Comprehensive macro-model for the U.S. economy

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Introduction

This paper introduces our macroeconomic concepts. It also summarizes general empirical findings related to the evolution of principal macroeconomic variables in the U.S. Thorough analysis and modeling of real GDP per capita, inflation, labor force participation rate, productivity and unemployment has revealed a number of (linear and nonlinear) relationships, often with time lags. The sequence of interaction between the aforementioned macroeconomic variables in the U.S. is as follows: the evolution of the number of 9-year-olds completely defines the fluctuations in the growth rate of real GDP per capita relative to its potential rate. The latter term is represented by a reciprocal function of the attained level of real GDP per capita itself. Real economic growth drives labor force participation rate with a two-year lag. Apparently, working age population is an exogenous variable and can be obtained by independent measurements. Therefore, the level of labor force is completely defined in the model. The change in the level of labor force represents the driving force of price inflation (as represented by GDP deflator or CPI) and unemployment rate with two- and five-year lags, respectively. Labor productivity is unambiguously derived from real GDP and the number of employed, i.e. the difference between the labor force and the unemployment rate times the labor force.

Hence, one can extrapolate the change in an estimated birth rate in a given year and predict unemployment rate at a 16-year horizon; inflation at a 13-year horizon; labor force participation at a 11-year horizon, and real GDP per capita at a 9-year horizon. Big changes in demographic structure, i.e. highly varying levels of migration and an elevated death rate, can introduce substantial bias in such predictions. Such processes have been not observed in the U.S. since the late 1950s, however.

The relationships compiling our macro-model of the U.S. economy have passed rigorous statistical testing, including tests for cointegration, in order to avoid spurious regressions. These tests demonstrated the presence of cointegrating relations, high level of statistical significance and goodness-of-fit. Moreover, similar cointegrating relations were obtained for the biggest developed countries. The predictive power is illustrated by a comparison of measured and predicted variables.

In this paper, we also validate previously obtained relationships using new data. The data were obtained from various sources: population estimates from the U.S. Census Bureau (2008); estimates of real GDP and GDP deflator - from the Bureau of Economic Analysis (2008); labor force level and participation rate, unemployment, and productivity - from the Bureau of Labor Statistics (2008). In some cases, we used data presented by the Conference Board (2008).

1. Real GDP

Real GDP is not a directly measured economic variable. It is a results of the correction of nominal (current dollar) GDP for GDP price deflator. This procedure leads to a somewhat elevated level of measurement errors, which can be seen in consequent revisions conduced by the Bureau of Economic Analysis. A conservative estimate of the accuracy of real GDP measurement is slightly below one percentage point. Such relatively low accuracy creates additional problems for modeling of corresponding growth rate - annual changes in real GDP are compatible to this accuracy.

The change rate of real GDP is defined by the evolution of two components: working age population, N, and real GDP per capita, G:

$$dGDP/GDP = d(GN)/GN = dN/N + dG/G, \tag{1}$$

where G is based on the working age population. The former term represents the extensive source of real economic growth: the working age population has been growing since the late 1950s at a rate of ~1 per cent per year in the U.S.

Our (empirically derived) model (Kitov, 2006a) stipulates that the growth rate of real GDP per capita is defined by the following relationship:

$$dG/G = A/G + 0.5dN_9/N_9, \tag{2}$$

where A=$398 (2002 U.S. dollars) is empirical constant, and N_9 is the number of 9-year-olds. The first term in (2) represents economic trend (potential), i.e. the growth rate that would be observed in the case of constant N_9. The second term introduces the fluctuations of the growth rate around its potential level. Asymptotically, the economic trend approaches the zero line. In 1975, the trend was ~2.4% per year, and it fell to 1.3% per year in 2005.

Equations (1) and (2) provide a complete description of the evolution of real GDP, when $N(t)$ and $N_9(t)$ are known. These demographic variables are exogenous ones and driven by many factors, likely including the history of real economic growth. In practice, both variables are enumerated during decennial population censuses and estimated between the censuses.

Reciprocally, one can use real GDP to recover the evolution of the number of 9-year-olds from the start of accurate population and GDP measurements. Such recovery method might potentially be of a higher accuracy than routine censuses. Reversing and integrating (2), one can obtain the following relationship for $N_9(t)$:

$$dN_9(t) = N_9(t) - N_9(t-1)$$
$$N_9(t) = N_9(t-1)[2*(dG/G - A/G) + 1] \qquad (3)$$

where $N_9(t-1)$ is the specific age population at time $t-1$; and by default, $\Delta t=1$. Relationship (3) can be interpreted in the following way - the deviation between the observed growth rate of real GDP per capita and that defined by the long-term trend is completely determined by the change rate of the number of 9-year-olds. A reversed statement is hardly to be correct - the number of people of some specific age can not be completely (or even in any significant part) defined by contemporary real economic growth. The causality principle prohibits any influence at the birth rate nine years ago.

In fact, relationship (3) provides a prediction for the number of 9-year-olds using only independent measurements of real GDP per capita. Therefore, amplitude and statistical properties of the deviation between measured and predicted number of 9-year-olds can be used for the validation of relationship (2). Figure 1 displays the measured and observed N_9 in the U.S. between 1960 and 2003. Both Engle-Granger and Johansen tests for cointegration (Kitov, Kitov, Dolinskaya, 2007a) confirmed the presence of a long-term equilibrium relation between the measured and predicted (i.e. derived from GDP) populations in Figure 1. The goodness-of-fit is (R^2=) 0.8 and the residual deviation between the curves in Figure 1 can be likely explained by errors in measurements. Effectively, the predicted curve lies practically inside the uncertainty bounds of the measured one, which are about ±300,000, i.e. the predicted curve might be the measured one with a high probability.

Hence, there is a one-to-one link between the number of 9-year-olds and real GDP per capita. This fact implies that real economic growth, as expressed in monetary units, is driven only by the evolution of age structure. (Same statement is valid for other developed countries.) An increasing number of 9-year-olds guarantees an elevated growth rate above that defined by constant annual increment of real GDP per capita.

The fluctuations of actual annual increment of real GDP per capita around a constant level represent a random process. This stochastic component is driven only by one force and can be actually predicted to the extent one can predict the number of 9-year-olds at various time horizons. The population estimates for younger ages in previous years provide an excellent source for such prediction. For example, the number of 6-year-olds today is a very good approximation of the number of 9-year-olds in three years, as Figure 2 demonstrates. The growth rate of a single year population can be predicted even with a higher accuracy because the levels of adjacent cohorts change practically in sync.

Our empirical analysis (Kitov, 2006a) also showed that the growth rate of real GDP in the U.S. can be split into another two components. First component is defined by the reciprocal value of the duration of the period of mean income growth with work experience, T_{cr}, (Kitov, 2005a). In 2005, T_{cr} in the U.S. was ~40 years, i.e. 55 years of age. The T_{cr} grows over time as the square root of real GDP per capita. Second component is again $0.5dN_9(t)/N_9(t)$. This finding, however, can be an artificial result of the functional dependence of T_{cr} on real GDP per capita and practically constant growth in working age population.

2. Labor force participation rate

The growth in real GDP drives the change in labor force supply through redistribution of personal incomes. Fluctuations in the number of 9-year-olds produce fluctuations in real GDP per capita relative to that defined by the potential economic growth and, thus, create variations in personal income relative to that associated with this "neutral" growth rate. The simplest assumption on the redistribution of an “excessive” (positive) amount of personal income consists in some increase in the fraction of population in labor force. At first glance, more people would be able to obtain paid jobs with extra money produced in a given economy.

