What Is Driving The TFP Slowdown? Insights From a Schumpeterian DSGE Model

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Abstract

In this paper, I incorporate a Schumpeterian mechanism of creative destruction in a medium-scale DSGE framework. In the model, a sector of profit-maximizing innovators invests in R&D and endogenously generates productivity gains, ultimately determining the economy's growth rate. I estimate the model using Bayesian methods on U.S. data of the last 25 years (1993q1-2018q4) in order to disentangle the key forces underlying the productivity slowdown experienced by the US economy since the early 2000s. In contrast with the previous literature, I exploit Fernakl (2014) data on TFP, factor utilization and labour quality to discipline the production function, and find that the bulk of the TFP slowdown is due to a decrease in innovation's ability to generate TFP gains. These findings challenge the view of a large part of the literature, according to which the recent TFP dynamics in the US are mostly driven by demand slumps and/or liquidity crunches.

Keywords: DSGE model, Endogenous TFP, Schumpeterian Growth, TFP Slowdown

JEL No: E5, E24, E32, O47

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1. Introduction

During the last decade, the US economy experienced a significant slowdown in the Total Factor Productivity growth rate (henceforth, TFP). This phenomenon is not limited to the US economy, but involves the bulk of the developed world economies. Figure 1 provides a snapshot of the global dimension of the TFP slowdown: in France, Germany, Japan, and the UK, TFP started decelerating in the mid-2000s and did not catch up with the pre-crisis trend since then. A number of influential empirical works attributes the cause of the occurrence of the TFP slowdown to the recent Global Financial Crisis. Ball (2014), Hall (2014), and Reifschneider et al. (2015) argue that potential output declined consequently to the Great Financial Crisis due to the decline in capital accumulation and slower TFP growth and estimate sizable potential output losses. This deep downturn of potential output produced a marked gap between data and the theoretical predictions provided by pre-crisis standard macroeconomic models. Whereas conventional DSGE models predicted a reversal of GDP to the pre-crisis trend, as Guillen-Quintana and Jiménez (2019) point out, in 2017 (10 years after the beginning of the Global Financial Crisis) there was still no sign of reversal to the pre-crisis trend in most of the major developed economies.

In the attempt of explaining the TFP slowdown within the context of standard macroeconomic models, an important, growing literature is developing models of the business cycle in which stochastic shocks to the fundamentals of the economy can permanently affect potential output by temporarily hampering the process of technological development. Amongst the most notable contributions, Bianchi, Kung, and Morales (2019), Guillen-Quintana and Jiménez (2019), and Anzogregui et al. (2019) develop DSGE models featuring mechanisms of endogenous TFP. In this class of models a demand slump, or a liquidity crunch, can dampen the innovation process and generate the TFP slowdown that we observe in the data. However, two crucial testable implications common to all the explanations provided by these models fail when the theoretical implications are confronted with the data. First, as argued in Fernald (2015), the TFP slowdown started before the beginning of the recession (see Figure 1). Hence, if the productivity slowdown had to be attributed to a demand or a liquidity contraction, the TFP slowdown should have followed, not preceded the Global Financial Crisis. Second, both the demand-side and the liquidity crunch explanation imply that R&D-based innovation proxies should have sharply (or persistently) dropped consequently to the Great Recession, in order to generate such a persistent slowdown. By contrast, as I show in Figure 2, R&D-to-GDP rose in the US during the Global Financial Crisis, and the R&D-to-TFP ratio exhibited only a short contraction1.

In order to rationalize these facts, in this work, I develop and estimate a model of the US business cycle featuring R&D efficiency and effectiveness frictions, and argue that the productivity slowdown is due to a decrease in the ability of innovations to produce aggregate TFP gains2.

1Typically, following Jones (1995), the semi-endogenous growth literature, adopted R&D-to-TFP as a standard proxy to evaluate the innovation effort with respect to the level of technological development of a country. Accordingly, in quantitatively realistic semi-endogenous Schumpeterian growth models, productivity gains are a function of the R&D-to-TFP ratio (in levels). Another parallel stream of literature, including Young (1998) and Peretto (1998) (scale-effect free fully-endogenous growth models) argue that TFP gains are a function of R&D investment levels. Nevertheless, the same considerations hold even when real R&D investment growth is considered.
2Several works in the empirical literature suggests that the recent productivity dynamics in the US might be driven by a
Total Factor Productivity for selected countries (1993-2016, 2008=100).


In blue: R&D-to-TPF ratio in the US (from the Federal Reserve Bank of St. Francisco, on the left-hand axis, 2008=100).

In red: R&D-to-GDP Ratio (rom the Bureau of Economic Analysis, on the right-hand axis).

Shaded regions are US NBER recessions.

decrease of innovation’s productivity potential, as for instance Gordon (2012) or Bloom (2017), although these two works study the phenomenon from a long-run perspective.
In the model, I introduce novel supply-side friction and innovation-specific shocks that affect the process of technological development of the economy at different stages. Namely, the model features R&D adjustment costs in terms of innovation probability and allows for shocks on a) the R&D investment-specific technology, that affects the capacity of R&D investment of generating innovation, and b) the innovation step i.e. the capability of innovation of generating productivity gains. The presence of this friction implies that the effectiveness of investments in technological activities is affected by sudden changes in the amount of R&D invested. Namely, when a firm decides to adjust the level of its investment in R&D, the marginal probability of innovation per unit of R&D investment decreases. This feature captures the idea, as in Terry (2015), that R&D is less effective when volatile. The presence of this friction generates very persistent R&D dynamics (relatively to GDP). Hence, this feature is able to rationalize why the R&D-to-GDP ratio rose during the Great Recession although the incentive to innovate collapsed. Relying on a more sophisticated identification structure, and imposing a greater amount of discipline on the production function, I find that the bulk of the TFP slowdown has to be attributed to a decrease in the ability of R&D investment to generate productivity gains, as in the narrative of Gordon (2012), who identifies the cause of the TFP slowdown in the scarcity of substantial, path-breaking innovations (unrelated to the Global Financial Crisis).

Whereas in standard DSGE models as Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005) potential output follows a predetermined exponential path\(^3\), in the model presented in Section 3, it can be affected by innovation-specific disturbances and by all the forces that conventionally in standard models drive the business cycle. In the model, I introduce a Schumpeterian growth engine in a standard DSGE model, by modeling that mechanism of creative destruction that, in the words of Schumpeter (1942), "is the essential fact about capitalism". The endogenous TFP mechanism provides a transmission channel from stochastic shocks hitting the economy to the innovation sector, in which endogenous entry and exit firm dynamics give rise to creative destruction cycles.

The introduction of a Schumpeterian growth engine in a DSGE model is appealing for three main reasons. First, Schumpeterian growth theory "can account for several interesting facts about competition and growth which no other growth theory can explain" (Aghion et al. 2014). Second, the Schumpeterian growth theory can rely on the support of a set of empirical micro-level studies including Liu (1993), Campbell (1998), and Brandt et al. (2011), that show the process of creative destruction to be one of the leading forces underlying productivity gains. Third, the recent US TFP slowdown was accompanied by a slowdown in business dynamism. Figure 3 shows that the rate of creation of new establishments and the productivity improvements in the US on the 1993-2018 period, slowed down in parallel to the TFP. This evidence is of great relevance for the validation of the Schumpeterian interpretation of the TFP slowdown, since Schumpeterian growth theory predicts that TFP gains should materialize jointly with the appearance of new players on the market. Deeper microfoundations based on creative destruction mechanisms might thus be essential for a more realistic characterization of recent potential output dynamics. Compared to the previous works as Anzoategui et al. (2019), this paper shifts the

\(^3\)Except for the presence of standard TFP shocks
focus on the innovation efficiency/effectiveness issue\(^4\), which is often neglected in the DSGE model literature, although regarded as a crucial issue in the empirical literature (as for instance in Bloom et al. (2018)). For these reasons, in the model, the development and the adoption of new technologies, as well as firms’ entry and exit dynamics, are endogenized in a Schumpeterian innovation sector, giving rise to creative destruction cycles. In this framework, therefore, standard DSGE theory is reconciled with the endogenous growth literature tradition.

\[\text{FIG.3 - NEW FIRM BIRTH RATES AND TFP DYNAMICS IN THE US (1993-2018)}\]

\[(a)\text{ Blue Solid Line: New Firm Birth Rates in the US (Source: BLS, on the left-hand axis, in \% on the total number of firms).}\]

\[(b)\text{ Black Dotted Line: Utilization-Adjusted \% TFP gains (From Fernald 2014, on the Right-Hand Axis)}\]

\[(c)\text{ Red Solid Line: Utilization-Adjusted \% TFP gains filtered with a HP-filter (\(\lambda = 100\), RHA, correlation with (a): 0.61)}\]

\[(d)\text{ Red Dashed Line: Utilization-Adj. \% TFP gains filtered with a 5-years centered moving average (RHA, corr. with (a): 0.61)}\]

The model I will present in Section 3 is based on the Smets and Wouters (2007) workhorse, and embeds a simple and highly tractable endogenous TFP growth mechanism à la Benigno and Fornaro (2018), that connects productivity gains to innovators’ incentive to innovate. In a nutshell, the model exhibits the following features:

(i) productivity is driven by research and development investment; (ii) the development and adoption of new technologies is determined by the optimizing behavior of a sector of innovators; (iii) price and wages dynamics are affected by nominal and real rigidities; (iv) stochastic shocks to fundamentals stochastically hit the economy.

