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Abstract

The equality between factor pay and marginal product is a major component of the neoclassical paradigm. The paper begins with a brief historical review of this principle. Follows a questioning about the relevance of this law as an argument in the social debates: does marginal product represent the very contribution of the agent and if so, is it a legitimate reference for the setting of remuneration? Our answer to the first part of the question is irresolute; to the second, it is negative.

But most of the article is devoted to analysing the economic realism of the said law, both empirically and theoretically. We review some statistical studies present in the literature, with particular attention for the debate regarding the regressions of Cobb and Douglas. Evidence does not strengthen the neoclassical law of retribution.

The paper analyses the factors that hinder either the determinateness of marginal product or the equalisation between it and factor's remuneration. Are analysed:
- the restrictions inherent in the law of marginal productivity: constant returns to scale and perfect competition
- an alternative explanation of interest: the Austrian theory
- incentive wage theories: efficiency wage and tournament theory.

The article then considers the particular case of the CEO's remuneration.

INTRODUCTION

Distribution of income is a sensitive subject, covering economic and social issues. In political economy, the general theories of distribution are not legion. Various explanations concern some specific aspects of wage, interest, rent or profit. Among general explanations, one has emerged, the theory of marginal productivity, pillar of neoclassical economics. This theory has buried the earlier explanations of classical economists and it seems to have dried up the inspiration of possible alternatives. The purpose of this article is to assess its relevance by identifying the economic facts likely to thwart its action.

Before getting into the subject, let's take a quick look at the teachings of the classics. **Smith** confusingly determines the value of a commodity from the amount of work incorporated; afterwards he considers that this value is obtained by adding rent, profit and wage, without explaining how these two operations lead to a single result. He gives separate explanations of the natural levels of remuneration of the three factors, which remain superficial. **Ricardo** constructs a more coherent theory that unites product value and factor remuneration through a rent theory that makes it an adjustment variable. As the price of wheat must allow all producers to be profitable, it equals the cost of production on the least fertile land, leaving a surplus profit to the infra-marginal farms, captured by the landowner in the form of land rent. Distribution between wages and profits is also rationalised by becoming a zero sum game. But his explanation relies by a great deal on the apocalyptic view of population growth of Malthus, a theory today widely discredited.

Marx follows on Ricardo's heels; here the distribution between wage and profit is analysed more finely. Labour being a commodity capable of producing more than its value, the capitalist who exploits it gets a gain which is the substance of its profit. Labour's value is the cost of production of the goods required for keeping the worker in life and supplying labour. But what are the necessary goods? Those who ensure survival? or well-being ? The normal standard of living of the worker is not defined; the rate of surplus-value therefore remains unexplained although analysed at length. Marxist theory has another weakness: whatever Marx says, the theory of the equalisation of the rate of profit, which he brings after having exposed the exploitation of labour, renders the theory of surplus value superfluous.

The book "Production of commodities by commodities" (1960) of **Sraffa** reformulates the Ricardian theory in a more rigorous and formal way. Sraffa proves the existence of a general equilibrium of the economy where each sector yields the same rate of profit and where product prices equal their costs of production while the remuneration of factors is determined beforehand, again within the framework of a zero sum game. An essential characteristic of this model is that one of the two variables, wage or profit, must be exogenous, the other being endogenous. This amounts to deny that economics alone can explain distribution; non-economic considerations should interfere.

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The theory of marginal productivity achieves the tour de force of determining precisely all remunerations by means of a single law which resorts to exclusively economic factors, hence the aura it enjoys among economists.

Consider a set of economic agents that cooperate in a production. Let us classify them into some relatively homogeneous groups that we call factors of production. The potential size of these groups depends on natural or historical availability and constitutes the factor's supply. Factors' demand is the doing of entrepreneurs; it obviously depends on their productivity. But the productivity of the agents in a group depends on the size of the group. The more numerous it is, the less unit productivity will be. Marginal productivity of a factor is the supplement of product due to the fact that its size is n agents and not $n - 1$. It is decreasing, because if one group of agents increases while the others remain constant, its members will individually have available a smaller amount of the other factors. Suppose then that the prices of the different factors and of the product are given; the entrepreneur maximises his profit by hiring from each of them the number for which its marginal product value[1] equals its price. This behaviour is rational: if he employed a smaller volume, the entrepreneur could increase his profit by increasing the recruitment; if he employed a larger number, the agents would pay him less than they cost him.

Given the interplay of supply and demand, under the assumption of competitive markets, we must infer from the above that a factor's remuneration necessarily equals the value of its marginal product. This does not mean that marginal productivity determines the factor's price. Endogenous determination of factor remuneration can be obtained only in the context of a system of simultaneous equations manipulating all the prices and quantities of the economy; the marginal productivity of a factor and its remuneration are then mutually determined. Nevertheless, according to this theory, we are all supposed to be paid depending on our productivity. Since we postulate the quasi-homogeneity of the factors, all the units have the same individual productivity, determined by the size of the workforce[2].

Based entirely on the rationality of agents, this theory has the appearance of evidence. After a historical note, this paper wonders about the social and the economic meaning of marginal productivity. Afterwards, it reviews the difficulties linked with the theory. In the following of the article, the theory that equates remuneration and marginal product value will simply be called "*the law*".

A BRIEF HISTORICAL NOTE: THE ORIGIN

The purpose of this section is not to go back to precursors, but to see how the early neoclassical theory formulated the law and made it one of its pillars. It was in 1889 that the law was first expressed in fullness and clarity, by John Bates **Clark,** who was its most tenacious promoter. But in 1894 it received a choice support, with the

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[1] The said value is the marginal product multiplied by the selling price of the product

[2] It is therefore not necessary that the quality of the units recruited decreases progressively like the fertility of the lands cultivated in Ricardo. Wicksteed distinguishes the DESCRIPTIVE marginal productivity curve where the quality of the units decreases on the abscissa and the FUNCTIONAL marginal productivity curve whose ordinate is a pure function of the quantity on the abscissa.

inauguration by Philip **Wicksteed** of the famous *production function*, in "An Essay on the Coordination of the Laws of Distribution":

$$P = F(A, B, C \ldots K \ldots N) \tag{1}$$

The quantity produced (P) is a function *F* of the quantities used of each of the *n* factors of production. Let the variables *A, B ... K ... N* represent these quantities[3]. The underlying idea is to establish between the quantity produced on the one hand and that of the inputs on the other hand a mathematical relation that can be put in the form of an equation.

F is a multivariate function. It is obvious that its partial derivatives give the variation of *P* generated by a marginal variation of the only quantity of the factor considered. They correspond to the marginal productivity of the factors. We can therefore designate the marginal product of a factor called *K* by the mathematical notation of the partial derivative $\partial P/\partial K$ or F_K.

If factor *K* units are all remunerated for their marginal product, we have:

$$\frac{\partial P}{\partial K}.K = \text{total remuneration of } K \tag{2}$$

The validity of the law implies that the sum of the remunerations calculated this way equals the quantity produced to be distributed. This problem is known as the *product exhaustion*. The equation of product exhaustion is:

$$P = A.\frac{\partial P}{\partial A} + B.\frac{\partial P}{\partial B} + \ldots + N.\frac{\partial P}{\partial N} \tag{3}$$

Wicksteed brings a long mathematical proof of this equality, but an important assumption is required : the returns to scale must be constant. The mathematical terminology says that function *F* is *homogeneous of degree one*, which, mathematically, is expressed like this:

$$\text{if } P = F(A, B, C), \text{ then :} \qquad m.P = F(m.A, m.B, m.C) \tag{4}$$

where *m* is any constant

Wicksteed analyses this condition: Is it realistic? His answer is built as follows:
1. He does not doubt that the physical product is a homogeneous function of degree one of the inputs. He seems to ignore Marshall's careful analysis of returns to scale at the same period.
2. But when *P* is no longer a simple physical quantity but the producer's income, the production function is homogeneous only if the elasticity of the demand for the product is infinite, which excludes monopolies. Finally, Wicksteed concludes: "the proposition is approximately true for small increments" (1932 p.36).

