Market Concentration and the Productivity Slowdown

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Abstract

Since around 2000, U.S. aggregate productivity growth has slowed and product market (sales) concentration has risen. At the same time, productivity differences among firms in the same sector appear to have risen dramatically. Sector-level data shows correlations between rising productivity gaps, concentration, and slowing productivity growth. I propose a rich model of competition and innovation to explain these findings. A key parameter governing all three phenomena is the probability that innovating firms make larger, more "radical" innovations. Thus one explanation for the coincidence of these three observations is that the incidence of radical innovations has slowed relative to the 1990s, when the internet and other information technology radically transformed production and sales technology in many sectors. The model also provides an explanation for what might be called the "superstar productivity puzzle": sales growth of the most productive firms has coincided with *slower* aggregate productivity growth.

1 Introduction

Among U.S. public companies, the largest firm's average market share of sales has risen significantly since the late 1990s (figure 1).[1] Both labor productivity and total factor productivity (TFP) differences between so-called "frontier" or "superstar" firms and their competitors have been growing since 2000, particularly in information and communications technology (ICT) intensive industries (figure 2). Despite optimists

[1]See Grullon et al. (2017) and Council of Economic Advisers (2016) for overviews of trends in market concentration. More than 75% of U.S. industries have experienced an increase in the Herfindahl-Hirschman index.

suggesting that the rise of superstar firms will improve allocative efficiency and productivity (Autor et al. (2017b)), aggregate productivity growth has fallen over the same period (figure 1).[2]

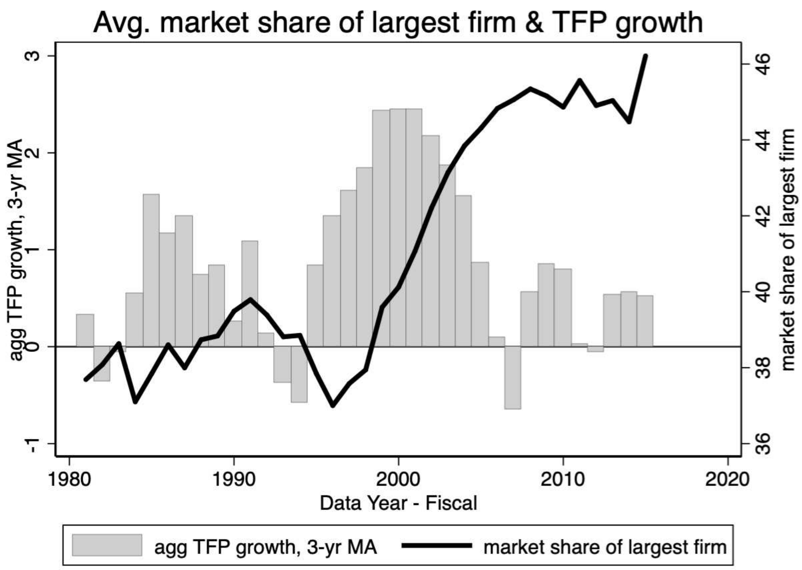


Figure 1: Source: Market share of largest firm (by sales) in 4-digit SIC industries from Compustat (weighted by industry sales); utilization-adjusted total factor productivity (TFP) growth from Fernald (2014), three year moving average.

To explain the coincidence of these three phenomena I propose a general equilibrium quality ladder model of innovation across many sectors. Within each sector, two firms produce products that are imperfect substitutes and interact strategically to set prices and invest in research and development. In the model I derive a mapping from "technology gaps" (quality or productivity differences between firms in the same sector) to market shares, with a firm's market share growing in its *relative* quality. A firm's optimal innovation rate is usually highest when competitors have the same quality and drops off for both the quality leader and the quality follower as quality differences grow. Aggregate productivity and output growth therefore depend on the distribution of sectors over technology gaps between competitors.

In the model, the size of quality improvements conditional on innovating is random.

[2]Some argue that productivity growth has not actually slowed, that it has just been persistently mismeasured recently. Syverson (2016) challenges these hypotheses' ability to explain the majority of the measured slowdown using four separate analyses.

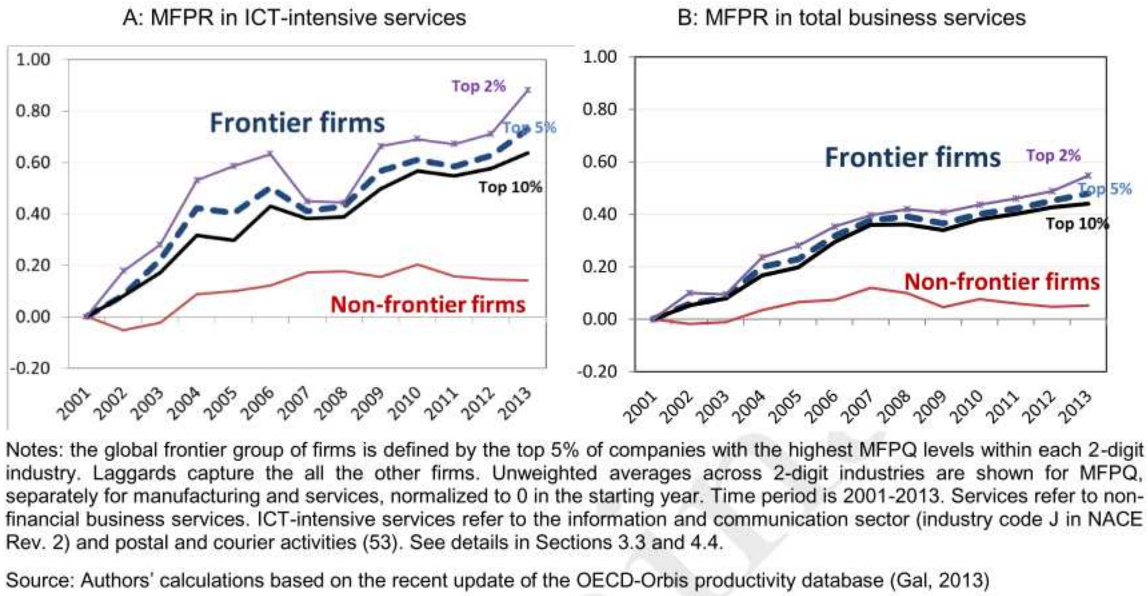


Figure 2: Source: Andrews et al. (2016). MFPR is revenue-based multi-factor productivity. MFPQ is markup-corrected multi-factor productivity.

After presenting the model I show that varying the parameter that governs the chance of making a radical innovation (large quality improvement)[3] can explain a significant share of the empirical changes in leader market share, the productivity growth rate, and productivity gaps between firms observed in the U.S. since 2000. Evidence from various patent quality measures suggests that this probability does indeed vary over time, coinciding with the arrival of general purpose technologies like the internet. In the model, lowering this probability results in endogenously lower innovation effort of firms and greater dispersion of sectors over technology gaps between competitors. Both of these forces contribute to lower aggregate productivity growth, higher leader market share on average, higher average markups, a larger profit share of GDP, and a lower real interest rate.

Going forward, I plan to use the model to address a variety of questions. First, computing transition dynamics from a steady state with higher probability of radical innovations to lower probability of radical innovations can potentially match the explosion of research and development, productivity, rising margins, and sales growth of

[3]The use of "radical innovation" in this paper to describe a relatively large quality improvement differs from some other papers in the literature such as Acemoglu & Cao (2015) who use "radical innovation" to refer to an entrant replacing an incumbent.

large firms in the 1990s and subsequent decline below trend in 2004-2018. Moreover, the model may be helpful in predicting the effects of potentially radical innovations due to artificial intelligence on firm dynamics, market structure, and aggregate productivity growth.

Understanding the causes of the productivity slowdown is critical to assessing prospects for future growth and the role that policy can play in alleviating the slowdown. Hall (2015) finds that output in 2013 was 13% below trend (based on 1990-2007) and decomposes this shortfall into various components. Below-trend business investment was the greatest contributor and has been studied by Alexander & Eberly (2018), Crouzet & Eberly (2018), Gutierrez & Philippon (2017), Gutierrez & Philippon (2016), and Jones & Philippon (2016), among others. The second largest contributor was a TFP shortfall that accounted for more than a third of the output shortfall and is less well understood.

What theory could connect rising market shares and technological advantages of the top firms with slowing productivity? Empirical evidence suggest that these large firms are often some of the most productive in their industry (hence the "superstar firm" label), so growth in sales of these firms should increase measured aggregate productivity, all else equal. However, there are also dynamic considerations: when the technology gap between the largest firm and its rivals widens, the large firm might "rest on its laurels" rather than invest in further productivity-enhancing technologies that simply replace its own technology (this is Arrow (1962)'s replacement effect). Thus a dynamic model of productivity growth at the firm level with multiple firms operating heterogeneous technologies in the same sector is needed to untangle the balance of these two forces.

The failure of productive technologies to diffuse to other firms is also a growing concern, according to Anzoategui et al. (2017) and Andrews et al. (2016). Diffusion is an important determinant of productivity growth in firms farther from the technology frontier. With wider technology gaps, smaller firms have a slimmer chance of closing the gap. If the definition of research and development in the model is expanded to include investments with uncertain outcomes, such as attempting to adopt a new technology, the model can also explain this development because it predicts that laggard firms will invest less in quality improvements when catching up to the leader is less likely.

The model is a tractable general equilibrium model of strategic interactions in both innovation and pricing decisions. Most neo-Schumpeterian growth models feature a single firm operating a product line as a monopolist at any given moment in time (see Klette & Kortum (2004), Lentz & Mortensen (2008), Acemoglu & Cao (2015), and Akcigit & Kerr (2018), for leading examples). Because of this, these models take matching firm-level moments seriously, but are unable to address industry-level moments.[4] Introducing a duopoly allows me to make unified predictions both about market concentration at the industry level and firm-level innovation rates.

This formulation brings together previously distinct strands of literature in macroeconomics concerned with (i) slowing growth (ii) changes in market structure and potentially market power and (iii) superstar firms. Many papers studying the recent rise of large firms have made passing references to the potentially harmful dynamic effects of these large firms on productivity growth but have failed to articulate this link theoretically (OECD (2018)). This model provides a theoretical foundation for the link between the two.

Strands (ii) and (iii) typically rely on opposing assumptions. According to the literature on rising market concentration, incumbent firms exercise greater market power now than in the past and this is reflected in rising markups and profitability (de Loecker & Eeckhout (2017)). On the other hand, the literature on superstar firms typically contends that greater import competition and greater consumer price sensitivity due to better search technology like online retail have *increased* competitive pressures and reduced the market power of incumbent firms, resulting in reallocation to the most productive (superstar) firms (Autor et al. (2017a)). The concept of "market power" is not always well-defined. If measured using markups, as is often done in the literature, the model demonstrates how markups can rise at the same time as there is reallocation to the most productive firms without any changes at all to consumer preferences.

Several recent papers have articulated the link between slowing business dynamism, rising profitability, and rising concentration. Liu et al. (2019) argue that declining interest rates are contributing to all of these phenomena. Their model features a differential response of market leaders and followers to a lower interest rate. Lower interest rates induce greater patience for leaders and followers, and encourage investment only for leaders that expect to eventually capture a larger share of industry profits. This is perhaps surprising in contrast to conventional wisdom that small firms are more credit-constrained and should benefit from lower-interest rate environments. In contrast to this paper, my model takes the interest rate as an endogenous object and shows

[4] Aghion et al. (2001) were the first to introduce a duopoly in a neo-Schumpeterian model with directed innovation. However, the paper does not quantify the model or attempt to match industry moments and there is no heterogeneity in innovation size.

that the declining real interest rate can be a by-product of changes to the innovation production function.

Two papers by Akcigit and Ates (Akcigit & Ates (2019a) and Akcigit & Ates (2019b)) explore the impact of slowing knowledge diffusion in a similar model to the one presented here and demonstrate that slower knowledge diffusion generates less business dynamism (though not necessarily slower aggregate productivity growth) and increased productivity differences between firms. Goods within sectors, however, are perfect substitutes in these models so it's not possible to explore the alternative hypotheses that market power has increased or decreased.

Finally, Aghion et al. (2019) consider an *undirected* model of innovation. In their model, ICT lowers the cost of operating multiple product lines and allows more productive firms to grow larger, generating a lower labor share, higher profits, and slower productivity growth as firms are discouraged by the possibility of taking over a product line already operated by a high-type firm (one dimension of productivity in the model is exogenous). Because, as in the Akcigit and Ates papers, just one firm operates in each sector, the notion of concentration in this model is the share of sectors where the high type firm produces the product. The mapping between this and sales-based concentration measures within sectors is unclear.

The rest of the paper is organized as follows. In section 2 I present additional empirical motivation and evidence of the correlation between productivity growth, productivity gaps, and concentration at the sector level. I also discuss changes in the patent quality distribution over time. Section 3 presents the model and section 4 lays out preliminary results from a numerical exercise comparing the growth rate and other features of the economy in two different steady states with higher and lower probabilities of radical innovations. In section 5 I discuss the role of the elasticity of substitution within sectors, comparing explanations for increased markups and profits in recent years to the superstar firm hypothesis that greater price sensitivity has driven the growth of large, productive firms. Within the model, neither story matches the data as well as a decrease in the probability of radical innovations.

2 Empirical Evidence

The main contribution of the paper is the model, which is presented in section 3. This section reviews the existing literature and presents some new evidence about the productivity slowdown, productivity gaps, and market concentration for the U.S., mainly using data from Compustat. I also present and discuss evidence of the correlation among these phenomena at the sector level. Finally, I discuss trends in patent quality over time using various measures of patent quality.

2.1 Productivity Slowdown

A variety of explanations for the productivity slowdown have been proposed. Most focus on the labor productivity slowdown rather than on total factor productivity. A few explanations are cyclical, such as Anzoategui et al. (2017), who argue that the negative liquidity demand shock that touched off the financial crisis also reduced firms' incentives to introduce new products and adopt existing productive technologies. Such cyclical explanations are unsatisfying for explaining the entire slowdown since the consensus is that the slowdown began well before the global financial crisis.[5]

Secular explanations include the aging workforce (Eggertsson & Mehrotra (2014)) and slowing business dynamism (Decker et al. (2016) and Decker et al. (2018)). Engbom (2017) studies the interactions of aging with innovation and business dynamism. Surprisingly little attention has been devoted to studying firm-level productivity patterns that could illuminate the causes of the productivity slowdown, as I do in this paper.

For example, according to the standard Olley & Pakes (1996) decomposition, aggregate total factor productivity growth could be slowing down for two reasons. First, average TFP growth across all firms could be slowing down. Second, reallocation to the most productive firms (i.e. the growth rate of productive firms) could be slowing down.[6] Appendix A provides details of the firm-level TFP estimation procedure for the

[5]In Anzoategui et al. (2017)'s estimated model of endogenous TFP from 1980 to 2015, they require a negative shock to the productivity of R&D expenditures beginning in the late 1990s to explain why measured R&D in the early 2000s fell below what the model would predict absent the negative R&D efficiency shock. This is similar to the change in the probability of radical innovations I explore in my model.

