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Bhan, Aditya and Kabiraj, Tarun

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By

Aditya Bhan and Tarun Kabiraj

Indian Statistical Institute, Kolkata

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Correspondence to: Tarun Kabiraj, Economic Research Unit, Indian Statistical Institute, 203 B. T. Road, Kolkata - 700108, India. Fax: (91) (33) 2577 8893.

E-mail: (Tarun Kabiraj): tarunkabiraj@hotmail.com; (Aditya Bhan): bhan.aditya@gmail.com.

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Abstract: We consider interaction of two terror outfits and study possible counter-terrorism (CT) measures, both in the absence and presence of external terror finance. In our paper, external sponsorship with proportional allocation rule, induces strategic interaction and incentivizes more attacks. We provide a theoretical foundation for the ubiquity of defensive counter-terrorism (CT) versus the limited applicability of offensive measures and confidence-building measures (CBMs). Curtailing external sponsorship is always effective in inhibiting terror activity. In fact, targeting external funding may be the most effective CT tool if terror activity is sufficiently low. While CBMs may be more effective in the absence of external sponsorship, defensive CT may be preferable in its presence. However, CBMs may not be as effective in the presence of external sponsorship, as in its absence.

Key words: Terror outfit; counter-terrorism; external sponsorship; defensive, offensive and confidence building measures.
JEL Classifications: D62, D74, F52, H89.

1. Introduction

Terrorism is a menacing problem afflicting large parts of the world. Terror events such as those of September 11, 2001 (United States), December 13, 2001 (New Delhi, India), October 12, 2002 (Bali, Indonesia), October 23, 2002 (Moscow, Russia), March 11, 2004 (Madrid, Spain), July 7, 2005 (London, United Kingdom), July 11, 2006 (Mumbai, India), November 26, 2008 (Mumbai, India), May 22, 2013 (London, United Kingdom), July 27, 2015 (Gurdaspur, India), January 2-5, 2016 (Pathankot, India), September 18, 2016 (Uri, India), February 14, 2019 (Pulwama, India) amongst many others; illustrate the magnitude of the threat posed by fundamentalist ideology driven terrorists.[1] The intolerance, which results from religious fundamentalism, is manifested in the dramatic increase in the number of casualties from terror strikes since the turn of the millennium. From 3,361 in 2000, the number of fatalities from terrorism has soared to 15,952 in 2018, at a compounded annual growth rate of 9%. Illustrating

[1] For detailed accounts on fundamentalism, see Gilling (1992) and Pratt (2006). Tibi (2002), on the other hand, views religious fundamentalism as a political doctrine rather than a spiritual faith.

the potency of religious ideology as a cause of terrorism, the Global Terrorism Index (GTI) (2019) report states that the majority of claimed deaths from terrorist attacks – 57.8 per cent in 2018 - are claimed by only four terrorist organizations, namely the Taliban, ISIL, the Khorasan Chapter of the Islamic State, and Boko Haram. Radical doctrines rooted in Wahhabi Islam provide the crucial common denominator for all four groups, even though their strategic objectives may vary.

Increasing terrorist activity, however, cannot solely be explained on the basis of rising religious fundamentalism. Acharya (2009) observes that "*if radical ideology and extremism are at the heart of terrorism today, finance is its lifeblood*" (p.7). Sources of terrorism finance can be classified as internal or external, depending on whether their origins lie within or outside the country to be targeted. It needs to be emphasized that there is a clear distinction between sources and channels of money transfer. External sources of terrorism finance include charities/NGOs, counterfeit currency, drug trafficking, and state funding; whereas internal sources primarily include extortion or taxation, criminal activities, Non-Governmental Organizations (NGOs), and Designated Non-Financial Businesses and Professions (DNFBPs). In this paper, we focus on external funding and study its impact on terror activity. We restrict our analysis to external sponsorship provided as a means of incentivizing terror activity. We demonstrate, in particular, the ability of an external sponsor to induce enhanced levels of terror activity through the strategic provision of funds to terror outfits.[2]

In India, a major part of external funding for terrorism comes through counterfeit currency, drug-trafficking, charities, NGOs, and finally due to alleged state sponsorship by Pakistan. In Pakistan, for instance, the government has limited control over charities and NGOs (Ghumman, 2012). Terrorism finance is therefore generated from NGOs and charities such as *Jamaat-ud-Dawa* (JuD) and *Falah-e-Insaniyat* Foundation (FeF) within Pakistan. Saudi Arabia has also emerged as a large source of funds for terrorist groups like the Lashkar-e-Taiba (LeT), which functions on an approximate annual budget of US $5.25 million (Walsh, 2010). Given the scale of money collected in the country, even a small fraction is adequate to support terrorism.

[2] Byman (2005) similarly argues that terrorist groups that enjoy state support have greater ability and inclination for large-scale bloodshed, than those without state support.

Further, counterfeiting of Indian currency is allegedly used by Pakistan not only to fund terrorism in India, but also to destabilize the Indian economy. Fake Indian currency is allegedly used to fund groups like LeT, Al-Badr, Harkat-ul-Jihad-e-Islami (HuJI), Khalistan Commando Force (KCF) and operations run by Dawood Ibrahim. Bangladesh and Nepal are amongst the most viable routes for inducting Fake Indian Currency Notes (FICN) (Chadha, 2015). Additionally, drugs are a major source of terrorism finance. Afghanistan emerged as the hub for the global production of opiates. In 2009, the Afghan Taliban was estimated to have earned around US $150 million from the opiate trade, Afghan drug traffickers US $2.2 billion, and Afghan farmers US $440 million (see United Nations Office on Drugs and Crime, 2011). Criminal and terrorist groups from Bangladesh have also allegedly exploited the drug trade to fund terrorism (Bhattacharya, 2012). Lastly, the Pakistan Government is often accused of employing its intelligence agency (the Inter Services Intelligence) to fund terrorist activities in India. Addressing the Hindustan Times Leadership Summit in 2014, Shri Rajnath Singh (then Home Minister, Govt. of India) said, "*Terrorism here is not home grown. It is externally aided. Pakistan blames non-state actors for it. I ask them whether the Inter Services Intelligence (ISI) is a non-state actor. If anyone is fully helping terrorists, it is the ISI*".[3]

It must be noted at this juncture, however, that Pakistan is far from being the only state-sponsor of terrorism. It is alleged, for instance, that Syrian sponsorship of the Palestinian Islamist group Hamas has greatly enhanced the outfit's operational capabilities. Since the mid-1990s, Syria and Syrian-occupied Lebanon had allegedly become prime conduits for channeling weapons and explosives to Hamas, and safe havens for training hundreds of its operatives. In addition to greatly augmenting the movement's ability to inflict casualties, alleged Syrian sponsorship had fueled its willingness to kill. The alleged weakening of the internal leadership of Hamas vis-à-vis the external leadership had allegedly made the group's military cells less sensitive to public disaffection with the costs of terror (Gambill, 2002).

Also consider the Abu Nidal Organisation (ANO), a terrorist organization infamous for having conducted deadly attacks on Western, Palestinian and Israeli targets in the 1980s. Since its inception in 1974, the ANO had allegedly received state support from Iraq, Syria and Libya

[3] *The Hindu* (November 23, 2014).

during different stages of its existence. The group moved to Syria after Iraq allegedly withdrew its support in 1983. Syria expelled the ANO in 1987, probably under U.S. pressure. However, the supply of external sponsorship almost vanished completely in 1999, after local authorities curbed the ANO's operations in Libya. Since then, the organization is considered largely inactive (Council of Foreign Relations, 2009). Attacks conducted by the ANO numbered 6 in the 1970s, 33 in the 1980s, and 12 in the 1990s. Since 1999, the outfit has not conducted any attack.[4]

The above discussion indicates that in addition to being a major determinant of terrorist activity, external sponsorship of terrorism is largely strategic in nature. This is because most of the major external sources of terrorism finance are sufficiently autonomous to operate as separate entities from the terror outfits that they support. This, in turn, implies that these strategic external sponsors are able to manipulate the behavior of the recipient terror outfits. In contrast, most of the major internal sources of terrorism finance such as extortion or taxation, crime, diversion of the funds of NGOs, and money laundering from DNFBPs, are largely controlled and managed by the recipient terror outfits themselves. Therefore, one would not expect a typical internal/domestic terrorism sponsor to be able to behave strategically vis-à-vis the terrorists it supports.

Siqueira and Sandler (2006) have argued that state sponsorship and franchising of terrorists augment violence. They demonstrate that the external sponsor can curb the abatement in hostilities due to (governmental) measures aimed at gaining widespread backing and strategic leadership, provided it is able to control and channel resources to the terror outfit. They hence claim that al-Qaida's sponsorship of contemporary outfits, and support to mature ones (e.g., Abu Sayyaf in the Philippines), fulfil a diabolical end by diminishing intrinsic curbs on conflict. In similar vein, our analysis demonstrates how external sponsorship can reduce the effectiveness of confidence-building measures (CBMs) in reducing violence.[5]

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[4] Global Terrorism Database:
(http://www.start.umd.edu/gtd/search/Results.aspx?start_yearonly=1974&end_yearonly=2013&start_year=&start_month=&start_day=&end_year=&end_month=&end_day=&asmSelect0=&asmSelect1=&perpetrator=275&dtp2=all&success=yes&casualties_type=b&casualties_max=).

[5] More details are provided in Section 4, which deals with CT strategy.

Byman and Kreps (2010) discuss the delegation problem involved in state sponsorship of terrorists, with tradeoffs between agent autonomy and agency losses. Analyzing state-sponsored terrorism as an illicit principal-agent issue, the authors suggest that through "*a disinformation campaign to increase the principal's suspicion of a group's competence and fidelity*", it is potentially fruitful for counter-terrorism (CT) officials to exploit the information gap between states and the terrorists they support.

The present paper characterizes the strategic interaction between terror outfits and the government of the country targeted by them - both in the presence and absence of a potential strategic external sponsor of terrorism - to establish and contrast the limited applicability of offensive CT and CBMs, with the widespread utility of defensive CT.[6] We specifically demonstrate that while offensive CT is effective only against resource-constrained outfits, CBMs are effective only against resource-abundant outfits. We also find that if defensive CT is more effective than CBMs in the absence of external sponsorship, then it must be more effective even in its presence.

Sandler and Siqueira (2006) demonstrate that nations confronted by a common terrorist threat, can never achieve the proper CT policy mix between deterrence and pre-emption through leadership. There are some sharp distinctions, however, between their analysis and ours. Firstly, our scenario involves more than one terror outfit (specifically two), and is therefore able to capture inter-outfit strategic interaction. Secondly, our analysis does not focus on co-ordination and externality-related issues between targeted countries when confronted by a common terrorist threat, and we therefore limit our structure to include only one targeted country. Our framework, in fact, explores the possibility and implications of externalities resulting from the terror activities of an outfit on another, induced via the introduction of strategic external sponsorship. Thirdly, in our analysis, all payoffs are derived endogenously from the respective utility functions of each group. And finally, our analysis addresses the possibility of external sponsorship, and is therefore able to derive additional and deeper insights in respect of CT policy.

