ is that $\frac{\partial^2 p_2^{*s}}{\partial \theta^2} \leq 0$. This is satisfied under fairly standard demand specifications, such as linear, exponential and iso-elastic demand functions. ■

4 Convex production costs

In the following remark, we derive the conditions under which limited commitment leads to lower prices in the presence of convex production costs. We denote by $C_\tau(\cdot)$ the total production costs in period $\tau \in \{1,2\}$, where $C'_\tau(\cdot) \geq 0$ and $C''_\tau(\cdot) > 0$. Moreover, we assume that $C'_2(\cdot) > C'_1(\cdot)$ at given quantities. We focus on the plausible situation where consumers store to a lower extent under limited commitment than under full commitment for given prices. Evaluating consumer storage at the equilibrium limited commitment prices, we have $D_s^{*c} < D_s^{cc*}$.

Remark 5 *Suppose $0 < D_s^{*c} < D_s^{cc*}$. Then, in each period the price under limited commitment is lower than the price under full commitment, i.e., $p_\tau^{*c} < p_\tau^{cc}$, $\tau \in \{1,2\}$, if $\frac{\partial D_2^N(p_1^{*c})}{\partial p_1} > 0$.*

Proof of Remark 5. Using the binding storability constraint $p_2 = p_1 + s_c$, the firm's aggregate profits are

$$\begin{aligned}\Pi^c = {} & p_1[D(p_1) + D_s(\cdot)] - C_1(D(p_1) + D_s(\cdot)) \\ & + (p_1 + s_c)[D(p_1 + s_c) - D_s(\cdot)] - C_2(D(p_1 + s_c) - D_s(\cdot)). \end{aligned} \tag{S10}$$

[3] As discussed in Section 8, the presence of arbitrageurs implies that $\mathbb{E}[p_2] = p_1 + s_c$. It can be easily shown that the results are qualitatively unaffected when the arbitrageurs do not operate and consumers directly engage in storage activities.

Under full commitment, differentiating (S10) with respect to p_1 and D_s yields the following first-order conditions

$$D(p_1) + D(p_1 + s_c) + p_1 D'(p_1) + (p_1 + s_c) D'(p_1 + s_c)$$
$$-C_1'(\cdot) D'(p_1) - C_2'(\cdot) D'(p_1 + s_c) = 0 \tag{S11}$$

$$-s_c - C_1'(\cdot) + C_2'(\cdot) \leq 0. \tag{S12}$$

Condition (S11) determines the equilibrium full commitment prices p_1^{cc} and $p_2^{cc} = p_1^{cc} + s_c$. It follows from (S12) that $D_s^{cc} > 0$ in equilibrium for $s_c < \widetilde{s}_c^{cc} \equiv C_2'\left(D\left(p_1^{cc}|_{D_s=0} + s_c\right)\right) - C_1'\left(D\left(p_1^{cc}|_{D_s=0}\right)\right)$. The assumption that $C_2'(\cdot) > C_1'(\cdot)$ at given quantities is a (necessary) condition for $\widetilde{s}_c^{cc} > 0$. This implies that (S12) holds with equality.

Under limited commitment, proceeding backwards, the firm faces the following second period maximization problem

$$\max_{p_2} p_2 [D(p_2) - D_s(p_1)] - C_2(D(p_2) - D_s(p_1)).$$

Using the binding storability constraint $p_2 = p_1 + s_c$, the first-order condition for p_2 is

$$D(p_1 + s_c) - D_s(p_1) + (p_1 + s_c) D'(p_1 + s_c) - C_2'(\cdot) D'(p_1 + s_c) = 0. \tag{S13}$$

