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Abstract

We explore patterns of price competition in an oligopoly where consumers vary in the set of suppliers they consider for their purchase. In the case of "nested reach" we find equilibria, unlike those in existing models, in which price competition is segmented: small firms offer only low prices and large firms only offer high prices. We characterize equilibria in the three-firm case using correlation measures of interaction between pairs of firms. We show how entry, merger and market expansion can affect patterns of price competition in novel ways.

1   Introduction

We study oligopoly pricing in a setting where consumers differ in their choice sets—that is, the set of firms they consider for their purchase—and buy from the firm in their choice set with the lowest price. Bertrand equilibrium then involves firms choosing their prices according to mixed strategies, and a firm chooses from a *range* of prices. The structure of price competition could take many forms. Firms might all choose from a similar range of prices, or competition might be more segmented with only a small subset of firms competing at a given price. Who competes with whom at each price is determined in equilibrium. How does the equilibrium structure of price competition depend on the underlying structure of consumer choice sets?

The simplest situation in which this question arises is a duopoly in which each firm has some captive customers, while non-captive customers are able to pay the lower of the two

firms' prices. A firm then has choice between "entering the fray", by competing against its rival for the contested consumer segment with a low price, or "retreating" back towards its captive base by setting a high price, and in equilibrium these strategies yield the same profit. Even if the firms are asymmetric, they use the same interval range of prices. With more than two firms, though, richer patterns of consumer consideration become possible. Taking the interaction between two firms to be the "overlap" in the sets of consumers who consider buying from them, different pairs of firms might have very different levels of interaction. With more than one rival a firm can compete on several fronts, and richer patterns of pricing also emerge. With a segmented pricing pattern, for instance, a firm might compete against one firm when it charges a low price and another firm when it charges a higher price.

The foundation of our model is the distribution of choice sets among consumers. There are various reasons why different consumers have different sets of choices open to them. Perhaps following a prior stage of advertising by firms or search by consumers, some consumers become aware of a different set of suppliers than other consumers. For instance, Draganska and Klapper (2011) document limited and heterogeneous consumer awareness of various brands of ground coffee, while Honka, Hortacsu, and Vitorino (2017) do the same for retail banks. Alternatively, as in Spiegler (2006), there might be horizontal product differentiation such that only a subset of products could meet a consumer's needs. The set of firms who are currently active in the market might be uncertain (Janssen and Rasmusen (2002)), as might be the set of firms who choose to post prices on a comparison website (Baye and Morgan (2001)). Some consumers might be constrained in their choices by location, transport costs or switching costs. For instance, some models of spatial competition, such as Smith (2004), suppose that a consumer considers buying from those firms located within a specified radius of her. Consumers might also differ in their ability to make comparisons between offers, with confused consumers choosing randomly between suppliers or buying from a default seller (Piccione and Spiegler (2012), Chioveanu and Zhou (2013)). Our analysis does not take a view on the underlying reason why consumers have different choice sets. Rather, it takes the distribution of choice sets in the consumer population as given, and explores the consequences for competition.

A considerable literature has explored aspects of this general framework, and some settings are now well understood: (i) the case with symmetric firms; (ii) the case with

independent reach, and (iii) the "one-or-all" case where consumers are either captive to one firm or can choose between all firms. (These special cases, which overlap, are discussed in more detail in section 2.) Within case (i), which covers the great majority of existing models, Rosenthal (1980) and Varian (1980) considered the situation in which some consumers are randomly captive to particular firms, while others compare the prices of all firms and buy from the cheapest. (Thus these papers also fall under case (iii).) In case (i) there is a symmetric equilibrium with price dispersion, in which all firms choose prices according to the same mixed strategy. Burdett and Judd (1983, section 3.3) analyze a more general symmetric model, in which arbitrary fractions of consumers consider one random firm, two random firms, and so on. Provided some consumers consider just one firm and some consider more than one, the symmetric equilibrium involves price dispersion, and industry profit is proportional to the number of captive consumers who consider just one firm. Johnen and Ronayne (2019) show that this symmetric equilibrium is the unique equilibrium if and only if there are some consumers who consider precisely two firms.[1]

In case (ii) with independent reach, the fact that a consumer considers one firm does not affect the likelihood she considers any other firm. Then the firm that reaches the most consumers also has the largest proportion of captive consumers among the consumers within its reach—i.e., it has the highest captive-to-reach ratio. This model was studied by Ireland (1993) and McAfee (1994), who show that in equilibrium all firms use the same minimum price, but the maximum price charged is lower for smaller firms. Thus price supports are nested, so that smaller firms only offer low prices while the largest firms offer the full range of prices. Since firms use the same minimum price, their profits are proportional to their reach.[2]

In case (iii), where consumers either consider just one firm or consider the whole set of firms, was fully solved by Baye, Kovenock, and De Vries (1992).[3] In the symmetric version of the model (which coincides with the models of Varian and Rosenthal), when there are more than two firms many asymmetric equilibria exist alongside the symmetric equilibrium.

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[1]Banerjee and Kovenock (1999) study a model with symmetric firms but where the set of firms considered by consumers is not random: firms are arranged on a circle, and if a consumer considers a given firm the only other firms she might consider are the two neighboring firms.

[2]This equilibrium was subsequently shown by Szech (2011) to be unique. Spiegler (2006) studies the special case of this framework where all firms are equally likely to be considered (which therefore also fits into case (i) with symmetric firms). Manzini and Mariotti (2014) study a choice model where an agent is aware of a particular option with specified independent probability. In an empirical study of the personal computer market, Sovinsky Goeree (2008) assumes that the reach of the various products is independent.

[3]This framework includes duopoly as a special case, which was studied by Narasimhan (1988).

In an asymmetric market where firms have different numbers of captive customers, all but the two smallest firms choose the monopoly price for sure, while the two smallest firms compete using mixed strategies. Intuitively, the two firms with the fewest captive customers have a strong interaction and compete against themselves, leaving firms with more captives with an incentive to retreat to their captive base. This is an extreme instance of the situation where large firms choose only high prices, which we will discuss further at several points in the analysis to follow. In this equilibrium, each firm except for the smallest obtains profit proportional to the number of its captive customers (rather than being proportional to its reach as with case (ii)).

While these three special cases are natural benchmarks, in practice patterns of consumer consideration will fall outside these cases. For example, in their study of ground coffee Draganska and Klapper (2011) document in their sample that firms are not close to being symmetric (Table 2), that consumer awareness is far from independent across brands (Table 5), and that the choice sets of many consumers consisted neither of a single firm nor of the whole set of firms (Figure 1). The aim of the present paper is to provide a unifying framework which encompasses special cases (i) to (iii), but which allows us to study richer situations outside these cases as well, and to discover new types of equilibrium interaction.

The analysis is organised as follows. In section 2 we present the general framework, and recapitulate the analysis for the special cases (i) to (iii). In section 3 we introduce and analyse *nested* reach, in which only the largest firm has any captive customers, and if the increments between successive firm sizes are non-decreasing we find equilibria with a form of segmented pricing which we term "overlapping duopoly": there is an increasing sequence of prices $\{p_k\}$ such that the range of prices that the kth smallest firm might charge is an interval $[p_{k-1}, p_{k+1}]$. Hence small firms charge low prices while large firms charge high prices, and firms compete against precisely one rival with any price they offer.

Section 4 then provides a general analysis of the three-firm case. Even with triopoly, a wide variety of patterns of consumer consideration is possible. We define a measure of the interaction between a pair of firms, which reflects *correlation* between consumer consideration of the two firms. When interactions between pairs of firms are similar, as with independent reach, we show that all firms use a common lowest price and hence have profit proportional to their reach. In some of these cases, however, we find that the price support of the least competitive firm might not be an interval—the firm might

price high and low but not in an intermediate range. By contrast, when one pair of firms has significantly more interaction than other pairs, the equilibrium has the "overlapping duopoly" property—one firm prices low, one high, and one across the full price range. Intuitively, this pair mostly compete with each other, leaving the remaining firm with an incentive to set high prices.

When the interaction increases between one pair of firms—e.g., if additional consumers consider both firms—this can induce the remaining firm to retreat towards its captive base. While entry into a duopoly market by a third firm often pushes down prices, there are natural patterns of interaction where, counter-intuitively, the opposite happens and consumers are harmed by entry. Again, the reason is that more intense competition in the contested segment induces incumbents to retreat towards their captive base. We also discuss the impact of mergers in our framework. It is common for profitable mergers to harm consumers (in the absence of cost synergies), which is always the case for three-to-two mergers, but we also describe situations where a profitable merger between two firms with a strong interaction can reduce industry profit. The reason is that such a merger opens up a profitable front for the non-merging firms, and induces these firms to "enter the fray" from their captive bases.

We conclude in section 5 by summarizing our main insights, and suggesting avenues for further research on this topic.

2   A model with consumer choice sets

There are n firms that costlessly supply a homogeneous product. There is a population of consumers of total measure normalized to 1, each of whom has unit demand and is willing to pay up to 1 for a unit of the product.[4] Consumers differ according to which firms they consider for their purchase, and for each subset $S \subset \{1, ..., n\}$ of firms (including the null set) suppose that the fraction of consumers who consider exactly the subset S is α_S. (We slightly abuse notation, and write α_1 for the fraction who consider only firm 1, $\alpha_{12} = \alpha_{21}$ for the fraction who consider only firms 1 and 2, and so on.) When there are only few firms the pattern of choice sets can be illustrated using a Venn diagram, and Figure 1 depicts

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[4]The positive analysis which follows is not affected if each consumer has a downward-sloping demand function $x(p)$, provided revenue $px(p)$ is an increasing function up to the monopoly price. However, welfare analysis (for instance in our discussion of entry) requires adjustment with downward-sloping demand.

a market with three firms.[5] Here, a consumer considers a particular subset of firms if she lies inside the "circle" of each of those firms. For instance, a fraction α_{12} of consumers consider the two firms 1 and 2.

A consumer is *captive* to firm i if she considers i but no other other firm, and there is a fraction α_i of such consumers. The *reach* of firm i is the set of consumers who consider the firm, and the fraction of such consumers is denoted σ_i, so that

$$\sigma_i = \sum_{S|i \in S} \alpha_S \ .$$

Finally, the *captive-to-reach* ratio of firm i is denoted ρ_i, where
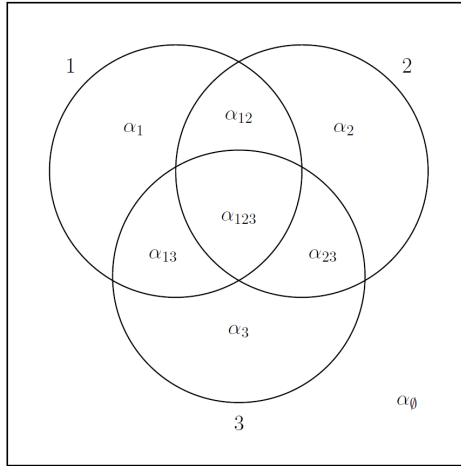
$$\rho_i = \frac{\alpha_i}{\sigma_i} \ .$$

Figure 1: Choice sets with three firms

Firms compete in a one-shot Bertrand manner, and a consumer buys from the firm she considers that has the lowest price (provided this price is no greater than 1). In particular, a firm offers a uniform price to all its potential customers, and cannot make its price to a consumer contingent on the choice set of that consumer.[6] If two or more firms choose the same lowest price, we suppose that the consumer is equally likely to buy from any

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[5]In a spatial context this Venn diagram has a more literal interpretation: if consumers only consider buying from a firm within a specified distance, then the locations of firms determine the centre of the circles on the diagram. With more firms (and a finite set of consumers), consideration sets can be conveniently depicted using a bipartite graph, where the two groups in the graph are the consumers and the firms, and a line connecting a consumer to a firm corresponds to the former considering the latter. In a very different context, Prat (2018) uses a model of consideration sets similar to that presented here.