Surprisingly, this assumption is wrong for the U.S. Correct intuition behind the mechanism of the reaction of labor force participation (LFP) to the redistribution is opposite - less people are forced to seek income through paid job because of the presence of some other channels (likely not included in the Current Population Survey's questionnaire) of personal income distribution (PID). A smaller part of working age population obtains larger personal income and somehow transfers it to the residual fraction of the population (not in labor force) to recover original PID (Kitov, 2007a). When the growth rate of real GDP per capita is below its potential value, the overall personal income grows at a rate below the neutral one and the lack of personal income earned by people in the labor force has to be compensated by some increase in the LFP. Figure 1 demonstrates that the N_9 was on a downward trend in the late 1960s and the 1970s. These years are characterized by the growth rate of real GDP per capita below its potential and, thus, by an increase in the measured LFP.

Quantitatively, the influence of the growth in real GDP on the LFP has to be affected by exponential distribution of personal inputs to real GDP – the number of people with given income (GDP portion) rolls-off exponentially as a function of income. If the effect of real growth is based on the excess of the total personal income above its potential level, then higher levels of the LFP are more sensitive to this real growth. It is reasonable to assume that the sensitivity of the LFP to the difference between actual and potential growth rates, $g(t)=dG/G–A/G$, increases exponentially with a growing LFP. Also, there might be a time delay between action and reaction and the LFP may lag behind the $g(t)$ (Kitov, Kitov, 2008a):

$$\{B_1 dLFP(t)/LFP(t) + C_1\}exp\{\ \alpha_1[LFP(t) - LFP(t_0)]/LFP(t_0) =$$
$$= \int\{dG(t-T))/G(t-T) - A/G(t-T)\}dt, \qquad (4)$$

where B_1 and C_1 are empirical constants, α_1 is an empirical exponent, t_0 is the start year (of modeling), T is the time lag, and $dt=t_2-t_1$, t_1 and t_2 are the start and the end time of integration of the $g(t)$ (one year in our model). The exponential term defines the change in the sensitivity due to the deviation of the LFP from its initial value $LFP(t_0)$. Effectively, the $LFP(t)$ is a nonlinear function of real economic growth.

A simple transformation of (4) using (3) provides another useful form of relationship (4), which relies on $N_9(t)$ instead of the integral of $g(t)$:

$$\{B_2 dLFP(t)/LFP(t) + C_2\} \exp\{\alpha_2[LFP(t) - LFP(t_0)]/LFP(t_0)\} = N_9(t-T) \quad (5)$$

where B_2 and C_2 are empirical constant different from B_1, C_1, and $\alpha_2=\alpha_1$.

Figure 3 depicts some results of the $N_9(t)$ prediction using original LFP time series from the BLS. Corresponding constants are as follows: t_0=1963; T=2 years, α_2=-1.85, B_2=-1.5E+8, C_2=4.94E+6. The predicted time series leads the observed one by two years, i.e. an accurate forecast at a two-year horizon is a natural feature of the model. Coefficient B_2 is negative and results in a declining rate of the LFP growth during the years of real growth above the potential one, for example, between 1983 and 2000. Exponential term in (5) provides a factor of 0.77 in 2000 (the largest LFP of 67.1%) relative to 1963, when the LFP was only 58.7%. This means that 1% change in the N_9 at the LFP level of 67.1% produces a larger change in the $dLFP/LFP$ by factor of 1/0.77=1.3 than 1% change at the level of 58.7%. Also displayed is the case without exponential weighting, α_2=0. This case demonstrates that the specific age population (N_9) is overestimated by the model.

Considering the uncertainty in the underlying time series – N_9 and *LFP,* the observed and predicted time series are in a good overall agreement: timing of main turns in both series is excellent and amplitudes of the largest changes are also practically coincide.

Historically, we first tried to model *dLFP/LFP* as a nonlinear function of G and tested a simple relationship similar to (3):

$$dLFP(t)/LFP(t) = D_1[dG(t-T)/G(t-T) - A_2/G(t-T)] + D_2 \quad (6)$$

where D_1 and D_2 are empirical constants, and A_2 is also an empirical constant different from A in (2). This model served as a workhorse for those countries, which do not provide accurate estimates of the specific age population. According to (4) one can rewrite (6) in the following (discrete) form:

$$Ns(t_2) = Ns(t_1)\{ 2[dG(t_2-T)/G(t_2-T) - A_2/G(t_2-T)] + 1\} \quad (7)$$

$$dLFP(t_2)/LFP(t_2) = Ns(t_2\text{-}T)/B + C \quad (8)$$

where $Ns(t)$ is the (formally defined) specific age population, as obtained using A_2 instead of A; B and C are empirical constants. Relationship (7) defines the evolution of some specific age population, which is different from actual one. The discrete form is useful for calculations.

Figure 4 depicts the observed and predicted relative change rate of the *LFP*. The latter is obtained from (7) and (8) with the following constants and coefficients: Ns(1959)=4.5E+6, A_3=\$350, B_3=-1.23E+8, C_3=0.04225. Notice that coefficient A_2 is smaller than A=\$398 in (2). Due to high volatility of the original *dLFP/LFP* time series we compare the predicted series to MA(5) of the observed one. The goodness-of-fit is high: R^2=0.73.

Labor force participation rate determones the level of labor force, *LF*, in an economy with a given population:

$$LF(t) = LFP(t)N(t) \quad (9)$$

By definition, the level of employment, $E(t)$, is the difference between labor force and the number of unemployed, $E(t) = LF(t) - UE(t)*LF(t)$. The link between unemployment, *UE*, and labor force is described in Section 4.

3. Labor productivity

Labor productivity, *P*, can be represented as a function of *LFP* and *G*, $P\text{\textasciitilde}G{\cdot}N/N{\cdot}LFP = G/LFP$. From (4), it follows that *P* is a function of *G* only. Therefore, the growth rate of labor productivity can be presented in the same way as labor force participation. Since the change in productivity is synchronized with *G* and labor force participation, the first useful relationship mimics (4):

$$dP(t)/P(t) = \{B_3 dLFP(t)/LFP(t) + C_3\}\cdot exp\{\ \alpha_3[LFP(t) - LFP(t_0)]/LFP(t_0)\} \quad (10)$$

Figure 5 depicts two curves reported by the BLS and those predicted with B_3=-5.0, C_3=0.040, and α_3=5.0; and B_3=-3.5, C_3=0.042, and α_3=3.8, respectively. Due to volatility in the original productivity and labor force (time derivative) series we replace them with their MA(5). A five-year time interval provides an increased resolution and allows smoothing measurement noise. As expected, coefficient B_3 is negative implying a decline in productivity with increasing labor supply. The goodness-of-fit for both observed time series is about (R^2=) 0.6. Moreover, principal features (troughs and peaks) of the observed series are similar in the predicted series, with slight time shifts, however.

Another relationship defines *dP/P* as a nonlinear function of *G*:

$$Ns(t_2) = Ns(t_1)\cdot\{ 2[dG(t_2-T)/G(t_2-T) - A_4/G(t_2-T)] + 1\} \quad (11)$$

$$dP(t_2)/P(t_2) = N(t_2-T)/B_4 + C_4 \quad (12)$$

where A_4, B_4, and C_4 are (country-specific) empirical constants.

