Measuring the Dynamic Cost of Living Index from Consumption Data

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Abstract

In the U.S., the objective of consumer price index (CPI) measurement is to measure the cost of living. However, the current CPI or, in other words, cost of living index (COLI) measures the cost of living in a static optimization problem. This paper proposes a new method to construct a dynamic cost of living index (DCOLI). Our method offers several advantages compared to other dynamic cost of living indices proposed in the literature. First, our measure is based on total wealth. Previous indices limited attention to financial wealth. Second, we consider an Epstein-Zin preference structure. Most previous

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literature has used log preferences. We derive formulas that relate our DCOLI to the COLI and derive conditions under which the two coincide. We also produce empirical measures of our DCOLI. We find that under standard assumptions on preferences, the volatility of our DCOLI is about the same as that of the COLI. In certain periods, e.g., 1977–1983, our measure differs sharply from the COLI.

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1 Introduction

The Consumer Price Index (CPI) is the most widely used measure of the general price level in the U.S. Taxes, welfare payments, retirement payments, and labor contracts are all indexed to the CPI. Stabilizing CPI growth is a central objective of monetary policy.

In the U.S., it is generally accepted that the objective of CPI measurement is to measure changes in the purchasing power of money (for details, see, e.g., the Bureau of Labor Statistics (2007)). That is, the CPI is a "cost of living" index (COLI). Changes in a cost of living index are defined as the ratio of the expenditure function evaluated at different prices. Current COLI measurement implicitly assumes that the expenditure function is associated with a static expenditure minimization problem.

It has been known for a long time that this assumption has some problems. If a household is active for more than one period, then the cost of living should reflect not just the price of today's goods but also that of future goods. Alchian and Klein (1973) point out this problem and propose a dynamic cost of life index (DCOLI) that recognizes that the money cost of goods includes the cost of future goods as well as that of current goods. Pollak (1975) provides a general theoretical treatment of intertemporal price indices.

A DCOLI has some very attractive properties. A DCOLI measures the money cost of yielding a reference level of lifetime utility. If a change in prices leads to an increase in the DCOLI, it implies that the money cost of goods has risen. In other words, the lifetime utility delivered by the reference level of nominal wealth has fallen. In situations where households are active for many periods, the properties of a DCOLI can, in principle, differ substantially from those of a COLI, which just focuses on current period utility.

However, there are some major obstacles to measuring a DCOLI. One obstacle emphasized by Alchian and Klein (1973) is measuring nominal wealth. In principle, one needs futures prices for each component of nominal wealth. Due to the absence of markets for these goods, one has to infer their prices indirectly by imposing restrictions on preferences and assuming complete markets.

Some of the first efforts to construct a DCOLI took place in Japan. The large swings in Japanese asset prices in the 1980s and 1990s precipitated a discussion about whether asset prices should be considered when setting monetary policy. Shibuya (1992) assumes that households have log utility, measures wealth as financial wealth, and assumes that the real return on wealth is constant. He finds that the money price of goods using a DCOLI differs significantly from the money price using a COLI before the first oil price shock in 1973 and also between 1985 and 1990. Shiratsuka (1999) relaxes the assumption of constant real returns and addresses the question of whether a DCOLI should be used when setting monetary policy. His answer to this question is negative: he suggests that a DCOLI is considerably more volatile than the GDP deflator; that the reliability of the measurement of certain assets used to construct the DCOLI in the previous literature, such as land and house prices that receive large weight in wealth, is low; and that asset prices may respond to variations in spurious variables (e.g., sunspots). Reis (2005) constructs a DCOLI using U.S. data and also finds that it is much more volatile than the COLI. These problems have led Bryan et al. (2001) to adopt an empirical approach for measuring the dynamic cost of life that combines some restrictions from theory with an econometric approach for identifying good indicators of future prices. One feature common to all this previous research is that human wealth is not used when constructing the measure of the DCOLI. Shiratsuka (1999) points out that the human wealth component is large but argues that it is hard to measure and only reports results for a DCOLI that uses financial wealth.

The measurement of wealth has received considerable attention in finance because wealth is important for asset pricing. Jagannathan and Wang (1996) emphasize the important role of accounting for human wealth in pricing the cross-section of returns. Campbell (1996) describes a methodology for deriving the dynamics of total wealth from a vector autoregression (VAR) and investigates the dynamics of asset pricing using Epstein-Zin preferences. Lustig et al. (2008) estimate that 85% of total wealth is human wealth. They also propose a strategy for measuring human wealth that is robust in the sense that they do not have to take an explicit position on the expected returns on human wealth or its growth rate. They find that the volatility of human wealth and thus total wealth is considerably lower than that of financial wealth. A common theme underlying this entire literature is that restrictions from preferences are not used to restrict the dynamics of human wealth.

One contribution of this paper is that we use restrictions from preferences to identify and estimate both human and total wealth. We adopt a specific preference structure, assume complete markets, and derive a stochastic pricing kernel. Then, we use this pricing kernel to value dividends on human and financial wealth.

We also consider a class of preferences that is more general than that used in the previous literature on DCOLI measurement. Shibuya (1992) and Shiratsuka (1999) both assume log preferences. Reis (2005) uses log preferences for most of his analysis but considers a generalization to Epstein-Zin preferences. His analysis of this case imposes the assumption that equity prices follow a random walk and that goods prices follow an AR(1) in first differences. In addition, he does not produce an empirical measure for this preference structure.

We assume Epstein-Zin preferences throughout. Research by Bansal and Yaron (2004) finds that this preference structure in conjunction with the assumption that consumption growth has a small long-run risk component can account for many key asset pricing anomalies. Using this preference structure, we are able to derive a representation that decomposes the growth rate of the DCOLI into two components: the growth rate of the COLI in a static problem and the real dynamic cost of living index (RDCOLI). We find that when the EIS (elasticity of intertemporal substitution) is very large, the DCOLI coincides with the COLI. Our DCOLI also has the property that its long-run growth rate coincides with that of the COLI.

We now summarize our empirical results. First, there are sharp differences between the COLI and DCOLI in 1973–1976 and 1977–1983, i.e., around the time of the first and second oil crises. During these periods, the RDCOLI, which is equal to the DCOLI minus the COLI, experienced the sharpest decline. This indicates that the prices of future goods sharply fell or, in other words, the expected future returns on total wealth increased. Second, contrary to the previous research, the volatility of our DCOLI is about the same as that of the COLI. We find that the difference between our result and those of the previous studies stems from the fact that we take into account human wealth; the volatility of the DCOLI that only takes into account dividends from financial wealth is about four to eight times higher than that of our DCOLI, which also takes into account dividends from human wealth in the log utility case.

The rest of the paper is organized as follows. Section 2 presents a representative consumer problem, defines the DCOLI and RDCOLI, and derives the formulas of the DCOLI and RDCOLI that can be measured from the data. Using these formulas, Section 3 constructs the DCOLI and RDCOLI from consumption data. Finally, Section 4 concludes the paper.

