Investigating double counting terms in the value-added decomposition of gross exports

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Abstract

Several papers using intercountry input-output tables have developed frameworks to decompose value added in gross exports and to remove potential double-counting in intermediate inputs. But these papers rely on different definitions for the domestic value added, foreign value added and double-counting terms, depending in particular on the perspective from which gross exports are decomposed (world level, country level or bilateral level). At this stage, it is very difficult for any user of value-added trade statistics to know what is calculated and which type of decomposition should be used. In this paper, we provide a general framework that relies on extraction matrices to unambiguously and consistently define domestic and foreign value-added terms in the world, country and bilateral perspective. This framework allows us to classify existing decompositions based on the perspective taken and their definition of double-counting. We also indicate the most relevant decompositions for different types of trade analysis.

1. Introduction

To better understand the fragmentation of production and trade in the context of global value chains (Gereffi and Fernandez-Stark, 2016), a series of papers have introduced frameworks for decomposing gross exports in inter-country input-output tables. These papers aim at measuring the value-added contribution of all countries involved in the production process (Daudin et al., 2011; Johnson and Noguera, 2012; Koopman et al., 2014; Foster-McGregor and Stehrer, 2013; Los et al., 2016; Miroudot and Ye, 2017; Borin and Mancini, 2017; Johnson, 2018; Arto et al., 2019). One motivation for developing value-added measures of trade is to remove the 'double-counting' in gross exports. In the input-output framework, the concept of double-counting comes from the measurement of intermediate inputs. Output is equal to (domestic) value-added plus intermediate inputs. But intermediate inputs are also produced with (domestic or

foreign) value-added and other intermediate inputs. Double-counting can be regarded as a subset of intermediate inputs in output decomposition.

Since gross exports correspond to the share of output sold to foreign consumers, there is also a double-counting involved. This double-counting in intermediate inputs can be removed by looking at net trade (Trefler and Zhu, 2010; Foster-McGregor and Stehrer, 2013) or by working with measures of value-added trade derived from final demand (Johnson and Noguera, 2012). But when authors start to introduce double-counting terms in the decomposition of gross exports, things become more complicated since intermediate inputs are both part of exported goods and foreign inputs used in their production. Moreover, the concept of 'foreign value added' (FVA) in trade, which is the variable of interest to understand global production, leads to further questions on what is double-counted. When looking at exports of all countries in the world, any foreign value added is by definition double-counted, since it is 'domestic value added' (DVA) in other countries. What authors try to define as double-counting is therefore no longer the intermediate inputs double-counted in output but some share of value added that would be counted several times from the point of view of the exporting economy, including in the FVA term (something sometimes referred to as 'pure double-counting').

Understanding double-counting in the value-added decomposition of gross exports is important for two reasons. First, it can amount to a significant share of gross exports, thus introducing a bias in empirical work relying on the measurement of domestic or foreign value added in trade, including when value-added is replaced by jobs, CO2 emissions or other variables in extensions of the intercountry input-output

model. Second, since the definition and calculation of double-counting is what distinguishes existing decomposition frameworks in the literature, a better understanding of the meaning of double-counting terms is also required to clarify the different metrics proposed so far and to create a general framework.

In the domestic content of gross exports, domestic double-counting (DDC) corresponds to domestic intermediate inputs that come back to the exporting economy embodied in foreign inputs and are exported again after further processing. An example would be steel produced by China and exported to Thailand to be incorporated in parts and components from the motor vehicles industry that are then exported back to China and used in exports of Chinese cars. Within Chinese gross exports, the value added related to this production of steel will be counted twice (in exports of steel and in exports of cars).

This double-counting can also happen with foreign inputs. The same Chinese steel, for example, could be used in Thailand to produce cylinders for an engine manufactured in Malaysia. This engine could then be exported back and incorporated in cars assembled in Thailand and exported to Japan. This kind of 'circular trade' is what creates double-counting. When decomposing gross exports of Thailand, the value added associated to Chinese steel (which is foreign) will be counted twice: first in exports of engines to Malaysia and then in exports of cars to Japan.

There is no consensus yet on the definition and calculation of this foreign double-counting (FDC) in gross exports decompositions. As a consequence, there is no consensus either on the correct measure for FVA since the two terms should sum to a

foreign content already defined as the difference between the domestic content (DVA+DDC) and gross exports.

Two papers offer more insights on what differs across the different decompositions. Los and Timmer (2018) introduce a unified framework and distinguish the value added in exports (what they call VAX-D) from the value added consumed abroad (VAX-C, corresponding to the framework first introduced by Johnson and Noguera, 2012) and from the value added used abroad in the final production stage (a new measure called VAX-P). Measures based on final demand or production (VAX-C and VAX-P) have no double-counting issue and are unambiguously defined in the traditional input-output model. In this paper, we are interested in VAX-D, which is the indicator for an analysis of trade flows in value-added terms matching existing gross exports.[1] The unified framework proposed by Los and Timmer (2018) follows the methodology of Los et al. (2016) for the calculation of VAX-D but does not cover the other decompositions found in the literature and does not address the question of the foreign value added in exports. However, an important insight from Los and Timmer (2018) with respect to VAX-D is that double-counting in gross exports also depends on whether aggregate or bilateral exports are decomposed. In particular, they identify a 'double count of domestic value added in summing bilateral measures' which is the difference between domestic value-added in aggregate exports (i.e. with partner world)

[1] As highlighted by Los and Timmer (2018), VAX-D is the metric to be used for the impact of trade barriers or trade agreements as it measures the value-added trade flows between countries that have put in place barriers or signed a trade agreement to remove them.

and the sum of bilateral domestic value-added across all partners.

A paper that further looks at value added in exports and provides a detailed analysis of what differs across existing decompositions for FVA is Borin and Mancini (2019). Building on Borin and Mancini (2017), the authors introduce an important distinction between measures taking a global (world) perspective from measures focusing on exports of a specific economy (country perspective), as well as bilateral measures. Their framework can be seen as a generalisation of Koopman et al. (2014) using gross exports accounting to derive different terms corresponding to different definitions of FVA and FDC with a different economic interpretation.

In this paper, we go one step further by introducing a general framework based on extraction matrices (as in Los et al., 2016) that unambiguously define double-counting terms and in which the same equations are used to derive DVA, DDC, FVA and FDC terms based on a world, country or bilateral perspective, resulting in consistent definitions for these terms. As in Los and Timmer (2018), the bilateral perspective provides DVA terms that do not sum to the DVA measured in countries' or world exports. Unlike previous papers, we argue that FVA in the world perspective is equal to zero and that authors that introduce a FVA term in this framework are in fact further decomposing the foreign double-counting (FDC). We also show that DVA and FVA have a defined origin and destination so that there is no 'source-based' or 'sink-based' approach when decomposing gross exports into DVA, DDC, FVA and FDC. These approaches are only introducing further sub-terms in the decomposition. Using numerical examples and calculations with the World Input-Output Database (WIOD), we explain the economic

interpretation of the different perspectives and how they can answer different types of questions in relation to global production and trade.

Section 2 discusses the concept of double-counting in gross exports decompositions and how it was dealt with in previous papers, introducing the three perspectives from which gross exports can be decomposed (world, country and bilateral perspectives). Section 3 presents a new input-output framework (consistent with Los et al., 2016) that allows us to clarify the definition of double-counting terms and to define with the same equations DVA, DDC, FVA and FDC in the world, country and bilateral perspective. Section 4 develops numerical examples and use WIOD data to illustrate how the different decompositions compare to each other and what we can learn through the double-counting terms. Section 5 concludes.

2. Defining double-counting in the decomposition of gross exports: three perspectives

In the framework developed by Koopman, Wang and Wei (2014), KWW hereafter, double-counting is defined as the value added that crosses international borders more than once. Therefore, all the FVA is already double-counted. It makes sense since the authors are interested in removing double-counting from aggregate world trade statistics. In this case, FVA in exports is by definition DVA in the exports of another country and double-counted. In order to decompose gross exports of a specific country and to introduce a FVA term, the authors then refer to a 'pure' double-counting, which is the difference between gross exports and the sum of DVA and FVA. This 'pure double-counting' is then split between a domestic and foreign component so that at the

end gross trade is decomposed into four terms: DVA, DDC, FVA and FDC.[2] Defined as a residual, this pure double-counting can be calculated but there is no clear interpretation of what it exactly measures, even if the intuition is that the double-counting terms correspond to value added coming back to the exporting economy.