Some results of productivity modeling by (11) and (12) are presented in Figure 6. (Model parameters are given in Figure captions.) Overall, 60% of variability in the observed curve is explained by the predicted one – same as explained by *G* itself. Timing of main turns in the curves is excellent. This is an expected effect, however, because productivity is essentially the same class variable as real GDP per capita. An important feature to predict is amplitude, as Figure 6 indicates – the productivity is not a scaled version of the real GDP per capita. So, the success of our model is related to a good prediction of the LFP.

As a validation of our model, we predicted the evolution of productivity for other developed countries using relevant GDP per capita data (Kitov, Kitov, 2008b). Figure 7 presents predicted and measured productivity in Canada. Overall, this is the best example we have obtained.

Productivity is a secondary (dependent) economic variable. The growth of real GDP per capita above or below its potential rate is transferred one-to-one in relevant changes in labor force participation and, thus, in employment and productivity. Since real economic growth depends only on the evolution of specific age population, one must control demographic processes in order to control productivity and stable economic growth.

One may also conclude that all attempts to place labor productivity in the center of conventional theories of real economic growth are practically worthless. Productivity is not an independent variable, which can be influenced and controlled by any means except demography.

4. Inflation and unemployment

According to our model (Kitov, 2006cd), inflation and unemployment are linear and lagged functions of labor force change as expressed by the following relationships:

$$\pi(t)=a_1dLF(t-t_1)/LF(t-T_1)+a_2 \quad (13)$$

$$UE(t)=b_1dLF(t-t_2)/LF(t-T_2)+b_2 \quad (14)$$

where $\pi(t)$ is the inflation rate at time t, $UE(t)$ is the unemployment rate at time t, $LF(t)$ is the level of labor force, T_1 and T_2 are the time lags between the inflation, unemployment and the labor force, respectively; a_1, b_1, a_2, and b_2 are country-dependent empirical coefficients. In Section 2, the level of labor force is wholly defined by relationship (9) as a lagged function of real GDP per capita.

Linear relationships (13) and (14) define inflation and unemployment separately as functions of labor force change. These two variables are indivisible sides of a unique process, however. The process is the labor force growth, which is accommodated in developed economies though two channels. (We always stress that these relationships are valid only for large developed economics implying that small developed, developing and emerging economies might be characterized by different links.) The first channel is the change in employment and relevant reaction of PID. All persons obtaining new paid jobs or their equivalents presumably change their incomes to some higher levels. There is a reliable empirical fact, however, that PID in the U.S. has not been changing over time in relative terms (Kitov, 2007a). The increasing number of people at higher income levels, as related to the new paid jobs, leads to a certain disturbance in the PID. This over-concentration must be compensated by such an extension in the income scale, which returns the PID to its original density. In other words, the economy demands an injection of some amount of money extra to that defined by real economic growth in order to recover the PID. As a result, prices in the economy grow at an elevated rate, i.e. are prone to inflation. This process is accompanied by corresponding stretch in the PID income scale. The mechanism responsible for the compensation and the scale stretching has some relaxation time, which effectively separates in time the source of inflation, i.e. the labor force change, and the reaction, i.e. price inflation.

The second channel is related to those who failed to obtain a new paid job, i.e. to enter employment. These people do not leave the labor force but join unemployment. Effectively, they do not change the PID because they do not change their incomes. So, the total labor force change (wholly defined by G) equals the unemployment change plus employment change. In the case of "normal" behavior of an economic system, the proportion between unemployment and inflation is retained through time and both linear relationships hold separately. There is always a possibility, however, to fix one of the two variables. For example, central banks are able to fix inflation by some monetary means. Such violations of the natural behavior will undoubtedly distort the partition of the labor force change – the portion previously accommodated by inflation will be redirected to unemployment, and vice versa. To account for this effect one should use a generalized relationship as represented by the sum of relationships (13) and (14):

$$\pi(t) + UE(t) = a_1 dLF(t-T_1)/LF(t-T_1) + b_1 dLF(t-T_2)/LF(t-T_2) + a_2 + b_2 \qquad (15)$$

Equation (15) balances labor force change, inflation and unemployment, the latter two variables potentially lagging by different times behind the labor force change. The importance of this generalized relationship is demonstrated by Kitov (2007b) for the case of France before and after joining the European Monetary Union.

For the U.S., there is no need (so far) to apply relationship (15). The changing monetary policy of the Federal Reserve has not affected the natural partition of labor force change, as has been observed since the late 1950s. Therefore, relationship (13) with a_1=4, a_2=-0.03, T_1=2 years (GDP deflator as a measure of inflation) provides the best fit between observed and predicted inflation, as presented in Figure 8 and 9 for annual and cumulative values. The best fit of the cumulative curves provides an accurate procedure for the estimation of the coefficients.

Negative constant a_2 makes some permanent increase in labor force of great importance for avoiding deflationary periods. Population growth rate of 0.01 to 0.015 per year, as has been observed in the U.S. during the last twenty years, completely compensates the effects of negative term a_2. With the boomers' retirement, however, the growth rate of labor force started to decelerate in 2005.

One can describe inflation in the U.S. with an uncertainty controlled by the accuracy of labor force estimates. Thus, a direct way to improve the predictive power of the inflation/labor force relationship is available. Only some simple arrangements are necessary. Moreover, one can easily introduce a target value for the inflation uncertainty and link it to the resources available and needed.

In our model, inflation forecasting is equivalent to the inflation regression against the change rate of labor force. In forecasting practice, the root mean square forecast error (RMSFE) is a standard measure of uncertainty. This term indicates that forecasted values of inflation are obtained in the framework of out-of-sample approach, i.e. using only past values of predictors. The best prediction obtained with our model for the period between 1960 and 2005 for the annual readings gives RMSFE of 0.008 (0.8%). This value is lower than any RMSFE at a two-year horizon we were able to find in literature for the same or comparable period.

Unemployment in the U.S. has been also predicted as a linear lagged function of the labor force change and is as follows: $UE(t)=0.023+2.1*dLF(t-5)/LF(t-5)$. The lag of the observed unemployment behind the change in labor fore is five years – the value obtained by simple visual fit of the smoothed curves as presented in Figure 10. Due to high volatility associated with measurement errors, there is some discrepancy between the two curves in Figure 10, however. Figure 11 displays the same curves smoothed with MA(7) for the period between 1960 and 2004. The predicted curve almost coincides with the observed one during the last 35 years and provides a prediction for the next five years.

Figure 12 presents a prediction for the unemployment according to (15), i.e. based on the labor force change and inflation. The following empirical version is obtained: $UE(t) = \pi(t-3) - 2.5dLF(t-5)/LF(t-5) + 0.0585$. The lags are three years for inflation and five years for labor force change. Figure 13 depicts corresponding MA(7) smoothed curves used to estimate corresponding coefficients.

5. Conclusion

In the U.S., the change in the specific age population drives such macroeconomic variables as real economic growth, labor force participation rate, productivity, inflation, and unemployment according to relationships (1) through (15). These relationships represent a comprehensive macro-model of the U.S. economy, i.e. its reaction to exogenous (demographic) forces and the interaction between principal macro-variables.

This conclusion is supported by corresponding tests for the presence of cointegrating relations and other statistical estimates (Kitov, Kitov, Dolinskaya, 2007abc). Moreover, our concept provides reliable relationships for the prediction of the studied macroeconomic variables at very large (more than 9 years) time horizons.