[6]Formally, as in Olley & Pakes (1996) let a_t be aggregate TFP, $a_{i,t}$ firm-level TFP, and $\bar{a}_t$ unweighted average TFP. Let s denote sales-based market share either individually or on average (with the same notation as for TFP), and N the total number of firms at time t:

$$a_t = \frac{1}{N}\sum_{i=1}^{N} a_{i,t} + \sum_{i=1}^{N}(s_{i,t} - \bar{s}_t)(a_{i,t} - \bar{a}_t)$$

The first term is the unweighted mean and the second is that allocative efficiency term that captures the covariance of size and productivity.

TFP estimates I refer to throughout the paper.[7] Figure 3 shows that the former story is much more important: the fact that the within-firm (unweighted mean component) is what shows a decline means that broad-based below-trend efficiency, not compositional change or misallocation among U.S. firms, is driving the aggregate slowdown, lending support to explanations focusing on the incentives of existing firms to improve productivity, like the hypothesis I propose here. An alternative decomposition by Baqaee & Farhi (2018) shows that allocative efficiency gains contributed around 50% of total TFP growth over the past 20 years.

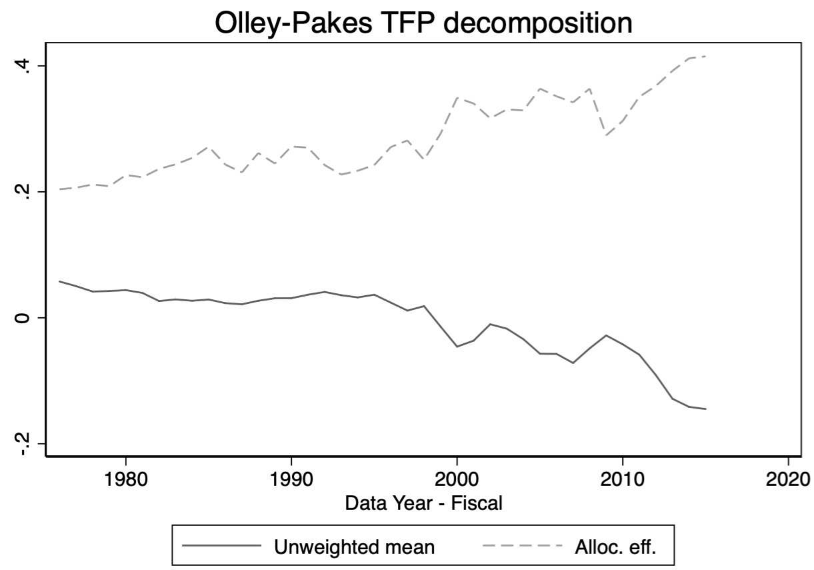


Figure 3: Source: Author's calculations from Compustat.

The consensus is that innovation and technology adoption drive productivity at the firm, in addition to random shocks (see Griliches (2001) for a survey of the relationship between R&D and productivity at the firm level and Zachariadis (2003) for a leading empirical test). Has the productivity slowdown been accompanied by a slowdown in research inputs to improve productivity? Aggregate R&D's share of sales in Compustat has been roughly flat since 1999, before which it had been rising steadily since 1980 (figure 4).

Similar to a firm's decision to invest in physical capital, many factors may influence the decision to invest in innovation. For investment, q-theory suggests that firms should

[7] I follow the estimation strategy of de Loecker & Warzynski (2012) to estimate TFP in Compustat, using the variable construction of de Loecker & Eeckhout (2017) in Compustat.

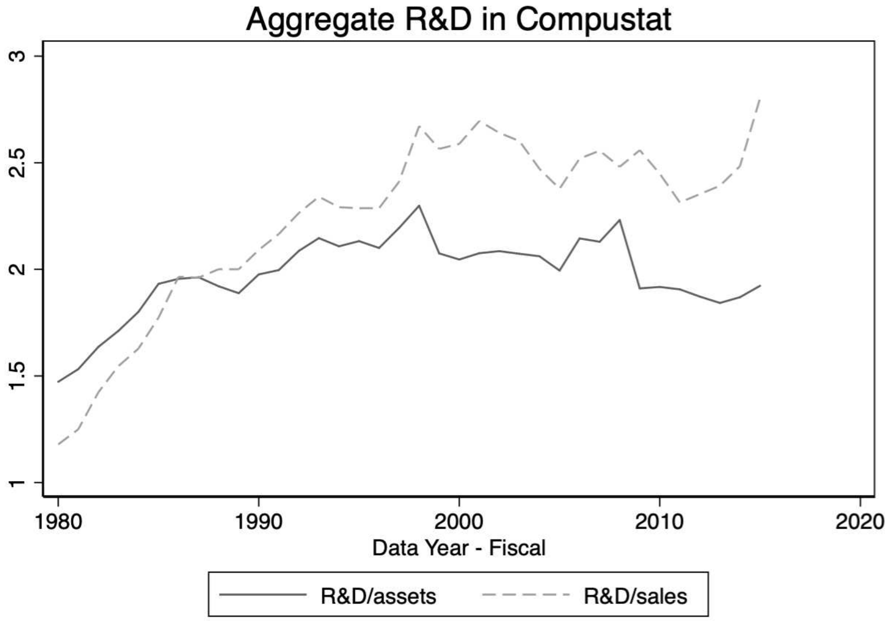


Figure 4: Source: Author's calculations from Compustat summing R&D (XRD) for all firms and dividing by the sum of sales (SALE) or assets (AT) for all firms.

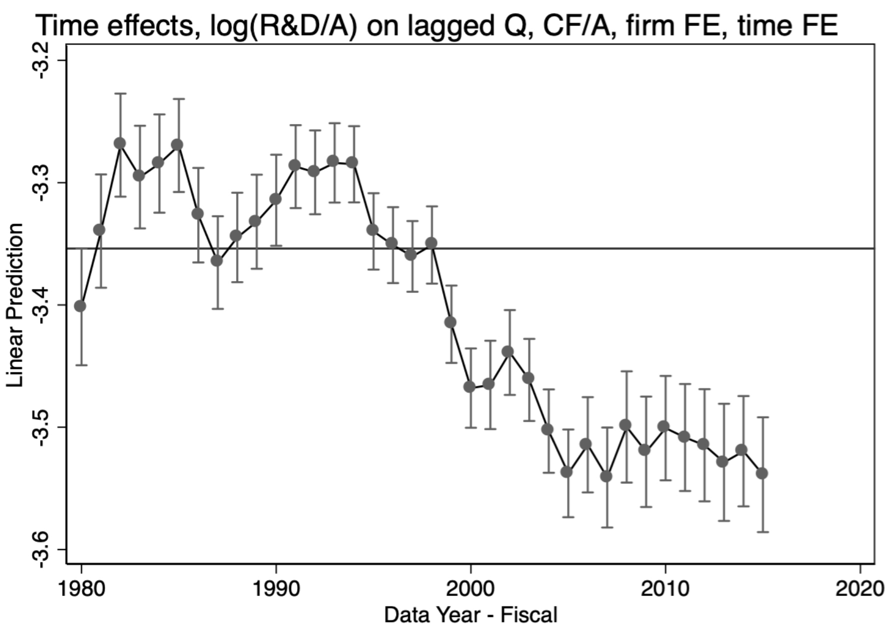


Figure 5: Source: Author's calculations from Compustat.

invest when the market value of their assets exceeds the book (replacement) value.[8] I replicate the exercise of Alexander & Eberly (2018) for intangible capital using R&D expenditure as the outcome variable to check whether R&D is slowing down *relative to what theory would predict.* The regression is:

$$\log\left(\frac{R\&D}{assets}_{i,t}\right) = \alpha + \eta_t + X_i + \beta_1 \log\left(\frac{cashflow}{assets}_{\ i,t}\right) + \beta_2 \log\left(q_{i,t}\right) + \varepsilon_{i,t}$$

I find that R&D has also declined relative to what would be predicted by the theory (figure 5).

In studying the investment slowdown that has partially driven the growth slowdown recently, Gutierrez & Philippon (2016) point out that net investment is low among U.S. public firms despite high value of Tobin's q and explore potential explanations. Increasing concentration appears to be one of the strongest factors correlated with the investment slowdown. Substituting intangible investment or R&D for physical capital investment doesn't change their results. In conclusion, both firm-level average productivity and R&D seem to be slowing down among public firms (with the caveat that q-theory may not be the ideal predictor for optimal R&D behavior).

2.2 Productivity Differences

In two companion papers Andrews et al. (2015) and Andrews et al. (2016) summarize characteristics of firms at the global productivity frontier (defined as either the top 100 or the top 5% of firms by estimated productivity in each industry-year). The first paper highlights the growing productivity gap between this frontier and other firms. Globally, frontier firms' productivity has grown at a rate of 3.6% per year while non-frontier firms' productivity grew at just 0.4% over the 2000s.[9] The authors identify two distinct periods: from 2001-2007, frontier firms' productivity grew 4-5% per year and other firms grew 1%, but since the global financial crisis frontier firms have seen productivity growth of just 1% per year while the productivity growth of non-frontier firms was flat.

To demonstrate that this global fact is also true within the U.S., figure 6 shows average productivity gaps for U.S. public firms in Compustat, either from the industry's

[8] Formally, I construct Tobin's q in Compustat as $\frac{\text{assets (AT)+shares outstanding (CSHO)*share price (PRCCF)-common equity (CEQ)}}{\text{assets (AT)}}$

[9] Productivity is measured in their papers using both labor productivity and multi-factor productivity to account for the fact that frontier firms tend to be more capital intensive. These numbers refer to labor productivity.

market leader in terms of sales or to the most productive firm within its industry. Gaps are defined as the difference between detrended log-TFP of the two firms. I then compute the (unweighted) average of these gaps for all firms within an industry and compute the economy-wide average weighted by industry size. Both measures of the gap are growing over time. The standard deviation of TFP in logs over the sample is 0.77. The average gap to the largest firm grew about 0.5 log points from 1990 to 2010, so about 65% of a standard deviation, and the widening of the gap to the most productive firm was even larger, around one standard deviation.

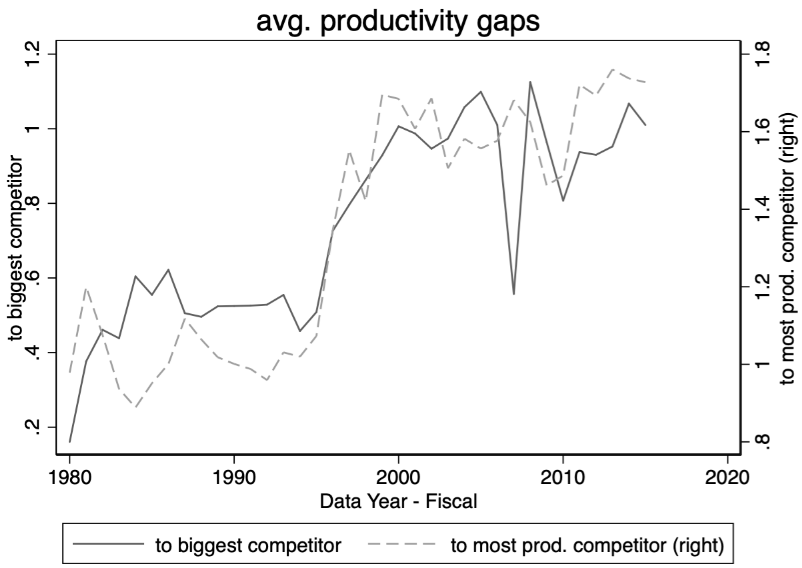


Figure 6: (Unweighted) average productivity gap to either the largest (left axis) or most productive (right axis) competitor in 4-digit SIC, sale-weighted across industries. Compustat.

Averages might not tell the whole story. Figure 7 shows the rightward shift in the distribution of firms over technology gaps to their largest or most productive competitor, verifying that more firms have large technology gaps in 2015 compared to 1995.

2.3 Product Market Concentration

The dramatic rise in the average market share of the largest firm in each industry shown in figure 1 is not just driven by large increases in a few sectors. Figure 8 shows how the entire distribution of industries over the leader's market share has changed from 1995 to 2015. Many more sectors now have just one very large public firm than in 1995,

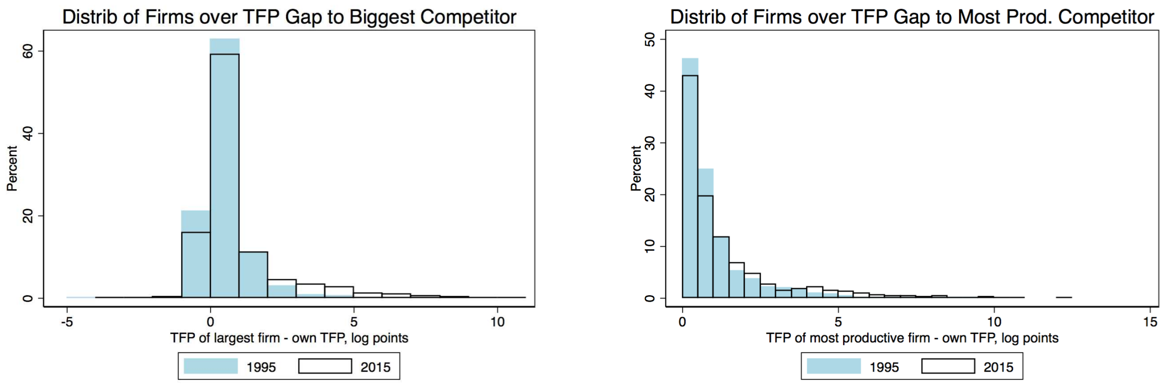


Figure 7: Source: Compustat. Gap is determined to largest (in terms of sales) or most productive firm in 4-digit SIC. A more positive gap means the firm is lagging further behind the leader.

and the peak of the distribution has shifted rightward significantly. This rise is not just due to the increasing market share of foreign firms and resultant mismeasurement of industry sales in Compustat: Gutierrez & Philippon (2017) construct an import-adjusted Herfindahl index for the U.S. and a similar rise can be seen in this metric.

