Divisibility and indivisibility of labor supply, and involuntary unemployment: A perfect competition model

Tanaka, Yasuhito

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Yasuhito Tanaka[a,*]

[a] Faculty of Economics, Doshisha University.

Abstract

We show the existence of involuntary unemployment without assuming wage rigidity. We derive involuntary unemployment by considering utility maximization of consumers and profit maximization of firms in an overlapping generations model under perfect competition with decreasing or constant returns to scale technology. Indivisibility of labor supply may be a ground for the existence of involuntary unemployment. However, we show that under some conditions there exists involuntary unemployment even when labor supply is divisible.

Key Words: involuntary unemployment, perfect competition, divisible labor supply

JEL Codes: E12, E24.

[*] Corresponding author.
E-mail: yatanaka@mail.doshisha.ac.jp.
Address: Faculty of Economics, Doshisha University, Kamigyo-ku, Kyoto, 602-8580, Japan.
Phone number: +81-75-251-3648.
Fax: +81-75-251-3648.

1 Introduction

According to Otaki (2009) the definition of involuntary unemployment consists of two elements.

1. The nominal wage rate is set above the nominal reservation wage rate.
2. The employment level and economic welfare never improve by lowering the nominal wage rate.

Umada (1997) derived an upward-sloping labor demand curve from mark-up principle for firms under increasing returns to scale technology, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firms' behavior is ad-hoc. In this paper we consider utility maximization of consumers and profit maximization of firms in an overlapping generations model under perfect competition according to Otaki (2010), Otaki (2011) and Otaki (2015) with decreasing or constant returns to scale technology, and show the existence of involuntary unemployment without assuming wage rigidity. In some other papers we have shown the existence of involuntary unemployment under perfect or monopolistic competition when labor supply by individuals is indivisible.

Indivisibility of labor supply means that labor supply of each individual can be 1 or 0. On the other hand, if labor supply is divisible, it is a variable in $[0,1]$. As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if labor supply is infinitely divisible, there exists no unemployment. However, if labor supply by each individual is not so small, there may exist involuntary unemployment even when labor supply is divisible. In this paper the first element of Otaki's two elements of involuntary unemployment should be

Labor supply of each individual is positive at the current real wage rate.

In the next section we analyze consumers' utility maximization in an overlapping generations model with two periods. We consider labor supply by individuals as well as their consumptions. In Section 3 we consider profit maximization of firms under perfect competition. In Section 4 we show the existence of involuntary unemployment when labor supply is divisible.

2 Consumers

We consider a two-period (young and old) overlapping generations model under perfect competition according to Otaki (2010, 2011 and 2015). There is one factor of production, labor, and there is one good which is produced under perfect competition. There is a continuum of firms. The volume of firms is one. Consumers are born at continuous density $[0,1] \times [0,1]$ in each period. They supply l units of labor when they are young (the first period), $0 \leq l \leq 1$.

[1] Lavoie (2001) presented a similar analysis.

We use the following notations.

c_i: consumption the good at period i, $i = 1,2$.
p_i: price of the good at period i, $i = 1,2$.
W: nominal wage rate.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual.
L: number of employment of each firm and that of total employment.
L_f: population of labor or employment at the full-employment state.
$y(Ll)$: labor productivity, which is decreasing or constant with respect to "employment $\times$ labor supply (Ll)", $y(Ll) \geq 1$, $y' \leq 0$.

We define the elasticity of the labor productivity with respect to Ll as follows,

$$\zeta = \frac{y'}{\frac{y(Ll)}{Ll}}.$$

We assume that $-1 < \zeta \leq 0$, and ζ is constant. Decreasing (constant) returns to scale means $\zeta < 0$ ($\zeta = 0$). If the good is produced under constant returns to scale technology, $\Pi = 0$.

The utility of a representative consumer of one generation over two periods is

$$U(c_1, c_2) = u(c_1, c_2) - G(l).$$

We assume that $u(c_1, c_2)$ is homogeneous of degree one (linearly homogeneous). $G(l)$ is a function of disutility of labor which is continuous, strictly increasing, differentiable and strictly convex, thus $G' > 0$, $G'' > 0$.