Pointing out the issue with KWW, Borin and Mancini (2017) propose a different definition for double-counted terms. From the point of view of a specific exporting economy, double-counting corresponds to the value added that has crossed the country's border more than once. It is a better starting point but the issue with a definition of double-counting based on the number of border crossings is that the input-output framework cannot tell us how many times value added has crossed borders. The input-output matrix identifies international and domestic transactions but there are many paths through which value added can reach final consumers and these paths are not known. They are summarized in a single input-output matrix that has collapsed the different production stages (Los and Timmer, 2018). As we will see, some assumptions have to be made to allocate value added across domestic and foreign terms, and to decide whether or not it is double-counted. This subtlety explains why there is no simple formula to calculate FVA in exports and why there is no consensus yet in the literature on how it can be done.

The definition that Borin and Mancini (2017) propose for double-counting in

[2] There are actually 9 terms in the KWW decomposition but the additional terms further decompose DVA and FVA based on where value added is absorbed and whether goods exported are intermediate or final.

the sense of value added coming (at least) twice to the same economy is conceptually sound. But its implementation in the input-output framework is problematic. As we will formally show in the next Section, value-added ratios multiplied by the Leontief inverse can be used to measure value added when it enters a specific country "for the first time". But before entering a specific country, this value added has already crossed all possible borders according to the input-output table. Therefore, there is no clarity in terms of how many times borders are crossed. Moreover, the concept of 'border' is not the same when dealing with global exports (exports to the world) and bilateral exports. This further complicates the reference to border crossings in the definition of double counting. What is the 'border' already depends on the initial assumptions and the setting of the decomposition, as explained in Borin and Mancini (2019).

An alternative way of defining double-counting without a reference to border crossings is found in Miroudot and Ye (2017) who rely on the supply-side input-output model. The Ghosh insight already refers to different rounds in the process of value generation. There is, embedded in the model, the concept of an initial round and value added measured in all later rounds is by definition double-counted. This framework provides a clear definition of double-counting and is straightforward when it comes to its implementation in the context of an intercountry input-output table. However, there are debates on the foundations and assumptions behind the supply-side input-output model (Oosterhaven, 1988; Dietzenbacher, 1997). Value added in this case is regarded as exogenous, which helps when it comes to its allocation to different countries, but one can question this assumption. Moreover, this approach also leads to a definition of

double-counting that assumes that there is a first country where value added is generated (and exported) and that any time this value added is measured somewhere else in the exports of another country, it has to be regarded as part of double-counting terms. As explained in Borin and Mancini (2019), such approach defines double counting based on world exports (world perspective). Referring to the work of Nagengast and Stehrer (2016), this approach is also presented as 'source-based', i.e. from the point of view of the (first) exporting economy.[3]

Lastly, the paper by Los et al. (2016) is the only one that does not introduce double-counting terms. It also has no explicit formula for FVA. Nevertheless, the methodology it applies to derive DVA in gross exports (a hypothetical extraction method) can also be used to estimate FVA. The difference between the sum of DVA and FVA in such framework also creates a residual that can be interpreted as double-counting. As we will show, this residual corresponds to the value added coming actually twice to the exporting economy (domestic or foreign). The framework of Los et al. (2016), further developed in Los and Timmer (2018), is also the one that explains why

[3] Nagengast and Stehrer (2016) distinguish a 'source-based' approach from a 'sink-based' approach in the value-added analysis of bilateral trade balances. The source-based approach takes the perspective of the exporting economy and the sink-based approach the perspective of the country of final absorption. When decomposing gross exports, the country of final absorption is not known, unless the decomposition introduces additional terms to identify it (as in KWW). As such, the distinction between a 'source-based' and 'sink-based' approach is an extension of the gross exports decomposition in the framework we introduce.

the sum of bilateral measures of value-added is different from the decomposition of aggregate measures (i.e. exports to the world). This is why we use it as a starting point in our analysis.

Looking at the above literature, we can identify three main approaches in the way authors allocate value added across countries when decomposing gross exports, in relation to the way they define double-counting (Table 1). To distinguish these approaches, we refer to the 'world perspective', 'country perspective' and 'bilateral perspective', as it is done in Borin and Mancini (2019).[4] We talk about a 'perspective' because each approach corresponds to a different definition of the 'border' or 'boundary of production' in the input-output model, leading to a different interpretation of double-counting. But the distinction is not based on the type of trade flow decomposed. For example (as seen in Table 1), bilateral exports can be decomposed with a world perspective or country perspective and not just with the bilateral perspective.[5] All

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[4] We use the terminology from Borin and Mancini (2019) and the decompositions they provide can be related to the ones we propose but our framework is different and leads to different interpretations of the FVA and FDC terms, as well as different results for the country perspective. Note also that a fourth approach could be introduced: the 'industry perspective'. But we argue in Section 4 that double-counting in this case becomes so small that it does not really matter.

[5] Authors following the world perspective or country perspective have bilateral measures but in this case what they provide is a mapping of the country's exports across different partners rather than a decomposition of bilateral gross exports. With such an approach, the sum of bilateral measures is equal to the aggregate value-added decomposition.

combinations are possible but not all make sense from an analytical point of view. We come back to this in Section 4.

Table 1. Classification of gross exports decompositions based on their definition of double-counting and type of exports decomposed

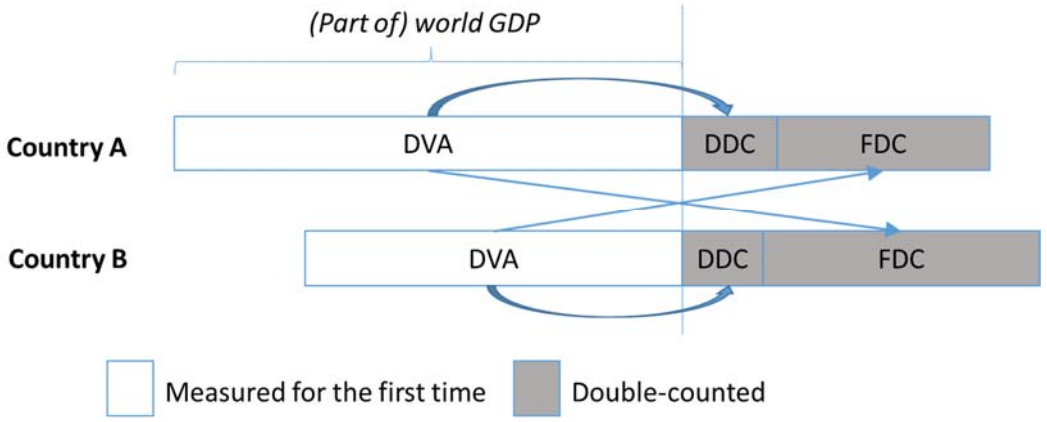
Definition of double-counting (explicit or implicit)	Decomposition of aggregate exports (partner world)	Decomposition of bilateral exports
World perspective: value-added that crosses borders more than once	Koopman, Wang and Wei (2014) (sink-based variation) Miroudot and Ye (2017) (source-based variation)	Borin and Mancini (2017, 2019) (source and sink-based variations) Wang, Wei and Zhu (2017) (sink-based variation)
	Johnson (2018) (2 countries)	
Country perspective: value-added that crosses the border of the exporting economy more than once	Los et al. (2016)	Arto, Dietzenbacher and Rueda-Cantuche (2019) Borin and Mancini (2017, 2019)
Bilateral perspective: value-added that crosses the bilateral border of the exporting economy and its partner more than once		Los et al. (2016) Borin and Mancini (2019)

With the world perspective, double-counting can be defined as the value added that crosses international borders more than once (the definition used by KWW). As previously pointed out, world exports can be decomposed into two terms: value added (in trade) and intermediate inputs (in trade). There is no concept of domestic or foreign at the world level. All intermediate inputs that are re-embodied in trade (i.e. not absorbed by the direct partner) are double-counted (or counted multiple times) in gross exports and the value added in exports is consistent with world GDP (the share of GDP going to exports).[6] World value added is DVA and can be allocated to the different

[6] See Miroudot and Ye (2019) for a discussion of GDP going to exports and the fact that it overlaps with GDP not going to exports (i.e. GDP going to domestic sales). Once again, there is no simple relationship between value-

countries where it was generated. Then, the value of intermediate inputs (already counted in DVA) can be split between different terms and this is where authors have diverging views. In our framework, to be consistent with the way each perspective is formally defined, FVA is equal to zero and the value of intermediate inputs re-embodied in trade is split between DDC and FDC (see Figure 1). DDC is DVA from the same exporting country counted multiple times, while FDC is DVA from other countries double-counted.