[6]In Armstrong and Vickers (2019) we investigate the impact of firms being able to offer different deals to captive and contested customers.

such firm. Since industry profit is a continuous function of the vector of prices chosen, Theorem 5 in Dasgupta and Maskin (1986) shows that an equilibrium exists. Since an individual firm's profit is usually discontinuous in the price vector, the equilibrium will usually involve mixed strategies for some firms. We make assumptions to rule out some extreme and uninteresting configurations. The first requires that there be some interaction between firms:

Assumption 1: Some consumers consider at least two firms.

(If all customers were captive, each firm chooses $p \equiv 1$ for sure.) The second assumption prohibits the possibility that a subset of firms choose the competitive price $p \equiv 0$ for sure, as such firms play no important role in the analysis:

Assumption 2: Every non-empty subset of firms S contains at least one firm with consumers within its reach who consider no other firm in S.

For instance, this assumption rules out the situation where two firms reach precisely the same set of consumers. Intuitively, Assumption 2 ensures that no subset S of firms will set $p \equiv 0$, since there is a firm in S which has some customers with no overlap with other firms in S, and this firm can profitably raise its price above zero. These two assumptions together imply that there is no equilibrium in pure strategies, and at least some firms choose their price according to a mixed strategy.

When firm i chooses price $p \leq 1$ it will sell to a consumer when that consumer is within its reach and when none of the other firms the consumer considers offers a lower price. Therefore, when rival firms $j \neq i$ choose price according to the cumulative distribution function (CDF) $F_j(p)$, firm i's expected demand with price $p \leq 1$ is

$$q_i(p) \equiv \sum_{S|i \in S} \alpha_S \left(\prod_{j \in S/i} (1 - F_j(p)) \right) . \tag{1}$$

Here, the sum is over all consumer segments which consider firm i, and for each such segment the product is over all rivals for firm i in that segment. (If there are no such rivals, i.e., when the segment comprises firm i's captive customers, we use the convention that this product equals 1.[7]) Equilibrium occurs when for each firm i there exists a profit

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[7]Expression (1) is written without taking into account the possibility of ties; however, Lemma 1 shows that ties do not occur with positive probability.

level π_i and a CDF $F_i(p)$ such that firm i's profit $pq_i(p)$ is equal to π_i for every price in firm i's support and no higher than π_i for any price outside its support.[8]

The following result collects a number of observations about the structure of price competition in equilibrium, some of which are familiar from the existing literature.[9]

Lemma 1 *In any equilibrium:*

(i) firm i obtains profit $\pi_i \geq \alpha_i$, with equality for at least one firm, and the minimum price in its support is no smaller than ρ_i;

(ii) each firm obtains positive profit (even if it has no captive customers) and p_0, the minimum price chosen by any firm, is positive;

(iii) each firm's price distribution is continuous (that is, has no "atoms") in the half-open interval $[p_0, 1)$;

(iv) each price in the interval $[p_0, 1]$ lies in the price support of at least two firms;

(v) if there are three or more firms, there is at least one price which lies in the support of three or more firms, and

(vi) p_0 lies weakly between the second lowest ρ_i and the highest ρ_i. If the firm with the highest ρ_i has p_0 in its support then p_0 is equal to the highest ρ_i.

Proof. All proofs are contained in the appendix. ∎

Comparative statics with regard to market changes can naturally be studied within this framework of limited choice sets. For instance, entry by a new firm can be modelled as a new "circle" superimposed onto the existing Venn diagram. That is, entry does not affect which consumers consider the incumbent firms, and the reach of an incumbent firm is unaffected by entry, although its number of captive customers will weakly fall.[10] Since welfare (consumer surplus plus industry profit) is the total number of consumers reached, it follows that entry (if it is costless) will weakly increase welfare. Likewise, if entry reduces industry profit it will benefit consumers. Mergers also have a natural set-theoretic interpretation in this framework: when two or more firms merge we assume that the merged entity sets the same price to all its customers, and that the set of consumers

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[8]As usual, the support of firm i's price distribution is defined to be the smallest closed set $\mathcal{P} \subset [0, 1]$ such that the probability that the firm chooses a price in $\mathcal{P}$ equals one.

[9]For instance, see McAfee (1994, page 28).

[10]In particular, there is no danger of "choice overload", whereby the number of consumers who compare prices falls when there are more firms, as discussed for instance in Spiegler (2011, page 150).

who consider the merged entity is the union of the sets of consumers who considered the separate firms.[11] Thus, a merger (with no accompanying cost synergies) has no impact on welfare, and harms consumers if and only if it increases industry profit. Note that the fraction of consumers reached by the merged firm is no greater than the sum of those reached by the separate firms, while the captive base of the merged firm is no smaller than the sum of captives of the separate firms. Finally, a market expansion can be modelled as an increase in the fractions of consumers in each non-empty segment of the Venn diagram (taken from the consumer segment $\alpha_\emptyset$ who previously had no choice).

As discussed in the introduction, previous work has studied the cases with symmetric firms, with independent reach, and where consumers are either captive to one firm or can choose from all firms, and we next describe those cases and add comments about entry and mergers.

Symmetric firms: Burdett and Judd (1983, section 3.3) study a market with $n \geq 2$ symmetric firms and where consumers consider firms at random (a specified fraction consider one random firm, a specified fraction consider two random firms, and so on). This model can be generalised somewhat so that firms are symmetric but choice sets need not be random. Specifically, suppose that each firm has a_1 captive customers, a_2 consumers who consider exactly one other firm (not necessarily random), and in general a_m consumers who consider $m - 1$ other firms for $m \leq n$. Let

$$\phi(x) \equiv a_1 + a_2 x + a_3 x^2 + ... + a_n x^{n-1}$$

be the probability generating function associated with the number of rivals faced by a firm. Here, $\phi(x)$ is convex and increasing, the number of captive customers for each firm is $\phi(0)$, each firm has reach $\sigma = \phi(1)$ and captive-to-reach ratio $\rho = \phi(0)/\phi(1)$. Assumptions 1 and 2 imply $0 < \phi(0) < \phi(1)$.

In a symmetric market, the unique symmetric equilibrium (which is not necessarily the only equilibrium) is derived as follows. Each firm obtains equilibrium profit $\pi_i \equiv \phi(0)$ and has the minimum price ρ. When each of its rivals uses the CDF $F(p)$, a firm's demand with price $p \leq 1$ in (1) is $q(p) = \phi(1 - F(p))$. Since each firm makes profit $\phi(0)$, the symmetric

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[11] An alternative approach would be for the merged entity to maintain separate brands and to be able to charge distinct prices for each brand.

equilibrium CDF satisfies

$$\phi(1 - F(p)) \equiv \frac{\phi(0)}{p} \ , \tag{2}$$

and the function $F(p)$ strictly increases from 0 to 1 as p increases from ρ to 1.

The models in Rosenthal (1980) and Varian (1980) are special cases of this framework, where consumers either consider one random firm or consider all firms, so that $a_m = 0$ for $1 < m < n$. With this pattern of consideration, Baye $et\ al.$ (1992) show that when $n \geq 3$ there are multiple equilibria (all of which involve the same profit for firms). For instance, one asymmetric equilibrium has all but two firms choosing $p = 1$ for sure, selling only to their captive customers, while the remaining two firms choose prices on the interval $[\rho, 1]$.

In general, entry by a new firm into a symmetric market has ambiguous effects on industry profit and consumer surplus, as we discuss in more detail in section 4. However, a merger between two or more firms in a symmetric market is always profitable. Before merger each firm obtained profit equal to its captive base, and a merger can only increase the merged entity's number of captive customers. A merger cannot decrease the profit of the non-merging firms (since they still obtain at least their captive profit), and so the merger increases industry profit and harms consumers. Finally, in this symmetric configuration there are "search externalities", in the sense that an increase in the number of consumers who consider more than one firm (i.e., an increase in a_m for some $m \geq 2$) will benefit all existing consumers (including those captive to a firm). To see this, note that an increase in a_m for $m \geq 2$ induces a rise in the $F(p)$ which solves (2), and so each firm will lower its price in the sense of first-order stochastic dominance.

$Independent\ reach$: Ireland (1993) and McAfee (1994) study the situation where each firm has an independent chance of being considered by a consumer. Specifically, firm i is considered by an independent fraction σ_i of the consumer population, where firms are labelled so that $0 < \sigma_1 \leq \sigma_2 \leq ... \leq \sigma_n \leq 1$. (Assumption 2 requires $\sigma_{n-1} < 1$.) The fraction of consumers captive to firm i is $\alpha_i = \sigma_i \Pi_{j \neq i}(1 - \sigma_j)$ and so this firm's captive-to-reach ratio is $\rho_i = \Pi_{j \neq i}(1 - \sigma_j)$. Thus the firm with the largest reach is also the firm with the highest captive-to-reach ratio $\rho_n = \Pi_{i=1}^{n-1}(1 - \sigma_i)$.

Firm i sells to a consumer when it chooses price p if it reaches that consumer (which occurs with probability σ_i) and no rival reaches that consumer with a lower price. If firm j chooses its price with the CDF $F_j(p)$, the probability that firm j reaches the consumer

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with a lower price is $\sigma_j F_j(p)$. Therefore, firm i's demand with price $p \leq 1$ in (1) takes the multiplicatively separable form

$$q_i(p) = \sigma_i \prod_{j \neq i}(1 - \sigma_j F_j(p)) . \tag{3}$$

Ireland (1993) and McAfee (1994) show that the equilibrium is such that all firms have the same minimum price $p_0 = \rho_n$, and the profit of firm i is $\pi_i = \sigma_i p_0$. Thus, firms' profits are proportional to their reaches, and the profit of the largest firm is equal to its number of captive consumers, while the profit of smaller firms is weakly greater than their number of captive consumers. The CDFs which support these equilibrium profits are such that firm i chooses its price with interval support $[p_0, p_i]$, where firm i's maximum price p_i is smaller for smaller firms. The two largest firms choose prices with support $[p_0, 1]$, so that the maximum prices satisfy $p_1 \leq p_2 \leq ... \leq p_{n-1} = p_n = 1$.

Industry profit is $\Pi = \left(\sum_{i=1}^n \sigma_i\right) p_0$, total welfare is the fraction of consumers who consider at least one firm which is $1 - (1 - \sigma_n)p_0$, and the difference between welfare and profit is consumer surplus

$$CS = 1 - \left(1 + \sum_{i=1}^{n-1} \sigma_i\right) p_0 . \tag{4}$$

Consumer surplus does not depend on the reach of the largest firm, σ_n, but increases with the reach of each smaller firm.

The model with independent reach has intuitive properties with respect to entry and mergers. Entry by a firm which also has independent reach will increase consumer surplus (4). If the entrant is not the largest firm in the market, so its reach is $\sigma_E < \sigma_n$, then the minimum price p_0 falls by the multiplicative factor $(1 - \sigma_E)$ which outweighs the impact of the additional σ_E in the sum in the term $(\cdot)$ in (4).[12] If two firms i and j merge, the merged entity has independent reach $\sigma_i + \sigma_j - \sigma_i \sigma_j$. Since this combined reach is lower than the sum of the pre-merger reaches, the only way the merger can be profitable is if the minimum price p_0 rises after the merger, in which case the non-merging firms also increase their profit after the merger.[13] A profitable merger must therefore increase industry profit, and so reduce consumer surplus.

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[12] If the entrant is the largest firm in the market, then the same analysis applies with σ_n replacing σ_E.

[13] It is only possible for a merger to raise the minimum price if the merged entity is the largest firm in the post-merger market. For instance, one can check that a merger between the two largest firms is always profitable.