There were several relationships between main macroeconomic variables revealed in our study. These relationships have been valid during the last several decades. (It should be notice here that one can not extend these relationships further in the past due to the absence of reliable demographic and economic data before 1960.) The relationships reflect inherent links between people, which had been established in the U.S. economy as a result of economic and social evolution. There was time, however, when these relationships were not valid. Also, it is possible that they will fail some time in the future due to the development of some new links. Therefore, we consider current macro-state of the U.S. economy as a temporary and transient one. In addition, the macroeconomic predictions we have given in the study are prone to corrections, as related to changes in monetary policy (shift in inflation/unemployment balance) and various demographic processes including fluctuations in immigration.

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Figures

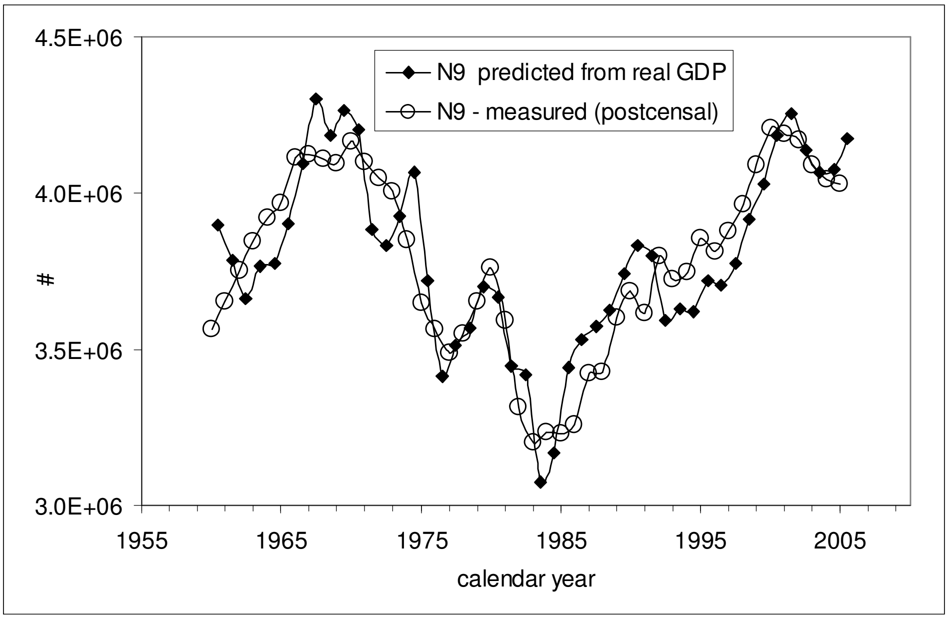
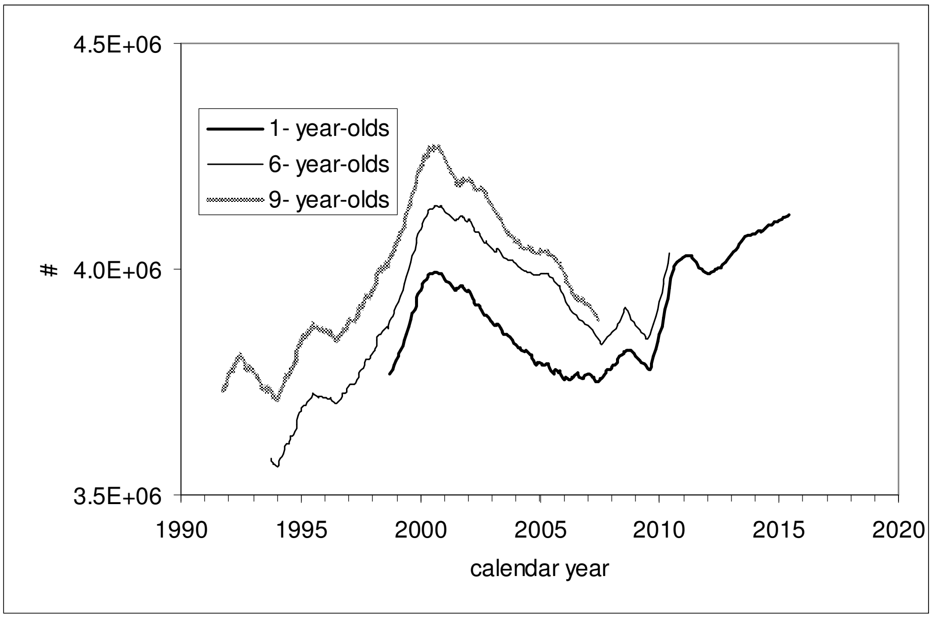


Figure 1. Measured and predicted number of 9-year-olds in the U.S. The predicted number is obtained from the estimates of real GDP per capita according to (3). Here, real GDP per capita was estimated for persons of 16 years of age and over, i.e. for the working age population. According to linear regression of the time series between 1962 and 2005, the goodness-of-fit is (R^2=) 0.81.



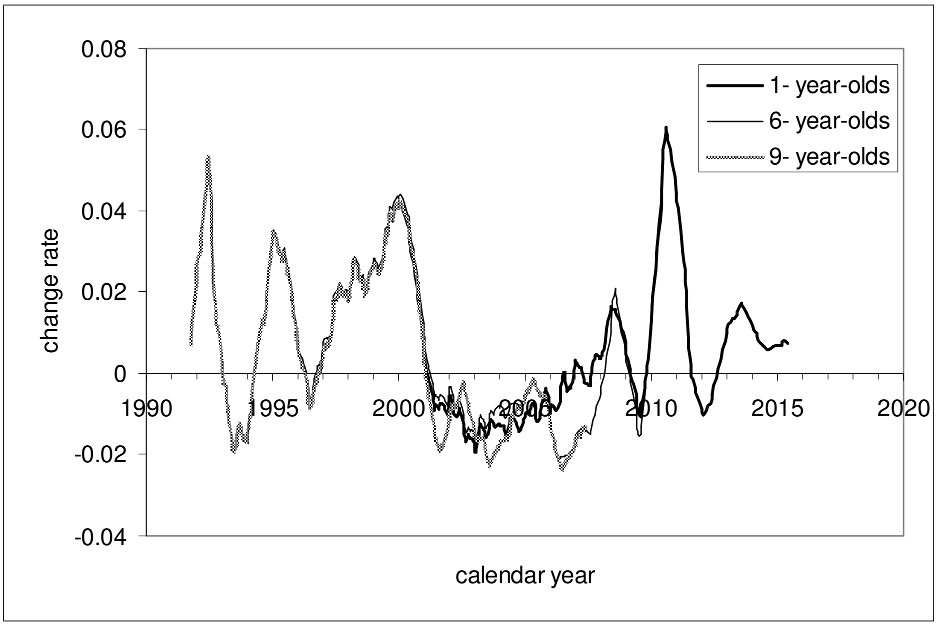


Figure 2. Prediction of the number of 9-year-olds by extrapolation of population estimates for younger ages (1- and 6-year-olds).

Upper panel: Population estimates of the number of 9-, 6, and 1-year-olds. The time series for younger ages are shifted ahead by 3 and 8 years, respectively.

Lower panel: The change rate of the population estimates, which is proportional to the growth rate of real GDP per capita. Notice the difference in the change rate provided by the 1-year-olds and 6-year-olds for the period between 2003 and 2010. This discrepancy is related to the age-dependent difference in population revisions.

Since 2002, the input of the population related component of the growth rate has been negative. It turns to a positive one near 2010. This also results in an elevated growth rate of real GDP per capita during the period between 2010 and 2017.