It follows from the implicit function theorem that consumer storage D_s is such that $\frac{\partial D_s}{\partial p_1} < 0$. Moving to the first period and differentiating (S10), where $D_s(p_1)$ satisfies (S13), the first-order condition for p_1 is given by

$$D(p_1) + D(p_1 + s_c) + p_1 D'(p_1) + (p_1 + s_c) D'(p_1 + s_c) - C_1'(\cdot) D'(p_1)$$
$$-C_2'(\cdot) D'(p_1 + s_c) - \left[s_c + C_1'(\cdot) - C_2'(\cdot)\right] \frac{\partial D_s(p_1)}{\partial p_1} = 0, \tag{S14}$$

which yields the equilibrium limited commitment prices p_1^{*c} and $p_2^{*c} = p_1^{*c} + s_c$. Substituting the first-order condition for p_1^{*c} in (S14) into the left-hand side of the first-order condition for p_1^{cc} in (S11) yields

$$D'(p_1^{*c}) \left[C_1'(D(p_1^{*c}) + D_s^{*c}) - C_1'(D(p_1^{*c}) + D_s^{cc*})\right]$$
$$+ D'(p_1^{*c} + s_c) \left[C_2'(D(p_1^{*c} + s_c) - D_s^{*c}) - C_2'(D(p_1^{*c} + s_c) - D_s^{cc*})\right]$$
$$+ \frac{\partial D_s(p_1^{*c})}{\partial p_1} \left[s_c + C_1'(D(p_1^{*c}) + D_s^{*c}) - C_2'(D(p_1^{*c} + s_c) - D_s^{*c})\right].$$

After some manipulation, this expression can be rewritten as

$$D'(p_1^{*c}) \left[C_1'(D(p_1^{*c}) + D_s^{*c}) - C_1'(D(p_1^{*c}) + D_s^{cc*})\right]$$
$$+ \frac{\partial D_2^N(p_1^{*c})}{\partial p_1} \left[C_2'(D(p_1^{*c} + s_c) - D_s^{*c}) - C_2'(D(p_1^{*c} + s_c) - D_s^{cc*})\right]$$
$$+ \frac{\partial D_s(p_1^{*c})}{\partial p_1} \left[s_c + C_1'(D(p_1^{*c}) + D_s^{*c}) - C_2'(D(p_1^{*c} + s_c) - D_s^{cc*})\right], \tag{S15}$$

where $\frac{\partial D_2^N(p_1^{*c})}{\partial p_1} = D'(p_1^{*c} + s_c) - \frac{\partial D_s(p_1^{*c})}{\partial p_1}$. Note from $D_s^{*c} < D_s^{cc*}$ and (S12) that the last expression in square brackets is negative. As $\frac{\partial D_s}{\partial p_1} < 0$, the entire expression (S15) is positive if $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} > 0$. As $p_2^{cc} = p_1^{cc} + s_c$ and $p_2^{*c} = p_1^{*c} + s_c$, it follows from Assumption 1 that $p_\tau^{*c} < p_\tau^{cc}$, $\tau \in \{1,2\}$, if $\frac{\partial D_2^N(p_1^{*s})}{\partial p_1} > 0$. ■

5 Durable goods

In the following remark, we derive some relevant results in a two-period framework where a durable good monopolist faces a linear demand of the form $D(p_\tau) = \alpha - \beta p_\tau$, $\tau \in \{1,2\}$, with $\alpha > 0$ and $\beta > 0$. We denote by p_τ^{cd} and p_τ^{*d} the equilibrium prices under full and limited commitment in period τ, respectively, and by q_τ^{cd} and q_τ^{*d} the corresponding quantities.

Remark 6 *It holds*
(i) $p_\tau^{cd} > p_\tau^{*d}$, $\tau \in \{1,2\}$;
(ii) $p_1^{*d} > p_2^{*d}$;
(iii) for a given c_1, $\frac{\partial p_1^{*d}}{\partial \Delta c} < 0$ *and* $\frac{\partial p_2^{*d}}{\partial \Delta c} > 0$, *with* $\frac{\partial p_2^{*d}}{\partial \Delta c} > \left|\frac{\partial p_1^{*d}}{\partial \Delta c}\right|$.