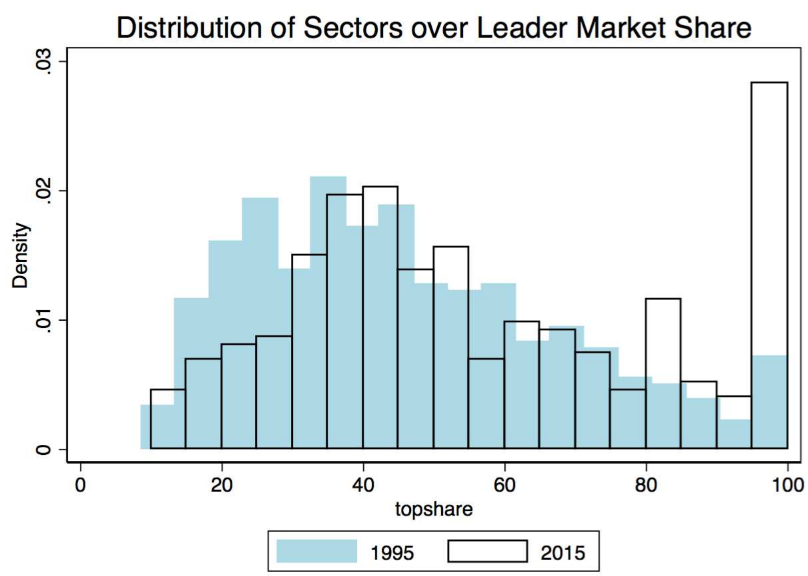


Figure 8: "Sector" refers to 4-digit SIC. Source: Compustat.

A variety of explanations for rising sales concentration have been proposed, from the introduction of ICT that creates winner-take-all markets in a wide variety of industrial classes (retail, entertainment, banking, etc.) and enables the growth of superstar firms (see for example Bessen (2017) and van Reenen (2018)), to excessive regulations that erect barriers to entry and create unnatural monopolies (Gutierrez & Philippon (2017)), to increased mergers and acquisitions activity, possibly due to weak antitrust enforcement (Grullon et al. (2017)).

2.4 Correlation Among Three Phenomena

An exploration of the causal relationships among productivity growth, productivity gaps, and concentration is beyond the scope of this paper, but some recent sector-level evidence suggests at least a correlation, consistent with the predictions of the model. Gutierrez & Philippon (2017) find that R&D expenditure has slowed down more in more concentrated sectors. Autor et al. (2017b) generally find that sectors with large superstar firms have higher productivity growth over long horizons, but don't investigate changes in the relationship between concentration and productivity growth over time. When taking these changes into account, Gutierrez & Philippon (2017) find positive correlations between concentration and TFP growth "only before 2002, but an insignificant and sometimes negative correlation after 2002." In another paper they find that the contribution of superstar firms to aggregate productivity growth has fallen by more than a third since 2000 (Gutiérrez & Philippon (2019)). Autor et al. (2017b) also find that technology diffusion is slower in more concentrated sectors. More concentrated industries have also seen the greatest slowdown in investment, which has accompanied the aggregate labor productivity slowdown that began in the early 2000s (Gutierrez & Philippon (2016); Hall (2015); Crouzet & Eberly (2018)).

To directly study the productivity *slowdown* rather than the correlation between productivity growth and concentration, I use data from the Bureau of Economic Analysis estimates of multifactor productivity[10] at the 3-digit NAICS level and data from Compustat to check the association between the change in the leader's market share in Compustat and the change in the sector's average productivity growth rate from 1994-2004 to 2005-2017 at the sector level. Figure 9 shows that sectors experiencing greater slowdowns in average productivity growth rates between 1994-2004 and 2005-2017 also saw greater increases in leader market share on average.

[10]https://www.bea.gov/data/special-topics/integrated-industry-level-production-account-klems

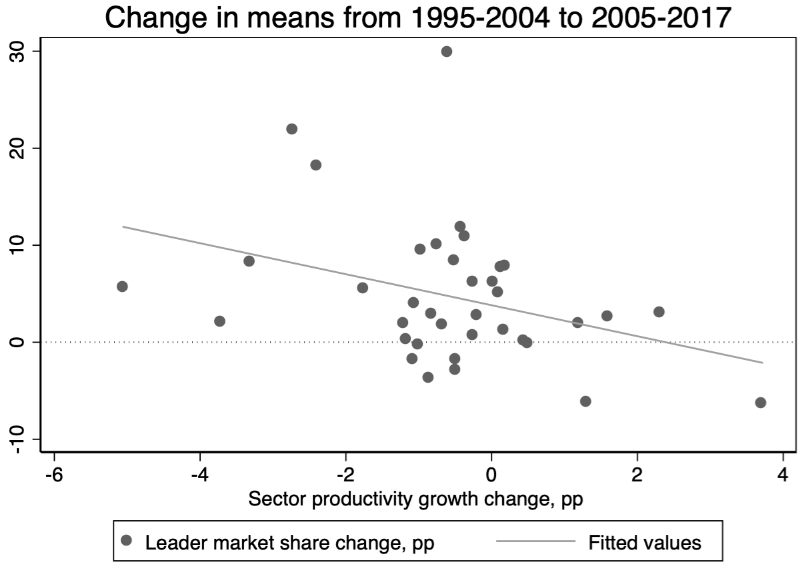


Figure 9: Source: Author's calculations from Compustat and BEA Integrated Industry-Level Production Accounts. 3-digit NAICS sectors, comparing 1994-2004 average to 2005-2017 average.

2.5 Trends in Patent Quality

Various measures of patent quality show substantial heterogeneity at any given time (Akcigit & Kerr (2018)). Recent evidence from patent data also points to changes in quality over time, particularly in the right tail of the quality distribution. The main exercise I conduct with the model is to vary the probability of large, breakthrough innovations over time, consistent with this fact.

Fluctuations in patent quality are often attributed to waves of innovation due to the arrival of general purpose technologies (GPTs). Bresnahan & Trajtenberg (1995) identify characteristics of GPTs. First, GPTs are *pervasive*, meaning they are applicable in a wide range of sectors. Second, GPTs involve *innovational complementarities*: the productivity of downstream research and development increases as a result of innovation in the GPT. Due to these complementarities, the gains of which are diffuse from the perspective of the sector creating the GPT, their model rationalizes the lags involved in the commercialization of GPTs. For example, the ICT revolution arguably began in the 1970s with the invention of the microprocessor but the biggest gains for productivity growth did not occur until the 1990s.

These innovational complementarities are apparent in patent data. Kelly et al.

(2018) create a text-based measure of patent quality, identifying "breakthrough" patents as those where the patent's text differs from past patents but is similar to future patents. Plotting the median, 75th, 90th, and 95th percentile of the patent quality distribution using this metric shows periods of high patent quality coincide with timelines of general purpose technologies, including the ICT revolution in the 1990s (figure 10).

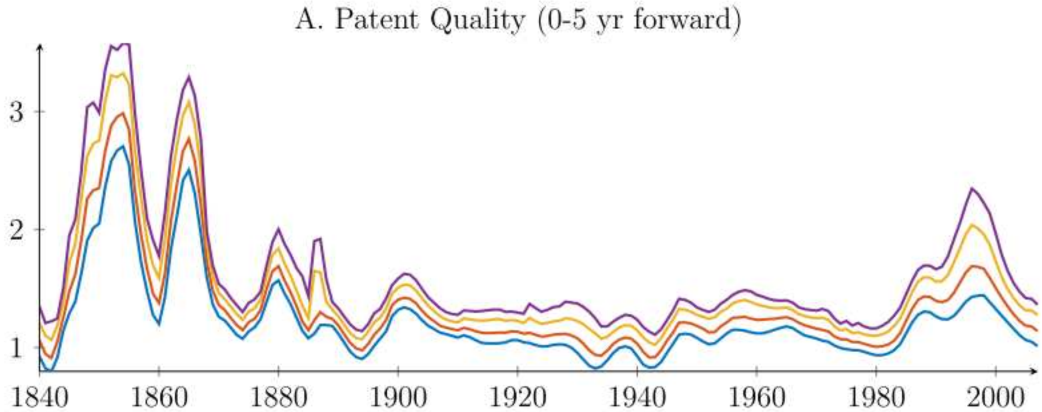


Figure 10: Blue = P50, Red = P75, Yellow = P90, Purple = P95. Source: Kelly et al. (2018) fig. 3A text-based patent quality measure.

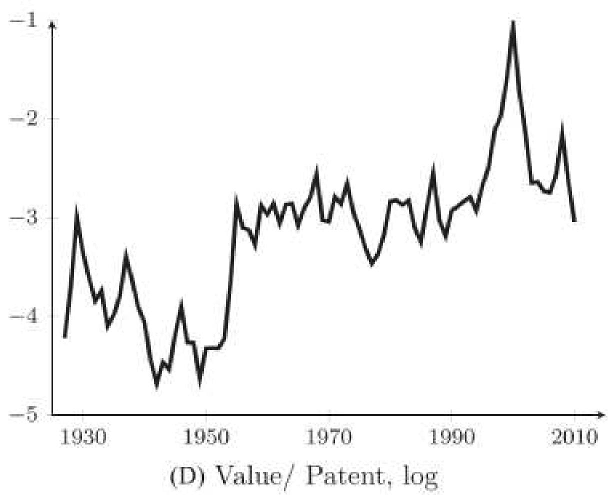


Figure 11: Source: Kogan et al. (2017) figure IV(d) market-based patent quality measure.

Consider also the measure of patent quality by Kogan et al. (2017) that estimates the market value of each patent issued in the U.S. and assigned to public firms from 1926-2010. The average patent value (in real terms) is plotted in figure 11. Kelly et al.

(2018) document the striking correlation between the market- and text-based measures at the patent level (as well as the correlation of these measures with forward citations). Both measures show a sharp uptick in average patent quality and in the right tail during the 1990s and a subsequent decline beginning in the late 1990s. Figure 12 plots a kernel density estimate of the distribution of patent values in millions of 1982 USD for 1995 and 2010. 2010 features many fewer cases of higher-value patents than 1995 (also robust to considering 2005 instead of 2010).

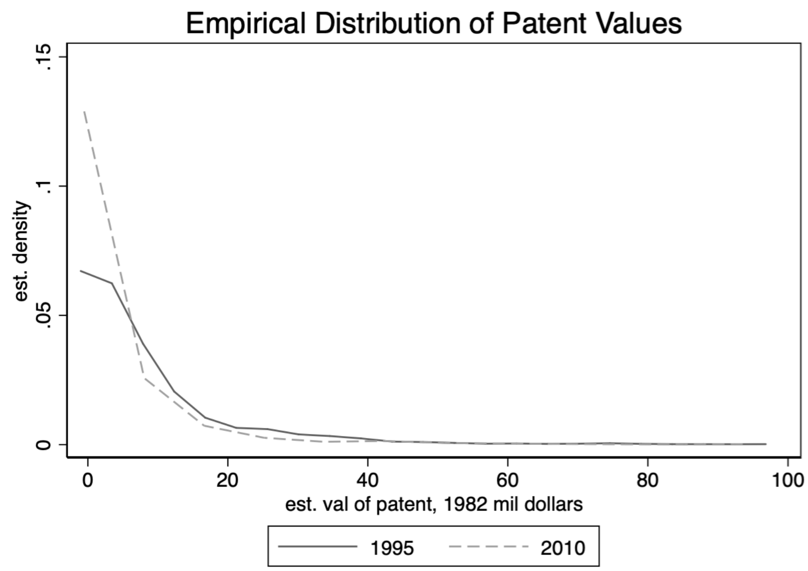
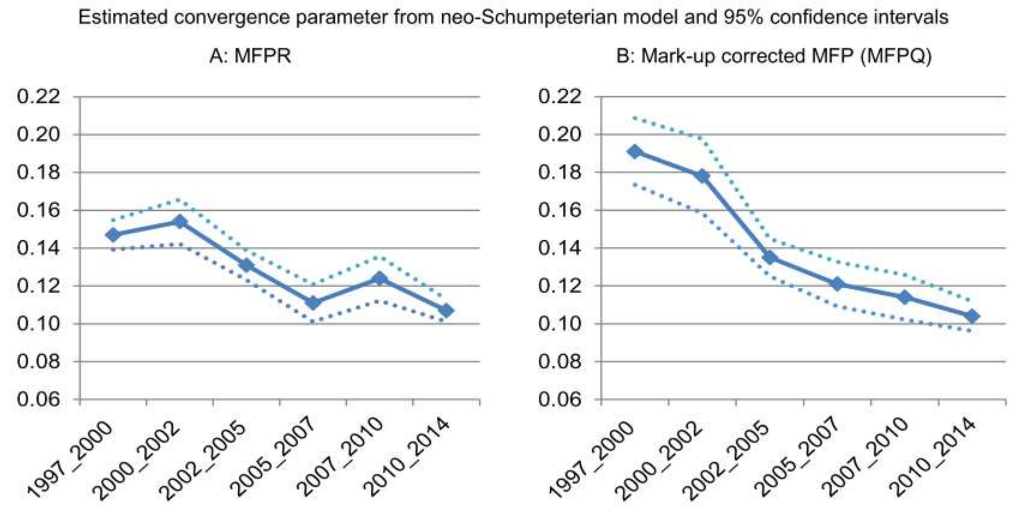


Figure 12: Source: Kernel density estimates based on Kogan et al. (2017) market-based patent quality estimates.

Firm-level data also shows that the "advantage of backwardness" has fallen relative to the 1990s, consistent with the idea that it is now harder to catch up through innovation than it was in the 1990s, though this evidence is by no means conclusive. Andrews et al. (2016) show that in a regression of firm-level productivity growth on a variety of explanatory variables, the coefficient on the lagged productivity gap to the technology frontier has been declining over the 2000s (figure 13), suggesting that distance to the productivity frontier is becoming a less important predictor of future growth. [11]

Finally, turnover at the technology frontier appears to be slowing. According to

[11] However, this empirical observation is endogenous according to the model, because it may be a result of both structural change to catchup speeds and to the endogenously lower innovation effort by laggard firms since their regression doesn't control for innovation effort (R&D investment).



Notes: The marker line shows the evolution over time of the estimated $GAP_{i,t-1}$ coefficient from the firm level MFP growth regression in Table B1 Appendix B (Column 1, Panel A), while the dashed line provides the 95% confidence interval around this coefficient estimate.

Figure 13: Speed of technological convergence is slowing down. Plotted coefficient is the coefficient on a firm's lagged multi-factor productivity (MFP) gap in a regression of current MFP on lagged MFP gap to the frontier. Source: Andrews et al. (2016).

Andrews et al. (2016), from 2011-2013, more frontier firms survived an additional year at the frontier, and those moving into the productivity frontier were more likely to already come from the top 10% or 20% of the productivity distribution in the previous year compared to 2001-2003. In Compustat, since 2000, the share of leading firms that have maintained their market leadership (in terms of sales) for at least 5 years has risen from 50% in 2002 to around 70% in 2015. I next present a model that can capture these findings parsimoniously.

3 Model

The model is of a closed economy in continuous time. There are three types agents: a representative household, a representative competitive final good firm, and intermediate goods firms producing capital goods. This section presents the model by going step-by-step through each type of agent in the economy, then analyzes the equilibrium of the model.

3.1 Households

A representative household consumes, saves, and supplies labor inelastically to maximize:

$$U_t = \int_t^\infty \exp(-\rho(s-t)) \frac{C_s^{1-\psi}}{1-\psi} ds$$

subject to:

$$r_t A_t + W_t L = P_t C_t + \dot{A}_t$$

I use $\dot{X}$ to denote the time derivative of the variable X.

