Divisibility and indivisibility of labor supply, and involuntary unemployment: A monopolistic competition model with homothetic preferences

Tanaka, Yasuhito

31 January 2020

manuscript No.
(will be inserted by the editor)

Divisibility and indivisibility of labor supply, and involuntary unemployment: A monopolistic competition model with homothetic preferences

Yasuhito Tanaka

Abstract We show the existence of involuntary unemployment without assuming wage rigidity. We derive involuntary unemployment by considering utility maximization of consumers and profit maximization of firms in an overlapping generations model under monopolistic competition with increasing or constant returns to scale technology and homothetic preferences of consumers. Indivisibility of labor supply may be a ground for the existence of involuntary unemployment. However, we show that under some conditions there exists involuntary unemployment even when labor supply is divisible.

Keywords involuntary unemployment, monopolistic competition, divisible labor supply.

JEL Classification No.: E12, E24.

1 Introduction

According to Otaki (2009) the definition of involuntary unemployment consists of two elements.

1. The nominal wage rate is set above the nominal reservation wage rate.
2. The employment level and economic welfare never improve by lowering the nominal wage rate.

Umada (1997) derived an upward-sloping labor demand curve from mark-up principle for firms under increasing returns to scale technology, and argued that such an upward-sloping labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firms' behavior is ad-hoc. In this paper we consider utility maximization of consumers and profit maximization of firms in an overlapping generations model under monopolistic competition according to Otaki (2007), Otaki (2009), Otaki (2011) and Otaki (2015) with increasing or constant returns to scale technology and homothetic preferences of consumers, and show the existence of involuntary unemployment without

This work was supported by Japan Society for the Promotion of Science KAKENHI Grant Number 18K01594.

Yasuhito Tanaka
Faculty of Economics, Doshisha University, Kamigyo-ku, Kyoto, 602-8580, Japan.
E-mail: yatanaka@mail.doshisha.ac.jp

[1] Lavoie (2001) presented a similar analysis.

assuming wage rigidity. In some other papers we have shown the existence of involuntary unemployment under perfect or monopolistic competition when labor supply by individuals is indivisible[2].

Indivisibility of labor supply means that labor supply of each individual can be 1 or 0. On the other hand, if labor supply is divisible, it is a variable in $[0, 1]$. As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if labor supply is infinitely divisible, there exists no unemployment. However, if labor supply by each individual is not so small, there may exist involuntary unemployment even when labor supply is divisible. In this paper the first element of Otaki's two elements of involuntary unemployment should be

Labor supply of each individual is positive at the current real wage rate.

In the next section we analyze consumers' utility maximization in an overlapping generations model with two periods. We consider labor supply by individuals as well as their consumptions. In Section 3 we consider profit maximization of firms under monopolistic competition. In Section 4 we show the existence of involuntary unemployment when labor supply is divisible.

2 Consumers

We consider a two-period (young and old) overlapping generations model under monopolistic competition according to Otaki (2007, 2009, 2011 and 2015). There is one factor of production, labor, and there is a continuum of goods indexed by $z \in [0, 1]$. Each good is monopolistically produced by Firm z. Consumers are born at continuous density $[0, 1] \times [0, 1]$ in each period. They supply l units of labor when they are young (the first period), $0 \leq l \leq 1$.

We use the following notations.

$c^i(z)$: consumption of good z at period i, $i = 1, 2$.
$p^i(z)$: the price of good z at period i, $i = 1, 2$.
$X^i = \left\{\int_0^1 c^i(z)^{1-\frac{1}{\eta}} dz\right\}^{\frac{1}{1-\frac{1}{\eta}}}$, $i = 1, 2$, $\eta > 1$.
W: nominal wage rate.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual.
L: employment of each firm and the total employment.
L_f: population of labor or employment at the full-employment state.
$y(Ll)$: labor productivity, which is increasing or constant with respect to "employment $\times$ labor supply (Ll)", $y(Ll) \geq 1$, $y' \geq 0$.

We define the elasticity of the labor productivity with respect to Ll as follows.

$$\zeta = \frac{y'}{\frac{y(Ll)}{Ll}}.$$

We assume that $0 \leq \zeta < 1$ and it is constant. Increasing returns to scale means $\zeta > 0$.