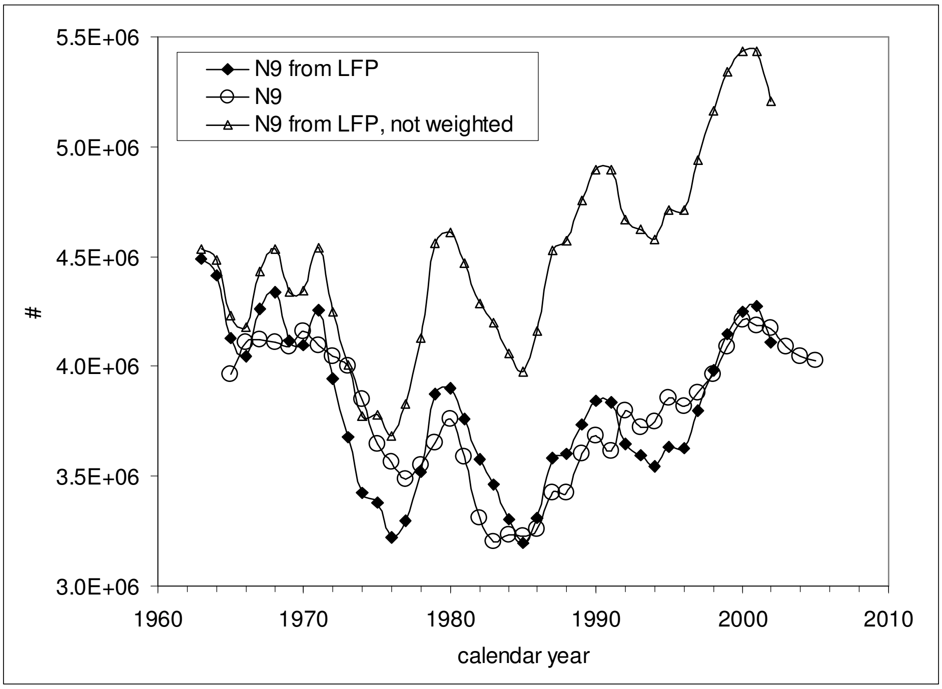


Figure 3. The number of 9-year-olds: the observed one and that obtained from the LFP with and without exponential weighting in (5). Constants t_0=1963; B_2=-1.5E+8, C_2=4.94E+6, α_2=-1.85. The case with α_2=0 in also shown: N_9 is highly overestimated.

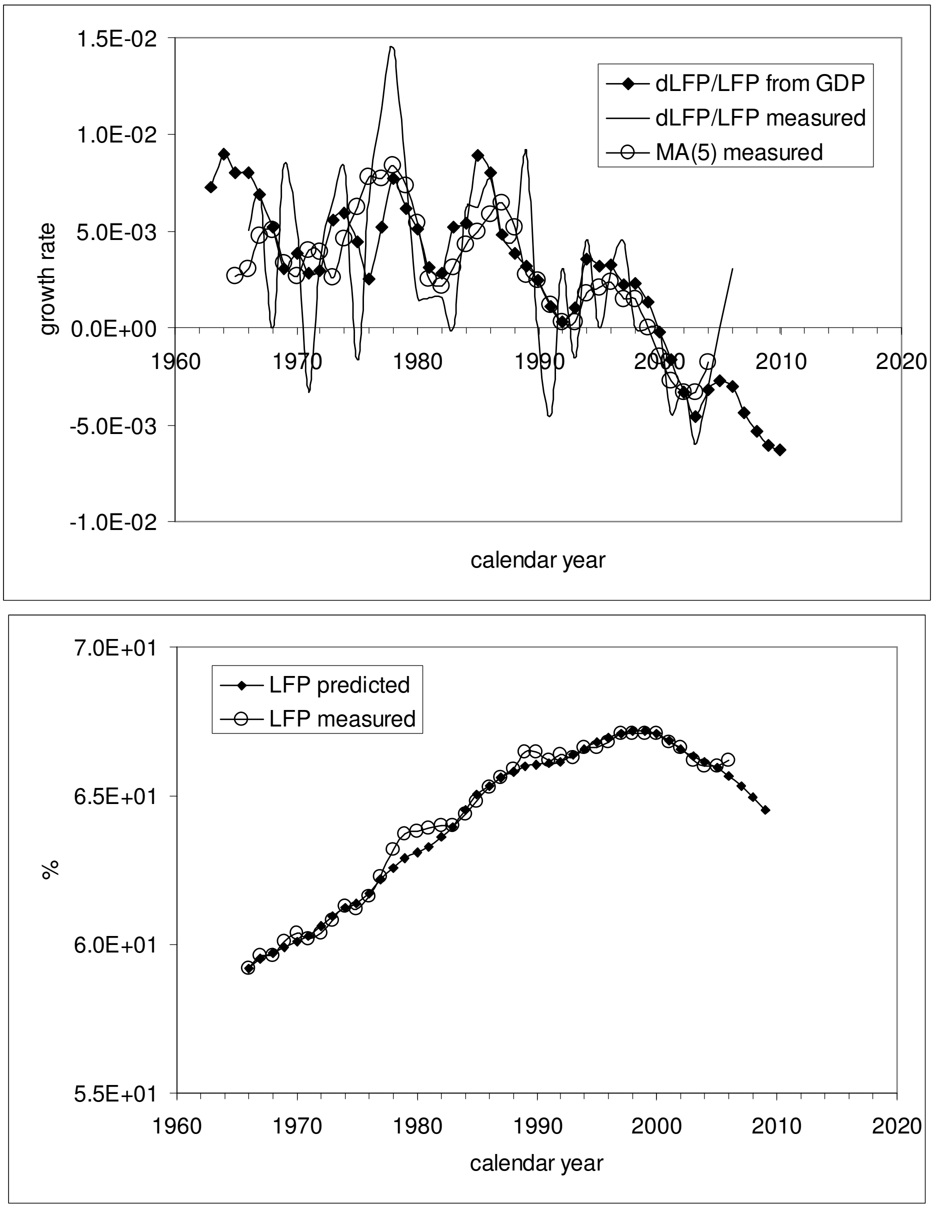


Figure 4. Upper panel: observed and predicted growth rate of LFP in the U.S. The predicted curve is obtained from real GDP per capita using (5) and (6) with *Ns*(1959)=4.5E+6, A_2=$350 (2002-dollars), *B*=-1.23E+8, *C*=0.04225. Linear regression gives R^2=0.73.

Lower panel: measured and predicted LFP for the growth rates in the upper panel. The LFP has been decreasing after 2000.