In the "Lectures on Political Economy" (1901), Knut **Wicksell** takes over Wicksteed's problem and brings the solution learned by generations of economists. He writes: "The scale on which an enterprise operates nearly always has some influence on its average product (…) as a rule the best returns are obtained at some particular scale of operation for the firm in question ; if this is exceeded, the advantages of centralization

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[3] For ease, I use the same letters to "label" the factors and to express the amount that is used.

are outweighed by the increased costs (…) This scale of operation is, under the given circumstances, the 'optimum' towards which the firm must always, economically speaking, gravitate: and as it lies at the point of transition from 'increasing' to 'diminishing returns' (relatively to the scale of production) the firm will conform to the law of constant returns" (1934 p.129). It should be noted, however, that the U-shape of the long-term average cost curve has been subjected to much criticism.

Wicksell also shows that in increasing returns to scale, the right-hand side of equation (3) is greater than the left-hand side and the inverse is true in decreasing scale-returns.

There is a difference in point of view between Clark and Wicksteed. For Wicksteed, there are a multitude of factors involved in the production function, from the types of human work to capital goods or natural goods. His conception is microeconomic. Clark always gives his examples with two factors of production: labour and capital; and he considers the economy as a whole, which he justifies by the levelling of marginal productivity through the whole economy by competition[4]. Clark sees capital as the permanent fund embodied in what he calls "capital goods", while these constitute the inputs of the production function of Wicksteed. Speaking of factor K, representative in equations (1) and (2), the latter writes : « Or K may be pick-axes, or any kind of machinery or tools (…) Observe too that it involves no pooling of unlike elements, and no expression of those unlike elements under a conventional unit, such as the £ sterling, and no artificial grouping of different factors of production. Each factor is expressed in its own unit and treated as having its independent influence, at the margin, on the increment or decrement of the product" (1932 p.13).

The article "A Theory of Production" (1928) by economist Paul **Douglas** and mathematician Charles **Cobb** combines both points of view. The production function remains the central part, but it applies to the global economy and has only two inputs: capital and labour. So was born the *aggregate production function*. The authors give the production function the form (previously designed by Wicksell):

$$P = b.K^{\alpha}.L^{\beta} \tag{5}$$

Their purpose is to prove that marginal productivity theory correctly explains the distribution of income in the real economy. Assuming product exhaustion ($\alpha+\beta = 1$), the authors carry out an econometric analysis of US industrial statistics from 1899 to 1922, more precisely the annual indices of the variables studied.

Function (5) has properties that make it perfectly suited to this analysis. On the one hand, the homogeneity of this function is easy to manage. On the other hand, its parameters are empirically observable data: α and β correspond to the respective relative shares of factors K and L in product P[5].

By the least squares method, equation (5) becomes:

$$P = 1{,}01. K^{1/4} .L^{3/4}. \tag{6}$$

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[4] Employment of the factor increases in firms where it is more productive and decreases in firms where it is less productive. This process leads to an equalisation of marginal productivity of each factor.

[5] The equalities $\alpha=(K.\partial P/\partial K)/P$ and $\beta=(L.\partial P/\partial L)/P$ are obtained from some manipulations on partial derivative.

Global payroll is worth 3/4 of national income and the marginal productivity of labour is equal to 0,75P/L. The good correlation between the index of *P* calculated using equation (6) and the index of real output allows Cobb and Douglas to conclude that the theory of marginal productivity is valid; they must, however, recognise that the correlation between the observed wage indices and the calculated marginal productivity of labour is rather low, which does not seems to upset their optimism. An additional reason for satisfaction: other statistical studies confirm the share of 3/4 going to labour.

A BRIEF HISTORICAL NOTE : FURTHER DEVELOPMENTS

An important step is the publication of "The Economics of Imperfect Competition" (1933) by Joan **Robinson**. This monumental and impressive work innovates in that it deals in a uniform and coherent way with perfect competition and imperfect competition as particular cases of the same general rule, both on the product market and on the factors markets. To this end, the analysis offers a range of linked concepts:
- The *marginal physical product* and the *value of the marginal physical product*, concepts already known, are here determined on the basis of a constant expenditure allocated to the other factors of production.
- The term *marginal product* is reserved for the *marginal physical product* multiplied by the *marginal revenue*.
- The *marginal net product*, unlike the *marginal product*, supposes that the quantity of other factors was adapted so as to keep it optimal. It equals the difference between the variation of total revenue and the variation of total cost ensuing[6]. This concept was already present - but rather inaccurately – in the earlier Cantabridgians Marshall and Pigou.
- The *average cost* of a factor is his unit pay (for example, hourly wage); the *marginal cost* of the factor is the extra cost of hiring an additional factor unit. These two variables[7] reflect the factor's supply within the firm ; they coincide when this supply is perfectly elastic.

The well-known rule for maximising profit is to equalise the marginal revenue with the marginal cost of the product. Robinson brings a new rule that leads to the same result: equalisation of the net marginal product of factors with their marginal cost[8].

The Paretian revival in the thirties does not revolutionise marginal productivity theory. The *marginal rate of technical substitution* between two factors brings nothing new since it simply equals the ratio between their marginal products. But the production function does not lose its audience, as the works of Hotelling and Samuelson attest.

A chapter of the "Foundations of Economic Analysis" of **Samuelson** (1947) rewrites some grand principles of Robinson's Theory on the basis of the production function, a tool she had completely ignored. Noting that many authors simplify outrageously the

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[6] Robinson shows that for the enterprise that controls and optimises the inputs, net marginal product and marginal product are the same. But this equality not necessarily obtains at the aggregate level.

[7] Average and marginal costs of the factor should not be confused with average and marginal costs of the quantity produced.

[8] "Full equilibrium", that is absence of pure profit because of competition in the industry, implies moreover the equality between *average net product* and *average factor cost*.

issue of product exhaustion and the annihilation of profit, Samuelson clarifies: "it should be reasonably clear that this cannot be determined by the properties of the production function alone, but must depend upon the marketing situation of the firm, which in turn depends on the competition of other firms. It is quite clear that as far as the single firm is concerned it is possible that it be making huge profits regardless of the homogeneity of the production function. This condition is neither necessary nor sufficient for the exhaustion of the product. If the production function were homogeneous, but demand were sufficiently favorable, of course product would not be exhausted- even under pure competition" (1966 p.83).

The neowalrasian wave of the post-war period seriously changes the way microeconomic theories are presented. But the crux remains. As Malinvaud has shown, to define a set of feasible outputs is tantamount to determining a coherent technological relationship between the inputs and the outputs of each activity vector. The production function is therefore underlying.

In its Paretian and neowalrasian versions, neoclassical theory outlines a general equilibrium. But Robinson's system is also, implicitly, an almost general equilibrium. The parameters related to the product market and to the inputs markets are indissolubly mixed. Robinson's method, however, remains that of *partial equilibrium*. Everything is linked with everything, but the mechanisms are analysed one by one. The risk is then to obtain a dubious general equilibrium. The NET *marginal product* assumes that the remunerations of other factors are determined beforehand. Robinson does not explain how she avoids the trap of circular reasoning[9]. This difficulty is likely to be solved within the framework of a system of simultaneous equations, but the partial equilibrium method prevents demonstrating that the problem has a solution.

It follows that the law is an indispensable part in the whole formed by the general neoclassical equilibrium; this one cannot do without it. An alternative theory of retribution is possible only within a fundamentally different paradigm, for example that of the *full cost pricing* developed during the marginalist controversy from 1939 to the end of the fifties.