Households own all the firms, and the total assets in the economy are:

$$A_t = \int_0^1 \sum_{i=1}^{2} V_{ijt} dj$$

Where V_{ijt} is the value of intermediate good firm i in sector j at time t. These value functions are explained in greater detail in section 3.3. The number of firms per sector (two) and the measure of sectors (one) are imposed exogenously. For a balanced growth path with constant growth rate of output g this yields the standard Euler equation $r = g\psi + \rho$.

3.2 Final Good Producers

The competitive final goods sector combines intermediate goods and labor to create the final output good which is used in consumption, research, and intermediate good production. The final good firm's technology is as follows:

$$Y = \frac{1}{1-\beta} \left(\int_0^1 K_j^{1-\beta} dj \right) L^\beta$$

where K_j is a composite of two intermediate good firms' products within sector j described below. β determines both the elasticity of substitution across sectors ($\frac{1}{\beta}$) and the labor share. For now consider the final good firm's problem of hiring sector composite goods K_j and labor:

$$\max_{K_j, L} P \frac{1}{1-\beta} \left(\int_0^1 K_j^{1-\beta} dj \right) L^\beta - P_j K_j - WL$$

where P is the price of the final good and P_j is the price of the sector j composite good and W is the nominal wage. The first order condition for sector j's composite good given sector j's composite price index P_j will give the demand for sector j's good:

$$K_j = \left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L$$

and the real wage is equal to the marginal product of labor:

$$\beta\frac{Y}{L} = \frac{W}{P}$$

Now to derive the demand curve for each firm i within sector j we need to define the sector composite K_j explicitly:

$$K_j = \left((q_{1j}k_{1j})^{\frac{\epsilon-1}{\epsilon}} + (q_{2j}k_{2j})^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}}$$

where q_{ij} is the quality of firm i's product (equivalently as firm i's productivity) and k_{ij} is the output of firm i purchased by the final good producer.[12] $\epsilon > \frac{1}{\beta}$ is the elasticity of substitution between products in the same sector.

Within each sector j the final good producer will seek to minimize (dropping the j subscript to focus on a single sector):

$$\min_{\{k_i\}_{i=1}^2} \sum_{i=1}^{2} p_i k_i$$

subject to:

$$K = \left((q_1k_1)^{\frac{\epsilon-1}{\epsilon}} + (q_2k_2)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \geq \underline{\mathrm{K}}$$

Taking the first order condition for either k_i yields:

$$p_i = P_j K_j^{\frac{1}{\epsilon}} q_i^{\frac{\epsilon-1}{\epsilon}} k_i^{-\frac{1}{\epsilon}}$$

So, plugging in for K_j we get inverse demand:

$$\begin{aligned} p_i &= P_j \left(\left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L\right)^{\frac{1}{\epsilon}} q_i^{\frac{\epsilon-1}{\epsilon}} k_i^{-\frac{1}{\epsilon}} \\ &= k_i^{-\frac{1}{\epsilon}} q_i^{\frac{\epsilon-1}{\epsilon}} P_j \left(\frac{P_j}{P}\right)^{-\frac{1}{\beta\epsilon}} L^{\frac{1}{\epsilon}} \end{aligned}$$

Rearranging, the demand function is:

$$k_i = q_i^{\epsilon-1}\left(\frac{p_i}{P_j}\right)^{-\epsilon}\left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L \qquad (1)$$

[12]This is the sense in which productivity and quality are equivalent: doubling the quality q_{ij} of both firms has the same effect on final output as doubling the output k_{ij} of both firms.

3.3 Intermediate Goods Producers

In this section I discuss production and competition in the intermediate goods market and then the innovation technology. In both cases I present the firms' problem first and then the solution. Each intermediate good sector is a duopoly and there is no entry margin. This assumption precludes the possibility of analyzing the contribution of entry to output and productivity growth. Empirical evidence summarized in Bartelsman & Doms (2000) suggests that incumbents are responsible for around 75% of industry-level TFP growth in the U.S. so the model still captures a large share of productivity growth dynamics.

3.3.1 Production and Price Setting

The intermediate goods producers purchase final goods to transform them into differentiated intermediate goods. Each unit of intermediate output requires $\eta < 1$ units of the final good to produce. There are no other inputs to intermediate good production.

Facing the demand for their product from the final good producer given in equation 1, I assume the technology *follower*, the firm with lower quality q_i, must set price equal to marginal cost. Understanding this, the leader chooses its optimal price in response to the price of the follower.[13] If the firms are neck-and-neck I assume both set price equal to marginal cost. Since this competition assumption plays an important role in the mechanisms of the model I take a quick detour to discuss it here.

First, this pricing assumption is actually similar to the assumptions made in other quality ladder models (Klette & Kortum (2004), Acemoglu & Cao (2015), Akcigit & Kerr (2018)), with the caveat that my model features the presence of a lower-quality substitute to the market leader's product. In those other models, quality improvement over a product line confers a fully enforceable patent on the product until the next innovation occurs. I similarly assume the firm with the higher quality has a fully enforceable patent on its product, but I introduce the notion of sectors and include a second lower quality product in each sector.[14]

[13] Solving the model where both firms set prices a la Bertrand is also possible. See Appendix B for a discussion and results.

[14] Another way to micro-found this assumption is by introducing a cost to filing a patent that is sufficiently high that only the leader, who exercises some additional market power by possessing the higher quality and thus earns higher profits, would be willing to pay. Most Schumpeterian growth models provide no micro-foundation for the monopolist pricing assumption.

If one further assumes that there is no patent protection for the lower quality product and there is free entry to the production of the follower's product, the pricing decisions are the fully optimal outcomes of Bertrand pricing. Under that assumption, the number of followers is indeterminate since the intermediate good production technology is constant returns to scale. The simplest way to resolve this indeterminacy is to assume the lower quality product is produced by a single firm with zero profits.

This assumption plays an important role in determining the shape of the innovation policy as a function of technology differences, specifically the hump shape. This shape has been suggested theoretically in the work of Aghion et al. (2005), Akcigit et al. (2018), and Schmidt (1997) and found in a variety of studies including Aghion et al. (2005) and Carlin et al. (2004). Intuitively the hump shape appears in this model because the pricing assumption means that the greatest incremental gain in flow profits comes from obtaining quality leadership, so innovation effort will be highest when firms have equal quality.

Finally, the assumption generates empirically plausible predictions about profit shares: the largest U.S. public firms (by sales) capture by far the largest share of industry profits (see figure 14).[15]

I now proceed to the solution of the leader's pricing problem. Some definitions are needed. First, sector j's price index:

$$P_j = \left(\sum_{i=1}^{2} q_{ij}^{\epsilon-1} p_{ij}^{1-\epsilon} \right)^{\frac{1}{1-\epsilon}}$$

Let s_i be firm i's market share of sales in sector j, plugging in the final good firm's demand for k_{ij} (again dropping js to focus on a single sector):

$$s_i = \frac{p_i k_i}{\sum_{i=1}^{2} p_i k_i} = \frac{q_i^{\epsilon-1} p_i^{1-\epsilon} P_j^{\epsilon-\frac{1}{\beta}} P^{-\frac{1}{\beta}} L}{\sum_{i=1}^{2} q_i^{\epsilon-1} p_i^{1-\epsilon} P_j^{\epsilon-\frac{1}{\beta}} P^{-\frac{1}{\beta}} L} = q_i^{\epsilon-1} \left(\frac{p_i}{P_j} \right)^{1-\epsilon} = \frac{p_i}{P_j} \frac{\partial P_j}{\partial p_i}$$

The final equality holds because:

$$\frac{\partial P_j}{\partial p_i} = \frac{1}{1-\epsilon} P_j^{\epsilon} (1-\epsilon) q_i^{\epsilon-1} p_i^{-\epsilon} = q_i^{\epsilon-1} \left(\frac{p_i}{P_j} \right)^{-\epsilon}$$

Now we are ready to consider the pricing problem of the technology leader i in sector j:

[15] TFP and size are highly correlated, and the figure looks similar if one uses a productivity ranking instead of sales-based ranks.

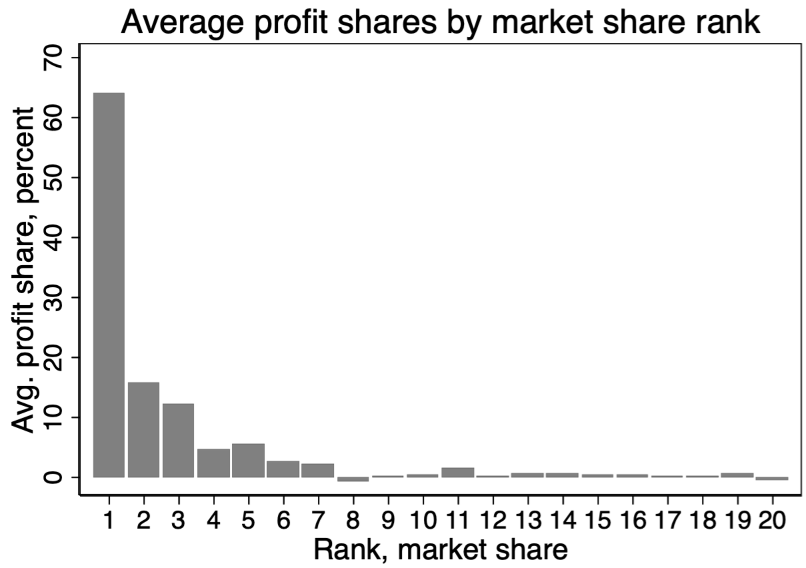


Figure 14: Source: Compustat, 1975-2015. Firms are ranked by market share (sales) within 4-digit SIC industries, and these ranks are compared to profit shares (firm's own operating income as a share of industry-total operating income). The figure averages across 4-digit sectors.

$$\max_{p_i} p_i k_i - \eta k_i$$

subject to the inverse demand:

$$k_i = q_i^{\epsilon-1}\left(\frac{p_i}{P_j}\right)^{-\epsilon}\left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L$$

Taking the first order condition for the price and using the definition of market share above yields the optimal pricing policy:

$$p_i = \frac{\epsilon - (\epsilon - \frac{1}{\beta})s_i}{\epsilon - (\epsilon - \frac{1}{\beta})s_i - 1}\eta$$

The optimal price is the standard one for two-layered constant elasticity of demand structures (nested CES): a variable markup that rises in market share. This is easiest to see for the two extreme cases where market share is 0 or 1. When market share is 0, the firm is atomistic with respect to the sector and charges a markup $\frac{\epsilon}{\epsilon-1}$, the CES solution for an elasticity of substitution equal to ϵ. On the other hand, if the market share is 1, the firm only weighs the elasticity of substitution across sectors and sets a markup $\frac{1}{1-\beta} > \frac{\epsilon}{\epsilon-1}$ since products are less substitutable across sectors than within sectors.

It will be important for the (tractable) solution of the model that firms' prices not depend on their quality level, only on their quality *relative* to their competitor, referred to as the *technology gap* between the two firms. The technology gap is defined by the ratio $\frac{q_1}{q_2}$ for firm 1 and $\frac{q_2}{q_1}$ for firm 2. Below I show that this is the case. First, this is clearly satisfied for the technology follower who always sets price equal to marginal cost η regardless of absolute quality.

Second, for the leader, use the definition of the market share and the price index to solve for the market share of the leader i in sector j ($-i$ denotes the follower):

$$\begin{aligned} s_i &= q_i^{\epsilon-1}\left(\frac{p_i}{P_j}\right)^{1-\epsilon} \\ &= \frac{q_i^{\epsilon-1}p_i^{1-\epsilon}}{q_i^{\epsilon-1}p_i^{1-\epsilon}+q_{-i}^{\epsilon-1}\eta^{1-\epsilon}} \\ &= \frac{1}{1+\left(\frac{q_{-i}}{q_i}\right)^{\epsilon-1}\left(\frac{p_i}{\eta}\right)^{\epsilon-1}} \end{aligned}$$

Now using the pricing decision of the leader $p_i = \frac{\epsilon-(\epsilon-\frac{1}{\beta})s_i}{\epsilon-(\epsilon-\frac{1}{\beta})s_i-1}\eta$:

$$s_i = \frac{1}{1+\left(\frac{q_{-i}}{q_i}\right)^{\epsilon-1}\left(\frac{\epsilon-(\epsilon-\frac{1}{\beta})s_i}{\epsilon-(\epsilon-\frac{1}{\beta})s_i-1}\right)^{\epsilon-1}}$$

Thus there is a mapping from technology gaps to market shares and prices that is independent of quality levels. The market shares and prices for firms with a given technology gap are shown in figure 15. The x-axis values m are a way of capturing the number of quality steps ahead/behind a firm is from its competitor described in detail in the next section. The particular parameterization used to generate the figure is given in table 1 in section 4. The slope of these figures is sensitive to ϵ, the elasticity of substitution between firms in the same sector. The leader's optimal price p_i rises as the technology gap widens (that is, as the leader's relatively quality improves). Most of the effect of increased quality appears in the leader's output k_i, so the market share of the leader $(\frac{p_i k_i}{\sum_{i=1}^{2} p_i k_i})$ rises more dramatically in quality than does the price. In this particular parameterization, market share rises from around 32% of sales with one quality step ahead to 70% of the market at 16 steps ahead. The follower, who must sell at $p = \eta$, has a large market share (due to their relatively low price) that also increases as the follower's relative quality improves.

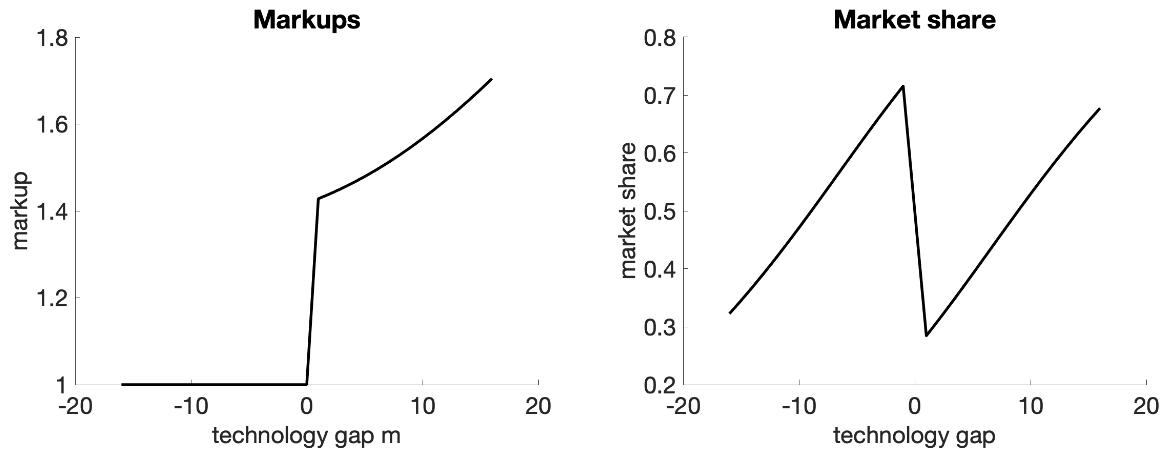


Figure 15: Markups and resulting market shares as a function of the technology gap (ratio of firm qualities).