Proof of Remark 6. Under full commitment, the price that the firm charges in the first period is $p_1 = \frac{2}{\beta}(\alpha - q_1)$. The firm's maximization problem is given by

$$\max_{q_1} \frac{2}{\beta}(\alpha - q_1) q_1 - c_1 q_1.$$

Taking the first-order condition for q_1 yields $q_1^{cd} = \frac{2\alpha - \beta c_1}{4}$, which implies $p_1^{cd} = \frac{2}{\beta}\left(\alpha - q_1^{cd}\right) = \frac{2\alpha + \beta c_1}{2\beta}$. Moreover, we have $q_2^{cd} = 0$ and $p_2^{cd} = \frac{\alpha}{\beta} - \frac{q_1^{cd} + q_2^{cd}}{\beta} = \frac{2\alpha + \beta c_1}{4\beta}$.

Under limited commitment, proceeding backwards, in the second period the firm solves

$$\max_{q_2} \left(\frac{\alpha}{\beta} - \frac{q_1 + q_2}{\beta}\right) q_2 - c_2 q_2,$$

where the expression in round brackets is the price in the second period. It follows from the first-order condition for q_2 that $q_2(q_1) = \frac{\alpha - q_1 - \beta c_2}{2}$ and $p_2(q_1) = \frac{\alpha}{\beta} - \frac{q_1 + q_2^{cd}}{\beta} = \frac{\alpha - q_1 + \beta c_2}{2\beta}$. Moving to the first period, we have

$$\max_{q_1} \frac{3(\alpha - q_1) + \beta c_2}{2\beta} q_1 - c_1 q_1 + \frac{(\alpha - q_1 - \beta c_2)^2}{4\beta},$$

where the first ratio corresponds to the first period price $p_1 = \frac{\alpha - q_1}{\beta} + p_2$. Taking the first-order condition for q_1 yields $q_1^{*d} = \frac{2(\alpha + \beta \Delta c)}{5}$, which implies $p_1^{*d} = \frac{3\left(\alpha - q_1^{*d}\right) + \beta c_2}{2\beta} = \frac{9\alpha + 6\beta c_1 - \beta c_2}{10\beta}$. Moreover, we have $q_2^{*d} = \frac{\alpha - q_1^{*d} - \beta c_2}{2} = \frac{3\alpha + 2\beta c_1 - 7\beta c_2}{10}$ and $p_2^{*d} = \frac{\alpha - q_1^{*d} + \beta c_2}{2\beta} = \frac{3\alpha + 2\beta c_1 + 3\beta c_2}{10\beta}$. Standard computations show that $p_\tau^{cd} > p_\tau^{*d}$, $\tau \in \{1,2\}$, and $p_1^{*d} > p_2^{*d}$. For a given c_1, we find that $\frac{\partial p_1^{*d}}{\partial \Delta c} = -\frac{1}{10} < 0$ and $\frac{\partial p_2^{*d}}{\partial \Delta c} = \frac{3}{10} > 0$, with $\frac{\partial p_2^{*d}}{\partial \Delta c} > \left|\frac{\partial p_1^{*d}}{\partial \Delta c}\right|$. ■

6 Discount factor

We now consider a general discount factor $\delta \in (0,1]$. Adopting the same rationale as in the baseline model, there exists a threshold $s_c^{l\delta}$ such that for $s_c < s_c^{l\delta}$ consumer storage occurs irrespective of the firm's commitment powers. The following remark extends our results to a discount factor $\delta \in (0,1]$ in the presence of consumer storage.

Remark 7 *Suppose $s_c < s_c^{l\delta}$, where $s_c^{l\delta} \leq \delta c_2 - c_1$. Then, in each period the price under limited commitment is lower than the price under full commitment, i.e., $p_\tau^{*\delta} < p_\tau^{c\delta}$, $\tau \in \{1,2\}$, if and only if $\frac{\partial D_2^N\left(p_1^{*\delta}\right)}{\partial p_1} > 0$.*

Proof of Remark 7. Given that the storability constraint is binding, i.e., $p_1 + s_c = \delta p_2$, the firm's aggregate profits correspond to

$$\Pi^\delta = (p_1 - c_1)\left[D(p_1) + D_s(p_1)\right] + \delta\left(\frac{p_1 + s_c}{\delta} - c_2\right)\left[D\left(\frac{p_1 + s_c}{\delta}\right) - D_s(p_1)\right]. \quad \text{(S16)}$$