Figure 1. Decomposition of gross exports from a world perspective

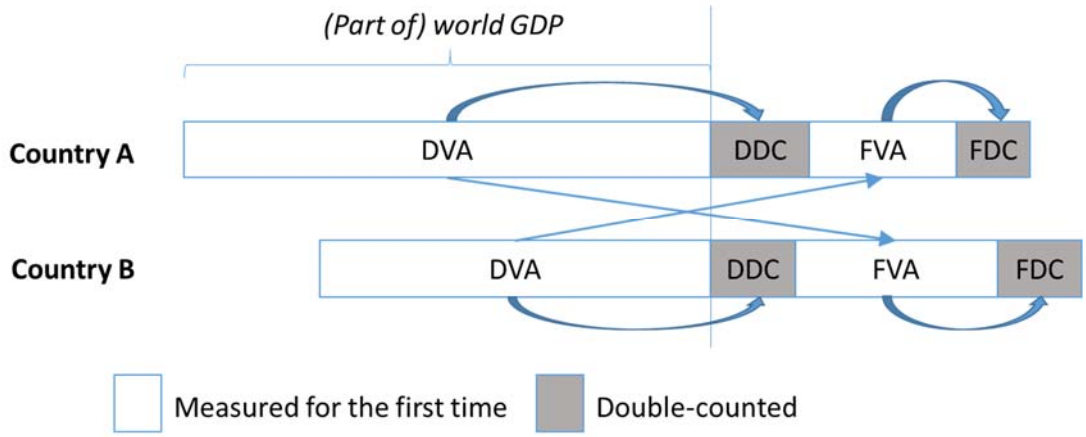


The country perspective is the approach where double-counting can be defined as value added (domestic or foreign) that crosses the border of the exporting economy more than once (the definition proposed by Borin and Mancini, 2017). When allocating value added found in exports across countries, this approach disregards what is measured in other countries and only takes the perspective of the exporting economy. In this case, FVA is not equal to zero but corresponds to value added that crosses the

added and gross concepts that include intermediate inputs such as gross exports or domestic sales.

border of the exporting economy for the first time while FDC corresponds to foreign value added coming back to the exporting economy multiple times (see Figure 2).

Figure 2. Decomposition of gross exports from a country or bilateral perspective



The bilateral perspective is similar to the country perspective (Figure 2), but double-counting is this time defined as the value added crossing the 'bilateral' border of the exporting economy more than once. With more than two countries, this double-counting is by definition smaller as value added coming back to the exporting economy as part of other bilateral trade flows is no longer regarded as double-counted from the perspective of this specific bilateral export flow. Therefore, the sum of bilateral measures (for all terms) are not consistent with the decomposition of aggregate exports of the country (i.e. with partner world), a property identified by Los and Timmer (2018).

3. Research approach

3.1 A general framework for the world, country and bilateral perspective

We start with the standard Leontief (1936) input-output framework extended to G countries and N sectors in an inter-country input-output (ICIO) table, as it is usually

done in the trade in value-added literature. The basic input-output relationship states that all gross output must be used either as an intermediate good or as a final good:

$$\mathbf{x} = \mathbf{A}\mathbf{x} + \mathbf{Y}\mathbf{d} \tag{1}$$

where, **x** is the $NG \times 1$ gross output vector, **Y** is the $NG \times G$ final demand matrix, in which **d** is a unit column $G \times 1$ vector, and **A** is the $NG \times NG$ I-O coefficients matrix. We define $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$ as the Leontief inverse matrix (with **I** the $NG \times NG$ identity matrix) and $\mathbf{v}$ ($1 \times NG$ vector) as the direct value-added coefficients in the ICIO. Each element of $\mathbf{v}_i$ ($1 \times N$ vector) gives the share of direct domestic value added in the output of country *i*.

In this sub-section, our objective is to recreate the basic equations of the Leontief model but separating out gross exports from the rest of gross output. As such, our methodology is close to the hypothetical extraction method proposed by Los et al. (2016). But the value-added decomposition (and definition of double-counting) depends on whether we separate out exports of all countries (world perspective), exports of a single country (country perspective) or exports to a single partner (bilateral perspective). The framework is 'general' in the sense that we can keep the same equations but with different extraction matrices when separating out exports. The fact that we have the same equations ensures the consistency of concepts and definitions of terms across the different approaches.

For the world perspective, we split the output vector into an exports vector **e=[** $\mathbf{e}_1$**,**

$\mathbf{e}_2$ **,…,** $\mathbf{e}_i$**,…,** $\mathbf{e}_G$**]** $(1\times NG)$ with the exports of all countries in the ICIO.[7] For the country perspective, the exports vector becomes the array of exports in country *i* corresponding to elements $\mathbf{e}_i$ and zeros elsewhere (**e=[0,…,** $\mathbf{e}_i$**,…,0]**). For the bilateral perspective, the elements are replaced by bilateral exports (e.g. bilateral exports from country *i* to *j*, **e=[0,…,** $\mathbf{e}_{ij}$**,…,0]**). In all cases, the remaining term is **h** with **x=e+h**.

Then, the following accounting equations can be obtained: $\mathbf{e} = \mathbf{A^I(e+h)} + \mathbf{Y^I d}$ and $\mathbf{h} = \mathbf{A}^*\mathbf{(e+h)} + \mathbf{Y}^*\mathbf{d}$, where $\mathbf{A^I}$ is the given export measurement matrix including the IO coefficients for the use of intermediate inputs from one country into another country. For the elements in matrix $\mathbf{A^I}$, we have:

$$\mathbf{A^I_{sr}} = \begin{cases} \mathbf{A}_{sr}\ if\ s \neq r, 0 \text{ otherwise (world perspective)} \\ \mathbf{A}_{sr}\ if\ s = i\ and\ r \neq i, 0 \text{ otherwise (country perspective for country } i) \\ \mathbf{A}_{sr}\ if\ s = i\ and\ r = j, 0 \text{ otherwise (bilateral perspective, exports from } i \text{ to } j) \end{cases}$$

$\mathbf{A}^*$ is the corresponding extraction matrix, so that we have $\mathbf{A} = \mathbf{A^I} + \mathbf{A}^*$. $\mathbf{Y^I}$ is the foreign final demand for the given exports and $\mathbf{Y}^*$ is the extraction final demand matrix, so that $\mathbf{Y} = \mathbf{Y^I} + \mathbf{Y}^*$.

For example, assuming that we have three countries *i*, *j* and *k*, the intermediate inputs coefficients matrix is:

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[7] We omit subscripts related to industries to simplify the presentation of the framework but all matrices and vectors refer to an inter-country input-output table with *G* countries and *N* industries, as previously indicated. Note that extracting exports of a single industry in the exporting country also leads to different results and can be used to obtain a decomposition of gross exports at the industry level (the fourth approach previously mentioned).

$$\mathbf{A} = \begin{pmatrix} \mathbf{A}_{ii} & \mathbf{A}_{ij} & \mathbf{A}_{ik} \\ \mathbf{A}_{ji} & \mathbf{A}_{jj} & \mathbf{A}_{jk} \\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{A}_{kk} \end{pmatrix}$$

With the world perspective where exports of all countries are extracted (**e=[e**$_i$,**e**$_j$,**e**$_k$,**]**), the corresponding matrices are:

$$\mathbf{A}^{\mathbf{I}}_{w} = \begin{pmatrix} \mathbf{0} & \mathbf{A}_{ij} & \mathbf{A}_{ik} \\ \mathbf{A}_{ji} & \mathbf{0} & \mathbf{A}_{jk} \\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{0} \end{pmatrix} \text{ and } \mathbf{A}^{*}_{w} = \begin{pmatrix} \mathbf{A}_{ii} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_{jj} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{A}_{kk} \end{pmatrix}$$

Such matrices define double-counting as the value added that crosses borders more than once, as in KWW. Note that borders are defined based on exports and the production system described in the A matrix.

With the country perspective, gross exports from country *i* to other countries are extracted (**e=[e**$_i$, **0**,**0]**) and the **A** matrix is split into:

$$\mathbf{A}^{\mathbf{I}}_{c} = \begin{pmatrix} \mathbf{0} & \mathbf{A}_{ij} & \mathbf{A}_{ik} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \text{ and } \mathbf{A}^{*}_{c} = \begin{pmatrix} \mathbf{A}_{ii} & \mathbf{0} & \mathbf{0} \\ \mathbf{A}_{ji} & \mathbf{A}_{jj} & \mathbf{A}_{jk} \\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{A}_{kk} \end{pmatrix}$$

With such matrices, double-counting is now defined as value added crossing the border of the exporting economy *i* more than once. This country perspective is originally in Los et al. (2016) –but without defining FVA and FDC- and also described in Borin and Mancini (2017) as one potential decomposition.

Finally, if we measure value added in bilateral exports between country *i* and *j* (**e=[e**$_{ij}$,**0**,**0]**), the matrices become:

$$\mathbf{A}_b^{\mathbf{I}} = \begin{pmatrix} \mathbf{0} & \mathbf{A}_{ij} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} \end{pmatrix} \text{ and } \mathbf{A}_b^{*} = \begin{pmatrix} \mathbf{A}_{ii} & \mathbf{0} & \mathbf{A}_{ik} \\ \mathbf{A}_{ji} & \mathbf{A}_{jj} & \mathbf{A}_{jk} \\ \mathbf{A}_{ki} & \mathbf{A}_{kj} & \mathbf{A}_{kk} \end{pmatrix}$$

This time, double-counting is the value added crossing the bilateral border of the exporting economy *i* and its trading partner *j* more than once.