2 Model

Conceptually, the DCOLI is the relative nominal expenditure necessary (and sufficient) to yield a certain utility level. We first formalize the concept of a DCOLI without specifying the preference structure and the dynamics of prices. We also introduce the RDCOLI (real DCOLI), the counterpart of the DCOLI in real terms. The growth rate of the DCOLI is decomposed into the usual inflation rate of the COLI and the growth rate of the RDCOLI (Equation (7)).

Then, we specify the preference structure as in Epstein and Zin (1991), which includes the standard expected utility specification as a special case, and which has been widely used in recent financial literature. The information regarding the growth rate of the RDCOLI is then reduced to the change in the (optimally chosen) wealth-consumption ratio, which is independently determined from the reference utility levels (Equation (14)).

Finally, we specify the dynamics of prices as being conditionally homoscedastic. Then, by applying Campbell's (1993) method of approximation, the change in the wealth-consumption ratio and hence the growth rate of the RDCOLI are shown to be approximately equal to the change in a linear combination of the expected future consumption growths (Equation (25)).

Thus, the measurement of the DCOLI is reduced to yielding the estimates of (the linear combination of) the expected consumption growth rates. Our empirical exercise in Section 3 utilizes this result.

2.1 Formulation of the DCOLI

First, we introduce our general settings. The (dynamic and possibly stochastic) expenditure minimization problem is formalized according to the settings, and this is followed by the formal definition of the DCOLI.

2.1.1 Settings

We consider a representative, infinitely-lived consumer who evaluates consumption streams at period t in state s_t by some utility function U as

$$U(C_t, \{C_{t+j}(\cdot)\}_{j=1}^{\infty} | m(s_t)), \tag{1}$$

where C_t is the current (period t) consumption, $C_{t+j}(\{s_i\}_{i=t+1}^{t+j})$ is the consumption at period $t+j$ given that states $\{s_i\}_{i=t+1}^{t+j}$ have been realized, and $m(s_t)$ is the probability measure regarding the future states $\{s_i\}_{i=t+1}^{\infty}$ conditional on the current state s_t. Note that the probability measure is completely determined by s_t. Thus, all information regarding the evolution of the future states is included in the current state.

In addition to the consumption good, there are K assets, $k \in \{1, \ldots, K\}$, including all the possible income sources as human capital. As is often assumed in the standard financial models, all assets are always tradable. All information regarding current prices is also included in the current state: the prices in state s of the consumption good, assets, and the dividends of the assets are $P(s)$, $Q(s) = (Q_1(s), \ldots, Q_k(s), \ldots, Q_K(s))$, and $D(s) = (D_1(s), \ldots, D_k(s), \ldots, D_K(s))$, respectively.

2.1.2 Expenditure Minimization

By this setting, the nominal expenditure necessary (and sufficient) to yield a utility level $\bar{U}$ at period t in state s_t, $E(\bar{U}|s_t)$, is formalized as follows:[1]

$$E(\bar{U}|s_t) = \min_{C_t, A_{t+1}, \{C_{t+j}(\cdot), A_{t+j+1}(\cdot)\}_{j=1}^{\infty}} P(s_t)C_t + Q(s_t) \cdot A_{t+1} \tag{2}$$

subject to

$$U(C_t, \{C_{t+j}(\cdot)\}_{j=1}^{\infty} | m(s_t)) = \bar{U},$$

and for all $\{s_i\}_{i=t+1}^{\infty}$,

$$(D(s_{t+1}) + Q(s_{t+1})) \cdot A_{t+1} = P(s_{t+1})C_{t+1}(s_{t+1}) + Q(s_{t+1}) \cdot A_{t+2}(s_{t+1}) \tag{3}$$

and for all $j \geq 2$,

$$(D(s_{t+j}) + Q(s_{t+j})) \cdot A_{t+j}(\{s_i\}_{i=t+1}^{t+j-1}) = P(s_{t+j})C_{t+j}(\{s_i\}_{i=t+1}^{t+j}) + Q(s_{t+j}) \cdot A_{t+j+1}(\{s_i\}_{i=t+1}^{t+j}), \tag{4}$$

where $A_{t+j}(\{s_i\}_{i=t+1}^{t+j-1}) = (A_{1,t+j}(\{s_i\}_{i=t+1}^{t+j-1}), \ldots, A_{k,t+j}(\{s_i\}_{i=t+1}^{t+j-1}), \ldots, A_{K,t+j}(\{s_i\}_{i=t+1}^{t+j-1}))$ denotes the portfolio chosen by the consumer at (the end of) the period $t+j$ given that states $\{s_i\}_{i=t+1}^{t+j}$ have been realized.

[1] In general, a consumption path that is optimal in the current point of view may not be so when considered from a future point of view (time-inconsistency). Thus, by formalizing the expenditure minimization problem as if the current consumer can (or believes that he can) implement any consumption path, we implicitly assume some time-consistency (at least in his current view) here.

2.1.3 DCOLI

Now, we can compare the monetary cost of living in any period, t, with that of any other period, τ, through their associated states, s_t and s'_τ, respectively, and a reference utility level, $\bar{U}$.[2] That is, the DCOLI, $\pi(s_t|s'_\tau, \bar{U})$, is defined as follows:

$$\pi(s_t|s'_\tau, \bar{U}) = \frac{E(\bar{U}|s_t)}{E(\bar{U}|s'_\tau)}. \tag{5}$$

Note that the two periods may differ not only in terms of the current prices (represented by the difference between $(P(s_t), Q(s_t))$ and $(P(s'_\tau), Q(s'_\tau))$ but also in terms of the (expected) conditions of the future prices (represented by the difference between $m(s_t)$ and $m(s'_\tau)$). Note also that all such differences are reduced to the difference in the associated states, s_t and s'_τ.

2.1.4 RDCOLI

In a similar manner, we can also define the RDCOLI (real DCOLI instead of the nominal one), the relative real expenditure necessary (and sufficient) to yield a certain utility level, by evaluating the expenditure not in monetary terms but in terms of the current consumption good, i.e., by changing the minimization problem by just dividing (2) by $P(s_t)$. Thus, the defined RDCOLI, $\pi_c(s_t|s'_\tau, \bar{U})$, clearly satisfies

$$\pi_c(s_t|s'_\tau, \bar{U}) = \frac{E(\bar{U}|s_t)/P(s_t)}{E(\bar{U}|s'_\tau)/P(s'_\tau)}. \tag{6}$$

[2]If $\bar{U}$ is equal to the realized (optimal) utility at period τ, then our DCOLI is equal to the dynamic price index (DPI) in Reis (2005).

