Distinctive numbers of production systems functioning description

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DISTINCTIVE NUMBERS OF PRODUCTION SYSTEMS FUNCTIONING DESCRIPTION

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The production system of an enterprise is represented as the system with large quantity of elements, which are the objects of one's labour. The distinctive numbers of the production system are introduced by means of statistical mechanics. This approach gives the possibility of qualitative estimation of production processes functioning, sound selection of the corresponding equations set of macroscopic parameters balances for description of real production object. The estimation of the model selection should be interpreted as the qualitative one. The approach has the advantage of easy comparison of the results, corresponding to different microscopic models.

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1. INTRODUCTION

Extensive categories of organization, planning and operation of production enterprise are developed within the limits of simple models [1-7]. However, not necessarily the real production systems functioning can be accurately described with the help of these simplest models [8-11]. Different production systems under the same external conditions conduct themselves in different ways. Thus, the same equations, even with addition of the corresponding boundary conditions, are not enough for the description of specific production system functioning [12]. This fact gets obvious if the number of equations is less than the number of constituent unknown values. The equation set is open. Construction of the closed equation set, showing the functioning of the production system under consideration, is connected with the definition of additional relationships among the parameters of the given production system. Construction of the closed equation set means construction of mathematical simulator of the production environment being studied.

Construction of new models of production systems is connected with experimental study of organization and production techniques [1,8], it is caused by the requirements of the fifth stage of economics [8]. To construct such models, the application of well-known general laws of mechanics and physics, e.g. thermodynamic relations [13] is necessary. The application of variational principles [14,15] is appeared to be useful. Large variety and complication of production method of the system final product requires construction of theory of production system functioning on the basis of representation of production system of an enterprise as the set of objects of one's labour, being in different stages of technological treatment [14]. However, it is impossible to follow the conduct of each object of labour (the base product of production system) because of their quite large quantity and probabilistic nature of influence of manufacturing equipment on the base product [12]. Statistical physics is one of the general approaches to analysis of large systems conduct. Here probabilistic approach to study of large systems is usually applied. Such approach allows to obtain the functioning model of production system with definite manufacturing method in the framework of manufacturing equipment at the enterprise by means of the aggregation of the microscopic parameters of the production under consideration. Having such approach allows to exclude selection of the model (one of the existing models for description of production systems) which is the closest to the object under consideration. At the same time from practical view point it is interesting to obtain distinctive numbers for production systems functioning, allowing to substantiate selection of the corresponding model of real production object description.

2. KINETIC EQUATION, DESCRIBING PRODUCTION SYSTEM FUNCTIONING

Description of functioning of contemporary mass production systems is represented as stochastic process [6, p.178]. The system state is defined as the state of the total number of base products N [7, p.183] of production system. The state of the base product is described by microscopic parameters (S_j, μ_j), here S_j (hrn) and $\mu_j = \lim_{\Delta t \to 0} \frac{\Delta S_j}{\Delta t}$ (hrn/hr) are correspondingly the sum of common expenses and expenses in a unit of time, transferred by production system to the j-th base product, $0 < j \le N$. Production system is characterized by the criterion function $J(t, S_j, \mu_j)$ [1,7]:

$$J(t, S_j, \mu_j) = \sum_{j=1}^{N_p} \frac{\mu_j^2}{2} + F_{1\psi}(S_{\psi V}) \cdot \sum_{j=1}^{N_p} \mu_j - W_{0\psi}(S_{\psi V}) \quad (1)$$

Function $W_{0\psi}(S_{\psi V})$ is the productive potential of an enterprise, it makes manufacturing field of production process, being assigned directly by technological potential $F_{0\psi V}(S_{\psi V})$, potential of overheads $F_{0\psi C}(S_{\psi V})$ and potential of interaction $F_{0\psi VC}(S_{\psi V})$. The system's state at any point of time is defined in the case when microscopic values $(S_1, \mu_1, ..., S_N, \mu_N)$ are defined, and at any other point of time the state equation of base products is obtained:

$$\frac{dS_j}{dt} = \mu_j,$$
$$\frac{d}{dt}\left(\frac{\partial J_{\Pi}(t,S_j,\mu_j)}{\partial \mu_j}\right) = \frac{\partial J(t,S_j,\mu_j)}{\partial S_j} = f_j(t,S), \quad (2)$$