Under full commitment, consumer storage occurs if and only if $s_c < \delta c_2 - c_1$. Specifically, the firm prefers to induce full consumer storage, i.e., $D_s(p_1) = D\left(\frac{p_1+s_c}{\delta}\right)$. The firm's problem of maximizing its profits in (S16) reduces to

$$\max_{p_1}\ (p_1 - c_1)\left[D(p_1) + D\left(\frac{p_1 + s_c}{\delta}\right)\right].$$

The first-order condition for p_1 is

$$D(p_1) + D\left(\frac{p_1 + s_c}{\delta}\right) + (p_1 - c_1)\left[D'(p_1) + D'\left(\frac{p_1 + s_c}{\delta}\right)\frac{1}{\delta}\right] = 0. \quad \text{(S17)}$$

Under limited commitment, it follows from the first-order condition for the second period profit maximization and from the binding storability constraint $p_1 + s_c = \delta p_2$ that $D_s(p_1) = \max\left\{0, D\left(\frac{p_1+s_c}{\delta}\right) + \left(\frac{p_1+s_c}{\delta} - c_2\right) D'\left(\frac{p_1+s_c}{\delta}\right)\right\}$. Let $s_c^{l\delta}$ be the threshold for s_c below which consumer storage occurs under the two commitment regimes. This implies that $s_c^{l\delta} \leq \delta c_2 - c_1$. For $s_c < s_c^{l\delta}$, we have $D_s(p_1) > 0$. Using (S16), the first-order condition for p_1 can be written as

$$\begin{aligned} &D(p_1) + D\left(\frac{p_1 + s_c}{\delta}\right) + (p_1 - c_1)D'(p_1) \\ &+ \left(\frac{p_1 + s_c}{\delta} - c_2\right) D'\left(\frac{p_1 + s_c}{\delta}\right) + \frac{\partial D_s(p_1)}{\partial p_1}(\delta c_2 - c_1 - s_c) = 0. \end{aligned} \quad \text{(S18)}$$

After substituting the first-order condition for $p_1^{*\delta}$ in (S18) into the left-hand side of the first-order condition for $p_1^{c\delta}$ in (S17) and using $D_2^N(p_1) \equiv D\left(\frac{p_1+s_c}{\delta}\right) - D_s(p_1)$, we find that

$$(\delta c_2 - c_1 - s_c)\frac{\partial D_2^N\left(p_1^{*\delta}\right)}{\partial p_1} > 0,$$

where the inequality holds if and only if $\frac{\partial D_2^N\left(p_1^{*\delta}\right)}{\partial p_1} > 0$ (recall $s_c < s_c^{l\delta} \leq \delta c_2 - c_1$). As $p_2^{c\delta} = \frac{p_1^{c\delta}+s_c}{\delta}$ and $p_2^{*\delta} = \frac{p_1^{*\delta}+s_c}{\delta}$, it follows from Assumption 1 that $p_\tau^{*\delta} < p_\tau^{c\delta}$, $\tau \in \{1,2\}$, if and only if

$\frac{\partial D_2^N(p_1^{*\delta})}{\partial p_1} > 0$. With a linear demand function of the form $D(p_\tau) = \alpha - \beta p_\tau$, $\tau \in \{1,2\}$, where $\alpha > 0$ and $\beta > 0$, the equilibrium prices are $p_1^{c\delta} = \frac{2\alpha\delta + \beta c_1(1+\delta) - \beta s_c}{2\beta(1+\delta)}$ and $p_2^{c\delta} = \frac{p_1^{c\delta}+s_c}{\delta}$ under full commitment, while they are $p_1^{*\delta} = \frac{2\alpha\delta + \beta c_1(2+\delta) - \beta\delta c_2}{2\beta(1+\delta)}$ and $p_2^{*\delta} = \frac{p_1^{*\delta}+s_c}{\delta}$ under limited commitment. Standard computations show that $\frac{\partial p_\tau^{c\delta}}{\partial\delta} > 0$ and $\frac{\partial p_\tau^{*\delta}}{\partial\delta} > 0$, $\tau \in \{1,2\}$. Taking the price difference between two commitment regimes yields $\Delta p^\delta \equiv p_1^{c\delta} - p_1^{*\delta} = p_2^{c\delta} - p_2^{*\delta} = \frac{\delta c_2 - c_1 - s_c}{2(1+\delta)} > 0$ (recall $s_c < s_c^{l\delta} \leq \delta c_2 - c_1$). Then, we find that $\frac{\partial \Delta p^\delta}{\partial\delta} > 0$. ■