Obtaining quality leadership in the model causes a drop in market share but, crucially, a rise in profits which is the payoff-relevant object of the firm. Growing market share itself is not an objective of the firm. I focus on the market share of the quality leader in the numerical results because the model's two-firm setup has no direct analogy to industry-level data with more firms per sector. From the discussion of the pricing assumption, an alternative interpretation is that a competitive mass of small firms produce the lower quality product, but a single firm, the quality leader, has the ability to produce the higher quality product.

3.3.2 Innovation

The innovation process for improving the quality of intermediate goods follows Akcigit et al. (2018). Intermediate good producers choose the amount of research spending R of the final good to maximize the discounted sum of expected future profits. Innovations arrive randomly at Poisson rate x which depends on research spending according to the function:

$$x = \left(\frac{\gamma R}{\alpha}\right)^{\frac{1}{\gamma}} q_i^{\frac{1-\frac{1}{\beta}}{\gamma}}$$

that is, since $\beta < 1$, at higher quality levels more research spending is needed to achieve the same arrival rate of innovations x. γ and α are R&D technology parameters.

Conditional on innovating the size of the quality improvement is random.[16] For-

[16] Akcigit & Kerr (2018) formalize measures of heterogeneous quality improvements due to innovations

mally, conditional on innovating,

$$q_{i,(t+\Delta t)} = \lambda^{n_{i,t}} q_{i,t}$$

where $\lambda > 1$ is the minimum quality improvement and $n_{i,t} \in \mathbb{N}$ is a random variable. Note that each competitor improves over their own quality when they innovate.[17] Initial qualities of all firms are normalized to 1. Let $N_{i,t} = \int_0^t n_{i,s} ds$ denote the total number of step size improvements over a product line i since the beginning of time. The technology gap from firm 1's in sector j's perspective at moment t is:

$$\frac{q_{1,j,t}}{q_{2,t}} = \frac{\lambda^{N_{1,t}}}{\lambda^{N_{2,t}}} \equiv \lambda^{m_{1,t}}$$

Given λ, $m_{i,j,t}$ parameterizes the technology gap between the two firms, representing the number of λ steps ahead or behind its competitor firm i is. For numerical tractability I impose a maximal technology advantage $\bar{m}$, but in calibrating the model I will set the parameters so that this maximal gap rarely occurs in steady state. I assume that the only spillover in the model between firms occurs when a firm at the maximal gap innovates. In that case, both the innovating firm and its competitor's quality grow by λ, keeping the technology gap unchanged but raising the absolute quality of the sector composite good.

The probability distribution of possible quality improvements depends on the firm's relative quality compared to its competitor. As in Akcigit et al. (2018), I assume there exists a fixed distribution $\mathbb{F}(n) \equiv c_0(n + \bar{m})^{-\phi}$ for all $n \in \{-\bar{m}+1, \ldots, \bar{m}\}$, shown in the left panel of figure 16, that applies to firms that are the furthest possible distance behind their competitor. The curvature parameter ϕ is critical in the model and determines the speed of catchup by increasing or decreasing the probability of larger innovations. A higher ϕ means a lower probability of these "radical" improvements.[18] c_0 is simply a shifter to ensure $\sum_n \mathbb{F}(n) = 1$.

Given this fixed distribution, the step size distribution specific to each technology gap $m > -\bar{m}$ is given by:

$$\mathbb{F}_m(n) = \begin{cases} \mathbb{F}(m+1) + \mathbb{A}(m) & \text{for } n = m+1 \\ \mathbb{F}(s) & \text{for } n \in \{m+2, \ldots, \bar{m}\} \end{cases}$$

using patent citations. They argue that quality improvements are empirically heterogeneous.

[17] Luttmer (2007) provides a rationale for this type of assumption: entrants are typically small and enter far from the productivity frontier, implying that imitation of other firms' technologies is difficult.

[18] As noted by Akcigit et al. (2018), this formulation converges to the less general step-by-step model as $\phi \to \infty$.

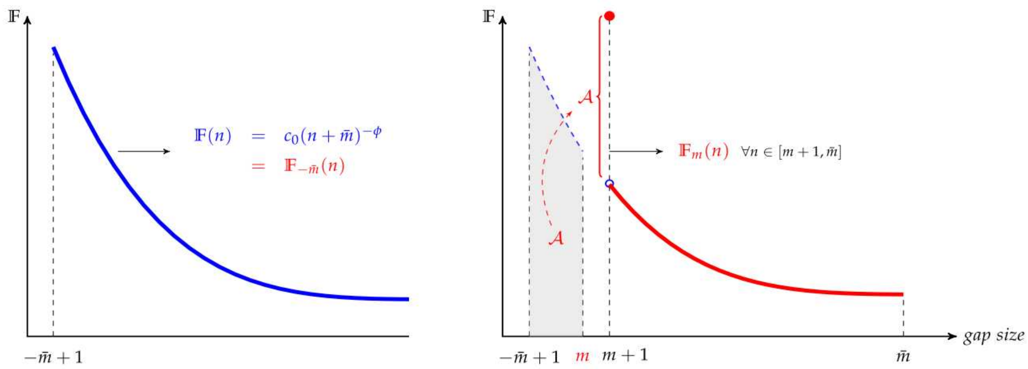


Figure 16: Source: Akcigit et al. (2018)

where $\mathbb{A}(m) \equiv \sum_{-\bar{m}+1}^{m} \mathbb{F}(n)$. This distribution is shown in the right panel of figure 16. Simply put, all the mass of the fixed distribution on steps down from current quality is instead put on one-step ahead improvements.

The decision to model R&D as a process of own-product quality improvement by incumbents is consistent with the evidence in Garcia-Macia et al. (2019) that: (i) incumbents are responsible for most employment growth in the U.S., and this share has increased in recent years; (ii) growth mainly occurs through quality improvements rather than new varieties; (iii) creative destruction by entrants and incumbents over other firms' varieties accounted for less than 25% of employment growth from 2003-2013.

3.3.3 Value Functions

An intermediate good firm's value function with quality q_t and gap to its rival m_t at moment t is defined as[19]:

$$
\begin{aligned}
r_t V_{mt}(q_t) - \dot{V}_{mt}(q_t) = \max_{x_{mt}} \{ & \pi(m, q_t) - \alpha \frac{(x_{mt})^{\gamma}}{\gamma} q_t^{\frac{1}{\beta}-1} \\
& + x_{mt} \sum_{n_t=m+1}^{\bar{m}} \mathbb{F}_m(n_t)[V_{nt}(\lambda^{n_t - m} q_t) - V_{mt}(q_t)] \\
& + x_{(-m)t} \sum_{n_t=-m+1}^{\bar{m}} \mathbb{F}_{-m}(n_t)[V_{(-n)t}(q_t) - V_{mt}(q_t)] \} \qquad (2)
\end{aligned}
$$

The firm chooses the arrival rate of innovations x_{mt}. The first line denotes the flow

[19] I give the slightly altered equations for firms at the minimum and maximum gaps in Appendix C.

profits and the research cost R_{mt} given the choice of x_{mt}. The second line denotes the probability the firm innovates and sums over the possible states the firm could move to using the distribution of steps sizes and the firm's new value function with higher quality and a larger quality advantage over its rival. The final line denotes the chance the firm's rival innovates and the change in the firm's value because its relative quality falls when the rival innovates.

Now consider the flow profits of the firm, denoting the optimal price of the leader at technology gap m as $p(m)$. We want to eliminate q_t from the value function for tractability, so that each technology gap is associated with a value and the specific firm value function scales in q_t or some function of q_t.

$$\pi(m, q_t) = \begin{cases} 0 & \text{if } m \leq 0 \\ (p(m) - \eta)k_i & \text{for } m \in \{1, \ldots, \bar{m}\} \end{cases}$$

Use the fact that $k_i = q_i^{\epsilon-1} p(m)^{-\epsilon} P_j^{\epsilon - \frac{1}{\beta}} P^{-\frac{1}{\beta}} L$. Normalize P and L to 1. Expanding the definition of the sectoral price index P_j: $k_i = q_i^{\epsilon-1} p(m)^{-\epsilon} (q_i^{\epsilon-1} p(m)^{1-\epsilon} + q_{-i}^{\epsilon-1} \eta^{1-\epsilon})^{\frac{\epsilon - \frac{1}{\beta}}{1-\epsilon}}$. This further simplifies (by factoring out $q_i^{\epsilon-1}$ from the price index) to:

$$\pi(m, q_t) = \begin{cases} 0 & \text{if } m \leq 0 \\ q_i^{\frac{1}{\beta}-1} (p(m) - \eta) p(m)^{-\epsilon} (p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} \eta^{1-\epsilon})^{\frac{\epsilon - \frac{1}{\beta}}{1-\epsilon}} & \text{for } m \in \{1, \ldots, \bar{m}\} \end{cases}$$

So $V_{mt}(q_t) = v_{mt} q_t^{\frac{1}{\beta}-1}$. It can be shown by a guess-and-check approach that this is the case.

The firm's optimal arrival rate x_{mt} (which I refer to as effort in subsequent discussions) is the solution to the first order condition of equation (2), which gives:

$$x_{mt} = \begin{cases} \left(\frac{\sum_{n=m+1}^{\bar{m}} \mathbb{F}_m(n_t)[(\lambda^{n_t - m})^{\frac{1}{\beta}-1} v_{nt} - v_{mt}]}{\alpha} \right)^{\frac{1}{\gamma-1}} & \text{for } m < \bar{m} \\ \left[\frac{1}{\alpha} (\lambda^{\frac{1}{\beta}-1} - 1) v_{\bar{m}t} \right]^{\frac{1}{\gamma-1}} & \text{for } m = \bar{m} \end{cases}$$

The model delivers the predictions that R&D intensity is independent of size (sales) and heterogeneous across firms in the same sector, consistent with the empirical evidence discussed in Klette & Kortum (2004).

3.4 Equilibrium Output

Below I solve for output Y plugging in the intermediate goods firms' output decisions to illustrate the components of output growth:

$$
\begin{aligned}
Y &= \frac{1}{1-\beta}\left(\int_0^1 K_j^{1-\beta} dj\right) L^\beta \\
&= \frac{1}{1-\beta}\left(\int_0^1 \left(\sum_{i=1}^{2} q_i^{\frac{\epsilon-1}{\epsilon}} (q_i^{\epsilon-1}\left(\frac{p_i}{P_j}\right)^{-\epsilon}\left(\frac{P_j}{P}\right)^{-\frac{1}{\beta}} L)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}1-\beta} dj\right) L^\beta \\
&= \frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}}\left(\int_0^1 P_j^{\epsilon(1-\beta)-\frac{1-\beta}{\beta}}\left(\sum_{i=1}^{2} q_i^{\epsilon-1} p_i^{1-\epsilon}\right)^{\frac{\epsilon(1-\beta)}{\epsilon-1}} dj\right) \\
&= \frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}}\left(\int_0^1 P_j^{-\frac{1-\beta}{\beta}} dj\right)
\end{aligned}
$$

The demand shifter $P^{\frac{1}{\beta}}L$ index is common to all firms and can be taken out entirely (and normalized to one since I assume zero population growth). The price index P_j of each sector falls as the qualities of the two firms in the sector grow, and the exponent is negative for all $\beta \in (0,1)$ so Y is growing in qualities.

Common to all firms with a particular technology gap m are the prices $p(m)$ of the firm at gap m and its competitor at $-m$, $p(-m)$. At time t, Y can be expressed as:

$$
Y_t = \frac{1}{2}\frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}} \sum_{m=-\bar{m}}^{\bar{m}} \left(\int_0^1 \left(q_{it}^{\epsilon-1} p_i(m)^{1-\epsilon} + q_{-it}^{\epsilon-1} p_{-i}(-m)^{1-\epsilon}\right)^{-\frac{(1-\beta)}{\beta(1-\epsilon)}} \mathbb{1}_{\{i\in\mu_{mt}\}} di\right)
$$

$$
Y_t \equiv \frac{1}{2}\frac{L}{1-\beta} P^{\frac{1-\beta}{\beta}} \sum_{m=-\bar{m}}^{\bar{m}} Q_{mt} \tag{3}
$$

Where Q_{mt} is defined as:

$$
\begin{aligned}
Q_{m,t} &= \int_0^1 \left(q_{it}^{\epsilon-1} p(m)^{1-\epsilon} + q_{-it}^{\epsilon-1} p(-m)^{1-\epsilon}\right)^{-\frac{(1-\beta)}{\beta(1-\epsilon)}} \mathbb{1}_{\{i\in\mu_{mt}\}} di \\
&= \left(p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} p(-m)^{1-\epsilon}\right)^{\frac{1-\beta}{\beta(\epsilon-1)}} \int_0^1 q_{i,j}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i\in\mu_{mt}\}} di
\end{aligned} \tag{4}
$$

Here, μ_{mt} is the measure of firms at each technology gap m at time t (normalizing measure of firms to one) and Q_{mt} is a particular index of the qualities of all firms at gap m. The change in output therefore depends on the changes $\dot{Q}_{mt}$ for each technology gap m which in turn depend on the innovation arrival rates x_{mt} chosen by firms and the exogenous distribution of quality improvement sizes $F(n)$. The final component determining output will be the measure of firms at each technology gap μ_{mt} that is itself an endogenous object. Appendix D gives formulas for the growth of the quality indexes Q_{mt} and the next section describes how to solve for the measures μ_{mt}.

3.5 Stationary Distribution Over Technology Gaps

Firms move to technology gap m through innovation from a lower technology gap, or because their competitor innovates to gap $-m$. The distributions $F_n(m)$ and $F_{-n}(-m)$ determine these probabilities, combined with the innovation efforts of firms at n and $-n$. The outflows from gap m are due to the firm at m or its competitor at $-m$ innovating. Putting this together into the Kolmogorov forward equations for the evolution of the mass of firms at each gap:

$$\dot{\mu}_{mt} = \sum_{n=-\bar{m}}^{m-1} x_n F_n(m)\mu_{nt} + \sum_{n=m+1}^{\bar{m}} x_{-n}F_{-n}(-m)\mu_{nt} - (x_m + x_{-m})\mu_{mt}$$

The highest and lowest technology gaps are special cases because of spillovers: if the firm at the highest gap innovates both firms remain at the same gap in the next instant.

$$\dot{\mu}_{-\bar{m}t} = \sum_{n=-\bar{m}+1}^{\bar{m}} x_{-n}F_{-n}(\bar{m})\mu_{nt} - x_{-\bar{m}}\mu_{-\bar{m}t}$$

$$\dot{\mu}_{\bar{m}t} = \sum_{n=-\bar{m}}^{\bar{m}-1} x_n F_n(\bar{m})\mu_{nt} - x_{-\bar{m}}\mu_{\bar{m}t}$$

On a balanced growth path, $\mu_{mt} = \mu_m$ for all m, t. Replacing the left hand side of the above equations with zero change in equilibrium and the measures on the right hand side with the constants μ_n, μ_m defines a system of $2\bar{m}+1$ equations in $2\bar{m}+1$ unknowns. There are several additional restrictions on the solution to this system. First, for each firm at m there is a firm at $-m$ (that is, the stationary distribution is symmetric). Second, I impose the restriction that the measure of all the firms sums to one.

