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# Dampened expectations in the Phillips Curve: a note

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## Abstract

Dampened inflation expectations have a significant impact on the New Keynesian Phillips Curve. This dampening not only flattens the long run Phillips Curve, but it can also lead to a bias in the estimation of its short run slope. It also affects the response of a small NK model to demand shocks, and affects the optimal monetary policy: in particular, the price targeting result of the Ramsey policy is violated when there is dampening.

JEL codes: E31, E52

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## 1 Introduction

The Phillips Curve is central to macroeconomics but its shape has been questioned recently. Blanchard (2016) argues against a vertical or near-vertical long-run Phillips curve: it has allegedly become flatter, largely due to inflation expectations anchored at zero or low levels. The inflation expectation used in the Phillips Curve is a long-run expectation which is anchored around a reference point, and only adjusts partially to changes in short-run expectations. As such, the effect of short-run inflation expectations is largely dampened, and this would imply a real trade-off between output and inflation in the long run.

The Phillips Curve is often assumed to be accelerationist or near accelerationist: if current inflation increases one-to-one with short-run inflation expectations, this implies that the output gap is related to the acceleration of inflation (in the New Keynesian Phillips Curve, inflation doesn't exactly increase one-to-one with expectations, but the pass-through is close to unity, implying a near accelerationist curve). But if short-run inflation expectations are dampened, or play a dampened role, in the sense that they matter less for agents (and hence current inflation) – then the Phillips Curve is no longer accelerationist. The rate of inflation – and not only its acceleration – matters for the output gap.

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When estimating a short run Phillips Curve, ignoring this dampening leads to a reduced coefficient on output/unemployment. Since current inflation is correlated with output (or unemployment), expectations of future inflation are correlated with future (and hence current) output (unemployment). Mismeasuring the role of expectations necessarily biases the slope coefficients.

Explanations for dampened expectations fall in two categories. One is behavioral: inflation expectations are either anchored around a reference (Blanchard, 2016), or agents are myopic about the future (Gabaix, 2018). The other relies on product creative destruction in the price Phillips Curve (see Bilbiie et al. (2008, 2014)) or job turnover in the wage Phillips Curve (Snower and Tesfaselassie (2017), Lepetit (2018)). These papers have looked at the long term consequences: the long-run Phillips Curve and the optimal inflation target. On the contrary, my paper focuses on the short run consequences of this dampening.

This paper also belongs to the stream of literature that reassesses the New Keynesian model in light of the Great Recession and the Zero Lower Bound. While this paper introduces an extra discount factor in the Phillips curve, other papers have introduced a discount factor in the Euler equation instead, to explain the *forward guidance puzzle* (see McKay et al. (2016, 2017) and Del Negro et al. (2015)). The interaction between a discounted Phillips curve and a discounted Euler equation has been partially studied by Gabaix (2018).

I first look at the interaction between dampened inflation expectations and the slope of the short run Phillips Curve. I then look at the consequence for a small New Keynesian model, in terms of the response to supply and demand shocks. Finally I revisit the Ramsey optimal stabilisation policy in such a setup.

## 2 Dampened inflation expectations

The New Keynesian Phillips Curve is typically written with inflation  $\pi_t$  and the output gap  $y_t$  (or wage inflation and cyclical unemployment):

$$\pi_t = \kappa y_t + \beta E_t[\pi_{t+1}] \tag{1}$$

In this setup  $\beta$  is the risk-less discount factor and  $\kappa$  the output coefficient. Suppose, however, that the true model features dampened inflation expectations:

$$\pi_t = \kappa y_t + \beta \delta E_t[\pi_{t+1}] \tag{2}$$

$\delta \in (0, 1)$  is the dampening factor. It can come from anchored expectations (Blanchard, 2016), a behavioral bias (Gabaix, 2018), product creative destruction (Bilbiie et al. (2014), or job turnover (Snower and Tesfaselassie, 2017)). In such cases the long run version of (2) implies a flatter long-run Phillips curve, and it is no longer vertical or nearly vertical as in the standard case:  $\bar{\pi} = \frac{\kappa}{1-\beta\delta} \bar{y}$ . Dampened inflation expectations do not directly affect  $\kappa$ , the slope of the short run PC. But it can nevertheless lead to a downward bias in its estimation.