ETHICAL MEANING OF MARGINAL PRODUCTIVITY

At this point, we have at our disposal a completely refined definition of marginal productivity, a pure abstraction, which holds in a simple mathematical formula: $\partial P/\partial K$, considering the volume K of factor K. Is it possible to give it a more concrete content?

It is fairly widely understood that marginal productivity is interpretable as the contribution of agents participating in social production. The competitive equilibrium leads to paying the economic agents in proportion to their contribution. The main promoter of this idea is **Clark** himself. According to him, the theory of production and that of distribution join together. The theory traces in production the origin of shares

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[9] Marshall refutes that the application of the law to the factor labour constitutes in itself a theory of wages, "since in order to estimate net product of his work, we have to take for granted all the expenses of production of the commodity on which he works, other than his own wage". Further on, about interest, he adds: "They cannot be made into a theory of interest, any more than into a theory of wages without reasoning in a circle" '(1920 pp.518-519).

distributed to agents, despite the fact that social institutions hide the natural law behind bargaining processes: "The entire study of distribution is in this view a study of *specific production*. It is an analysis of the wealth-creating operation, and a tracing to each of the three[10] agencies that together bring wealth into existence of the part which it separately contributes to the joint result" (1908 p.12).

Going even further, he concludes to the legitimacy of the social system that remunerates the factors on this basis. He does not conceal his ideological preoccupation: "The right of Society to exist in its present form, and the probability that it will contribute to exist, are at stake" (1908 p.12). Clark even claims that his law provides the reason for not being socialist.

Clark then considers the question of whether to give each one the equivalent of his product is a fair rule and answers: "The entire question whether it is just or not lies outside our inquiry, for it is a matter of pure ethics. Before us, on the other hand, is a problem of economic fact" (1908 p.14). This statement is quite contradictory with the multiple allusions to the legitimacy of this distribution. Admittedly, the word "merit" does not appear in his text. Clark, however, approaches a moralising conception. This is evidenced by his insistence on the concept of *abstinence* to characterise saving. In the controversy between the two men, Böhm Bawerk challenged the relevance of this concept, giving the example of the almost automatic saving of very rich persons. Clark retorted: "By extending the lines of railroad he owns, the multi-millionaire denies himself the enlarging of his palatial residences" (1895 p.98).

Clark seems to be the great economist who has gone the farthest on this path that could be described as justificatory. Other neoclassical economists generally distinguish contribution and merit. Here are two examples that both bring a close and powerful argument:
- Frank **Knight** : "The income doesn't go to factors but to their owners and can in no case have more ethical justification than has the fact of ownership. The ownership of personal or materiel productive capacity is based upon a complex mixture of inheritance, luck and effort, probably in that order of relative importance. What is the ideal distribution from the standpoint of absolute ethics may be disputed, but of the three considerations named certainly none but the effort can have ethical validity" (1923 p.598).
- **Cobb and Douglas**: "…even if there were precise correspondence, it (Edn: the result of their analysis) would not furnish any light upon the question as to whether capital for example should be privately owned to the degree to which it is in our society. For while capital may be "productive", it does not follow that the capitalist always is" (1928 p.164).

With regard to labour, the disconnect between productivity and merit is less obvious than for capital. However, equal opportunities never existed: Children from privileged backgrounds are significantly more likely to get a diploma that opens the door to better paying jobs.

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[10] According to Clark, land is part of the capital factor. The third agent here is the entrepreneur. In the rest of the presentation, Clark will only consider capital and labor.

Marginal productivity also depends on the abundance or scarcity of a factor. While there are examples where scarcity can be explained by superior skill, abundance or scarcity are often a matter of luck, for example when they come from demographic factors.

At a more philosophical level, it is permissible to question the merit of being born intelligent or strong, the two main individual qualities favourable to productivity.

As Clark admitted with little conviction, science cannot distinguish merit or justice.

ECONOMIC MEANING OF MARGINAL PRODUCTIVITY

Marginal productivity seems to have a mathematical significance $\partial P/\partial K$ but to have no ethical significance[11]. The next question is whether it has an economic significance. The usual association with the agent's contribution to the common work, is it justified? Is the expression $\partial P/\partial K$ really representative of the individual contributions or is it just a mathematical object?

A first approach is validation by contradiction from the mathematical condition. For marginal productivity to reflect the agent's contribution, an obvious condition is that the sum of the marginal productivities of all agents of the various factors of production equals exactly the total product. We know that this condition is satisfied only in the case of constant returns to scale and provided that competition plays its role. Thus, marginal productivity does not reflect the contribution when returns to scale are not constant. The fact that the returns to scale are constant or not is random in relation to the meaning of individual contribution to production ; so, if the marginal product does not represent the contribution in all types of returns to scale, it does not represent it in any of them.

The other approach is more concrete. One argument consistently advanced against the law is that the production process combines all inputs so indistinctly that the result is indecomposable. At its origin lies the thesis of "organic cooperation" put forward by the English heterodox economist John Atkinson **Hobson** who was regularly in rather vehement controversy with Marshall and his followers. In "The Industrial System" (1909), he writes: "So intimate is the interdependence of the factors upon one another, and of the several parts of each factor upon the other parts of its own factor and upon all the parts of the other factors, that no separate productivity can rightly be attributed to any factor, still less to any part of a factor" (1910 p.106).

If marginal productivity is reduced to the simple formula $\partial P/\partial K$, Hobson's argument is not correct. As long as the function *F* is differentiable, it will always be possible to determine, at least in theory, the variation of the production due to the last unit of the factor put into activity. But what if we want to charge $\partial P/\partial K$ with economic significance?

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[11] I do not write "no normative sense", because it could be argued that marginal productivity pay has an incentive effect.

This question has the level of abstraction of metaphysical questions. In my opinion, the problem of the contribution belongs to this class of questions about which one can talk without limit but whose answer simply does not exist.

MARGINAL PRODUCTIVITY AND REMUNERATION : EMPIRICAL ASPECT

Before analysing the possible causes thwarting the law, let us see the results of empirical studies regarding the relationship between remuneration and marginal productivity.

<u>Studies specifically dedicated to the validation of the law</u>

The most famous study is that of **Gottschalk** (1978). He himself reminds some previous studies that indicated that capital was overpaid relative to its productivity and labour underpaid. After this reminder, he gives the results of two tests:

A. <u>Productivity of eight categories of workers and capital across the economy</u>.

So we have nine factors. Gottschalk considers that industrial production results from the combination of two complementary processes: actual production and sales administration. The first one uses five occupations plus capital, the second six occupations. There are thus two mixed occupations. Both processes have a production function ensuring a certain substitution between the inputs; on the other hand, the two processes are associated in a purely complementary way. The eight occupations are qualified either white collar or blue collar.

The methodology includes the following steps:

1- Determination of the *production functions* for manufacturing and for administration
2- The nine marginal revenue product functions are derived.
3- Estimation of the production functions using cross-sectional data relating to manufacturing production.
4- Calculation of marginal productivities from these estimated functions
5- Comparison with observed wages.

The double conclusion of this test is:

- o Capital received 24% of the total value added while its productivity would have allowed 19%. This confirms the results of previous studies.
- o The ratio of remuneration to marginal productivity varies from 2.64 for commercials to 0.35 for craftsmen. In general, it is significantly higher for white-collar workers than for blue-collar workers, and there is an inverse correlation between this ratio and the productivity calculated by category.

B. <u>Internal analysis for each of the eight occupations</u>

Direct examination of the data relating to each occupation has the advantage of not requiring the specification of a production function, an operation which is always delicate. Here, the necessary assumption is that marginal productivities are proportional to average productivities.

In the above conditions, "the marginal productivity theory can be tested by regressing earnings against average productivity, which is directly observable" (1978 p.376).

The correlation proves bad for all categories of workers except one.