Using these different extraction matrices for the decomposition of value added in gross exports, our framework provides a clear definition of the 'boundaries' of exports and the meaning of double-counting with the world, country and bilateral perspective. It is also intuitive that the Leontief inverse based on these different matrices of inputs coefficients cannot lead to the same value-added decomposition.

After re-arrangement, the accounting relationship between exports and final demand in destination countries in the ICIO model can be expressed as:

$$\mathbf{e} = \widetilde{\mathbf{A}}\mathbf{e} + \widetilde{\mathbf{Y}}\mathbf{d} \tag{2}$$

with $\widetilde{\mathbf{Y}} = \mathbf{Y}^{\mathbf{I}} + \widetilde{\mathbf{A}}\mathbf{Y}^{*}$ and $\widetilde{\mathbf{A}} = \mathbf{A}^{\mathbf{I}}(\mathbf{I} - \mathbf{A}^{*})^{-\mathbf{1}}$.

Each element of the $\widetilde{\mathbf{A}}$ matrix describes how domestic intermediate goods are sent abroad (or sold domestically) to produce one unit of exports in foreign countries (or sales in the domestic economy). For example, the element $\widetilde{\mathbf{A}}_{ji}$ ($N \times N$ matrix) means that in order to produce one unit of exports in country *i*, country *j* needs to produce $\widetilde{\mathbf{A}}_{ji}$ units of intermediate inputs that are then embodied in exports in country *j*. $\widetilde{\mathbf{A}}_{ji}\mathbf{e}_i$ ($N \times 1$ vector) means that country *j* needs to produce $\widetilde{\mathbf{A}}_{ji}\mathbf{e}_i$ intermediate inputs for exports $\mathbf{e}_i$ ($N \times 1$ vector) in country *i*. We can regard $\widetilde{\mathbf{A}}$ as the 'direct exports requirements matrix'. Re-arranging equation (2) above, we obtain $\mathbf{e} = \widetilde{\mathbf{B}}\widetilde{\mathbf{Y}}\mathbf{d}$,

and $\widetilde{\mathbf{B}} = (\mathbf{I} - \widetilde{\mathbf{A}})^{-1}$, similar to $\mathbf{B} = (\mathbf{I} - \mathbf{A})^{-1}$ in the IO model. We can define matrix $\widetilde{\mathbf{B}}$ as the 'total exports requirements matrix'. Still we have $\widetilde{\mathbf{B}} = \mathbf{I} + \mathbf{A^I B}$.

3.2 Value added in exports

Before decomposing gross exports, we need to introduce an important equation clarifying the relationship between the Leontief inverse and the extraction matrix. For any extraction matrix $\mathbf{A}^*$, if we define $\mathbf{B}^* = (\mathbf{I} - \mathbf{A}^*)^{-1}$, we can show that:

$$\mathbf{B} = \mathbf{B}^* \widetilde{\mathbf{B}} = \mathbf{B}^* (\mathbf{I} + \mathbf{A^I B}) \tag{3}$$

Merging with the total value-added coefficients matrix (**vB**) (or total value-added multipliers), the equation can be re-written as:

$$\mathbf{ue} = \mathbf{vBe} = \mathbf{vB}^* (\mathbf{I} + \mathbf{A^I B})\mathbf{e} = \mathbf{vB}^* \mathbf{e} + \mathbf{vB}^* \mathbf{A^I Be} \tag{4}$$

Where $\mathbf{u}$ is a $1 \times NG$ unit vector. Equation (4) splits gross exports into two parts: $\mathbf{vB}^* \mathbf{e}$ and $\mathbf{vB}^* (\mathbf{A^I B})\mathbf{e}$, as discussed below.

With respect to $\mathbf{vB}^* \mathbf{e}$ and coming back to equation (2), for each element $\mathbf{e}_i$ ($N \times 1$ vector) that are exports from country *i*, all the intermediate inputs needed are $\sum_j^G \widetilde{\mathbf{A}}_{ji} \mathbf{e}_i$. We can thus calculate the value added in exports of country *i* as $\mathbf{w}(i)^{\mathrm{T}} = \mathbf{e}_i - \sum_j^G \widetilde{\mathbf{A}}_{ji} \mathbf{e}_i$ (where $\mathbf{w}(i)$ is a $1 \times N$ vector). This value added does not only include country *i*'s value-added (DVA) but also other countries' value-added (FVA). We can then express the value added in exports in the form of a $1 \times NG$ vector $\widetilde{\mathbf{V}}$, defined as:

$$\widetilde{\mathbf{V}} \mathbf{u}^{\mathrm{T}} = \mathbf{u}\big(\mathbf{I} - \widetilde{\mathbf{A}}\big)\mathbf{e} = \mathbf{u}(\mathbf{I} - \mathbf{A})(\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{e} = \mathbf{v}(\mathbf{I} - \mathbf{A}^*)^{-1}\mathbf{e} = \mathbf{vB}^* \mathbf{e} \tag{5}$$

where $\mathbf{u}^{\mathrm{T}}$ is the transpose of vector **u**, i.e. a column $NG \times 1$ unit vector. This equation

explicitly measures value added when it enters a specific 'border' and is embodied in exports for the 'first time'. But the definition of the 'border' depends on the initial extraction matrix.

For the other term in equation (4), $\mathbf{vB}^{*}(\mathbf{A}^{\mathbf{I}}\mathbf{B})\mathbf{e}$, $\mathbf{A}^{\mathbf{I}}$ is the matrix that was used to identify exports and separate them from the rest of gross output. This matrix has the ICIO coefficients for the use of intermediate inputs from one country into another country. It defines the concept of 'border' when decomposing exports. With the country perspective, it is the border between the specific exporting economy and other countries, while for the bilateral perspective it corresponds to the border between the exporting economy and its partner country. With the world perspective, the concept of 'border' encompasses all exporting economies so that once value added leaves the country of origin, it is recorded as DVA in exports and then as double-counting when crossing more borders and leaving other countries.

Coefficients $\mathbf{A}^{\mathbf{I}}\mathbf{B}$ in equation (4) point at flows of value added crossing the 'border' twice and we can interpret $\mathbf{vB}^{*}(\mathbf{A}^{\mathbf{I}}\mathbf{B})\mathbf{e}$ as the expression for double-counting. It measures value added (domestic or foreign) that has crossed the given border (as defined by $\mathbf{A}^{\mathbf{I}}$) more than once and which is already accounted for in the $\mathbf{vB}^{*}$ expression. As we can see, this double-counting depends on the definition of $\mathbf{A}^{\mathbf{I}}$.

In a nutshell, our framework starts with the definition of an identification matrix $\mathbf{A}^{\mathbf{I}}$ and corresponding extraction matrix $\mathbf{A}^{*}$ and provides the following decomposition for exports of country *i*:

$$\mathbf{te}_i = \mathbf{v}_i \mathbf{B}_{ii}^* \mathbf{e}_i + \mathbf{v}_i [\mathbf{B}^* \mathbf{A}^{\mathbf{I}} \mathbf{B}]_{ii} \mathbf{e}_i + \sum_{j \neq i}^{G} \mathbf{v}_j \mathbf{B}_{ji}^* \mathbf{e}_i + \sum_{j \neq i}^{G} \mathbf{v}_j [\mathbf{B}^* \mathbf{A}^{\mathbf{I}} \mathbf{B}]_{ji} \mathbf{e}_i \quad (6)$$

where $\mathbf{t}$ is a $1 \times N$ unit vector. On the right-hand side of equation (6), the first term is DVA, the second term is DDC, the third term is FVA and the last term is FDC. The equation does not change for the different perspectives, except that when decomposing bilateral exports one needs to replace $\mathbf{e}_i$ with $\mathbf{e}_{ij}$ and corresponding identification and extraction matrices with bilateral form matrices. Equation (6) does not change but $\mathbf{B}^*$ and $\mathbf{A}^{\mathbf{I}}$ are different matrices across the different perspectives, based on how the extraction matrix $\mathbf{A}^*$ was defined. We can already see in equation (6) that $\mathbf{B}_{ii}^*$ does not change between the country and world perspective but is a different matrix in the bilateral perspective. As for FVA, it has different values with the world, country and bilateral perspective based on $\mathbf{B}_{ii}^*$.