## 2.1 Short run bias

Suppose that we are estimating eq (1) but eq (2) is the true model. Even though it is not the right model, estimating eq (1) will provide an unbiased estimate of  $\kappa$  if the output and inflation expectation coefficient are jointly estimated (of course the estimated coefficient for expectations will estimate  $\beta\delta$  not  $\beta$ ). But eq (1) is often estimated with a calibrated  $\beta$ , which doesn't account for any dampening. This leads to a biased estimation of  $\kappa$  if the true model is eq (2) and the output gap (or cyclical unemployment) is serially correlated.

For example, let us estimate a reduced form Phillips Curve featuring only the output gap and current inflation  $\pi_t = \tilde{\kappa}y_t$ , and then back out the structural parameters by relying on the auto-regressive properties of the output gap. This approach has been followed, eg, in Gali (2011).<sup>1</sup> Assume that output is serially correlated:<sup>2</sup>  $y_t = \rho_y y_{t-1} + u_t$ , with  $u_t$  a mean-zero shock. Iterate eq (2) forward:

$$\pi_t = \kappa y_t + \beta\delta E_t[\pi_{t+1}] = \kappa \sum_{k \geq 0} (\beta\delta)^k E_t y_{t+k} = \frac{\kappa}{(1 - \rho_y \beta\delta)} y_t$$

Estimating this reduced-form equation provides an estimate of  $\tilde{\kappa} = \frac{\kappa}{(1 - \rho_y \beta\delta)}$  from which  $\kappa$  can be uncovered if  $\beta$ ,  $\delta$  and  $\rho_y$  are known. But if the dampening factor is not accounted for (assuming  $\delta = 1$ ), the estimate of  $\kappa$  will be biased.

**Property 1** *If  $\kappa$  is the true output coefficient, the estimated  $\kappa^*$  is smaller*

$$\kappa^* = \frac{(1 - \beta\rho_y)}{(1 - \rho_y\beta\delta)} \kappa < \kappa$$

The dampening factor  $\delta$  affects the slope of a reduced-form, Phillips Curve displaying only current inflation and output,  $\pi_t = \tilde{\kappa}y_t$ . A non-linear estimation using the reduced form equation above and the auto-regressive properties of the output gap (or cyclical unemployment) to back out the slope of the true Phillips Curve will lead to a biased estimate if the wrong model without  $\delta$  is estimated.<sup>3</sup>

The same bias occurs if equation (1) is directly estimated and a wrong restriction is imposed for the coefficient  $\beta$ . This is the case in the empirical estimates of Gali and Gertler (1999), where they use marginal costs instead of the output gap. They estimate  $\pi_t = \lambda mc_t + \beta E\pi_{t+1}$ . The estimated coefficient of marginal costs,  $\lambda$ , depends on the assumption about the coefficient of future inflation,  $\beta$ . When this coefficient is restricted to  $\beta = 1$ , the estimated value of  $\lambda$  is smaller than when there is no restriction and  $\beta$  takes a lower value.

<sup>1</sup>Gali (2011) estimates a hybrid wage Phillips Curve with unemployment. As unemployment is assumed to be AR(2), a reduced form Phillips Curve without inflation expectations is first estimated, before backing out the structural parameters of the true hybrid PC.

<sup>2</sup>Assuming an AR(1) process allows for simple expressions for the source and amplitude of the bias. Assuming a more sophisticated process changes the expressions but not the logic.

<sup>3</sup>The same drawback would obviously apply to any joint estimation of a larger model, if inflation expectations are dampened but the estimated model doesn't account for it.

## 2.2 Demand and supply shocks

I will now examine how this modified Phillips Curve changes the response of output and inflation to shocks in a simple New Keynesian framework.

Let me look at a standard Euler equation: in log-linear form,

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1})$$

It can be combined with a Taylor rule

$$i_t = \phi_\pi \pi_t + \phi_y y_t$$

$\phi_\pi$  and  $\phi_y$  are the inflation and output coefficients. While McKay et al. (2016, 2017) or Del Negro et al. (2015) have a modified Euler equation, I do not look at this modification here because this equation becomes isomorphic to a standard Euler equation with modified parameters, once combined with a Taylor rule.<sup>4</sup> Similarly, if one looks at dampened inflation expectations in the Euler equation,<sup>5</sup> it is also isomorphic to a more standard one, once combined with a Taylor rule.<sup>6</sup> This also has limited quantitative implications, hence I can assume it away.