The budget constraint for an employed individual is

$$p_1 c_1 + p_2 c_2 = Wl + \Pi.$$

p_2 is the expectation of the price at period 2. The Lagrange function is

$$\mathcal{L} = u(c_1, c_2) - G(l) - \lambda[p_1 c_1 + p_2 c_2 - (Wl + \Pi)].$$

λ is the Lagrange multiplier. The first order conditions are

$$\frac{\partial u}{\partial c_1} - \lambda p_1 = 0,$$

and

$$\frac{\partial u}{\partial c_2} - \lambda p_2 = 0.$$

They are rewritten as

$$\frac{\partial u}{\partial c_1} c_1 = \lambda p_1 c_1,$$

$$\frac{\partial u}{\partial c_2} c_2 = \lambda p_2 c_2.$$

Since $u(c_1, c_2)$ is homogeneous of degree one,

$$\frac{\partial u}{\partial c_1} c_1 + \frac{\partial u}{\partial c_2} c_2 = u(c_1, c_2) = \lambda(p_1 c_1 + p_2 c_2) = \lambda(Wl + \Pi),$$

Also, since $u(c_1, c_2)$ is homogeneous of degree one, λ is a function of p_1 and p_2, and $\frac{1}{\lambda}$ is homogeneous of degree one because proportional increases in p_1 and p_2 reduce c_1

and c_2 at the same rate given $Wl+\Pi$. Then, we obtain the following indirect utility function.

$$V=\frac{1}{\varphi(p_1,p_2)}(Wl+\Pi)-G(l).$$

$\varphi(p_1,p_2)$ is a function of p_1 and p_2. It is positive, increasing in p_1 and p_2[2], and homogeneous of degree one. Maximization of V with respect to l implies

$$W=\varphi(p_1,p_2)G'(l). \tag{1}$$

Let $\rho=\frac{p_2}{p_1}$. From (1)

$$\omega=\frac{W}{p_1}=\varphi(1,\rho)G'(l). \tag{2}$$

ω is the real wage rate. If the value of ρ is given, l is obtained from (2) as a function of ω. Since $G''>0$, labor supply l is increasing in the real wage rate ω.

For an unemployed individual the indirect utility is

$$\frac{1}{\varphi(p_1,p_2)}\Pi.$$

3 Firms

Let

$$\alpha=\frac{p_1c_1}{Wl+\Pi},$$

and

$$1-\alpha=\frac{p_2c_2}{Wl+\Pi}.$$

We have $0<\alpha<1$. Demand for the good of consumers of younger generation is

$$c_1=\frac{\alpha(Wl+\Pi)}{p_1}.$$

His demand in the second period is

$$c_2=\frac{(1-\alpha)(Wl+\Pi)}{p_2}.$$

Let $\bar{c}_2$, $\bar{l}$, be demand for the good and labor supply of an older generation consumer, $\bar{W}$ and $\bar{\Pi}$ be the nominal wage rate and the profit in his first period. Then

$$\bar{c}_2=\frac{(1-\alpha)(\bar{W}\bar{l}+\bar{\Pi})}{p_1}.$$

$(1-\alpha)(\bar{W}\bar{l}+\bar{\Pi})$ is his saving carried over from his first period. Let M be the income of an individual of older generation including the saving. Then, his demand for the good is

$$\frac{M}{p_1}.$$

The government expenditure as well as consumptions of younger and older generations constitutes the national income. The total demand for the good is

$$c=\frac{Y}{p_1}.$$

Y is the effective demand defined by

$$Y=\alpha(WLl+\Pi)+G+M.$$

G is the government expenditure (about this demand function please see Otaki (2007), Otaki

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[2] λ is decreasing in p_1 and p_2.

(2009), Otaki (2015)).

Let x and z be the output and "employment $\times$ labor supply" of a firm. We have $x = y(z)z$ and

$$\zeta = \frac{y'}{\frac{y(z)}{z}}.$$

Thus,

$$\frac{dz}{dx} = \frac{1}{y(z) + y'z} = \frac{1}{(1+\zeta)y(z)}.$$

The profit of a firm is

$$\pi = p_1 x - \frac{x}{y(z)} W.$$

p_1 is given for each firm. The condition for profit maximization under perfect competition is

$$p_1 - \frac{y(z) - xy'\frac{dz}{dx}}{y(z)^2} W = p_1 - \frac{1 - y'z\frac{dz}{dx}}{y(z)} W = p_1 - \frac{1}{y(z)+y'z} W = p_1 - \frac{1}{(1+\zeta)y(z)} W = 0.$$

Therefore,

$$p_1 = \frac{1}{(1+\zeta)y(z)} W.$$

This means the marginal cost pricing. Since at the equilibrium $x = c$ and $z = Ll$, we obtain

$$p_1 = \frac{1}{(1+\zeta)y(Ll)} W. \quad (3)$$

With decreasing (constant) returns to scale $-1 < \zeta < 0$ ($\zeta = 0$).