One-or-all choice: Suppose there are $n \geq 2$ firms, where $\alpha > 0$ consumers consider all firms and α_i consumers consider only firm i. No consumers consider intermediate numbers of firms, and so the reach of firm i is $\sigma_i = \alpha + \alpha_i$. We have already discussed the symmetric case $\alpha_1 = ... = \alpha_n$, so as in Baye *et al.* (1992, Section V) suppose that $\alpha_1 < \alpha_2 < ... < \alpha_n$. Suppose first that $n = 2$, which is the situation studied by Narasimhan (1988). Lemma 1 then determines the unique equilibrium, which is that both firms have the same support for prices, $[p_0, 1]$, where $p_0 = \alpha_2 / \sigma_2$ is the larger captive-to-reach ratio, and firm $i = 1, 2$ has profit $\pi_i = \sigma_i p_0$. To maintain indifference for firm j, the CDF used by firm i in equilibrium, F_i, satisfies

$$p[\sigma_j - \alpha F_i(p)] \equiv \sigma_j p_0 \ . \tag{5}$$

Similarly to independent reach, the smaller firm's profit exceeds its captive profit α_1 while the larger firm obtains exactly its captive profit.

Industry profit in equilibrium is

$$\Pi = (\sigma_1 + \sigma_2)p_0 = \sigma_1 + \sigma_2 - \alpha - \alpha\frac{\sigma_1}{\sigma_2} \ . \tag{6}$$

One can check that industry profit increases with each portion in the Venn diagram (i.e., with α_1, α_2 and α), so that any market expansion boosts industry profit. Total welfare is the total number of consumers reached, $W = \sigma_1 + \sigma_2 - \alpha$, and consumer surplus is therefore

$$CS = \alpha\frac{\sigma_1}{\sigma_2} \ .$$

Thus, keeping reaches constant, consumer surplus increases when the overlap α is larger, even though fewer consumers are then served. Likewise, consumer surplus decreases when the larger firm's set of captive customers expands, keeping the other regions of the Venn diagram unchanged, even though more consumers are served.

To extend this analysis to more than two firms, introduce additional firms $i = 3, ..., n$, all with $\alpha_i > \alpha_2$. If the smallest firms, 1 and 2, continue to use the price strategies (5), one can check that each firm $i \geq 3$ is better off choosing the monopoly price $p = 1$ than to offer any lower price. Thus it is an equilibrium for the two smallest firms to follow the above duopoly strategies, and for all larger firms to serve only their captive base and choose the monopoly price for sure.[14] The result is that all firms except the smallest one obtain their captive profit, and only the two smallest firms ever choose prices below the monopoly level.

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[14]Baye *et al.* (1992) show this to be the unique equilibrium.

In this framework a merger between firms, falling short of a merger to monopoly, leaves the number of captive customers unchanged. As a result, almost all pair-wise mergers leave the merged entity's profit unchanged or cause it to fall. In fact, in contrast to independent reach, the only way a pair of firms could profitably merge is if the merged firm is the smallest firm in the post-merger market. Such a merger has no impact on the non-merging firms' profits and so increases industry profit and harms consumers.

In the remainder of the paper we show that other, richer possibilities exist outside these special cases. We start in the next section by describing how a radically different kind of equilibrium can arise when firms have *nested* reach.

3   Nested reach

The situation with independent reach has all consumers being equally likely to be reached by a firm, regardless of which other firms they consider. At the other extreme one could envisage consideration sets as being *nested*, in the sense that if firm i reaches a greater fraction of consumers than firm j, all firm j's consumers also consider firm i. For example, an entrant's reach lies inside an incumbent's reach if only a subset of latter's existing customers are willing to consider buying from the entrant. Likewise, if consumers consider options in an ordered fashion, as may be the case with internet search results (where some consumers just consider the first result, others consider the first two, and so on), then the reach of a lower ranked option is nested inside that of a higher ranked option. Alternatively, if consumers only consider the firms whose product they find suits their tastes, then low-quality firms might be considered by only a subset of the consumers who consider a higher-quality firm. With nested reach, only the largest firm has any captive customers, and a smaller firm has positive demand only if its price is below all the prices of larger firms.

As depicted in Figure 2, suppose there are $n \geq 3$ firms with nested reach. Let firm i have reach σ_i, where firms are ordered as $0 < \sigma_1 < \sigma_2 < ... < \sigma_n$, and for $i \geq 2$ write $\beta_i = \sigma_i - \sigma_{i-1}$ for the incremental reach of firm i. While it is hard to find the equilibrium in all nested situations, the following result describes equilibrium in those cases where incremental reach is larger for larger firms.

Proposition 1 *Suppose $n \geq 3$ firms have nested reach such that*

$$0 < \beta_2 \leq \; ... \; \leq \beta_n \; . \tag{7}$$

Then there is an equilibrium with price thresholds $p_1 < p_2 < ... < p_{n-1} < p_n = 1$ such that the price support of firm 1 is $[p_1, p_2]$, the support of firm n is $[p_{n-1}, p_n]$, and the support of firm $1 < i < n$ is $[p_{i-1}, p_{i+1}]$. Thus, only firms i and $i+1$ (where $1 \leq i < n$) choose prices in the interval (p_i, p_{i+1}). The thresholds are determined recursively by $p_2 = \frac{\sigma_1 + \beta_2}{\beta_2} p_1$ and for $1 < i < n$

$$p_{i+1} = p_i + \frac{\beta_i}{\beta_{i+1}} p_{i-1} \; , \tag{8}$$

where p_1 is chosen to make $p_n = 1$. The profit of firm 1 is $\pi_1 = \sigma_1 p_1$ and the profit of firm $i > 1$ is $\pi_i = \beta_i p_i$.

The format of this equilibrium consists of "overlapping duopolies", where each price is in the support of exactly two firms,[15] and where smaller firms only choose low prices while larger firms only choose high prices.[16] In this sense there is segmented price competition rather than head-to-head price competition, even though there is head-to-head competition in terms of consumer consideration (as firm 1's potential customers consider all firms). Nevertheless, the presence of large firms affects the profits of smaller firms, and (except for the very largest firm) *vice versa*.

To illustrate, consider the case where reach decays with a constant rate of attrition, so that the reach of firm $i = 1, ..., n$ is $\sigma_i = \delta^{n-i}$. In this case $\beta_i = \delta^{n-i}(1 - \delta)$ which increases with i as required for Proposition 1, and equation (8) becomes $p_{i+1} = p_i + \delta p_{i-1}$. When $n = 2$ the two firms have reaches $\sigma_1 = 1 - \delta$ and $\sigma_2 = 1$, and the duopoly analysis in section 2 shows that the minimum price is $1 - \delta$ and industry profit is $\Pi_2 = 1 - \delta^2$. When $n = 3$, Proposition 1 implies that the two threshold prices are $p_1 = \frac{1-\delta}{1+\delta(1-\delta)}$ and $p_2 = \frac{1}{1+\delta(1-\delta)}$ and that industry profit is $\Pi_3 = 1 - \frac{\delta^2}{1+\delta(1-\delta)}$. Both Π_2 and Π_3 decrease from 1 to 0 as δ increases from 0 to 1. Perhaps surprisingly, though, when $0 < \delta < 1$ the profit with three firms is

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[15] With the exception of the threshold prices $p_2, ..., p_{n-1}$, which are in the support of three firms.

[16] A similar pattern of segmented pricing is seen in Bulow and Levin (2006). They study a matching model where n heterogeneous firms each wish to hire a single worker from a pool with n heterogeneous workers, where the payoff from a match is (in the simplest version of their model) the product of qualities of the firm and worker. Firms choose wages which they must pay regardless of the quality of the worker eventually hired, workers care only about their wage, and higher quality workers choose their employer first. In equilibrium, firms offer wages according to mixed strategies, where higher quality firms offer wages in a higher range than lower quality firms.

strictly higher than with two firms. Since the change from $n = 2$ to $n = 3$ corresponds to entry by a third firm with reach δ^2 into the duopoly market, this implies that entry of this form *raises* industry profit.[17] Since no new consumers are served by the entrant, it follows that entry harms consumers in aggregate, even though the minimum price offered in the market is lower after entry.
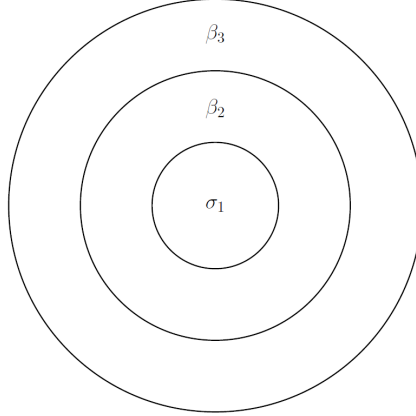

Figure 2: Three firms with nested reach

Proposition 1 can be used to obtain an expression for industry profit with any n. However, the analysis simplifies in the limit with many firms, when one can show that the threshold prices are given by the geometric progression $\kappa, \kappa^2, ...$, where $\kappa = \frac{2}{1+\sqrt{1+4\delta}} \leq 1$, and the minimum price offered converges to zero. Industry profit is $\Pi_\infty = (1 - \delta)(1 + \delta\kappa + (\delta\kappa)^2 + ...) = \frac{1-\delta}{1-\delta\kappa}$. This limit profit also decreases from 1 to 0 as δ increases, and satisfies $\Pi_2 < \Pi_\infty < \Pi_3$ when $0 < \delta < 1$. Thus entry has a non-monotonic effect on industry profit, first increasing and then decreasing profit (although the limit profit with many firms remains above that with duopoly). We discuss the possibility that entry can harm consumers in more detail in the next section, using a more transparent framework with symmetric incumbents.[18]

Proposition 1 describes equilibrium only for cases where incremental reach weakly increases. In the next section we thoroughly analyse our general framework in the case of triopoly, and find that overlapping duopoly pricing is by no means special to the nested configuration. We will also obtain results that imply for the case of nested reach that

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[17]Entry does not affect the profit of the largest firm, which obtains its captive profit in either case, but it reduces the profit of the smaller incumbent. However, the profit obtained by the entrant outweighs the lost profit of this incumbent.

[18]Another case which is easily solved is when incremental reach β_i is constant, in which case (8) entails $p_{i+1} = p_i + p_{i-1}$. It follows that threshold prices are proportional to the Fibonacci sequence.

(i) when $\beta_3 > \beta_2$ the equilibrium in Proposition 1 is unique and (ii) when $\beta_3 < \beta_2$ the equilibrium instead has all three firms using the same minimum price. However, in the latter case we will see that the largest firm can sometimes have a gap in its price support, so that it charges high and low prices but not intermediate prices.

4   The three-firm problem

In all the asymmetric configurations considered so far (independent reach, "one-or-all" choice, and nested reach) there is a clear-cut ordering of the firms, in the sense that a firm with a larger reach also has a weakly higher captive-to-reach ratio. More generally, though, these two ways to order firms need not coincide. For instance, a "niche" firm could have limited reach but have a high proportion of its reach being captive. In this section we allow for general patterns of consumer consideration in the context of triopoly.

Consider the triopoly market shown on Figure 1. For each pair of firms i and j define

$$\gamma_{ij} = \frac{\alpha_{ij} + \alpha}{\sigma_i \sigma_j} \ ,$$

where to simplify notation we have written $\alpha = \alpha_{123}$. The parameter γ_{ij} reflects *correlation* in the reach of firms i and j: σ_i and σ_j are the respective probabilities that a consumer considers firm i and firm j while $(\alpha_{ij} + \alpha)$ is the probability she considers both firms, and so γ_{ij} is above or below 1 according to whether consideration of firm i is positively or negatively correlated with consideration of firm j. With independent reach we have all $\gamma_{ij} = 1$, while if the reach of firms i and j is disjoint then $\gamma_{ij} = 0$. The pair of firms with the largest γ_{ij} can be thought of having the "strongest interaction" in the market. As we will see, if only two firms choose the lowest price p_0 in equilibrium, while the third firm only uses higher prices, they will be the pair of firms with the largest γ_{ij}.