A more recent study is given by **Kampelmann and Rycx** (2011), who analysed data on Belgian firms between 1999 and 2006. Workers were grouped into 7 categories. There is a clear correlation between these categories and earnings but not between categories and marginal productivity. The authors conclude: "the occupations at the top of the wage hierarchy ('Managers' and 'Professionals') appear to be overpaid, while 'Service and Sales workers' as well as all blue-collar occupations come out as being significantly underpaid with respect to their relative marginal productivities" (2011 p.126).

Busness cycle, productivity and wage

Giving blind faith to marginal productivity theory, **Keynes**, in The *General Theory* (1936), adhered to the dominant concept that real wages fluctuate contracyclically unlike nominal wages; it seemed logical to the economists of the time that the marginal productivity of labour and with it the real wage evolve in the opposite direction of employment. From 1938 and 1939, empirical studies have been published (Dunlop in Great Britain and Tarshis in the United States) which contradicted this creed. The real wage would be procyclical too. The clarification of Keynes in the article "Relative Movements of Real Wages and Output" (1939), stating that his demonstration of the General Theory would have been easier if these elements had been known at the time, is the first act of a revision of the economic theory in this matter.

The behaviour of productivity over the business cycle has long been debated; it is generally agreed that it is procyclical. The analysis is complex because, in addition to unemployment, there are two types of underemployment difficult to observe and to measure as the factors remain within the firm:
- The underemployment of machines
- The phenomenon known as *labour hoarding*: during recessions, enterprises prefer to retain employees by imposing a less intensive work rather than dismissing them with the risk of lack of qualified arms when comes the recovery.

Many recent empirical studies show that the wages of newly recruited workers are highly procyclical, much more than the wages of insiders. **Carneiro, Guimaraes** and **Portugal** write : "Exploring the idea that in the search and matching model job creation is driven by the difference between the expected productivity and the expected cost of labor in new matches, Pissarides shows that in equilibrium the wages negotiated in new matches are about as cyclical as productivity. This prediction of the model seems to be consistent with the empirical evidence that wages in new matches are much more procyclical than wages in continuing jobs" (2009 p.3). These new jobs would subsequently be subject to corrective wage variations inducing alignment with older contracts. Traditional neoclassical theory views the labour market as a day-to-day market. In this case, equalisation between productivity and wages is almost automatic. In reality, contracts run for longer periods; the link between productivity and wages can be managed over the long term. **Bellou** and **Kaymak** are interested in this phenomenon and conclude : "This pattern of wage adjustments is consistent with a contractual labor market, where employers and workers partake in an implicit agreement to shield wage payments from fluctuations in a worker's marginal product, without fully committing themselves to future payments and work. Our results, therefore, indicate a decoupling of the marginal product from wage payments

providing a potential explanation for the low elasticity of wages over the business cycle" (2011 p.26).

ECONOMETRICS AND COBB-DOUGLAS FUNCTION

Another way to validate the law is the one initiated by **Douglas** and **Cobb**, consisting in empirically testing the production function. We had seen that Douglas and Cobb were quite satisfied with the degree of confirmation obtained. After their 1928 article, with other economists, they persevered on the path traced, by improving their method and by applying it to other data. Since then, analyses of this type have multiplied. In general, the results are rather encouraging for the theory of marginal productivity.

A first improvement was to drop the assumption of constant returns to scale. The α and β exponents both become endogenous. If the sum α+β approaches unity, it is seen as an additional confirmation to the results of 1928. Recognizing that "The use of time-series data in computing the production functions carried with it a host of technical problems", Douglas explains in a retrospective article published in 1976: "The second important change introduced was to substitute cross-section studies of separate industry observations for the previous time series" (1976 p.905).

In this same article, Douglas congratulates himself about the result of these studies. They agree on a sum α+β close to unity. In addition, the values of α and β are always close to the relative shares of national income that evidence grants to capital and labour. As another reason for satisfaction, the sector residues calculated with respect to α+β have the positive or negative sign where this result is expected, given the characteristics in terms of competition of the sector considered.

The Cobb-Douglas function seems thus to be the most favourable way for empirical validation of the law. But these results are controversial. The disagreement here is not about the conformity of the calculated data with the observed data: it is indisputable. Critics have pointed some shortcomings of the work of Cobb and Douglas since the first publication in 1928. Especially, they omitted to isolate technological progress.

$$P(t) = A(t).f(K,L) \qquad\qquad (7)$$

This is the first production function integrating technical progress, introduced by Solow in 1957. The variable $A(t)$ captures the effect of technical progress on production.

On a personal note, I would add this double criticism against the statistical work of 1928:
- *Capital* input is reduced to *fixed capital*. The *circulating capital* is deliberately excluded. Yet interest remunerates total capital composed of both parts.
- *Labour* input includes only blue collars; clerical employees are excluded, whose number has increased twice more, according to the authors themselves, who seem aware of this weakness of their analysis.

It is also worth mentioning a more theoretical criticism: aggregation of microeconomic functions into a macroeconomic production function is valid only under very restrictive conditions. Franklin Fisher demonstrates this mathematically in the article

"The Existence of Aggregate Production Functions" (1969), but to develop this aspect would take us beyond our subject.

Let us come to the main criticism, which denies any significance to the good performance of the Cobb-Douglas function in empirical tests. Its robustness could be explained by a reason other than coincidence between the function and the real economy. This argument is expressed by great economists like **Phelps-Brown**, **Simon** and **Samuelson**. The paper "The Estimation of the Cobb-Douglas Function: A Retrospective View" by **Felipe** and **Adams** synthesises this problem very well.

Let us write the tautological identity between, on the one hand, the product-national income and, on the other hand, the sum of the unit wage multiplied by the amount of labour plus the rate of profit multiplied by the capital. This last term includes both the normal cost of capital and the pure profit:

$$P \equiv a.L + r.K \qquad\qquad (8)$$

Let us call α' the relative share of wages in income deduced from (8). We have α' = a.L/P.
As we have seen, from the partial derivative of the Cobb-Douglas function with respect to *L*, it comes that α = a.L/P (where a = $\partial P/\partial L$).
So, the α of Cobb-Douglas is similar to the α' of the accounting identity.

And Felipe and Adams to conclude: "As Samuelson (1979) conjectured, this explanation is that all the aggregate Cobb-Douglas function regression captures is the path of the value added accounting identity according to which value added equals the sum of the wage bill plus total profits" (2005 p.430). Identity (8) can always be transformed into a function P = F(K,L,t). The authors add: "since what has been estimated is simply an identity, or a very good approximation of it, nothing can be inferred" (2005 p.434). Similarly, the estimates result inescapably in a sum α+β close to one.

Reworking the data of the 1928 article, Felipe and Adams also make the surprising finding that the correlation of the Cobb-Douglas function with actual data worsens when they add the variable *t* to account for technical progress: β then takes negative values.

FOR MARGINAL PRODUCTIVITY TO BE DEFINED

For the law to be valid, its central concept, marginal productivity, must be defined. The existence of F_K supposes continuity and differentiability of the production function. Jumps in production resulting from the removal or addition of a factor unit are not allowed. Isoquants must not include linear segments forming angles because different isocosts could be tangent to a same point at the corner. The production function must provide sufficient freedom for substitution between factors, which excludes fixed factors, fixed proportion factors, complementary factors[12]. Let's look at the conditions for these requirements to be met.

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[12] **Pareto** was in favour neither of the production function nor of the marginal productivity theory precisely because, in his opinion, all productions combine fixed-coefficient factors with variable-coefficient factors and because variable coefficients are not necessarily independent amongst

The production function is a very flexible concept. It can be applied to a production process in a workshop, a factory, an enterprise that has multiple production units, a sector with many businesses or the economy as a whole. Similarly, a factor can be examined at various levels; for instance, a unit of labour could be a welder, a blue-collar, an employee.