The model is estimated with Bayesian techniques using data on seven standard observables of the US business cycle (GDP, consumption, physical capital investment, investment in R&D, inflation, worked hours, and the policy rate) and three observables from the Fernald (2014) database (TFP, labour quality, and factor utilization). The use of the last three series is crucial in the exercise as they allow to impose a rigorous discipline on the

\(^4\text{I refer here to amount of innovation produced per unit of R&D as innovation efficiency (number of patents/R&D) and to the amount of TFP gains produced with a given amount of innovation as innovation effectiveness (\(\Delta \text{TFP} / \text{number of patents}\)).}
production function, and assure that the concept of productivity in the model, is the same adopted by the empirical literature. The model does not feature any kind of financial frictions\(^5\) due to two main reasons. First, as documented by Veugelers and Kalck (2018), the bulk of the R&D activities are typically performed by large firms\(^6\), which are only marginally affected by liquidity constraints. Second, studies like Hao and Hafe (1993) or Altomonte et al. (2016), show that financial conditions do not affect the amount of R&D investment performed by large firms.

The remainder of the paper is organized as follows: in Section 2 I review the literature connected to the paper, in Section 3 I describe the general equilibrium conditions that characterize the economy, in Section 4 I describe the set of measurement equations, the solution, the identification, and the estimation techniques, in Section 5 I discuss the main results, and in Section 6 I conclude and discuss the main policy implications.

2. Related Literature

This paper builds on the modern endogenous TFP literature, which typically borrows endogenous productivity growth mechanisms from the theoretical endogenous growth literature of the early 1990s and incorporates them into DSGE models. This paper, in particular builds on the seminal contribution of Aghion and Howitt (1992), who develop a theoretical framework in the spirit of the Schumpeterian idea of creative destruction.


Another strand of literature focuses on the policy implications of endogenous TFP. Annicchiarico et al. (2011), Annicchiarico and Rossi (2013), Annicchiarico and Pelloni (2019) analyze the implications for optimal monetary policy of the presence of an endogenous TFP channel. Ikedo and Kurozumi (2019) study optimal operational interest rules in an endogenous TFP model augmented with financial frictions. Cozzi et al. (2017) show that a fully-fledged DSGE model augmented with a creative destruction engine can empirically outperform standard models. Bondiani and Oh (2019) show that uncertainty shocks might affect medium-term TFP dynamics by dampening investment in R&D. Eventually, this paper relates to the news shocks literature follow-

\(^5\)Note that this is not equivalent to say that in the model financial shocks are irrelevant for TFP dynamics determination.

\(^6\)According to Veugelers and Kalck (2018), in 2015, the firms within the top 1% of global R&D spenders accounted for the 27% of global R&D expenditure, the top 10% accounted for the 71%.
ing Beaudry and Portier (2006). R&D based models with endogenous TFP, similarly to news shocks, produce expectations of future productivity gains once a unit of R&D is invested (see Miranda-Agrippino et al. (2018) for an application with R&D and patent application data). The core mechanism of this paper is inspired by the work of Benigno and Fornaro (2018), who build on Aghion and Howitt (1992) in order to develop a stagnation trap model combining elements from the Zero Lower Bound and the Schumpeterian Growth literature.

This paper methodologically contributes to the literature by developing and estimating a Schumpeterian DSGE with detailed production function data, such as the TFP, utilization, and labour quality. This novel identification strategy allows disentangling for the effect of technological and non-technological TFP shocks. No previous study in the literature about DSGE models with endogenous TFP is concerned about comparing and matching the identified productivity series with the empirical estimates, and disentangling the technological component of TFP improvements from non-technological factors at play. This paper contributes to the literature by analyzing the role of R&D efficiency frictions within the TFP slowdown process within the context of a full-blown DSGE model, and showing that the ability of innovation to generate sustained TFP gains played an important role in the TFP slowdown.

3. The Model

In this section, I describe a medium-sized Schumpeterian DSGE model whose core structure is to a large extent based on Smets and Wouters (2007, henceforth SW). The SW structure is extended to allow for endogenous productivity dynamics, through the introduction of a sector of innovators à la Benigno and Fornaro (2018). On one hand, the Smets and Wouters (2007) structure allows giving the model an important quantitatively realistic dimension, so that the model is of straightforward interest for policymakers willing to implement endogenous TFP considerations into their policy evaluation process. On the other hand, the introduction of an endogenous TFP engine à la Benigno and Fornaro (2018) provides theoretical microfoundations useful to evaluate the impact of standard Smets and Wouters (2007) shocks and frictions on TFP dynamics, and to disentangle the sources of TFP cyclical fluctuations.

The SW building blocks are equation wise preserved in their original fashion, so that the model here presented nests the SW economy. This approach allows for straightforward comparison with the baseline SW during empirical exercises. The economy features four different categories of producers: (i) intermediate goods producers, (ii) semi-finished good producers, (iii) innovators, and (iv) final good producers. This sophistication of the market structure aims to mimic the complexity of the present-day supply chain, in which good production is the result of the assembling of several technological and non-technological components. Intermediate producers operate in a monopolistically competitive setup and use labour and capital services in order to produce a continuum of varieties of the intermediate good. Semi-finished good producers are in perfect competition and aggregate intermediate goods in order to sell them to the innovators. Innovators operate in a monopolistically competitive environment with free access to R&D and produce a continuum of varieties of the innovative
good. Eventually, a perfectly competitive sector of good producers aggregate innovative goods and sell them to households.

### 2.1 Households

Households maximize an intertemporal utility function in consumption $C_{i,t}$, hours worked $L_{i,t}$, bonds $B_{i,t}$, investment $I_{i,t}$, the capital utilization rate $U_{i,t}^h$, and the technology adoption rate $U_{i,t}^g$. As standard in medium scale DSGE exhibiting wage rigidities, each household is a monopolistically competitive supplier of a differentiated type of labour. $h$ is the external habit parameter, $R_t$ the rate of return of bonds, $R_i^c$ the capital rental rate, $R_i^g$ the technology adoption cost, $P_t$ the price level, $T_i$ is a lump sum tax, $T_i^c$ is a lump-sum transfer to the innovators, $W_t^h$ the hourly wage, $K_i$ the stock of physical capital, $A_t$ the stock of adopted technologies, $a_t$ is the rental rate respectively of the capital and the technology stock, $D_{i,t}^{\text{firms}}$ and $D_{i,t}^{\text{inn}}$ are the amount of dividends distributed by the firms and the innovators, $\delta$ is the capital depreciation rate and $S$ is the investment convex adjustment cost function, where $S(1)=1$ and $S''>0$. The households’ intertemporal decision problem can be formulated as:

$$
\max_{\mathcal{E}_t} E_t \sum_{s=0}^{\infty} \beta^s \left[ \frac{1}{1-\sigma_c} (C_{i,t+s} - hC_{i,t+s-1})^{1-\sigma_c} \right] \exp \left( \frac{\sigma_c - 1}{1+\sigma_t} I_{i,t+s}^{1+\sigma_t} \right)
$$

subject to

$$(1)$$

$$
C_{i,t+s} + I_{i,t+s} + \frac{B_{i,t+s}}{\epsilon_i^t R_{i,t+s} P_{t+s}} - T_{t+s} - T_i^c \leq \frac{B_{i,t+s-1}}{P_{t+s}} + \frac{W_{t+s}^h L_{i,t+s}}{P_{t+s}} + \frac{R_{i,t+s} K_{i,t+s-1}^{1+\delta}}{P_{t+s}}
$$

$$
+ \frac{R_{i,t+s} U_{i,t+s}^a A_{i,t+s-1}}{P_{t+s}} - a (U_{i,t+s}) K_{i,t+s-1} - a (U_{i,t+s}) A_{i,t+s-1} + \frac{D_{i,t}^{\text{firms}}}{P_{t+s}} + \frac{D_{i,t}^{\text{inn}}}{P_{t+s}}
$$

$$
K_{i,t} = (1-\delta) K_{i,t-1} + \epsilon_i^t \left[ 1 - S \left( \frac{I_{i,t}}{I_{i,t-1}} \right) \right] I_{i,t}
$$

The shock on bonds’ risk premia $\epsilon_i^b$ captures inefficiencies in credit supply or temporary fluctuations in agents’ risk aversion. $\epsilon_i^t$ is an investment-specific technology shock, which leads to fluctuations in physical capital investment adjustment cost. Both $\epsilon_i^b$ and $\epsilon_i^t$ follow an AR(1) process in logs such that $\ln \epsilon_i^b = \rho_b \ln \epsilon_i^{b,-1} + \eta_i^b$ and $\ln \epsilon_i^t = \rho_t \ln \epsilon_i^{t,-1} + \eta_i^t$ where $\eta_i^b \sim N(0, \sigma_b)$ and $\eta_i^t \sim N(0, \sigma_t)$. Similarly, the utilization rate $\epsilon_i^u$ is subject to an exogenous disturbance so that $\ln \epsilon_i^u = \rho_u \ln \epsilon_i^{u,-1} + \eta_i^u$ where $\eta_i^u \sim N(0, \sigma_u)$. This feature aims to capture the effect of productive capacity under-utilization due to exogenous reasons, such as strikes, malfunctions, or production scheduling inefficiencies.