Thus, the growth rate of the DCOLI is decomposed into the growth rates of COLI in a static problem (i.e., $p_t - p'_\tau$, where $p_t = \ln P(s_t)$), and the RDCOLI:

$$\ln \pi(s_t|s'_\tau, \bar{U}) = \{p_t - p'_\tau\} + \ln \pi_{\mathrm{c}}(s_t|s'_\tau, \bar{U}). \quad (7)$$

The first term is conceptually the usual inflation rate of the COLI, which is directly available from the data. Thus, by leaving the problem regarding the COLI to the vast existing COLI literature, our focus in the following sections shifts to developing a way to induce the second term, the change in the RDCOLI.

Finally, note that if the preference has a homothetic structure (i.e., $U(\alpha C_t, \{\alpha C_{t+j}\}_{j=1}^{\infty}|m) = \alpha U(C_t, \{C_{t+j}\}_{j=1}^{\infty}|m)$), then reference utility levels can be ignored in the calculation of the DCOLI and RDCOLI.[3] That is,

$$\pi(s_t|s'_\tau, \bar{U}) = \pi(s_t|s'_\tau), \text{ and } \pi_{\mathrm{c}}(s_t|s'_\tau, \bar{U}) = \pi_{\mathrm{c}}(s_t|s'_\tau). \quad (8)$$

In fact, we assume the Epstein and Zin (1991) utility in the following sections, which satisfies the homotheticity.

2.2 Preference Specification: Epstein-Zin Utility

We use the recursive utility proposed by Epstein and Zin (1991):

$$U(C_t, \{C_{t+j}(\cdot)\}_{j=1}^{\infty}|m(s_t)) = \{(1-\delta)C_t^{1-\frac{1}{\psi}} + \delta(E_t[[U_t^1]^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\}^{\frac{1}{1-\frac{1}{\psi}}}, \quad (9)$$

[3] In this case, our DCOLI is exactly equal to the DPI in Reis (2005).

where

$$U_t^j(\{s_i\}_{i=t+1}^{t+j}) = \{(1-\delta)C_{t+j}(\{s_i\}_{i=t+1}^{t+j}) + \delta(E_{t+j}[[U_t^{j+1}]^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\}^{\frac{1}{1-\frac{1}{\psi}}}$$

for all $j \geq 1$ and $\{s_i\}_{i=t+1}^{t+j}$, and $E_{t+j} = E_{m(s_{t+j})}$ for all $j \geq 0$ and s_{t+j}. $\frac{1}{\delta} - 1$ is the rate of time preference, ψ is the EIS, and γ is the coefficient of the risk aversion. If $\gamma = \frac{1}{\psi}$, then (9) collapses to the standard expected utility specification:

$$[U(C_t, \{C_{t+j}(\cdot)\}_{j=1}^{\infty}|m(s_t))]^{1-\frac{1}{\psi}} = (1-\delta)E_t \sum_{j=0}^{\infty} \delta^j C_{t+j}^{1-\frac{1}{\psi}}.$$

Epstein and Zin (1991) consider the utility maximization problem with initial (real) wealth W_t and state s_t:

$$V(W_t|s_t) = \max_{C_t, A_{t+1}, \{C_{t+j}(\cdot), A_{t+j+1}(\cdot)\}_{j=1}^{\infty}} U(C_t, \{C_{t+j}(\cdot)\}_{j=1}^{\infty}|m(s_t)) \qquad (10)$$

subject to (3), (4), and the initial budget constraint,

$$P(s_t)W_t = P(s_t)C_t + Q(s_t) \cdot A_{t+1}. \qquad (11)$$

By the homotheticity, the optimally chosen wealth-consumption ratio, $W_t/C^*(W_t|s_t)$, where $C^*(W|s)$ denotes the optimally chosen initial consumption with initial wealth W and state s, does not depend on W_t. Let $WC(s_t)$ denote the ratio. Epstein and Zin (1991) show that the optimal value can be decomposed as follows:

$$V(W_t|s_t) = [(1-\delta)^{\psi} WC^*(s_t)]^{-1/(1-\psi)} W_t. \tag{12}$$

By the duality (i.e., $E(\bar{U}|s) = P(s)W$ for $V(W|s) = \bar{U}$),

$$E(\bar{U}|s_t) = \frac{\bar{U}}{[(1-\delta)^{\psi} WC^*(s_t)]^{-1/(1-\psi)}}, \tag{13}$$

and hence, by (5) and (6),

$$\ln \pi_c(s_t|s'_\tau) = \frac{1}{1-\psi}\{wc_t - wc'_\tau)\}, \tag{14}$$

and

$$\ln \pi(s_t|s'_\tau) = \{p_t - p'_\tau\} + \frac{1}{1-\psi}\{wc_t - wc'_\tau\}, \tag{15}$$

where $wc_t = \ln WC^*(s_t)$.

Thus, the change in the RDCOLI is reduced to that in the (optimally chosen) wealth-consumption ratio.

2.3 Loglinear Approximations

We apply Campbell's (1993) loglinear approximation. First, by approximating the budget constraints, we decompose wc into weighted sum of future consumption growth rates and returns on total wealth. Second, by approximating the Euler equation, each expected return is decomposed into the corresponding expected consumption growth rate and the variance regarding them, the conditional homoscedasticity implying that the variance regarding the future return and future consumption growth rate is constant.

2.3.1 Budget Constraint Approximation

By the homotheticity, the realized real gross return on the total wealth at period $t+1$ in state s_{t+1} given that the consumer has chosen the optimal portfolio $A^*(W_t|s_t)$ with the initial wealth W_t and the state s_t at the previous period t (i.e., RHS below) depends only on s_t and s_{t+1}. Let $R^*(s_{t+1}|s_t)$ denote the return:

$$R^*(s_{t+1}|s_t) = \frac{\frac{(Q(s_{t+1})+D(s_{t+1}))\cdot A^*(W_t|s_t)}{P(s_{t+1})}}{\frac{Q(s_t)\cdot A^*(W_t|s_t)}{P(s_t)}}. \tag{16}$$

Then, by letting $W^*_{t+1}(W_t|s_t, s_{t+1}) = ((Q_{t+1}(s_{t+1})+D_{t+1}(s_{t+1}))\cdot A^*(W_t|s_t))/P(s_{t+1})$ denote the realized initial total wealth at period $t+1$ given that the consumer has chosen the optimal portfolio in period t, it follows that

$$W^*_{t+1}(W_t|s_t, s_{t+1}) = R^*(s_{t+1}|s_t)(W_t - C^*(W_t|s_t)), \tag{17}$$

which can be rearranged as

$$\frac{W^*_{t+1}(W_t|s_t, s_{t+1})}{C^*(W^*_{t+1}(W_t|s_t, s_{t+1})|s_{t+1})} \frac{C^*(W^*_{t+1}(W_t|s_t, s_{t+1})|s_{t+1})}{C^*(W_t|s_t)} = R^*(s_{t+1}|s_t)\left(\frac{W_t}{C^*(W_t|s_t)} - 1\right). \tag{18}$$