With my Phillips Curve, the dynamics, incorporating simple shocks, write:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(\phi_\pi \pi_t + \phi_y y_t - E_t \pi_{t+1}) + u_t$$

$$\pi_t = \beta \delta E_t \pi_{t+1} + \kappa y_t + v_t$$

The first equation is often thought as a demand equation, while the second is a supply equation.  $u_t$  is a demand shock – coming either from a shock to the natural rate of interest, or from a monetary policy shock – while  $v_t$  is a supply shock – generated either by a productivity or by a cost-push (markup) shock.

<sup>4</sup>In their modified Euler equation,  $y_t = \gamma E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1})$  with  $\gamma \in [0, 1]$  the dampening factor of the Euler equation. While it has strong implications at the Zero Lower Bound (where the Taylor rule does not apply), it is easy to see that when  $\gamma \neq 1$ , the modified equation can still be rearranged to *appear* as a standard Euler equation, without any dampening:  $y_t = E_t y_{t+1} - \frac{1}{\sigma \gamma}(\phi_\pi \pi_t + (\phi_y + (1 - \gamma)\sigma)y_t - E_t \pi_{t+1})$

Hence, when a modified Euler equation (with dampening factor  $\gamma < 1$ ) is combined with a Taylor rule, the behavior is isomorphic to one with a lower elasticity of intertemporal substitution ( $\gamma\sigma$  instead of  $\sigma$ ) and a higher output coefficient in the Taylor rule ( $\phi_y + (1 - \gamma)\sigma$ , instead of  $\phi_y$ ). Once this equation is combined with a standard Phillips Curve into the basic New Keynesian model, the dampening has the same effect as increasing the output coefficient: The model would show very similar responses to interest rate shocks or cost push shocks.

<sup>5</sup>Whether the inflation dampening only applies to the Phillips Curve, or the Euler equation as well, depends on the roots behind this dampening. If it is caused by firm/product entry and exit, it applies only to the PC, while the Euler equation of consumption is unaffected. If it is a behavioral bias, or expectation anchoring, firms and consumers probably use a similar cognitive process (see e.g. Coibion et al., 2018), with the same dampening.

<sup>6</sup>A modified Euler equation  $y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - \tilde{\delta} E_t \pi_{t+1})$  (with possibly  $\tilde{\delta} \neq \delta$ ) can be combined with a Taylor rule  $i_t = \phi_\pi \pi_t + \phi_y y_t$  as  $y_t = E_t y_{t+1} - \frac{\tilde{\delta}}{\sigma}(\frac{\phi_\pi}{\tilde{\delta}} \pi_t + \frac{\phi_y}{\tilde{\delta}} y_t - E_t \pi_{t+1})$ . It is isomorphic to a standard model where  $(\sigma, \phi_\pi, \phi_y)$  are replaced with  $(\frac{\sigma}{\tilde{\delta}}, \frac{\phi_\pi}{\tilde{\delta}}, \frac{\phi_y}{\tilde{\delta}})$

### The role of persistence

It is only with persistent shocks that dampening brings significantly different impulse responses to supply and demand shocks. It is easy to see why. If there is a transitory (white noise) supply or demand shock today, future expected variables are not affected,  $E_t\pi_{t+1} = E_t y_{t+1} = 0$ . Whether there is a coefficient  $\beta$  or  $\beta\delta$  in the Phillips Curve is irrelevant, as the expected inflation is zero. On the other hand, if the supply or demand shock is  $AR(1)$  with  $\rho \approx 1$ , then  $E_t\pi_{t+1} \approx \pi_t$ . The Phillips Curve becomes  $(1 - \beta\delta)\pi_t \approx +\kappa y_t + v_t$  and is flatter with  $\delta < 1$  than normally. Output becomes more sensitive to demand shocks.