### 2.2 Intermediate Good Producers

The standard intermediate good is produced by a sector of firms using labour and capital:

$$
Y_{i,t} = \epsilon_i^f A_{i,t}^{1-\alpha} K_{i,t}^\alpha \left( Q_{i,t}^L \right)^{1-\alpha}
$$

I assume the absence of a fixed cost and introduce a labour quality adjustment $Q_{i,t}^L$ in order to bring the
specification of the production function as close as possible to Fernald (2014). The stochastic disturbance \( \epsilon_t \) is an exogenous productivity shock that aims at capturing non-technological, unexplained business cycle frequency fluctuations in TFP, as cross-sectoral reallocation or measurement errors. \( A_t \) is the endogenous, technological component of TFP, which is determined by the stock of technology \( \hat{A}_t \) and its adoption rate \( U_t^a \).

The labour quality disturbance captures the idea that TFP is affected by exogenous variations in workers’ skills and education and finds its one-to-one empirical counterpart in the Aaronson and Sullivan (2001) labour quality series used in Fernald (2014), i.e. \( Q_t^i = \epsilon_t^q \). The exogenous TFP shock and the labour quality shock follow AR(1) exogenous processes such that where \( \ln \epsilon_t^i = \rho_x \ln \epsilon_{t-1}^i + \eta_t^i \) and \( \eta_t^i \sim N(0, \sigma_i) \) where \( i = x, q \).

As standard in literature, the intermediate good producers fix prices according to Calvo pricing with partial indexation. Let \( \tilde{P}_t \) be the newly set price, and \( X_{t,s}^p \) a state variable that assumes value 1 when \( s = 0 \) and value \( \Pi_{t=s+1}^{t-s} \pi_{t-1}^{s+1} \) when \( s > 0 \). Firms optimize prices according to the following objective function:

\[
\max \ E_t \sum_{s=0}^{\infty} \xi_s \beta^s \Lambda_{t,t+s} \left[ \frac{\tilde{P}_t}{P_{t+s}} \left( \Pi_{t+1}^{t+s} \pi_{t+s}^{t-s} - MC_{t+s} \right) \right] Y_{t+s} \tag{5}
\]

\[
s.t. \ Y_{t,t+s} = Y_{t+s} G^{-1} \left( \frac{P_t X_{t,s}^p}{P_{t+s} \tau_{t+s}} \right) \tag{6}
\]

Let \( \xi_s \) be the probability of being allowed to reoptimize prices, \( \tau_t \) the price indexation coefficient and \( \tau_t = \int_0^1 G' \left( \frac{Y_t}{Y_t} \right) \frac{Y_t}{Y_t} \text{d}i \), the Calvo pricing scheme implies the following equation for the aggregate price index:

\[
P_t = (1 - \xi_p) P_t G^{-1} \left[ \frac{P_t \tau_t}{P_t} \right] + \xi_p \pi_{t-s}^{s+1} \pi_{t-1}^{s+1} P_{t-1} G^{-1} \left[ \frac{\pi_{t-1}^{s+1} \pi_{t-1}^{s+1}}{P_t} \right] \tag{7}
\]

### 2.3 Employment Agencies

As standard in the literature, a labour union sets wages and sells labour to an employment agency. The employment agency acquires the differentiated labour services from the workers' union and supplies labour to the standard intermediate good producers by maximizing the following objective function:

\[
\max \ W_i L_i - \int_0^1 W_{i,i} L_{i} \text{d}i \tag{8}
\]

\[
s.t. \ \left[ \int_0^1 H \left( \frac{L_{i,i}}{L_{i,i}} ; \xi_i \right) \text{d}i \right] = 1 \tag{9}
\]

\( W_i \) and \( W_{i,i} \) are the prices of the composite and intermediate labour, and \( H \) is the Kimball aggregator, strictly increasing and concave with \( H(1) = 1 \). The stochastic process \( \epsilon^w_t \) captures changes in the elasticity of demand for labour which result in a wage markup shock and follows an exogenous process such that \( \ln \epsilon^w_t = \rho_w \ln \epsilon^w_{t-1} + \eta^w_t \) where \( \eta^w_t \sim N(0, \sigma_w) \). Let \( \tilde{W}_{i,i} \) be the newly set price, and \( X_{t,s}^w \) a state variable that assumes value 1 when \( s = 0 \) and value \( \Pi_{t=s+1}^{t-s} \pi_{t-1}^{s+1} \) when \( s > 0 \), the labour union's optimization problem under Calvo prices with partial indexation assume the following formulation:
\[
\max \ E_t \sum_{s=0}^{\infty} \xi_w^{s} \beta^{s} \Lambda_{t,t+1} \frac{W_t}{W_{t+s}} \left[ \tilde{W}_{s} \left( H_{t+1}^{\prime} \pi_{t+1}^{w} \right) - W_{t+s}^{h} \right] L_{t+s} \\
\text{s.t. } L_{t+s,i} = L_{t+s} H_{t+1} \left( \frac{W_t X_{t,s}^{w}}{W_{t+s}^{h}} \right) 
\]

Let \( \xi_p \) be the probability of being allowed to reoptimize wages, \( \iota_w \) the indexation coefficient the resulting aggregate wage index is:

\[
W_{t} = (1 - \xi_w) \tilde{W}_{t} H_{t+1}^{-1} \left[ \frac{W_t \pi_{t}^{w}}{W_{t}} \right] + \xi_w \gamma \pi_{t-1}^{w} \pi_{t}^{1-r_{p}} W_{t-1} H_{t}^{-1} \left[ \frac{\gamma \pi_{t-1}^{w} \pi_{t}^{1-r_{p}} W_{t-1} \pi_{t}^{w}}{W_{t}} \right] 
\]

### 2.4 Semi-Finished Good Producers

The semi-finished good producers manufacture a unit of the intermediate good \( \tilde{Y}_{t} \) using a continuum of \( i \) varieties. They maximize profits according to the following objective function:

\[
\max \ P_{t} \tilde{Y}_{t} - \int_{0}^{1} P_{i} Y_{i} di \\
\text{s.t. } \int_{0}^{1} G \left( \frac{Y_{i}}{\tilde{Y}_{t}}, \epsilon_{i}^{p} \right) di = 1 
\]

In the constraint of the optimization problem, \( G \) is the Kimball aggregator, which guarantees that the demand for the standard intermediate good \( \tilde{Y}_{t} \) is decreasing in its relative price, while the elasticity of demand is a positive function of the relative price. \( G \) has the properties of being strictly increasing and concave, with \( G(1)=1 \). The stochastic process \( \epsilon_{i}^{p} \) captures changes in the elasticity of demand which result in a mark-up shock and follow an AR(1) process such that \( \ln \epsilon_{i}^{p} = \rho_{p} \ln \epsilon_{i-1}^{p} + \eta_{i}^{p} \) where \( \eta_{i}^{p} \sim N(0, \sigma_{p}) \). The semi-finished goods are then sold to the innovation sector.

### 2.5 Innovators

A continuum of innovators produces the technological good by employing one unit of the semi-finished good, i.e. \( Y_{jt} = Y_{t} \). One might think of technological goods as "patents" developed by innovators in order to improve the production technology used by the intermediate producers. As in Benigno and Fornaro (2018), one innovator emerges as a leader in every period for each sector \( j \) and holds the monopoly position for a single period. Therefore, only those who developed a successful innovation are allowed to produce and sell to the final good producers. The technological good will be priced with a mark-up \( \mu_{j} > 1 \) upon the cost of the final good, where \( \mu_{j} = \frac{1}{\sigma} \) and \( \sigma \) is the elasticity of substitution of the final good producers' sector. The price of the technological component \( P_{jt} \) will be therefore equal to \( \mu_{j} \tilde{Y}_{t} \) and the innovators' profit in nominal terms can be computed as:

\[
\Pi_{jt} = P_{jt} Y_{jt} - \tilde{P}_{j} \tilde{Y}_{t} = (1 + \mu_{j}) \tilde{P}_{j} \tilde{Y}_{t} - \tilde{P}_{j} \tilde{Y}_{t} = \mu_{j} \tilde{P}_{j} \tilde{Y}_{t} 
\]

The innovation sector operates in two different stages in each period. During the first one, all players invest a

\( ^{7} \text{Remind that the representative household is the owner of both type of firms, and therefore allocates resources among the different kinds of firms as a global planner.} \)
given amount of R&D according to their incentive to innovate. With probability \( \Psi_{jt} \), each player will develop an innovation and emerge as a leader in the sector \( j \). In the second phase, in each sector, an innovator is randomly selected according to the distribution of the individual probability of innovations and acts as a competitive monopolist. I assume the probability of innovation to depend on the amount of R&D expenditure invested by the innovator \( j \) normalized by the current stock of technology, as a variant of the semi-endogenous growth literature tradition, following Jones (1995):

\[
\Psi_{jt} = \left( \frac{J_{jt}}{\mathcal{A}_t} \right)^{\phi_{RD}} Z \left( \frac{J_{jt}}{J_{jt-1}} \right) \epsilon_{t}^{RD}
\]

(16)

