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Abstract

The stabilization effect of nonlinear income taxation is addressed in the standard model of small open economy. It is shown that if income taxation schedule is progressive, the small open economy tends hold saddle-point stability. On the other hand, if taxation on the interest income is regressive, then the small open economy may exhibit sunspot-driven fluctuations or it displays a diverging behavior.

Keywords: Taxation Rule, Indeterminacy, Small Open Economy

JEL Classification: E62, O41

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1 Introduction

In a well-cited article, Guo and Lansing (1998) reveal that progressive income taxation may contribute to stabilizing closed economy models with production externalities. Those authors introduce a nonlinear taxation rule into the model of Behhabib and Farmer (1994), and show that the progressive tax schedule narrows the parameter space in which equilibrium indeterminacy emerges. Many authors have re-examined Guo and Lansing's finding in alternative settings[1]. The foregoing studies, however, have focused on closed economy models, and the stabilization effect of taxation rule in open economies has not been explored well[2]. In this paper, we introduce nonlinear taxation schemes into an otherwise standard, one-sector model of small open economy. We show that under progressive taxation, equilibrium indeterminacy will not hold. In contrast, if a regressive taxation rule is applied to the interest income of financial asset, then the small open economy exhibits sunspot-driven fluctuations or it displays a diverging behavior.

2 Model

Consider a small open economy in which the representative firm produces a homogeneous good according to the production function such as

$$y_t = Ak_t^a \bar{k}_t^{\varepsilon}, \qquad 0 < a < 1, \quad \varepsilon > 0,$$

where y_t and k_t respectively denote output and capital per labor, and $\bar{k}_t$ is the average capital labor ratio. Here, $\bar{k}_t^{\varepsilon}$ represents external effects generated by the social level of capital in the sense of Romer (1986). We normalize the total labor to one, so that in equilibrium it holds that $\bar{k}_t = k_t$. Hence, the social production function is $y_t = Ak_t^{\alpha}$, where $\alpha = a + \varepsilon$. We assume that $\alpha = a + \varepsilon < 1$, meaning that the social level of capital exhibits diminishing returns. The factor markets are competitive, so that the rate of return to private capital, r_t, and the real

[1] A sample includes Dromel and Pintus (2008), Guo and Harrison (2015), Chen and Guo (2016), and Chen, Hus and Hus (2018).

[2] A recent contribution by Huang, Meng and Xue (2017) examine stabilization effect of the balanced-budget rule in a small open economy. Chapter 6 of Mino (2017) presents an overview of equilibrium indeterminacy in open economy models.

wage, w_t, are respectively given by

$$r_t = aAk_t^{\alpha-1}, \quad w_t = (1-a)\, Ak_t^{\alpha}. \tag{1}$$

There is a continuum of identical households with a unit mass. The representative household freely lends to or borrows from the foreign households. The household's optimization problem is as follows:

$$\max \int_0^{\infty} e^{-\rho t} \log c_t dt$$

subject to

$$\dot{b}_t = (1-\tau_{y,t})(r_t k_t + w_t) + (1-\tau_{b,t})\, Rb_t - c_t - \left(\frac{i_t}{k_t} + \frac{\theta}{2} \left(\frac{i_t}{k_t} \right)^2 \right) k_t, \quad \theta > 0, \tag{2}$$

$$\dot{k}_t = i_t - \delta k_t, \tag{3}$$

together with given initial levels of k_0 and b_0. In the above, b_t = holding of foreign bond (net asset position) , R = a given world interest rate, i_t = investment spending on capital, δ = depreciation rate of capital, and $(\theta/2)\,(i_t/k_t)_t^2\, k_t$ represents adjustment costs of capital. In addition, $\tau_{y,t}$ and $\tau_{b,t}$ denote the rates of tax on domestic income and interest income earned from bond holding, respectively. In this paper, we assume that the domestic households are creditors, and thus, b_t has a non-negative value.