The more general the analysis, the more the continuity of the function is plausible; the more it turns towards the specific, the more discontinuities are likely. If I carry out Cobb and Douglas's exercise that considers two factors, capital and labour, in the American manufacturing industry as a whole, the size of the indivisibilities remains minimal in relation to the volume of factors of production. On the other hand, if I am interested in the productivity of the various functions in a workshop, the observation will necessarily occur that for some of them, if I remove one unit, production stops.

A conclusion emerges: the concept of marginal product seems sufficiently defined to determine remunerations, only in the scope of GENERAL analyses. The disadvantage is that general analyses can explain only general remunerations; for example the average remuneration of employees in France. An internal analysis inside a firm will hardly lead to significant results. The more the results are mathematically significant, the less they allow understanding wages, considering the triple heterogeneity that characterises labour in the real economy: of employers, of tasks and of individual qualities of employees.

To this distinction between the general and the specific echoes a distinction between the long term and the short term. Consider an increase in the volume of a factor of production: this QUANTITATIVE variation affects output differently depending on the time left to the factor to reorganise itself QUALITATIVELY. With more workers, it may be necessary to alter the division between occupations. With more capital invested, technological choices need maybe to be revised. But these adjustments take time.

Short-term analysis leads to a discontinuity problem. If I add a unit of a factor without rearranging anything, in many cases it will be of no use. If I remove one, in many cases the whole production will be disorganised or even stopped. Obviously, the short-term concept is a stalemate. This is not the way how **Clark** conceives of marginal productivity. He writes: "The final increment of the capital of this railroad corporation is, in reality, a difference between two kinds of plants for carrying goods and passengers. One of these is the railroad as it stands, with all its equipment brought up to the highest pitch of perfection that is possible with the present resources. The other is the road built and equipped as it would have been if the resources had been by one

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themselves. He gives these two examples: to produce iron, at a given technological level, the quantity of ore is fixed. Increasing land area of a firm does not compensate for reductions in any other factor. To the production function, he prefers the Walrasian-type equations:

$$p_a = a_t.p_t + a_p.p_p + a_k.p_k + ... \qquad (n1)$$

The cost *p* of product *A* is the sum of the prices of factors *t* , *p* , *k* , ... weighted by the respective technical coefficients. Factors prices being imposed by competition, the entrepreneur minimises *p* by choosing the technical coefficients so as to have the differential of equation (n1) equal to zero. Pareto shows that this method is capable of handle fixed or interdependent coefficients.

degree less (…) the final increment of the capital of this industry is not one that can be physically taken out of it" (1908 p.140).

Arthur Cecil **Pigou** (1920) confirms Clark's conception. Before quoting the same passage from Clark's work as above, he states: "For the addition or subtraction of a small increment can be accomplished in several different ways with correspondingly different results. We are here concerned with a particular way. For us the marginal net product of any flow of resources employed in any use or place is equal to the difference between the aggregate flow of product for which that flow of resources, *when appropriately organised*, is responsible and the aggregate flow of product for which a flow of resources differing from that flow by a small (marginal) increment, *when appropriately organised*, would be responsible. In this statement the phrase *when appropriately organised* is essential. If we were thinking of marginal net product in the sense of the difference between the products of two adjacent *quantities* of resources, we should normally imagine the resources to be organised suitably to one of these quantities and, therefore, not to the other. Since, however, our interest is in the difference between the products of two adjacent *flows* of resources, it is natural to conceive each of the two flows as organised in the manner most appropriate to itself. This is the conception we need. It is excellently illustrated by Professor J. B. Clark"[13] (1932 pp.132-133).

The difficulty is that, conceived in this way, marginal product is the difference between two situations that are far apart in time and not clearly identifiable. How does the end of the adjustment process become manifest? Having gained credibility over the short-run version, the long-run marginal product has lost in precision. In the "when appropriately organised" approach of Clark and Pigou, it is even unobservable and immeasurable.

FIRST CONDITION OF VALIDITY : HOMOGENEOUS PRODUCTION FUNCTION[14]

Let us now examine what happens when the assumptions of the theory are not satisfied; we begin with homogeneity of the production function.

Entrepreneur, marginal productivity and residue

In my opinion, the theory of entrepreneur and profit is one of the main stumbling blocks of the neoclassical economy. It is characterised by cacophony; divergences on it go back to the first neoclassical generation. **Edgeworth** devoted an interesting article to it in 1904, which reviews the differences of interpretation. The crux of the problem

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[13] Pigou refers to the Cantabridgian concept of NET marginal product (see above: Marshall and Robinson), in which all factors undergo an adjustment (both quantitative and qualitative). I think Clark stays loyal to the restrictive concept of one factor that varies while all others remain constant.

[14] From Nomidis's analysis (2018), it emerges that more than constant returns to scale, it is the straightness of the expansion path that matters. The two conditions are close but do not coincide. The expansion path can be straight for a convex or a concave production hill. Moreover the neoclassical definition of constant returns to scale, which implies this rectilinear character, seems to me open to criticism in the sense that it is justified only by the search for mathematical ease. If we expressed the firm's size by the total cost or by the fixed factor's cost, it would allow variations in the proportion between factors.

is to reconcile the RESIDUE tending to zero due to competition with the need for entrepreneurs to gain an income.

Clark argues that the residue tends to zero. An innovation rewards the entrepreneur with a high profit. But competition shifts factors to where they are most profitable and increases their marginal product. Profit, evanescent, is gradually transformed into additional interest and wage. The existence of profit is due only to friction: "if competition worked without let or hindrance, pure business profit would be annihilated as fast as it could be created - entrepreneur as such could never get and keep any income" (1908 p.219). Edgeworth contrasts this conception with that of Mangoldt, from whom he cites these two quotations:
- "We must suppose the existence of undertaker's gain (Unternehmergewinn), otherwise what object has the entrepreneur to increase his business?"
- The undertaker's remuneration is "not simply something transitory" but "a permanent species of income" (1925 p.25).

It is common to link the entrepreneur function with risk-taking. But, as Edgeworth notes, "in Professor Clark's nomenclature, this risk is borne by the capitalist", which he illustrates by the quote: "Business repays men not only for their labours, but their fears" (1925 p.24). Clark is right. Contrary to a wide belief, pure profit (positive or negative - let's not forget it) is not the reward of risk but the cause of this reward, which takes the form of a *risk premium* contained in interest or even in wage or rent.

In no author, is the entrepreneur so disembodied as in Walras. The product is shared among the many factors belonging to the three major groups: labour, land and capital. The entrepreneur does not earn anything if he is not himself in one of these groups. But Walras avoids the problem of the residue by making it impossible: its object is a competitive general equilibrium, of which a class of equations equalises the price of the product with its cost of production representing the total of the rewards.

Other economists consider that the function of entrepreneur is in itself a factor of production, remunerated like others according to its marginal productivity. This is the case of Wicksteed, who like many authors, sometimes has difficulty distinguishing between entrepreneur and management. Within this conception, the necessary reconciliation between residue and marginal productivity generally leaves to be desired: for instance, Barone evacuates the problem of reconciliation with too much ease. Edgeworth cites him : "In such conditions, the law of marginal productivity extends to the remuneration of the entrepreneur ; and after having remunerated all the factors (the work of the entrepreneur included) in proportion to their marginal productivity (…), there remains no undistributed residue" (1925 pp.27-28). Fairly sceptical, Edgeworth remarks: "It is only with respect to factors of production which are articles of exchange that the proposed law of remuneration, the law of marginal productivity, is fulfilled in a regime of competition" (1925 p.28).

Résidue and factor remuneration
Let us return to **Wicksell**'s reasoning where we left it, when he was analysing remunerations in case of non-constant returns to scale. Let k be the quotient of total product over total factor cost. Whatever the returns to scale, equilibrium is given by equation (9) :

$$\frac{F_A}{W_A} = \frac{F_B}{W_B} = \dots = \frac{F_N}{W_N} = k \qquad (9)$$

Where W is the remuneration of the factor in subscript. For each factor, we have[15]:

$$W_A = F_A/k \; ; \; W_B = F_B/k \; ; \; \dots W_N = F_N/k \qquad (10)$$

If $k > 1$, the factor's pay is less than its marginal product; if $k < 1$, it is higher. The remuneration of the factor is no longer necessarily equal to the value of the marginal product, but the ratio of marginal product to remuneration is still the same for all the factors.