The normalization by the productivity level implies that R&D investment will be less productive as the economy advances on the quality ladder - and allows to rule out the R&D scale effect, assuring the respect of the balanced growth path conditions. I assume decreasing returns to R&D, where \( \phi_{RD} \) represents a "stepping-on-toes" parameter as in Jones and Williams (1998), and convex R&D investment adjustment cost, where \( Z \) is a convex adjustment cost function. The "stepping-on-toes" effect captures inefficiencies due to the congestion effects, research duplication, knowledge theft, etc. The R&D adjustment cost instead captures the idea, as in Terry (2015), that R&D investment is less effective when volatile. Furthermore, I assume the innovation probability to be affected by a stochastic shock to the adjustment cost \( \epsilon_{t}^{RD} \), which follows an AR(1) process such that \( \ln \epsilon_{t}^{RD} = \rho_{RD} \ln \epsilon_{t-1}^{RD} + \eta_{t}^{RD} \). This shock aims to capture variations in the amount of innovation produced per unit of R&D. Due to the assumption of free access to R&D, the level of R&D investment is determined by the following zero-profit condition:

\[
\Psi_{jt} \Pi_{jt} = J_{jt}
\]

(17)

Similarly as in Benigno and Fornaro (2018), this condition states that the expected profit of innovating and becoming a leader is equal to the amount of R&D invested by the firm. By replacing the individual components, one can obtain that:

\[
\mu_j P_t Y_t \left( \frac{J_{jt}}{\mathcal{A}_t} \right)^{\phi_{RD}} Z \left( \frac{J_{jt}}{J_{jt-1}} \right) \epsilon_{t}^{RD} = P_t J_{jt}
\]

(18)

This equation states that the greater will be the profit innovators can achieve by successfully developing a new vintage of the technological good, the greater will be the amount of R&D they perform. As standard in the literature, I impose the symmetric equilibrium, i.e. every firm invests the same amount in R&D, and through

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8 A more natural alternative might be here assuming a fixed number of lags. For instance, Terry (2015) assume a lag of 1 period. Nevertheless, assuming a number \( k \) of lags delivers unrealistic responses exhibiting kinks at the \( k \)-th period, whereas assuming contemporaneous productivity gains and R&D adjustment frictions delivers very gradual, smoothed productivity gains as in the reduced form evidence by Miranda-Agrippino et al. (2019).

9 Adjustment costs here take the form of \( \left( 1 + \gamma - \frac{J_{jt}}{J_{jt-1}} \right)^{2 \psi_{RD}} \) with \( \psi_{RD} > 1/2 \). This functional form in equilibrium yields \( S(.) = 1, S'(.) > 1, \) and \( S''(.) > 1, \) and implies that economies characterized by higher equilibrium R&D-to-GDP ratios, will incur in higher marginal adjustment costs.

10 In order to preserve tractability, I assume the innovation sector to be financed via a frictionless lump-sum transfer \( T_j^I \) from the household sector.

11 For simplicity, I assume here that the price of the R&D investment good maps one-to-one with the consumption good.
the symmetry assumption, obtaining:

\[
\left( \frac{J_t}{A_t} \right)^{1-\phi_{RD}} = \frac{Y_t}{A_t} \left( \frac{J_t}{J_{t-1}} \right) \epsilon_t^{RD}
\]  

(19)

This condition establishes the aggregate amount of R&D expenditure, which in turn determines the probability of innovation in each sector \( \Psi_t \). By the law of large numbers, in a fraction \( \Psi_t \) of the sectors a player will develop an innovation at every period. Each innovation will allow for a productivity gain \( \gamma_a \) in a given sector, while the non-innovating sectors will maintain the productivity level of the previous period. The term \( \epsilon_t^a \) represents a stochastic shock to the innovation step - i.e. the productivity gain triggered by the occurrence of an innovation. This disturbance, therefore, aims to capture fluctuations in the impact in terms of productivity of technological innovations. The technological stock will thus evolve according to the following law:

\[
\bar{A}_{t+1} = (1 - \Psi_t)(1 - \delta_a) \bar{A}_t + \Psi_t (1 + \gamma_a \epsilon_t^a) \bar{A}_t
\]  

(20)

The first term at the right-hand side of the equation represents the structural TFP level that will characterize the \((1 - \Psi_t)\) sectors that will not innovate, whose productivity level will deteriorate by \((1 - \delta_a)\). The second term of the sum represents the structural productivity level which will characterize the \( \Psi_t \) sectors that will innovate, whose productivity level will increase by \((1 + \gamma_a \epsilon_t^a)\). The latter will be determined by a fixed innovation step \( \gamma_a \) and by a stochastic shock \( \epsilon_t^a \) that follows an AR(1) process in logs so that \( \ln \epsilon_t^a = \rho_a \ln \epsilon_{t-1}^a + \eta_t^a \). By rearranging the previously described terms, I obtain the productivity growth path to be defined by this simple law:

\[
\frac{\bar{A}_{t+1}}{\bar{A}_t} = (1 - \delta_a) + \Psi_t (\delta_a + \gamma_a \epsilon_t^a)
\]  

(21)

2.6 Final Good Producers

A perfectly competitive sector of final good producers operates aggregating a continuum of varieties of the innovative good according to the following production function:

\[
Y_t = \left( \int_0^1 Y_t \sigma dj \right) \sigma \frac{\sigma - 1}{2} \chi
\]  

(22)

Final good producers maximize their profits subject to the previously stated production function, taking as given all intermediate goods prices \( P_{jt} \) and the final good price \( P_t \). Hence, their maximization problem is:

\[
\max P_t Y_t^d \int_0^1 P_{jt} Y_{jt} dj
\]  

(23)

From the maximization problem, one can find the associated input demand functions, i.e.:

\[
Y_{it} = \left( \frac{P_{it}}{P_t} \right) Y_t^d
\]  

(24)

where \( Y_t^d \) is the aggregate demand. Combining with the zero profit condition \( P_t Y_t = \int_0^1 P_{it} Y_{it} di \), one can obtain
the aggregate price index:

\[ P_t = \left( \int_0^1 P_{jt}^{1-\sigma} \, dj \right)^{\frac{1}{1-\sigma}} \]  

(25)

2.7 Government Policies

As standard in the literature, the central bank follows a nominal interest rate rule in order to stabilize output and inflation, such that:

\[ \frac{R_t}{R^*} = \left( \frac{R_{t-1}}{R^*} \right)^{\rho r} \left[ \left( \frac{\pi_t}{\pi^*} \right)^{\rho r} \left( \frac{Y_t}{Y^*} \right)^{\rho r} \right]^{1-\rho r} \left( \frac{Y_t / Y_{t-1}}{Y^* / Y^*_{t-1}} \right)^{r_y} \epsilon_t^r \]  

(26)

The stochastic process \( \epsilon_t^r \) captures the monetary policy shock and follows an exogenous process such that \( \ln \epsilon_t^r = \rho_r \ln \epsilon_{t-1}^r + \eta_t^r \) where \( \eta_t^r \sim N(0, \sigma_r) \).

The government budget constraint is expressed as:

\[ P_t G_t + B_{t-1} = T_t + \frac{B_t}{R_t} \]  

(27)

Government spending as a ratio of the steady-state output \( \epsilon_t^g = \frac{G_t}{Y_t} \) is a random shock which captures the impact of fiscal policy shock and can be described with \( \ln \epsilon_t^g = \rho_g \ln \epsilon_{t-1}^g + \rho_{TFP} TFP_t + \eta_t^g \) where \( \eta_t^g \sim N(0, \sigma_g) \) and \( TFP_t \) is defined as the Solow residual.\(^{12}\)

2.8 Aggregate Resource Constraints

Finally, the aggregate resource constraint assures that the amount of resources produced in the economy equals the total amount of resources demanded by the agents:

\[ Y_t = C_t + I_t + J_t + G_t + a (u_k^t) \bar{K}_{t-1} + a (u_a^t) \bar{A}_{t-1} \]  

(28)

\(^{12}\)The adjustment for \( TFP \) is due to the fact that, as in Smets and Wouters (2007), the shock to the aggregate resource constraint has to capture the (unmodeled) additional contribution to aggregate demand given by the net export component in the data.
4. Solution, Calibration, and Estimation

The model is stationarized by dividing the trending variables by the stock of available technologies $\dot{A}_t$. Therefore, in the system of equilibrium conditions (in Appendix B), all the trending variables are normalized by the technology $\dot{A}_t$, which represents the potential level of TFP. As standard in the literature, I solve the model by computing a first-order approximate solution around the deterministic steady-state and estimate the model using Bayesian techniques. The estimation database is composed of a standard set of quarterly time series for the U.S. economy, augmented with the production function series constructed as in Fernald (2014): real GDP, real consumption, real gross capital formation (except R&D expenditures), worked hours, real wages, the GDP deflator, the Wu-Xia (2016) Federal Funds Rate, real investment in R&D, TFP, factor utilization, and labour quality. The GDP, consumption, gross capital formation, and R&D investment have been extracted from the Bureau of Economic Analysis database. Net investment is defined as gross fixed capital formation less investment in R&D. Consumption and investment are normalized by the GDP deflator. Inflation is computed as the first difference of the log of the GDP deflator. Hours worked, real wages are extracted from the Bureau of Labour Statistics database for the Non-Farm Business Sectors. Additionally to the standard adjustment by BLS average weekly hours, I adjust by labour effort as in Basu et al. (2006). Therefore, both labour utilization and effort are jointly observed with labour supply. Real wages are obtained by deflating nominal wages by the GDP price deflator. Aggregate variables are normalized by the working-age population over 16. Instead of the standard Federal Funds Rate, I estimate the model using the Wu-Xia (2016) Shadow Rate, which has the perk of integrating: additional information about unconventional monetary policy measures implemented by the Federal Reserve. As the Shadow Rate is not constrained at the Zero Lower Bound, its use (i) allows to circumvent the estimation bias implied by the ZLB, (ii) provides relevant information regarding the conditions of the credit market, and (iii) prevents the onset of a stagnation trap à la Benigno and Fornaro (2018). I eventually discipline the production function using the 3 series constructed using Fernald (2014) methodology. The use of these series is crucial for the paper. In the rest of the literature concerning general equilibrium models with endogenous TFP, no other paper disciplines the production function. Without using those, there is no guarantee that the productivity estimates are empirically realistic. By contrast, this paper uses the most conventional TFP estimates available in the data and treats the estimation dataset in order to maximize the compatibility between the productivity concept in the empirical and the DSGE models literature.