Similarly, write

$$\gamma = \frac{\alpha}{\sigma_1 \sigma_2 \sigma_3} \ ,$$

which is again equal to 1 with independent reach. Note that $\sigma_k \gamma \leq \gamma_{ij}$ for distinct i, j and k, with equality if and only if $\alpha_{ij} = 0$. For simplicity, if $F_i(p)$ is firm i's CDF for price in equilibrium write $G_i(p) \equiv \sigma_i F_i(p)$, so that G_i increases from zero to σ_i. Using this notation, firm i's demand at price p in (1) is

$$q_i = \alpha_i F_j F_k + \sigma_i(1 - F_j)(1 - F_k) + (\alpha_i + \alpha_{ij})(1 - F_j)F_k + (\alpha_i + \alpha_{ik})F_j(1 - F_k)$$

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$$= \sigma_i + \alpha F_j F_k - (\alpha + \alpha_{ij}) F_j - (\alpha + \alpha_{ik}) F_k \tag{9}$$

$$= \sigma_i [1 + \gamma G_j G_k - \gamma_{ij} G_j - \gamma_{ik} G_k] \ . \tag{10}$$

Our main result in this section shows that the form of equilibrium depends on whether or not the interactions between firms, measured by γ_{ij}, are similar or asymmetric.

Proposition 2 *Suppose that firms are labelled so that firms 2 and 3 have the strongest interaction, i.e., $\gamma_{23} \geq \max\{\gamma_{12}, \gamma_{13}\}$.*
(i) If

$$\gamma \min\{\sigma_2, \sigma_3\} < \gamma_{12} + \gamma_{13} - \gamma_{23} \tag{11}$$

then in equilibrium all firms have the same minimum price p_0, which is the highest captive-to-reach ratio among the firms;
(ii) If

$$\gamma \min\{\sigma_2, \sigma_3\} > \gamma_{12} + \gamma_{13} - \gamma_{23} \tag{12}$$

then equilibrium takes the form of "overlapping duopoly": if firms 2 and 3 are labelled so $\sigma_3 \leq \sigma_2$, then there are prices p_0 and p_1, with $p_0 < p_1 \leq 1$, such that firm 3 has price support $[p_0, p_1]$, firm 2 has support $[p_0, 1]$ and firm 1 has support $[p_1, 1]$. (If $\sigma_2 = \sigma_3$ then $p_1 = 1$ and firm 1 chooses $p \equiv 1$ for sure.) Explicit expressions for the thresholds p_0 and p_1, as well as for the profits of the three firms, are given in the proof.

This result shows that only limited kinds of pricing patterns can emerge in equilibrium. For example, it cannot be that two firms choose prices over a range $[p_0, 1]$ while the third firm only chooses from an intermediate or upper range of prices.

Part (i) of this result applies when interactions are similar across pairs of firms (and where some consumers consider exactly two firms so that $\gamma \sigma_k < \gamma_{ij}$), as is the case with independent reach. Indeed, part (i) applies if the two pairs with the greatest interaction have a similar interaction: if say $\gamma_{23} = \gamma_{13} \geq \gamma_{12}$ and there are some consumers who consider exactly two firms then condition (11) is satisfied. In particular, if in the statement of Proposition 2 there is a "tie" for which pair of firms has the strongest interaction, then part (i) must apply. With nested reach the two smallest firms have the strongest interaction and condition (11) requires that incremental reach is smaller for larger firms. Thus with three nested firms, the cases not covered by Proposition 1 have all firms using the same minimum price.

Part (ii) applies when one pair of firms has significantly stronger interaction than other pairs. For instance, if firms 2 and 3 are considered by almost the same set of consumers (so their circles on the Venn diagram almost coincide), and if $\alpha_1 > 0$, then firms 2 and 3 have the greatest interaction and condition (12) is satisfied, and firm 1 chooses price $p \approx 1$. Intuitively, when two firms reach nearly the same set of consumers, they compete fiercely between themselves, leaving the remaining firm to price at or near the monopoly level. Likewise, if firm 1 has a large captive base so that α_1 is large (and when firms 2 and 3 have some overlap), then firms 2 and 3 have the greatest interaction and condition (12) is satisfied. With nested reach, condition (12) requires that incremental reach is larger for larger firms, thus verifying Proposition 1. Another situation where (12) holds is the "one-or-all" specification in Baye *et al.* (1992, Section V), where no consumer considers exactly two firms and $\sigma_1 > \sigma_2 \geq \sigma_3$, in which case $\gamma_{ij} = \alpha/(\sigma_i \sigma_j)$ and the two smallest firms 2 and 3 have the greatest interaction. Yet another configuration where part (ii) applies is when two firms have disjoint reach, so that $\gamma_{13} = \gamma = 0$ say, in which case (12) holds whenever $\gamma_{12} \neq \gamma_{23}$. Thus the only way that two firms can have overlapping price supports is if they have overlapping reach.

In the knife-edge case where

$$\gamma \min\{\sigma_2, \sigma_3\} = \gamma_{12} + \gamma_{13} - \gamma_{23} \, , \tag{13}$$

which is not covered by Proposition 2, there is the possibility that both kinds of equilibrium coexist. For instance, this is so in the symmetric Varian-type market where $\alpha_{12} = \alpha_{13} = \alpha_{23} = 0$ and $\alpha_1 = \alpha_2 = \alpha_3$, where there is a symmetric equilibrium where all firms price low and also asymmetric equilibria where one of the firms chooses $p \equiv 1$. (See Baye *et al.* (1992) for the full range of equilibria in this market.)

Equilibrium strategies when all firms use the same minimum price: Proposition 2 provided much information about equilibria in this model—it characterises equilibrium profit and consumer surplus in the two regimes, and it describes equilibrium strategies when part (ii) applies. However, it does not describe equilibrium pricing strategies for part (i), and the equilibrium patterns of prices turn out to have interesting economic properties.

In the earlier version of this paper (Armstrong and Vickers, 2018, Proposition 2) we calculated an equilibrium whenever part (i) applied (without showing if it was unique), and this took one of two forms: either (a) the three firms were active in a lower price range

and then two were active in a range of higher prices, or (b) the three firms were active in a lower price range, then only the most competitive pair were active in an intermediate price range, and then another pair of firms were active in a higher range. In particular, in situation (b) one firm (firm 1 using the labelling in Proposition 2) chose low and high prices, but not intermediate prices.
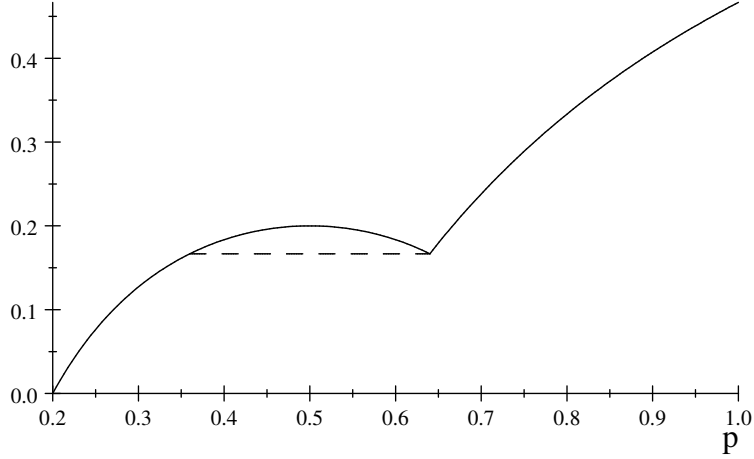

Figure 3: "Ironing" in a nested market with $\sigma_1 = 1/2$, $\sigma_2 = 4/5$, $\sigma_3 = 1$

The general analysis was complicated, and here we merely report an example to show the possibility. Suppose three firms have nested reach, where $\sigma_3 = \frac{1}{2}$, $\sigma_2 = \frac{4}{5}$ and $\sigma_1 = 1$. We show in the appendix that equilibrium with this pattern of choice sets has all firms choosing prices in the range $[\frac{1}{5}, \frac{9}{25}]$, firms 2 and 3 choosing prices in the range $[\frac{9}{25}, \frac{16}{25}]$ and firms 1 and 2 choosing prices in the range $[\frac{16}{25}, 1]$. The reason why the largest firm has non-convex price support can be explained as follows. When all firms price low in equilibrium, so that part (i) of Proposition 2 applies, one can calculate that the three CDFs increase in p for prices just above p_0, the minimum price. (This is ensured by condition (11).) One can also calculate the smallest price, p_1 say, at which some CDF reaches 1 and above which the two remaining firms compete as duopolists for prices up to 1. (In the nested case, it is the smallest firm's CDF which first reaches 1, although in the general model more detailed analysis is required to determine which firm first drops out.)

However, in some cases—as in this example—firm 1's candidate CDF (i.e., when we ignore the monotonicity constraint on the CDF) starts to *decrease* in p before the largest CDF reaches 1, which cannot therefore be a valid CDF. Figure 3 illustrates firm 1's candidate CDF if we ignored its monotonicity constraint. The correct CDF for this firm is

then obtained by "ironing" this curve as shown on the figure, so that the largest firm does not choose prices in the interval denoted by the dashed line, which in this example is the interval $(\frac{9}{25}, \frac{16}{25})$. (This CDF does not reach 1 since this firm has an atom at $p = 1$ in equilibrium.)

The equilibria with ironing—when one firm's price support has a gap in the middle—provide insight into the relationship between the two seemingly contrasting parts of Proposition 2. A configuration which is "well inside" the parameter space defined by (11) will have a pattern of prices similar to that with independent reach: all three firms choose low prices, then firm 3 drops out leaving firms 1 and 2 to compete in the range with high prices. As parameters change to approach the boundary (13), the candidate CDF for firm 1 will start to decrease before firm 3's CDF reaches 1. In this case, the "ironing" procedure is used so that firm 1's price support has a gap in the middle. As the boundary (13) is reached, the lower price range where all three firms are active shrinks and ultimately vanishes, leaving an equilibrium of the overlapping duopoly form when parameters lie in the region (12).

The impact of entry: As an application of this analysis, consider the impact of entry by a third firm into a duopoly market. If the three firms have independent reach, then as discussed in section 2 entry will always increase consumer surplus. Beyond this case, however, the analysis is less clear cut. Entry might induce an incumbent to retreat towards its captive base by raising its price, thereby harming its captive customers. This is the case, for example, when the set of consumers reached by the entrant approximately coincides with the set reached by one of the incumbents. Then these firms will set prices $p \approx 0$, while the other incumbent chooses $p \approx 1$ and almost fully exploits its captive customers. Nevertheless, since entry of this form reduces industry profit, consumers overall will benefit.

In section 3 we have already seen examples where entry harms consumers overall. These involved nested reach with a constant rate of decay in consideration, where entry by a third smaller firm induced overlapping duopoly pricing with the result that the minimum price fell after entry. This kind of nested entry does not affect the number of captive customers in the market. More generally, when entry only occurs within contested segments there is a tendency for entry to harm consumers overall.