[6] Arce and Sandler (2005), in similar flavour, conclude that governments tend to prefer deterrence over preemption, as a result of coordination failure in the provision of CT effort.

The present analysis also interestingly indicates that targeting external funding may be more fruitful than both CBMs and defensive CT measures, if the terror outfits do not conduct too many terror attacks in initial interior equilibrium, both individually and collectively. But this does not seem to hold true if terror activity is sufficiently high. What is established beyond doubt, however, is the importance for counter-terrorists to know whether the outfits targeted by them are externally sponsored.[7]

The next section presents the basic model involving terror outfits and the targeted country's government. Section 3 introduces external sponsorship to the structure described in Section 2. Section 4 addresses the problem facing the targeted country, and discusses the ramifications of the results derived in Sections 2 and 3 on its CT policy. Section 5 concludes the paper by summarizing the analysis, and providing potential directions for future research.

2. Model

In this section we provide, as a benchmark, a model of interaction between two terror outfits which internally finance their activities. In this paper, we are more interested in closed form solutions, and therefore take specific forms of the different functions.[8]

Suppose there are two independent terror outfits - T_1 and T_2 - operating in a country, whose government aims to minimize the total number of terror attacks the two groups conduct. We assume that a terror group draws utility from a basket of consumption goods and terror strikes. Let the payoff function of the i^{th} terror group ($i = 1, 2$) be

$$U_i = X_i + \alpha_i A_i \tag{1}$$

where X_i is level of consumption (of the numeraire good) over and above subsistence consumption, A_i is the number of terror strikes it conducts, and α_i (≥ 0) is the parameter representing its intrinsic propensity of violence. A larger α_i implies that the i^{th} terror group is

[7] For an alternative framework (without external sponsorship) demonstrating the need for countries to invest in their intelligence apparatus as an essential part of their CT efforts, in order to understand whether the terror threat faced by them is political or militant, see Arce and Sandler (2007).
[8] A discussion with general functional forms can be found in Bhan and Kabiraj (2019).

more hardline. Hence, α_i captures the fundamentalism which drives terrorism. Note that in our formulation we have assumed that the utility function is separable in its two arguments, X_i and A_i. This also means that the marginal utility with respect to either argument is independent of the other argument, which is reasonable to expect. Both X_i and A_i are assumed to be continuous.

Let the associated cost to terror group i of conducting A_i terror strikes be

$$C_i(A_i) = \frac{1}{2}\beta_i A_i^2 \quad (2)$$

where β_i is the cost-efficiency parameter of terror outfit T_i, such that lower (higher) β_i represents higher (lower) efficiency. The cost function is increasing and convex, reflecting the increased difficulty in conducting each successive attack. This can be driven by the increased alertness and enhanced response of the governmental authorities and security forces, after each successive terror strike. In this sense, β_i represents a counter-terrorism parameter. If R_i be the initial resource endowment of T_i, net of expenditure on subsistence consumption and measured in terms of the numeraire good, its budget constraint is,

$$X_i + \frac{1}{2}\beta_i A_i^2 = R_i \quad (3)$$

Therefore, T_i's optimization problem is to maximize its objective function (1), subject to the constraint (3). Hence, its maximization problem is

$$Max_{A_i} U_i = R_i - \frac{1}{2}\beta_i A_i^2 + \alpha_i A_i \quad (4)$$

When an interior optimum exists, the first order condition is given by

$$-\beta_i A_i + \alpha_i = 0 \quad (5)$$

This solves for

$$A_i = \frac{\alpha_i}{\beta_i} \equiv A_i^0 \text{ and } X_i = R_i - \frac{1}{2}\frac{\alpha_i^2}{\beta_i} \equiv X_i^0 \quad (6)$$

It is also easy to see that the second order sufficient condition for utility maximization is satisfied, because $\frac{\partial^2 U_i}{\partial A_i^2} = -\beta_i < 0$. It is then easy to check that interior equilibrium exists if and only if $R_i \geq \frac{1}{2}\beta_i \left(\frac{\alpha_i}{\beta_i}\right)^2 = \frac{1}{2}\frac{\alpha_i^2}{\beta_i}$. Further, note that each terror outfit's problem is solved independently of other's problem.

From (6), we observe that a terror group which is more violent tends to conduct more attacks. On the other hand, if the government steps up its counter-terrorism efforts against a terror outfit, it increases the marginal cost of conducting a terror strike for that outfit. Consequently, it reduces the optimal number of terror strikes. But the number of terror strikes an outfit conducts is independent of the size of its initial resource endowment. So in interior equilibrium, any variation in R_i will lead to a corresponding equivalent variation in X_i.

Now consider the case when a terror group is resource constrained, and is therefore unable to conduct A_i^0 attacks.[9] In this situation, the marginal benefit from terrorism exceeds its marginal cost, i.e., $-\beta_i A_i + \alpha_i > 0$. Then the optimal number of attacks by an outfit will be solved from the budget constraint (3) subject to $X_i = 0$. We call this a corner solution. Under this case, the entire initial resource endowment is spent on terrorism, and hence the optimal number of attacks is given by,

$$A_i = \sqrt{\frac{2R_i}{\beta_i}} \tag{7}$$

So the terror outfit consumes the numeraire good only at the subsistence level.[10]

Note that in interior equilibrium, the level of terror strikes does not depend on the resources the outfit holds. In contrast, in corner equilibrium, the level of terror strikes optimally conducted by the outfit will depend positively on the level of resources it has initially, but is independent of its inherent propensity for violence. However, the parameter β_i has a similar effect in both the cases.

3. External Sponsorship

In this section, we examine the role of external sponsorship in inducing terror activities, and also determine its optimal size.

[9] This is the case when $R_i < \frac{1}{2}\beta_i\left(\frac{\alpha_i}{\beta_i}\right)^2$.

[10] Note that we have interpreted R_i to be the initial resource endowment of the terror outfit i $(= 1, 2)$ net of its expenditure on subsistence consumption, and X_i to be T_i's consumption of the numeraire good over and above the subsistence level.

Suppose there is an external sponsor (S) having an amount $F > 0$, in units of the numeraire good, to induce terror attacks by the outfits. It distributes the funds between the terror outfits based on some allocation mechanism, which is common knowledge at the beginning of the game.[11] Having observed A_1 and A_2, suppose S rewards terror outfits T_1 and T_2 with amounts F_1 and F_2 such that $F_1 + F_1 = F$. As earlier, the structure of the game is assumed to be common knowledge.

The above structure captures the role of external sponsorship as an inducement for violence. Terror attacks by an outfit are restricted by the resources available to it, *a priori*. This is not to say that in reality, terror outfits are only provided external sponsorship as a reward or inducement. For example, external sponsorship may be provided to a terror outfit even before it has conducted any terror strike. In our analysis, such sponsorship would be captured by a higher initial resource endowment (R_i). However, as demonstrated earlier, if a terror outfit having an interior solution in the absence of external sponsorship was initially provided with more resources, it would use these additional resources only for increased consumption of the numeraire good. This would leave the level of terror strikes unchanged. Although, if there existed a corner solution for the terror outfit in the absence of external sponsorship, provision of more resources initially would raise the number of terror strikes.

With sponsor money, the payoff function of the i^{th} terror group ($i = 1, 2$) is modified to be[12]

$$U_i = X_i + \alpha_i A_i + F_i \tag{8}$$

Correspondingly, T_i's payoff maximization problem becomes

$$Max_{A_i} U_i = R_i - \frac{1}{2}\beta_i A_i^2 + \alpha_i A_i + F_i \tag{9}$$

We shall now introduce a rule or mechanism by which the sponsor allocates its fund between the outfits, to incentivize more attacks.

[11] The sponsor which seeks to maximize the number of terror strikes conducted by the terror outfits, may appropriately decide the allocation rule. In the present paper, however, we restrict our analysis to proportional allocation rule only defined in subsection 3.1, although we have mentioned the other possible allocation rules in subsection 3.3.

[12] Because the sponsorship amounts (F_1 and F_2) provided to the terror outfits are in units of the numeraire good, it is reasonable to assume that they would enter the utility functions of the terror outfits in the same way that X_1 and X_2 enter.

3.1 Proportionate Rule and Terror Activity

Under this allocation mechanism, the external sponsor awards each terror outfit a fraction of F equal to the fraction of total terror strikes carried out by that outfit. So the fraction is endogenous. Then terror outfit T_i $(i = 1, 2)$ receives[13]

$$F_i = \frac{A_i}{A_i + A_j} F \tag{10}$$

Substituting Equation (10) in Equation (9), we can rewrite T_i's utility maximization problem as[14]

$$Max_{A_i} U_i = R_i - \frac{1}{2}\beta_i A_i^2 + \alpha_i A_i + \frac{A_i}{A_i + A_j} F \tag{11}$$

If an interior optimum exists, the first order condition for terror outfit T_i $(i = 1, 2)$ is

$$\alpha_i + \frac{A_j}{(A_i + A_j)^2} F - \beta_i A_i = 0 \tag{12}$$

Clearly, $\alpha_i + \frac{A_j}{(A_i + A_j)^2} F$ is the marginal benefit and $\beta_i A_i$ is the marginal cost of an additional attack. Comparing (12) with (5), we can see that marginal benefit in the presence of sponsorship is larger than that in its absence. Hence, the optimal number of attacks under sponsorship exceeds that in the absence of external sponsorship. This is the inducement effect of sponsorship.

The second order condition for T_i's optimization problem under external sponsorship is

$$\frac{\partial^2 U_i}{\partial A_i^2} = -\beta_i - 2\frac{A_j}{(A_i + A_j)^3} F < 0 \tag{13}$$

which is satisfied. Further note that

$$D \equiv \frac{\partial^2 U_i}{\partial A_i^2}\frac{\partial^2 U_j}{\partial A_j^2} - \frac{\partial^2 U_i}{\partial A_i \partial A_j}\frac{\partial^2 U_i}{\partial A_j \partial A_i} = \left(\beta_i + 2\frac{A_j}{(A_i + A_j)^3} F\right)\left(\beta_j + 2\frac{A_i}{(A_i + A_j)^3} F\right) + \frac{\left(A_i - A_j\right)^2}{\left(A_i + A_j\right)^6} F^2 > 0 \tag{14}$$

Therefore, the equilibrium is unique and stable. Equation (12) generates the terror outfit i's $(i = 1, 2)$ best-response (reaction) function $A_i = A_i\left(A_j\right)$, with intercept $A_i(0) = \frac{\alpha_i}{\beta_i} \equiv \underline{A}_i > 0$, and slope

[13] Alternatively, we may assume that T_i $(i = 1, 2)$ will receive the entire sponsorship fund F with probability $\frac{A_i}{A_i + A_j}$ $(i \neq j = 1, 2)$. This, then, becomes similar to the Tullock (1980) game, where the probability of winning depends on the relative investments or efforts of the players. We call this the probabilistic allocation mechanism. Note that our analysis will remain unchanged in this case, if both terror outfits are assumed to be risk-neutral.