The productivity concept used for estimation is a variant of the one used by Fernald (2014) with constant labour and capital shares, as these are typically fixed in general equilibrium models. As I show in Section C of the Appendix (Figure 8), the fixed share assumption has a negligible impact on the productivity estimates. The capital utilization series is constructed following the Basu, Fernald, and Kimball (2013) methodology, which includes adjustments in capital’s workweek and workers’ effort. Finally, labour quality series is constructed following Aaronson and Sullivan (2001) in order to account for fluctuations in workers’ skills and education.

\textsuperscript{13} Typically DSGE model estimation is performed with hours worked series adjusted for labour utilization only. See for instance Smets and Wouters (2007) or Araoztegui et al. (2019).

\textsuperscript{14} Labour quality, utilization, and TFP series are available on the website of the Federal Reserve Bank of St. Francisco.
Further details about the sources of the data and the construction of the dataset are available in Section A of the Appendix.

The TFP model-consistent definition, therefore, includes (a) an exogenous disturbance $\varepsilon_t^x$, accounting for measurement mismatches, reallocation effects, exogenous disturbances not captured by the model, (b) the technology adoption rate $U^n$, (c) the stock available technologies $A$, (d) capital utilization $U^k$, (e) labour quality $Q^l$, and reads as follows:

$$ TFP_t = \varepsilon_t^x (U^n_t A_t)^{1-\alpha} (U^k_t)^{\alpha} (Q^l_t)^{1-\alpha} $$

As standard in the literature, I calibrate one sub-set of structural parameters and estimate the remainder.

I calibrate $\alpha = 0.35$, i.e. the empirical mean of the labour share of output between 1993Q1 and 2018Q4. I calibrate the coefficient of intertemporal risk-aversion and the inverse of the Frisch elasticity to 1. The physical capital stock quarterly depreciation rate $\delta$ is calibrated to 2.5% as in Smets and Wouters (2007). Concerning the knowledge obsolescence parameter $\delta_k$, there is a fundamental disagreement in the empirical estimates. Conventionally, the literature assumes an annual R&D depreciation of 15% (see Hall et al. 2010). Nevertheless, Hall (2010) shows that estimates can range from 0% (when using a production function approach) to between 20 and 40% (when using a market valuation approach). Diewert and Huang (2011) obtain sector-specific estimates ranging from 1% for the chemical sector to 29% for the manufacturing sector, whereas Warusawitharana (2015) estimates a depreciation rate of 32% using a structural estimation methodology. I adopt a conservative approach and assume an annual depreciation of 15%, which corresponds to a quarterly R&D depreciation of 3.6%. The steady-state government spending to GDP ratio $G/Y$ is calibrated to 18%, the steady-state price mark-up $\phi_p$ to 1 (consistently with the absence of fixed costs in the production function), the steady-state wage mark-up $\phi_w$ to 1.5, the curvature of the Kimball aggregator for the non-intermediate goods and the labour market to 10, and the steady-state of R&D investment to GDP ratio to its historical mean on the sample 1993Q1-2018Q4 (0.027). Additional details about the calibration are provided in the Appendix (Table 1).

In the Appendix, I present the prior distributions, and the estimated posterior distributions of the structural parameters in Table 2, and the parameter estimates relative to the shock processes in Table 3. Prior distributions are borrowed from Smets and Wouters (2007), except for the newly estimated parameters. For the stepping-on-toes parameter in the innovation probability, I set the prior mean equal to 0.35, following Jones and Williams (1998). For the R&D adjustment cost curvature parameter, the empirical literature provides little guidance\(^{15}\). I thereby set the prior mean to 2 in order to have a prior featuring quadratic adjustment costs.

The structural parameters estimates are generally in line with the literature. Few differences emerge when the estimates are compared to the baseline Smets and Wouters (2007) estimates. I find a high degree of capital investment adjustment costs ($\psi_k = 5.78$). The Calvo price updating probability $\zeta_p$ is 0.92, signaling stronger nominal rigidities with respect to SW, while the wage updating probability $\zeta_w$ is 0.36. The price inflation indexation coefficient $\pi_p$ is 0.35, whereas the wage indexation coefficient $\pi_w$ is 0.50. There are not many surprises

\[^{15}\text{Note that, as the functional form of the R&D adjustment cost is not equivalent to physical capital adjustment costs, and as they are a cost in terms of innovation performance, the physical capital adjustment parameter can not represent a benchmark.}\]
among the Taylor Rule parameter estimates: the Federal Reserve responds vigorously to inflation ($\rho_I = 1.54$), mildly to output (0.21 in levels and 0.02 in differences), and is highly inertial ($\rho_r = 0.89$) in setting the Federal Funds Rate. Eventually, the steady-state growth rate of the economy $\gamma$ is strikingly lower than in previous works reflecting the presence of a protracted TFP slowdown in the sample.

Concerning the newly estimated parameters, the stepping-on-toes parameter $\phi_{RD}$ is 0.34, in line with the empirical estimates, the R&D adjustment cost curvature parameter $\psi_1$ is 1.06 suggesting a mild degree of non-linearity in the adjustment cost function. In the Appendix, in Table 2, I present the estimates for the shock process parameters. The exogenous TFP, and the utilization shocks are found to be very persistent. I find a high persistence of government spending shocks, as standard in the literature, and a moderate persistence of risk premium shocks. I find labour quality fluctuations to be very persistent, reflecting the fact that changes in workers’ skills and education occur at a relatively low frequency. As for the capital utilization shock, the high persistence might be reflecting that capital utilization is highly affected by demand fluctuations whereas, in the model, it is mostly exogenous\textsuperscript{16}. A potentially promising explanation for the high-persistence of exogenous TFP shocks is that they could mostly capture the effect of sectoral reallocations, which induce persistent changes in productivity. Mark-ups, monetary policy, and the innovation step shocks are instead not particularly persistent.

5. Results

In Figure 4, I show that the smoothed innovation probability closely tracks the number of PCT patent applications in the US (correlation is 0.70). This result suggests that the (unobserved) concept of probability of innovation in the model, is strongly related to the amount of innovation produced by the US economy, which suggests that the innovation dynamics are well identified. The model is thereby able to correctly predict the number of patents developed in the US in the economy on the base of the investment in R&D and aggregate TFP gains. The probability of innovation $\Psi_t$, in the model, is the probability with which in a given sector, a player will emerge as a technological leader. As explained in Section 3, by the law of large numbers, this implies that $\Psi_t$ is also equal to the fraction of sectors in the economy in which an innovation is developed. Given that the economy features a fixed number of sectors, the innovation probability is thereby in the model a proxy for the amount of innovation produced in the economy and finds its natural empirical counterpart in the number of patents developed in the US economy. Patent application data are thus an appealing empirical proxy for the amount of innovation developed in the economy. Furthermore, Kogan et al. (2016) show patents to be an important predictor of future productivity gains and Miranda-Agrippino et al. (2019) estimate a VAR using the number of patent applications as a proxy of news about productivity. In order to isolate the sub-sample of high-potential patents only, in this paper I make use of the PCT (Patent Cooperation Treaty) applications originated in the US between 2000 and 2019 rather than the US Patent Office patent application data. A PCT application assures the inventor the priority to register an item in each of the 152 national patent offices of

\textsuperscript{16}Utilization fluctuations in the data are much wider than those implied by the baseline SW endogenous utilization feature, hence the bulk of the utilization fluctuations is explained by the utilization shock. I leave to future research the challenge of providing a data-consistent theory of endogenous fluctuations in capital utilization.
the contracting states. Hence, a PCT patent application is a signal of the intention of the inventor to enforce the patent on a global scale, or at least in several countries. As enforcing a patent at an international level requires a much greater investment (in each country the patent should undergo a standard approval process at standard fees), these applications are likely to concern high-profitability patents only, and therefore high productivity potential patents only. It is important here to note that, despite the profitability of a patent might not be directly related to its innovative content\textsuperscript{17}, standard TFP accounting techniques imply that higher profits mechanically translate into higher productivity in the statistics.