To illustrate, suppose the incumbents are symmetric and the entrant is considered only

by those consumers who already consider both incumbents, as shown on Figure 4. This pattern of consideration is reasonable if only "savvy" consumers consider buying from the entrant, and these are the consumers who are already able to consider both incumbents. In this case part (i) of Proposition 2 applies to the post-entry market (provided the entrant's reach lies strictly inside the incumbents' overlap). The minimum price is equal to an incumbent's captive-to-reach ratio, which is unchanged with entry. Thus, entry of this form leaves welfare and incumbent profit unaffected, increases industry profit due to the profit obtained by the entrant, and so harms consumers. In fact, it is perfectly possible that even the consumers who consider all three firms are harmed by this form of entry, despite being able to choose among more firms, as the higher prices offered by incumbents leave the entrant relatively free to set high prices too.
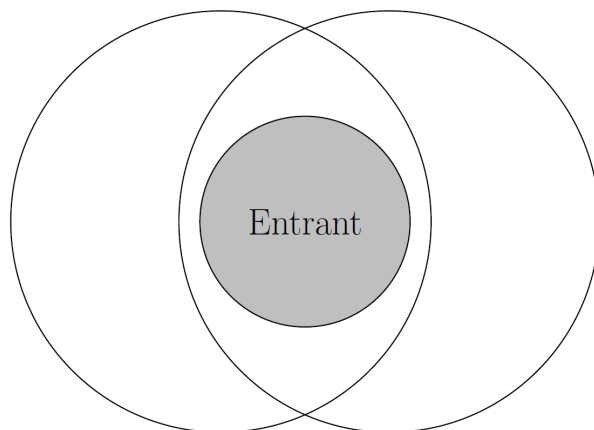

Figure 4: Entry into the contested market

This result is related to Rosenthal (1980), where entry by a new firm causes the average price paid by both captive and informed consumers to rise. However, in his model the entrant arrives with its own new pool of captive customers, thus raising welfare, whereas the effect arises in our scenario despite the entrant having none.[19]

The impact of market expansion: Another informative comparative statics exercise is to consider the impact of a market expansion. An old intuition is that an increase in the num-

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[19]Relatedly, in a setting with differentiated products, Chen and Riordan (2008) show how entry to a monopoly market can induce the incumbent to raise its price. For instance, entry by generic pharmaceuticals might cause a branded incumbent to raise its price, as it prefers to focus on those "captive customers" who care particularly about its brand. Closer to the consideration set framework is Chen and Riordan (2007), who study a model with symmetric firms, where consumers either consider a single random firm or consider a random pair of firms. Among other results, they show that the equilibrium price can increase when an additional firm enters.

ber of comparison shoppers—consumers who compare prices from several firms—induce firms to lower their prices, which benefits all consumers including captives. As discussed in section 2, this is true in a symmetric market where an increase in the number of consumers who consider $m \geq 2$ firms induces all firms to reduce their prices. However, this is less clear more generally. If the interaction between one pair of firm increases disproportionately, this could give a third firm an incentive to *raise* its price, thereby harming its captive customers. To illustrate, starting from a symmetric triopoly market, if we increase α_{23} then part (ii) of Proposition 2 will eventually apply, in which case firm 1 will focus on exploiting its captive base and choose $p \equiv 1$. Thus, increased interaction between two firms can harm the captives of a third firm.[20]

Consider next the impact of a market expansion on industry profit. With duopoly, we have seen that an increase in any or all of the three parameters α_1, α_2 and α_{12} must increase industry profit (although it might reduce one firm's profit). With duopoly, increasing the size of the overlap region α_{12} will intensify competition (in the sense that the minimum price p_0 is reduced), but this is outweighed by impact on each firm's reach so that $(\sigma_1+\sigma_2)p_0$ rises. With triopoly, by contrast, increasing the fractions in some regions of the Venn diagram can intensify competition to an extent that outweighs the market expansion effect, so that industry profit falls. To see this, consider a triopoly market where part (i) of Proposition 2 applies, in which case industry profit is

$$\Pi = (\sigma_1 + \sigma_2 + \sigma_3)p_0 \ , \tag{14}$$

where p_0 is the highest captive-to-reach ratio. If firm 1 has the highest captive-to-reach ratio, then a small increase in that firm's overlap regions α_{12}, α_{13} or α will keep the form of the equilibrium unchanged, but the minimum price p_0 will fall. Firm 1's profit is unchanged (since it obtains its captive profit regardless), and one can calculate that the impact on industry profit (14) of an increase in α_{12} or α_{13} is negative if $\sigma_1 < \sigma_2+\sigma_3$, while an increase in α reduces profit if $2\sigma_1 < \sigma_2 + \sigma_3$.

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[20]A similar effect can occur when the fraction of consumers who consider all three firms rises. For instance, suppose consumer segments are (proportional to) $\alpha_1 = 3$ and $\alpha_2 = \alpha_3 = \alpha_{12} = \alpha_{13} = \alpha_{23} = 1$, then for any α firms 2 and 3 are the most competitive pair, and for small α part (i) of the proposition applies, while if α is increased part (ii) eventually applies in which case firm 1 chooses $p \equiv 1$. Here, an increase in α affects the interaction between firms 2 and 3 disproportionately, and pushes the market towards segmented pricing.

The impact of a merger: We discussed in section 2, within the three special patterns of consideration described there, how a profitable merger harmed consumers overall. We now show that the same is always true in the three-firm case. Specifically, we show that a profitable merger between two firms necessarily increases the third firm's profit. Suppose that firm N is the one not merging. If N's pre-merger profit was equal to its captive profit α_N, the merger of the other two firms clearly cannot reduce that. If N's pre-merger profit was equal to $\sigma_N p_0$—i.e., its reach times the minimum price—then the merger between the other two will increase N's profit because the minimum price must rise for the merger to be profitable.[21] The only remaining possibility is a merger between firms 2 and 3 in the conditions of part (ii) of Proposition 2 where, moreover, firm 2 has an atom at $p = 1$. We show in the appendix that a profitable merger increases 1's profit in this case too. Therefore profitable mergers in the three-firm case never reduce the profit of the non-merging firm, in which case industry profit rises. We deduce that any profitable merger is detrimental to consumers.

Figure 5: A profitable merger which benefits consumers

However, it is not true in general, with more than three firms, that profitable mergers harm consumers. It may be, for example, that a merger between two firms with a strong interaction—which is therefore likely to be profitable—might induce non-merging firms to "enter the fray" and compete for the newly-profitable consumer segment, with the result that overall industry profits might fall and consumers are made better off. To illustrate

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[21]Before the merger, the combined profit of the merging firms, say firms A and B, was at least $(\sigma_A + \sigma_B)p_0$, and since their combined reach falls after the merger, for the merger to be profitable the minimum price must rise.

this possibility consider the following example, which draws from our analysis of triopoly together with the symmetric case from section 2. Figure 5 shows the pattern of consumer consideration. There are initially five firms, where firms 4 and 5 reach precisely the same set of consumers (depicted as the shaded set) and hence set price $p = 0$ in equilibrium. Firms 1, 2 and 3 each have a single captive customer, a single consumer considers each set of firms $\{1, 2\}$ and $\{3, 4, 5\}$, while four consumers consider each set of firms $\{2, 3\}$ and $\{1, 4, 5\}$. (No consumers consider more than three firms.)

Since firms 4 and 5 set price zero, firms 1, 2 and 3 compete as triopolists as if the shaded row on Figure 5 was eliminated. Here, firms 2 and 3 have the greatest competitive interaction in this triopoly, and Proposition 1 implies that equilibrium takes the overlapping duopoly form with firms 2 and 3 setting low prices and firms 1 and 2 setting high prices. The proof of part (ii) of the Proposition shows that firm 1 obtains its captive profit ($\pi_1 = 1$), while firms 2 and 3 obtain respective profits $\pi_2 = 6p_0$ and $\pi_3 = 5p_0$, where $p_0 = \frac{2}{7}$ is the minimum price. Since firms 4 and 5 make zero profit industry profit is $\frac{29}{7}$. Due to the asymmetry between firms, industry profit substantially exceeds the captive profit, 3.

Now suppose firms 4 and 5 merge. (Clearly this is a profitable merger, as before the firms obtained no profit.) The symmetry of the market implies that each firm now obtains its captive profit ($\pi_i = 1$), so that industry profit falls to 4 after the merger, and consumers overall are better off. Intuitively, before the merger the market was highly asymmetric, which allowed firms to enjoy high profits, and the merger brings more intense symmetric competition to the market. This example shows that not all profitable mergers in our setting are detrimental to consumers, but such competition-enhancing mergers appear to be relatively rare.

5   Conclusions

The aim of this paper has been to explore, in a parsimonious framework with price-setting firms and homogeneous products, how the structure of consumer choice sets matters for the nature of equilibrium price dispersion. The analysis has yielded a number of results that we did not initially expect.

First, we found equilibria with segmented pricing patterns, i.e., with some firms only pricing high and others only pricing low. Second, in the three-firm case we established generically either that all firms set the same minimum price (in which case their profit

was proportional to reach), or that pricing was segmented (so that one firm only set low prices and one set only high prices). In prior literature multiplicity of equilibria has gained considerable attention, and all such cases lie on the knife edge between these two regimes. Third, the key to determining which of the two regimes applies was found to be the proximity or otherwise of the correlation measures of pairwise interaction, and when one pair of firms had significantly stronger interaction than other pairs then segmented pricing ensued. Fourth, for some parameter configurations we found equilibria with a gap in one firm's price support, so that that firm sometimes prices high, and sometimes low, but never in between. Fifth, we found plausible patterns of consumer consideration in which entry is detrimental to consumers because it softens competition between incumbents, leading them to retreat towards their captive base. Likewise, there were situations where an increase in the number of consumers who consider one pair of firms causes a third firm to retreat towards its captive base, showing that search externalities need not benefit all consumers. Sixth, profitable mergers were shown always to be detrimental to consumers in the three-firm case, as in the special cases discussed in section 2, but not more generally.

The analysis could be extended in a number of directions. One would be to settings beyond nested reach and the three-firm case that we have analysed in detail. For example, one could seek more general conditions for equilibrium to take the overlapping duopoly form, or one could try to establish that all firms use the same minimum price when (appropriately generalised) competitive interactions are similar enough. Second, one could investigate policy interventions in these kinds of markets. For example, when would the imposition of a price cap on a large firm induce other firms to lower or raise their prices? A third extension would be to endogenise the pattern of choice sets, beyond our analysis of entry and mergers, by introducing search by consumers, word-of-mouth communication between consumers, or advertising by firms.[22] For instance, one could study a model of non-sequential search where a consumer can determine her choice set S by incurring a specified up-front search cost (increasing in S). Such a framework would generalize Burdett and Judd (1983, section 3.2) to allow firms to be asymmetric and for consumers to target specific firms for consideration. Alternatively, word-of-mouth communication could mean that a firm's reach was influenced by the price it offers.

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[22] For instance, in the context of advertising, Ireland (1993) and McAfee (1994) study a sequential model where firms first invest in reach and then compete in price, while Butters (1977) studies the situation where firms choose their reach and price simultaneously. (In each case reach is assumed to be independent.)

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Technical Appendix

Sketch proof of Lemma 1: We first discuss arguments to do with deletion of dominated prices. In any equilibrium we have $\pi_i \geq \alpha_i$, since firm i can ensure at least this profit by choosing price equal to 1 and serving its captive customers. For this reason, no firm would ever offer a price below ρ_i, its captive-to-reach ratio, since if it did so it would obtain profit below α_i even if it supplied its entire reach.

To see that every firm makes positive profit we invoke Assumption 2. There is at least one firm i which has captive customers, and which will not set price below $\rho_i > 0$. (Clearly this firm makes positive profit.) From the remaining firms, at least one firm j has captive customers in the subset of firms excluding i, and so this firm can set price ρ_i and be sure to obtain positive profit. Firm j therefore also has a positive lower bound on its prices. Following the same argument, a firm in the subset of firms excluding both i and j can obtain positive profit, and so on until the set of firms is exhausted. In particular, each firm's minimum price is strictly above zero and hence so is p_0. This proves part (ii).

If price $p < 1$ is in firm i's support then $q_i(\cdot)$ in (1) cannot be flat for prices just above p, for otherwise the firm would obtain strictly greater profit by raising its price above p. This implies that this price must be in the support of at least one other firm. More precisely, if price $p < 1$ is in firm i's support it must be in the support of at least one of its "potential competitors", where in a given equilibrium we say that firm j is a "potential competitor" for firm i at price p if firm i's expected demand falls when j slightly undercuts i at price p given the equilibrium strategies followed by firms other than i and j. (This then implies that i is a potential competitor for j.) If for all duopoly segments we have $\alpha_{ij} > 0$, then every firm is a potential competitor for every other firm. However, two firms might have disjoint reaches, and so cannot be potential competitors. More generally,

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the overlap between i and j might be contained within a third firm's reach, and if in the equilibrium the third firm always chooses price below p, then i and j do not compete at price p. If price p in firm i's support was *not* in the support of at least one of its potential competitors, firm i's demand would be flat (and positive) in this neighbourhood of p, which is not compatible with p maximizing the firm's profit.