The fiscal authority sets the rates of tax in such a way that

$$\begin{aligned} \tau_{y,t} &= 1 - \eta_y \left(r_t k_t + w_t \right)^{-\phi_y}, \quad \phi_y < 1, \quad \eta_y > 0, \\ \tau_{b,t} &= 1 - \eta_b \left(Rb_t \right)^{-\phi_b}, \quad \phi_b < 1, \quad \eta_b > 0. \end{aligned}$$

As a result, the after-tax incomes are expressed in the following manner:

$$(1-\tau_{y,t})\,(r_t k_t + w_t) = \eta_y \left(r_t k_t + w_t \right)^{1-\phi_y} \tag{4}$$

$$(1-\tau_{b,t})\, Rb_t = \eta_b R^{1-\phi_b} b_t^{1-\phi_b}. \tag{5}$$

Note that if $0 < \phi_y < 1$ and $0 < \phi_b < 1$, then the marginal tax rate exceeds the average tax rate, so that tax schedules are progressive and the after-tax marginal incomes decrease with

$r_t k_t + w_t$ and b_t. On the other hand, if $\phi_y < 0$ and $\phi_b < 0$, then the marginal rates of tax are smaller than the average rates of tax, implying that tax schedules are regressive[3].

Given above assumptions, the Hamiltonian function for the household's problem is given by

$$\begin{aligned} H_t &= \log c_t + \lambda_t \left[\eta_y (r_t k_t + w_t)^{1-\phi_y} + \eta_b R^{1-\phi_b} b_t^{1-\phi_b} - c_t - \left(\frac{i_t}{k_t} + \frac{\theta}{2} \left(\frac{i_t}{k_t} \right)^2 \right) k_t \right] \\ & \quad + q_t \left(i_t - \delta k_t \right), \end{aligned}$$

where λ_t and q_t denote the utility prices of capital and financial asset, respectively. The necessary conditions for an optimum include the following:

$$c_t = \frac{1}{\lambda_t}, \tag{6}$$

$$q_t = \lambda_t \left(1 + \theta \frac{i_t}{k_t} \right), \tag{7}$$

$$\dot{\lambda}_t = \lambda_t \left[\rho - (1 - \phi_b) \eta_b R^{1-\phi_b} b_t^{-\phi_b} \right], \tag{8}$$

$$\dot{q}_t = (\rho + \delta) q_t - \lambda_t \left[\left(1 - \phi_y\right) \eta_y \left(r_t k_t + w_t\right)^{-\phi_y} r_t + \frac{\theta}{2} \left(\frac{i_t}{k_t} \right)^2 \right], \tag{9}$$

together with the transversality conditions: $\lim_{t \to \infty} e^{-\rho t} \lambda_t b_t = 0$ and $\lim_{t \to \infty} e^{-\rho t} q_t k_t = 0$.

It is to be noted that if a flat rate of tax is applies to the interest income, that is, $\phi_b = 0$, then (8) becomes $\dot{\lambda}_t = \lambda_t (\rho - \eta_b R)$. Therefore, we should assume that $\rho = \eta_b R$ in order to define the steady state equilibrium. In this standard case, the steady state level of b_t depends on its initial level of b_0. Such a 'zero-root problem' (Schmitt-Grohé and Uribe 2003) can be avoided in our setting[4].

[3]Noting that $r_t k_t + w_t = y_t$, we see that the government's tax revenue on domestic income is $\tau_{y,t} y_t = y_t - \eta_y y_t^{1-\phi_y}$ and the marginal tax revenue is $\frac{d}{dy_t} (\tau_{y,t} y_t) = 1 - \eta_y \left(1 - \phi_y\right) y_t^{-\phi_y}$. Thus, if $0 < \phi_y < 1$ $\left(\text{resp. } \phi_y < 0\right)$, then the marginal tax revenue is higher (resp. lower) than the average tax revenue $\left(= \tau_{y,t} = 1 - \eta_y y_t^{-\phi_y}\right)$, so that taxation scheme is progressive (resp. regressive). The same property holds for the taxation on the interest income.

[4]In discrete time models, the non-stationarity of small open economy models with free capital mobility is called the unit-root problem. See also Lubick (2007).