3.3 Looking more closely at the world perspective

With the world perspective, we can also use equation (6) to decompose the exports of specific countries (i.e. map world trade to the country level). However, it should be noted that in the world perspective, the third term is: $\sum_{j \neq i}^{G} \mathbf{v}_j \mathbf{B}_{ji}^* \mathbf{e}_i = 0$. There is no FVA with the world perspective, only DVA and double-counting (DDC and FDC). What authors do when they introduce a FVA term with the world perspective is actually a decomposition of FDC into two terms based on whether the foreign double-counted value added is coming for the first time to the exporting economy or for the second time (or multiple times). If we split the fourth term in equation (6) into

$\sum_{j\neq i}^{G}\mathbf{v}_j[\mathbf{B}^*\mathbf{A}^{\mathbf{I}}\mathbf{B}^*]_{ji}\mathbf{e}_i+\sum_{j\neq i}^{G}\mathbf{v}_j[\mathbf{B}^*\mathbf{A}^{\mathbf{I}}(\mathbf{B}-\mathbf{B}^*)]_{ji}\mathbf{e}_i$, we can find that these sub-terms match what is called FVA and FDC in Borin and Mancini (2017) or in Miroudot and Ye (2017). This can be regarded as a source-based decomposition of FDC.

However, we believe that it would clarify the understanding of the gross exports decomposition and help users navigate across the different frameworks by not referring to FVA and FDC for these two terms but by introducing them as an additional decomposition of FDC in the world perspective. This way, a single and consistent definition can be kept for what is called FVA across the different perspectives.

Finally, we can also show with our framework that the KWW decomposition is based on the world perspective and just has a different way of decomposing the FDC term. Going back to equation (2), we can expand this equation for the exports of a specific country *i*;

$$\mathbf{e}_i=\sum_{j}^{G}\widetilde{\mathbf{A}}_{ij}\mathbf{e}_j+\sum_{j}^{G}\widetilde{\mathbf{Y}}_{ij}=\sum_{j\neq i}^{G}\mathbf{A}_{ij}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{e}_j+\sum_{j\neq i}^{G}[\mathbf{Y}_{ij}+\mathbf{A}_{ij}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{Y}_{jj}]$$

Using this equation, we can re-write the fourth term in equation (6) as follows:

$$\begin{aligned}&\sum_{j\neq i}^{G}\mathbf{v}_j[\mathbf{B}^*\mathbf{A}^{\mathbf{I}}\mathbf{B}]_{ji}\mathbf{e}_i=\sum_{j\neq i}^{G}\mathbf{v}_j\mathbf{B}_{ji}\mathbf{e}_i\\&=\sum_{s\neq i}^{G}\sum_{j\neq i}^{G}\mathbf{v}_s\mathbf{B}_{si}\mathbf{Y}_{ij}+\sum_{s\neq i}^{G}\sum_{j\neq i}^{G}\mathbf{v}_s\mathbf{B}_{si}\mathbf{A}_{ij}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{Y}_{jj}+\sum_{s\neq i}^{G}\mathbf{v}_s\mathbf{B}_{si}[\sum_{j\neq i}^{G}\mathbf{A}_{ij}(\mathbf{I}-\mathbf{A}_{jj})^{-1}\mathbf{e}_j]\end{aligned}\quad(7)$$

Equation (7) provides a decomposition similar to KWW with the first term being ‘foreign value in final goods exports’ (term 7 in KWW), the second term ‘foreign value in intermediate goods exports’ (term 8 in KWW) and the third term ‘double counted intermediate exports produced abroad’ (term 9 in KWW). There is nothing

wrong with such a decomposition but from our point of view it is a decomposition of FDC with the world perspective, splitting the value added already measured in DVA of other countries (and thus double-counted) on the basis of whether it transits in the exporting economy as part of a final good directly absorbed in the partner country, as part of an intermediate good absorbed in the partner country or as part of an (intermediate) good that will be exported to third countries. Going back to equation (2) in our framework, we could also say that the two first terms in equation (7) can be interpreted as double-counted value going into exports for final demand in the destination country and the last term double-counted value going into exports for intermediate demand. And this would be a way to define a sink-based decomposition of the FDC term with the world perspective. But in our framework, FVA and FDC are calculated the same way across the different perspectives and there is no source-based or sink-based version of FVA and FDC (as opposed to Borin and Mancini, 2019). Only further decompositions of these terms introduce a source-based and sink-based approach, as the origin and destination of value added is unambiguously defined in our framework for DVA, DDC, FVA and FDC.

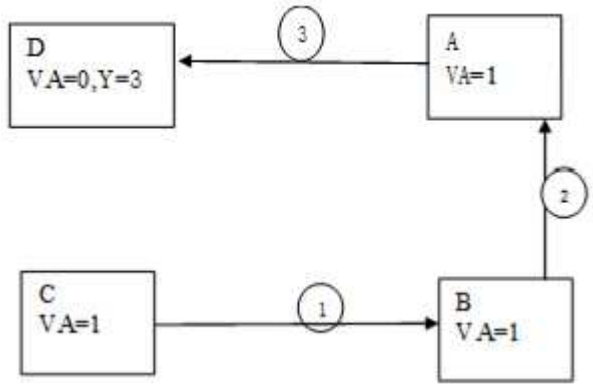
4. Numerical examples and empirical analysis

To further understand the difference between the world perspective and country perspective, as well as between aggregate and bilateral measures, we first develop in this Section three simple numerical examples that illustrate differences in double-counting and what they mean. For each example, we show both the ‘global value chain’

(GVC) and the corresponding ICIO as already the information in the ICIO has collapsed the different stages of production. This is where we can see the assumptions made to go from the ICIO (which is the only empirical information we have) to an allocation of value added that could recreate the GVC (something we do not observe in aggregate statistics but that authors try to recreate on the basis of the ICIO when decomposing gross exports).

4.1 Numerical examples illustrating the difference between the country perspective and world perspective

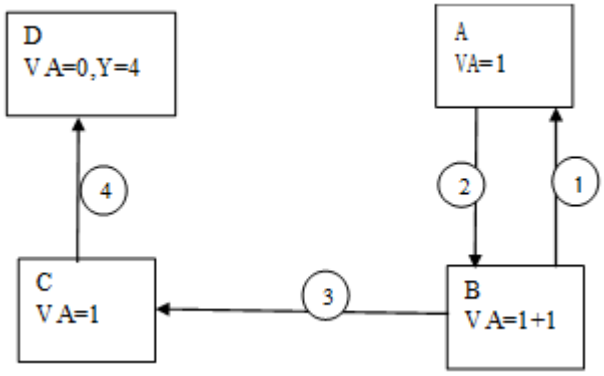
Case 1: country C exports 1 unit of intermediate inputs to country B, then B exports 2 units to country A (using as input the production of country C), then A exports 3 units to country D (using as input the production of country B) that are finally absorbed by D. The value chain and the corresponding ICIO table can be represented as below[8]:



	A	B	C	D	A	B	C	D
A	0	0	0	0	0	0	0	3
B	2	0	0	0	0	0	0	0
C	0	1	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0
VA	1	1	1	0				

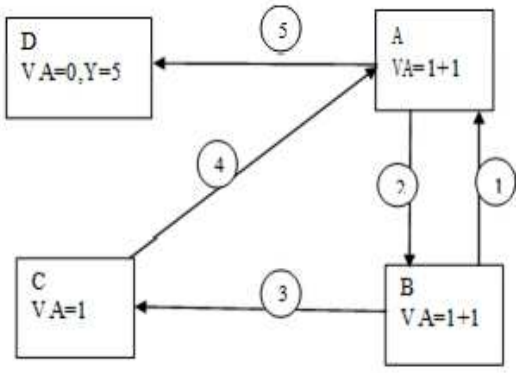
[8] These examples omit industries for simplicity. The first block in the ICIO is the intermediate consumption matrix and the second block the final demand. VA is at the bottom of the intermediate consumption matrix. VA = value-added, Y = final demand.

Case 2: country B exports 1 unit to country A at the beginning, then A exports 2 units back to country B, then B re-exports 3 units to country C, then C exports 4 units to country D, finally absorbed by D.



	A	B	C	D	A	B	C	D
A	0	2	0	0	0	0	0	0
B	1	0	3	0	0	0	0	0
C	0	0	0	0	0	0	0	4
D	0	0	0	0	0	0	0	0
VA	1	2	1	0				

Case 3: this case is similar to the previous one but with a simple modification. For the fourth step in the value chain, country C now exports 4 units back to country A again, then A exports 5 units to country D, finally absorbed by D.