[14] The problem formulated here matches, in some sense, very closely with the generalized Tullock contest game of Chowdhury and Sheremeta (2011), but in contrast to them we are implementing a quadratic cost function.

$$\frac{dA_i}{dA_j} = -\frac{\frac{\partial^2 U_i}{\partial A_i \partial A_j}}{\frac{\partial^2 U_i}{\partial A_i^2}} = \frac{(A_i - A_j)F}{\beta_i (A_i + A_j)^3 + 2A_j F} \gtreqless 0 \tag{15}$$

The best-response function is initially increasing, since $\left(\frac{dA_i}{dA_j}\right)_{A_j=0} = \frac{F}{\beta_i A_i^2} > 0$, and reaches its maximum at $A_i = A_j = \overline{A}_i$ (say) at which it intersects the line of equality, $A_i = A_j$. From the reaction function, we have $\overline{A}_i = \frac{\alpha_i + \sqrt{\alpha_i^2 + \beta_i F}}{2\beta_i}$. Then $\overline{A}_1 \gtreqless \overline{A}_2$ according as $\frac{\alpha_1 + \sqrt{\alpha_1^2 + \beta_1 F}}{\alpha_2 + \sqrt{\alpha_2^2 + \beta_2 F}} \gtreqless \frac{\beta_1}{\beta_2}$. In the presence of external sponsorship, the reaction function of each outfit is therefore positively sloped till it reaches its maximum at its intersection with the line of equality, and is thereafter negatively sloped.[15] Finally, the optimum number of terror strikes (A_1^*, A_2^*) in an interior equilibrium (where neither outfit is resource-constrained) can be obtained at the intersection of the reaction functions of the terror outfits.

Therefore, if both terror outfits are characterized by interior optima, then $A_1^* \gtreqless A_2^*$ according as $\overline{A}_1 \gtreqless \overline{A}_2$. When $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$, we must have $A_1^* = A_2^* = \overline{A}_1 = \overline{A}_2$ in equilibrium. When $\alpha_i > \alpha_j$ but $\beta_i = \beta_j$, we have $\overline{A}_i > \overline{A}_j$ and hence, $A_i^* > A_j^*$. This is shown in Figure 1 below. Note that a sufficient (but not necessary) condition for the existence of an interior optimum is $R_i \geq \frac{1}{2}\beta_i \overline{A}_i^2 \ \forall i = 1, 2$.

[15] For similar reaction functions in contexts other than external sponsorship of terror outfits, see Chowdhury and Sheremeta (2011) and Dixit (1987). In contrast, when there are operational externalities in the absence of external sponsorship, the reaction functions of the outfits may either slope upwards throughout or downwards throughout, depending on whether there are positive externalities or negative externalities respectively (Bhan and Kabiraj, 2019). Therefore, the CT policy implications under external sponsorship depend, to a large extent, on whether an initial equilibrium occurs at the rising portion or falling portion of the reaction functions.

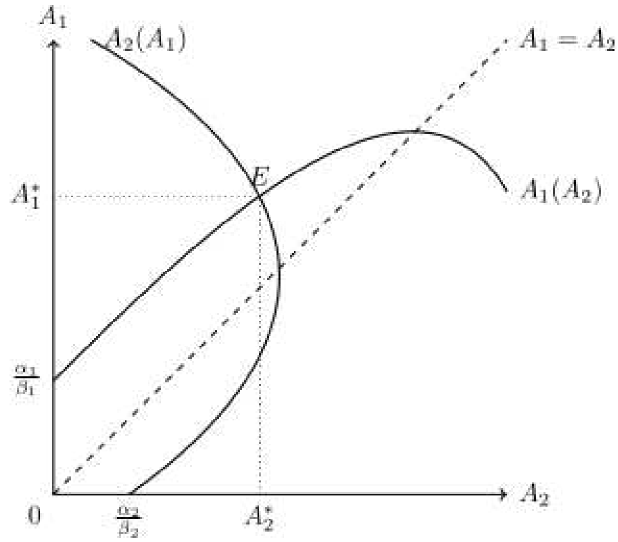

Figure 1: Interior optimum with external sponsorship

Now consider the scenario of corner solution. A necessary (though not sufficient) condition for the existence of a corner solution is $R_i < \frac{1}{2}\beta_i \overline{A}_i^2$ for some $i \in \{1, 2\}$. If terror outfit T_i's resource constraint binds, A_i^* satisfies $\frac{1}{2}\beta_i {A_i^*}^2 = R_i$. Clearly, if R_i is small enough, the optimal number of attacks may even go below $\underline{A}_i \left(= \frac{\alpha_i}{\beta_i}\right)$. Figure 2 illustrates the case where T_1 alone is characterized by a corner solution. One interesting observation that follows in this case is that if only one terror outfit is resource-constrained, the other terror outfit may conduct a higher number of attacks compared to the interior optimum.[16] To write it more formally,

Proposition 1: *Under proportionate external sponsorship, if only T_i $(i = 1, 2)$ is resource-constrained in the vicinity of the interior equilibrium, T_j's terror activity exceeds that in interior optimum whenever $\overline{A}_i > \overline{A}_j$.*

The reason is that when $\overline{A}_i > \overline{A}_j$, in an interior equilibrium we have $A_i^* > A_j^*$, and T_i's reaction function intersects T_j's reaction function in the latter's negatively sloping section. Now if T_i be resource-constrained in the vicinity of equilibrium, its reaction function becomes horizontal and will continue to intersect at the negatively sloped portion of the T_j's reaction

[16] It is easy to understand that if both terror outfits are resource-constrained, the terror activity conducted by each will be less than that in interior optimum.

function, but below the interior equilibrium. Hence the result. In fact, if the resource-constrained outfit (T_i) is not too handicapped, that is, if the resource constraint is not too severe, then the resource-abundant outfit (T_j) would find it optimum to conduct more attacks than that under interior equilibrium.. This is because at the number of attacks corresponding to the interior equilibrium, the resource-abundant outfit's marginal benefit from conducting more attacks would exceed the marginal cost of the same, thereby making it beneficial for this outfit to grab an even greater share of the external sponsorship on offer by conducting more attacks than in interior optimum. The result is portrayed in *Figure* 2.

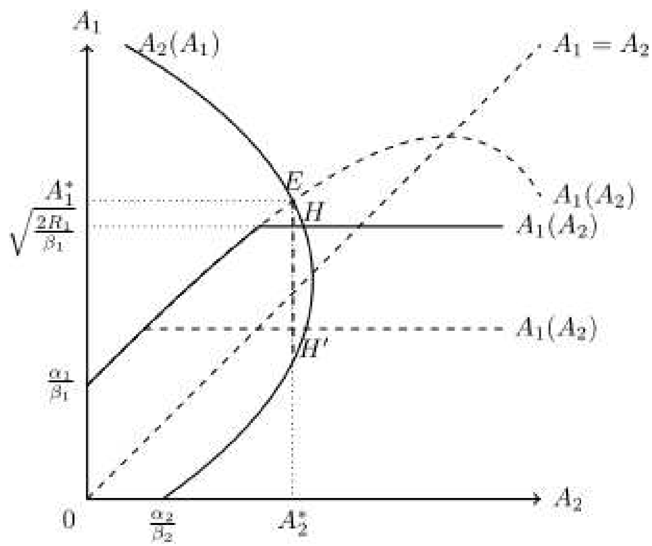

Figure 2: Equilibrium under external sponsorship when terror outfit 1 is resource-constrained and terror outfit 2 is not

If the resource-constrained outfit faces a sufficiently severe resource-crunch however, then at the level of attacks corresponding to the interior equilibrium, the resource-abundant outfit's marginal cost of conducting more attacks would exceed the marginal benefit from the same, thereby making it optimal for this outfit to conduct fewer attacks than in interior equilibrium.

We are now in a position to explain the role of sponsorship in the context of our model. We have already seen that in the absence of sponsorship (i.e., $F = 0$) the optimal attacks conducted by T_i is $\frac{\alpha_i}{\beta_i}$ if it is not resource-constrained (i.e., $R_i \geq \frac{1}{2}\frac{\alpha_i^2}{\beta_i}$). and it is $\sqrt{\frac{2R_i}{\beta_i}}$ if T_i is resource constrained.. Moreover, each outfit's decision is independent of the other. But when $F > 0$, each outfit's reaction function first rises, and then falls if it is not resource-constrained; it becomes a horizontal or vertical line (in (A_i, A_j) space) at the terror activity satisfying the budget with strict equality when the respective outfit becomes resource-constrained. This means, external sponsorship with proportional rule of allocation makes the outfits' decisions interdependent. Therefore, sponsorship with proportional allocation rule forces the outfit to behave strategically. The inter-outfit competition for a larger share of external sponsorship causes the outfits to conduct a higher number of terror attacks, exceeding that in the absence of such funding. This is the inducement effect of external finance. By committing to reward an outfit in proportion to its attack, the sponsor incentivizes each outfit to conduct more attacks. In subsection 3.4 we have derived the optimal level of sponsorship from the perspective of the sponsor which seeks to maximize the total number of attacks (i.e., $A = A_1 + A_2$). It may, however, be noted that in our structure an outfit's attacks are restricted by its initial resource, that is, $A_i \leq \sqrt{\frac{2R_i}{\beta_i}}$. Therefore, the induced effect will work only up to that level. Hence we can write the following proposition.

Proposition 2: *If at least one outfit is not resource-constrained, external sponsorship will induce more attacks.*

In the next section we study comparative static effects of the change of different parameters, There, we show that a sponsor, by means of increasing sponsorship can further induce attacks till both outfits become resource-constrained.

3.2 Comparative Static Results

In our structure, counter-terrorism (CT) policy will affect either one or the other parameter underlying the model. Therefore, to understand the impact of any CT policy, it is necessary to understand the effect of the change of a parameter in the model on A_i and A_j. The effect actually depends on the initial equilibrium, i.e., whether $\overline{A}_i \gtreqless \overline{A}_j$ and whether any outfit is resource-constrained or not. For the following analysis, we continue to assume the proportionate rule to allocate sponsorship, and discuss the effect of the change of a parameter in the vicinity of the initial equilibrium. Note that when $\overline{A}_i > \overline{A}_j$ and none of the outfits is resource constrained, then in the interior equilibrium we must have $A_i^* > A_j^*$.