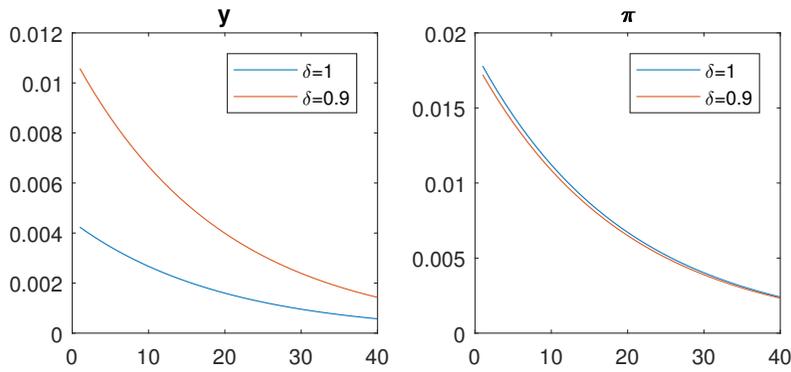


Figure 1:  $y$  and  $\pi$  in response to a persistent demand shock

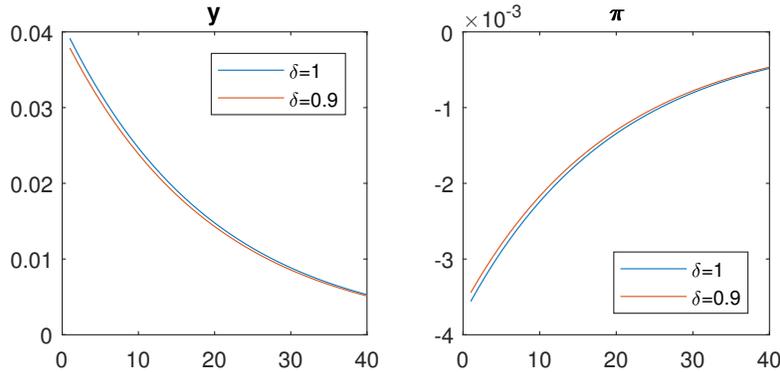


Figure 2:  $y$  and  $\pi$  in response to a persistent supply shock

Figures 1 and 2 display the impulse response functions of output and inflation to persistent demand and supply shocks respectively. The model is calibrated with  $\sigma = 1$ ,  $\kappa = 0.025$ ,  $\phi_\pi = 1.5$  and  $\phi_y = 0$ . The shocks are very persistent ( $\rho = 0.95$ ), and the dampening factor is set to 1 and 0.9 respectively. One can see how the dampening only has a significant impact on the response of

output to persistent demand shocks. As hinted before, the dampening makes the supply curve flatter in the presence of persistent shocks. A demand shock has a bigger impact on output when the supply curve is flatter. This implies that for an observed fluctuation in output, the underlying fluctuation in the natural rate of interest is not as high as what the standard model would predict. Hence the dampening significantly matters when the natural rate of interest is depressed for a while (a persistent negative demand shock) or when monetary policy persistently undershoots as compared to its baseline Taylor rule.<sup>7</sup>

### 2.3 Price targeting

Dampened expectations have strong implications for the optimal Ramsey policy in a standard New Keynesian model. Normally, price targeting is optimal for the Ramsey policy: even with steady state distortions, the long run optimal level of inflation is zero; while inflation reacts to cost push shocks in the short run, this is accompanied by deflation in the future, so that there is full mean reversion of the price level. But with dampening, price targeting is no longer optimal: long run inflation is non zero if there are steady state distortions; in response to cost push shocks, some deflation in the future offsets the initial response of inflation, but there is no longer full mean reversion of the price level.

**Lemma 1** *As in Gali (2008), the approximation of the planner's objective is*

$$U = - \sum_{t \geq 0} \beta^t \left[ \kappa \frac{(y_t - \bar{y})^2}{2} + \epsilon \frac{\pi_t^2}{2} \right]$$

I also assume an  $AR(1)$  cost push shock  $u_t$  in the Phillips curve:

$$\pi_t = \kappa y_t + \beta \delta E_t \pi_{t+1} + u_t$$

Denoting  $\lambda_t$  the Lagrange multiplier of the Phillips Curve at time  $t$ , the Lagrangian of the optimal Ramsey policy is

$$L = -E_0 \sum_{t=0}^{+\infty} \beta^t \left( \frac{1}{2} \left[ \epsilon \pi_t^2 + \kappa (y_t - \bar{y})^2 \right] + \lambda_t [\pi_t - \beta \delta E_t \pi_{t+1} - \kappa y_t - u_t] \right)$$