4 Involuntary unemployment

From (3) the real wage rate is

$$\omega = \frac{W}{p_1} = (1+\zeta)y(Ll). \quad (4)$$

It is determined by firms' behavior. Under decreasing (constant) returns to scale, since ζ is constant, ω is decreasing (constant) with respect to Ll. From (2) and (4) we get

$$(1+\zeta)y(Ll) = \varphi(1,\rho)G'(l). \quad (5)$$

From (5) labor supply of an individual l is obtained as a function of L. Denote it by $l(L)$. Since $G'' > 0$ (convex disutility of labor) and $y' \leq 0$ (decreasing or constant returns to scale), we have

$$\varphi(1,\rho)G''(l) - (1+\zeta)y'L > 0.$$

This guarantees that $l(L)$ is decreasing and $Ll(L)$ is strictly increasing with respect to L because

$$\frac{dl(L)}{dL} = \frac{(1+\zeta)y'l(L)}{\varphi(1,\rho)G''(l) - (1+\zeta)y'L} \leq 0,$$

and

$$\frac{dLl(L)}{dL} = \frac{\varphi(1,\rho)G''(l)l(L)}{\varphi(1,\rho)G''(l) - (1+\zeta)y'L} > 0.$$

Then, the real wage rate ω is decreasing in L because $y' < 0$.

Alternatively, from (5) l is obtained as a function of Ll. Denote it by $l(Ll)$. Then,

$$\frac{dl(Ll)}{d(Ll)} = \frac{(1+\zeta)y'}{\varphi(1,\rho)G''} < 0.$$

The aggregate supply of the good is equal to

$$WLl + \Pi = p_1 Lly(Ll).$$
Ll is an abbreviation of $Ll(L)$ or $Ll(Ll)$. The aggregate demand is
$$\alpha(WLl + \Pi) + G + M = \alpha p_1 Lly(Ll) + G + M.$$
Since they are equal,
$$p_1 Lly(Ll) = \alpha p_1 Lly(Ll) + G + M,$$
or
$$p_1 Lly(Ll) = \frac{G+M}{1-\alpha}.$$
In real terms[3]
$$Lly(Ll) = \frac{1}{1-\alpha}(g+m), \quad (6)$$
or
$$Ll = \frac{1}{(1-\alpha)y(Ll)}(g+m),$$
where
$$g = \frac{G}{p_1},\ m = \frac{M}{p_1}.$$
(6) means that "employment $\times$ labor supply" Ll is determined by $g+m$. $Lly(Ll)$ is strictly increasing in Ll because
$$\frac{d(Lly(Ll))}{d(Ll)} = y(Ll) + Lly' = y(Ll)\left(1 + \frac{Lly\prime}{y(Ll)}\right) = y(Ll)(1+\zeta) > 0.$$
Therefore, there exists the unique value of Ll which satisfies (6) given $g+m$. It is strictly increasing in $g+m$. From (5) we obtain the value of $l(Ll)$, and the value of L is determined by $L = \frac{Ll}{l(Ll)}$. Ll can not be larger than $L_f l(L_f)$. However, it may be strictly smaller than $L_f l(L_f)$. Then, there exists *involuntary umemployment*, that is, $L < L_f$ because Ll is strictly increasing in L.

If we consider the following budget constraint for the government with a lump-sum tax T on the younger generation consumers,
$$G = T,$$
the aggregate demand and the aggregate supply are
$$\alpha(WLl + \Pi - G) + G + M = \alpha(p_1 Lly(Ll) - G) + G + M = p_1 Lly(Ll).$$
Then, we get[4]
$$Ll = \frac{1}{(1-\alpha)y(Ll)}[(1-\alpha)g + m],$$
If labor supply of each individual is small, there exists no unemployment. If it is not small, however, it is likely that there exists involuntary unemployment without sufficiently large value of $g+m$.

If
$$(1+\zeta)y(Ll) > \varphi(1,\rho)G'(l) \text{ for any } 0 < l < 1, \text{given } L,$$
individuals choose $l = 1$, and then the labor supply is indivisible.

On the other hand, if
$$\lim_{Ll\to 0}(1+\zeta)y(Ll) < \varphi(1,\rho)G'(0),$$

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[3] $\frac{1}{1-\alpha}$ is a multiplier.

[4] This equation means that the balanced budget multiplier is 1.

individuals choose $l = 0$. However, if $G'(0)$ is sufficiently small, $l > 0$.