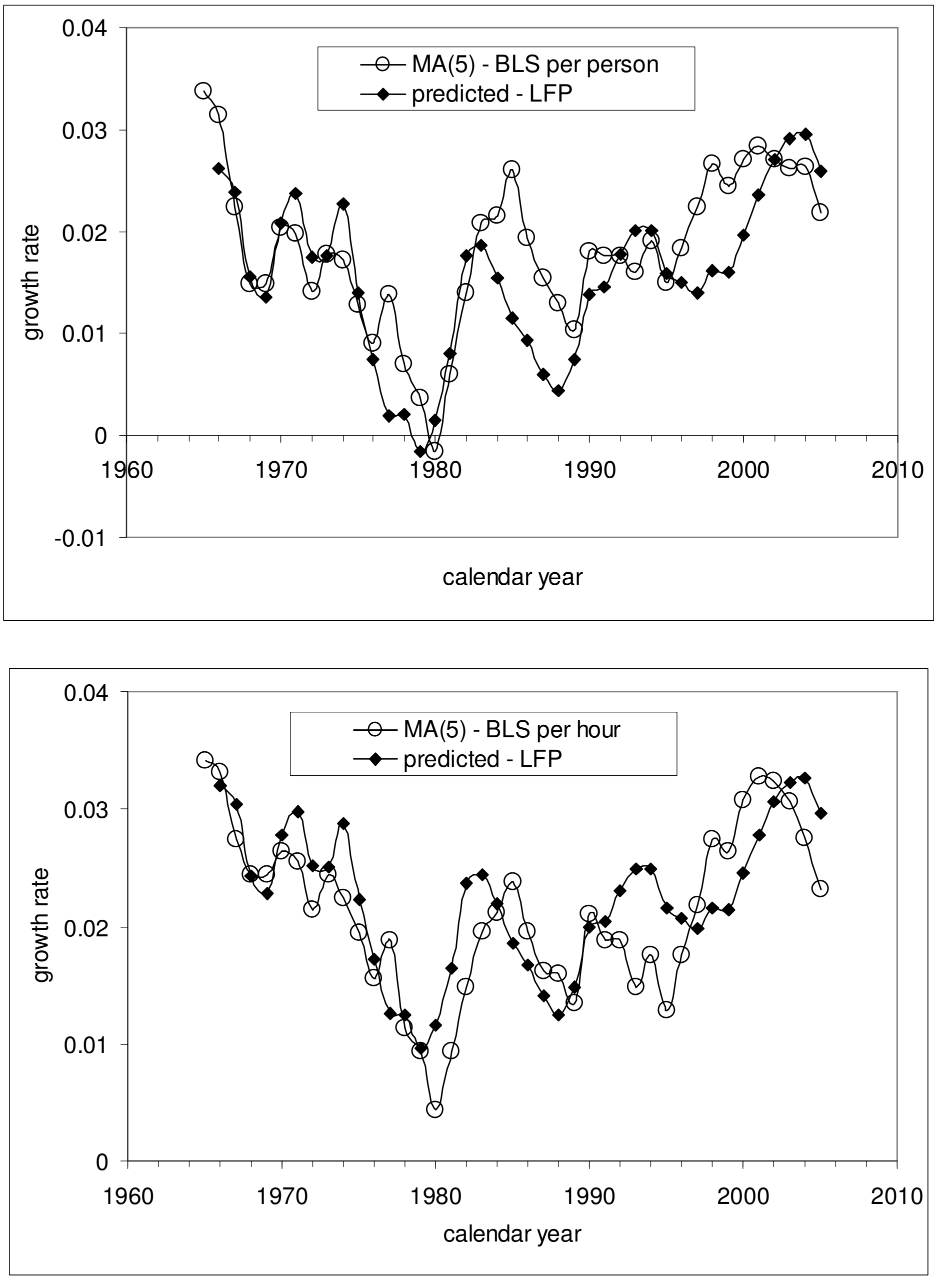


Figure 5. Observed and predicted growth rate of labor productivity. Two BLS measures of productivity are presented: upper panel - output ($) per person; lower panel - output ($) per hour. Linear regression gives close results - R^2=0.6 in both cases.

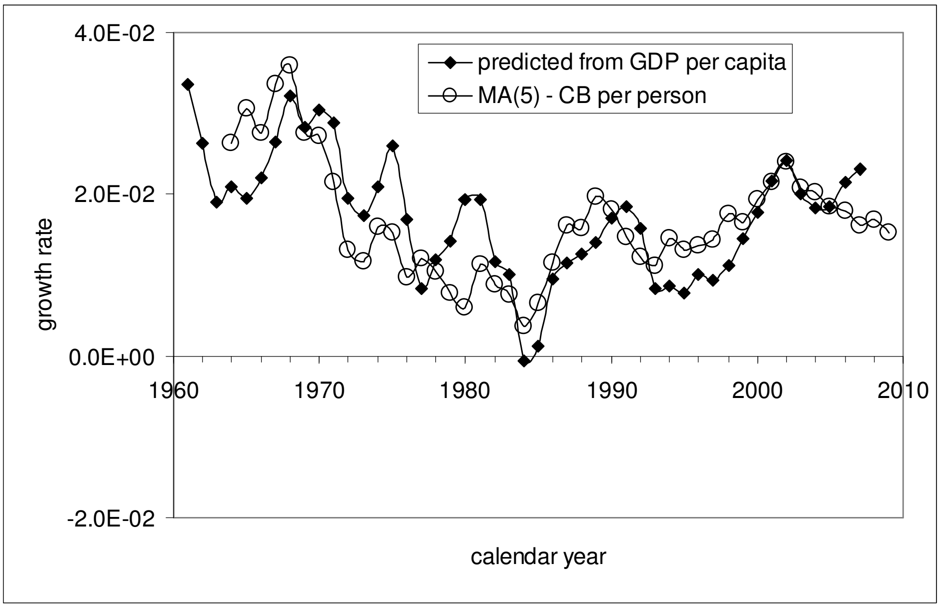


Figure 6. Observed and predicted change rate of productivity (Conference Board - GDP per person employed). The observed curve is represented by MA(5) of the original one. Linear regression gives R^2=0.6. Model parameters are as follows: Ns(1959)=4500000, A_4=\$420 (2002-dollars), B_4=3500000, C_4=-0.095.

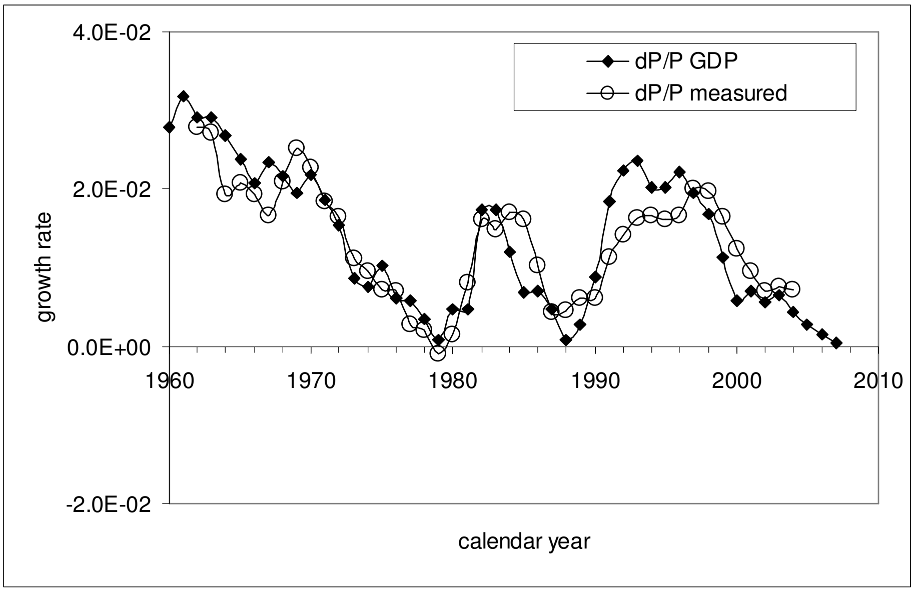


Figure 7. Observed and predicted productivity in Canada: $Ns(1959)=270000$, $A_4=\$300$ (1990 US dollars) , $B_4=-3200000$, $C_4=0.108$. $R^2=0.8$.