here $f_j(t,S)$ is the production-engineering function. Instead of considering the state of production system with microscopic values $(S_1,\mu_1,...,S_N,\mu_N)$, we introduce normalized function of distribution $\chi(t,S,\mu)$ of the base products number N in phase space (S,μ) [16], satisfying the kinetic equation:

$$\frac{\partial\chi}{\partial t} + \frac{\partial\chi}{\partial S}\cdot\mu + \frac{\partial\chi}{\partial\mu}\cdot f(t,S) = J(t,S,\mu), \quad (3)$$

$$\frac{dS}{dt} = \mu,$$
$$\frac{d}{dt}\left(\frac{\partial J_{\Pi}(t,S,\mu)}{\partial\mu}\right) = \frac{\partial J_{\Pi}(t,S,\mu)}{\partial S} = f(t,S). \quad (4)$$

Generating function $J(t,S,\mu)$ is assigned by compactness of equipment arrangement λ_{eq} lengthwise technological chain and its features [12]:

$$J(t,S,\mu) = \lambda_{eq} \times\left\{\int_0^\infty [\psi[\tilde{\mu}\to\mu]\cdot\tilde{\mu}\cdot\chi(t,S,\tilde{\mu})]\cdot d\tilde{\mu} - \mu\cdot\chi\right\}, \quad (5)$$

here $\psi[\mu\to\tilde{\mu}]$ is a function, being defined by equipment certificate. The total probability of the base product transfer into any state equals to one:

$$\int_0^\infty \psi[\mu\to\tilde{\mu}]\cdot d\tilde{\mu} = 1 \quad (6)$$

(the zero moment of the function $\psi[\mu\to\tilde{\mu}]$), and productivity of equipment functioning $[\chi]_{1\psi} = \mu_\psi\cdot[\chi]_0$ and the mean-square deviation $\sigma_\chi^2 = \sigma_\psi^2\cdot[\chi]_0^2$ can be defined with the help of the first and second moments of the function of manufacturing equipment operating $\psi[\mu\to\tilde{\mu}]$:

$$\int_0^\infty \psi[\mu\to\tilde{\mu}]\cdot\tilde{\mu}\cdot d\tilde{\mu} = \mu_\psi \quad (7)$$

(the first moment of the function $\psi[\mu\to\tilde{\mu}]$),

$$\int_0^\infty \psi[\mu\to\tilde{\mu}]\cdot\tilde{\mu}^2\cdot d\tilde{\mu} = \mu_\psi^2 + \sigma_\psi^2 \quad (8)$$

(the second moment of the function $\psi[\mu\to\tilde{\mu}]$).

The first moment of the function of manufacturing equipment operating $\psi[\mu\to\tilde{\mu}]$ characterizes the dependence of the rate of expense change when the base product passes the unit of manufacturing equipment, the second moment is the mean-square deviation of the rate of expense change when the base product passes the unit of manufacturing equipment from its mean value μ_ψ, being defined by the equipment features and peculiarities of manufacturing process.

3. DIMENSIONLESS DISTINCTIVE FEATURES OF PRODUCTION SYSTEM

The solution of the equation relative to function of the base products distribution in the rates of expenses change $\chi(t,S,\mu)$ in the phase space (s,μ) is connected with some difficulties. The first step in the integral-differential analysis of the equation (3) has to contain analysis of order of values of different summands.

Let τ, η, ξ be correspondingly the distinctive time, rate of expenses change and step of the variable S. Let us input dimensionless variables $\hat{t}$, $\hat{S}$, $\hat{\mu}$, connected with the variables τ, η, ξ as follows:

$$t = \tau\cdot\hat{t};\ S = \xi\cdot\hat{S};\ \mu = \eta\cdot\hat{\mu};$$
$$J(\chi,\chi) = \langle\lambda_{eq}\rangle\cdot\eta\cdot\hat{J}(\chi,\chi), \quad (9)$$

here $\langle\lambda_{eq}\rangle$ is the distinctive compactness of equipment arrangement lengthwise technological chain of production process. Then the kinetic equation (3) of production system looks like:

$$\left[\frac{\partial\chi}{\tau\cdot\partial\hat{t}} + \frac{\partial\chi}{\xi\cdot\partial\hat{S}}\cdot\eta\cdot\hat{\mu} + \frac{\partial\chi}{\eta\cdot\partial\hat{\mu}}\cdot\frac{\eta\cdot d\hat{\mu}}{\tau\cdot\partial\hat{t}}\right] = \langle\lambda_{eq}\rangle\cdot\eta\cdot\hat{J}(\chi,\chi). \quad (10)$$