We next turn to arguments concerning the possibility of "atoms" in the price distributions. First observe that two firms cannot both have an atom at price p if they are potential competitors at this price (for otherwise each would have an incentive to undercut the price p and gain a discrete jump in demand).

To see that each firm's price distribution is continuous in the interval $[p_0, 1)$, suppose by contrast that firm i has an atom at some price $0 < p < 1$ in its support. We claim that firm's i demand in (1) must then be locally flat above p. As noted above, there cannot be a potential competitor to i at price p which also has an atom at p, and so q_i does not jump down discretely at p. In addition, any potential competitor to i at p obtains a discrete increase in demand if it slightly undercuts p, and so such a firm would never choose a price immediately above p. Since no potential competitor chooses a price immediately above p, firm i loses no demand if it raises its price slightly above p, which is not compatible with p maximizing the firm's profit. Therefore, firm i cannot have an atom below 1, and this completes the proof of part (iii). This implies that each firm's demand (1) is continuous in the interval $[p_0, 1)$.

Similarly, if p_0 is the minimum price ever chosen in the market, then all prices in the interval $[p_0, 1]$ are sometimes chosen. If p is in firm i's support but no firm is active in an interval (p, p') above p, then firm i has flat demand over the range (p, p'), and this cannot occur in equilibrium. This completes the proof of part (iv).

Suppose now that there are at least three firms. Let P_{ij} denote the set of prices in $[p_0, 1]$ which are in the supports of both firm i and firm j, which is a closed set. Part (iv) implies that the collection $\{P_{ij}\}$ covers the interval $[p_0, 1]$, and since each firm participates, at least two of the sets in $\{P_{ij}\}$ are non-empty. If there were no price in the support of three or more firms then the collection $\{P_{ij}\}$ would consist of disjoint sets. However, since $[p_0, 1]$ is connected it cannot be covered by two or more disjoint closed sets, and we deduce that at least two sets in $\{P_{ij}\}$ must overlap, which proves part (v).

Firms can have an atom at the reservation price $p = 1$. However, as noted above, if

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firm i has an atom at $p = 1$ then no potential competitor can also have an atom at 1, which implies that when firm i chooses $p = 1$ it sells *only* to its captive customers and so its profit is precisely $\pi_i = \alpha_i$. If no firm has an atom at $p = 1$ then any firm with $p = 1$ in its support (and there must be at two such firms from part (iv)) has profit equal to α_i. This completes the proof for part (i).

Let firm j be a firm which obtains profit equal to α_j. Then the minimum price ever chosen, p_0, must be no higher than ρ_j (for otherwise firm j could obtain more profit by choosing $p = p_0$), and so p_0 cannot exceed the highest ρ_i. Since no firm sets a price below its ρ_i, the minimum price p_0 (which from part (iv) is sometimes chosen by at least two firms) must be weakly above the second lowest ρ_i. Finally, if the firm with the highest ρ_i has p_0 in its support, then p_0 cannot be strictly lower than this highest ρ_i, and so must equal this highest ρ_i. This completes the proof for part (vi).

Proof of Proposition 1: We construct an equilibrium of the stated form. The profit of the largest firm n is $\pi_n = \beta_n$, its number of captive customers, and denote the profit of smaller firms by π_i. In the highest interval $[p_{n-1}, 1]$ used by the two largest firms, these firms are sure to be undercut by all smaller rivals, and so in this price range their CDFs must satisfy

$$\beta_n + \beta_{n-1}(1 - F_{n-1}(p)) = \frac{\beta_n}{p} \; ; \; \beta_{n-1}(1 - F_n(p)) = \frac{\pi_{n-1}}{p} \; .$$

Since $F_n(p_{n-1}) = 0$ it follows that p_{n-1} and π_{n-1} are related as

$$\pi_{n-1} = \beta_{n-1} p_{n-1} \; .$$

We have $F_{n-1}(1) = 1$, while the largest firm has an atom at $p = 1$ with probability $1 - F_n(1) = \pi_{n-1}/\beta_{n-1} = p_{n-1}$.

In the lowest interval $[p_1, p_2]$ used by the two smallest firms, these firms are sure to undercut all larger rivals, and so in this range their CDFs must satisfy

$$\beta_2 + \sigma_1(1 - F_1(p)) = \frac{\pi_2}{p} \; ; \; \sigma_1(1 - F_2(p)) = \frac{\pi_1}{p}$$

and since $F_1(p_1) = F_2(p_1) = 0$ it follows that

$$\pi_1 = \sigma_1 p_1 \; ; \; \pi_2 = (\sigma_1 + \beta_2)p_1 \; .$$

Since $F_1(p_2) = 1$ we have $\pi_2 = \beta_2 p_2$, which combined with the previous expression for π_2 implies that

$$p_2 = \frac{\sigma_1 + \beta_2}{\beta_2}p_1 \; . \tag{15}$$

If there are just three firms, these are the two price intervals in the equilibrium. With more than three firms there are intermediate intervals, and in the interval $[p_i, p_{i+1}]$, where $1 < i < n - 1$, firms i and $i + 1$ are active and will be undercut by smaller rivals and undercut their larger rivals. Therefore, in this range their CDFs must satisfy

$$\beta_{i+1} + \beta_i(1 - F_i(p)) = \frac{\pi_{i+1}}{p} \; ; \; \beta_i(1 - F_{i+1}(p)) = \frac{\pi_i}{p} \; . \tag{16}$$

Since $F_{i+1}(p_i) = 0$ it follows that

$$\pi_i = \beta_i p_i \; .$$

An intermediate firm i, where $2 \leq i \leq n - 1$, is active in both the intervals $[p_{i-1}, p_i]$ and $[p_i, p_{i+1}]$, and its CDF F_i needs to be continuous across the threshold price p_i. At the price p_i we therefore require that

$$\frac{\pi_{i-1}}{\beta_{i-1} p_i} = 1 - F_i(p_i) = \frac{1}{\beta_i} \left(\frac{\pi_{i+1}}{p_i} - \beta_{i+1} \right) \; , \tag{17}$$

where in the case of $i = 2$ we have written $\beta_1 = \sigma_1$. If we write $p_n = 1$ then we have $\pi_i = \beta_i p_i$ for all firms $1 \leq i \leq n$, and so for $2 \leq i \leq n - 1$ expression (17) entails expression (8). This is a second-order difference equation in p_i where p_1 is free, p_2 is given in (15), and the terminal condition $p_n = 1$ serves to pin down p_1. It is clear from (15) and (8) that the sequence $p_1, p_2, p_3, ...$ is an increasing sequence of price thresholds. This completes the description of the candidate equilibrium.

We next show that no firm has an incentive to deviate from its described strategy. By construction, firm i is indifferent between choosing any price in the interval $[p_{i-1}, p_{i+1}]$, assuming its rivals follow the stated strategies. We need to check that a firm's profit is no higher if it chooses a price outside this interval. Consider first an upward price deviation, which is only relevant if $i < n - 1$. If $i < n - 2$ and firm i chooses a price above p_{i+2} is has no demand since firm $i + 1$ is sure to set a lower price and all firm i's potential customers also consider firm $(i + 1)$'s price. Suppose then that $i < n - 1$ and firm i chooses a price $p \in [p_{i+1}, p_{i+2}]$, in which case it has demand β_i if its price is below the prices of both rivals $i + 1$ and $i + 2$. Therefore, from (16) its profit with such a price is

$$p\beta_i[1 - F_{i+1}(p)][1 - F_{i+2}(p)] = \frac{\beta_i \pi_{i+1}}{\beta_{i+1}^2} \left(\frac{\pi_{i+2}}{p} - \beta_{i+2} \right) = p_{i+1} \frac{\beta_i \beta_{i+2}}{\beta_{i+1}} \left(\frac{p_{i+2}}{p} - 1 \right) \; .$$

This profit decreases from $\pi_i = \beta_i p_i$ at $p = p_{i+1}$ to zero at $p = p_{i+2}$. We deduce that firm i cannot increase its profit by choosing a price above p_{i+1}.

Next consider a downward price deviation, so that firm i chooses a price below p_{i-1} (which is only relevant when $i > 2$). Suppose that this firm chooses a price in the interval $[p_j, p_{j+1}]$, where $j \leq i - 2$. The firm will undercut all firms larger than firm $j + 1$, and so obtain demand at least $\beta_{j+2} + ... + \beta_i$. It will also serve the segment β_{j+1} if it undercuts firm $j + 1$ and it will additionally serve the segment β_j if it undercuts both firms j and $j + 1$. Putting this together implies that the firm's profit with price $p \in [p_j, p_{j+1}]$ is

$$p \left\{ \beta_{j+2} + ... + \beta_i + (1 - F_{j+1}(p))(\beta_{j+1} + \beta_j(1 - F_j(p))) \right\} . \tag{18}$$

Given the CDFs in (16), this profit is a convex function of p and so must be maximized in this range either at p_j or at p_{j+1}. Therefore, we can restrict our attention to deviations by firm $i > 2$ to the threshold prices $\{p_1, p_2, ..., p_{i-2}\}$. If it chooses price p_j where $2 \leq j \leq i - 2$, expression (18) implies its profit is

$$p_j \left\{ \beta_{j+1} + ... + \beta_i + \beta_j(1 - F_j(p_j)) \right\} .$$

Expression (17) implies that $\beta_j(1 - F_j(p_j))$ is equal to $\beta_{j+1}(\frac{p_{j+1}}{p_j} - 1)$, in which case the above deviation profit with price p_j is

$$p_j \left(\beta_{j+1} + ... + \beta_i + \beta_{j+1}(\frac{p_{j+1}}{p_j} - 1) \right) = \beta_{j+1}p_{j+1} + (\beta_{j+2} + ... + \beta_i)p_j . \tag{19}$$

One can check that expression (19) holds also for $j = 1$. We need to show that (19) is no higher than firm i's equilibrium profit, which is $\pi_i = \beta_i p_i$. We do this in two steps: (i) we show that (19) is increasing in j given i, so that $j = i - 2$ is the most tempting of these deviations for firm i, and (ii) we show (19) is below $\beta_i p_i$ when $j = i - 2$.

To show (i), suppose that $i \geq 4$, which is the only relevant case, and suppose that $1 \leq j \leq i - 3$. Then firm i's deviation profit with price p_{j+1} from (19) is

$$\beta_{j+2}p_{j+2} + (\beta_{j+3} + ... + \beta_i)p_{j+1} = \beta_{j+1}p_j + \beta_{j+2}p_{j+1} + (\beta_{j+3} + ... + \beta_i)p_{j+1}$$

$$\geq \beta_{j+1}p_j + \beta_{j+2}p_{j+1} + (\beta_{j+3} + ... + \beta_i)p_{j+1} - (\beta_{j+2} - \beta_{j+1})(p_{j+1} - p_j)$$

$$= \beta_{j+1}p_{j+1} + \beta_{j+2}p_j + (\beta_{j+3} + ... + \beta_i)p_{j+1} \geq \beta_{j+1}p_{j+1} + (\beta_{j+2} + ... + \beta_i)p_j$$

where the final expression is the firm's deviation profit with price p_j, which proves claim (i). (Here, the first equality follows from (8), the first inequality follows from (7) and the fact that $\{p_j\}$ is an increasing sequence, while the final inequality follows from $\{p_j\}$ being an increasing sequence.)

To show claim (ii), suppose that $i \geq 3$ which is the only relevant case, and observe that

$$
\begin{aligned}
\beta_i p_i &= \beta_{i-1} p_{i-2} + \beta_i p_{i-1} \\
&\geq \beta_{i-1} p_{i-2} + \beta_i p_{i-1} - (\beta_i - \beta_{i-1})(p_{i-1} - p_{i-2}) \\
&= \beta_{i-1} p_{i-1} + \beta_i p_{i-2}
\end{aligned}
$$

where the final expression is (19) when $j = i - 2$. (Here, the first equality follows from (8) and the inequality follows from $\{\beta_i\}$ being an increasing sequence.) This completes the proof that the stated strategies constitute an equilibrium.