An increase in the intrinsic propensity of violence

Consider an increase in T_i's intrinsic propensity of violence (α_i). One can see from (15) that the slope of its reaction function remains unchanged, but the intercept ($\frac{\alpha_i}{\beta_i}$) increases. Therefore, if T_i is not resource constrained, its reaction function will shift up by an equal amount corresponding to each level of terror strikes conducted by the other terror outfit. Hence, A_i^* will increase. But whether A_j^* will increase or decrease in the vicinity of the initial equilibrium, depends on whether $\overline{A}_i < \overline{A}_j$ or $\overline{A}_i > \overline{A}_j$. However, if T_i is resource-constrained, then both A_i^* and A_j^* will remain unchanged. On the other hand, if T_j is resource-constrained but T_i is not, then A_i^* will go up but A_j^* will remain unchanged. When both outfits are resource-constrained, there will be no effect on the number of attacks. To conclude, if there is an increase in propensity of violence of an outfit, generally it would tend to increase the total number of attacks. In particular, given the second order and stability conditions, if none or only T_j is resource constrained, the total number of terror attacks ($A^* = A_i^* + A_j^*$) must increase if the intrinsic propensity of violence of any outfit increases. The formal proof of the result is given in *Appendix 1*.

An increase in cost inefficiency

Consider an increase in β_i (i.e., the cost inefficiency of T_i). This will shift down the reaction function of T_i such that both the intercept and the absolute slope will fall. If T_i is resource constrained, the horizontal segment of its reaction function will undergo a downward shift. Therefore, if β_i increases, T_i's equilibrium number of attacks (A_i^*) must fall irrespective of whether one or the other outfit is initially resource-constrained. If neither outfit is resource-constrained or T_i alone is resource-constrained, A_j^* will increase or decrease according as whether T_i's reaction function intersects the T_j's reaction function on the latter's falling or rising portion in the initial equilibrium (i.e., whether $A_i^* > A_j^*$ or $A_i^* < A_j^*$ initially). On the other hand, if T_j alone or both outfits are initially resource constrained, A_j^* will remain unchanged. However, even when A_j^* goes up, it will be dominated by the fall in A_i^*, hence the total number of attacks will fall. This happens because the direct effect of an increase in β_i, will dominate its indirect effect. To summarize, an increase in the inefficiency of any outfit will necessarily lead to a lower total attacks (see *Appendix 1*).[17]

An increase in external sponsorship

An increase in sponsorship F, ceteris paribus, leaves the intercepts of the reaction functions unchanged, although the absolute slope of each reaction function increases (see Equation (15)). This will lead to an increase in the number of terror strikes conducted by each outfit, provided that none is resource-constrained (such that (12) is satisfied). Hence, the new equilibrium lies to the north-east of the original equilibrium. If only one terror outfit is resource-constrained, it is unable to increase its number of attacks in response to a higher F. However, the other outfit increases its number of terror strikes. To summarize, if at least one outfit is resource-abundant, the total number of attacks must increase with an increase in F. In *Appendix 2* we have provided the formal proof.

An increase in resources

Suppose resource-endowment R_i of outfit T_i increases. Then it will have no effect on the number of attacks of any outfit if neither outfit, or only outfit T_j, is resource constrained at the initial equilibrium. On the other hand, there will be an equivalent increase in X_i. However, if T_i alone is initially resource-constrained and R_i increases, then it will enhance A_i^*. But whether A_j^* will increase or decrease depends on whether at the initial equilibrium, $A_i^* < A_j^*$ or $A_i^* > A_j^*$. If both

[17] Note that when T_i is resource-constrained and β_i increases, A_j^* will fall if $A_i^* < A_j^*$ in the initial equilibrium, although A_i^* must fall.

outfits are resource-constrained initially, then an increase in the resources of one outfit will raise its number of attacks, although the other outfit's attacks will remain unchanged. Thus, an increase in the resources of an outfit may not necessarily increase the total number of attacks. For details, see *Appendix 2*.

To interpret the effects very briefly, note that an increase in propensity of violence of an outfit, say T_i, or increasing efficiency (i.e., lowering of β_i) will induce the outfit to enhance its terror activities, and the number of attacks will go up if it is not already resource constrained. Now as A_i^* goes up, it will induce terror outfit T_j to change its optimal attacks A_j^* along its reaction function. If T_j was initially conducting more attacks (i.e., $A_j^* > A_i^*$), then in the vicinity of the initial equilibrium it would also optimally raise its optimal number of attacks. This captures the competition for external sponsorship F, which leads T_j to raise its terror attacks in order to neutralize the negative impact of the increase in T_i's terror attacks on its share of external sponsorship. If, on the other hand, T_j was conducting fewer attacks to begin with (i.e. $A_j^* < A_i^*$), then T_j optimally reduces its terror attacks in the vicinity of the initial equilibrium in response to an increase in the number of other outfit's attacks. This is because the benefit from cost-savings due to lower number of terror strikes dominates the loss from obtaining a reduced fraction of F. Similarly, a higher F implies a higher prize to be divided between the outfits on the basis of their fractions of the total number of terror strikes and, therefore, a fiercer competition between the outfits. This induces both outfits to conduct more attacks relative to the initial equilibrium. The effect of the change of resource can similarly be interpreted. The comparative static results of this section are summarized in the following Table.

Table 1: Comparative static results under sponsorship: [Here, "↑" denotes `increase', "↓" `decrease' and "↔" `remain unchanged'; $A^* = A_1^* + A_2^*$.]

Parameter	Both T_i and T_j are Unconstrained	Only T_i is resource-constrained	Only T_j is resource-constrained	Both T_i and T_j are resource-constrained
α_i ↑	A_i ↑ A_j ↑ or ↓ acc. as $A_j^* \gtreqless A_i^*$ A ↑	A_i ↔ A_j ↔ A ↔	A_i ↑ A_j ↔ A ↑	A_i ↔ A_j ↔ A ↔
β_i ↑	A_i ↓ A_j ↑ or ↓ acc. as $A_j^* \lesseqgtr A_i^*$ A ↓	A_i ↓ A_j ↑ or ↓ acc. as $A_j^* \lesseqgtr A_i^*$ A ↓	A_i ↓ A_j ↔ A ↓	A_i ↓ A_j ↔ A ↓
F ↑	A_i ↑ A_j ↑ A ↑	A_i ↔ A_j ↑ A ↑	A_i ↑ A_j ↔ A ↑	A_i ↔ A_j ↔ A ↔
R_i ↑	A_i ↔ A_j ↔ A ↔	A_i ↑ A_j ↑ or ↓ acc. as $A_j^* \gtreqless A_i^*$ A ↑	A_i ↔ A_j ↔ A ↔	A_i ↑ A_j ↔ A ↑

3.3 Alternative Sponsorship Mechanisms

In the above discussion, it has been assumed that the total amount of external sponsorship F, is fixed and committed before the game. Even if however, the external sponsorship amount is drawn from some probability distribution such that its expected value is F, the number of attacks optimally conducted by the terror outfits will be the same as before if the terror outfits are risk-neutral. If the outfits are risk-averse, however, then each will conduct fewer attacks. However, the number of attacks under risk-aversion would still exceed the number of attacks in the absence of external sponsorship. This is because the realized or *ex post* value of F can never be negative and hence, neither terror outfit can be worse off than in the absence of external sponsorship, despite being risk-averse.

It may also be possible that S grants a per-attack (constant) reward of $\gamma > 0$ to each terror outfit, that is, $F_i = \gamma A_i$ $(i = 1, 2)$. In this case, the total external sponsorship is not fixed but proportional to the total number of attacks, that is, $F = \gamma(A_i + A_j)$. In this case, the solution to the relevant first order condition yields the following optimum level of attacks for T_i $(i = 1, 2)$:

$$A_i = \frac{\alpha_i + \gamma}{\beta_i} > A_i^0 \tag{16}$$

It is easy to check that the second order condition for optimization holds. Hence, as in the case of proportionate or probabilistic allocation, external sponsorship in the form of per-attack reward once again results in a higher optimal number of attacks. This is due to the higher marginal benefit $\alpha_i + \gamma$ from each terror strike, compared to α_i in the absence of external sponsorship. There is, however, a marked similarity between this case and that without external sponsorship in that the optimal number of terror strikes conducted by one terror outfit is independent of the number of attacks conducted by the other outfit, hence there is no strategic interaction between the terror outfits in this case.

It is also possible that S may fix for T_i a sponsorship amount $\widetilde{F}_i > 0$ $(i = 1, 2)$ and some level of attacks $\widetilde{A}_i > A_i^0$ such that if $A_i \geq \widetilde{A}_i$, then $F_i = \widetilde{F}_i$; and if $A_i < \widetilde{A}_i$, then $F_i = 0$. Note that such an inducement for additional terror strikes can only work if $\widetilde{A}_i$ is not too high. Specifically, $\widetilde{A}_i$ must satisfy $\frac{1}{2}\beta_i \widetilde{A}_i^{\,2} \leq R_i$, which can be interpreted as a participation constraint for T_i. A necessary condition for this is that T_i must not be resource-constrained initially.

For this sponsorship mechanism to successfully induce T_i to conduct $\widetilde{A}_i$ attacks instead of only A_i^0, however, it must also satisfy incentive-compatibility.[18] The necessary and sufficient condition for this is $U_i(\widetilde{A}_i) \geq U_i(A_i^0)$, where $U_i(\widetilde{A}_i) = R_i - \frac{1}{2}\beta_i \widetilde{A}_i^{\,2} + \alpha_i \widetilde{A}_i + \widetilde{F}_i$ and $U_i(A_i^0) = R_i + \frac{1}{2}\frac{\alpha_i^2}{\beta_i}$. This condition entails that $\widetilde{F}_i$ be large enough to compensate for the marginal disutility to T_i, of conducting $(\widetilde{A}_i - A_i^0)$ additional attacks. Specifically, it can be shown that this translates to $\widetilde{F}_i \geq \frac{1}{2}\frac{\alpha_i^2}{\beta_i} - \left(\alpha_i \widetilde{A}_i - \frac{1}{2}\beta_i \widetilde{A}_i^{\,2}\right)$. The first and the second[19] terms on the right-hand side represent

[18] Note that T_i will not conduct more than $\widetilde{A}_i$ attacks because any additional attacks above this level would leave F_i unchanged, but would increase the cost incurred on terror strikes.
[19] The second term refers to the term within parentheses.

the net benefits from conducting A_i^0 and $\widetilde{A_i}$ attacks respectively, in the absence of external sponsorship. Hence, the right-hand side as a whole represents the marginal disutility of increasing the number of attacks from A_i^0 to $\widetilde{A_i}$. This is positive because A_i^0, being the optimal number of terror strikes conducted by T_i in the absence of external sponsorship, must necessarily generate a higher net benefit than that achieved by conducting $\widetilde{A_i}$ attacks. Therefore, for incentive-compatibility to hold, the external sponsor must compensate T_i for the marginal disutility incurred by the latter in increasing its number of terror strikes above its interior optimum to the level desired by the former. Expectedly, it is easier to induce an inherently more violent group to conduct a given number of additional attacks, because the minimum compensation required for such inducement varies inversely with an outfit's intrinsic propensity for violence. Similarly, it is harder to induce a group to conduct a fixed number of additional terror strikes if the government's counter-terrorism efforts are more focused towards it.

In the sponsorship mechanism described above, we observe that because S commits F_1 and F_2 (and not F), there is once again no strategic interaction between the terror outfits, and A_1 and A_2 are thus mutually independent.

