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Abstract

This paper examines the stability of a small open economy under alternative income taxation rules. Using a one-sector real business cycle model with external increasing returns, we show that if the income taxation is progressive, the small open economy will not generate equilibrium indeterminacy, but it exhibits a diverging behavior if the degree of external increasing returns is sufficiently large. In this case, a progressive tax schedule on the factor income may recover saddle stability. We also reveal that if the taxation on the interest income from financial assets is regressive, then the small open economy may exhibit equilibrium indeterminacy. In this situation, progressive taxation is also useful for eliminating sunspot-driven fluctuations.

Keywords: Income taxation rule, Equilibrium indeterminacy, Built-in stabilizer, small open economy

JEL Classification: E62, O41

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1 Introduction

Does the income taxation rule act as a built-in stabilizer? This long-standing question has attracted a renewed interest in public finance, ever since Guo and Lancing (1998) revealed that progressive income taxation contributes to stabilizing an economy in the presence of sunspot-driven business fluctuations. Using a one-sector real business cycle model with external increasing returns, Guo and Lansing (1998) demonstrated that progressive income taxation narrows the parameter space in which equilibrium indeterminacy emerges. They also confirmed that regressive income taxation enhances the possibility of equilibrium indeterminacy. Subsequent studies have reconsidered Guo and Lansing's finding in alternative settings such as two-sector real business cycle models, models with productive public investment, models with utility-enhancing public spending, and models of endogenous growth.[1] Those studies have shown that the taxation rule may play a decisive role in stabilizing the economy in various settings.

So far, the research on the stabilization effect of income taxation rules has focused on closed economies, and the role of taxation schemes for the stabilization effect in open economies has not yet been explored well. The purpose of this paper is to investigate the relation between income tax schedules and the stability of a small-open economy. We introduce the nonlinear taxation rule formulated by Guo and Lansing (1998) into a prototype model of a one-sector, small open economy with free capital mobility. Based on this analytical framework, we investigate which type of taxation rule contributes to stabilizing the small open economy.

We obtain two main findings. First, if the income taxation schedule is progressive, the small open economy will not yield equilibrium indeterminacy, regardless of the degree of external effects associated with aggregate labor and capital. However, if the aggregate production function holds a high level of external increasing returns, then the equilibrium path of the small open economy diverges from the steady-state equilibrium.

[1] A sample includes Ben-Gad (2003), Chen, Hsu and Hsu (2018), Chen and Guo (2015, 2016, 2017), Dromoel and Pintus (2007, 2008), Gokan (2013), Greiner (2006), Guo and Harrison (2001, 2015), Lloyd-Braga, Modesto and Seegmuller (2008), and Zhang (2000).

In this case, progressive income taxation contributes to stabilizing the economy in the sense that it recovers saddle-point stability of the steady-state equilibrium.

Our second finding is that regressive taxation on the interest income from foreign bonds may generate equilibrium indeterminacy, regardless of the taxation scheme applied to the domestic factor income. This means that, as far as the taxation on the interest income is concerned, progressive taxation would act as a built-in stabilizer in the sense that it may eliminate sunspot-driven fluctuations. Therefore, our paper shows that the main conclusion of Guo and Lansing (1998) generally holds in the open economy counterpart as well.

Besides the literature on the stabilization effect cited in Footnote 1, our paper is closely related to the studies by Weder (2001), Lahiri (2001), Meng (2003) and Meng and Velasco (2003, 2004), who examined equilibrium indeterminacy in small open economies. Those early contributions utilized two sector models in which consumption goods are traded, while investment goods are not traded. They showed that small open economies tend to be volatile, because indeterminacy holds under weaker conditions than in closed economies. We find that such a conclusion does not hold in the one-sector, small open economy model that is the standard analytical framework in the open economy macroeconomics literature.[2]

We should point out that a few authors have examined the stabilization effect of fiscal policy rules in small open economies. Among others, Huang, Meng and Xue (2017) introduced the balanced-budget rule à la Schmitt-Grohé and Uribe (1997) into a two-sector small-open economy model with variable labor supply. These authors revealed that the destabilizing effect of the balanced-budget rule emphasized by Schmitt-Grohé and Uribe (1997) does not necessarily hold in their small-open economy model. These authors focused on the role of the balanced-budget rule and did not consider nonlinear taxation. To the best of our knowledge, Zhang (2015) is the most closely related study to our paper. By use of a two-sector small open economy model in which capital goods

[2]See Chapter 6 in Mino (2017) for a detailed discussion on equilibrium indeterminacy in open economy models.

are not traded, Zhang (2015) examined the stabilization effect of the balanced budget rule under Guo and Lansing's (1998) taxation scheme. Although the research concern of Zhang's study overlaps with our paper, Zhang (2015) did not analyze the role of the taxation rule on the interest income from financial assets. Therefore, Zhang (2015) and our study are complements rather than substitutes.

The rest of the paper is organized as follows. The next section constructs the baseline model. Section 3 characterizes the equilibrium dynamics and inspects the stabilization effect of the taxation rule. Section 4 modifies the base model by considering alternative tax schedules. Section 5 concludes.

2 Model

In this paper, we use the one-sector real business cycle model with investment adjustment costs that has been frequently used in open economy macroeconomics[3]. We introduce production externalities and nonlinear taxation into the standard setting.