7 Linear demand

In the following remarks, we characterize the main results of the paper in a framework with a linear demand of the form $D(p_\tau) = \alpha - \beta p_\tau$, $\tau \in \{1,2\}$, where $\alpha > 0$ and $\beta > 0$. The threshold values are defined in the proofs.

Remark 8 *Suppose $s_c < \bar{s}_c^*$. Then,*

(i) under full commitment, consumer storage is $D_s^{cs} = \frac{2\alpha - 2\beta c_1 - 3\beta s_c}{4}$, and prices are $p_1^{cs} = \frac{2\alpha + 2\beta c_1 - \beta s_c}{4\beta}$ and $p_2^{cs} = \frac{2\alpha + 2\beta c_1 + 3\beta s_c}{4\beta}$;

(ii) under limited commitment, consumer storage is $D_s^{*s} = \frac{3}{2}\beta\Delta c - 2\beta s_c$, and prices are $p_1^{*s} = \frac{2\alpha + 2\beta c_1 - \beta\Delta c}{4\beta}$ and $p_2^{*s} = \frac{2\alpha + 2\beta c_1 - \beta\Delta c + 4\beta s_c}{4\beta}$.

Consumer surplus is higher under limited commitment than under full commitment. Total welfare is higher under limited commitment than under full commitment if and only if $\Delta c > \widetilde{\Delta c}$.

Proof of Remark 8. As $\Delta\mu^m = -\frac{\Delta c}{2} < 0$ and $\Delta c + \Delta\mu^m = \frac{\Delta c}{2} < \tilde{s}_c^* = \frac{3}{4}\Delta c$, it follows from the proof of Proposition 3 that case (I) applies and s_c^l in (A14) corresponds to $\bar{s}_c^*$. The results in points (i) and (ii) of the remark are a direct application of Corollaries 1 and 3 (recall from the proof of Proposition 3 that $\bar{s}_c^* < \bar{s}_c^c$). Under full commitment, consumer surplus and the firm's profits are respectively

$$\Psi^{cs} = \frac{4(\alpha - \beta c_1)^2 - 4\beta(\alpha - \beta c_1)s_c + 5\beta^2 s_c^2}{16\beta} \tag{S19}$$

and

$$\Pi^{cs} = \frac{[2\alpha - \beta(2c_1 + s_c)]^2}{8\beta}. \tag{S20}$$

Under limited commitment, consumer surplus and the firm's profits are respectively

$$\Psi^{*s} = \frac{[2\alpha - \beta(2c_1 - \Delta c)]^2 - 4\beta[2\alpha - \beta(2c_1 - \Delta c)]s_c + 8\beta^2 s_c^2}{16\beta} \tag{S21}$$

and

$$\Pi^{*s} = \frac{4\alpha^2 - 4\alpha\beta(2c_1 + \Delta c) + \beta^2\left(4c_1^2 + 4c_1\Delta c + 9\Delta c^2 - 16 s_c\Delta c + 8s_c^2\right)}{8\beta}. \tag{S22}$$

When the static monopoly prices are charged, consumer surplus and the firm's profits are respectively

$$\Psi^{m}=\frac{2\left(\alpha-\beta c_{1}\right)^{2}-2\beta\left(\alpha-\beta c_{1}\right)\Delta c+\beta^{2}\Delta c^{2}}{8\beta} \tag{S23}$$

and

$$\Pi^{m}=\frac{2\left(\alpha-\beta c_{1}\right)^{2}-2\beta\left(\alpha-\beta c_{1}\right)\Delta c+\beta^{2}\Delta c^{2}}{4\beta}. \tag{S24}$$