\textbf{FIG.4 - INNOVATION PROBABILITY AND PATENT APPLICATIONS}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\end{figure}

Solid line: Smoothed Innovation Probability in % deviations from the deterministic steady state (left-hand axis), Dotted Line: number of PCT applications originated in the USPTO (in tens of thousands, right-hand axis). Correlation is 0.70.

In Figure 5, I consider three kinds of shock affecting TFP in the model: an exogenous TFP shock, an innovation step shock, and a R&D investment technology shock. Interestingly, the innovation step shock has very similar dynamics to the VAR evidence presented in Miranda-Agrippino et al. (2019): the TFP gain following an innovation shock is smooth and concentrated during the first 5 quarters, GDP rises during the first 12 quarters and experiences a slow contraction since then, consumption rises slowly and steadily, hours worked shortly contract in the beginning and then recover exhibiting hump-shaped dynamics, and investment and R&D

\textsuperscript{17}As for instance, in the notorious cases of Amazon’s "one-click purchase" (1999) or Apple’s iPhone "round corner" (2012) patents.
response is smooth and persistent. The main finding here is that different kinds of innovation shock imply a different balance between wealth effects on consumption and labour, and can accordingly be contractionary or expansionary on the labour market. In particular, I show that static shocks acting on the production function imply labour market contractions, whereas technological shocks acting through the endogenous TFP channel produce labour market expansion. This happens because R&D technology and innovation step shocks produce large and persistent TFP which, in addition to consumption wealth effects, trigger sustained increases in investment that more than counter the negative labour wealth effect. Therefore these results are of potential interests for the debate about the labour market response to technological shocks, featuring papers like Gali (1999) or Basa et al (2006).

The endogenous TFP channel represents an additional source of persistence in the business cycle. In order to show this property of the model, in Figure 6, I consider a dynamic impulse response to a 0.1 standard deviation monetary policy shock for the endogenous TFP model described in Section 3 and the corresponding exogenous TFP counterpart\textsuperscript{18}. The TFP response to a 0.1 standard deviations shock is sizable and relatively persistent. It follows that the model suggests the presence of strong spillovers to from the business cycle to TFP. The variable which appears being most affected is inflation. This occurs because TFP drops are deflationary in the model, as they imply a negative wealth effect in consumption and reduce the marginal productivity of physical capital, therefore dampening investment. Due to this amplification effect, monetary policy is found to be a much more powerful stabilization tool compared to standard models, with very persistent effects. In Figure 7, I show the unconditional variance decomposition analysis of the core variables of the business cycle. I group the shocks featured in the model in 4 classes: (i) demand shocks (risk premium, government spending, investment-specific technology and monetary policy shocks), (ii) technological shocks (exogenous TFP, innovation step, and R&D technology shocks), (iii) mark-up shocks (on wages and prices), (iv) labour quality and utilization shocks. Output fluctuations in the model are mostly driven by demand shocks, as in the New Keynesian models tradition. Demand shocks account for about the 70\% of output fluctuations. Technological shocks for about the 15\%, and another 15\% is explained by labour quality, utilization, and mark-up shocks. The relevance of technological shocks is instead greater for consumption, due to the presence of large wealth effects, whereas investment is mostly driven by demand shocks. Wages are largely explained by mark-up shocks, whereas prices mostly by demand shocks. In the model, the missing deflation puzzle is significantly reduced. The reason is that many of the newly introduced shocks affect at once demand and supply with the same sign, therefore they can produce large fluctuations in aggregate fluctuations with limited inflation response. Hence, a smaller, more realistic amount of price mark-up shocks is required in order to explain price dynamics in the aftermath of the Great Financial Crisis.

\textsuperscript{18}The exogenous TFP version is a version of the model presented in Section 3 without R&D and an endogenous TFP channel. I calibrate the estimated parameters to the values presented in Tables 1-2-3 of the Appendix.
FIG. 5 - DYNAMIC RESPONSE OF THE ECONOMY TO SEVERAL TYPE OF INNOVATION SHOCKS


FIG. 6 - DYNAMIC RESPONSE TO A MONETARY POLICY SHOCK WITH AND WITHOUT ENDOGENOUS TFP

Impulse response function to a monetary policy shock of 0.1 standard deviations. Values in percentage deviations from their deterministic steady-state level. Solid line: endogenous TFP model, Dotted Line: exogenous TFP model.
The labour market and the interest rate are primarily driven by demand shocks, and only marginally affected by technological shocks. Eventually, TFP dynamics are mostly driven by technological shocks (responsible for about two-thirds of their fluctuations), with only a limited contribution of demand shocks (about 10%). This result is at the core of the paper, being consistent on many dimensions with the empirical observations (as explained in Section 1) but in stark contrast with the rest of the recent literature concerning DSGE model with endogenous TFP. In the Figures from 10 to 12, in Section D of the Appendix, I show the historical decompositions of different classes of shocks. In Figure 10, I show the contributions of demand shocks to TFP dynamics in the US. I find that the effect of demand and financial shocks, like risk premium and investment-specific shocks, is sizable in the short-run. In particular, I find financial shocks to be more important to explain productivity dynamics during recessions.

**FIG. 7 - UNCONDITIONAL VARIANCE DECOMPOSITION (in percent)**

I find the investment-specific shock to be important factors underlying the productivity contraction that preceded the dot-com recession in 2001 and approximately one-third of the sharp TFP contraction occurred in 2008. Nevertheless, the risk premium and investment-specific shocks do not have the persistence necessary to explain the persistent stagnation of TFP. Furthermore, financial shocks do not appear being able to explain productivity during expansions, and do not explain TFP dynamics in the aftermath of the Great Financial Crisis. In Figure 11, I show the contributions of technological shocks to TFP dynamics. As previously mentioned, the bulk of the TFP slowdown is explained by a persistent drop by in the ability of innovation of producing TFP gains. This idea was explored in works like Gordon (2012) and Bloom et al. (2017), both identifying the loss of productivity-enhancement potential of innovation as the cause of the productivity slump. I obtain that the R&D specific shock is providing a positive contribution in the last years of the sample, meaning that more
innovation is being produced for the same amount of R&D investment, which is in turn consistent with the fact that the number of patents in the US has a positive trend. The contribution of exogenous TFP shocks is also quite relevant, reflecting the presence of strong sectoral reallocation phenomena, as extensively documented by the empirical literature.

6. Conclusion

In this paper, I developed a New Keynesian Dynamic Stochastic General Equilibrium model featuring a creative destruction based mechanism of endogenous TFP growth. I estimated the model with U.S. data (1993q1-2016q4) using Bayesian methods and performed several empirical exercises in order to shed light on the TFP slowdown that occurred in the U.S. in the 2000s. I showed that the model is able to match the number of PCT patent applications that originated in the US from 2000 to 2019, a fact that would suggest that the model is quantitatively and qualitatively realistic. Furthermore, the innovation step shock exhibits dynamics consistent with the patent application VAR evidence from Miranda-Agrippino et al. (2019).

With respect to other studies relying on DSGE frameworks (as for instance Anzaegui et al. 2019), the introduction of the creative destruction based endogenous-TFP mechanism implies quantitatively similar TFP responses and provides a quantitatively similar amplification effect to stochastic shocks hitting the fundamentals of the economy. The sophistication of the innovation process allows to shed light on the heterogeneous sources of the productivity dynamics and to illustrate the effect of their spillovers on the macro aggregates. In particular, it allows explaining why some classes of productivity shocks have an expansionary effect on the labour market and others have a contractionary effect. The answer provided by this paper is that shocks to the innovation sector are expansionary because they imply persistent investment increase which adds up to large consumption wealth effects. By contrast, exogenous TFP shocks contract the labour market, being the negative wealth effect prevalent due to the lack of persistence of TFP. On the other hand, the presence of supply spillovers from demand shocks, implies that inflation is less volatile and that the missing deflation puzzle is less marked than in standard models.

In the historical and conditional variance decomposition analysis, I show that technological shocks are responsible for the bulk of TFP fluctuations, whereas demand shocks play a limited role. In particular, demand shocks are important to explain TFP contraction during recessions, but are not able to explain productivity booms and too short-lived to explain persistent slowdowns. These striking findings are in stark contrast with the rest of the literature although being consistent with many stylized facts and empirical works: (i) the TFP slowdown began before the Great Financial Crisis and is still currently ongoing, (ii) R&D-to-GDP ratio and R&D-to-TPF ratios did not drastically drop during and after the financial crisis yet TFP did, and (iii) a big role of the slowdown is played by the inability of investment in new technologies of generating sustained TFP gains (widely documented by Gordon 2012 and Bloom et al. 2017).

The results of the analysis performed in this paper challenge the view that the TFP slowdown was generated
by demand or financial shocks and support the hypothesis that innovation specific factors played a crucial role instead. The estimates suggest that innovation-specific shocks to the technological sector are more important to interpret TFP dynamics and typically generate much more persistent TFP dynamics compared to demand shocks. The interpretation provided by this paper suggests that the TFP slowdown might be much more long-lasting than what predicted by previous models, and raises significant concerns about the sustainability of the US growth path in the coming years.