Again, by the homotheticity, the second term in the LHS, which is the gross growth rate of the optimally chosen consumption, does not depend on W_t. Let $GC^*(s_{t+1}|s_t)$ denote the rate. Then, we can rewrite the above equation as

$$WC^*(s_{t+1})GC^*(s_{t+1}|s_t) = R^*(s_{t+1}|s_t)(WC^*(s_t) - 1), \tag{19}$$

or in logs, by letting $\Delta c_{t+1} = \ln GC^*(s_{t+1}|s_t)$ and $r_{t+1} = \ln R^*(s_{t+1}|s_t)$,

$$wc_{t+1} + \Delta c_{t+1} = r_{t+1} + \ln(\exp(wc_t) - 1). \tag{20}$$

Campbell (1993) approximates the second term in the RHS around the long-run average log wealth-consumption ratio $\overline{wc}$:

$$\ln(\exp(wc_t) - 1) \simeq \ln(\exp(\overline{wc}) - 1) + \frac{\exp(\overline{wc})}{\exp(\overline{wc}) - 1}(wc_t - \overline{wc}).$$

Then,

$$wc_t \simeq \rho(\Delta c_{t+1} - r_{t+1}) - \rho\kappa + \rho wc_{t+1},$$

where

$$\rho \equiv \frac{\exp(\overline{wc}) - 1}{\exp(\overline{wc})} \text{ and } \kappa \equiv \ln(\exp(\overline{wc}) - 1) - \frac{1}{\rho}\overline{wc},$$

and hence, by assuming $\lim_{j\to\infty} \rho^j wc_j = 0$,

$$wc_t \simeq \sum_{j=1}^{\infty} \rho^j (\Delta c_{t+j} - r_{t+j}) - \frac{\rho\kappa}{1-\rho}. \tag{21}$$

2.3.2 Euler Equation Approximation

Under Epstein-Zin preferences, the Euler equation becomes

$$1 = E_t\left[\exp\left(\frac{1-\gamma}{1-\frac{1}{\psi}}\left(\ln\delta - \frac{1}{\psi}\Delta c_{t+1} + r_{t+1}\right)\right)\right].$$

Then, the second-order Taylor approximation around $E_t[-\frac{1}{\psi}\Delta c_{t+1} + r_{t+1}]$ as in Campbell (1993) yields

$$0 \simeq \ln \delta - \frac{1}{\psi} E_t[\Delta c_{t+1}] + E_t[r_{t+1}] + \frac{1}{2} \frac{1-\gamma}{1-\frac{1}{\psi}} \operatorname{var}_t \left(-\frac{1}{\psi} \Delta c_{t+1} + r_{t+1} \right), \quad (22)$$

where var_t is the conditional variance at period t.

2.4 Price Dynamics Specification: Conditional Homoscedasticity

As in Campbell (1993), we assume that the consumption growth rates and returns on total wealth are jointly conditionally homoscedastic. That is,

$$\operatorname{var}_t \left(-\frac{1}{\psi} \Delta c_{t+1} + r_{t+1} \right) = \text{const.}$$

Then, (22) implies

$$E_t[\Delta c_{t+1} - r_{t+1}] \simeq \text{const.} + \left(1 - \frac{1}{\psi}\right) E_t[\Delta c_{t+1}]. \quad (23)$$

By substituting these equations into (21), we obtain

$$wc_t \simeq \text{const.} + \left(1 - \frac{1}{\psi}\right) \sum_{j=1}^{\infty} \rho^j \, \mathrm{E}_t[\Delta c_{t+j}]. \quad (24)$$

Note that by the conditional homoscedasticity, the constant term in (24) is equal between different states, and it disappears when we take the difference of wcs. Thus, the growth rate of the RDCOLI is expressed as the change in the linear combination of expected consumption growth rates: that is,

$$\ln \pi_{\mathrm{c}}(s_t|s'_\tau) \simeq -\frac{1}{\psi}\sum_{j=1}^{\infty}\rho^j\{E_t[\Delta c_{t+j}] - E'_\tau[\Delta c'_{\tau+j}]\}, \tag{25}$$

and hence, the DCOLI is also expressed as

$$\ln \pi(s_t|s'_\tau) \simeq \{p_t - p'_\tau\} - \frac{1}{\psi}\sum_{j=1}^{\infty}\rho^j\{E_t[\Delta c_{t+j}] - E'_\tau[\Delta c'_{\tau+j}]\}. \tag{26}$$

Thus, yielding the estimates of the expected consumption growth rates suffices for (approximately) measuring the DCOLI. We actually apply the formula to develop empirical exercises in the next section.

Finally, we note some properties obtained from (26). First, given the estimates of the expected consumption growth rates, the higher the EIS, the closer to the COLI is the induced DCOLI. Second, as long as the consumption growth rate is stationary, the long-run growth rate of the DCOLI coincides with that of the COLI. Third, let P^t_{t+j} be the price of period $t+j$ consumption in the dynamic budget constraint (i.e., $P^t_t = P_t$, $P^t_{t+1} = P_t/R^*_{t+1}$, $P^t_{t+2} = P_t/(R^*_{t+1}R^*_{t+2}), \dots$), and $p^t_{t+j} = \ln P^t_{t+j}$. Then, by using (23), (26) can be rewritten as

$$\ln \pi(s_t|s'_\tau) \simeq \{p_t - p'_\tau\} - \sum_{j=1}^{\infty}\rho^j\{E_t[r_{t+j}] - E'_\tau[r'_{\tau+j}]\} = \sum_{j=0}^{\infty}\rho^j(1-\rho)\{E_t[p^t_{t+j}] - E'_\tau[p'^{\tau}_{\tau+j}]\}, \tag{27}$$

which is exactly the log of the geometric average of P^t_{t+j}s.[4] The above equation also indicates that if expected future returns decrease, the current prices

[4]In the actual values case, the above equation coincides with what Shibuya (1992) derives in his theory, if ρ is equal to time preference δ.

of future goods will increase, and so will the cost of living (and vice versa).

3 Measuring the DCOLI and RDCOLI from the Data

In this section, we measure the DCOLI and RDCOLI using the U.S. quarterly data from 1959:4 to 2003:1, and our formula in (25) and (26). We measure the DCOLI growth rates, Δdcoli_t, and the RDCOLI growth rates, Δrdcoli_t (we mainly focus on Δdcoli_t). Δdcoli_t is defined by $\ln \pi(s_t|s_{t-1})$, and Δrdcoli_t is defined by $\ln \pi_{\text{c}}(s_t|s_{t-1})$, where s_t represents the realized state variables at period t. Because the measurement of the RDCOLI term is crucial for our DCOLI measurement, we also report demeaned log RDCOLI, rdcoli_t, which is equal to demeaned $wc_t/(1-\psi)$.