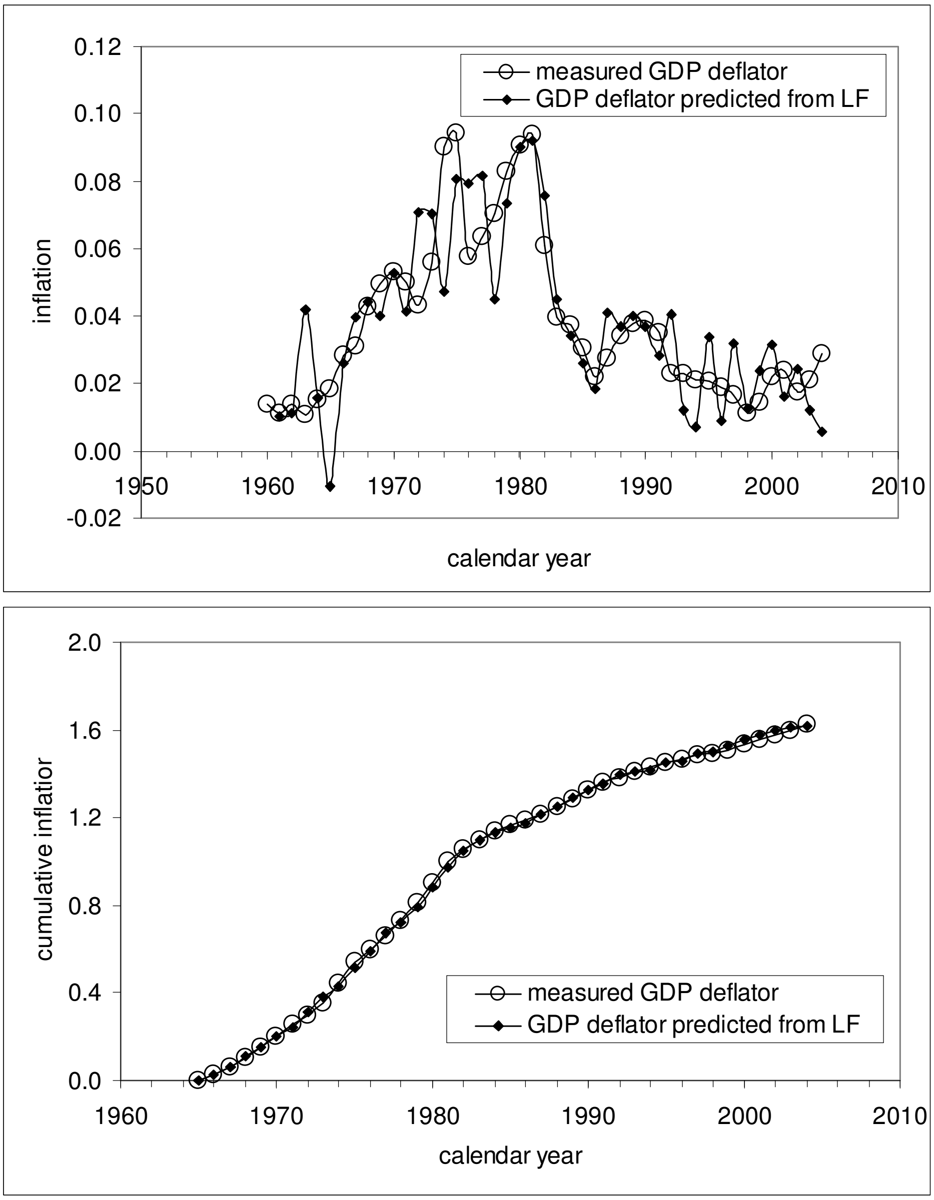


Figure 8. Observed and predicted inflation (GDP deflator). The predicted values are obtained using relationship (13) with a_1=4.0, a_2= -0.3, and T_1=2 years. The upper panel compares annual readings and the lower one – cumulative values.

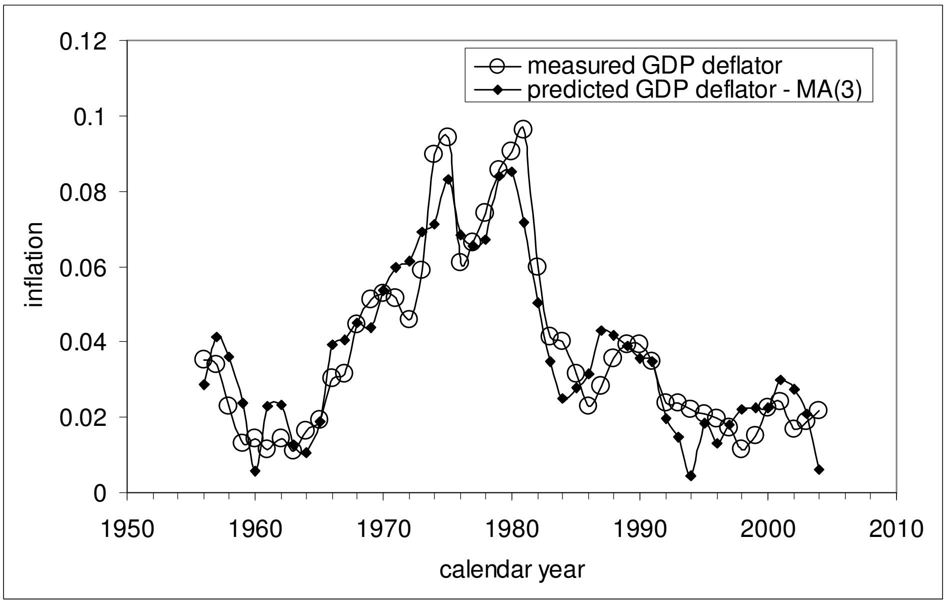


Figure 9. Measured and predicted inflation, The latter is represented by MA(3). Linear regression is characterized by R^2=0.88 and Standard Error of 0.0057, i.e. RMSFE is only 0.6% at a 1 year horizon.

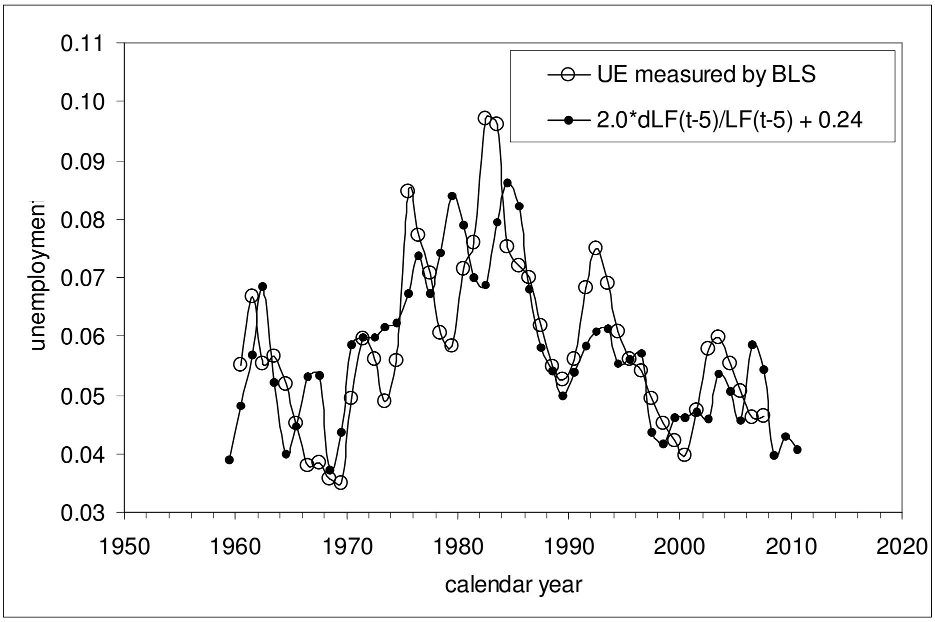


Figure 10. Observed and predicted unemployment rate. The latter is obtained from the estimates of the labor force in the U.S. between 1960 and 2006. The 5-year lag of the unemployment behind the labor force allows a prediction of unemployment up to 2011. Notice a local peak in the predicted curve near 2008.