References


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Appendix

A. Data and Sources \(^{19}\)

**Real GDP:** Real Gross Domestic Product, Billions of Chained 2009 Dollars, Quarterly, Seasonally Adjusted Annual Rate in the United States. Normalized by the population over 16 years and expressed in \(100\times\Delta(\log)\). Source: Bureau of Economic Analysis.

**Real Consumption:** Real Personal Consumption Expenditures, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate in the United States. Normalized by the population over 16 years and expressed in \(100\times\Delta(\log)\). Source: Bureau of Economic Analysis.

**Real Net Investment:** Real Fixed Private Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate in the United States less Research and Development Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate in the United States. Normalized by the population over 16 years and expressed in \(100\times\Delta(\log)\). Source: Bureau of Economic Analysis.

**Real R&D Investment:** Real Research and Development Investment, Billions of Dollars, Quarterly, Seasonally Adjusted Annual Rate. Normalized by the population over 16 years and expressed in \(100\times\Delta(\log)\). Source: Bureau of Economic Analysis.

**Real Wages:** Real Non-Farm Business Sector, Compensation Per Hour, Index 2009=100, Quarterly, Seasonally Adjusted. Deflated via the GDP deflator. Expressed in \(100\times\Delta(\log)\). Source: Bureau of Labour Statistics.

**Inflation:** Implicit Price Deflator, Index 2009=100, Quarterly, Seasonally Adjusted. Expressed in \(\Delta(\log)\). Source: Bureau of Economic Analysis.


**Federal Funds Rate:** Wu-Xia (2016) Shadow Federal Funds Rate. Source: Federal Reserve Bank of Atlanta.

**Capital Utilization:** From Fernald (2014). Follows Basu, Fernald, Fisher, and Kimball (2013) and includes adjustments in workers’ effort and capital’s workweek (BLS). Sources: Federal Reserve Bank of St. Francisco, BLS.


\(^{19}\)All the real variables are deflated using the GDP deflator.
B. General Equilibrium Conditions

1 - Intermediate Good Producers' Marginal Cost
\[ \tilde{mc}_t = (1 - \alpha)(\tilde{w}_t - \tilde{c}_t^a) + \alpha(\tilde{r}_t^k - \tilde{c}_t^m) - \tilde{c}_t^a \]

2 - Capital Utilization
\[ \tilde{u}_t^k = \frac{1 - \tau_k^g}{\tau_k^g} \tilde{r}_t^k \]

3 - Technology Adoption
\[ \tilde{a}_t^a = \frac{1 - \tau_a^g}{\tau_a^g} \tilde{r}_t^a \]

4 - Cost of Capital
\[ \tilde{r}_t^k = \tilde{w}_t + \tilde{t}_t - \tilde{k}_t \]

5 - Cost of Adoption
\[ \tilde{r}_t^a = \tilde{w}_t + \tilde{t}_t - \tilde{a}_t^{gap} \]

6 - Households' FOC w.r.t Capital Utilization
\[ \tilde{k}_t = \tilde{k}_{t-1} - \tilde{g}_t^a \]

7 - Households' FOC w.r.t Technology Adoption
\[ \tilde{a}_t^{gap} = \tilde{z}_t^a - \tilde{a}_t^a \]

8 - Households' FOC w.r.t Investment
\[ \tilde{r}_t = \frac{1}{1 + \beta_\gamma} \left[ \tilde{t}_{t-1} - \tilde{g}_t^a + \beta_\gamma \left( \tilde{t}_{t+1} + \tilde{g}_t^{k+1} \right) \right] + \beta_\gamma \tilde{Q}_t^k \]

9 - Households' FOC w.r.t Capital
\[ \tilde{Q}_t^k = \left( \tilde{H}_{t+1} - \tilde{R}_t - \tilde{c}_t^k \right) - \sigma \tilde{g}_{t+1} + \frac{\sigma^k}{\tau_{1+1} + \eta_1} \tilde{t}_{t+1} + \frac{1 - \theta_1}{\tau_{1+1} + \eta_1} \tilde{Q}_{t-1} \]

10 - Households' FOC w.r.t Consumption
\[ \tilde{c}_t = \frac{h/\gamma}{1 + h/\gamma} \left( \tilde{c}_{t-1} - \tilde{g}_t^a \right) - \sigma \tilde{g}_{t+1} + \frac{1}{1 + h/\gamma} \left( \tilde{c}_{t+1} + \tilde{g}_{t+1}^a \right) + \frac{\sigma - 1}{\sigma(1 + h/\gamma)} \left( \tilde{t}_t - \tilde{t}_{t+1} \right) - \frac{1 - h/\gamma}{\sigma(1 + h/\gamma)} \left( \tilde{H}_{t+1} - \tilde{R}_t + \tilde{c}_t^k \right) \]

11 - Production Function
\[ \tilde{g}_t = \alpha(\tilde{a}_t^k + \tilde{k}_t) + (1 - \alpha)(\tilde{a}_t^{gap} + \tilde{g}_t^a + \tilde{t}_t + \tilde{c}_t^k) \]

12 - Phillips Curve
\[ (1 + \beta_\gamma \eta_0) \tilde{P}_t = \eta_0 \tilde{P}_{t-1} + \beta_\gamma \tilde{P}_{t+1} + \frac{(1 - \zeta_0)(1 - \beta_\gamma \zeta_0)}{\zeta_0(\phi_0 - 1) \lambda_0 + 1} \tilde{mc}_t + \tilde{c}_t^f \]

13 - Wage Setting
\[ (1 + \beta_\gamma) \tilde{w}_t = (\tilde{w}_{t-1} - \tilde{g}_t^a) + \beta_\gamma (\tilde{w}_{t+1} - \tilde{g}_{t+1}^a) + t \tilde{w}_{t-1} - (1 + \beta_\gamma \gamma) \tilde{P}_t + \beta_\gamma \beta_\gamma (\tilde{P}_{t+1} + \tilde{Q}_t^k) \]

\[ + \frac{(1 - \zeta_0)(1 - \beta_\gamma \zeta_0)}{\zeta_0(\phi_0 - 1) \lambda_0 + 1} \left[ \sigma \tilde{u}_t + \frac{1}{1 + h/\gamma} \tilde{c}_t^k - \frac{h/\gamma}{1 + h/\gamma} \left( \tilde{c}_{t+1} - \tilde{g}_{t+1}^a \right) - \tilde{w}_t + \tilde{c}_t^w \right] \]

In the system, all the tending variables are normalized by the stock of technology \( \tilde{A}_t \), i.e., \( y_t = Y_t / \tilde{A}_t, c_t = C_t / \tilde{A}_t, \) and \( \Lambda_t, \lambda_{t+1} = \lambda_{t+1}/\tilde{A}_t, \) and \( \lambda_{t+1} = \lambda_{t+1}/\tilde{A}_t, \) \( a_{gap} = \tilde{A}_t / \tilde{A}_t, \) and \( a_{gap} = \tilde{A}_t / \tilde{A}_t. \)
14 - R&D Investment Euler Equation
\[ \hat{\gamma}_t = \frac{1}{1-\phi_{RD}+2\psi\gamma} \left[ \hat{y}_t + \gamma^2 \psi \left( \hat{\gamma}_{t-1} - \hat{\gamma}_t^a \right) + \hat{\epsilon}_t \right] \]

15 - Probability of Innovation
\[ \hat{\Psi}_t = \phi_{RD} \hat{\gamma}_t + 2\psi \gamma \left( \hat{\gamma}_t - \hat{\gamma}_{t-1} \right) + \hat{\epsilon}_t^\Psi \]

16 - Productivity Law of Motion
\[ \hat{g}_t = \frac{\gamma(1-\delta_a)}{\gamma} \hat{\Psi}_t + \frac{\gamma(1-\delta_a)(1-\Phi_{RD})}{\gamma} \hat{\epsilon}_t^g \]

17 - Aggregate Resource Constraint
\[ \hat{y} = \frac{c}{y^*} \hat{c} + \frac{i}{y^*} \hat{i} + \frac{r^a_k}{y^*} \hat{z}^a_k + \hat{\epsilon}^y_i \]

18 - Taylor Rule
\[ \hat{R}_t = \rho_r \hat{R}_{t-1} + \psi_p (1-\rho_p) \hat{p}_t + \psi_y (1-\rho_y) (\hat{y}_t - \hat{y}_t^f) + \psi_{dy} (\hat{y}_t - \hat{y}_t^f - \hat{y}_{t-1} + \hat{y}_{t-1}^f) + \hat{\epsilon}^m_{ts} \]

C. TFP Series Used in the Estimation

FIG. 8 - FIXED SHARES TFP (as in the estimation) AND VARIABLE SHARES TFP (as in Fernald 2014)
E. Which shocks drive TFP?

FIG. 9 - CONTRIBUTION OF DEMAND SHOCKS TO TFP DYNAMICS (1993-2018)

[Graph showing the contribution of demand shocks to TFP dynamics from 1993 to 2018.]

- Black solid line: Smoothed TFP 1993-2018, annual data.
- Dashed blue line: investment technology shock.
- Dashed red line: risk premium shock.
- Dashed green line: government spending shock.

FIG. 10 - CONTRIBUTION OF TECHNOLOGICAL SHOCKS TO TFP DYNAMICS (1993-2018)

[Graph showing the contribution of technological shocks to TFP dynamics from 1993 to 2018.]