	A	B	C	D	A	B	C	D
A	0	2	0	0	0	0	0	5
B	1	0	3	0	0	0	0	0
C	4	0	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0
VA	2	2	1	0				

Next, we show results for the decomposition of gross exports using the country perspective and the world perspective. For the world perspective and to facilitate the comparison with results from other authors, we then further decompose the FDC term into FDC1 and FDC2, based on a source-based approach similar to Borin and Mancini

(2017) or Miroudot and Ye (2017) and a sink-based approach similar to KWW. FDC1 in this case corresponds to what these authors have defined as FVA and FDC2 is similar to their FDC.[9] For the country perspective, results are consistent with Los et al. (2016)[10] and the equations proposed by Arto et al. (2019) lead to the same results.[11]

Table 2. Decomposition of gross exports for case 1

		Country and world perspective		Country perspective		World perspective		World perspective source-based		World perspective sink-based, KWW	
	Gross exports	DVA	DDC	FVA	FDC	FVA	FDC	FDC1	FDC2	FDC1	FDC2
A	3	1	0	2	0	0	2	1	1	2	0
B	2	1	0	1	0	0	1	1	0	0	1
C	1	1	0	0	0	0	0	0	0	0	0
D	0	0	0	0	0	0	0	0	0	0	0

[9] There are obviously many ways to decompose FDC. KWW is just one example of a potential sink-based approach. Borin and Mancini (2017 and 2019) introduce another type of sink-based approach. To calculate FDC1 and FDC2, we use the equations provided in Section 3 where the source-based and sink-based (KWW) decomposition of FDC were introduced.

[10] We thank Bart Los for having provided information on the calculation of FVA according to this framework, something that was not included in the published paper but developed by the authors.

[11] Note however that the country perspective already included in Borin and Mancini (2017) and further explained in Borin and Mancini (2019) has equations not providing the same results, with possibly a mixed approach between the country and world perspectives. With respect to Johnson (2018), equations are specified only for two countries and therefore there is no difference between the country and world perspective.

Table 3. Decomposition of gross exports for case 2

		Country and world perspective		Country perspective		World perspective		World perspective source-based		World perspective sink-based, KWW	
	Gross exports	DVA	DDC	FVA	FDC	FVA	FDC	FDC1	FDC2	FDC1	FDC2
A	2	1	0.33	0.5	0.17	0	0.67	0.5	0.17	0	0.67
B	4	2	0.67	1	0.33	0	1.33	1	0.33	0	1.33
C	4	1	0	3	0	0	3	1.5	1.5	3	0
D	0	0	0	0	0	0	0	0	0	0	0

Table 4. Decomposition of gross exports for case 3

		Country and world perspective		Country perspective		World perspective		World perspective source-based		World perspective sink-based, KWW	
	Gross exports	DVA	DDC	FVA	FDC	FVA	FDC	FDC1	FDC2	FDC1	FDC2
A	7	2	0.8	3	1.2	0	4.2	1.5	2.7	3	1.2
B	4	2	0.8	0.86	0.34	0	1.2	0.57	0.63	0	1.2
C	4	1	0.3	2.08	0.62	0	2.7	1.5	1.2	0	2.7
D	0	0	0	0	0	0	0	0	0	0	0

The results illustrate the advantages and disadvantages of the various approaches. First, it should be noted that there is no difference in the way DVA and DDC are measured with the world perspective and the country perspective. There is also no difference related to the source-based or sink-based approach for the four main terms (DVA, DDC, FVA and FDC). However, we see important differences in the way value added is allocated to FVA and FDC between the country perspective and world perspective and how FDC is further split between the source-based and sink-based version of the world perspective (FDC1 and FDC2).

Based on our framework, the world perspective and the country perspective provide clearly distinct results, with FVA always equal to zero in the case of the world

perspective. We believe this is the right way to introduce a world perspective, consistent with the definition of double-counting as value added crossing more than one border. Still, FVA can also be equal to zero with the country perspective when the exporting economy does not use any foreign input to produce its exports (as country C in case 1 and country D in all cases). A positive FVA term different from zero means that exports are produced with foreign value added and it conceptually makes sense to start distinguishing FVA from FDC only when taking the perspective of a specific country where foreign value added that enters the production of exports for the first time needs to be distinguished from FVA coming back several times.

We can nonetheless look at world exports from the point of view of the contribution of a specific country and find within the FDC term, the concept of 'first time' measurement of foreign value added, leading to FDC1 measured from a source-based perspective. One can also be interested in value added absorbed by trade partners or further transiting in global value chains, leading to a different type of FDC1 as in KWW. As illustrated in the above tables, the measurement of FDC1 and FDC2 with this latter approach depends more on the country of absorption. But the sum of FDC1 and FDC2 is the same for the source-based and sink-based version of the world perspective. And it is equal to the foreign content (i.e. FVA+FDC in the country perspective).

When value-added crosses more than one country and is still not finally absorbed, KWW counts this value added as 'pure' foreign double-counting (i.e. FDC2). This is reflected in the gross exports decomposition of country B in all cases and of

country C in case 3. Because the export flow is not absorbed by the direct importer, the value of FDC1 is 0 in the KWW framework. This approach has often been criticised as leading to counter-intuitive results with high values for FDC. But it can be understood in our framework where all the foreign content is in FDC and where its further allocation has nothing 'right' or 'wrong' or 'counter-intuitive'. With the KWW decomposition, the foreign value added that 'continues to travel' after the next partner country is passed to the FDC2 term. It is consistent with the definition of double-counting in the KWW paper (the world perspective). Crossing the border of the exporting economy plus the border of another country downstream before final absorption also qualifies to become part of FDC.

The world perspective approach in its source-based version can lead to high values for FDC2 for a different reason. This time, it is based on the source country and the fact that when value added has already crossed a border and is measured a second time in the exports of another country upstream, this value added contributes to FDC2. This is illustrated with the decomposition of gross exports of country A in case 1, country C in case 2, or countries B and C in case 3. If we look at country A in case 1, it exports 3 units of value-added: one unit is domestic, one unit is from country B and one from country C. In the world perspective, the 1 unit of value added from country C is measured as DVA in C's exports and measured as FDC1 in B's exports because it is the first time it leaves its originating country and the foreign country where it was embodied. Since this value added was already recorded by country B as FDC1, it goes to FDC2 when it leaves country A, even if it has not crossed twice the border of country A and

leaves it for the first time. This is different with the country perspective where from the point of view of A, the same unit of value added from country C becomes part of FVA. Because in the country perspective, double-counting starts only when value added crosses twice the border of the same (exporting) economy.

We can therefore suggest using a country perspective when trying to disentangle DVA from FVA in exports of a specific country and trying to remove only double-counting related to inputs coming back to the same exporting economy. While the world perspective was introduced in KWW and the early literature on the measurement of trade in value-added terms, its usefulness might be limited to an analysis of what causes double-counting in world trade and further analysis of GVCs at the world level. As soon as some country-level analysis is needed and a FVA term becomes useful, there is no reason to resort to the world perspective. A source-based or sink-based decomposition of FVA and FDC can also be done with the country perspective to get further insights on what happens to value added in third countries before entering the FVA or FDC terms. There is no reason either to use the world perspective to derive some country-specific GVC indicators based on the decomposition of gross exports.

4.2 Numerical example illustrating the difference between the bilateral perspective and the country perspective

Using case 3 as a numerical example, Table 5 provides the decomposition of bilateral exports for country A. It illustrates how in the bilateral analysis the concept of border becomes the bilateral border. For example, in bilateral trade between country A and D, since there is no value added crossing the border between A and D more than once,

there is no double-counting.

Table 5. Decomposition of bilateral exports of country A in case 3 with the bilateral perspective

	Exports	DVA	DDC	FVA	FDC
A_B	2	0.57	0.23	0.86	0.34
A_C	0	0	0	0	0
A_D	5	2	0	3	0
Sum	7	2.57	0.23	3.86	0.34
Aggregate	7	2	0.8	3	1.2

Table 5 also confirms that the sum of bilateral measures is not equal to the aggregate value-added decomposition, as pointed by out by Los and Timmer (2018). It can be seen in the last row of Table 5 where the aggregate figures are reproduced from Table 4 and are not equal to the sum of DVA, DDC, FVA and FDC across partners. Since the border in bilateral exports has been changed, we can easily explain within our framework why the decomposition of bilateral exports is not the mapping of the aggregate decomposition across partners.[12] For some analysis, it might be useful to have bilateral values that sum to the aggregate ones. But in this case authors should be clear about the fact that they map the aggregate measure across partners or that they

[12] Going back to equation (6), the fact that DVA and FVA are higher with the bilateral perspective as compared to the country perspective is also intuitive since $\mathbf{B}^*_{ii}$ and $\mathbf{B}^*_{ji}$ have more zeroes in the off-diagonal elements of the country's row in the $\mathbf{A}^*$ matrix of the country perspective.

have defined double-counting on the basis of the country border and not the bilateral border (i.e. that they use a country perspective). For most of bilateral trade analysis, whether it is to assess the impact of trade barriers, the effect of trade agreements or calculating the jobs or CO2 embodied in exports to a specific partner, the bilateral perspective seems to us the most appropriate, as in this case the DVA term matters and one does not want to exclude from this DVA term the domestic value added already accounted for in other bilateral relationships. For example, when assessing DVA in exports of A to D in case 3, it makes more sense to work with DVA=2 (as in Table 5) rather than DVA=(2/7)*5=1.43, which would be the DVA measured when applying the country perspective to bilateral exports. For the same reason, a bilateral measure based on the world perspective would not be the most appropriate metric either.