2.1 Production and Consumption

The analytical framework of our study is a small open economy version of the model of Benhabib and Farmer (1994) which introduced production externalities into an otherwise standard baseline model of real business cycles. The home country and the rest of the world produce homogeneous goods. The aggregate production function of the home country is given by

$$Y_t = AK_t^a N_t^{1-a} \bar{K}_t^{\alpha-a} \bar{N}_t^{\beta-(1-a)} \quad A > 0, \quad 0 < a < 1, \quad a < \alpha \leq 1, \quad \beta > 1 - a,$$

where Y_t is output, K_t is capital, N_t is labor, and $\bar{K}_t$ and $\bar{N}_t$ represent country-specific, external effects associated with the aggregate levels of capital and labor. In

[3]See Schmitt-Grohé,and Uribe (2017) for detailed discussion on this prototype model in open economy macroeconomics.

our representative-agent economy, the mass of agents is normalized to one, and, thus in equilibrium, $\bar{K}_t = K_t$ and $\bar{N}_t = N_t$ hold for all $t \geq 0$. Therefore, the social production function is written as

$$Y_t = AK_t^{\alpha} N_t^{\beta}. \quad (1)$$

The final good and factor markets are assumed to be competitive, and the factor prices are given by

$$r_t = aAK_t^{\alpha-1} N_t^{\beta}, \quad w_t = (1-a)\, AK_t^{\alpha} N_t^{\beta-1}, \quad (2)$$

where r_t is the rate of return to capital and w_t is the real wage rate.

Our formulation of a small open economy is the conventional one: domestic households freely lend to or borrow from foreign households, and international lending and borrowing are carried out by trading foreign bonds under a given world interest rate. The objective function of the representative household is the following lifetime utility:

$$U = \int_0^{\infty} e^{-\rho t} \log \left(C_t - \frac{N_t^{1+\gamma}}{1+\gamma} \right) dt, \quad \rho > 0, \quad \gamma > 0,$$

where ρ denotes a given time discount rate. In this paper, we assume that the representative household has the Greenwood–Hercowitz–Huffman (GHH) preferences (Greenwood, Hercowitz and Huffman, 1988). Under GHH preferences, there is no wealth effect on labor supply, so the labor supply solely depends on the real wage. As is well known, the emergence of equilibrium indeterminacy in the Benhabib-Farmer model stems from the wealth effect on labor supply, coupled with the presence of strong externality that makes the labor demand curve steeper than the Frisch labor supply curve. In this paper, we exclude the wealth effect to focus on income taxation rules rather than on production and preference structures in discussing the equilibrium (in)determinacy problem.

The household's flow budget constraint is

$$\dot{B}_t = (1-\tau_{y,t})\,(r_t K_t + w_t N_t) + (1-\tau_{b,t})\, RB_t - \left[\frac{I_t}{K_t} + \frac{\theta}{2} \left(\frac{I_t}{K_t} \right)^2 \right] K_t - C_t, \quad \theta > 0, \quad (3)$$

where B_t denotes the stock of foreign bond (net asset position) held by domestic households, R is a given world interest rate, $\tau_{b,t}$ is the rate of tax on interest income, $\tau_{y,t}$ is the rate of factor income tax, and I_t denotes gross investment on capital. Here, the term $(\theta/2)\,(I_t/K_t)^2\,K_t$ represents the adjustment costs of investment. In this paper. we assume that the home country is a lender to foreign households, so B_t has a positive value and the taxation on interest income is available. The capital stock changes according to

$$\dot{K}_t = I_t - \delta K_t, \quad 0 < \delta < 1, \tag{4}$$

where δ denotes the rate of the depreciation of capital.

The household maximizes the lifetime utility U by controlling C_t, N_t and I_t subject to (3) and (4) together with the initial condition on K_t and B_t as well as with the no-Ponzi-game condition:

$$\lim_{t\to\infty} e^{-(1-\tau_b)Rt} B_t \geq 0. \tag{5}$$

2.2 Taxation Rules

Following Guo and Lansing (1998), we assume that the fiscal authority adjusts each rate of income taxes according to the following manner:

$$\tau_{y,t} = 1 - \eta_y \left(\frac{Y^*}{Y_t}\right)^{\phi_y}, \quad 0 < \eta_y < 1, \quad \underline{\phi_y} < \phi_y < 1, \qquad \underline{\phi_y} < 0, \tag{6}$$

$$\tau_{b,t} = 1 - \eta_b \left(\frac{B^*}{B_t}\right)^{\phi_b}, \quad 0 < \eta_b < 1, \quad \underline{\phi_b} < \phi_b < 1, \quad \underline{\phi_b} < 0. \tag{7}$$

In the above, $Y_t = r_t K_t + w_t N_t$ denotes domestic factor income of the household in period t and Y^* is a reference level of factor income, which is represented by the steady-state level of factor income. Similarly, the reference income in the case of the taxation on the interest income is its steady- state level, RB^*. Hence, the taxation rule is written as (7) on the interest income, RB_t. In the above, the restrictions on η_y and η_b mean that when $Y_t = Y^*$ holds, the average tax rates are between 0 and 1. In addition, parameters $\underline{\phi_y}$ and $\underline{\phi_b}$,are given by

$$\underline{\phi_i} = \frac{\eta_i - 1}{\eta_i}, \quad i = y,\ b.$$

In this taxation scheme, the rate of income tax is endogenously determined out of the steady state, but it becomes an exogenously given flat rate, $1-\eta_y$ and $1-\eta_b$, at the steady state. The restriction on ϕ_i $(i = y, b)$ ensures that the marginal tax revenue of the government increases with households' incomes at the steady state even if the taxation schedule is regressive $\left(\phi_y,\ \phi_b < 0\right)$[4].