3.6 Equilibrium

Let $R_t = \int_0^1 \sum_{i=1}^2 R_{ijt}dj$ denote total research and development spending, C_t total consumption, and $K_t = \int_0^1 \sum_{i=1}^2 \eta k_{ijt}dj$ total purchases of final goods for production of intermediate goods.

An equilibrium is an allocation $\{k_{ijt}, K_t, x_{ijt}, R_t, Y_t, C_t, L, \mu_{mt}, Q_{mt}, A_t\}_{i\in\{1,2\},j\in[0,1],m\in[-\bar{m},\bar{m}]}^{t\in(0,\infty)}$ and prices $\{r_t, W_t, p_{ijt}\}_{i\in\{1,2\},j\in[0,1]}^{t\in(0,\infty)}$ such that for all t:

1. Intermediate goods firms solve their innovation and price-setting problems (price-setting optimally for the leader only)
2. Final goods firms solve their problem to hire labor and intermediate goods
3. Households solve their consumption-savings problem
4. Goods market clears: $Y_t = C_t + R_t + K_t$
5. Asset market clears, pinning down r_t via the household's Euler equation
6. Labor market clears
7. μ_{mt}, Q_{mt} are consistent with firms' optimal innovation decisions

I focus on a balanced growth path where the measure of sectors with each technology gap μ_{mt} is constant and the growth rate of output is constant. I describe the solution algorithm in the next section.

4 Results

In this section I present preliminary results from a numerical exercise with the model. I briefly describe the solution algorithm for finding the steady state and the choice of the parameters for the numerical exercise before presenting the results. The exercise compares properties of the economy in two different steady states of the model with different values of the probability of radical innovations parameter ϕ. This is meant to capture the fact that radical innovation probabilities change over time depending on general purpose technologies like the internet and other ICT. Changing ϕ is an appropriate representation of the changing impact of a GPT since it affects research productivity in all sectors of the economy. The 1990s are often thought of as a time of "disruptive" innovations where new innovators made large quality improvements relative to existing products.

The two values for ϕ are chosen to match average TFP growth rates in 1994-2003 and from 2004-2017. I show that the leader's market share, productivity gaps, markups, and the profit share are all higher when radical innovations are less likely. The growth rate, R&D as a share of GDP, and the real interest rate are lower.

I then decompose the difference in growth rates in the two steady states into the contributions from the exogenous effect of reducing the average size of quality improvements and the endogenous effect of reduced effort by firms and the resulting change in the distribution of sectors over technology gaps. I find that the endogenous forces account for between a quarter to a half of the total reduction in the growth rate between the two steady states depending on the order of the decomposition.

4.1 Solution Algorithm

The solution algorithm involves first guessing a steady state interest rate. Given this interest rate, solve the value functions for each technology gap by policy function iteration using the fact that $\dot{v}_{mt} = 0$ on a balanced growth path. This process yields the optimal innovation decisions of firms at each technology gap. Given the policy functions the stationary distribution of firms over technology gaps can be obtained by solving the system of equations described in section 3.5. To obtain the growth rate of GDP g, simulate a panel of firms (3000 for the results presented here) beginning with initial quality levels of 1 for all firms until the growth rate of the economy stabilizes to a constant value. Finally, check whether this growth rate is consistent with the interest rate guess using the household's Euler equation: $r = g\psi + \rho$. Update the guess of the interest rate and repeat until the interest rate guess and the interest rate implied by the resulting growth rate and the Euler equation are consistent.

4.2 Calibration

For now I take the parameters of the model from the literature. Eventually I plan to estimate the innovation parameters using a method of simulated moments approach with data on U.S. firms in the two periods of study, using moments like R&D as a share of sales, average TFP differences among firms in the same sector, patenting rates, leader's market share, and the persistence of market leadership at the firm level, plus aggregate growth rates. Table 1 shows the parameterization I use for the current results.

Parameter	Value	Meaning/source
ψ	1	Intertemporal elasticity of substitution
ρ	0.05	Rate of time preference (annual)
β	0.6	Labor share/Nechio & Hobijn (2017)
ϵ	4	Elasticity of substitution within sectors
η	0.4	Marginal cost of intermediate producers
λ	5%	Min. qual. improvement
γ	2	Curvature of R&D fn., Akcigit et al. (2018)
α	1.3	R&D technology
$\bar{m}$	16	Maximum number of steps ahead

Table 1: Parameters used in the calibration

4.3 Numerical Results

I solve for the steady state twice under two different values of the radical innovation parameter ϕ.[20] As a baseline, I use $\phi = 1.25$, which results in a steady state (TFP) growth rate of 1.6% annually, approximately matching average annual TFP growth from 1994-2003. I call this the "quick catchup" regime because under this value of ϕ, firms make larger quality improvements on average conditional on innovating and thus innovating is more likely to result in the laggard catching up to or overtaking the leader. For the "slower catchup" regime I use $\phi = 1.75$ which results in a steady state annual growth rate of 0.8% to match average TFP growth from 2004-2017.

Table 2 compares features of the equilibrium in the two steady states. A rise in the average productivity/technology gap (plotted in figure 17) drives a significant rise in leader market share and markups[21] in the slower catchup steady state. For market concentration, the 4.3 percentage point rise in leader market share is about 54% of the total rise in the data from 1998 to 2017. The magnitude of changes in leader market share and markups are sensitive to the choice of substitutability ϵ between goods in the same sector, and grow as ϵ grows. The profit share of GDP rises about 1.3 percentage points.[22]

[20] I plot the step size distribution $F(n)$ for quality improvements under the two values of ϕ in Appendix E.

[21] The average markup for U.S. public firms estimated in de Loecker & Eeckhout (2017) went from around 1.4 in the late 1990s to 1.67 in 2014, so the model explains only a small share of the total change.

[22] Barkai & Benzell (2018) document a rise in the profit share in the U.S. from around 4% in 2000 to

	$\phi = 1.25$	$\phi = 1.75$
Annual growth rate, %	1.64	0.79
Interest rate, %	6.64	5.79
Avg. leader market share, %	43.03	47.34
Profit share of GDP	5.50	6.88
Average markup	1.24	1.27
R&D/GDP, %	1.51	0.93
Decomposition, role of effort, %	-	46.22
Decomposition, role of ϕ, %	-	71.60

Table 2: Steady state comparison. Parameterization given in table 1.

The lower probability of radical innovation induces lower total innovation effort by firms: total R&D expenditure as a share of GDP falls from 1.5% to 0.9%. Finally, consistent with the lower growth rate, r, the risk-free rate of return/rate of return on the portfolio of firms, falls from 6.6% annually to 5.8%.

In figure 17 I plot the firm innovation policy functions under the two regimes as a function of the technology gap to the firm's competitor. The main difference between the two steady states is the innovation effort of the follower, which is slightly lower in the slow catchup regime. This can also be seen as corresponding to slower technology adoption and diffusion of laggard firms observed in the data. Because the chance of being overtaken is now lower for market leaders and the equilibrium interest rate is lower, they discount the future less and innovate more. This also applies to firms at the neck-and-neck state.

These changes in effort result in the spreading out of the stationary distribution of firms over technology gaps shown in the second panel of the figure. Many fewer sectors are near the neck-and-neck state at any given time in the slower catchup regime. This change is entirely due to the firms' endogenous responses, since lowering ϕ but holding the firm policy functions fixed from the baseline equilibrium results in a *tighter* distribution around the neck-and-neck state (plotted in figure 23 in Appendix F). This endogenous change in the distribution, a composition effect, is fully responsible for the increase in concentration and markups.

The growth rate falls in the slow catchup regime for several reasons. First, the level

around 15% in 2014 (see Figure 1a in the paper).

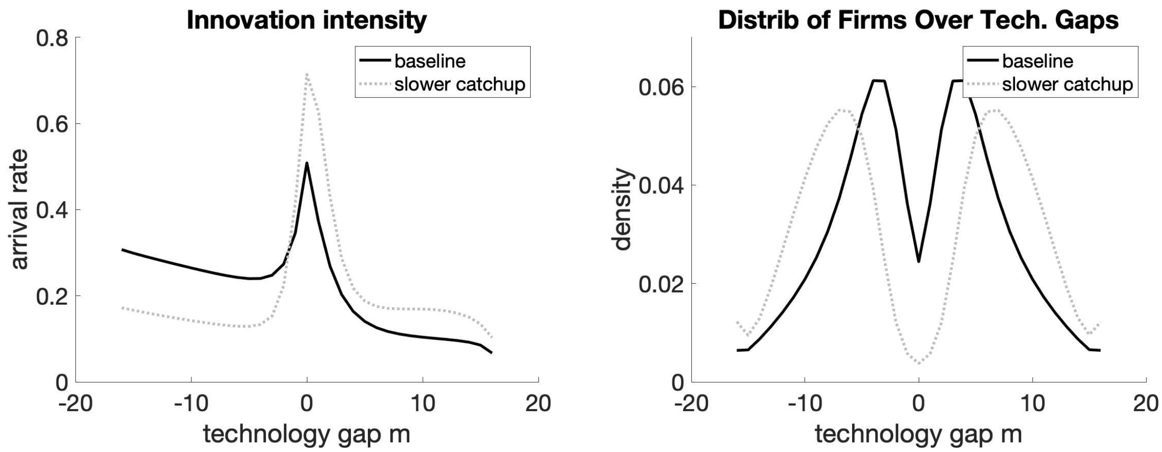


Figure 17: Firm policy functions depending on technology gap (a) and stationary distribution of firms over technology gaps (b), two catchup regimes.

and location of effort changes endogenously, slowing the growth of the quality indexes described in equation 4, even if ϕ is held fixed. The "role of effort" row in table 2 gives the share of the change in the growth rate that results from using the policy functions from the slow catchup regime in simulation with $\phi = 1.25$. According to this experiment, nearly half of the change in the growth rate from the baseline to the slow catchup regime is due to the endogenous response of firms' innovation effort, including the composition effect that more sectors have wider technology gaps and thus innovate less in aggregate than the sectors closer to the neck-and-neck state.

However, an alternative decomposition considers the first order effect of raising ϕ, which means firms make smaller quality improvements conditional on innovating, keeping the innovation policies of the firms fixed at their baseline equilibrium values. This alone can account for around 70% of the decline in the growth rate ("role of ϕ" row in table 2).

4.4 Discussion of Results

In addition to the evidence on changing patent quality, one reason to believe this exercise comparing two steady states with different probabilities of radical innovations is the correct one to fit the situation in the U.S. since the 1990s is simply that the results are consistent with the data. Targeting just the growth rate delivers the correct direction of the non-targeted moments including leader market share and productivity gaps. Under the slow catchup regime, the stationary distribution of firms has fatter tails and less mass around the neck-and-neck state. This generates the model's analogy to figures 7 and 8, where more sectors have larger technology gaps and higher leader market share. Recall also that figure 2 shows that productivity divergence is particularly pronounced in ICT intensive sectors, pointing to ICT as a potential cause. Bessen (2017) also shows that ICT intensity is correlated with rising market concentration.

5 Role of Elasticity of Substitution

The elasticity of substitution within sectors ϵ turns out to play an important role in the determination of the growth rate and to a lesser extent productivity gaps. In this section I explore both directions of change in ϵ compared to the baseline exercise in section 4, as each can capture (in a very reduced form) different structural changes in the U.S. economy that have been suggested in the literature recently. Neither change matches the direction of all moments of interest that the baseline exercise does.

5.1 Increasing Market Power?

Recent research has focused on the potential costs of rising market power and markups (see de Loecker & Eeckhout (2017), Eggertsson et al. (2018) and Edmond et al. (2018) for example) for growth and welfare. Can an increase in market power generate the same predictions for the macroeconomic changes experienced in the U.S. in recent years as a change in the probability of radical innovations? I model an increase in market power as a decrease in the substitutability of products in the same sector, ϵ.

I keep the calibration the same as in table 1 and set $\phi = 1.25$ (1990s case). I decrease ϵ from 4 in the baseline to 3. Average markups rise by 7 percentage points from 1.24 to 1.33, about a quarter of the total rise estimated by de Loecker & Eeckhout (2017). With the exception of markups and the profit share, the results are the opposite of what has happened in the data. Because of greater market power, markups and profits are higher when the firm has market leadership and this induces higher innovation effort for laggard firms as they try harder to overtake the market leader (R&D/GDP rises from 1.5% to 2.1%, see table 3 and figure 18). This results in a higher growth rate. There is also greater turnover in market leadership and average productivity differences go down (figure 18).

	$\epsilon = 4$	$\epsilon = 3$
Annual growth rate, %	1.64	1.96
Interest rate, %	6.64	6.96
Avg. leader market share, %	43.03	40.20
Profit share of GDP, %	5.50	6.12
Average markup	1.24	1.33
R&D share of GDP, %	1.51	2.10

Table 3: Steady state comparison, market power. Parameterization given in table 1.

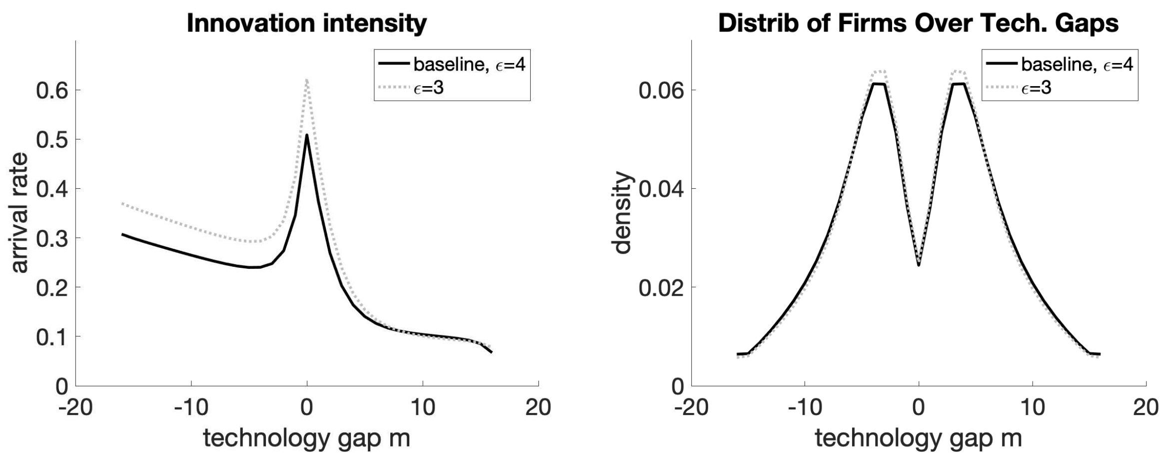


Figure 18: Firm policy functions depending on technology gap (a) and stationary distribution of firms over technology gaps (b), two market power regimes.