Summary of discussions

The real aggregate demand and "employment $\times$ labor supply" (Ll) are determined by the real value of $g + m$. Labor supply of each individual is determined by Ll according to (5), and the employment L is determined by

$$L = \frac{Ll}{l(Ll)}.$$

The employment may be smaller than the population of labor, then there exists involuntary unemployment. The real wage rate is determined by Ll according to (4). There exists no mechanism to reduce involuntary unemployment unless $g + m$ is increased.

Comment on the nominal wage rate

The reduction of the nominal wage rate induces a proportionate reduction of the price even when there exists involuntary unemployment, and it does not rescue involuntary unemployment (please see Chapter 2 of Otaki (2016)[5].

In the model of this section no mechanism determines the nominal wage rate. When the nominal value of $G + M$ increases, the nominal aggregate demand and supply increase. If the nominal wage rate rises, the price also rises. If the rate of an increase in the nominal wage rate is smaller than the rate of an increase in $G + M$, the real aggregate supply and the employment increase. Partition of the effects by an increase in $G + M$ into a rise in the nominal wage rate (and the price) and an increase in the employment may be determined by bargaining between labor and firm[6].

Full-employment case

If $L = L_f$, full-employment is realized. Then, (6) is written as

$$L_f l(L_f) y(L_f l(L_f)) = \frac{1}{1-\alpha}(g + m). \quad (7)$$

$l(L_f)$ is obtained from

$$(1 + \zeta) y(L_f l) = \varphi(1, \rho) G'(l).$$

$L_f l(L_f) > Ll(L)$ for any $L < L_f$ because $Ll(L)$ is strictly increasing in L. Since $L_f l(L_f)$ is constant, (7) is an identity not an equation. On the other hand, (6) is an equation not an identity. (7) should be written as

$$L_f l(L_f) y(L_f l(L_f)) \equiv \frac{1}{1-\alpha}(g + m). \quad (8)$$

This defines the value of $g + m$ which realizes the full-employment state.

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[5] However, there is room for improvement of employment by the real balance effect if the nominal values of government expenditure and consumption by older generation are maintained.

[6] Otaki (2009) has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and R. M. Solow (1981). The arguments of this paper, however, do not depend on bargaining.

From (8) we have

$$p_1 = \frac{1}{(1-\alpha)L_f l(L_f) y(L_f l(L_f))}(G+M),$$

where

$$g = \frac{G}{p_1},\ m = \frac{M}{p_1}.$$

Therefore, the price level p_1 is determined by $G+M$, which is the sum of nominal values of government expenditure and consumption by older generation. Also the nominal wage rate is determined by

$$W = (1+\zeta)y(L_f l(L_f))p_1.$$

Steady state

At the steady state $\rho = 1$. If $g+m$ is constant, the employment is constant.

5 Concluding Remark

In this paper we have examined the existence of involuntary umemployment using a simple perfect competition model with decreasing or constant returns to scale technology. It seems to be possible to extend the analyses in this paper to monopolistic competition with increasing or constant returns to scale technology without changing main conclusions.

References

Lavoie, M. Efficiency wages in Kaleckian models of employment. *Journal of Post Keynesian Economics*, 23:449–464, 2001.

McDonald, I. M. and R. M. Solow. Wage barganing and employment. *American Economic Review*, 71:896–908, 1981.

Otaki, M. The dynamically extended Keynesian cross and the welfare-improving fiscal policy. *Economics Letters*, 96:23–29, 2007.

Otaki, M. A welfare economics foundation for the full-employment policy. *Economics Letters*, 102:1–3, 2009.

Otaki, M. *A pure theory of aggregate price determination*. DBJ Discussion Paper Series, No. 906, 2010.

Otaki, M. *Fundamentals of the Theory of Money and Employment (Kahei-Koyo Riron no Kiso (in Japanese))*. Keiso Shobo, 2011.

Otaki, M. *The Aggregation problem in employmnet theory*. DBJ Discussion Paper Series, No. 1105, 2012.

Otaki, M. *Keynsian Economics and Price Theory: Re-orientation of a Theory of Monetary Economy*. Springer, 2015.

Otaki, M. *Keynes's general theory reconsidered in the context of the Japanese economy*. Springer, 2016.

Umada, T. On the existence of involuntary unemployment (hi-jihatsuteki-shitsugyo no sonzai ni tsuite (in japanese)). *Yamaguchi Keizaigaku Zasshi*, 45:61–73, 1997.