To compute the threshold $\bar{s}_c^*$, we use (S22) and (S24), which yields $\Pi^{*s}>\Pi^m$ if and only if $s_c<\bar{s}_c^*$, where $\bar{s}_c^*=\left(1-\frac{1}{2\sqrt{2}}\right)\Delta c$ (see Corollary 3). Now, we turn to the welfare analysis. Taking the difference between (S21) and (S19) yields $\Delta\Psi\equiv\Psi^{*s}-\Psi^{cs}=\frac{4\alpha-\beta(4c_1-\Delta c+3s_c)}{16}\left(\Delta c-s_c\right)>0$, where the inequality follows from the assumptions on the parameters of the model. Taking the difference between (S22) and (S20) yields $\Delta\Pi\equiv\Pi^{*s}-\Pi^{cs}=-\frac{4\alpha-\beta(4c_1+9\Delta c-7s_c)}{8}\left(\Delta c-s_c\right)<0$ (the limited commitment profits are lower than the full commitment profits). Then, we obtain $\Delta W\equiv\Delta\Psi+\Delta\Pi=-\frac{4\alpha-\beta(4c_1+19\Delta c-17s_c)}{16}\left(\Delta c-s_c\right)$. It holds $\Delta W>0$ if and only if $\Delta c>\widetilde{\Delta c}$, where $\widetilde{\Delta c}\equiv\frac{4\alpha-4\beta c_1+17\beta s_c}{19\beta}$, with $\frac{\partial\widetilde{\Delta c}}{\partial\beta}<0$. Note that $\widetilde{\Delta c}<\widehat{\Delta c}\equiv\frac{2\alpha-2\beta c_1+4\beta s_c}{5\beta}$, which ensures that the second period profit margin is positive. ■

Remark 9 *A. Suppose* $\bar{s}_c^*\leq s_c<\bar{s}_c^c$. *Then,*

(i) under full commitment, consumer storage is $D_s^{cs}=\frac{2\alpha-2\beta c_1-3\beta s_c}{4}$, *and prices are* $p_1^{cs}=\frac{2\alpha+2\beta c_1-\beta s_c}{4\beta}$ *and* $p_2^{cs}=\frac{2\alpha+2\beta c_1+3\beta s_c}{4\beta}$;

(ii) under limited commitment, consumer storage is $D_s^m=0$, *and prices are* $p_1^m=\frac{\alpha+\beta c_1}{2\beta}$ *and* $p_2^m=\frac{\alpha+\beta c_1+\beta\Delta c}{2\beta}$.

Consumer surplus and total welfare are lower under limited commitment than under full commitment.

B. Suppose $s_c\geq\bar{s}_c^c$. *Then, the static monopoly solution in point (ii) applies under the two commitment regimes.*

Proof of Remark 9. A. It follows from the proof of Proposition 4 that s_c^h in (A15) corresponds to $\bar{s}_c^c$. Using (S20) and (S24), we obtain $\Pi^{cs}>\Pi^m$ if and only if $s_c<\bar{s}_c^c$, where $\bar{s}_c^c=\frac{2(\alpha-\beta c_1)}{\beta}-\frac{\sqrt{2}}{\beta}\sqrt{2\left(\alpha-\beta c_1\right)^2-\beta\left(2\alpha-2\beta c_1-\beta\Delta c\right)\Delta c}$ (see Corollary 1). The results in points (i) and (ii) of the remark are a direct application of Corollaries 1 and 3. Taking the difference between (S23) and (S19) yields $\Delta\Psi\equiv\Psi^m-\Psi^{cs}=-\frac{4\alpha(\Delta c-s_c)-\beta\left[2\Delta c(2c_1+\Delta c)-4c_1s_c-5s_c^2\right]}{16}<0$, where the inequality follows from the assumptions on the parameters of the model. As $\Delta\Pi\equiv\Pi^m-\Pi^{cs}<0$, it holds $\Delta W\equiv\Delta\Psi+\Delta\Pi<0$.

B. The proof follows from Corollaries 1 and 3. ■