Profit, which will be a loss if $k < 1$, is equal to the sum of the marginal productivities of factors i minus the sum of the remunerations:

$$\pi = \sum_{i=1}^{N} F_i - \sum_{i=1}^{N} W_i \qquad (11)$$

It is clear that this profit is a pure residue and that it does not pay a service necessary for the accomplishment of the production.

Who gets the profit? Marginalist theory answers: the entrepreneur. We have seen the vagueness about him in neoclassical theory.

In my opinion, a clear understanding of the entrepreneur question involves distinguishing between the function and the organ. As a function, he is isolated, particular. But it is impossible for a person to be invested with this single function except any other. The undertaker is none other than one of the factors of production, invested with a special status. As a function, it is distinct from the factors; as an agent, he is one of them. Depending on the type of enterprise, the status of entrepreneur goes to one factor or another, possibly to several ones or to only a specific part of one of them. Under capitalism, this function rests most often on the shoulders of the shareholders, who may delegate its exercise to a single person, inside or outside the factor[16], as the CEO.

As a result, with factor K as the entrepreneur, ordinary factors' pay is worth F_i/k ($i = A\dots J,L,M,N$) and that of the factor-entrepreneur is worth $F_K/k + \pi$. So, the dividend of shareholders includes indistinctly F_K and π. There is no longer a single proportion between factors' remuneration and their marginal product. With profit, the law is inoperative. In the neoclassical theory which disembodies him, the entrepreneur intervenes opportunely as a deus ex machina that saves the appearances of the law.

SECOND CONDITION OF VALIDITY : COMPETITION

After returns to scale, the other fundamental assumption of the law is perfect competition in both the factors and product markets. Historically, this matter has mostly been treated by Pigou and by Robinson. Both have used the concept of labour *exploitation* to describe a situation where employees are paid below the value of their marginal physical product.

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[15] In fact, Wicksell considers only two factors, labour and land; capital does not intervene before the next chapter of the "Lectures", where it is handled in the Austrian way.

[16] I consider as essential this distinction between the status of entrepreneur and the exercise of the function.

Exploitation of labour

Mobility of labour , **Pigou** explains, normally ensures the employee to get a wage up to his marginal product (in value). But, he adds, "In so far, however, as movements of workpeople are hampered by ignorance and costs, a monopolistic element is introduced into the wage bargain. Consequently, there is created *a range of indeterminateness,* within which the wages actually paid to any workman can be affected by individual 'higgling and bargaining'" (1932 p.557). As stated by Pigou, if bargaining becomes decisive, employers are favoured because they are more accustomed to negotiate and because the disadvantage suffered by not hiring a worker is less than that of not being engaged. Collective organisation of workers can compensate this handicap.

In the opinion of **Robinson**, exploitation of labour may have its origin in the imperfection of the product market (inelastic demand) as well as in the imperfection of the factor market (inelastic supply). The firm optimises its situation by hiring labour as long as its *net marginal product* covers its *marginal cost.*

- Exploitation due to imperfections in the <u>product market</u>, In this case, marginal revenue is lower than price and therefore the *net marginal product* is less than the *value of the marginal physical product.* Wage, equalised with the net marginal product, will therefore be lower than its normal value.

- Exploitation due to imperfections of the <u>labour market</u>. Several cases are possible, the simplest of which is the following: the wage paid by the firm (monopsony) follows a curve that increases with the number of employees hired. *Marginal cost of labour* is higher than its average cost, hence higher than wage, which therefore will be less than the value of marginal physical product.

Robinson's conception seems dogmatic. She considers that the value of the marginal physical product remains the norm in an imperfectly competitive environment. As this rule is based on the play of perfect competition, this amounts to saying that only perfect competition is admissible.

Monopoly rent

When the firm is a monopolist or an oligopolist in the product market, it gets a rent in the form of a surplus profit. Many economists consider that the factors of production have potential access to it. Rent sharing is a matter of bargaining and therefore of power relations. It is obvious that under such conditions, our law cannot be de rigueur.

Van Biesebroeck (2015) is interested in the sharing of this type of rent. It is usual to see sectors or firms with above-average profits paying higher wages. Kalecki is the first great economist to have considered this observation. The question is: « Are unobserved worker qualities the source of higher profits or are workers able to appropriate some of the profits that originate from uncompetitive product markets" (2015 p.19). The author mentions two studies that try to answer:

- **Krueger** and **Summers** (1988) : "Even after controlling for workers observable and unobservable characteristics and institutional features that are likely to explain sectoral wage differences, such as fringe benefits, demand shocks, actual or

threatened unionization, they find a positive association between wages and industry profits for equally-skilled workers" (2015 p.19). High wages are then the result of appropriating a part of the rent.

- **Gibbons** and **Katz** (1992) study the wage consequences of job changes as a result of plant closings. Constancy of the wage level of the worker, when it beforehand included a "premium", would be an indication that his personal qualities (eventually unmeasured) interfere. The conclusion of their study is mixed. The attribution of cross-industry wage differences to a single cause seems very difficult.

Many studies have focused on cross-industry wage gaps. **Du Caju, Rycx** and **Tojerow** (2011) mention some of them, which concern various countries. The phenomenon of high salaries in the most profitable industries is very widespread and persistent. The authors analyse the data of the Belgian private sector from 1999 to 2005. Their result shows a considerable influence of rent-sharing but confirms that it does not explain by itself the whole dispersion of wages of apparently similar occupations.

SOME THEORIES CONTRADICTING THE LAW

Let us come to a few causes that invalidate the law, because they involve factors that the law did not take into account. This review is not intended to be exhaustive.

The Austrian theory of capital

Chronologically, the first blow came from the Austrian school. At about the same time as Clark deduced wage and interest of the marginal productivities associated, **Böhm Bawerk** distinguished between capital and what he called the *primary factors*, land and labour. The remunerations of the primary factors is explained by their marginal productivity but interest of capital is not. The interest rate is equalised with the marginal productivity of the lengthening of the production period. This is the famous formula $f'(t)/f(t)$ already conceived by Jevons, where *t* is the *production period* and $f(t)$ is the quantity produced being an increasing concave function of this period.

Wicksell goes further into this idea. Capital consists in the "produced means of production", the best definition according to him. But, unlike land, measurable in hectares, and unlike labour, measurable in hours/men, capital has no inherent measure. Its heterogeneity is much greater than that of labour and land. The only practical measure is its value expressed in any kind of numeraire. But this measure is imperfect. Consider a situation where savings increase the value of invested capital: this results in an increase in the demand for labour and land and hence an increase in wages and rent. Precisely, wage and rent constitute the substance of circulating capital. Part of the increase in capital does not therefore contribute to lengthen the production process, but is absorbed by the increase in the cost of primary factors. The change in the value of capital is therefore not a good indicator of the change in physical capital.

In his "Lectures on Political Economy", simplifying the problem to retain only circulating capital, Wicksell algebraically demonstrates that the marginal product of capital in value is necessarily less than interest. In a later article that includes fixed capital (Akerman's problem) , he comes to the conclusion that the marginal product of capital in value may be lower or higher than the interest.