We impose two different assumptions on households' expectations. In the first case, we assume that the households' expectations for future consumption growth rates (approximately) coincide with the actual values. In the second case, the households form their expectations for future consumption growth rates based on the VAR model (which we discuss later). In the following two sections, we consider these two cases.

In order to measure the DCOLI, we also need to specify the parameter of the EIS, ψ, and long-run average log wealth-consumption ratio, $\overline{wc}$ (or ρ). For the EIS, we try several values from 0.2 to 2.0. For $\overline{wc}$, we set the value of the long-run average log price-dividend ratio on households' financial wealth, which is 4.627 in the U.S. quarterly data.[5] Then, $\rho \approx 0.9902$.

[5] We measure households' financial wealth as in Lettau and Ludvigson (2001). For

3.1 Actual Values Case

In this section, we measure the DCOLI and RDCOLI based on the assumption that the expected values of future consumption growth rates coincide with the actual values, at least on the aggregated level of the RDCOLI (i.e., $\sum_j \rho^j E_t[\Delta c_{t+j}] = \sum_j \rho^j \Delta c_{t+j}$). Of course, the actual values are only available for the period before 2003. Thus, for the values after 2003, we use the average value over the sample periods as proxy. In summary, we assume that

$$\sum_{j=1}^{\infty} \rho^j E_t[\Delta c_{t+j}] = \sum_{j=1}^{2003:1-t} \rho^j \Delta c_{t+j} + \sum_{j=2003:2-t}^{\infty} \rho^j g, \tag{28}$$

where g is the average consumption growth rate over 1959:4 to 2003:1. In order for consumption values to construct the growth rates, we use per capita real consumption data (for details, see Appendix A.2.2).

We first look at the demeaned rdcoli. Figure 1 plots the demeaned rdcoli.[6] The demeaned rdcoli captures the economic boom from the latter half of the 1960s to the former half of the 1970s, stagnation after the first and second energy crises, and boom around 2000.

Figures 2 and 3 plot Δcoli and Δdcolis. The property in (26) that Δdcoli converges to Δcoli as the EIS becomes larger is confirmed in the figures. Since it might be difficult to see the differences between the COLI and DCOLIs in these figures, in Figure 4, we also plot the three-years moving averages of Δcoli, Δdcoli, and Δrdcoli (EIS = 0.5), which is calculated as the three-years

details, see Appendix A.4.1.

[6]Note that although the shape of fluctuations in wc inverts at $\psi = 1.0$ (at $\psi = 1.0$, wc becomes constant), the shape of the fluctuation in demeaned rdcoli (i.e., demeaned $wc/(1-\psi)$) does not depend on ψ. We can confirm this property from (25).

average of inflation before and after the period, as in Reis (2005). During the first and second oil crises (i.e., 1973–1976 and 1977–1983), Δrdcoli, which is equal to the difference between Δdcoli and Δcoli, was the lowest. Based on the final remark in Section 2.4, it can be interpreted that the current prices of future consumption decreased, or in other words, future returns increased during the periods.[7] On the other hand, Δrdcoli reached its highest level around 1965 and 1985.

Table 1 reports the standard deviation and autocorrelation of Δcoli and Δdcoli, and the correlation between Δcoli and Δrdcoli.[8] Except for the case where EIS = 0.2, the standard deviations of Δdcolis are close to that of Δcoli.[9] This is different from the results of the previous studies. The autocorrelation of Δdcoli is lower than that of Δcoli. Thus, Δdcoli is less persistent. The correlation between Δcoli and Δrdcoli is negative. This means that when the price of current goods increases, the prices of future goods decrease, or, in other words, expected future returns increase.

In order to consider how the result is affected by human wealth, we also calculate two types of Δdcoli that use data on households' financial wealth instead of total wealth (we refer to them as financial Δdcolis). One is the Δdcoli that is calculated by (15) using the price-dividend ratio data of households' financial wealth instead of the wealth-consumption ratio.[10] Note that

[7] During the same periods, equity prices were relatively low.

[8] We also calculate the average and standard deviation of Δcoli and Δdcoli, and the correlation between Δcoli and Δrdcoli for every ten years in Table 6.

[9] In the table, when EIS $\geq$ 1.0 (EIS = 1.0 corresponds to the log utility case), the volatility of Δdcoli is less than that of Δcoli. This is because the covariance between Δcoli_t and Δrdcoli_t is negative (note that $\text{var}(\Delta\text{dcoli}_t) = \text{var}(\Delta\text{coli}_t) + \text{var}(\Delta\text{rdcoli}_t) + 2\,\text{cov}(\Delta\text{coli}_t, \Delta\text{rdcoli}_t)$ and that $\text{corr}(\Delta\text{coli}_t, \Delta\text{rdcoli}_t)$ is negative in the data).

[10] For the definition and construction of households' financial wealth, see footnote 5 and Appendix A.4.1.

because the data price-dividend ratio does not become constant at EIS = 1.0 (log utility case), it cannot be calculated at this EIS value. The other financial Δdcoli is calculated using dividend growth rates on financial wealth instead of consumption growth rates in (26).[11] Tables 2 and 3 report the standard deviations of the former and latter cases of financial Δdcoli. These are highly volatile compared with the Δdcolis that take into account human wealth. In particular, the latter financial Δdcoli is about eight times more volatile than the Δdcoli calculated from consumption data at an EIS of 1.0.

3.2 VAR Case

We measure the DCOLI and RDCOLI, where households (rationally) forecast the future using a VAR model. We assume that the expectation of a household is formed by the following VAR:

$$\mathbf{z}_{t+1} = \mathbf{A}\mathbf{z}_t + \epsilon_t,$$

where $\mathbf{z}_t$ is the vector of state variables, and ϵ_t is i.i.d. with mean zero. The components of $\mathbf{z}_t$ are basically the same as those of Lustig and Van Nieuwerburgh (2006), and $\mathbf{z}_t = (\Delta c_t, \Delta y_t, lis_t, r^a_t, pd^a_t, YK_t, rtb_t, ysps_t)'$, where Δc_t is the per capita real consumption growth, Δy_t is the per capita real labor income growth, lis_t is the labor income share, r^a_t is the real return on households' financial wealth, pd^a_t is the log price-dividend ratio of households' financial wealth, YK_t is the output-(physical) capital ratio, rtb_t is the rela-

[11] We assume that the expected values of future dividend growth rates coincide with the realized values before 2003 and that they are equal to the average dividend growth rate over the sample periods after 2003.

tive T-bill return, and $ysps_t$ are yield spreads of several bonds. For details regarding these data, see the Appendix. We include real return r_t^a, because the return is related to consumption growth Δc_t through the Euler equation. We also include YK_t, because YK_t times capital intensity is the return on aggregate capital under the Cobb-Douglas aggregate production function. Then, for example,

$$E_t[\Delta c_{t+j}] = E_t[\mathbf{e}_1 \mathbf{z}_{t+j}] = \mathbf{e}_1 \mathbf{A}^j \mathbf{z}_t,$$

where $\mathbf{e}_1 = (1, 0, \ldots, 0)$. Matrix $\mathbf{A}$ is estimated from data using ordinary least squares (OLS).