S may also subsidize the cost to terror outfits of conducting attacks by providing a per-attack subsidy to the terror outfits.[20] The outcome would be identical to that under per-attack reward, if the per-attack subsidy is set equal to the per-attack reward ($\gamma > 0$). This is because the budget constraint under per-attack subsidy is $X_i + \frac{1}{2}\beta_i A_i^2 = R_i + \gamma A_i$ and hence, the optimization problem of T_i is given by $Max_{A_i} U_i = R_i - \frac{1}{2}\beta_i A_i^2 + \gamma A_i + \alpha_i A_i$, the solution to which is $A_i = \frac{\alpha_i + \gamma}{\beta_i}$, $(i = 1, 2)$. This, of course, is identical to the number of terror strikes under an equivalent per-attack reward.

It must also be noted that if it was possible to provide external sponsorship to an outfit at the beginning of the game, then it would be equivalent to a higher initial resource endowment for

[20] Note that a lump-sum subsidy would fail to induce additional terror strikes if the outfit is not-resource constrained, because it would be equivalent to a higher R_i, and therefore leave the marginal cost of a terror strike unaltered. If however, the outfit is resource-constrained to begin with, then the optimal number of terror strikes would increase due to a lump-sum subsidy.

that outfit. Therefore, if the outfit is not resource-constrained, the external sponsorship would fail to induce additional terror strikes. The external sponsorship would only result in higher consumption. A resource-constrained outfit, however, would optimally conduct a higher number of terror strikes if provided with such sponsorship. Here, too, there is no strategic interaction between the outfits.

In the present paper we are focusing on external sponsorship which leads to strategic interaction between the outfits, hence we restrict ourselves to the proportionate allocation rule as given by (10), for the remaining analysis.

3.4 *Optimal Sponsorship*

In subsection 3.1 we have assumed that before the outfits choose non-cooperatively the number of the attacks they will conduct, the sponsors commit to pay a sum of money $F > 0$ to the outfits once the attacks take place, in proportion to their respective attacks, i.e., $F_i = \frac{A_i}{A_i+A_j}F$; $i \neq j$. We have further shown that an $F > 0$ will induce more terror activities compared to the no-sponsor case if and only if at least one outfit is not resource-constrained in the no-sponsor case i.e., $R_i \leq \frac{\alpha_i^2}{2\beta_i}$ at least for one i. On the other hand, if $R_i \leq \frac{\alpha_i^2}{2\beta_i}$ for both $i = 1, 2$, sponsorship cannot induce more terror attacks compared to no-sponsorship, but only causes consumption of the outfits to adjust. Under the proportionate rule, since sponsorship accrues to the outfits only after terror activities have taken place, each outfit's terror activity is restricted by the size of its resource endowment, i.e., $A_i \leq \sqrt{\frac{2R_i}{\beta_i}}$, $i = 1, 2$. The implication is that under proportionate rule, sponsorship may induce terror attacks at most up to that level.

Let us assume that unlimited funds are available with the external sponsor, who wants to determine optimally how much funds to provide for sponsoring terror activities with the objective of maximizing the total number of terror attacks. We continue to assume that the sponsorship will be divided between the outfits as per the proportionate rule. We shall discuss the problem under the following assumptions:

Assumption (A1): $R_i > \frac{\alpha_i^2}{2\beta_i}$, $i = 1, 2$

Assumption (A2): $R_1 > \frac{\alpha_1^2}{2\beta_1}$ *and* $R_2 \leq \frac{\alpha_2^2}{2\beta_2}$

Assumption (A3): $R_i \leq \frac{\alpha_i^2}{2\beta_i}$ *with strict inequality at least for one* i

If external sponsorship is unavailable (i.e., $F = 0$), then the optimal number of attacks under assumption (A1) is $A_i = A_i^0 \left(= \frac{\alpha_i}{\beta_i}\right)$, $i = 1, 2$. Then it follows from subsection 3.1 that for any $F > 0$, the optimal number of attacks will be given by the solution to the FOC (12), i.e., $\alpha_i + \frac{A_j}{(A_i + A_j)^2} F - \beta_i A_i = 0$, $i \neq j$ subject to $A_i \leq \sqrt{\frac{2R_i}{\beta_i}}$. Therefore, under proportionate external sponsorship, the optimal number of attacks under non-cooperative competition will be given by

$$A_i^*(F) = \min\{A_i(F), \sqrt{\frac{2R_i}{\beta_i}}\}, i = 1,2 \tag{17}$$

where $A_i(F)$ is obtained by simultaneously solving the FOCs (as given in (12)). Further, for any F, if $A_i(F) < \sqrt{\frac{2R_i}{\beta_i}}$, then as F is increased, $A_i(F)$ will go on increasing up to the level $\sqrt{\frac{2R_i}{\beta_i}}$. If F further increases, $A_i(F)$ will be pegged at $\sqrt{\frac{2R_i}{\beta_i}}$. Therefore, the maximum number of attacks that can be induced by sponsor money will be $A_i(F) = \sqrt{\frac{2R_i}{\beta_i}}$, $i = 1,2$. Then plugging $A_i(.) = \sqrt{\frac{2R_i}{\beta_i}}$ in the FOCs we shall get,

$$\alpha_i + \frac{\sqrt{\frac{2R_j}{\beta_j}}}{\left(\sqrt{\frac{2R_i}{\beta_i}} + \sqrt{\frac{2R_j}{\beta_j}}\right)^2} F - \beta_i \sqrt{\frac{2R_i}{\beta_i}} = 0; i \neq j$$

Summing over $i = 1,2$, we shall get the solution for optimal sponsor money $F = F^*$ given by,

$$F^*(R_1, R_2) = \left(\sqrt{\frac{2R_1}{\beta_1}} + \sqrt{\frac{2R_2}{\beta_2}}\right)\left(\beta_1 \sqrt{\frac{2R_1}{\beta_1}} + \beta_2 \sqrt{\frac{2R_2}{\beta_2}} - \alpha_1 - \alpha_2\right) \tag{18}$$

Now consider assumption (A2). As shown in section 2, when $F = 0$, we must have $A_1^0 = \frac{\alpha_1}{\beta_1}$ and $A_2^0 = \sqrt{\frac{2R_2}{\beta_2}}$.[21] When $F > 0$, as follows from subsection 3.1, we have $A_2(F; R_2) = \sqrt{\frac{2R_2}{\beta_2}}$, and then $A_1 = A_1(F; A_2(F; R_2))$ is solved from the FOC: $\alpha_1 + \frac{A_2}{(A_1+A_2)^2}F - \beta_1 A_1 = 0$ subject to $A_1(F; A_2(F; R_2)) \leq \sqrt{\frac{2R_1}{\beta_1}}$. Therefore, given $F > 0$ the optimal number of attacks that T_1 will conduct is:

$$A_1^*(F) = \min\{A_1(F; A_2(F; R_2)), \sqrt{\frac{2R_1}{\beta_1}}\} \tag{19}$$

Therefore, the maximum number of attacks by T_1 that can be induced by appropriate choice of F will be $A_1(F; A_2(F; R_2)) = \sqrt{\frac{2R_1}{\beta_1}}$. Finally, plugging $A_i(.) = \sqrt{\frac{2R_i}{\beta_i}}$, $i = 1,2$, in the FOC of T_1, we shall solve for optimal F under assumption (A2), i.e.,

$$\alpha_1 + \frac{\sqrt{\frac{2R_2}{\beta_2}}}{(\sqrt{\frac{2R_1}{\beta_1}} + \sqrt{\frac{2R_2}{\beta_2}})^2} F - \beta_1 \sqrt{\frac{2R_1}{\beta_1}} = 0$$

Therefore, the optimal $F = F^*$ under assumption (A2) is given by

$$F^*(R_1, R_2) = (\beta_1 \sqrt{\frac{2R_1}{\beta_1}} - \alpha_1) \frac{[\sqrt{\frac{2R_1}{\beta_1}} + \sqrt{\frac{2R_2}{\beta_2}}]^2}{\sqrt{\frac{2R_2}{\beta_2}}} \tag{20}$$

Finally, consider assumption (A3). We have already mentioned that if any $F > 0$ is committed before attacks are conducted, then F will have no impact on optimal A_i. This means if $R_i \leq \frac{\alpha_i^2}{2\beta_i}$ $\forall i = 1,2$, then F will fail to induce A_i using the proportionate rule. Summarizing the above analysis, we can write the following proposition:[22]

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[21] Actually, if $R_2 = \frac{\alpha_2^2}{2\beta_2}$, then $A_2^0 = \frac{\alpha_2}{\beta_2} = \sqrt{\frac{2R_2}{\beta_2}}$.

[22] By leaving aside the proportionate rule one may suggest to provide T_i a priori, in case (A3), a lump sum sponsorship F_i such that T_i goes for attacks up to $\frac{\alpha_i}{\beta_i}$ $(i = 1, 2)$. In that case the optimal F_i will be solved from $\sqrt{\frac{2(R_i+F_i)}{\beta_i}} = \frac{\alpha_i}{\beta_i}$, hence $F_i = \frac{\alpha_i^2}{2\beta_i} - R_i$. Such F_i enables the outfit to conduct more attacks, but this is not via inducement.

Proposition 3: *If the sponsor can provide unlimited finance, then given either of assumptions (A1) through (A3), the amount of sponsorship can always be determined optimally so as to maximize the total number of attacks using the proportionate rule. Under assumptions (A1) and (A2), F will provide an inducement for attacks whereas under assumption (A3), proportionate external sponsorship is ineffective and hence none is provided optimally*

To complete the analysis, consider the situation when the sponsor has limited fund in the sense that $F = \bar{F} < F^*$. In this case we are back to the analysis of subsection 3.1 with $F = \bar{F}$, and then we must have $A_1^* + A_2^* < \sqrt{\frac{2R_1}{\beta_1}} + \sqrt{\frac{2R_2}{\beta_2}}$ (under assumption (A1) and (A2)). Note that we have restricted our analysis to the case of proportionate rule and demonstrates the situations when the sponsor may utilize external sponsorship to induce terror attacks. Naturally, counter-terrorism policy will attempt to block this flow of money and thereby minimize incremental terror activity.

4. Counter-Terrorism

We now turn our attention to the implications of the above discussion and results, for the counter-terrorism policy of the targeted country's government. Various categories of CT policies available to a country are discussed in Bhan and Kabiraj (2019). In the present paper the CT parameters that can be impacted by one or the other CT measure are β_is, α_is, R_is and F. For instance, an increase in β_i could be achieved through "hardening" of potential targets of T_i,[23] or by deploying governmental intelligence agencies against the outfit on a priority basis. Such CT efforts attempt to reduce the optimal number of terror strikes by reducing the (net) operational efficiency with which a terror outfit can attack certain targets. To illustrate, consider the Anti-Infiltration Obstacle System (AIOS). A double-row concertina wire fence was constructed by the Indian Army along the Line of Control (LoC) to inhibit the infiltration of terrorists from Pakistan-occupied Kashmir (PoK). Such CT measures can broadly be viewed as defensive in nature.