5.2 Superstar Firms?

Seminal work on the macroeconomic effects of superstar firms is Autor et al. (2017b). The main focus of their work is to point out that reallocation to more productive, already large firms can cause a decline in the labor share, as these firms produce more with fewer workers than their competitors. In their static industry model, firms produce products that are imperfect substitutes within an industry. Firms draw labor productivities from a distribution and then make an entry decision. Upon entering, firms set prices a la Bertrand.

The force for reallocation to more productive firms in their model is an increase in product substitutability within sectors. The authors argue that this increase could represent more fierce import competition from abroad, particularly China, in recent years or increased price sensitivity due to better search technology such as online shopping. Keeping the exogenous productivity distribution fixed, an ancillary result is that a sector's measured TFP will rise unambiguously when substitutability increases because of two forces: first, the minimum productivity threshold for entrants rises, and second, more productive firms grow their sales share.

I first show that this static result is usually true in my two-firm model without entry. Because there is no entry, sector TFP depends on the covariance between relative quality and leader's market share. Figure 19 plots market shares as a function of the technology gap for different values of ϵ for the baseline parameterization of the model given in table 1. For most values of the technology gap, increasing the substitutability of products statically increases the leader's market share which increases sector TFP using market shares.[23]

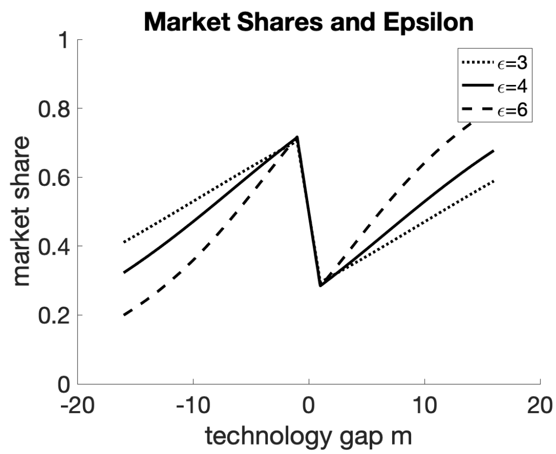


Figure 19: Market shares in leader-follower model with different values of elasticity of substitution within sectors.

On impact, therefore, increasing the substitutability of the firms' products will raise measured TFP as in Autor et al. (2017b). However, dynamically, this change reduces the markup that the leader charges and thus reduces both firms' expected future profits. The effect of this change on TFP *growth* is unclear ex-ante.

To answer this question I compare the steady state of the model with higher ϵ to the baseline. Under this parameterization, raising ϵ lowers the growth rate while

[23]With Bertrand pricing this is always true. But with the assumption that the follower sets price equal to marginal cost, if relative quality differences are small, increasing ϵ can cause a drop in the leader's market share.

increasing concentration dramatically. The rise in concentration comes from two forces. First, the static reallocation force operates: even if technology gaps were unchanged from one steady state to another, these same gaps would generate a higher average leader market share according to figure 19. Second, changes in effort shown in figure 20 cause the average technology gap to grow. With higher ϵ the value of overtaking the leader falls since leader markups are lower so followers and firms in the neck-and-neck position innovate less (figure 20). On the other hand, markups and profits become more elastic in the technology gap when ϵ is higher, while the likelihood of being overtaken falls along with the interest rate so leading firms discount the future less and leaders innovate more than before. In combination these changes in effort cause technology gaps to get larger on average.

	$\epsilon = 4$	$\epsilon = 6$
Annual growth rate, %	1.64	1.39
Interest rate, %	6.64	6.39
Avg. leader market share, %	43.03	50.26
Profit share of GDP, %	5.50	5.35
Average markup	1.24	1.18
R&D share of GDP, %	1.51	1.07

Table 4: Steady state comparison, superstar firms. Parameterization given in table 1.

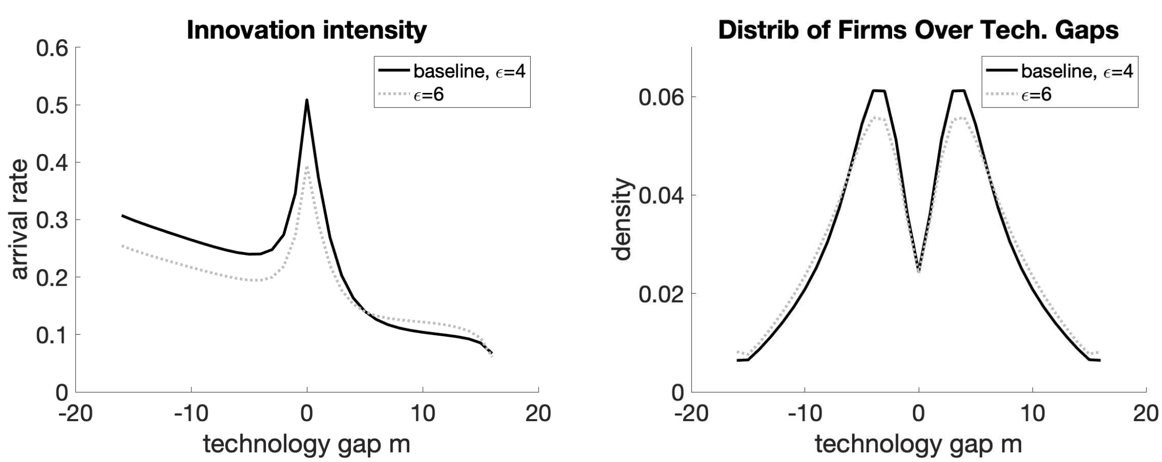


Figure 20: Firm policy functions depending on technology gap (a) and stationary distribution of firms over technology gaps (b), superstar firms.

This experiment demonstrates how the rise of superstar firms can coincide with slowing productivity growth through an increase in product substitutability. The dynamic dimension with endogenous productivity overturns the Autor et al. (2017b) result that superstar firms necessarily raise measured TFP. Transition dynamics following a permanent shock to ϵ would likely show first an increase in TFP due to greater allocative efficiency then a decline as the within-firm contribution to growth slows, similar to the pattern in figure 3. However, non-targeted moments go in the wrong direction: the profit share and average markups fall, unlike in the data.

6 Conclusion

I have presented a general equilibrium model of innovation and growth where multiple firms are active in a sector in each period and goods within sectors are imperfect substitutes. Through the lens of the model I unify the Schumpeterian endogenous growth literature with the growing literature on rising concentration, markups and market power in the U.S. Future work will involve the estimation of the model for U.S. firm-level data and computation of transition dynamics of the economy transitioning from high to low radical innovation steady states.

The model is able to match changes in the distribution of sectors over technology gaps and leader market shares from the 1990s to the 2010s with a change in the probability of radical innovations, supported by evidence on changes in the patent quality distribution over time associated with the arrival of general purpose technologies. A change in the elasticity of substitution cannot generate the same fit for data. The model jointly explains rising concentration, increasing productivity differences between firms in the same sector, and the productivity slowdown, which appear to be correlated at the sector level and more pronounced in ICT-intensive sectors. It can account for what might be called the "superstar productivity puzzle": despite the rapid growth of a few highly productive and already large firms over the 2000s, *aggregate* productivity growth has slowed down rather than sped up. The model predicts that this is because of the dynamic effects of a dominant leader on the innovation decisions of laggard competitors.

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A TFP Estimation

I use Compustat data on U.S. public firms from 1975-2015 to estimate total factor productivity (TFP) at the firm level. I focus on the non-financial sector and exclude utilities and firms without an industry classification. I keep only those companies that are incorporated in the U.S. The sample includes around 3,000 firms per year, though this number varies over time.

I construct each firm's capital stock $K_{i,t}$ by initializing the capital stock as PPEGT (total gross property, plant, and equipment) for the first year the firm appears. I then construct $K_{i,t+1}$ recursively:

$$K_{i,t+1} = K_{i,t} + I_{i,t+1} - \delta K_{i,t}$$

where PPENT (total net property, plant, and equipment) is used to capture the last two terms (net investment). I deflate the nominal capital stock using the Bureau of Economic Analysis (BEA) deflator for non-residential fixed investment.

In de Loecker & Warzynski (2012) the authors show that under a variety of pricing models the firm's markup can be computed as a function of the output elasticity θ_{it}^V of the variable input and the variable input's cost share of revenue[24] :

$$\mu_{it} = \theta_{it}^V \frac{P_{it} Q_{it}}{P_{it}^V V_{it}}$$

Following de Loecker & Eeckhout (2017) I use COGS (cost of goods sold) deflated by the BEA's GDP deflator series as the real variable input cost $M_{i,t}$ of the firm. While the number of employees is well measured in Compustat and would be sufficient to estimate productivity, the wage bill is usually not available and would be needed to compute the labor cost share needed to compute the markup simultaneously with productivity. I don't use this markup information in the current draft but plan to use it to differentiate between TFPR (revenue-based TFP) and TFPQ (markup-corrected TFP).

For the results presented in this paper, I assume a Cobb-Douglas production function[25] for firm i in 2-digit SIC sector s in year t so that factor shares may vary across

[24]This approach requires several assumptions. First, the production technology must be continuous and twice differentiable in its arguments. Second, firms must minimize costs. Third, prices are set period by period. Fourth, the variable input has no adjustment costs. No particular form of competition among firms must be assumed.

[25]Below I show the correlation of these firm-level TFP estimates with estimates from a model assuming a translog production function.

sectors:

$$Y_{i,s,t} = A_{i,s,t} M_{i,s,t}^{\beta_{M,s}} K_{i,s,t}^{\beta_{K,s}}$$

I use the variable SALE to measure firm output $Y_{i,s,t}$. I deflate SALE using the GDP deflator series to obtain real output at the firm level. I use time dummies η_t to detrend the TFP estimates and obtain TFP in logs (lower case variables denote variables in logs) by computing the residual of the following regressions for each 2-digit sector:

$$y_{i,t} = \alpha + \eta_t + \delta_i + \beta_{M,s} m_{i,t} + \beta_{K,s} k_{i,t} + \varepsilon_{i,t}$$

There seems to be a high correlation among various estimates obtained using different assumptions on the production function. Table 5 displays these correlations. The translog specification simply includes second order terms for each of the inputs.[26]

Table 5: Pairwise Correlations Among TFP Estimates

	C-D, sect.	C-D, agg.	Translog, sect.	Translog, agg.
Cobb-Douglas, sectors	1			
Cobb-Douglas, agg.	0.904	1		
Translog, sectors	0.805	0.719	1	
Translog, agg.	0.899	0.979	0.736	1

[26]The translog version allowing different elasticities across 2-digit sectors is

$$y_{i,t} = \alpha + \eta_t + \delta_i + \beta_{M1,s} m_{i,t} + +\beta_{M2,s} m_{i,t}^2 + \beta_{K1,s} k_{i,t} + \beta_{K2,s} k_{i,t}^2 + \varepsilon_{i,t}$$

as before, TFP is simply the residual of this equation.

B Alternate Model with Bertrand Pricing

It's also possible to solve the full dynamic model under the assumption that both firms set prices a la Bertrand, rather than requiring that the follower set price equal to marginal cost. The analogy from the model to the data becomes less obvious under this assumption, since the laggard firm can no longer be thought of representing a competitive fringe composed of many firms producing generic products that are perfectly substitutable with other generic products but imperfectly substitutable with the brand produced by the leader. This assumption also gives empirically counterfactual predictions that the profit shares of total industry profits of the market leader and the other firm in the industry are relatively similar, contradicting the pattern shown in figure 14.

Nonetheless, many of the main results carry through under this alternate assumption. Before describing these alternate results, I return to the pricing problem of the firms assuming the follower can now choose its optimal markup. Using the same derivation as in section 3.3.1 it can be shown that both firms follow the pricing policy the leader follows in the baseline model:

$$p_i = \frac{\epsilon - (\epsilon - \frac{1}{\beta})s_i}{\epsilon - (\epsilon - \frac{1}{\beta})s_i - 1}\eta$$

where

$$s_i = q_i^{\epsilon-1}\left(\frac{p_i}{P_j}\right)^{1-\epsilon}$$

I look for a Markov perfect equilibrium with balanced growth where each firm within a sector's price is the best response to its competitor's price at time t. The algorithm for finding the steady state remains the same, plugging in the pricing functions of the firms, plotted in figure 21.

Table 6 gives the results of the same experiment as in section 4.3 under the alternate pricing strategies with the same parameters as in table 1 and figure 22 shows the policy functions and stationary distributions. Note that innovation intensity is greatest for the most laggard firm under this pricing assumption. As before, changing ϕ has a level effect on total innovation effort but also changes the location of R&D from laggard firms to leading firms.

The level of the growth rates and the change in the growth rate from one steady state to the other under Bertrand pricing are very similar to the baseline model with marginal cost pricing of the follower. The change in concentration is much smaller, as

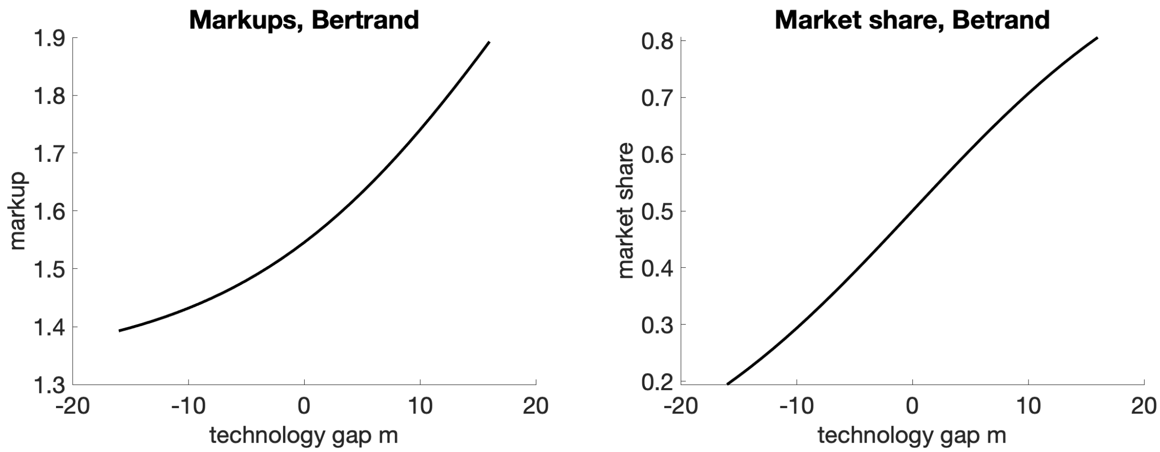


Figure 21: Markups and resulting market shares as a function of the technology gap (ratio of firm qualities), Bertrand pricing.

the change in technology gaps is not as dramatic as in the main case (figure 22), though technology gaps do increase modestly. Markups are basically unchanged. This is partly due to the fact that as the leader's markup rises when the gap gets larger, the follower's markup now falls whereas before it would have been unchanged (at zero). The profit share rises, though more modestly than before. As for the growth decomposition, the effects of the firms' innovation responses is smaller, and the first order effect of lowering the probability of radical innovations dominates.