η is (the inverse of) the degree of differentiation of the goods. At the limit when $\eta \to +\infty$, the goods are homogeneous. We assume

$$\left(1 - \frac{1}{\eta}\right)(1 + \zeta) < 1$$

[2] Tanaka (2019a), Tanaka (2019b), Tanaka (2019c).

so that the profits of firms are positive.

We assume that the utility function a consumer is homothetic. This means that his utility function is a strictly monotonic transformation of a function which is homogeneous of degree one. The utility of consumers of one generation over two periods is

$$U(X^1, X^2, l) = F(u(X^1, X^2)) - G(l).$$

F is a strictly increasing and differentiable function, thus $F' > 0$. $u(X^1, X^2)$ is homogeneous of degree one. $G(l)$ is disutility of labor. It is continuous, strictly increasing, differentiable and strictly convex, thus $G' > 0$ and $G'' > 0$.

The budget constraint is

$$\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz = Wl + \Pi.$$

$p^2(z)$ is the expectation of the price of good z at period 2. The Lagrange function is

$$\mathcal{L} = F(u(X^1, X^2)) - G(l) - \lambda\left(\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz - Wl - \Pi\right).$$

λ is the Lagrange multiplier. The first order conditions are

$$F'\frac{\partial u}{\partial X^1}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^1(z)^{-\frac{1}{\eta}} = \lambda p^1(z), \tag{1}$$

and

$$F'\frac{\partial u}{\partial X^2}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^2(z)^{-\frac{1}{\eta}} = \lambda p^2(z). \tag{2}$$

They are rewritten as

$$F'\frac{\partial u}{\partial X^1}X^1\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1} c^1(z)^{1-\frac{1}{\eta}} = \lambda p^1(z)c^1(z), \tag{3}$$

$$F'\frac{\partial u}{\partial X^2}X^2\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1} c^2(z)^{1-\frac{1}{\eta}} = \lambda p^2(z)c^2(z). \tag{4}$$

Let

$$P^1 = \left(\int_0^1 p^1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}},\ P^2 = \left(\int_0^1 p^2(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}.$$

They are price indices. By some calculations we obtain (please see Appendix)

$$u(X^1, X^2) = \left(\frac{\lambda}{F'}\right)\left[\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz\right] = \left(\frac{\lambda}{F'}\right)(Wl + \Pi), \tag{5}$$

$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}}, \tag{6}$$

$$P^1X^1 + P^2X^2 = Wl + \Pi. \tag{7}$$

The indirect utility of consumers is written as follows

$$V = F\left(\frac{Wl + \Pi}{\varphi(P^1, P^2)}\right) - G(l). \tag{8}$$

$\varphi(P^1, P^2)$ is a function of P^1 and P^2. It is positive, increasing in P^1 and P^2, and homogeneous of degree one. Maximization of V with respect to l implies

$$F'W = \varphi(P^1, P^2)G'(l). \tag{9}$$

Let $\rho = \frac{P^2}{P^1}$. From (9)

$$F'\omega = F'\frac{W}{P^1} = \varphi(1, \rho)G'(l). \tag{10}$$

ω is the real wage rate. F' is a function of $\frac{Wl+\Pi}{\varphi(P^1,P^2)}$ such that

$$F' = F'\left(\frac{Wl + \Pi}{\varphi(P^1, P^2)}\right) = F'\left(\frac{\omega l + \pi}{\varphi(1, \rho)}\right),$$

where

$$\pi = \frac{\Pi}{P^1}.$$

If the value of ρ is given, l is obtained from (10) as a function of ω.

From (10)

$$\frac{dl}{d\omega} = \frac{F' + F''\frac{\omega l}{\varphi(1,\rho)}}{\varphi(1, \rho)G'' - F''\frac{\omega^2}{\varphi(1,\rho)}}.$$

We assume

$$\varphi(1, \rho)G'' - F''\frac{\omega^2}{\varphi(1, \rho)} > 0,$$

and

$$F' + F''\frac{\omega l}{\varphi(1, \rho)} > 0.$$

Then, $\frac{dl}{d\omega} > 0$, and labor supply l is increasing in the real wage rate ω. If $F(u(X^1, X^2))$ is homogeneous of degree one, $F' = 1$ and $F'' = 0$.