The tax schedule given by (6) means that the marginal tax rate on the domestic income is given by

$$\frac{d}{dY_t}\left(\tau_{y,t}Y_t\right) = 1 - \left(1-\phi_y\right)\eta_y\left(\frac{Y_t^*}{Y_t}\right)^{\phi_y},$$

which is higher (lower) than the average tax rate, $\tau_{y,t}$ if $0 < \phi_y < 1$ $\left(\underline{\phi_y} < \phi_y < 0\right)$. Thus, the taxation is progressive (regressive) if $0 < \phi_y < 1$ $\left(\underline{\phi_y} < \phi_y < 0\right)$. The same argument holds for the taxation on the interest income. Note that under (6) and (7), the after-tax total income of the household is

$$\left(1-\tau_{y,t}\right)Y_t + \left(1-\tau_{b,t}\right)RB_t = \eta_y Y^{*\phi_y}\left(r_tK_t + w_tN_t\right)^{1-\phi_y} + \eta_b RB^{*\phi_b}B_t^{1-\phi_b}.$$

Denoting the government consumption as G_t, the flow budget constraint for the government is

$$G_t = \tau_{y,t}Y_t + \tau_{b,t}RB_t = \left[1-\eta_y\left(\frac{Y^*}{Y_t}\right)^{\phi_y}\right]Y_t + \left[1-\eta_b\left(\frac{B^*}{B_t}\right)^{\phi_b}\right]RB_t \qquad (8)$$

[4] The government's revenue from factor income taxation is $T_y = \tau_{y,t}Y_t = \left[1-\eta_y\left(\frac{Y^*}{Y_t}\right)^{\phi_y}\right]Y_t$. Thus

$$\frac{dT_y}{dY_t} = 1 - \eta_y\left(1-\phi_y\right)\left(\frac{Y^*}{Y_t}\right)^{\phi_y},$$

which shows that the marginal tax revenue is positive at the steady state if $1 > \left(1-\phi_y\right)\eta_y$, which gives the minimum level of $\underline{\phi_y}$. The same argument is applied to the taxation on the interest income.

We assume that the government simply consumes its tax revenue, so that the government spending affects neither households' welfare nor firms' production activities.

2.3 The Optimal Conditions

To derive the optimization conditions for the household, we set up the following Hamiltonian function:

$$\begin{aligned} H_t &= \log\left(C_t - \frac{N_t^{1+\gamma}}{1+\gamma}\right) + q_t\left(I_t - \delta K_t\right) \\ &+\lambda_t\left[\eta_y(Y^*)^{\phi y}\left(r_t K_t + w_t N_t\right)^{1-\phi_y} + \eta_b R\left(B^*\right)^{\phi_b} B_t^{1-\phi_b} - \left[\frac{I_t}{K_t} + \frac{\theta}{2}\left(\frac{I_t}{K_t}\right)^2\right]K_t - C_t\right], \end{aligned}$$

where q_t and λ_t respectively denote the utility prices of K_t and B_t.

Remember that when selecting $C_t,\ N_t$ and I_t, the representative household takes sequences of $\{r_t,\ w_t\}_{t=0}^{\infty}$ as given. Therefore, noting that $r_t K_t + w_t N_t = Y_t$, we find that the first-order conditions for an optimum include the following:

$$\max_{C_t} H_t \Longrightarrow \left(C_t - \frac{N_t^{1+\gamma}}{1+\gamma}\right)^{-1} = \lambda_t, \tag{9a}$$

$$\max_{N_t} H_t \Longrightarrow \left(C_t - \frac{N_t^{1+\gamma}}{1+\gamma}\right)^{-1} N_t^{\gamma} = \lambda_t \eta_y\left(1-\phi_y\right)\left(\frac{Y_t^*}{Y_t}\right)^{\phi_y} w_t, \tag{9b}$$

$$\max_{I_t} H_t \Longrightarrow q_t = \lambda_t\left[1 + \theta\frac{I_t}{K_t}\right], \tag{9c}$$

$$\dot{q}_t = (\rho+\delta)q_t - \lambda_t\left[(1-\phi_y)\eta_y\left(\frac{Y_t^*}{Y_t}\right)^{\phi_y} r_t + \frac{\theta}{2}\left(\frac{I_t}{K_t}\right)^2\right], \tag{9d}$$

$$\dot{\lambda}_t = \lambda_t\left[\rho - \left(1-\phi_b\right)\eta_b\left(\frac{B^*}{B_t}\right)^{\phi_b} R\right], \tag{9e}$$

together with the transversality condition: $\lim_{t\to\infty} e^{-\rho t} q_t K_t = 0$ and $\lim_{t\to\infty} e^{-\rho t}\lambda_t B_t = 0$.

3 Equilibrium Dynamics

3.1 Dynamic System

We find that, using (2), conditions $(9a)$ and $(9b)$ yield

$$N_t^{\gamma} = \left(1-\phi_y\right)\eta_y \left(\frac{Y^*}{Y_t}\right)^{\phi_y} (1-\alpha)\,\frac{Y_t}{N_t}.$$

Substituting (1) into the above and solving it with respect to N_t, we obtain

$$N_t = \Omega K_t^{\omega}, \tag{10}$$

where

$$\Omega = \left[\left(1-\phi_y\right)\eta_y (1-a)\, A^{1-\phi_y} Y^{*\phi_y}\right]^{\frac{1}{1+\gamma-\left(1-\phi_y\right)\beta}}, \quad \omega = \frac{\alpha\left(1-\phi_y\right)}{1+\gamma-\left(1-\phi_y\right)\beta}. \tag{11}$$

Equation (10) gives the equilibrium level of hours worked. Substituting (10) into (1) presents a reduced form of aggregate production function in such a way that

$$Y_t = A\Omega^{\beta} K_t^{\alpha+\beta\omega}. \tag{12}$$

As a result, the pre-tax real rate of return to capital is expressed as

$$r_t = a\frac{Y_t}{K_t} = aA\Omega^{\beta} K_t^{\alpha+\beta\omega-1}. \tag{13}$$

In what follows we impose the following restrictions on the parameter values:

$$1+\gamma > \left(1-\phi_y\right)\beta \text{ and } \alpha+\beta\omega < 1. \tag{14}$$

In (14), the former condition makes the labor demand curve less steep than the Frisch labor supply curve, and it ensures that $\alpha+\beta\omega > 0$. Under the latter condition, the reduced-form aggregate production function exhibits decreasing returns to capital.