- Black solid line: Smoothed TFP 1993-2018, annual data.
- Dashed blue line: innovation step shock.
- Dashed red line: exogenous TFP shock.
- Dashed green line: RD efficiency shock.

FIG. 11 - CONTRIBUTION OF UTILIZATION AND LABOUR QUALITY TO TFP DYNAMICS (1993-2018)
Black solid line: Smoothed TFP 1993-2018, annual data.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
<th>Target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Capital Share</td>
<td>0.35</td>
<td>Empirical Mean</td>
</tr>
<tr>
<td>$\sigma_c$</td>
<td>Coefficient of Relative Risk Aversion</td>
<td>1.5</td>
<td>Standard</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>Inverse of Labour Frisch Elasticity</td>
<td>1</td>
<td>Standard</td>
</tr>
<tr>
<td>$\delta_k$</td>
<td>Capital Depreciation</td>
<td>2.5%</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>$\delta_a$</td>
<td>Knowledge Depreciation</td>
<td>3.6%</td>
<td>Standard (Annual 15%)</td>
</tr>
<tr>
<td>$\lambda_p$</td>
<td>Curvature Kimball Aggregator (Prices)</td>
<td>10</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>$\lambda_w$</td>
<td>Curvature Kimball Aggregator (Wages)</td>
<td>10</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>$\phi_p$</td>
<td>Steady-State Price Mark-Up</td>
<td>1</td>
<td>No Fixed Costs</td>
</tr>
<tr>
<td>$\phi_w$</td>
<td>Steady-State Wage Mark-Up</td>
<td>1.5</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>Exogenous Govt Spending-to-GDP in Steady State</td>
<td>18%</td>
<td>Smets and Wouters (2007)</td>
</tr>
<tr>
<td>$J/Y$</td>
<td>R&amp;D-to-GDP in Steady State</td>
<td>2.7%</td>
<td>Empirical Mean</td>
</tr>
</tbody>
</table>
### Table 2: Structural Parameter Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Posterior</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean 90% CI</td>
<td>Distribution</td>
</tr>
<tr>
<td>Consumption Habits</td>
<td>( h ) 0.853 [0.815 ; 0.884]</td>
<td>B 0.7 0.1</td>
</tr>
<tr>
<td>Capital Adjustment Costs</td>
<td>( Z'' ) 5.781 [4.303 ; 6.977]</td>
<td>N 4 1</td>
</tr>
<tr>
<td>Price Updating Probability</td>
<td>( \zeta_p ) 0.924 [0.905 ; 0.941]</td>
<td>B 0.5 0.1</td>
</tr>
<tr>
<td>Wage Updating Probability</td>
<td>( \zeta_w ) 0.361 [0.301 ; 0.417]</td>
<td>B 0.5 0.1</td>
</tr>
<tr>
<td>Wage Indexation</td>
<td>( \iota_w ) 0.501 [0.283 ; 0.727]</td>
<td>B 0.5 0.15</td>
</tr>
<tr>
<td>Price Indexation</td>
<td>( \iota_p ) 0.348 [0.107 ; 0.535]</td>
<td>B 0.5 0.15</td>
</tr>
<tr>
<td>Utilization Cost</td>
<td>( z^k ) 0.792 [0.738 ; 0.838]</td>
<td>B 0.5 0.15</td>
</tr>
<tr>
<td>Adoption Cost</td>
<td>( z^a ) 0.465 [0.387 ; 0.534]</td>
<td>B 0.5 0.15</td>
</tr>
<tr>
<td>Agg. Demand Response to TFP</td>
<td>( \rho_{TFP} ) 0.036 [0.010 ; 0.072]</td>
<td>N 0.5 0.25</td>
</tr>
<tr>
<td>MP Response to Inflation</td>
<td>( \rho_{\pi} ) 1.544 [1.241 ; 1.901]</td>
<td>N 1.5 0.25</td>
</tr>
<tr>
<td>MP Inertia</td>
<td>( \rho_{\tau} ) 0.886 [0.859 ; 0.916]</td>
<td>B 0.750 0.1</td>
</tr>
<tr>
<td>MP Response to Output (Level)</td>
<td>( \rho_{y} ) 0.206 [0.159 ; 0.270]</td>
<td>N 0.125 0.05</td>
</tr>
<tr>
<td>MP Response to Output (Diff)</td>
<td>( \rho_{dy} ) 0.022 [0.002 ; 0.039]</td>
<td>N 0.125 0.05</td>
</tr>
<tr>
<td>TFP Growth Rate in S.S.</td>
<td>( \gamma ) 0.186 [0.114 ; 0.252]</td>
<td>N 0.4 0.1</td>
</tr>
<tr>
<td>Stepping-on-Toes Parameter</td>
<td>( \phi_{RD} ) 0.343 [0.279 ; 0.401]</td>
<td>B 0.35 0.1</td>
</tr>
<tr>
<td>Curvature R&amp;D Adj Costs</td>
<td>( 2\psi_{RD} ) 1.062 [1.000 ; 1.162]</td>
<td>N 2 1</td>
</tr>
</tbody>
</table>

**Legend:**  
N=Normal, B=Beta, G=Gamma, IG=Inverse Gamma
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Posterior</th>
<th>Prior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean 90% Confidence Interval</td>
<td>Distribution</td>
</tr>
<tr>
<td>Risk Premium AR(1)</td>
<td>$\rho_b$ 0.398 [0.283; 0.533]</td>
<td>B</td>
</tr>
<tr>
<td>Go&amp;x Spending AR(1)</td>
<td>$\rho_g$ 0.935 [0.910; 0.954]</td>
<td>B</td>
</tr>
<tr>
<td>Investment Tech AR(1)</td>
<td>$\rho_{qs}$ 0.867 [0.819; 0.926]</td>
<td>B</td>
</tr>
<tr>
<td>Monetary Policy AR(1)</td>
<td>$\rho_{m_s}$ 0.643 [0.555; 0.715]</td>
<td>B</td>
</tr>
<tr>
<td>Price Mark-Up AR(1)</td>
<td>$\rho_p$ 0.306 [0.105; 0.558]</td>
<td>B</td>
</tr>
<tr>
<td>Wage Mark-Up AR(1)</td>
<td>$\rho_w$ 0.752 [0.626; 0.883]</td>
<td>B</td>
</tr>
<tr>
<td>R&amp;D Efficiency AR(1)</td>
<td>$\rho_j$ 0.811 [0.743; 0.863]</td>
<td>B</td>
</tr>
<tr>
<td>Exogenous TFP AR(1)</td>
<td>$\rho_x$ 0.922 [0.861; 0.962]</td>
<td>B</td>
</tr>
<tr>
<td>Innovation Step AR(1)</td>
<td>$\rho_{a}$ 0.481 [0.404; 0.571]</td>
<td>B</td>
</tr>
<tr>
<td>Labour Quality Shock AR(1)</td>
<td>$\rho_{q_0}$ 0.808 [0.706; 0.904]</td>
<td>B</td>
</tr>
<tr>
<td>Utilization Shock AR(1)</td>
<td>$\rho_u$ 0.861 [0.818; 0.903]</td>
<td>B</td>
</tr>
<tr>
<td>Innovation Step St.Dev</td>
<td>$\sigma_{e_a}$ 2.64 [2.153; 2.971]</td>
<td>IG</td>
</tr>
<tr>
<td>Exogenous TFP St.Dev</td>
<td>$\sigma_{e_q}$ 0.736 [0.662; 0.804]</td>
<td>IG</td>
</tr>
<tr>
<td>Risk Premium St.Dev</td>
<td>$\sigma_{e_m}$ 0.176 [0.144; 0.219]</td>
<td>IG</td>
</tr>
<tr>
<td>Go&amp;x Spending St.Dev</td>
<td>$\sigma_{e_g}$ 0.619 [0.530; 0.702]</td>
<td>IG</td>
</tr>
<tr>
<td>Investment Tech St.Dev</td>
<td>$\sigma_{e_{qs}}$ 0.382 [0.314; 0.436]</td>
<td>IG</td>
</tr>
<tr>
<td>Monetary Policy St.Dev</td>
<td>$\sigma_{e_{qs}}$ 0.382 [0.314; 0.436]</td>
<td>IG</td>
</tr>
<tr>
<td>Price Mark-Up St.Dev</td>
<td>$\sigma_{e_p}$ 0.106 [0.077; 0.122]</td>
<td>IG</td>
</tr>
<tr>
<td>Wage Mark-Up St.Dev</td>
<td>$\sigma_{e_w}$ 0.535 [0.461; 0.641]</td>
<td>IG</td>
</tr>
<tr>
<td>R&amp;D Efficiency St.Dev</td>
<td>$\sigma_{e_{j}}$ 2.91 [2.805; 2.998]</td>
<td>IG</td>
</tr>
<tr>
<td>Labour Quality St.Dev</td>
<td>$\sigma_{e_{q_0}}$ 0.414 [0.368; 0.450]</td>
<td>IG</td>
</tr>
<tr>
<td>Capital Utilization St.Dev</td>
<td>$\sigma_{e_u}$ 0.573 [0.481; 0.651]</td>
<td>IG</td>
</tr>
</tbody>
</table>

**Legend:** N=Normal, B=Beta, G=Gamma, IG=Inverse Gamma