Figure 5 plots the demeaned rdcoli. Compared with the actual values case, different values of the rdcoli are observed after 2000. After 2000, the demeaned rdcoli of the VAR case is higher than that of the actual values case. This difference might be because in the actual values case, we assume that after 2003, the expected consumption growth rate is equal to the average growth rate of consumption over the sample periods.

Next, we look at growth rates. Figure 6 plots Δdcoli in the VAR case. Since the basic tendencies are the same for different EISs, we only plot the EIS = 0.5 and 1.0 cases. As in the actual values case, we also plot the three-years moving average in Figure 7. The basic tendencies are similar to those in the actual values case, except for after 2000. After 2000, the three-years moving average Δdcoli of the VAR case is higher than that of the actual values case. Table 4 reports the standard deviation and autocorrelation of

Δcoli and Δdcoli, and the correlation between Δcoli and Δrdcoli.[12] The volatilities of VAR Δdcolis are more volatile but still close to those in the actual values case.[13] The properties on the low persistency of Δdcoli and negative correlation between Δcoli and Δrdcoli are the same as those in the actual values case. As in the actual values case, we compare these Δdcolis with financial wealth versions of Δdcoli. The standard deviation of financial Δdcoli consisting of the price-dividend ratio data of financial wealth is reported in Table 2 and that of another financial Δdcoli calculated from expected dividend growth rates is reported in Table 5.[14] The latter financial Δdcolis are less volatile than the actual values version. Nonetheless, Δdcolis calculated from expected consumption growth rates are less volatile than the latter financial Δdcolis (the latter VAR version of financial Δdcoli is around four times more volatile than our Δdcoli at an EIS of 1.0).

4 Concluding Remarks

This paper develops a practical method to construct the DCOLI from consumption data, and using this method, measures the DCOLI. Compared with previous studies, our method has three advantages: (1) our DCOLI can

[12] As in the actual values case, we calculate the average and standard deviation of Δcoli and Δdcoli, and the correlation between Δcoli and Δrdcoli for every ten years in Table 6.

[13] The standard deviations are on average about 30% higher than those of the actual values case.

[14] The expected dividend growth rate is calculated by using the following relation:

$$E_t[\Delta d_{t+j}] = (\mathbf{e}_4 + \mathbf{e}_5)\mathbf{A}^j \mathbf{z}_t - \rho^{-1}\mathbf{e}_5 \mathbf{A}^{j-1}\mathbf{z}_t,$$

where Δd_{t+j} is the dividend growth rate, and $\mathbf{e}_i$ is a row vector with i-th element unity and other elements being zero. This relation holds because $r^a_{t+j} = \Delta d_{t+j} + pd^a_{t+j} - \rho^{-1}pd^a_{t+j-1}$ holds (where variables are demeaned).

capture contribution from change in human wealth, (2) our DCOLI is less volatile, and (3) the assumption on consumer preference is less restrictive.

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A Data Appendix

This appendix describes the data sources. We use quarterly data and the sample periods are 1959:4 to 2003:1.

A.1 Population and per capita hours worked

We take the working-age population (16–64 years old) and per capita hours worked data from Prescott et al. (2005).

A.2 Consumption

A.2.1 COLI

We construct Fisher's version of COLI (or, in other words, CPI) using the formula

$$\sqrt{\frac{\sum P_t Q_{t-1}}{\sum P_{t-1} Q_{t-1}}}\sqrt{\frac{\sum P_t Q_t}{\sum P_{t-1} Q_t}}.$$

We chain the indices to derive the price level of consumption. To construct the indices, we use the price data of "nondurable goods" (line 6) and "services" (line 13) in Table 2.3.4 and their quantity data in Table 2.3.3 in the National Income and Product Accounts (NIPA).

A.2.2 Per capita real consumption

In order to obtain real consumption data, we divide the nominal consumption by Fisher's version of the COLI explained above. We further divide the real consumption by the population explained above.

Nominal consumption data are from Table 2.3.5 in the NIPA. Our nominal consumption data are the sum of nondurable goods (line 6) and services (line 13). These data are seasonally adjusted at annual rates. Thus, we divide the values by 4.

A.3 Labor income

A.3.1 Labor income share

Data on labor income share are taken from Table 2.1 in the NIPA. The labor income share is calculated by the nominal labor income explained below / nominal "disposable personal income" (line 26).

We construct nominal labor income from "compensation of employees, received" (line 2) + "government social benefits to persons" (line 17) − "contributions for government social insurance" (line 24) − labor taxes. As in Lettau and Ludvigson (2001), the labor taxes are imputed from the labor tax ratio × the nominal labor income, where the labor tax ratio is calculated as the ratio of "wage and salary disbursements" (line 3) to "wage and salary disbursements" + "proprietors' income with inventory valuation and capital consumption adjustments" (line 9) + "rental income of persons with capital consumption adjustment" (line 12) + "personal income receipts on assets" (line 13).

A.3.2 Per capita real labor income

Basically, data on per capita real labor income are taken from Table 2.1 in the NIPA. We obtain real labor income from the labor income share defined above

$\times$ real disposable personal income. The real disposable personal income is obtained by "disposable personal income" (line 26) / the COLI explained above.[15] In order to obtain the per capita real labor income, we divide it by the population explained above. These data are seasonally adjusted at annual rates. Thus, we divide the values by 4.

A.4 Households' financial wealth

A.4.1 Price-dividend ratio of households' financial wealth pd^a

In order to obtain the price-dividend ratio of households' financial wealth, pd^a, we divide the nominal financial wealth by nominal dividends minus savings, both of which are explained below.

Nominal financial wealth data are obtained from the balance sheet of households and non-profit organizations, Flow of Funds Accounts Table B-100, provided by the Federal Reserve Board System. This wealth measure is on an end-of-period basis. Therefore, we use the $t-1$ value of the data for period t wealth. Our measure of households' financial wealth consists of net worth (line 41) $-$ consumer durable goods (line 7). Basically, our definition of nominal financial wealth is the same as that of Lettau and Ludvigson (2001) except that we exclude durable consumption from nominal financial wealth (because we exclude durable consumption from our consumption data).