Starting from the non-measurability of *physical capital* and the impossibility of determining *value capital* independently of the interest rate, the Cantabridgians sparked a controversy with the defenders of the neoclassical school. They proved that the demand curve for capital which monotonically declines with the interest rate is only valid under an unrealistic assumption: capital intensity must be the same in all sectors. In this very perspective, Bhaduri (1969) demonstrated that the same condition is necessary for the interest rate to the equal marginal productivity of capital:
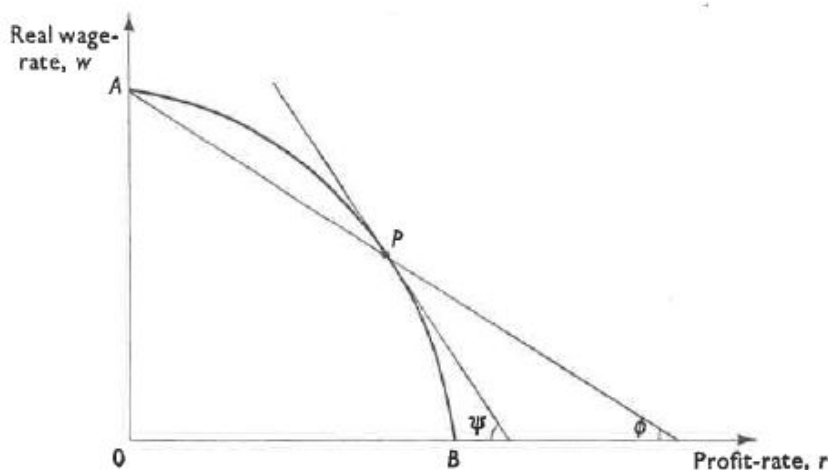
$$Y = K.r + L.w$$
$$y = k.r + w \quad \text{where} \quad y = Y/L \text{ et } k = K/L$$
$$dy = r.dk + k.dr + dw$$

To get dy/dk = r, the well-known marginal condition, implies that k.dr + dw = 0. So :

$$-\frac{dw}{dr} = k \qquad \qquad (11)$$

This graph shows the *factor price frontier* that has proven very useful in the *capital controversy*. Suppose we are at point P on this curve which indicates the combinations between *r* and *w* accessible for a given technique of production. According to equation (11), *k* corresponds to the tangent of angle Ψ.

At the same time, the value of capital equals the present value of profits:

$$k = \frac{y - w}{r}, \quad r > 0 \qquad \qquad (12)$$

In this equation, *k* corresponds to the tangent of angle ϕ, different from angle Ψ. To satisfy equations (11) and (12) simultaneously, the two angles must be equal; this will happen only if the factor price frontier is a straight line. This is precisely the characteristic when the production of the final good and the one of the capital good have the same capital intensity.

Theories of incentive wage

The reason why the competitive entrepreneur pays his factors of production at the value of their marginal product is that it leads him to maximise profit. In parallel, profitability also requires that wages stimulate effort. A priori, the wage determined on the basis of productivity satisfies this requirement. The problem is that individual productivity of employees is difficult to control. Wage methods that circumvent this difficulty may ultimately be more incentive than the productivity wage. Economists impute to enterprises various wage systems aimed at incentive. Let us see the two best known :

1- The *theory of efficiency wages*. **Shapiro and Stiglitz** (1984) explain: "…the inability of employers to costlessly observe workers' on-the-job effort can explain involuntary unemployment as an equilibrium phenomenon. » (1984 p.433). In perfect Walrasian competition, involuntary unemployment never happens. Employers, therefore, have no way of exerting pressure on shirkers. Dismissed employees will immediately find a new job of equal level. Shapiro and Stiglitz's idea is that employers deliberately offer a real wage higher than the one which clears the market. With such a wage, supply is superior to the demand, which implies involuntary unemployment. So, the threat of unemployment serves to dissuade employees of shirking[17]. This system is not necessarily more expensive than a system of permanent observation of the effort provided by each worker. "The equilibrium unemployment must be sufficiently large that it pays workers to work rather than to take the risk of being caught shirking" (Ibid).

2- *The theory of tournaments.* This theory was first expounded in 1981 by **Lazear** and **Rosen**. The idea is that the salary grid based on the hierarchy of functions rewards the ordinal variable that is the place in the ranking of employees rather than the cardinal variable which are the marginal products, because it is easier to observe. The better salary of the higher position is designed as the incentive for lower-level workers to work harder in order to be selected for promotion when the position is vacant. Such a system tends to widen the range of wages in relation to the range of productivities.

   Lazear and Rosen conclude: « This results in paying wages which resemble prizes : wages which differ from realized marginal products » (1981 p.863).

THE PRODUCT EXHAUSTION AGAIN

As neoclassical theory recognises, the firm has its full equilibrium at the minimum of its long-term average cost curve only if demand and marginal revenue curves are horizontal. This condition may turn out to be more restrictive than it seems.

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[17] In another article (2001), Stiglitz identifies four reasons motivating enterprises to pay an *efficiency wage* :
   - decrease staff turnover ;
   - attract to her the best workers ;
   - induce the workers to work harder (the case presented here) ;
   - provide the workers with the moral satisfaction of obtaining a fair wage.

Problem 1: the marginal revenue curve is not horizontal

Marginalist theory asserts that in perfect competition, the firm's demand curve is horizontal. But is that correct? In his book "Debunking Economics", Keen shows that it is not, considering a footnote in Stigler's article "Perfect Competition, Historically Contemplated" (1957). This note gives the formula of *marginal revenue* by differentiating the *total revenue*:

$$\frac{d\,(p\,q_i)}{d\,q_i} = p + q_i\,\frac{d\,p}{d\,Q}\,\frac{d\,Q}{d\,q_i}, \qquad (13)$$

Where p is the price, q_i is the quantity supplied by the firm, Q is the quantity supplied on the market. With the usual neoclassical assumption that the rival firms do not react, we have $dQ/dq_i = 1$. The expression dp/dQ indicates the slope of the market demand curve, which is declining. It is therefore inevitable that even in perfect competition, marginal revenue is declining, unless by a happy coincidence, dQ is zero. The average revenue and average cost curves are tangent to the left of the minimum cost, at a point where returns to scale are decreasing. There is no precise product exhaustion.

Problem 2 : effective competition or perfect competition ?

In addition to this formal reason, there are more economical arguments to challenge the horizontal shape of demand, even in a fully competitive environment. In two articles available on the MPRA website, I criticise the neoclassical conception of competition and profit maximization. I developed the idea that real world competition involves a market characterised by a small number of *price maker* firms that seek to maximize their long-term profit. *Perfect competition* as described by neoclassical economics is only representative of organised markets, a very minority part of the economy. The theory presented in these two articles admits what empirical studies generally indicate: returns to scale are generally constant or increasing.

Enterprises working in such a context face a decreasing demand. It is demand, not technology that limits the size of the business. In this environment, nothing entails equality between remuneration and marginal productivity.

DOES THE LAW APPLY TO CEO REMUNERATION?

This question is widely debated and is the subject of a lot of literature, especially in the United States, where there has been a striking increase in CEO compensation over the last forty years. **Shorter** and **Labonte** observe this increase with these words : "According to one estimate, between 1994 and 2005, the ratio of annual median CEO pay to median production worker pay nearly doubled, growing from 90 to 1 to 179 to 1. Indeed, for many workers, the perceived excessiveness of executive pay has become the most visible embodiment of growing pay inequality, contributing to a feeling that workers have not shared in the gains from economic growth. For example, adjusted for inflation, average worker pay rose 8% from 1995 to 2005, median CEO pay at the 350 largest firms rose about 150% over the same period" (2007 p.2).

Some economists consider that these pay gaps reflect productivity gaps between the agents concerned, but others are perplexed in front of these staggering figures. So was born a theory, called "managerial power critic". The article by Shorter and Labonte

summarises it. I will not develop it here because the arguments are more sociological and legal than economic, which does not correspond to the nature of the present paper. The authors detail the institutional weaknesses in the corporate world that allow CEOs to at least partially free themselves from the control of shareholders and directors.