4.3 Empirical results using the WIOD database

Numerical examples are useful to understand differences across decompositions, but one could argue that actual GVCs are more complex and that maybe differences are exaggerated using these simple examples. In Table 6, we provide results based on the World Input-Output Database (WIOD) tables (Timmer et al., 2015) for 44 countries for the year 2014 (aggregating across industries).

Table 6. Decomposition of gross exports, % (WIOD, 2014)

Country	Gross exports (million USD)	Country and world perspective		Country perspective		World perspective		World perspective source-based		World perspective sink-based, KWW	
		DVA	DDC	FVA	FDC	FVA	FDC	FDC1	FDC2	FDC1	FDC2
AUS	287,162	85.83	0.14	14.01	0.02	0	14.03	10.47	3.56	10.08	3.95

AUT	210,995	63.86	0.29	35.65	0.21	0	35.85	24.7	11.15	23.24	12.61
BEL	383,014	53.96	0.39	45.21	0.44	0	45.65	32.71	12.94	30.81	14.84
BGR	31,698	61.81	0.03	38.13	0.03	0	38.16	28.02	10.14	25.51	12.65
BRA	270,262	87.16	0.06	12.77	0.01	0	12.78	9.69	3.09	9.69	3.09
CAN	563,511	75.77	0.42	23.68	0.12	0	23.8	19.03	4.77	20.29	3.52
CHE	352,570	74.48	0.2	25.23	0.09	0	25.32	18.29	7.03	19.96	5.37
CHN	2,425,464	83.15	0.94	15.69	0.23	0	15.91	11.68	4.23	12.69	3.22
CYP	9,347	71.94	0.04	28	0.02	0	28.02	20.12	7.9	17.14	10.87
CZE	161,570	54.02	0.33	45.36	0.29	0	45.65	30.73	14.92	30.34	15.31
DEU	1,682,253	71.85	1.39	26.12	0.63	0	26.75	18.77	7.98	19.22	7.53
DNK	170,293	62.47	0.17	37.26	0.1	0	37.36	27.31	10.05	28.99	8.37
ESP	389,005	68.87	0.26	30.71	0.16	0	30.86	22.56	8.3	23.02	7.84
EST	18,266	56.55	0.09	43.28	0.08	0	43.36	28.83	14.53	30.77	12.59
FIN	100,453	64.97	0.12	34.82	0.09	0	34.9	25.83	9.07	24.01	10.9
FRA	759,654	72.28	0.46	27.06	0.2	0	27.26	19.44	7.82	19.96	7.3
GBR	751,599	80.74	0.29	18.89	0.08	0	18.97	13.84	5.13	13.7	5.27
GRC	56,261	69.58	0.04	30.35	0.02	0	30.38	23.19	7.19	22.61	7.77
HRV	23,269	72.68	0.05	27.25	0.02	0	27.27	19.37	7.9	19.36	7.91
HUN	116,445	48.13	0.16	51.51	0.2	0	51.71	35.46	16.25	35.84	15.87
IDN	210,599	82.74	0.11	17.13	0.02	0	17.15	12.61	4.54	13.15	3.99
IND	369,456	79.28	0.11	20.57	0.04	0	20.6	16.13	4.47	15.78	4.82
IRL	262,751	50.65	0.13	49.12	0.1	0	49.23	41.7	7.53	39.39	9.83
ITA	588,585	73.63	0.32	25.91	0.14	0	26.06	18.5	7.56	18.94	7.11
JPN	817,514	76.41	0.32	23.15	0.12	0	23.27	17.89	5.38	17.19	6.09
KOR	697,935	64.79	0.35	34.65	0.22	0	34.87	26.74	8.13	26.03	8.84
LTU	32,722	64.29	0.05	35.61	0.05	0	35.66	27.42	8.24	24.9	10.76
LUX	118,439	33.96	0.08	65.79	0.16	0	65.95	57.23	8.72	49.29	16.67
LVA	14,719	68.98	0.1	30.87	0.05	0	30.92	20.78	10.14	21.87	9.04
MEX	368,185	66.44	0.26	33.17	0.12	0	33.29	25.43	7.86	29.7	3.59
MLT	13,420	34.51	0.03	65.39	0.07	0	65.46	44.67	20.79	51.53	13.93
NLD	575,068	63.15	0.8	35.6	0.45	0	36.05	26.22	9.83	23.84	12.2
NOR	188,131	82.96	0.25	16.75	0.04	0	16.8	12.16	4.64	10.88	5.91
POL	251,642	69.04	0.27	30.56	0.13	0	30.7	21.52	9.18	20.82	9.87
PRT	76,633	68.84	0.09	31.01	0.06	0	31.07	21.47	9.6	22.42	8.65
ROU	77,648	73.31	0.07	26.59	0.03	0	26.63	18.35	8.28	18.17	8.46
RUS	493,789	92.36	0.14	7.49	0.01	0	7.49	5.27	2.22	4.86	2.64
SVK	82,119	51.86	0.2	47.72	0.22	0	47.93	30.87	17.06	33.75	14.18
SVN	30,812	62.63	0.08	37.24	0.05	0	37.3	25.15	12.15	25.29	12
SWE	235,354	71.2	0.28	28.38	0.14	0	28.52	20.75	7.77	19.81	8.71
TUR	249,783	71.47	0.13	28.35	0.06	0	28.41	19.31	9.1	22.02	6.39
TWN	369,923	58.17	0.4	41.15	0.29	0	41.43	29.87	11.56	28.08	13.35
USA	1,927,091	87.15	0.7	12.04	0.12	0	12.16	9.45	2.71	8.84	3.32
ROW	3,833,149	73.53	1.68	24.24	0.55	0	24.79	20.83	3.96	17.88	6.91

Table 6 highlights that in the actual world and despite the proliferation of GVCs, the value added in exports is mostly domestic. DDC is generally a small percentage of gross exports (most of the time below 1%) and here there is no difference between the country and world perspective. Very few domestic intermediate inputs exported come back to the exporting economy. When it comes to FVA and FDC, we find important differences across the different approaches, as it was the case with the simple numerical examples. FDC in the world perspective gives an idea of how important is double-counting in world gross trade statistics (and how the different countries contribute to this overall double counting). FDC with the country perspective is small and can be seen as the symmetric of DDC for foreign inputs. FDC is generally smaller than DDC, indicating that FVA coming back to the same exporting economy is something even less common than DVA coming back.

Results for FDC1 and FDC2 with the source-based and sink-based approach in the world perspective also confirm the intuition provided by the numerical examples. High figures for FDC terms can only be understood as part of the world perspective where all the foreign content is double-counted. For example, with the KWW approach, FDC1 is equal to about 30% for the Czech Republic and FDC2 is equal to about 15%. As noted by Borin and Mancini (2017) when they suggest a country perspective, it would be unsatisfactory to regard FDC1 as FVA and FDC2 as foreign double-counting as in KWW. But understood as two components of FDC with a sink-based approach, one can regard this result for the Czech Republic as the country being part of complex value chains. The high FDC2 highlights that a high share of the foreign content comes

from vertical trade downstream in the value chain.

Finally, in Table 7, we can see the decomposition of bilateral exports of China with all WIOD partners in 2014 using the bilateral perspective. DDC and FDC become very small at the bilateral level, suggesting that it might not even be worth trying to identify these double-counted terms when working with bilateral data.[13] While also small, the difference between the sum of bilateral measures and the aggregate measure confirms that decompositions where they are by definition equal tend to overestimate DDC and FDC and as such could be less accurate than decompositions simply omitting DDC and FDC and just using a foreign content and domestic content.

Table 7. Decomposition of Chinese bilateral exports, % (WIOD, 2014)

Bilateral relationship	Gross exports (million USD)	DVA	DDC	FVA	FDC
CHN_AUS	48,459	83.22	0.011	16.77	0.002
CHN_AUT	4,242	83.99	0.000	16.01	0.000
CHN_BEL	11,804	84.93	0.002	15.07	0.000
CHN_BGR	1,029	83.77	0.000	16.23	0.000
CHN_BRA	38,988	82.92	0.004	17.08	0.001
CHN_CAN	49,636	83.65	0.003	16.35	0.001
CHN_CHE	7,293	82.67	0.001	17.33	0.000
CHN_CYP	583	85.03	0.000	14.97	0.000
CHN_CZE	8,898	79.04	0.002	20.95	0.001
CHN_DEU	88,465	83.31	0.013	16.67	0.003
CHN_DNK	6,199	85.45	0.001	14.55	0.000

[13] China was picked as one country where circular trade is more pronounced. When going at the industry level, double counting becomes even more marginal. The same logic applies as when going from the extraction of exports with world to bilateral exports. Extracting a single industry allocates to this industry the FVA that would otherwise be regarded as double counted across different industries.