Nominal dividends minus savings are obtained from Table 2.1 in the NIPA. It is constructed by "proprietors' income with inventory valuation and capital consumption adjustments" (line 9) + "rental income of persons

[15] The reason that it is divided by the COLI is that in our model, real terms are expressed in consumption good units.

with capital consumption adjustment" (line 12) + "personal income receipts on assets" (line 13) − "other current transfer receipts, from business (net)" (line 23) − capital taxes − "personal saving" (line 33). Similar to the labor taxes above, the capital taxes are imputed from capital tax ratio × the nominal capital income, where the capital tax ratio is calculated as the ratio of "proprietors' income with inventory valuation and capital consumption adjustments" + "rental income of persons with capital consumption adjustment" + "personal income receipts on assets" to "wage and salary disbursements" (line 3) + "proprietors' income with inventory valuation and capital consumption adjustments" + "rental income of persons with capital consumption adjustment" + "personal income receipts on assets."

A.4.2 Real return of households' financial wealth r^a

We obtain the real return on households' financial wealth, r^a from $r^a_{t+1} \equiv \ln R_{t+1} = \ln(\frac{P^a_{t+1}}{P^a_t - D^a_t})$, where P^a is the per capita real financial wealth, and D^a is the per capita real dividends minus savings. P^a is calculated from the nominal financial wealth explained above divided by the COLI and population. D^a is calculated from nominal dividends minus savings divided by the COLI and population.

A.5 Relative T-bill return rtb_t and yield spreads $ysps_t$

Relative T-bill return rtb_t and the yield spreads of several bonds $ysps_t$ used in the VAR case are taken from Van Nieuwerburgh's website. Precisely, rtb_t corresponds to relTbill and $ysps_t$ corresponds to defsprBaaAAA, lefsprBaaTbond, and termspread in `quarterly_data_WSMS.xls` on his website.

A.6 Output-(physical) capital ratio YK_t

We calculate the output-(physical) capital ratio YK_t of the U.S. from Braun et al.'s (2006) dataset. The dataset is available from Braun's website. Note that we do not use $\ln(YK_t)$ but YK_t in the regression.

	std[Δcoli]	AC(1)[Δcoli]	AC(4)[Δcoli]	AC(8)[Δcoli]	
	0.71%	0.82	0.63	0.37	
EIS	std[Δdcoli]	AC(1)[Δdcoli]	AC(4)[Δdcoli]	AC(8)[Δdcoli]	corr[Δcoli, Δrdcoli]
0.2	2.35%	0.21	0.02	-0.16	-0.52
0.5	0.91%	0.27	0.09	0.01	–
1.0	0.62%	0.61	0.44	0.36	–
2.0	0.61%	0.81	0.64	0.47	–

Table 1: Standard deviation and autocorrelation of Δcoli and Δdcoli, and correlation between Δcoli and Δrdcoli of the actual values case. Notes: AC(d) represents the autocorrelation of d lags. corr[Δcoli, Δrdcoli] does not depend on the EIS value.

EIS	std[Δdcoli]
0.2	6.47%
0.5	10.34%
1.0	n.a.
2.0	5.28%

Table 2: Standard deviation of the financial Δdcoli calculated using the price-dividend ratio data of broad financial wealth instead of the wealth-consumption ratio. Note: For details regarding the price-dividend ratio data, see appendix A.4.1.

EIS	std[Δdcoli]
0.2	24.01%
0.5	9.59%
1.0	4.81%
2.0	2.45%

Table 3: Standard deviation of another financial Δdcoli of the actual values case. Note: This financial Δdcoli is calculated assuming that households earn income only from financial wealth.

EIS	std[Δdcoli]	AC(1)[Δdcoli]	AC(4)[Δdcoli]	AC(8)[Δdcoli]	corr[Δcoli, Δrdcoli]
0.2	3.16%	0.06	-0.05	-0.14	-0.29
0.5	1.30%	0.07	-0.02	-0.11	–
1.0	0.81%	0.31	0.21	0.06	–
2.0	0.69%	0.62	0.48	0.26	–

Table 4: Standard deviation and autocorrelation of Δdcoli and correlation between Δcoli and Δrdcoli of the VAR case. Notes: AC(d) represents the autocorrelation of d lags. corr[Δcoli, Δrdcoli] does not depend on the EIS value.

EIS	std[Δdcoli]
0.2	15.85%
0.5	6.33%
1.0	3.19%
2.0	1.67%

Table 5: Standard deviation of another financial Δdcoli of the VAR case. Note: This financial Δdcoli is calculated assuming that households earn income only from financial wealth.

mean	1960s	1970s	1980s	1990s
mean[Δcoli]	0.62%	1.73%	1.10%	0.61%
mean[Δdcoli] (AV EIS=0.5)	0.95%	1.39%	1.26%	0.65%
mean[Δdcoli] (AV EIS=1.0)	0.78%	1.56%	1.18%	0.63%
mean[Δdcoli] (VAR EIS=0.5)	0.90%	1.56%	1.12%	0.91%
mean[Δdcoli] (VAR EIS=1.0)	0.76%	1.64%	1.11%	0.76%
std	1960s	1970s	1980s	1990s
std[Δcoli]	0.40%	0.74%	0.61%	0.33%
std[Δdcoli] (AV EIS=0.5)	1.05%	0.80%	0.90%	0.73%
std[Δdcoli] (AV EIS=1.0)	0.61%	0.49%	0.54%	0.38%
std[Δdcoli] (VAR EIS=0.5)	1.17%	1.46%	1.54%	0.97%
std[Δdcoli] (VAR EIS=1.0)	0.65%	0.85%	0.88%	0.57%
corr	1960s	1970s	1980s	1990s
corr[Δcoli, Δrdcoli] (AV)	-0.14	-0.78	-0.58	-0.51
corr[Δcoli, Δrdcoli] (VAR)	-0.19	-0.41	-0.21	0.03

Table 6: Average and standard deviation of Δcoli and Δdcoli, and correlation between Δcoli and Δrdcoli for every ten years. Notes: AV and VAR denote the actual values and VAR cases. corr[Δcoli, Δrdcoli] does not depend on the EIS value.

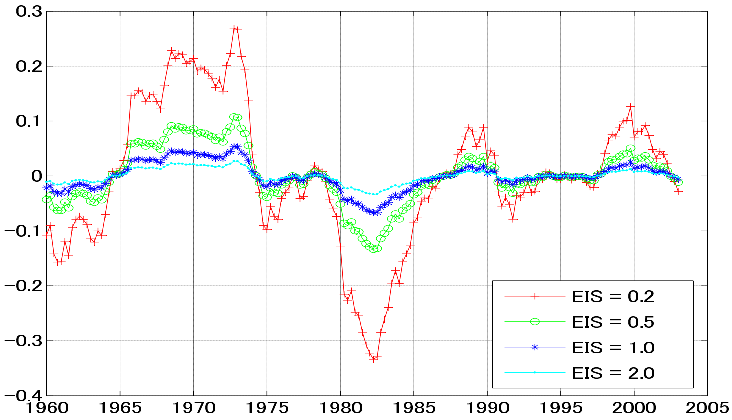

Figure 1: Demeaned rdcolis of the actual values case.