For an unemployed individual the indirect utility is

$$F\left(\frac{\Pi}{\varphi(P^1, P^2)}\right).$$

3 Firms

Let

$$\alpha=\frac{P^1X^1}{P^1X^1+P^2X^2}=\frac{X^1}{X^1+\rho X^2},\ 0<\alpha<1.$$

From (3) ~ (7),

$$\alpha(Wl+\Pi)\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c^1(z)^{-\frac{1}{\eta}}=p^1(z).$$

Since

$$X^1=\frac{\alpha(Wl+\Pi)}{P^1},$$

we have

$$\left(X^1\right)^{\frac{1}{\eta}-1}=\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}=\left(\frac{\alpha(Wl+\Pi)}{P^1}\right)^{\frac{1}{\eta}-1}.$$

Therefore,

$$\alpha(Wl+\Pi)\left(\frac{\alpha(Wl+\Pi)}{P^1}\right)^{\frac{1}{\eta}-1}c^1(z)^{-\frac{1}{\eta}}=\left(\frac{\alpha(Wl+\Pi)}{P^1}\right)^{\frac{1}{\eta}}P^1c^1(z)^{-\frac{1}{\eta}}=p^1(z).$$

Thus,

$$c^1(z)^{\frac{1}{\eta}}=\left(\frac{\alpha(Wl+\Pi)}{P^1}\right)^{\frac{1}{\eta}}P^1\left(p^1(z)\right)^{-1}.$$

Hence,

$$c^1(z)=\frac{\alpha(Wl+\Pi)}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

This is demand for good z of an individual of younger generation. Similarly, his demand for good z in the second period is

$$c^2(z)=\frac{(1-\alpha)(Wl+\Pi)}{P^2}\left(\frac{p^2(z)}{P^2}\right)^{-\eta}.$$

Let $\bar{c}_2(z)$, $\bar{l}$, be demand for good z and labor supply of an older generation consumer, $\bar{W}$ and $\bar{\Pi}$ be the nominal wage rate and the profit in his first period. Then

$$\bar{c}^2(z)=\frac{(1-\alpha)(\bar{W}\bar{l}+\bar{\Pi})}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

$(1-\alpha)(\bar{W}\bar{l}+\bar{\Pi})$ is his saving carried over from his first period. Let M be the saving. Then, his demand for good z is

$$\frac{M}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

The government expenditure constitutes the national income as well as consumptions of younger and older generations. The total demand for good z is written as

$$c(z)=\frac{Y}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

Y is the effective demand defined by

$$Y = \alpha(WLl + \Pi) + G + M.$$

G is the government expenditure (about this demand function please see Otaki (2007), Otaki (2009)). The total employment, the total profits, the total government expenditure and the total consumption by the older generation are

$$\int_0^1 Ldz = L,\ \int_0^1 \Pi dz = \Pi,\ \int_0^1 Gdz = G,\ \int_0^1 Mdz = M.$$

We have

$$\frac{\partial c(z)}{\partial p^1(z)} = -\eta \frac{Y}{P^1}\frac{p^1(z)^{-1-\eta}}{\left(P^1\right)^{-\eta}} = -\eta \frac{c(z)}{p^1(z)}.$$

From $c(z) = Lly(Ll)$,

$$\frac{\partial(Ll)}{\partial p^1(z)} = \frac{1}{y(Ll) + Lly'}\frac{\partial c(z)}{\partial p^1(z)}.$$

The profit of Firm z is

$$\pi(z) = p^1(z)c(z) - \frac{W}{y(Ll)}c(z).$$

P^1 is given for Firm z. $y(Ll)$ is the productivity of labor, which is increasing with respect to Ll.

The elasticity of the labor productivity with respect to Ll is

$$\zeta = \frac{y'}{\frac{y(Ll)}{Ll}}.$$

The condition for profit maximization with respect to $p^1(z)$ is

$$\begin{aligned}
&c(z) + \left[p^1(z) - \frac{y(Ll) - c(z)y'\frac{1}{y(Ll)+Lly'}}{y(Ll)^2}W\right]\frac{\partial c(z)}{\partial p^1(z)} \\
=&c(z) + \left[p^1(z) - \frac{1 - Lly'\frac{1}{y(Ll)+Lly'}}{y(Ll)}W\right]\frac{\partial c(z)}{\partial p^1(z)} \\
=&c(z) + \left[p^1(z) - \frac{W}{y(Ll) + Lly'}\right]\frac{\partial c(z)}{\partial p^1(z)} = 0.
\end{aligned}$$

From this

$$p^1(z) = \frac{W}{y(Ll) + Lly'} - \frac{c(z)}{\frac{\partial c(z)}{\partial p^1(z)}} = \frac{W}{(1+\zeta)y(Ll)} + \frac{1}{\eta}p^1(z).$$

Therefore, we obtain

$$p^1(z) = \frac{W}{\left(1 - \frac{1}{\eta}\right)(1+\zeta)y(Ll)}.$$

With increasing returns to scale, since $\zeta > 0$, $p^1(z)$ is lower than that in a case of constant returns to scale given the value of W.