3 Tax Schedule and Equilibrium (In)determinacy

Define $v_t = q_t/\lambda_t$. Then, using (1), (2), (3), (6), (7), (8), (9) and $r_t k_t + w_t = y_t = Ak_t^{\alpha}$, we can derive a complete dynamic system as follows:

$$\dot{b}_t = \eta_b R^{1-\phi_b} b_t^{1-\phi_b} + \eta_y A^{1-\phi_y} k_t^{\alpha\left(1-\phi_y\right)} - c_t - \left[\frac{1}{\theta}\left(v_t - 1\right) + \frac{1}{2\theta}\left(v_t - 1\right)^2\right] k_t, \tag{10}$$

$$\dot{v}_t = \left[\delta + \left(1-\phi_b\right)\eta_b R^{1-\phi_b} b_t^{-\phi_b}\right] v_t - \frac{1}{2\theta}\left(v_t - 1\right)^2 - a\left(1-\phi_y\right)\eta_y A^{1-\phi_y} k_t^{\alpha\left(1-\phi_y\right)-1}, \tag{11}$$

$$\dot{k}_t = \left[\frac{1}{\theta}\left(v_t - 1\right) - \delta\right] k_t, \tag{12}$$

$$\dot{c}_t = c_t\left[\left(1-\phi_b\right)\eta_b R^{1-\phi_b} b_t^{-\phi_b} - \rho\right]. \tag{13}$$

In the steady state, it holds that $\dot{b}_t = \dot{v}_t = k_t = \dot{c}_t = 0$. We denote the steady state values of endogenous variables by b^*, v^*, k^* and c^*. Those steady state values fulfill the following:

$$\eta_y A^{1-\phi_y} k^{*\alpha\left(1-\phi_y\right)} + \eta_b R^{1-\phi_b} b^{*1-\phi_b} = c^* + \left[\frac{1}{\theta}\left(v^* - 1\right) + \frac{1}{2\theta}\left(v^* - 1\right)^2\right] k^*, \tag{14}$$

$$\left[\delta + \left(1-\phi_b\right)\eta_b R^{1-\phi_b} b^{*-\phi_b}\right] v^* = \frac{1}{2\theta}\left(v^* - 1\right)^2 + a\left(1-\phi_y\right)\eta_y A^{1-\phi_y} k^{*\alpha(1-\phi_y)-1}, \tag{15}$$

$$\frac{1}{\theta}\left(v^* - 1\right) = \delta, \tag{16}$$

$$\left(1-\phi_b\right)\eta_b R^{1-\phi_b}\left(b^*\right)^{-\phi_b} = \rho. \tag{17}$$

Conditions (16) and (17) respectively give the unique steady state levels of v_t and b_t. Then (16) determines a unique steady state level of k_t. Finally, the steady state level of c_t is given by (14). As mentioned above, the steady state value of b_t is independent of its initial value.

We conduct linear approximation of the dynamic system at the steady state equilibrium.

Then we find that the coefficient matrix of the linearized system is written as follows:

$$J = \begin{bmatrix} \rho & -\left(\frac{1}{\theta}+\delta\right)k^* & \alpha\left(1-\phi_y\right)\eta_y A^{1-\phi_y}(k^*)^{\alpha\left(1-\phi_y\right)-1} - \delta\left(1+\frac{\theta\delta}{2}\right) & -1 \\ -\phi_b \frac{\rho}{b} v^* & \rho & -a\left(1-\phi_y\right)\left[\alpha\left(1-\phi_y\right)-1\right]\eta_y A^{1-\phi_y}(k^*)^{\alpha\left(1-\phi_y\right)-2} & 0 \\ 0 & \frac{k^*}{\theta} & 0 & 0 \\ -\phi_b \frac{\rho}{b} c^* & 0 & 0 & 0 \end{bmatrix}.$$

It is straightforward to show that the determinant of J is given by

$$\det J = -\phi_b \frac{\rho}{b} c^* \frac{k^*}{\theta} a\left(1-\phi_y\right)\left[\alpha\left(1-\phi_y\right)-1\right]\eta_y A^{1-\phi_y}\left(k^*\right)^{\alpha\left(1-\phi_y\right)-2}.$$

Since $\phi_y < 1$, the above expression leads to

$$\text{sign } \det J = \text{sign } \phi_b\left[1-\alpha\left(1-\phi_y\right)\right]. \tag{18}$$

Therefore, if taxations on the domestic income and the interest income are progressive, i.e., $0 < \phi_y,\ \phi_b < 1,$ then $\det J > 0$. Alternatively, if tax schemes are regressive (i.e., $\phi_y < 0$ and $\phi_b < 0$) and if $\alpha\left(1-\phi_y\right) > 1$, then it also holds that $\det J > 0$. Keeping in mind that the product and the sum of eigenvalues are respectively equal $\det J$ and the trace of J, in those alternative cases, the number of stable root is either zero or two. Since the dynamic system involves two jump variables, c_t and v_t, if $\det J > 0,$ indeterminacy will not arise. On the other hand, either if $\phi_b < 0$ and $1 > \alpha\left(1-\phi_y\right)$ or if $\phi_b > 0$ and $1 < \alpha\left(1-\phi_y\right)$, then $\det J < 0$, meaning that the number of stable roots is either one or three. In the former case, there is no converging path towards the steady state equilibrium, while the latter case means that equilibrium indeterminacy arises.