CHN_ESP	21,496	84.48	0.001	15.52	0.000
CHN_EST	1,073	81.97	0.000	18.03	0.000
CHN_FIN	6,870	83.64	0.001	16.36	0.000
CHN_FRA	41,291	83.97	0.004	16.03	0.001
CHN_GBR	51,850	83.11	0.003	16.89	0.001
CHN_GRC	4,190	83.53	0.000	16.47	0.000
CHN_HRV	714	83.63	0.000	16.37	0.000
CHN_HUN	5,396	78.66	0.002	21.34	0.000
CHN_IDN	34,969	83.46	0.005	16.54	0.001
CHN_IND	44,869	82.48	0.004	17.51	0.001
CHN_IRL	3,471	82.75	0.001	17.25	0.000
CHN_ITA	28,865	84.17	0.002	15.83	0.000
CHN_JPN	172,861	82.77	0.051	17.17	0.013
CHN_KOR	101,924	81.78	0.145	18.04	0.039
CHN_LTU	947	83.87	0.000	16.13	0.000
CHN_LUX	911	79.94	0.000	20.06	0.000
CHN_LVA	654	84.60	0.000	15.40	0.000
CHN_MEX	38,330	81.47	0.002	18.53	0.001
CHN_MLT	455	84.06	0.000	15.94	0.000
CHN_NLD	42,640	81.94	0.007	18.06	0.001
CHN_NOR	4,563	84.41	0.000	15.59	0.000
CHN_POL	14,316	82.28	0.001	17.72	0.000
CHN_PRT	2,251	83.47	0.000	16.53	0.000
CHN_ROU	2,614	81.64	0.000	18.35	0.000
CHN_RUS	65,198	87.73	0.003	12.26	0.001
CHN_SVK	2,002	81.47	0.000	18.53	0.000
CHN_SVN	1,369	84.46	0.000	15.54	0.000
CHN_SWE	11,173	85.80	0.001	14.20	0.000
CHN_TUR	23,149	81.88	0.002	18.12	0.000
CHN_TWN	43,622	80.29	0.167	19.50	0.051
CHN_USA	347,311	82.36	0.013	17.62	0.003
CHN_ROW	1,038,525	85.14	0.501	14.25	0.111
Sum	**2,425,464**	**83.86**	**0.230**	**15.86**	**0.052**
Aggregate	**2,425,464**	**83.15**	**0.938**	**15.69**	**0.225**

5. Concluding remarks

This paper has further investigated the concept of double-counting in the decomposition of gross exports and found that differences in definitions and approaches to the measurement of double-counting can explain why several decompositions are proposed

in the literature with results that are different for the four main terms of the decomposition: DVA, DDC, FVA and FDC.

When looking at world exports, the concept of 'border' between countries does not exist and world gross exports can be decomposed into value-added (equal to world GDP in exports) and intermediate inputs used to produce exports (that are the double counting part). FVA is equal to zero with this world perspective. What authors are doing is further decomposing the FDC term based on a variety of approaches, which can be source-based (i.e. referring to the origin of value added the first time it is measured) or sink-based (i.e. referring to the destination of value added and how it is used in third countries).

When looking at exports of specific countries, one can start to introduce a FVA term (which is DVA in other countries) and double-counting is defined as the value added coming more than once to the same exporting economy. For FVA, it implies that some value added could have already been measured as FVA somewhere else (with some potential global FVA double counting) but the perspective is the exporting economy.

The country perspective is conceptually closer to what seems to be the objective of the trade in value-added literature in the analysis of GVCs, i.e the identification of the foreign contribution in exports removing double-counting related to inputs coming back to the exporting economy. Such an approach was proposed by Los et al. (2016) but without deriving the formulas for FVA and FDC. As such, our paper offers a useful complement to calculate these additional terms and to fully decompose gross exports

with the country perspective.

In addition, our framework allows the decomposition of bilateral exports in a consistent way and confirms that double-counting in bilateral exports should not be regarded as a bilateral mapping of double-counting in exports with world. When introducing bilateral borders, one needs to redefine what is double-counted as crossing the same border twice (or more) no longer has the same meaning.

Empirically, we find small values for double-counted terms with the country perspective approach and even smaller in bilateral exports. For some analysis and for countries not too much involved in 'circular trade', we could argue that papers that have used a simple approach in decomposing value added in trade with just a domestic content and foreign content (such as the decomposition provided in the OECD Trade in Value-Added database) have used rather good estimates. Analysis based on the KWW framework or the world perspective approach with high shares of FDC are not wrong but it is important for users to understand how double counting is defined in these approaches and the difference with the country perspective approach.

As illustrated by the different frameworks found in the literature, the decomposition of value added in gross export is highly technical. The challenge is to provide measures of trade in value added based on ICIOs that do not overlook this complexity but are more accessible and clearer in terms of what is measured for researchers and policy-makers. Our framework has the advantage of proposing a relatively simple approach that accommodate the world, country and bilateral perspective using the same equations (but with different extraction matrices) and

providing consistent definitions for DVA, DDC, FVA and FDC.

References

Arto, I., E. Dietzenbacher and J. M. Rueda-Cantuche (2019) Measuring bilateral trade in terms of value added. *JRC Technical Reports,* European Commission.

Borin, A. and M. Mancini (2019) Measuring What Matters in Global Value Chains and Value-Added Trade. *World Bank Policy Research Working Paper,* No. 8804.

Borin, A. and M. Mancini (2017) Follow the Value Added: Tracking Bilateral Relations in Global Value Chains. *MPRA Paper*, No. 82692.

Daudin, G., C. Rifflart and D. Schweisguth (2011) Who Produces for Whom in the World Economy? *Canadian Journal of Economics*, 44, 1403–37.

Dietzenbacher, E. (1997) In vindication of the Ghosh Model: A reinterpretation as a price model. *Journal of Regional Science*, 37, 629–651.

Foster-McGregor, N. and R. Stehrer (2013) Value Added Content of Trade: A Comprehensive Approach. *Economics Letters*, 120, 354–357.

Gereffi, G. and K. Fernandez-Stark (2016) *Global Value Chain Analysis: A Primer*. 2nd edition, Duke Center on Globalization, Governance and Competitiveness.

Johnson, R. C. (2018) Measuring Global Value Chains. *Annual Review of Economics*, 10, 207–236.

Johnson, R. C. and G. Noguera (2012) Accounting for Intermediates: Production Sharing and Trade in Value Added. *Journal of International Economics*, 86, 224–36.

Koopman, R., Z. Wang and S.-J. Wei (2014) Tracing value-added and double counting in gross exports. *American Economic Review*, 104, 459–494.

Leontief, W. (1936) Quantitative Input and Output Relations in the Economic System of the United States. *Review of Economic and Statistics*, 18, 105–125.

Los, B. and M. P. Timmer (2018) Measuring Bilateral Exports of Value Added: A Unified Framework. *NBER Working paper*, No. 24896.

Los, B., M. P. Timmer and G. J. de Vries (2016) Tracing Value-Added and Double

Counting in Gross Exports: Comment. *American Economic Review*, 106, 1958–1966.

Miroudot, S. and M. Ye (2017) Decomposition of Value-Added in Gross Exports: Unresolved Issues and Possible Solutions. *MPRA Paper*, No. 83273.

Miroudot, S. and M. Ye (2019) Multinational production in value-added terms. *Economic Systems Research*, DOI: 10.1080/09535314.2019.1701997.

Nagengast, A. J. and R. Stehrer (2016) Accounting for the Differences Between Gross and Value Added Trade Balances. *World Economy*, 39, 1276–1306.

Oosterhaven, J. (1988) On the Plausibility of the Supply-Driven Input-Output Model. *Journal of Regional Science*, 28, 203–217.

Timmer, M. P., E. Dietzenbacher, B. Los, R. Stehrer and G. J. de Vries (2015) An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production. *Review of International Economics*, 23, 575–605.

Trefler, D. and S. C. Zhu (2010) The Structure of Factor Content Predictions. *Journal of International Economics*, 82, 195–207.

Wang, Z., Wei, S. J. and K. Zhu (2013) Quantifying International Production Sharing at the Bilateral and Sector Levels. *NBER Working Paper*, No. 19677, revised in February 2018.