Equation $(9a)$ gives

$$C_t = \frac{1}{\lambda_t} + \frac{N_t^{1+\gamma}}{1+\gamma}, \tag{15}$$

Using $(9c)$, $(9d)$ is written as follows:

$$\dot{K}_t = K_t \left[\frac{1}{\theta} \left(\frac{q_t}{\lambda_t} - 1 \right) - \delta \right], \tag{16}$$

Denoting $q_t/\lambda_t = v_t$ and using $(9e)$, (3), (15), (16) and $(9d)$, we obtain the following complete dynamic system with respect to B_t, v_t, K_t and λ_t :

$$\begin{aligned} \dot{B}_t &= \eta_y \left(\frac{Y^*}{A\Omega^\beta K_t^{\alpha+\beta\omega}} \right)^{\phi_y} A\Omega^\beta K_t^{\alpha+\beta\omega} + \eta_b \left(\frac{B^*}{B_t} \right)^{\phi_b} RB_t \\ &\quad - \left[\frac{1}{\theta}(v_t - 1) + \frac{1}{2\theta}(v_t - 1)^2 \right] K_t - \frac{1}{\lambda_t} - \frac{(\Omega K_t^\omega)^{1+\gamma}}{1+\gamma}, \end{aligned} \tag{17a}$$

$$\begin{aligned} \dot{v}_t &= \delta v_t - \left[(1-\phi_y)\eta_y \left(\frac{Y^*}{A\Omega^\beta K_t^{\alpha+\beta\omega}} \right)^{\phi_y} aA\Omega^\beta K_t^{\alpha+\beta\omega-1} + \frac{1}{2\theta}(v_t - 1)^2 \right] \\ &\quad + (1-\phi_b)\eta_b \left(\frac{B^*}{B_t} \right)^{\phi_b} Rv_t, \end{aligned} \tag{17b}$$

$$\dot{K}_t = K_t \left[\frac{1}{\theta}(v_t - 1) - \delta \right], \tag{17c}$$

$$\dot{\lambda}_t = \lambda_t \left[\rho - (1-\phi_b)\eta_b \left(\frac{B^*}{B_t} \right)^{\phi_b} R \right]. \tag{17d}$$

3.2 Steady-State Equilibrium

In the steady-state equilibrium, it holds that $Y_t = Y^*$ and $B_t = B^*$. In $(17d)$, the steady-state condition, $\dot{\lambda}_t = 0$, holds if and only if

$$\rho = (1-\phi_b)\eta_b R. \tag{18}$$

We assume that (18) is fulfilled in order to define a feasible steady-state equilibrium. The steady-state condition for the aggregate capital stock, $\dot{K} = 0$ in $(17c)$ gives

$$v^{*} = \theta\delta + 1, \tag{19}$$

which determines the steady-state level of relative utility price between the financial asset and the physical capital. Then the condition $\dot{v}_t = 0$ in $(17b)$, together with (18) and (19), leads to

$$(\rho + \delta)(\theta\delta + 1) = \eta_y(1 - \phi_y)aA\Omega^{\beta}K^{*\alpha+\beta\omega-1} + \frac{\theta\delta^2}{2}.$$

Note that $A\Omega^{\beta}(K^{*})^{\alpha+\beta\omega-1} = Y^{*}/K^{*}$. Hence, the steady-state level of output-capital ratio is determined by

$$\frac{Y^{*}}{K^{*}} = \frac{1}{a}\left[\frac{2(\rho + \delta)(\delta\theta + 1) - \theta\delta^2}{2\eta_y(1 - \phi_y)}\right]. \tag{20}$$

In view of (11), we see that the the steady-state level of Y^{*} satisfies

$$Y^{*} = A\left[\left(1 - \phi_y\right)\eta_y\left(1 - a\right)A^{1-\phi_y}Y^{*\phi_y}\right]^{\frac{\beta}{1+\gamma-(1-\phi_y)\beta}}\left(K^{*}\right)^{\alpha+\beta\omega}.$$

This means that the relation between K^{*} and Y^{*} is given by

$$Y^{*} = A^{\frac{1+\gamma-(1-\phi_y)\beta}{1+\gamma-\beta}}\left[\left(1 - \phi_y\right)\eta_y\left(1 - a\right)A^{1-\phi_y}\right]^{\frac{\beta}{1+\gamma-\beta}}\left(K^{*}\right)^{\frac{1+\gamma-(1-\phi_y)\beta}{1+\gamma-\beta}(\alpha+\beta\omega)}. \tag{21}$$

Using (20) and (21), we can express the steady-state levels of K_t and Y_t in terms of the parameters involved in the model. When K^{*} is expressed as a function of the parameters, from (10) the steady-state level of hours worked is determined by $N^{*} = \Omega\left(K^{*}\right)^{\beta}$.