Distributions of securities and stock options, which account for a significant portion of the CEO's compensation, reward their ABSOLUTE performance, which integrates general market growth, and not RELATIVE performance, which would take into account a comparison to other firms in the sector.

Other studies address the problem in its economic dimension. Mayer (2018) notes three explanations, alternative to that of productivity:
- The CEO market is not a <u>competitive market</u>. This argument is in accordance with the "managerial power critic". By freeing themselves from control, the CEO's exercise a kind of monopoly power.
- The CEO market is characterized by <u>inelastic supply</u>, for talent is scarce. A similarity is sometimes alluded to the case of stars of the show business. In my opinion, this comparison does not hold because it is precisely the notoriety of the stars that stimulates the sales of the entertainment industry. CEO's activate sales not by their notoriety but by their skill.
- The CEO market is characterized by <u>inelastic demand</u>. This is Mayer's thesis. He explains it so: "A firm typically employs only one CEO. Thus, neither an increase nor a decrease in CEO pay may affect the number of CEOs employed. Accordingly, the theory of marginal productivity may not apply to CEOs" (2018 p.15). Two of the factors mentioned by Marshall in his analysis of elasticity account for this inelasticity: the absence of substitutes and the relatively low share of CEO remuneration in total cost, precisely because it is unique.

This argument of inelastic demand seems to me rather convincing; it must however be put into perspective owing to a counter-argument. Considering the link between price and marginal utility, Wicksell drew attention to a particular aspect: the independent variable can be either the quality level or the quantity consumed. He gives the example of a buyer facing three horses. Given their age, endurance and strength, the prices are respectively 500, 550 and 575 shillings. The buyer will choose not the quantity of horses but the quality of the horse so as to equalise the price and the marginal utility. There is an underlying variable quality level that is less observable and less objective than quantity but nevertheless influential. An analogous application of this principle to the CEO's marginal productivity would be to rank the CEOs on the market by skill level and to consider that companies choose according to their budget the number of competence units which equalises remuneration with marginal product value. Unlike the number of CEO positions in the company, the competence units would be a continuous variable over a sufficient interval. A shortcoming of such a conception is the immeasurability of the decision variable. One can also doubt the ability of all firms to develop this ranking of potential CEOs and to appreciate precisely the adequacy between their qualities and the specificities of the firm.

MARGINAL SOCIAL PRODUCT AND MARGINAL PRIVATE PRODUCT

As the name suggests, marginal product is a product. The concept of product is to be taken in a broad sense. It includes both services and physical products, both services to persons (hairdressing, leisure ...) and to business (transport, storage, distribution etc.). This product also includes the contribution of support functions within the enterprise. If you cannot run a production without computers, without accounting, without catering, without security, these functions add value to their enterprise.

Normally, labour enriches society. By adding value at the enterprise level, it increases the national income which is the sum of these added values. In parallel to this very common form of productive occupations, some functions add value to their enterprise, but entirely or partially to the detriment of competing firms or other economic agents. Let's mention three examples:
- CEO's, executives
- Lawyers
- Advertising agencies

Let us look at this particular type of productivity in the case of CEOs. To a large extent, TOTAL demand on the market for its product is exogenous to the action of the firm, so it is rather on the market SHARE that it will act to maximise the sales volume. To succeed, the firm counts on the qualities of its CEO. The more efficient he is, the lower competitors' sales will be. Possibly, the employees may be another losing group, because the efficient CEO could get more out of his employees than competitors while paying the same remuneration.

The work of the CEO is of a special nature. Efficiency of other functions is absolute and not only relative, while, to a certain extent, the CEO is inefficient if he is less efficient than the CEOs of competing companies.

The case of advertisers is quite similar. Advertising is effective only if it is more convincing than that of competitors, in which case, ceteris paribus, it increases the market share of the firm at their expense.

The present argument distinguishing the functions that handle profits and those that produce is simply a consequence of the fact that competition is the mode of organisation of the economy.

What most matters in this distinction is its relation to the distribution of income. It is not surprising that a company is more motivated to pay well for functions perceived as having a more direct impact on its profit. Gaining one or two percent of market share will appreciably enhance the shareholders' s position. It would be difficult for the men behind the scene to achieve an equally profitable result.

An interpretative framework of this problem is offered by the welfare economics of **Pigou**, more precisely the distinction between the *marginal PRIVATE net product* and the *marginal SOCIAL net product*. He explains: "For complete accuracy it is necessary to distinguish between two senses in which the term marginal increment of resources may be employed. It may be conceived either as being added, so to speak, from outside, thus constituting a net addition to the sum total of resources in existence, or as being transferred to the particular use or place we are studying from some other use or

place (1932 p.131) The marginal social net product is the total net product of physical things or objective services due to the marginal increment of resources in any given use or place, no matter to whom any part of this product may accrue (p.132) The marginal private net product is that part of the total net product of physical things or objective services due to the marginal increment of resources in any given use or place which accrues in the first instance- i.e. prior to sale-to the person responsible for investing resources there" (p.134). In normal situations, both types of marginal product coincide.

Suppose that all the other difficulties about the marginal productivity concept are resolved. It could then be said that the CEO's labour has a positive and high marginal private product, but that his marginal social product is significantly less (but certainly not zero).

Which of the two marginal products, the private or the social, is to be considered as the agent's CONTRIBUTION? Here is one more difficulty related to the concept of contribution[18].

CONCLUSION

The law of retribution according to marginal product arouses doubt or even suspicion; some find it easy to see through in order to justify the existing distribution of income.

The most dazzling enrichments are often the case of great innovators. These innovations, which cannot be copied for legal or other reasons, are incompatible with perfect competition. The innovative firm that pays its factors at their marginal productivity then benefits from a significant residue that takes the form of a profit accruing to the entrepreneur-innovator. It must not be expected that marginal productivity explains the whole of the question.

Empirical studies do not seem to support the law. Their reliability is however questionable, because unlike average productivity, marginal productivity, an abstract concept, is not directly observable. No time series will ever show the extra output due to the marginal increase of one factor when all others remain constant. As for the estimation of the production function, the approach seems optimistic; the difficulties are numerous: human capital, technical progress per se, incorporation of a part of technical progress…

We have identified a number of specific aspects of economic reality that may either impede the determinateness of marginal product or divert earnings from this norm:

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[18] After having written this section, I found the article by Gottschalk and Tinbergen (1982), which relates **Tinbergen**'s theory of *counterproductivity*. Here are a few citations: "… concept of counterproductivity which suggests that marginal revenue products may not be a useful concept for setting the wages of certain types of labour. Counterproductivity is the phenomenon that it is some individuals' task to partly or wholly destroy somebody else's product (…) the total productivity of all the sales workers of firms *A* and *B* is small. The same applies to their marginal productivity. Does this mean that these workers should be paid almost no income? Not at all: their social function is to maintain a competitive system, which produces significantly more than a monopolistic system, where all product groups are monopolized" (p.332).

- The delay inherent in the marginal product if it is conceived in the Clark-Pigou way implying optimal adaptation of technology to the quantitative variation of the factor.
- The indivisibilities that make the production function non differentiable if the analysis relates to microeconomic entities
- The existence of rents due to monopoly or imperfect competition, gained by the factor-entrepreneur which he eventually shares with other factors as a result of bargaining
- Dominant positions in the factors markets
- Non-constant returns to scale that generate a profit or a loss that affects the remuneration of the factor-entrepreneur
- The doubt induced by the Austrian theory of capital that interest equals marginal productivity of capital
- Business practices in human resources management, including efficiency wages and tournaments
- The sensitivity of the recruitment wage to the business cycle, higher than that of the current wage. Both cannot be equal to the marginal productivity of labour
- The significant gap between private and social marginal productivity that characterises several important functions.

In itself, like many laws of economics, the one explaining retribution by marginal productivity results from an impeccable logical reasoning. But this logic handles very few variables in comparison to the complexity of the real economy. And its assumptions are quite restrictive.

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