[23] That is, by increasing the security levels of potential targets, thereby rendering them more difficult or costly for a terror outfit to attack.

If the targeted country's government takes the more pro-active/offensive approach of imposing financial and other sanctions or even conducting pre-emptive strikes to destroy the assets of terror outfits, then this would result in lower R_is. Consider for example, the American campaign against the Afghan Taliban, post the attacks conducted by al Qaeda in the United States on 11 September, 2001. Sustained airstrikes aimed at degrading the Taliban's assets and resources, were at the core of the war effort.

The government may alternatively attempt to win the *hearts and minds* of the alienated population living in a terror affected geographical area, in order to reduce the support for the terror outfit(s) operating in that area. Operation Sadbhavana, launched by the Indian Army in rural areas adjoining the LoC in 1998, is a case in point. Incentives may also be given to members/functionaries of a terror outfit in order to induce them to surrender. To this end, so-called *confidence-building measures* (CBMs) may be undertaken by the government. Further, the government may try to nudge religious institutions of learning to accept greater state regulation and to modify their curriculum and academic discourse. All such measures would tend to lower the intrinsic propensity of violence of an outfit active in that area.

Finally, economic sanctions may be imposed on institutions, individuals and countries sponsoring terrorism. The assets of such entities - financial and physical - may be frozen and confiscated, and the associated individuals jailed, thereby inhibiting their capacity to sponsor terror activities. All such measures would tend to reduce the external sponsorship available to terror outfits.

One of the most salient consequences of the analysis in Sections 2 and 3, is the effectiveness of defensive CT both in the absence and presence of external sponsorship, irrespective of whether the targeted terror outfit is resource-constrained or not. This is because as brought out by Equations (6), (7) and (12); the optimal number of terror attacks varies negatively with outfit-inefficiency β_i; which increases as a result of defensive CT. Equation (7), in fact, also has an important bearing on the applicability of offensive CT. Equation (7) gives the optimal number of terror attacks conducted by a resource-constrained terror outfit, both in the absence and presence of external sponsorship, and shows it to be an increasing function of resource-endowment R_i. Given that offensive CT causes R_i to decrease, it is effective against resource-constrained terror outfits. A similar result can be obtained for resource-abundant outfits, in terms of the applicability of CBMs, using Equations (6) and (12). This leads us to the proposition below.

Proposition 4: *Irrespective of the absence or presence of proportionate external sponsorship:*
a) *Defensive CT is effective against all terror outfits,*
b) *Offensive CT is effective against resource-constrained terror outfits, and*
c) *CBMs are effective against resource-abundant terror outfits*.

It is also noteworthy that in the vicinity of an interior equilibrium in the absence of external sponsorship, the marginal impact of defensive CT measures is greater or lesser than that of a decrease in the outfit's intrinsic propensity for violence through CBMs, etc., according as the intrinsic propensity for violence is greater or lesser than the outfit's inefficiency in interior equilibrium.[24] This is because from Equation (6), for $i = 1, 2$ we get

$$\left|\frac{\partial A_i^*}{\partial \beta_i}\right| = \frac{\alpha_i}{\beta_i^2} \gtrless \frac{1}{\beta_i} = \frac{\partial A_i^*}{\partial \alpha_i} \text{ according as } \alpha_i \gtrless \beta_i \quad (21)$$

This explains why victim countries often deal with low-intensity conflicts (LICs) with *kid gloves*, unless and until they evolve over time into violent insurgencies that threaten the very political stability of the region and the government's administrative machinery. It is only under such situations, that the government feels compelled to suppress the terrorists with an *iron fist*.

In the vicinity of corner equilibrium, the marginal impact of defensive CT measures is greater (lesser) than that of offensive CT measures, if the outfit's resource base is higher (lower) than its inefficiency in interior equilibrium.[25] This is because from Equation (7), for $i = 1, 2$ we get

[24] Or according as the number (or intensity, more generally) of terror strikes conducted by the outfit $(= \alpha_i/\beta_i)$ exceeds or falls short of unity. More generally, in interior equilibrium, the defensive bias of an ad hoc CT response versus CBMs is positively associated with the number/intensity of terror activity of the concerned outfit(s).
[25] Hence, in interior equilibrium, the offensive versus defensive bias of ad hoc CT response varies negatively with the outfit's resource base as well as the outfit's operational efficiency.

$$\left|\frac{\partial A_i^*}{\partial \beta_i}\right| = \frac{1}{\beta_i^{3/2}}\sqrt{\frac{R_i}{2}} \gtrless \frac{1}{\sqrt{2\beta_i R_i}} = \frac{\partial A_i^*}{\partial R_i} \text{ according as } R_i \gtrless \beta_i \qquad (22)$$

This illustrates why even while dealing with resource-constrained outfits, the CT response in many countries prioritizes offense over defense if and only if the outfit's resources are sufficiently low (or existing surveillance measures and/or security of potential high-value targets are adequate to begin with). Hence, governments often act the toughest against those resource-constrained outfits which are the easiest to counter. The following proposition summarizes the above discussion on ad hoc CT:

Proposition 5: *In the absence of external sponsorship, the CT response in the vicinity of the initial equilibrium tends to prioritize:*

1. *CBMs, if and only if the terror outfit is resource-rich and sufficiently inefficient (or insufficiently violent),*
2. *Offensive measures, if and only if the terror outfit is sufficiently resource-constrained (or sufficiently inefficient), and*
3. *Defensive measures for all other outfits.*

The above proposition is logical. Firstly, a resource-rich outfit has a stronger (weaker) incentive to respond to CBMs if it is unable (able) to carry out attacks with sufficient impunity, say due to high-value targets being sufficiently secure (insecure); or if it is not too violent intrinsically, say because its objectives are political rather than ideological. Hence, follows the first statement. Secondly, an ad hoc CT response tends to be predominantly offensive if and only if governmental efforts to neutralize/squeeze the outfit's assets has an immediate impact on the ability of the outfit to conduct attacks, without threatening to draw the government into a long drawn military campaign. This rationalizes the second statement. All other outfits are sufficiently efficient and are not highly resource-constrained. Hence, CBMs are not very effective and offensive measures threaten to snowball into a long-drawn and expensive military campaign, or are simply ineffective given that the outfit has sufficient resources at its disposal. Hence, the government tends to focus primarily on hardening potential targets. This justifies the third statement.

Let us address external sponsorship. In the context of an outfit that is not resource-constrained, there are some obvious CT implications that flow from the third section. As in the absence of external sponsorship, CBMs and defensive CT measures are effective in inhibiting the outfit's terror activities while offensive measures are ineffective. Moreover, measures effectively targeting the sponsor would inhibit the sponsorship available to the outfits, and thereby reduce the optimal number of attacks. This follows from Proposition 1. A prime example of such an outfit is Boko Haram which, despite its decline since 2015, continues to remain a potent regional threat (Thurston, 2017). In fact, Boko Haram's decline can be attributed, at least partially, to that of one of its principal sponsors, al Qaeda.[26] There can, however, be additional *indirect* impacts of CT measures targeting the other outfit, due to the inter-outfit strategic interaction induced by proportionate external sponsorship.

If the other outfit T_j $(j \neq i = 1, 2)$ is resource-rich as well, then as a consequence of the nature of the reaction functions derived in subsection 3.4, the optimal number of attacks conducted by the outfit under consideration T_i $(i = 1, 2)$ will be indirectly impacted by CBMs and defensive measures targeting T_j. If T_j conducts more attacks in the initial equilibrium, then these measures tend to increase A_i in the vicinity of the initial equilibrium. The converse is true if the other outfit conducts fewer attacks in initial equilibrium. If T_j is resource-constrained on the other hand, the optimal number of attacks conducted by T_i will be indirectly impacted by both defensive as well as offensive measures targeted at T_j. If T_j conducts more attacks in the initial equilibrium, then these measures tend to increase A_i in the vicinity of the initial equilibrium. The converse is true if the other outfit conducts fewer attacks in initial equilibrium.

For resource-constrained outfits, as in the absence of external sponsorship, the optimal number of attacks can be inhibited only by defensive and offensive CT, e.g., the Jammu and Kashmir Liberation Front (JKLF) in the Indian State of Jammu and Kashmir, failed to resurrect itself as a terror outfit after outfit head Shabbir Siddiqui and the 37 remaining members of the

[26] See the 2015 report by the United States Army for a discussion al Qaeda's declining influence over Boko Haram, and Byman (2017) for an overall discussion of al Qaeda's decline.

Amanullah Khan faction were eliminated in two separate encounters in Hazratbal, in March 1996.[27]

It is also interesting to note that the criteria for comparing the effectiveness of defensive CT measures versus CBMs against a resource-rich outfit, in terms of the optimal number attacks conducted by the targeted outfit, remains the same as that in the absence of external sponsorship.

Lemma 1: *Irrespective of the presence or absence of a strategic external sponsor, defensive CT is more or less effective than CBMs against a resource-rich outfit according as the optimal number of attacks conducted by the targeted outfit,* $A_i^* \gtrless 1$.

The proof of the lemma follows from the results in Appendix 1. In essence, the lemma states that if the targeted outfit's attacks are sufficiently high in number or intensity, the best CT approach involves defending against such attacks. Only if the outfit's activities are under control, can CBMs be used effectively. The intuition is that an outfit which can carry out terror strikes with impunity has little incentive to arrive at the negotiating table, and *vice versa*. What is of greater interest, however, is that the threshold which determines the effectiveness of defensive CT relative to CBMs remains the same, both in the presence and absence of external sponsorship. This allows for the possibility of different optimal policy choices under the two regimes, as stated in the proposition below.

Proposition 6: *It is possible that defensive CT is more effective in the presence of proportionate external sponsorship, while CBMs are more effective in its absence. If defensive CT is more effective in the absence of external sponsorship, however, then it must be more effective even in its presence. Conversely, if CBMs are more effective in the presence of external sponsorship, then they must be more effective even in its absence.*

To explain, first note that the optimal number of attacks conducted by a targeted resource-rich outfit in the presence of external sponsorship exceeds the optimal number of

[27] See Vembu (September, 8, 2011) and GlobalSecurity.org (November 7, 2011) for instances of the muscular approach adopted by the Indian State in countering the JKLF.

attacks in its absence. Further, from Lemma 1, we know that the threshold for comparing the effectiveness of defensive CT relative to CBMs is unity. So there are three possibilities. If the optimal number of attacks conducted by the targeted outfit under both regimes exceeds unity, defensive CT is more effective under both regimes. If the optimal number of attacks conducted by the targeted outfit under both regimes falls short of unity, CBMs are more effective under both regimes. However, if the optimal number of attacks in the absence of external sponsorship falls short of unity while that in the presence of external sponsorship exceeds unity, then CBMs are more effective under the former regime while defensive CT is more effective in the latter.