On the other hand, given (18), the steady-state levels of λ_t and C_t are not determined by the conditions $\dot{B}_t = \dot{v}_t = \dot{\lambda}_t = \dot{K}_t = 0$. As usual in the standard small-open economy model with free capital mobility, the steady state levels of λ_t and C_t are pinned down by use of the intertemporal budget constraint for the household under given initial values of K_0 and B_0. Note that the condition $\acute{B}_t = 0$ means that

$$\eta_y A \Omega^\beta K^{*\alpha+\mu\omega} + \eta_b R B^* = \left[\delta + \frac{\delta^2\theta}{2}\right] K^* + C^*, \tag{22}$$

where $C^* = \frac{1}{\lambda^*} + \frac{N^{*1+\gamma}}{1+\gamma}$. Since the magnitude of C^* depends on the initial levels of K_t and B_t, the steady-state level of asset holding, B^* also depends on the initial conditions.

3.3 Taxation Rules and Equilibrium (In)determinacy

We linearize the dynamics system consisting of $(17a)$, $(17b)$, $(17c)$ and $(17d)$ at the steady-state equilibrium. The coefficient matrix evaluated at the steady state is given by

$$J = \begin{bmatrix} \rho & -\left(\frac{1}{\theta}+\delta\right)K^* & S & \frac{1}{(\lambda^*)^2} \\ -\frac{\phi_b \rho v^*}{B^*} & \rho & T & 0 \\ 0 & \frac{K^*}{\theta} & 0 & 0 \\ \phi_b \frac{\rho}{B^*}\lambda^* & 0 & 0 & 0 \end{bmatrix}, \tag{23}$$

where

$$\begin{aligned} S &= \left(\frac{\partial \dot{B}_t}{\partial K_t}\right)^* = \eta_y\left(1-\phi_y\right)(\alpha+\beta\omega)\frac{Y^*}{K^*} - \delta\left(1+\frac{\theta\delta}{2}\right) - \omega\Omega^{1+\gamma}\left(K^*\right)^{\omega(1+\gamma)-1}, \\ T &= \left(\frac{\partial \dot{v}_t}{\partial K_t}\right)^* = -a\left(1-\phi_y\right)\eta_y\left[\left(1-\phi_y\right)(\alpha+\beta\omega)-1\right]A\Omega^\beta\left(K^*\right)^{(\alpha+\beta\omega)-2}. \end{aligned}$$

Note that T has a positive value, under (14).

Let the eigenvalues of J be μ_i $(i = 1, 2, 3, 4)$. Then we find the following:

$$\mu_1 + \mu_2 + \mu_3 + \mu_4 = \text{trace } J = \rho + \delta > 0, \tag{24}$$

$$\mu_1\mu_2\mu_3\mu_4 = \det J = \phi_b \frac{\rho K^* T}{B^* \theta \lambda^*}. \tag{25}$$

Since $T > 0$, if the tax schedule on the interest income is progressive $\left(0 < \phi_y < 1\right)$, then $\det J > 0$, meaning that either J has two stable roots or it has no stable root. Since the dynamic system involves two jump variables, v_r and λ_r, the former shows that equilibrium determinacy holds, whereas the latter means that there is no equilibrium path converging to the steady-state equilibrium.

On the other hand, if the tax scheme on the interest income is regressive $\left(\underline{\phi_b} < \phi_b < 0\right)$, then $\det J < 0$, and, hence, J has either one or three stable roots. If J has one stable root, the economy exhibits a diverging behavior unless it stays in the steady-state equilibrium at the outset. If J has three stable roots, then equilibrium intermediacy emerges at least around the steady state.

Since we cannot derive the exact analytical conditions that reveal the sign of each eigenvalue of J, we examine numerical examples. In so doing, we set the baseline parameter values (except for ϕ_b) in the following way:

$$\begin{aligned}\rho &= 0.02, \quad A = 1, \quad R = 0.03, .\delta = 0.1, \quad \theta = 1, \quad a = 0.35, \quad \alpha = 0.4, \quad \beta = 0.8, \\ \phi_y &= 0.3, \quad \eta_y = \eta_b = 0.7, \quad \gamma = 0.5.\end{aligned}$$

In the above, the magnitudes of ρ, δ, a and γ are conventional ones. To satisfy (14), we assume that there is a mild degree of externalities in aggregate production by setting $\alpha + \beta = 1.2$. Note that under our specifications, $\omega = 0.2978$ in (11) so $\alpha + \beta\omega = 0.638\,2$. Additionally, (20) yields $Y^*/K^* = 0.647$, which is not an unrealstic magnitude at least for the US economy. Then we can derive the steady-state values of K_t, Y_t, N_t and v_t as follows:

$$K^* = 0.9878, \quad Y^* = 0.5394, \quad N^* = 0.1959, \quad v^* = 1.1$$

We also find that $\Omega = 0.348$, $S = 0.1971$ and $T = 0.0346$. Since the steady-state levels of λ_t, C_t and B_t depend on the initial conditions, we assume that the initial levels of K_0 and B_0 are set to satisfy $C^* = 0.7Y^* = 0.376$, meaning that the income share of the private consumption is 0.7. Given this assumption, we find that and $B^* = 3.171$ and it holds that $\frac{1}{\lambda^*} = C^* - \frac{(N^*)^{1+\gamma}}{1+\gamma} = 0.316$.

We first assume that the taxation on the interest income is progressive by setting $\phi_b = 0.3$. Then we evaluate J based on the numerical values derived so far. We find that J has two positive and two negative real eigenvalues[5]. We change ϕ_b between 0.1 and 0.4 to see that the number of stable root remains the same[6]. Therefore, if a progressive taxation rule is applied to the interest income, the small open economy tends to hold equilibrium determinacy around the steady-state equilibrium.