Let us divide the above summands by $\eta\cdot\xi^{-1}$:

$$\frac{\eta}{\xi\langle\lambda_{eq}\rangle}\cdot\left[\frac{\xi\cdot\partial\chi}{\eta\cdot\tau\cdot\partial\hat{t}} + \frac{\partial\chi}{\partial\hat{S}}\cdot\hat{\mu} + \frac{\xi\cdot\partial\chi}{\eta\cdot\tau\cdot\partial\hat{\mu}}\cdot\frac{d\hat{\mu}}{\partial\hat{t}}\right] = \eta\cdot\hat{J}(\chi,\chi), \quad (11)$$

and, after reduction, we obtain

$$\frac{1}{\xi\cdot\langle\lambda_{eq}\rangle}\cdot\left[\frac{\xi\cdot\partial\chi}{\eta\cdot\tau\cdot\partial\hat{t}} + \frac{\partial\chi}{\partial\hat{S}}\cdot\hat{\mu} + \frac{\xi\cdot\partial\chi}{\eta\cdot\tau\cdot\partial\hat{\mu}}\cdot\frac{\partial\hat{\mu}}{\partial\hat{t}}\right] = \hat{J}(\chi,\chi). \quad (12)$$

Let us input the symbols

$$K_v = \frac{\left[\frac{1}{\langle\lambda_{eq}\rangle}\right]}{\xi},\quad P_m = \frac{\xi}{\tau\cdot\eta}, \quad (13)$$

$$P_0 = K_v\cdot P_m = \frac{1}{\langle\lambda_{eq}\rangle\cdot\tau\cdot\eta}. \quad (14)$$

Kinetic equation of production system (3) taking into account the symbols (13,14) looks as follows

$$K_v\cdot\left[P_m\cdot\frac{\partial\chi}{\partial\hat{t}} + \frac{\partial\chi}{\partial\hat{S}}\cdot\hat{\mu} + P_m\cdot\frac{\partial\chi}{\partial\hat{\mu}}\cdot\frac{d\hat{\mu}}{\partial\hat{t}}\right] = \hat{J}(\chi,\chi). \quad (15)$$

Multiplying kinetic equations correspondingly by 1, μ, $\frac{\mu^2}{2}$ and integrating them by the whole μ range, we obtain balances equation of macroscopic parameters of production system [16] in the zero approximation according to the small parameter $\varepsilon(K_v, P_m) \to 0$ relative to equilibrium position, that the equations of macroscopic parameters of production system, describing functioning of manufacturing process, depend on distinctive numbers of production system.

As τ, ξ, η (distinctive time, step of variable S, and rate of expenses change) we can take the time of production cycle T_d, $\tau = T_d$, the average cost price of base product S_d, $\xi = S_d$, and average rate of expenses change for one period of production cycle η_d, $\eta_d = \eta$. The value

$$\frac{1}{\langle \lambda_{eq} \rangle} = L_d \qquad (16)$$

is the average conversion of expenses to base product among equipment units (or the length of base product free path between manufacturing influences).

Then distinctive numbers of production system will look as follows:

$$K_v = \frac{L_d}{S_d},\ P_m = \frac{S_d}{T_d \cdot \eta_d},\quad P_0 = K_v \cdot P_m = \frac{L_d}{T_d \cdot \eta_d}. \qquad (17)$$

Substitution of production cycle time values T_d, the average cost price of base product S_d, average rate of expenses change for one period of production cycle η_d and average compactness of equipment arrangement lengthwise technological chain $\langle \lambda_{eq} \rangle$ in the expressions for distinctive numbers of production system (17) gives the possibility of justification of selection the model of production system functioning description. The given estimation would rather be taken as qualitative one than quantitative. However, such an approach has the advantage, allowing compare the results, corresponding to different microscopic models, easy, because the equation relative to distribution function of base products on the rates of expenses change $\chi(t,S,\mu)$ in the phase space (S,μ), expressed with the help of values τ, η, ξ being measured macroscopically, does not depend on the integral