To conduct a further investigation on the stability conditions, we inspect some numerical examples. We set baseline parameters in the following manner:

$$\rho = 0.02, \quad R = 0.03, \quad \delta = 0.1, \quad \theta = 1, \quad A = 1.0, \quad a = 0.3, \quad \alpha = a + \varepsilon = 0.4.$$

The magnitudes of the parameters shown above are standard ones. We first assume that taxation is progressive and set the degree of progressiveness of taxation as $\phi_b = \phi_y = 0.3$. We also set $\eta_y = 1.0$ and $\eta_b = 1.0$ to make the steady state levels of average tax, $\tau_v^* = 1 - \eta_y (y^*)^{-\phi_y}$ and $\tau_b^* = 1 - \eta_b (Rb^*)^{-\phi_b}$ feasible ones. Given our specification, we find:

$$b^* = 39.22, \; v^* = 1.1, \quad k^* = 2.991, \quad c^* = 0.953, \quad \tau_y^* = 0.123, \quad \tau_b^* = 0.667.$$

Evaluating J based on the parameter values and the steady state values of endogenous variables listed above, we find that J has four real eigenvalues and two of them are negative[5]. Hence, in the base line case, there is a unique converting path towards the steady state equilibrium, so that determinacy is established. We change ϕ_y and ϕ_b within the rage of $[0.1, 0.5]$ and adjust η_y and η_b to hold $\tau_y^*, \; \tau_b^* \in (0,1)$. We see that there always exists a two-dimensional stable manifold around the steady state. Therefore, it is safe to state that under progressive income taxation, the small open economy is generally free from sunspot-driven business cycles.

Next, let us consider the case in which the tax schemes are slightly regressive. In so doing, we set $\phi_y = \phi_b = -0.1, \; \eta_y = 0.8$, and $\eta_b = 0.7$. Here, we we obtain:

$$b^* = 0.228, \quad v^* = 1.1, \quad k^* = 2.941, \quad c^* = 0.595, \quad \tau_y^* = 0.164, \quad \tau_b^* = 0.576.$$

In this case, we find that J has one positive and one negative real eigenvalues. In addition, J has conjugate complex eigenvalues with negative real parts[6]. Hence, there are three stable roots, which gives rise to local indeterminacy. We change ϕ_y and ϕ_b within the range of $[-0.4, \; -0.1]$ and adjust η_y and η_b to hold $\tau_y^*, \tau_b^* \in (0,1)$, which leads to the same outcome. However, we also find that if $\phi_b < 0$ but $\phi_y > 0$ $\left(\text{for example, } \phi_b = -0.1 \text{ and } \phi_y = 0.3\right)$, then J generally has three unstable eigenvalues, meaning that there is no converging path leading to the steady state.

[5] The eigenvalues of J in the baseline case of progressive taxation are: $0.324\,26$, 0.0254, $-0.05\,4$, and $-0.304\,25$

[6] In the case of regressive tax on the intret income , the eigenbalues of J are: 0.00916, $-0.01039 + 053626i$, $-0.01039 - 053626i$, and $-0.0.03049$.

4 Conclusion

This paper addresses the stabilization effect of nonlinear income taxation in a small open economy with free capital mobility. We show that as confirmed by Guo and Lansing (1998) in the context of a closed economy model, progressive taxation contributes to establishing equilibrium determinacy in our small open economy. In contrast, when a regressive tax scheme is applied to the interest income, the small open economy may hold sunspot-driven fluctuations. Otherwise, the economy exhibits a diverging behavior. Therefore, a regressive tax schedule on the interest income makes the small open economy unstable in both senses[7].

[7] Chen, Hu and Mino (2018) examine the stabilization effect of nonlinear income taxation in a wide class of small open economy models such as models with variable labor supply and models with endogenous growth.

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