Next, we set $\phi_b = -0.3$, that is, the tax schedule on the interest income is regressive. In this case, we find that J has one positive and one negative real eigenvalue. In addition J also has conjugate complex eigenvalues with negative real parts[7]. Consequently, under a regressive tax schedule on the interest income, the economy exhibits local indeterminacy of equilibrium. We change ϕ_b between -0.1 and -0.4 and obtain the same outcome. However, we also find that if the taxation on the interest income is too regressive, for example $\phi_b = -0.6$, then J has one positive and one negative eigenvalue, together with conjugate complex eigenvalues with positive real parts. In this case, the small open autonomy shows a diverging behavior, unless the economy stays at the steady state at the outset.

The intuition behind the fact that regressive taxation on the interest income may cause equilibrium indeterminacy is the following. Suppose that the small open economy stays at the steady state in the initial period. Suppose further that a positive sunspot shock raises the household's expected permanent income and the household increases consumption. This leads to a negative current account, so the net asset position of the household, B_t, starts declining. Since the tax scheme on the interest income is regressive, a lower B_t decreases the marginal after-tax interest income, $\eta_b (1 - \phi_b) \left(\frac{B^*}{Bt}\right)^{\phi_b} R$,

[5] In this specific example, the eigenvalues are:$0.252\,49,\ 0.04.\,823\,,\ -0.008\,923\,,\ -0.159\,83$.

[6] We also adjust η_b to hold $(1 - \phi_b)\,\eta b R = \rho$.

[7] Specifically, the eigenvaues are:

$$0.241\,07,\ -0.02\,865\,1,\ -0.04.\,910\,1 + 0.010\,19i,\ -0.04\,910\,1 - 0.010\,19i.$$

and thus, from $(17d)$, the utility price of the financial asset, λ_t, starts rising. Hence, consumption, C_t, decreases, by which the level of B_t will return to the original steady state position. Such a self-stabilizing behavior of the economy allows the presence of sunspot equilibria.

4 Alternative Taxation Rules

In this section, we examine two alternative taxation rules in order to confirm that the dynamic behavior of the small open economy is sensitive to the tax schedule adopted by the fiscal authority.

4.1 Linear Tax on the Interest Income

If the fiscal authority applies a linear taxation scheme on the interest income from holding the financial asset, we set $\phi_b = 0$ and $\eta_b = 1 - \tau_b$, where $\tau_b \in (0, 1)$ denotes a flat rate of tax on the interest income. In this case, we obtain the following dynamic system:

$$\begin{aligned} \dot{B}_t &= \eta_y \left(\frac{Y^*}{A\Omega^\beta K_t^{\alpha+\mu\omega}} \right)^{\phi_y} A\Omega^\beta K_t^{\alpha+\mu\omega} + (1-\tau_b) RB_t \\ &\quad - \left[\frac{1}{\theta}(v_t - 1) + \frac{1}{2\theta}(v_t - 1)^2 \right] K_t - \frac{1}{\lambda_t} - \frac{(\Omega K_t^\omega)^{1+\gamma}}{1+\gamma}. \end{aligned} \tag{26a}$$

$$\begin{aligned} \dot{v}_t &= \delta v_t - \left[(1-\phi_y)\eta_y \left(\frac{Y^*}{A\Omega^\beta K_t^{\alpha+\mu\omega}} \right)^{\phi_y} aA\Omega^\beta K_t^{\alpha+\beta\omega-1} + \frac{1}{2\theta}(v_t - 1)^2 \right] \\ &\quad + (1-\tau_b) Rv_t \end{aligned} \tag{26b}$$

$$\dot{K}_t = K_t \left[\frac{1}{\theta}(v_t - 1) - \delta \right], \tag{26c}$$

$$\dot{\lambda}_t = \lambda_t \left[\rho - (1-\tau_b) R \right]. \tag{26d}$$

As usual, we should assume that $(1-\tau_b)\, R = \rho$, so that λ_t stays constant overtime.

It is to be noted that (26b) and (26c) constitute a complete dynamic system with respect to K_t and v_t. Linearizing (26b) and (26c) at the steady state in which $\dot{v}_t = \dot{K}_t = 0$ holds. we obtain the following coefficient matrix:

$$M = \begin{bmatrix} \rho & [(\alpha+\beta\omega)\, 1-\phi_y)-1]\Psi \\ \frac{K^*}{\theta} & 0 \end{bmatrix},$$

where $\Psi \equiv -(1-\phi_y)\eta_y a A\Omega^{\beta}(K^*)^{\alpha+\beta\omega-2}$ is a negative constant. Hence, we see that

$$\text{sign } \det M = \text{sign } \left[(\alpha+\beta\omega)\left(1-\phi_y\right)-1\right].$$

Under our assumption the right hand of the above has a negative sign, so that (26b) and (26c) establish a local saddle point stability. If we denote the stable arms on the $K-v$ plane as $v_t = \xi\left(K_t\right)$, we can confirm that $\xi'\left(v_t\right) < 0$. Thus, the dynamic system is reduced to the following:

$$\begin{aligned} \dot{B}_t \; &= \; \eta_y \left(\frac{Y^*}{A\Omega^{\beta}K_t^{\alpha+\mu\omega}}\right)^{\phi_y} A\Omega^{\beta}K_t^{\alpha+\mu\omega} + (1-\tau_b)\, RB_t \\ &\quad - \left[\frac{1}{\theta}\left(\xi\left(K_t\right)-1\right) + \frac{1}{2\theta}\left(\xi\left(K_t\right)-1\right)^2\right] K_t - \frac{1}{\bar{\lambda}} - \frac{\left(\Omega K_t^{\omega}\right)^{1+\gamma}}{1+\gamma} \end{aligned} \quad (27a)$$

$$\dot{K}_t = K_t\left[\frac{1}{\theta}\left(\xi\left(K_t\right)-1\right)-\delta\right] \quad (27b)$$