Proof of Proposition 2: Lemma 1 shows that in any equilibrium each price in the range $[p_0, 1]$ is chosen by at least two firms, where p_0 denotes the minimum price offered by any firm in the equilibrium. In particular, either two or all three firms have p_0 in their supports. The lemma also shows that there is at least one price in all three price supports. Let L and H denote respectively the lowest and highest price among the prices in all three supports. (The set of prices in all three supports is closed.)

(i) Suppose that an equilibrium has $L > p_0$, so that only two firms, say firms i and j, offer the minimum price p_0. These firms obtain profit $\pi_i = \sigma_i p_0$ and $\pi_j = \sigma_j p_0$ and in the interval $[p_0, L]$ where $G_k(p) = 0$ expression (10) implies

$$
\gamma_{ij} G_j(p) = \gamma_{ij} G_i(p) = 1 - \frac{p_0}{p} . \tag{20}
$$

This implies that $G_i \equiv G_j$ in this interval and let $\delta = G_i(L) = G_j(L) > 0$.

Firm k weakly prefers price L to price p_0, and so (10) implies

$$
\sigma_k p_0 \leq \sigma_k L[1 - \gamma_{ik} G_i(L) - \gamma_{jk} G_j(L) + \gamma G_i(L) G_j(L)] . \tag{21}
$$

(Here, the left-hand side is its profit if it chooses p_0, when it will serve its entire reach, while the right-hand side is its profit with the higher price.) This inequality can be written as

$$
\gamma_{ik} \delta + \gamma_{jk} \delta - \gamma \delta^2 \leq 1 - \frac{p_0}{L} = \gamma_{ij} \delta
$$

where the equality follows from (20). We can divide by $\delta > 0$ to obtain

$$
\gamma_{ij} \geq \gamma_{ik} + \gamma_{jk} - \gamma \delta . \tag{22}
$$

Since $\delta = G_i(L) \leq \sigma_i$ and $\delta = G_j(L) \leq \sigma_j$, the term $\gamma \delta$ is weakly below both γ_{ik} and γ_{jk}. Expression (22) therefore implies that γ_{ij} is weakly greater than both γ_{ik} and γ_{jk}, and so

using the stated labelling for firms we have $k = 1$ and the two low-price firms are firms 2 and 3. Since $\delta \leq \min\{\sigma_2, \sigma_3\}$, expression (22) then implies

$$\gamma_{23} \geq \gamma_{12} + \gamma_{13} - \gamma\delta \geq \gamma_{12} + \gamma_{13} - \gamma\min\{\sigma_2, \sigma_3\} \ . \tag{23}$$

Therefore, if (11) holds the equilibrium cannot take the form where $L > p_0$, and the only alternative is that all three firms use the same minimum price p_0. Lemma 1 (vi) shows that this minimum price must then be the highest captive-to-reach ratio.

(ii) If condition (12) holds we will show that $L = H$ so there is only one price in all three supports. Either all three firms have the same minimum price p_0 or only two firms do, and in the latter case the proof for part (i) shows that it must be firms 2 and 3 which price low. In either case firms 2 and 3 use p_0, and in either case we have $G_2(L) = G_3(L) = \delta \geq 0$. Suppose by contradiction that in equilibrium we have $H > L$. Let i and j label firms 2 and 3 such that $G_i(H) \geq G_j(H)$. Then since we cannot have only firm 1 active in the open interval (L, H), one or both of 2 and 3 must choose prices in (L, H), and so $\delta = G_i(L) < G_i(H) \equiv g$.

Firms 2 and 3 obtain respective profits $p_0\sigma_2$ and $p_0\sigma_3$, and let π_1 denote firm 1's profit. Expression (10) shows that a price p in firm 1's support satisfies

$$\pi_1 = \sigma_1 p[1 - \gamma_{12}G_2(p) - \gamma_{13}G_3(p) + \gamma G_2(p)G_3(p)] \ ,$$

and setting $p = L, H$ in the above and subtracting implies that

$$\begin{aligned}
\frac{\pi_1}{\sigma_1}\left(\frac{1}{L} - \frac{1}{H}\right) &= \gamma_{12}G_2(H) + \gamma_{13}G_3(H) - \gamma G_2(H)G_3(H) \\
&\quad - \gamma_{12}G_2(L) - \gamma_{13}G_3(L) + \gamma G_2(L)G_3(L) \\
&\leq \gamma_{12}g + \gamma_{13}g - \gamma g^2 - \gamma_{12}\delta - \gamma_{13}\delta + \gamma\delta^2 \\
&= (g - \delta)(\gamma_{12} + \gamma_{13} - \gamma(g + \delta)) \ . \tag{24}
\end{aligned}$$

Here, the inequality follows since $\gamma_{12} \geq \gamma G_3(H)$ and $\gamma_{13} \geq \gamma G_2(H)$, and so the initial expression is weakly increased if we replace $G_j(H)$ by $g = G_i(H) \geq G_j(H)$. Likewise, and using that fact that $G_1(L) = 0$, for firm j we have

$$\begin{aligned}
p_0\left(\frac{1}{L} - \frac{1}{H}\right) &= \gamma_{23}G_i(H) + \gamma_{1j}G_1(H) - \gamma G_1(H)G_i(H) - \gamma_{23}G_i(L) \\
&= \gamma_{23}g + \gamma_{1j}G_1(H) - \gamma g G_1(H) - \gamma_{23}\delta \\
&\geq \gamma_{23}(g - \delta) \ .
\end{aligned}$$

Since $\pi_1 \geq \sigma_1 p_0$ (as firm 1 weakly prefers any price in its support to p_0) and $g - \delta > 0$, it follows that

$$\gamma_{23} \leq \gamma_{12} + \gamma_{13} - \gamma(g + \delta) \ . \tag{25}$$

If $\gamma = 0$ (so no consumers consider all three firms) this inequality contradicts (12), so we deduce that it is not possible to have $H > L$ when (12) holds and $\gamma = 0$. Therefore, suppose henceforth that $\gamma > 0$. Then since $g > 0$ the inequality (25) contradicts the first inequality in (23) which holds whenever $L > p_0$. We deduce that if $H > L$ then all three firms must have the same minimum price p_0 and hence $\delta = 0$.

We show next that if all three firms have the same minimum price, then (12) cannot hold. First suppose that $H < 1$, so that only two firms are active in the upper range $(H, 1]$. If firm 1 uses $p = 1$, then one of firms 2 or 3 has its maximum price at H, so that $G_2(H) = \sigma_2$ or $G_3(H) = \sigma_3$. Therefore $g = G_i(H) \geq \min\{\sigma_2, \sigma_3\}$, in which case (25) is inconsistent with (12).

Continue with the assumption that $H < 1$, but now suppose it is firms 2 and 3 which are active above H, so that $G_1(H) = \sigma_1$. Since all three firms have profit equal to p_0 multiplied by their reach, (10) implies that for firm 1 and firm j we have respectively

$$
\begin{aligned}
p_0 &= H\left[1 - \gamma_{12}G_2(H) - \gamma_{13}G_3(H) + \gamma G_2(H)G_3(H)\right] \\
p_0 &= H\left[1 - \gamma_{1j}\sigma_1 - \gamma_{23}G_i(H) + \gamma\sigma_1 G_i(H)\right] \ ,
\end{aligned}
$$

and combining these yields

$$(\gamma_{23} - \gamma_{1i})G_i(H) = (\gamma_{1j} - \gamma G_i(H))(G_j(H) - \sigma_1) \ . \tag{26}$$

However, condition (12) implies $\gamma_{23} > \max\{\gamma_{12}, \gamma_{13}\}$, and since $G_i(H) > 0$ it follows that the right-hand side above is strictly positive, and in particular we have

$$\sigma_1 < \min\{G_2(H), G_3(H)\} \ . \tag{27}$$

Since firms 2 and 3 both use $p = H$ and $p = 1$, while $G_1(H) = \sigma_1$, for each $k = 2, 3$ we have

$$p_0\left(\frac{1}{H} - 1\right) = (\gamma_{23} - \sigma_1\gamma)(G_k(1) - G_k(H)) \ . \tag{28}$$

Write $\eta \equiv G_2(1) - G_2(H) = G_3(1) - G_3(H) > 0$. Note (28) implies that $\gamma_{23} > \sigma_1\gamma$ so that $\alpha_{23} > 0$ and there are some consumers who consider firms 2 and 3. As such, at most one

of these firms can have an atom at $p = 1$. Since firm 1 weakly prefers $p = H$ to $p = 1$, we have

$$p_0 \left(\frac{1}{H} - 1 \right) \leq \gamma_{12} G_2(1) + \gamma_{13} G_3(1) - \gamma G_2(1) G_3(1)$$
$$- \gamma_{12} G_2(H) + \gamma_{13} G_3(H) - \gamma G_2(H) G_3(H)$$

$$= \eta \left[\gamma_{12} + \gamma_{13} - \gamma(G_2(H) + G_3(H) + \eta) \right] .$$

Since $\eta > 0$, combining this inequality with (28) implies

$$\gamma_{23} - \sigma_1 \gamma \leq \gamma_{12} + \gamma_{13} - \gamma(G_2(H) + G_3(H) + \eta) ,$$

or

$$\gamma \min\{\sigma_2, \sigma_3\} \leq \gamma_{12} + \gamma_{13} - \gamma_{23} - \gamma(G_2(H) + G_3(H) + \eta - \sigma_1 - \min\{\sigma_2, \sigma_3\}) . \quad (29)$$

At most one of firms 2 and 3 has at atom at $p = 1$, so suppose that firm $k \in \{2, 3\}$ has no atom, so that

$$G_k(H) + \eta = G_k(1) = \sigma_k \geq \min\{\sigma_2, \sigma_3\} .$$

Combining this inequality with (27) and (29) then contradicts condition (12).

The final case to consider is when $H = 1$, so that all three firms use the highest price. If at most one of firms 2 and 3 has an atom at $p = 1$ then either $G_2(1) = \sigma_2$ or $G_3(1) = \sigma_3$ (or both). Therefore $g \geq \min\{\sigma_2, \sigma_3\}$, in which case (25) is inconsistent with (12). If both firms 2 and 3 have an atom at $p = 1$ then we must have $\alpha_{23} = 0$ otherwise the firms have an incentive to undercut one another at $p = 1$. It follows that $\gamma\sigma_1 = \gamma_{23}$, in which case (12) implies

$$\gamma(\sigma_1 + \min\{\sigma_2, \sigma_3\}) > \gamma_{12} + \gamma_{13} \geq \gamma(\sigma_2 + \sigma_3)$$

and so $\sigma_1 > \max\{\sigma_2, \sigma_3\}$. Since not all consumers are captive, when firms 2 and 3 each have an atom at $p = 1$, firm 1 cannot do so and $G_1(1) = \sigma_1$. Then the argument leading to the previous expression (27) applies, with $H = 1$, which contradicts our finding that $\sigma_1 > \max\{\sigma_2, \sigma_3\}$.

In sum, we have shown that when (12) holds, there is only one price in the support of all three firms, say p_1. In particular, only two firms offer the minimum price p_0, and these are firms 2 and 3. Clearly $p_0 < p_1$ and only firms 2 and 3 are active in the range $[p_0, p_1)$. If $p_1 = 1$ then the proof is complete. If $p_1 < 1$ then there is no price in $(p_1, 1]$ in

the support of all firms, and so only two firms are active in this range, one of which must be firm 1. The remaining issue is which of firms 2 and 3 is the other firm active above p_1. Suppose henceforth that firms 2 and 3 are labelled so $\sigma_2 \geq \sigma_3$. Expression (20) implies that $\sigma_2 F_2(p) = \sigma_3 F_3(p)$ in the range $[p_0, p_1]$. If $\sigma_2 = \sigma_3$ then $F_2 = F_3$, and so one of these firms cannot drop out before the other and we must have $p_1 = 1$. If $\sigma_2 > \sigma_3$ then in the range $[p_0, p_1]$ we have $F_3 > F_2$ and so it is firm 3 which drops out first.