	$\phi = 1.25$	$\phi = 1.75$
Annual growth rate, %	1.73	0.95
Interest rate, %	6.73	5.95
Avg. leader market share, %	63.66	64.30
Profit share of GDP	14.71	14.84
Average markup	1.56	1.56
R&D/GDP, %	1.81	1.40
Decomposition, role of effort, %	-	25.66
Decomposition, role of ϕ, %	-	92.05

Table 6: Steady state comparison, Bertrand. Parameterization given in table 1.

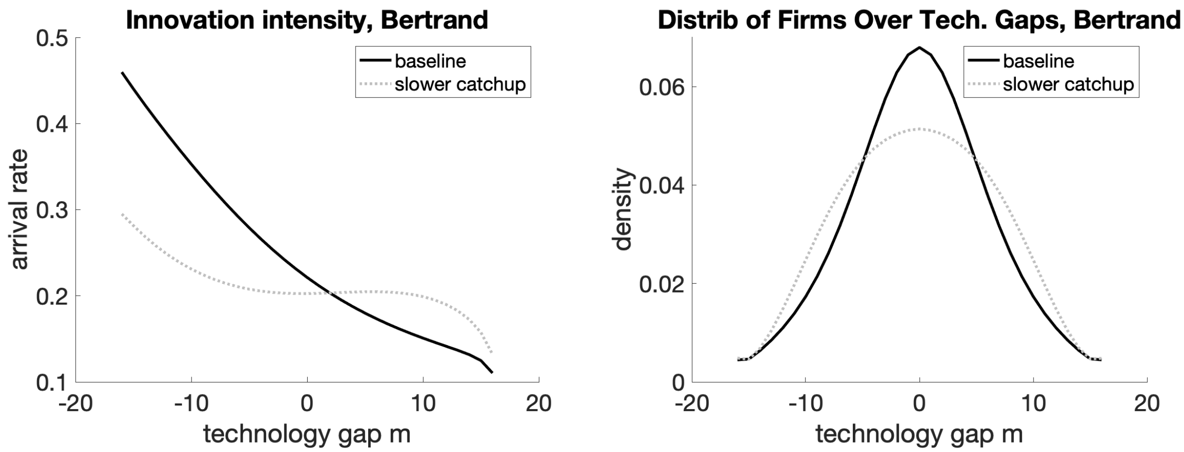


Figure 22: Firm policy functions depending on technology gap (a) and stationary distribution of firms over technology gaps (b), Bertrand pricing.

C Value Function Boundary Equations

For the firm that's furthest behind (at gap $-\bar{m}$ with quality q_t):

$$\begin{aligned} r_t V_{-\bar{m},t}(q_t) - \dot{V}_{-\bar{m},t}(q_t) = \max_{x_{-\bar{m},t}} \{ & 0 - \alpha \frac{(x_{-\bar{m},t})^\gamma}{\gamma} q_t^{\frac{1}{\beta}-1} \\ & + x_{-\bar{m},t} \sum_{n_t=-\bar{m}+1}^{\bar{m}} \mathbb{F}_m(n_t)[V_{nt}(\lambda^{n_t+\bar{m}} q_t) - V_{-\bar{m},t}(q_t)] \\ & + x_{\bar{m},t}(V_{-\bar{m},t}(\lambda q_t) - V_{-\bar{m},t}(q_t)) \} \end{aligned}$$

The difference between this and equation 2 is in the last line, where if the firm's competitor innovates, there is a spillover that causes the firm at gap $-\bar{m}$ to improve its quality by λ.

For a firm at gap $\bar{m}$ the value function is:

$$\begin{aligned} r_t V_{\bar{m},t}(q_t) - \dot{V}_{\bar{m},t}(q_t) = \max_{x_{\bar{m},t}} \{ & \pi(\bar{m}, q_t) - \alpha \frac{(x_{\bar{m},t})^\gamma}{\gamma} q_t^{\frac{1}{\beta}-1} \\ & + x_{\bar{m},t}(V_{\bar{m},t}(\lambda q_t) - V_{\bar{m},t}(q_t)) \\ & + x_{-\bar{m},t} \sum_{n_t=-\bar{m}+1}^{\bar{m}} \mathbb{F}_{-\bar{m}}(n_t)[V_{nt}(q_t) - V_{\bar{m},t}(q_t)] \} \end{aligned}$$

Where:

$$\pi(m, q_t) = \begin{cases} 0 & \text{if } m \leq 0 \\ q_i^{\frac{1}{\beta}-1}(p(m)-\eta)p(m)^{-\epsilon}(p(m)^{1-\epsilon}+(\lambda^{-m})^{\epsilon-1}\eta^{1-\epsilon})^{\frac{\epsilon-\frac{1}{\beta}}{1-\epsilon}} & \text{for } m \in \{1, \ldots, \bar{m}\} \end{cases}$$

D Growth of Quality Indexes

Recall from equation 3 that output can be written:

$$Y_t = \frac{1}{2}\frac{1}{1-\beta}\sum_{m=-\bar{m}}^{\bar{m}} Q_{mt}$$

What are these quality indexes? From the derivation of aggregate output:

$$\begin{aligned} Q_{m,t} &= \int_0^1 \left(q_{it}^{\epsilon-1} p_(m)^{1-\epsilon} + q_{-it}^{\epsilon-1} p(-m)^{1-\epsilon}\right)^{-\frac{(1-\beta)}{\beta(1-\epsilon)}} \mathbb{1}_{\{i\in\mu_{mt}\}} di \\ &= \left(p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} p(-m)^{1-\epsilon}\right)^{\frac{1-\beta}{\beta(\epsilon-1)}} \int_0^1 q_{i,j}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i\in\mu_{mt}\}} di \end{aligned}$$

Use the fact that:

$$Y_t = \frac{1}{2}\frac{1}{1-\beta}\sum_{m=-\bar{m}}^{\bar{m}} Q_{mt}$$

and

$$Y_{t+dt} = \frac{1}{2}\frac{1}{1-\beta}\sum_{m=-\bar{m}}^{\bar{m}} Q_{mt+dt}$$

so

$$\dot{Y}_t = \frac{1}{2}\frac{1}{1-\beta}\sum_{m=-\bar{m}}^{\bar{m}} \dot{Q}_{mt}$$

Dividing by Y_t:

$$\frac{\dot{Y}_t}{Y_t} = g_Y = \frac{\frac{1}{2}\frac{1}{1-\beta}\sum_{m=-\bar{m}}^{\bar{m}} \dot{Q}_{mt}}{Y_t}$$

Do a guess and check that $\frac{\dot{Q}_{mt}}{Y_t} = g_m$ for all m, t. In that case g, the growth rate of Y, is constant, as are the ratios: $\frac{\dot{Q}_m}{\dot{Q}_n}$ for all m, n. It's useful to define:

$$\tilde{Q}_{mt} = \int_0^1 q_{m,t,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i\in\mu_{mt}\}} di$$

So that:

$$Q_{mt} = \left(p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} p(-m)^{1-\epsilon}\right)^{\frac{1-\beta}{\beta(\epsilon-1)}} \tilde{Q}_{mt}$$

And $\dot{Q}_{mt} = \left(p(m)^{1-\epsilon} + (\lambda^{-m})^{\epsilon-1} p(-m)^{1-\epsilon}\right)^{\frac{1-\beta}{\beta(\epsilon-1)}} \dot{\tilde{Q}}_{mt}$

Assuming fixed distribution:

$$\dot{\tilde{Q}}_{mt} = \int_0^1 q_{m,t+dt,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i\in\mu_m\}} di - \int_0^1 q_{m,t,i}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{i\in\mu_m\}} di$$

Consider an arbitrary m ($-\bar{m}$ and $\bar{m}$ are special cases because of spillovers). For a share $1 - x_m - x_{-m}$ of them neither they nor their competitor innovate and thus the difference for that portion of firms from t to $t+dt$ is 0. However, a portion of firms at m at t innovate to a different gap, and another portion leave gap m because their competitor innovates. These are a random sample of the firms at gap m at time t. So the outflows from $\dot{\tilde{Q}}_m$ are:

$$-(x_m + x_{-m}) \int_0^1 q_{m,t,j}^{\frac{1-\beta}{\beta}} \mathbb{1}_{\{j\in\mu_m\}} dj = -(x_m + x_{-m})\tilde{Q}_m$$

The inflows come from two sources. First, some firms innovate into position m from a lower position n, improving their quality by $m-n$. The probability they innovate and reach gap m is given by $x_n F_n(m)$. Some firms fall back to m from a higher gap n because their competitor innovates to $-m$. The probability their competitor reaches $-m$ is given by $x_{-n}F_{-n}(-m)$. So cumulative inflows are:

$$\sum_{n=-\bar{m}}^{m-1} x_n F_n(m)(\lambda^{(m-n)})^{\frac{1-\beta}{\beta}}\tilde{Q}_n + \sum_{n=m+1}^{\bar{m}} x_{-n}F_{-n}(-m)\tilde{Q}_n$$

So, putting it together:

$$\dot{\tilde{Q}}_{mt} = \sum_{n=-\bar{m}}^{m-1} x_n F_n(m)(\lambda^{(m-n)})^{\frac{1-\beta}{\beta}}\tilde{Q}_n + \sum_{n=m+1}^{\bar{m}} x_{-n}F_{-n}(-m)\tilde{Q}_n - (x_m + x_{-m})\tilde{Q}_m$$

For lowest gap there are spillovers when competitor innovates:

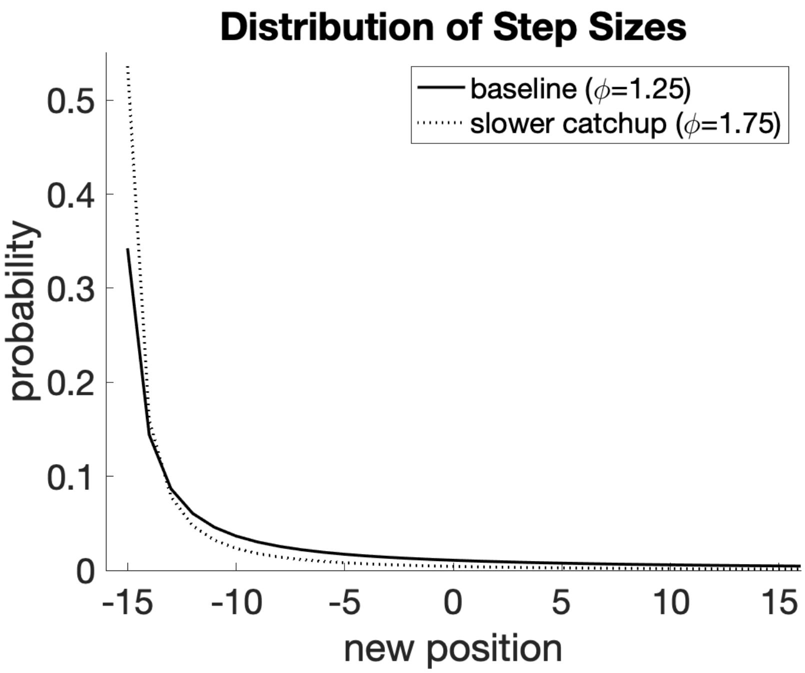
$$\dot{\tilde{Q}}_{-\bar{m}t} = \sum_{n=-\bar{m}+1}^{\bar{m}} x_{-n}F_{-n}(\bar{m})\tilde{Q}_n + x_{\bar{m}}(\lambda^{\frac{1-\beta}{\beta}} - 1)\tilde{Q}_{-\bar{m}} - x_{-\bar{m}}\tilde{Q}_{-\bar{m}}$$

For highest gap you do not exit that gap if you innovate:

$$\dot{\tilde{Q}}_{\bar{m}t} = \sum_{n=-\bar{m}}^{\bar{m}-1} x_n F_n(\bar{m})(\lambda^{(m-n)})^{\frac{1-\beta}{\beta}}\tilde{Q}_n + x_{\bar{m}}(\lambda^{\frac{1-\beta}{\beta}} - 1)\tilde{Q}_{\bar{m}} - x_{-\bar{m}}\tilde{Q}_{\bar{m}}$$

E Step Size Distributions for Results

Here I include a plot of the fixed distributions $F(n)$ discussed in section 3.3.2 under the two values of the radical innovation parameter ϕ used for the steady state comparisons in section 4.3. Recall that for a particular step size (size of quality improvement) $F(n) = c_0(n + \bar{m})^{-\phi}$. For the most backward firm at position $-\bar{m}$, the probability of getting a larger than one step improvement is about 20 percentage points higher in the baseline case than in the higher ϕ case.



F Role of Effort

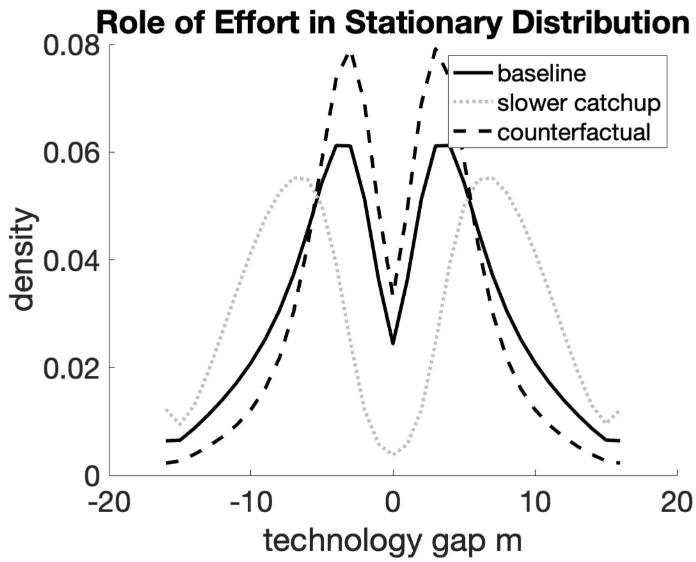


Figure 23: Stationary distribution of firms over technology gaps in two equilibria, plus distribution assuming firm innovation effort is fixed at the baseline equilibrium while ϕ changes to its value in the slow catchup equilibrium.