Equation (27b) indicates that K_t exhibits a self-stabilizing behavior, and it converges to its steady-state level, K^*. When $K_t = K^*$ (so that $Y_t = Y^*$), (27a) is written as

$$\dot{B}_t^* = \eta_y Y^* + \rho B_t^* - \delta\left(1+\frac{\theta\delta}{2}\right)K^* - \frac{1}{\bar{\lambda}} - \frac{\left(N^*\right)^{1+\gamma}}{1+\gamma}, \quad (28)$$

where B_t^* denotes the level of B_t when $K_t = K^*$ and $Y_t = Y^*$. In this case, the intertemporal budget constraint for the household is

$$B_0^* + \frac{1}{\rho}\eta_y Y^* = \frac{1}{\rho}\left[\frac{1}{\bar{\lambda}} + \frac{(N^*)^{1+\gamma}}{1+\gamma} + \delta\left(1 + \frac{\delta\theta}{2}\right)K^*\right]. \tag{29}$$

If $\bar{\lambda}$ is selected to satisfy (29) , then(28) becomes

$$\dot{B}_t^* = \rho\left(B_t^* - B_0^*\right),$$

which means that $\dot{B}_0 = 0$. As a result, an appropriate choice of $\bar{\lambda}$ fixes B_t at a steady-state level when K_t converges to K^*. Therefore, as far as $(\alpha + \beta\omega)\left(1 - \phi_y\right) < 1$ is fulfilled, the small open economy holds a unique stable equilibrium path that converges to the steady state. However, either if the external effects are strong (i,e. α and β are large) or if the degree of regressiveness of taxation is high enough, then it holds that $(\alpha + \beta\omega)\left(1 - \phi_y\right) > 1$. As shown above, in this case, the economy diverges from the-steady state equilibrium. Such an unstable behavior of the economy can be avoided if the fiscal authority adopts progressive taxation that satisfies $(\alpha + \beta\omega)\left(1 - \phi_y\right) < 1$.

To sum up, if the fiscal authority fixes the tax rate of the interest income and applies nonlinear taxation to the factor income, then the small open economy will not exhibit equilibrium indeterminacy. However, the economy would be totally unstable under the presence of strong external effects. In this case, a progressive tax schedule may recover saddle stability of the economy. Hence, the progressive taxation contributes to stabilizing the economy in the sense that is different from eliminating sunspot-driven fluctuations emphasized by Guo and Lansing (1998).

4.2 Tax Schedules without the Reference Income

So far, we have used the Guo-Lansing formulation of nonlinear taxation. Their formula is convenient for model manipulation, because the average tax rate is exogenously specified in the steady-state equilibrium. On the other hand, we should assume that

$(1-\phi_b)\,\eta_b = R$ to define a feasible steady- state equilibrium. This restriction yields a 'zero-root problem', which makes the steady-state levels of foreign bonds and consumption depend on the initial conditions.[8] To confirm that our main findings have nothing to do with the zero-root problem, we examine an alternative formulation of nonlinear income taxes under which the steady-state level of foreign bonds is uniquely determined. We now assume that the fiscal authority adjusts the average tax rates according to the following rules:

$$\tau_{y,t} = 1 - \eta_y Y_t^{-\phi_y}, \qquad \phi_y < 1, \tag{30}$$

$$\tau_{b,t} = 1 - \eta_b \left(RB_t\right)^{-\phi_b}, \qquad \phi_b < 1. \tag{31}$$

In this case, the tax rules are progressive if $0 < \phi_y,\ \phi_b < 1$, while they are regressive if $\phi_y,\ \phi_b < 0$.[9] The after-tax incomes of the household are $(1-\tau_y)\,(r_t K_t + w_t N_t) = \eta_y\,(r_t K_t + w_t N_t)^{1-\phi_y}$ and $(1-\tau_b)\,(RB_t)^{1-\phi_y}$, and thus the flow budget constraint for the household is

$$\dot{B}_t = \eta_y\,(r_t K_t + w_t N_t)^{1-\phi y} + \eta_b\,(RB_t)^{1-\phi_b} - \left[\frac{I_t}{K_t} + \frac{\theta}{2}\left(\frac{I_t}{K_t}\right)\right] K_t - C_t. \tag{32}$$

The optimization conditions for the household's problem involve the following:

$$\left(C_t - \frac{N_t^{1+\gamma}}{1+\gamma}\right)^{-1} = \lambda_t, \tag{33a}$$

$$\left(C_t - \frac{N_t^{1+\gamma}}{1+\gamma}\right)^{-1} N_t^{\gamma} = \lambda_t \eta_y \left(1-\phi_y\right) (Y_t)^{-\phi_y} w_t, \tag{33b}$$

$$q_t = \lambda_t \left[1 + \theta \frac{I_t}{K_t}\right], \tag{33c}$$

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[8] Given $\eta_b\,(1-\phi_b)\,R = \rho$, one of the eigenvalues of the coefficient matrix of the linearized dynamic system is zero. See Schmitt-Grohé and Uribe (2003).