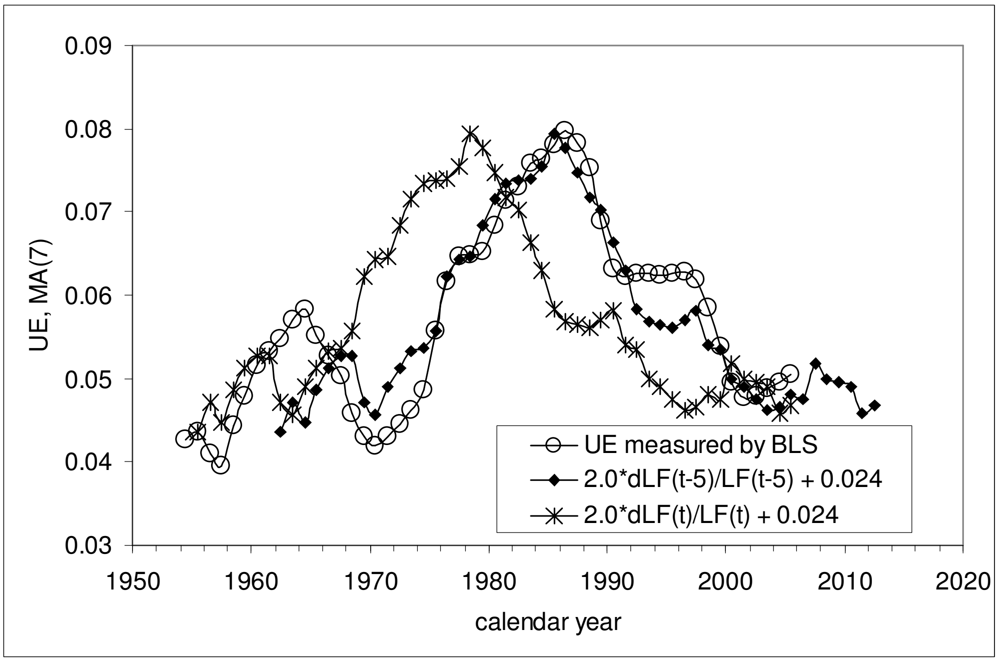


Figure 11. Predicted and observed unemployment rate smoothed by MA(7). The original predicted curve and that shifted by five years back are presented in order to illustrate synchronization process and the lag estimation.

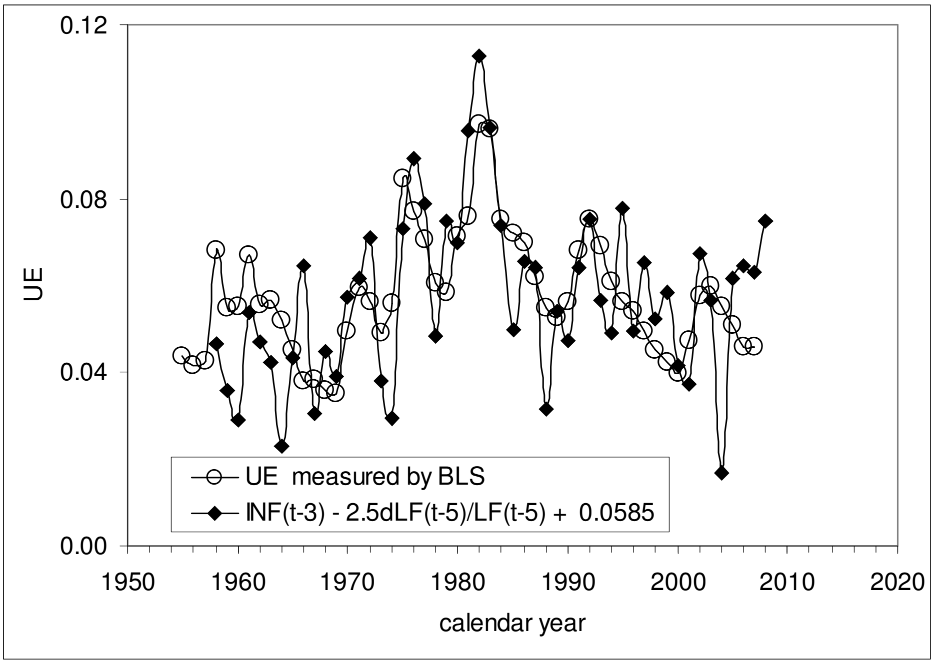


Figure 12. Unemployment measured by the BLS and that predicted as a lagged linear function of labor force change rate (*dLF/LF*) and inflation (*INF*) represented by GDP deflator: $UE(t) = \pi(t\text{-}3) - 2.5dLF(t\text{-}5)/LF(t\text{-}5) + 0.0585$.

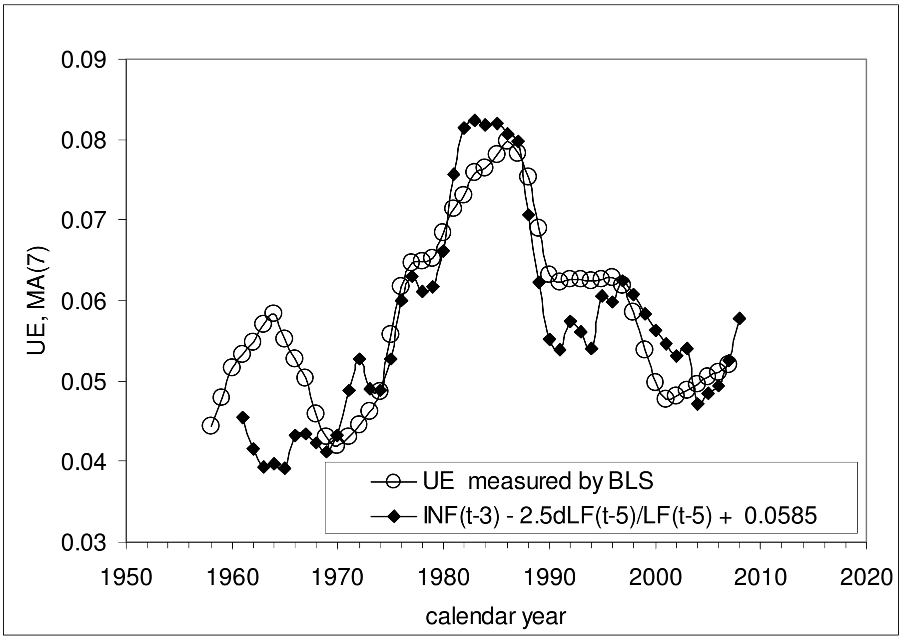


Figure 13. Same as in Figure 12, but smoothed by MA(7). There are some weak deviations (approximately 0.5 %) between the smoothed curves in the 1990s and the beginning of the 2000s.