Taking first order conditions and simplifying  $\lambda_t$ ,  $\epsilon \kappa \pi_0 = \kappa (\bar{y} - y_0)$  and  $\epsilon \kappa \pi_t = \kappa ((\bar{y} - y_t) - \delta (\bar{y} - y_{t-1}))$ . Used in the Phillips Curve, this shows that output  $y_t$  follows a second order difference equation:

$$(1 + \kappa \epsilon + \beta \delta^2) y_t = \beta \delta E_t y_{t+1} + \delta y_{t-1} + (1 - \delta)(1 - \beta \delta) \bar{y} - \epsilon u_t$$

The steady state of output and inflation are zero only if  $\delta = 1$ :

$$y^* = \frac{(1 - \delta)(1 - \beta \delta)}{(1 - \delta)(1 - \beta \delta) + \epsilon \kappa} \bar{y} \quad \pi^* = \frac{(1 - \delta) \kappa}{(1 - \delta)(1 - \beta \delta) + \epsilon \kappa} \bar{y}$$

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<sup>7</sup>The case of the Zero Lower Bound is different as the Taylor rule no longer applies there.

When  $\delta = 1$ ,  $\epsilon\pi_0 = (\bar{y} - y_0)$  and  $\epsilon\pi_t = -(y_t - y_{t-1})$ , so we can integrate  $\epsilon(w_t - w_{-1}) = (\bar{y} - y_t)$ . Thus  $w_t$  also follows a stationary difference equation (level targeting). With  $\delta < 1$ , the price level can no longer be integrated as a stationary variable. There is only partial mean reversion of the price level.

**Property 2** *Long run level targeting is the optimal Ramsey policy only when  $\delta = 1$ . When  $\delta < 1$ , targeting the price level is no longer optimal. Long run inflation is non zero if there are steady state distortions. And in response to cost push shocks, some deflation in the future offsets the initial response of inflation, but there is no longer full mean reversion of the price level (see fig 3).*

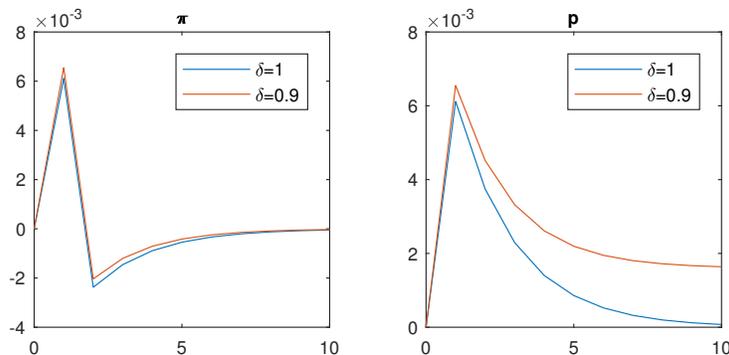


Figure 3: Ramsey policy in response to a cost-push shocks

The intuition is as follows: in the benchmark, by committing to give up some discretion in the future, the planner has some extra discretion in the present to offset cost push shocks, or an inefficient steady state. So that price (or wage) stability is optimal from today's perspective, but there is an incentive to renege tomorrow. With dampened expectations, firms are less responsive to commitments, so that the current gain in terms of commitment no longer offsets the inefficiency in the future. Thus, even with a credible commitment, inflation will always be used to offset cost push shocks or steady state inefficiencies.

### 3 Conclusion

In this note, I investigated some of the effects of dampened inflation expectations in the Phillips Curve. If the coefficient of future inflation is restricted in a standard NK Phillips Curve, this creates a bias on the estimate of the slope of the Phillips Curve. This dampening has important implications when one looks at the effect of persistent demand shocks in a small NK model: while the effect on inflation is very small, the response of output is magnified. I also showed how the dampening breaks the optimality of price stability. Optimal Ramsey policy no longer targets the price level in response to cost push shocks.

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