We now consider the special case where both outfits are resource-rich and equally efficient, and compare the effectiveness of the different categories of ad hoc CT measures.

<u>Proposition 7</u>: *If both terror outfits are resource-rich and equally efficient a priori, then in the presence of external sponsorship:*

1. $\left|\frac{\partial A^*}{\partial \beta_i}\right| \geq \frac{\partial A^*}{\partial \alpha_i} > \frac{\partial A^*}{\partial F}$ *if* $A_i^* \geq 1$,
2. $\frac{\partial A^*}{\partial \alpha_i} > max\left\{\left|\frac{\partial A^*}{\partial \beta_i}\right|, \frac{\partial A^*}{\partial F}\right\}$ *if* $1 - A_j^* < A_i^* < 1$, *and*
3. $\frac{\partial A^*}{\partial F} \geq \frac{\partial A^*}{\partial \alpha_i} > \left|\frac{\partial A^*}{\partial \beta_i}\right|$ *if* $A_i^* \leq 1 - A_j^*$.

Proof: From the expression of for $\frac{dA}{dF}$ and $\frac{dA}{d\alpha_i}$ (see Appendix 1 and 2), it is evident that $\frac{dA}{dF} \gtrless \frac{dA}{d\alpha_i}$ according as $(\beta_i - \beta_j)A_i \gtrless (A-1)\left(\beta_j A + \frac{F}{A}\right)$. If both outfits are equally efficient *a priori*, i.e., $\beta_i = \beta_j = \beta$ (say), then this condition reduces to $(A-1)\left(\beta A + \frac{F}{A}\right) \lesseqgtr 0$. This, combined with Lemma 1, proves Proposition 7.

It seems logical that if the targeted outfit is characterized by a high-enough activity level, then defensive CT measures should be most effective, while targeting the sponsor should be least effective. The first part follows from Lemma 1. On the other hand, if the targeted outfit is characterized by a sufficiently low activity level, then the converse is expectedly true. Finally, if the activity level of the targeted outfit is neither too high nor too low, then CBMs constitute the policy of choice. It goes without saying that as in the absence of external sponsorship, the activity level of neither outfit can be impacted by offensive CT measures. Similar analysis is possible in case one outfit is resource-constrained while the other is not.

Proposition 7 gives rise to the following mutually exclusive and collectively exhaustive scenarios, if both outfits are resource-rich and equally efficient *a priori*:

1. $A_i^* \geq 1 \; \forall i = 1, 2$: In this case, defensive CT measures are optimal against both outfits,
2. $A_i^* \geq 1, \; A_j^* < 1, \; i \neq j = 1, 2$: Here, defensive measures are optimal against T_i, while CBMs are optimal against T_j,
3. $A_i^* < 1 \; \forall i = 1, 2$, but $A_1^* + A_2^* > 1$: Here, CBMs are optimal against both outfits, and
4. $A_1^* + A_2^* \leq 1$: Here, it is most effective to target the sponsor.

A straight-forward corollary is that if targeting the sponsor or targeting an outfit with CBMs is optimal in the presence of external sponsorship, then CBMs against that outfit must be optimal in the absence of external sponsorship, given lower terror activity in the absence of external finance.

Finally, counter-terrorism policy must take into consideration the different impacts of given policy interventions under each of the two regimes of *sponsorship* and *no sponsorship*. Proposition 8 below compares the impact of CBMs under the two regimes.

<u>Proposition 8</u>: *If both terror outfits are resource-rich and equally efficient a priori, then CBMs are more effective in the absence of external sponsorship, than in its presence.*

Proof: If both outfits are resource-rich, the impact of CBMs in the absence of external sponsorship is $\left|\frac{dA}{d\alpha_i}\right|_{NS} = \frac{1}{\beta_i}$, and that in its presence is $\left|\frac{dA}{d\alpha_i}\right|_S = \frac{\beta_j + \frac{F}{A^2}}{D}$. Now supposing $\beta_i = \beta_j = \beta$, and then substituting for D using Equation (14), we get $\left|\frac{dA}{d\alpha_i}\right|_{NS} \gtrless \left|\frac{dA}{d\alpha_i}\right|_S$ according as $\beta A^2 + F \gtrless 0$. Since $\beta A^2 + F > 0$, therefore $\left|\frac{dA}{d\alpha_i}\right|_{NS} > \left|\frac{dA}{d\alpha_i}\right|_S$. Hence the proof.

The logic appeals to intuition. An outfit, which is responsive to the government's overtures in the absence of external funding, may not display the same urgency towards a negotiated settlement once it becomes a recipient of external funding. In the latter situation, the balance of power to *pull strings* with the outfit would likely be tilted in favour of the external sponsor. This is similar to the results obtained by Siqueira and Sandler (2006).

5. Conclusion

The present work explores the role of external sponsorship of terror outfits in augmenting violence, its potential to alter the behavior of terrorists, and the consequent impact on CT dynamics. Our very first result expectedly bears out the ability of proportionate and probabilistic external sponsorship to augment terror activity, by engendering competition between terror outfits for the reward of funds. Although this is in line with the arguments presented in Byman (2005), Siqueira and Sandler (2006) are only able to prove this result under the assumption that the terrorist's support base is strong. The present analysis is able to establish the robustness of this result by demonstrating that it holds true as long as at least one of the terror outfits is resource-rich. Hence, the result holds even if one of the outfits is resource-constrained, which would be likely if that outfit does not have a strong support base.

Also in line with Byman (2005) and Siqueira and Sandler (2006), we show that governmental efforts at outreach via CBMs may not be as effective in the presence of external sponsorship, as in its absence. This is because the negative impact of CBMs on the terrorists' intrinsic proclivity for violence is negated by the increased motivation for terror attacks due to increased sponsorship.

Another striking inference borne out of our effort is that defensive measures are a ubiquitous constituent of CT. This is because such measures reduce the efficiency with which terror outfits can use their scarce resources, thereby making them a safe choice in the context of any CT effort. And in contrast to the ubiquity of defensive measures, we find CBMs and offensive CT measures to have limited applicability, as these are demonstrated to only be of any use against a resource-rich and resource-constrained terror outfit respectively.

Further, it must be noted that since any terror outfit would conduct at least as many terror strikes in the presence of external sponsorship than in its absence, it is obvious that if an outfit is resource-abundant in the presence of external sponsorship, then it must be resource-abundant even in its absence. In fact, this paper establishes that if CBMs are more effective than defensive CT under external sponsorship, than they must be more effective even in its absence. As a corollary, if defensive CT is more effective than CBMs in the absence of external funding, then it must be more effective even in its presence.

CBMs can inhibit the activity of resource-rich terrorists by reducing their inherent propensity for violence, while offensive measures can serve to curtail the activity of a resource-constrained terror group by causing the degradation of its resources. This contributes to the existing literature which only demonstrates the general *over-investment* in defensive measures and *under-investment* in offensive measures (eg., Sandler (2005)), the inability of countries to arrive at the optimal CT mix between offence and defence when faced with a common terrorist threat (eg., Sandler and Siqueira (2006)), and the inability of the State to win a *war-on-terror* using preemption alone as long as the marginal cost of preemptive measures is increasing (Das and Lahiri (2019). The importance of our finding lies in its rationalization of the omnipresence of defensive CT on the basis of the above-mentioned *efficiency-of-resource-use* hypothesis, a novelty. This is demonstrated under constant marginal costs of preemption and defence, and both in the presence and absence of a strategic external sponsor, thereby establishing the inherent robustness of the result.

Finally, if finance is the *lifeblood* of terrorism (Acharya (2009)), curbing it must prove effectual in severing the terrorists' lifeline. The present work demonstrates that curtailing external sponsorship, if present, is always effective in reducing terror activity. Choking such funding is shown to successfully reduce the attacks conducted by each outfit. Moreover, our analysis illustrates that targeting external finance if present, may be the most effective means to reduce terror attacks if terror activity is sufficiently low. The decline and eventual disappearance of the Abu Nidal Organisation (ANO), is a prime example of the efficacy of constraints on external finance. The impact of curbed external funding results in reduced size of the reward, which inhibits the incentive of the terror outfits to compete as aggressively by conducting more attacks.

A lot remains to be explored about the external sponsorship of terror outfits, however. It would be meaningful to compare the effectiveness of alternative mechanisms of terror finance, both as an incentivizing device (as in the present work), and as an enabler of terror activity. In fact, it would be particularly meaningful to explore the latter in the context of resource-constrained outfits. Moreover, it is also important to delve into the impact of inter-outfit cooperation between terrorists, on the effectiveness of external funding.

Appendix

Appendix 1: Effect of change of α_i and β_i

Under external sponsorship, the problem of T_i is given by Eqn. (11). When interior optimum exists, A_i and A_j are solved from (12). The solutions are unique and stable, given (13) and (14) are satisfied. The equilibrium solutions are functions of the parameters in the model. We can further derive $\frac{\partial^2 U_i}{\partial A_i \partial A_j} = \frac{A_i - A_j}{(A_i + A_j)^3} F$. For any parameter, θ, the comparative static results in general are:

$$\frac{dA_i}{d\theta} = \frac{1}{D}\left[-\frac{\partial^2 U_i}{\partial A_i \partial \theta}\frac{\partial^2 U_j}{\partial A_j^2} + \frac{\partial^2 U_j}{\partial A_j \partial \theta}\frac{\partial^2 U_i}{\partial A_i \partial A_j}\right] \text{ and } \frac{dA_j}{d\theta} = \frac{1}{D}\left[-\frac{\partial^2 U_j}{\partial A_j \partial \theta}\frac{\partial^2 U_i}{\partial A_i^2} + \frac{\partial^2 U_i}{\partial A_i \partial \theta}\frac{\partial^2 U_j}{\partial A_j \partial A_i}\right]$$

Using the above, we shall now get the following comparative static results under the following situations:

Situation 1: No outfit is resource constrained.

Change of α_i:

$$\frac{dA_i}{d\alpha_i} = \frac{1}{D}\left(\beta_j + 2\frac{A_i}{(A_i + A_j)^3}F\right) > 0, \text{ and } \frac{dA_j}{d\alpha_i} = \frac{1}{D}\left(\frac{A_i - A_j}{(A_i + A_j)^3}\right)F; \text{ hence } \frac{dA_j}{d\alpha_i} \gtreqless 0 \iff A_j \gtreqless A_i.$$

Defining $A = A_i + A_j$, we have $\frac{dA}{d\alpha_i} = \frac{1}{D}\left(\beta_j + \frac{F}{(A_i + A_j)^2}\right) > 0$.

Change of β_i:

$$\frac{dA_i}{d\beta_i} = -\frac{A_i}{D}\left(\beta_j + 2\frac{A_i}{(A_i + A_j)^3}F\right) < 0 \text{ and } \frac{dA_j}{d\beta_i} = -\frac{A_i}{D}\left(\frac{A_j - A_i}{(A_i + A_j)^3}\right)F; \text{ hence } \frac{dA_j}{d\beta_i} \gtreqless 0 \iff A_j \lesseqgtr A_i$$

So, $\frac{dA}{d\beta_i} = -\frac{A_i}{D}\left(\beta_j + \frac{F}{(A_i + A_j)^2}\right) < 0$; $\frac{dA}{d\beta_i} = -A_i \frac{dA}{d\alpha_i}$.