$$\lambda_{eq} \cdot \int_0^\infty [\psi[\tilde{\mu} \to \mu] \cdot \tilde{\mu} \cdot \chi(t,S,\tilde{\mu}) - \psi[\mu \to \tilde{\mu}] \cdot \mu \cdot \chi(t,S,\mu)] \cdot d\tilde{\mu}, \qquad (18)$$

and can be represented as the equation relative to distribution function of base products in the rates of expenses change by means of values τ, η, ξ, being measured macroscopically:

$$\frac{\partial \chi}{\partial t} + \frac{\partial \chi}{\partial S} \cdot \mu + \frac{\partial \chi}{\partial \mu} \cdot f(t,S) \approx \lambda_{eq} \cdot \eta \cdot [\chi - \chi_0]. \qquad (19)$$

If $[\chi - \chi_0] = 0$, we have the case of production system equilibrium position, which is described by the identity

$$J(\chi_0, \chi_0) = 0. \qquad (20)$$

The value of the distinctive number K_v varies from zero to infinity, and there are two extreme cases for it $K_v \to 0$ and $K_v \to \infty$. These two cases describe the situations, which are related to extremely small and extremely large expenses changes of base product between two main operations.

Production systems with qualitative estimation of state having the factors values $K_v << 1$, $P_m \approx 1$ correspond to a compact flow of base products lengthwise technological chain with high concentration of technological equipment. The case, when $K_v >> 1$, $P_m \approx 1$, corresponds to production process with, as a rule, small compactness of manufacturing equipment $(\lambda_{eq} \to 0)$ lengthwise technological chain of base product production. Thereby, the way of base products among the main operations is long enough. When it is in the "free", unmanufactured, state, base product moves lengthwise technological chain without any interceptions. The free motion is the motion of base products lengthwise technological chain of production process, when the conversion of expenses to base product realizes by means of definite way, which is defined by engineering-production function of production process $f(t,S)$ without the expense change rate abmodality. Such conversion is characterized by the function $\psi[\mu \to \tilde{\mu}] = \psi[\mu \to \mu]$, i. e. after manufacturing the rate of expenses change of base product can take only the values, defined by equipment certificate without any deviations.

4. CONCLUSIONS

The model of production system functioning can be estimated by means of distinctive numbers. Distinctive numbers provide qualitative estimation of production process functioning, allow to select the appropriate set of balances equations of production system macroscopic parameters for description of real production system. Definite generating function of operating production system, when the system's position is close to the equilibrium one, can get obvious with the help of the values of production cycle time T_d, average cost price of base product S_d, average rate of expenses change in unit of production cycle period η_d and average compactness of equipment arrangement lengthwise technological chain $\langle \lambda_{eq} \rangle$ of production system distinctive numbers. Such an approach of model selection would rather be taken as qualitative one than quantitative. However, it has the advantage, allowing to compare the results, corresponding to different microscopic models.

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ХАРАКТЕРНЫЕ ЧИСЛА ОПИСАНИЯ ФУНКЦИОНИРОВАНИЯ ПРОИЗВОДСТВЕННЫХ СИСТЕМ

О.М. Пигнастый

Производственная система предприятия представлена в виде системы с большим количеством элементов — предметов труда. Посредством аппарата статистической механики введены характерные числа производственной системы. Данный подход дает возможность провести качественную оценку функционирования производственного процесса, обоснованно подобрать для описания реального производственного объекта соответствующую систему уравнений балансов макроскопических параметров. Оценку выбора модели следует воспринимать как качественную. Подход обладает тем преимуществом, что позволяет легко сравнивать результаты, соответствующие различным микромоделям.

ХАРАКТЕРНІ ЧИСЛА ОПИСУ ФУНКЦІОНУВАННЯ ВИРОБНИЧИХ СИСТЕМ

О.М. Пігнастий

Виробничу систему підприємства представлено у вигляді системи з великою кількістю елементів — предметів праці. За допомогою апарату статистичної механіки введено характерні числа виробничої системи. Цей підхід дає можливість провести якісну оцінку функціонування виробничого процесу, підібрати для опису реального виробничого об’єкту відповідну систему рівнянь балансів макроскопічних параметрів. Оцінку вибору моделі слід сприймати як якісну. Підхід має перевагу, що дозволяє легко порівнювати результати, що відповідають різноманітним мікромоделям.