The final step in the proof is to determine the profits of the three firms, as well as the price thresholds p_0 and p_1. Since firms 2 and 3 have p_0 as their minimum price in this equilibrium, their profits are $\pi_2 = \sigma_2 p_0$ and $\pi_3 = \sigma_3 p_0$. In the range $[p_0, p_1]$ their CDFs are given by (20), and firm 3 drops out at price p_1, so that the ratio p_0/p_1 satisfies

$$\gamma_{23}\sigma_3 = 1 - \frac{p_0}{p_1} \ . \tag{30}$$

Expression (20) then implies that

$$G_2(p_1) = \sigma_3 \ . \tag{31}$$

Either firm 1 or 2 (or both) obtains exactly its captive profit.[23] Suppose first that firm 1 obtains its captive profit, so that $\pi_1 = \alpha_1$. For prices in the upper range $[p_1, 1]$ firms 1 and 2 compete and are sure to be undercut by firm 3, so from (10) firm 2's CDF satisfies

$$1 - \gamma_{12}G_2 - \gamma_{13}\sigma_3 + \gamma\sigma_3 G_2 = \frac{\rho_1}{p} \ ,$$

where recall that ρ_1 is firm 1's captive-to-reach ratio. In order for $G_2(\cdot)$ to be continuous at the threshold price p_1, (31) implies that

$$1 - \gamma_{12}\sigma_3 - \gamma_{13}\sigma_3 + \gamma\sigma_3^2 = \frac{\rho_1}{p_1} \ ,$$

which determines p_1. Expression (30) in turn implies that

$$p_0 = p_1(1 - \gamma_{23}\sigma_3) = \frac{\rho_1(1 - \gamma_{23}\sigma_3)}{1 - \gamma_{12}\sigma_3 - \gamma_{13}\sigma_3 + \gamma\sigma_3^2} \ . \tag{32}$$

It is convenient to write P for the right-hand side above, so that

$$P = \frac{\rho_1(1 - \gamma_{23}\sigma_3)}{1 - \gamma_{12}\sigma_3 - \gamma_{13}\sigma_3 + \gamma\sigma_3^2} = \frac{\alpha_1(\alpha_2 + \alpha_{12})}{\alpha_1\sigma_2 + \alpha_{12}(\sigma_2 - \sigma_3)} \ , \tag{33}$$

---

[23]If one of these firms has no atom at $p = 1$ then the other obtains its captive profit when it chooses $p = 1$. If both have an atom at $p = 1$ then for neither to have an incentive to undercut the other we must have $\alpha_{12} = 0$, in which case both firms obtain their captive profit at $p = 1$.

where the second equality follows by routine manipulation. Note from the first expression for P above that the condition $P > \rho_1$ is equivalent to (11), and $P < \rho_1$ corresponds to (12). Note also that $P \le (\alpha_2 + \alpha_{12})/\sigma_2$, and so a sufficient condition for overlapping duopoly to be the equilibrium is that

$$\frac{\alpha_2 + \alpha_{12}}{\sigma_2} < \rho_1 \ .$$

In words, this condition states that the higher captive-to-reach ratio in the duopoly market with just firms 2 and 3 present is below firm 1's captive-to-reach ratio in the triopoly market. Expression (32) implies

$$p_1 = \frac{\alpha_1 \sigma_2}{\alpha_1 \sigma_2 + \alpha_{12}(\sigma_2 - \sigma_3)} \ . \tag{34}$$

Alternatively, suppose firm 2 obtains its captive profit, so that $\pi_2 = \alpha_2$. Since the firm has p_0 as its lowest price it follows that

$$p_0 = \rho_2 \ . \tag{35}$$

Expression (30) then implies that

$$p_1 = \frac{\alpha_2}{\alpha_2 + \alpha_{12}} \ . \tag{36}$$

For prices in the upper range $[p_1, 1]$ firm 2's CDF now satisfies

$$1 - \gamma_{12} G_2 - \gamma_{13}\sigma_3 + \gamma\sigma_3 G_2 = \frac{\pi_1}{\sigma_1 p} \ ,$$

where π_1 is firm 1's profit (to be determined). For G_2 to be continuous at $p = p_1 = \alpha_2/(\alpha_2 + \alpha_{12})$, (31) implies that

$$1 - \gamma_{12}\sigma_3 - \gamma_{13}\sigma_3 + \gamma\sigma_3^2 = \frac{\alpha_2 + \alpha_{12}}{\alpha_2} \cdot \frac{\pi_1}{\sigma_1} \ ,$$

which determines π_1. This can be expressed as

$$\pi_1 = \frac{\alpha_1 \rho_2}{P} \tag{37}$$

where P is given in (33).

We next determine when it is that firm 1 or firm 2 obtains its captive profit. When firm 1 obtains its captive profit, firm 2's minimum price is P in (33), which must be no lower than ρ_2 if firm 2 is willing to offer this price. Therefore, if $P < \rho_2$ the equilibrium

must instead have firm 2 obtaining its captive profit, in which case the threshold prices and firm 1's profit are given respectively by (35), (36) and (37). Conversely, when firm 2 obtains its captive profit, firm 1's profit is given in (37). This profit cannot be below its captive profit α_1, which therefore requires $P \leq \rho_2$. Therefore, if $P > \rho_2$ the equilibrium must involve firm 1 obtaining its captive profit, so $\pi_1 = \alpha_1$, and the threshold prices p_0 and p_1 are given respectively by (33) and (34). Finally, in the knife-edge case where $P = \rho_2$ the two equilibria coincide, and firms 1 and 2 each obtain their captive profit. This completes the proof.

Details for the nested example in section 4: Recall that the example in the text has nested reach with $\sigma_3 = \frac{1}{2}$, $\sigma_2 = \frac{4}{5}$ and $\sigma_1 = 1$. Then part (i) of Proposition 2 applies, and all firms use the same minimum price $p_0 = \frac{1}{5}$ (which is firm 1's captive-to-reach ratio) and have profits $\sigma_i p_0$. In this example we have $\gamma_{12} = \gamma_{13} = 1$ and $\gamma_{23} = \gamma = \frac{5}{4}$, and so expression (10) implies that for any price in the support of all three firms we have

$$1 + \frac{5}{4}G_1 G_2 - \frac{5}{4}G_2 - G_1 = 1 + \frac{5}{4}G_1 G_3 - \frac{5}{4}G_3 - G_1 = 1 + \frac{5}{4}G_2 G_3 - G_2 - G_3 = \frac{1}{5p} . \quad (38)$$

These simultaneous equations can be solved to give each G_i as a function of p:

$$G_1(p) = 1 - \frac{2}{5}\sqrt{\frac{1}{p(1-p)}} \ ; \ G_2(p) = G_3(p) = \frac{4}{5} - \frac{2}{5}\sqrt{\frac{1-p}{p}} \ . \quad (39)$$

These adjusted CDFs are each zero at $p = p_0$ and G_2 and G_3 increase with p for prices above p_0.

A first "guess" at the solution would be that all three firms choose prices in the range $[p_0, p_1]$, then firm 3 drops out leaving firms 1 and 2 active in the range $[p_1, 1]$. Here F_3 reaches 1, i.e., G_3 reaches $\sigma_3 = \frac{1}{2}$, i.e., , at $p_1 = \frac{16}{25}$. For prices above p_1 firms 1 and 2 compete alone, with firm 3 sure to undercut them, in which case the required adjusted CDFs in (38) are given by

$$G_1(p) = 1 - \frac{8}{15p} \ ; \ G_2(p) = \frac{4}{3} - \frac{8}{15p} \ .$$

The problem with this candidate solution, however, is that $G_1 = F_1$ in (39) increases with p only for prices below $\frac{1}{2}$, and thereafter it decreases with p as depicted as the solid curve on Figure 3 in the text. The correct solution is then obtained by "ironing" this CDF as shown as the dashed line on the figure so that F_1 is flattened to be no greater than the level $F_1(p_1) = \frac{1}{6}$ for prices below p_1. The smaller root of $G_1 = \frac{1}{6}$ in (39) is $\hat{p} = \frac{9}{25}$.

In this example, all three firms are active in the price range $[\frac{1}{5}, \frac{9}{25}]$, only firms 2 and 3 are active in the interior range $[\frac{9}{25}, \frac{16}{25}]$, and then only firms 1 and 2 are active in the range $[\frac{16}{25}, 1]$. In the interior range $[\frac{9}{25}, \frac{16}{25}]$, the adjusted CDFs G_2 and G_3 need modifying from (39) to reflect that they will be undercut by firm 1 with the constant probability $F_1(p_1) = \frac{1}{6}$ in this range (in which case they have no demand), so that

$$G_2(p) = G_3(p) = \frac{4}{5} - \frac{24}{125p} \ .$$

(Again, G_3 reaches $\sigma_3 = \frac{1}{2}$ at $p_1 = \frac{15}{25}$.) With these CDFs, one can check that firm 1 does not gain by choosing a price in the interior range $[\frac{9}{25}, \frac{16}{25}]$, and that firm 3 has no incentive to choose a price above $p_1 = \frac{16}{25}$, so that is indeed an equilibrium.

Proof that a profitable merger increases firm N's profit in section 4: Recall that the case requiring analysis was the effect on the profit of firm 1 of a merger between firms 2 and 3 starting from the conditions of part (ii) of Proposition 2 and with $P < \rho_2$ so that firm 1 obtained more than its captive profit as in (37). Using (33) we can bound firm 1's pre-merger profit by

$$\pi_1 = \frac{\alpha_1 \rho_2}{P} = \rho_2 \frac{\alpha_1 \sigma_2 + \alpha_{12}(\sigma_2 - \sigma_3)}{\alpha_2 + \alpha_{12}} \leq \frac{\alpha_1 \alpha_2}{\alpha_2 + \alpha_{12}} + \rho_2 \alpha_{12}$$

because $\sigma_2 - \sigma_3 \leq \alpha_2 + \alpha_{12}$. As $P < \rho_2$ is equivalent to

$$\alpha_1 < \rho_2(\sigma_2 - \sigma_3) \ , \tag{40}$$

it is apparent that $\alpha_1 < \alpha_2 \leq \alpha_M$, and so firm 1's post-merger profit will be $\sigma_1 \rho_M$, where $\rho_M > \rho_2$ is the captive-to-reach ratio of the merged firm and the inequality follows from the merger being profitable. The merger will therefore increase 1's profit if

$$
\begin{aligned}
0 \ &< \ \rho_M \sigma_1 - \left(\frac{\alpha_1 \alpha_2}{\alpha_2 + \alpha_{12}} + \rho_2 \alpha_{12} \right) \\
&= \ \left(\rho_M - \frac{\alpha_2}{\alpha_2 + \alpha_{12}} \right) \alpha_1 + (\rho_M - \rho_2) \alpha_{12} + \rho_M (\alpha_{13} + \alpha) \\
&= \ \left(\frac{(1 - \rho_M)\sigma_3 - (\alpha_{13} + \alpha)}{\alpha_2 + \alpha_{12}} \right) \alpha_1 + (\rho_M - \rho_2) \alpha_{12} + \rho_M (\alpha_{13} + \alpha) \ . \tag{41}
\end{aligned}
$$

This condition is met if if $\rho_M > \frac{\alpha_1}{\alpha_2 + \alpha_{12}}$ because the factors multiplying α_1, α_{12} and $(\alpha_{13} + \alpha)$ in (41) are then all positive. This last inequality does hold when $P < \rho_2$ because

$$\frac{\alpha_1}{\alpha_2 + \alpha_{12}} < \frac{\alpha_1}{\sigma_2 - \sigma_3} < \rho_2 < \rho_M$$

using (40) and the assumption that the merger is profitable. This completes the proof.