Situation 2: Only T_i is resource-constrained.

Here A_i is solved from $R_i = \frac{1}{2}\beta_i A_i^2$, hence $A_i = \sqrt{\frac{2R_i}{\beta_i}}$. But A_j is solved from the FOC: $\alpha_j + \frac{A_i}{(A_i + A_j)^2}F - \beta_j A_j = 0$.

Change of α_i:

Here we have: $\frac{dA_i}{d\alpha_i} = 0 = \frac{dA_j}{d\alpha_i}$, hence $\frac{dA}{d\alpha_i} = 0$.

Change of β_i:

$\frac{dA_i}{d\beta_i} = -\frac{1}{2\beta_i}\sqrt{\frac{2R_i}{\beta_i}} < 0$. From the FOC we can derive $\frac{dA_j}{d\beta_i} = \frac{(A_j - A_i)F}{\beta_j(A_i+A_j)^3 + 2A_iF}\frac{dA_i}{d\beta_i}$.

Hence, $\frac{dA_j}{d\beta_i} \gtrless 0 \iff A_j \lesseqgtr A_i$. Therefore, $\frac{dA}{d\beta_i} = \frac{\beta_j(A_i+A_j)^3 + (A_i+A_j)F}{\beta_j(A_i+A_j)^3 + 2A_iF}\frac{dA_i}{d\beta_i} < 0$.

Situation 3: Only T_j is resource-constrained

In this case A_j is solved from $R_j = \frac{1}{2}\beta_j A_j^2$, hence $A_j = \sqrt{\frac{2R_j}{\beta_j}}$. Now given A_j, A_i is solved from the FOC: $\alpha_i + \frac{A_j}{(A_i+A_j)^2}F - \beta_i A_i = 0$.

Change of α_i

$\frac{dA_j}{d\alpha_i} = 0$, and from FOC, $\frac{dA_i}{d\alpha_i} = \frac{1}{\beta_i + 2\frac{A_j}{(A_i+A_j)^3}F} > 0$, hence $\frac{dA}{d\alpha_i} = \frac{dA_i}{d\alpha_i} > 0$.

Change of β_i

$\frac{dA_j}{d\beta_i} = 0$, and using FOC, $\frac{dA_i}{d\beta_i} = -\frac{A_i}{\beta_i + 2\frac{A_j}{(A_i+A_j)^3}F} < 0$, hence $\frac{dA}{d\beta_i} = -A_i\frac{dA}{d\alpha_i} < 0$.

Appendix 2: Effect of change of F

When neither outfit is resource constrained, the optimal attacks are solved from the FOC, SOC and stability and uniqueness condition as given by (12), (13) and (14) respectively. Then we can derive:

$$\frac{dA_i}{dF} = \frac{1}{D(A_i+A_j)^5}\left[A_i(A_i - A_j)F + A_j\left\{\beta_j(A_i + A_j)^3 + 2A_iF\right\}\right] > 0,$$

$$\frac{dA_j}{dF} = \frac{1}{\Delta(A_i+A_j)^5}\left[-A_j(A_i - A_j)F + A_i\left\{\beta_i(A_i + A_j)^3 + 2A_jF\right\}\right] > 0,$$

Therefore,

$$\frac{dA}{dF} = \frac{1}{D(A_i+A_j)^3}\left[F + (A_i + A_j)(\beta_i A_i + \beta_j A_j)\right] > 0.$$

Now, if T_j is resource constrained but T_i not, then we have $A_j(F; R_j) = \sqrt{\frac{2R_j}{\beta_j}}$, and A_i is finally solved from T_i's FOC, $\alpha_i + \frac{A_j}{(A_i+A_j)^2}F - \beta_i A_i = 0$, subject to $A_i \leq \sqrt{\frac{2R_i}{\beta_i}}$. Then clearly $\frac{dA_i}{dF} > 0$, hence $\frac{dA}{dF} > 0$ for all $A_i \leq \sqrt{\frac{2R_i}{\beta_i}}$.

Appendix 3: Effect of change of R_i

When neither outfit or T_j alone is resource constrained, it is easy to understand that neither outfit's terror activity changes. When both are constrained, A_i rises because $A_i = \sqrt{\frac{2R_i}{\beta_i}}$. Therefore, when T_i alone is resource constrained, A_i goes up for the same reason, but in this case A_j is solved from the FOC: $\alpha_j + \frac{A_i}{(A_i+A_j)^2}F - \beta_j A_j = 0$. Using this we can derive $\frac{dA_j}{dR_i} = \frac{(A_j-A_i)F}{\beta_j(A_i+A_j)^3+2A_iF}\frac{dA_i}{dR_i}$. Hence, $\frac{dA_j}{dR_i} \gtreqless 0 \iff A_j \gtreqless A_i$. Finally, $\frac{dA}{dR_i} = \frac{\beta_j(A_i+A_j)^3+(A_i+A_j)F}{\beta_j(A_i+A_j)^3+2A_iF}\frac{dA_i}{dR_i} > 0$.

References

1. Acharya, Arabinda (2009): *Targeting Terrorism Financing: International Cooperation and New Regimes, Routledge*, New York.
2. Arce, Daniel G. and Sandler, Todd (2005): "Counterterrorism: A Game-Theoretic Analysis", *Journal of Conflict Resolution* 49 (2), 183-200.
3. Arce, Daniel G. and Sandler, Todd (2007): "Terrorist Signalling and the Value of Intelligence", *British Journal of Political Science* 37 (4), 573-586.
4. Bhan, Aditya and Tarun Kabiraj (2019): "Operational externalities and counter-terrorism." *Indian Economic Review* 54 (1): 171-187.
5. Bhattacharya, Pallab (2012): "Dhaka, Delhi Agree to Curb Drug Trafficking", *The Daily Star*, http://www.thedailystar.net/newDesign/news-details.php?nid=252651.
6. Byman, Daniel (2005): *Deadly Connections: States that Sponsor Terrorism*, Cambridge, UK, *Cambridge University Press*.
7. Byman, Daniel (2017): "Explaining Al Qaeda's Decline", *Journal of Politics* 79 (3), 1106-1117.
8. Byman, Daniel and Kreps, Sarah E. (2010): "Agents of Destruction? Applying Principal-Agent Analysis to State-Sponsored Terrorism", *International Studies Perspectives* 11, 1-18.
9. Chadha, Vivek (2015): *Lifeblood of Terrorism – Countering Terrorism Finance*, *Bloomsbury*.
10. Chowdhury, S.M. and R. M. Sheremeta. 2011. "A Generalized Tullock Contest." *Public Choice* 147 (3): 413-420.
11. *Council on Foreign Relations* (2009): "Abu Nidal Organization (ANO) aka Fatah Revolutionary Council, the Arab Revolutionary Brigades, or the Revolutionary Organization of Socialist Muslims", http://www.cfr.org/israel/abu-nidal-organization-ano-aka-fatah-revolutionary-council-arab-revolutionary-brigades-revolutionary-organization-socialist-muslims/p9153.
12. Das, Satya P. and Sajal Lahiri (2019): "Why Direct Counter-Terror (CT) Measures Only May Fail: An Analysis of Direct and Preventive CT Measures." *International Journal of Economic Theory*. Online publication: doi: https://doi.org/10.1111/ijet.12235.
13. Dixit, A. 1987. "Strategic Behavior in Contests." *American Economic Review* 77 (5): 891-898.
14. Gambill, Gary C. (2002): "Sponsoring Terrorism: Syria and Hamas", *Middle East Intelligence Bulletin* 4 (10), http://www.meforum.org/meib/articles/0210_s1.htm.
15. Ghumman, Khawar (2012): "Government Advised to Scrutinize NGO Funds", *Dawn*, http://dawn.com/2012/12/12/govt-advised-to-scrutinise-ngofinances/.
16. Gilling, Bryan (1992): *Be Ye Separate: Fundamentalism and the New Zealand Experience*, Red Beach, *Colcom Press*.
17. GlobalSecurity.org (2011): "Jammu and Kashmir Liberation Front (JKLF)", https://www.globalsecurity.org/military/world/para/jklf.htm.

18. Global Terrorism Index 2019: "Measuring the Impact of Terrorism. Sydney: Institute for Economics and Peace". http://visionofhumanity.org/app/uploads/2019/11/GTI-2019web.pdf.
19. Pratt, Douglas (2006): "Terrorism and Religious Fundamentalism: Prospects for a Predictive Paradigm", *Marburg Journal of Religion* 11 (1).
20. Sandler, Todd (2005): "Collective versus Unilateral Responses to Terrorism", *Public Choice* 124 (1/2), 75-93.
21. Sandler, Todd and Siqueira, Kevin (2006): "Global Terrorism: Deterrence versus Pre-Emption", *Canadian Journal of Economics* 39 (4), 1370-1387.
22. Siqueira, Kevin and Sandler, Todd (2006): "Terrorists versus the Government: Strategic Interaction, Support, and Sponsorship", *Journal of Conflict Resolution* 50 (6), 878-898.
23. *The Hindu* (2014): "Pakistan Aiding Terrorism: Rajnath", http://www.thehindu.com/news/national/dawood-is-in-afghan-border-rajnath/article6625240.ece.
24. Thurston, Alex (2017): "Boko Haram Is Weakening, but Its Decline Will Be Violent and Uneven", *World Politics Review* (March 14). https://www.worldpoliticsreview.com/articles/21517/boko-haram-is-weakening-but-its-decline-will-be-violent-and-uneven.
25. Tibi, Bassam (2002): "The Challenge of Fundamentalism: Political Islam and the New World Disorder", Updated Edition, Los Angeles, *University of California Press*.
26. Tullock, G. 1980. "Efficient Rent Seeking." in J.M. Buchanan, R..D Tollison and G. Tullock (eds.), *Toward a Theory of the Rent-Seeking Society*. College Station, TX. Texas. *A&M University Press*: 97-112.
27. *United Nations Office on Drugs and Crime* (2011): "The Global Afghan Opium Trade: A Threat Assessment", http://www.unodc.org/documents/data-and-analysis/Studies/Global_Afghan_Opium_Trade_2011-web.pdf, p.8.
28. United States Army (2015): "Threat Tactics Report: Boko Haram", *TRADOC G-2 ACE Threats Integration*.
29. Vembu, Venky (2011): "No compromise with terror: Indira's lesson for Manmohan", *Firstpost* (September 8). http://www.firstpost.com/politics/no-compromise-with-terror-indiras-lesson-for-manmohan-78935.html.
30. Walsh, Declan (2010): "Wikileaks cables portray Saudi Arabia as cash machine for terrorists", *The Guardian*, http://www.theguardian.com/world/2010/dec/05/wikileaks-cables-saudi-terrorist-funding.