[9] The amount of tax on the factor income is $T_{y,t} = \tau_{y,t} Y_t = Y_t - \eta_y Y_t^{1-\phi_y}$, so that the marginal tax payment is $dT_{y,t}/dY_t = 1 - \left(1-\phi_y\right)\eta_y Y_t^{-\phi_y}$, which is larger (smaller) than the average tax, $\tau_{y,t}$. Thus taxation is progressive (regressive) if $0 < \phi_y < 1$ $\left(\phi_y < 0\right)$. The same result holds for ϕ_b.

$$\dot{q}_t = (\rho + \delta) q_t - \lambda_t \left[\eta_y (1 - \phi_y) (Y_t)^{-\phi_y} r_t + \frac{\theta}{2} \left(\frac{I_t}{K_t} \right)^2 \right], \tag{33d}$$

$$\dot{\lambda}_t = \lambda_t \left[\rho - (1 - \phi_b) \eta_b (RB_t)^{-\phi_b} R \right]. \tag{33e}$$

Since the before-tax real wage satisfies $w_t = (1 - a) \frac{Y_t}{N_t}$, conditions $(33a)$ and $(33b)$ yield

$$N_t = \left[\left(1 - \phi_y \right) \eta_y \right]^{\frac{1}{1+\gamma}} Y_t^{\frac{1-\phi_y}{1+\gamma}}$$

Substituting the above into the production function (1) and solving it with respect to Y_t, we obtain the reduced form of the aggregate production function in such a way that

$$Y_t = \Gamma K_t^{\chi}, \tag{34}$$

where

$$\Gamma = \left\{ A \left[\left(1 - \phi_y \right) \eta_y \right]^{\frac{\beta}{1+\gamma}} \right\}^{\frac{1+\gamma}{1+\gamma-(1-\phi_y)\beta}}, \quad \chi = \frac{\alpha (1 + \gamma)}{1 + \gamma - \left(1 - \phi_y \right) \beta}.$$

Using (34), we find that a complete dynamic system is given by the following set of differential equations:

$$\begin{aligned} \dot{B}_t &= \eta_y \left(\Gamma K_t^{\chi} \right)^{1-\phi_y} + \eta_b \left(RB_t \right)^{1-\phi_b} - \left[\frac{1}{\theta} (v_t - 1) + \frac{1}{2\theta} (v_t - 1)^2 \right] K_t, \\ &\quad - \frac{1}{\lambda_t} - \frac{(N_t)^{1+\gamma}}{1 + \gamma}, \end{aligned} \tag{35a}$$

$$\begin{aligned} \dot{v}_t &= \delta v_t - \left[(1 - \phi_y) \eta_y \left(\Gamma K_t^{\chi} \right)^{-\phi_y} a \Gamma K_t^{\chi - 1} + \frac{1}{2\theta} (v_t - 1)^2 \right] \\ &\quad + (1 - \phi_b) \eta_b (RB_t)^{-\phi_b} R v_t, \end{aligned} \tag{35b}$$

$$\dot{K}_t = K_t \left[\frac{1}{\theta} (v_t - 1) - \delta \right], \tag{35c}$$

$$\dot{\lambda}_t = \lambda_t \left[\rho - (1 - \phi_b) \eta_b (RB_t)^{-\phi_b} R \right]. \tag{35d}$$

A distinctive feature of this model is that the steady-state levels of B_t, λ_t and C_t are independent of the initial conditions on the non-jump variables.[10] First, $\dot{\lambda}_t = 0$ in $(35d)$ gives the unique steady-state level of B_t as

$$B^* = \left[\frac{\rho}{\left(1-\phi_y\right)\eta_b R^{1-\phi_y}}\right]^{-\frac{1}{\phi_y}}.$$

Then, conditions $\dot{v}_t = \dot{K}_t = 0$ in $(35a)$ and $(35c)$ determine the unique levels of K^* and v^*. Finally, the steady state level of λ_t is determined by $\dot{B}_t = 0$ condition in $(35a)$, which gives $C^* = \frac{1}{\lambda^*} + \frac{(N^*)^{1+\gamma}}{1+\gamma}$, where $N^* = N(K^*)$ depends on the level of K^*.

Concerning the equilibrium dynamics of the model, we obtain the same outcome as that of the model with the reference income. We find that the coefficient matrix evaluated at the steady state may have two stable roots if the taxation scheme on income is progressive, that is, $0 < \phi_y,\ \phi_b < 1$. In this case, there exists a unique path converging to the steady state. If the taxation scheme on the interest income, RB_t is regressive $(\phi_b < 1)$, then the coefficient matrix may have three stable roots, which leads to equilibrium indeterminacy. In this policy regime, although we cannot obtain the analytical solution of K^*, we can find plausible numerical examples that establish equilibrium (in)determinacy. As a consequence, other than the fact that the steady-state levels of net asset positions and consumption do not depend on the initial values of B_t and K_t, the alternative tax formula provides us with essentially the same outcome as those obtained in the model with the reference income.

5 Conclusion

This paper addresses the stabilization effect of income taxation rules in a small open economy with free capital mobility. We have shown that in our small open economy, if

[10]Schmitt-Grohé. and Uribe (2003) presented some modifications of the model in which the zero-root problem is avoided. They proposed debt elastic world interest rates, endogenous time preference rates, portfolio adjustment costs, etc. Our formulation of nonlinear taxation on the interest income is an alternative idea to resolve the zero root problem. See also Lubick (2007) for this issue.

the tax schedule on the interest income from financial assets is progressive, equilibrium indeterminacy will not emerge, even though the taxation rule on the factor income is mildly regressive. However, if there is a strong degree of external increasing returns or the degree of regressiveness of factor income taxes is high enough, then the economy displays a diverging behavior from the steady state. Therefore, progressive tax rules contribute to stabilizing the economy in the sense that it may recover the saddle stability of an otherwise diverged equilibrium path. We have also confirmed that if the tax schedule on the interest income is regressive, then equilibrium indeterminacy may emerge. In this case, the progressive tax rule on the interest income acts as a built-in stabilizer in the sense that it eliminates sunspot-driven fluctuations. Hence, progressive income taxation is a useful automatic stabilizer in both senses. As in the closed economy counterpart, those outcomes demonstrate that income taxation rules would play a relevant role for the stability of small open economies.[11]

[11] Chen, Hu and Mino (2018) examine the stabilization effect of nonlinear income taxation in a wide class of small open economy models.

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