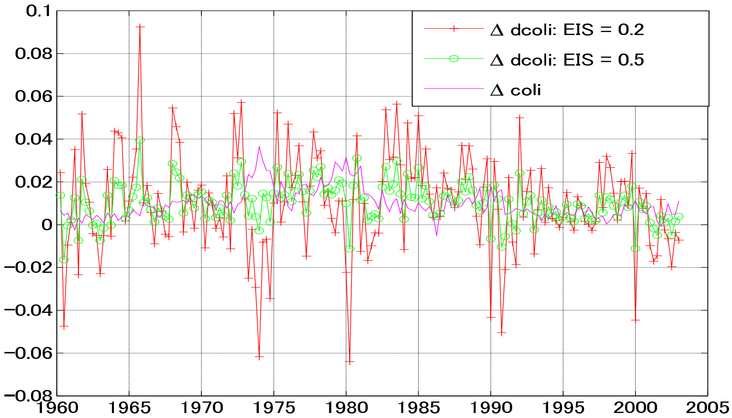

Figure 2: Δcoli and Δdcolis (for DCOLI, EIS = 0.2 and 0.5) of the actual values case

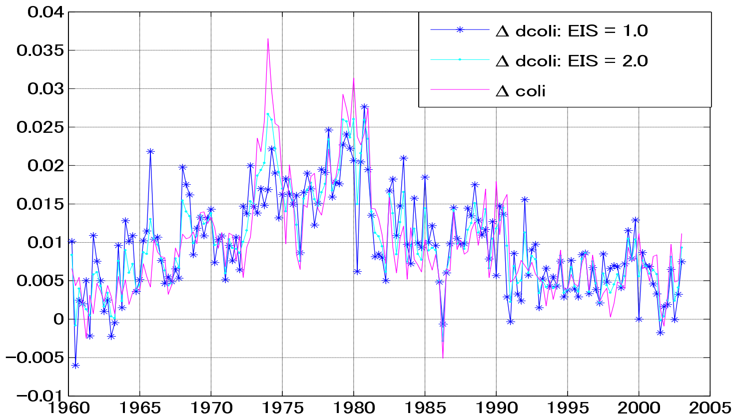

Figure 3: Δcoli and Δdcolis (for DCOLI, EIS = 1.0 and 2.0) of the actual values case

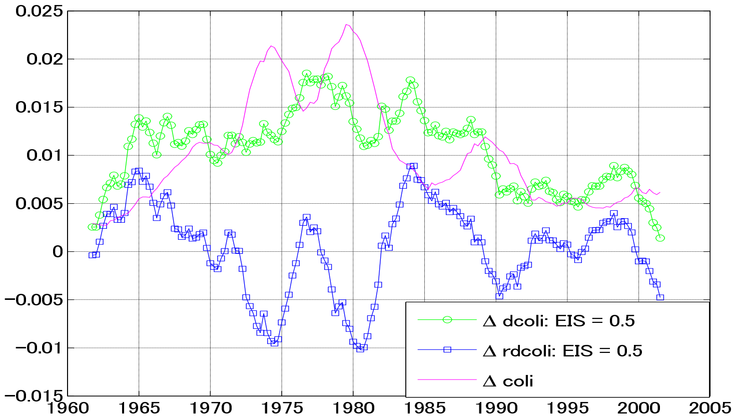

Figure 4: Three-years moving average of Δcoli, Δdcoli, and Δrdcoli (for Δdcoli and Δrdcoli, EIS = 0.5) of the actual values case

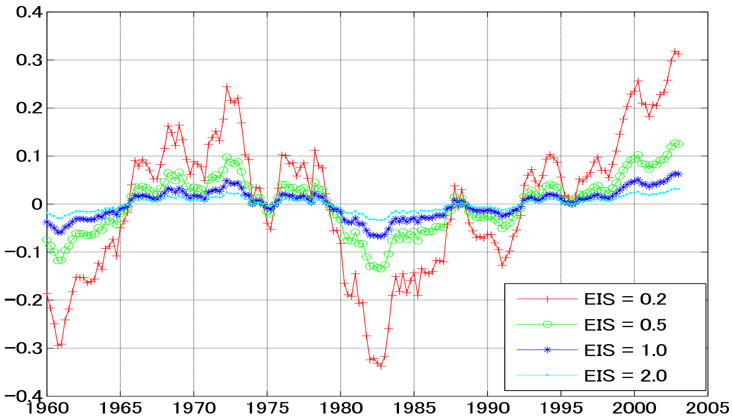

Figure 5: Demeaned rdcolis of the VAR case

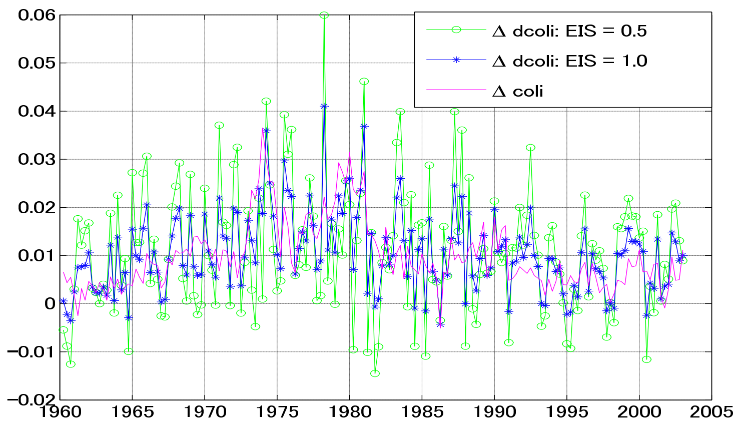

Figure 6: Δcoli and Δdcolis (for DCOLI, EIS = 0.5 and 1.0) of the VAR case

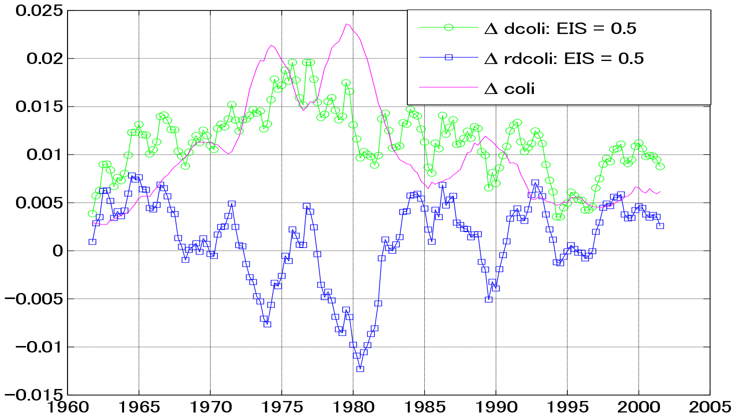

Figure 7: Three-years moving average of Δcoli, Δdcoli, and Δrdcoli (for Δdcoli and Δrdcoli, EIS $= 0.5$) of the VAR case