4 Involuntary unemployment

Since the model is symmetric, the prices of all goods are equal. Then,

$$P^1 = p^1(z).$$

Hence

$$P^1 = \frac{W}{\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)}.$$

The real wage rate is

$$\omega = \frac{W}{P^1} = \left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll). \tag{11}$$

It is determined by firms' behavior. Under increasing (constant) returns to scale, since ζ is constant, ω is increasing (constant) with respect to Ll. From (10) and (11) we get

$$F'\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll) = \varphi(1,\rho)G'(l). \tag{12}$$

From (12) labor supply of an individual is obtained as a function of L. Denote it by $l(L)$. We assume

$$\varphi(1,\rho)G''(l) - F'\left(1-\frac{1}{\eta}\right)(1+\zeta)y'L - F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{\omega}{\varphi(1,\rho)} > 0. \tag{13}$$

This means

$$\varphi(1,\rho)G''(l) - F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{\omega}{\varphi(1,\rho)} > 0. \tag{14}$$

(13) and (14) guarantee that $l(L)$ is increasing and $Ll(L)$ is strictly increasing with respect to L because

$$\frac{dl(L)}{dL} = \frac{F'\left(1-\frac{1}{\eta}\right)(1+\zeta)y'l(L)}{\varphi(1,\rho)G''(l) - F'\left(1-\frac{1}{\eta}\right)(1+\zeta)y'L - F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{\omega}{\varphi(1,\rho)}} \geq 0,$$

and

$$\frac{d(Ll(L))}{dL} = l(L)+L\frac{dl(L)}{dL} = \frac{\left[\varphi(1,\rho)G''(l) - F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{\omega}{\varphi(1,\rho)}\right]l(L)}{\varphi(1,\rho)G''(l) - F'\left(1-\frac{1}{\eta}\right)(1+\zeta)y'L - F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{\omega}{\varphi(1,\rho)}} > 0.$$

Then, the real wage rate ω is increasing in L because $y' \geq 0$.

Alternatively, from (12) l is obtained as a function of Ll. Denote it by $l(Ll)$. Then,

$$\frac{dl(Ll)}{d(Ll)} = \frac{F'\left(1-\frac{1}{\eta}\right)(1+\zeta)y'}{\varphi(1,\rho)G'' - F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{\omega}{\varphi(1,\rho)}} \geq 0.$$

The aggregate supply of the good is equal to

$$WLl + \Pi = P^1 Lly(Ll).$$

Ll is an abbreviation of $Ll(L)$ or $Ll(Ll)$. The aggregate demand is

$$\alpha(WLl + \Pi) + G + M = \alpha P^1 Lly(Ll) + G + M.$$

Since they are equal,

$$P^1 Lly(Ll) = \alpha P^1 Lly(Ll) + G + M,$$

or

$$P^1 Lly(Ll) = \frac{G + M}{1 - \alpha}.$$

In real terms[3]

$$Lly(Ll) = \frac{1}{1-\alpha}(g + m), \tag{15}$$

or

$$Ll = \frac{1}{(1-\alpha)y(Ll)}(g + m),$$

where

$$g = \frac{G}{P^1},\ m = \frac{M}{P^1}.$$

(15) means that "employment × labor supply" Ll is determined by $g + m$. $Lly(Ll)$ is strictly increasing in Ll because

$$\frac{d(Lly(Ll))}{d(Ll)} = y(Ll) + Lly' = y(Ll)\left(1 + \frac{Lly'}{y(Ll)}\right) = y(Ll)(1+\zeta) > 0.$$

Therefore, there exists the unique value of Ll which satisfies (15) given $g + m$. It is strictly increasing in $g + m$. From (12) we obtain the value of $l(Ll)$, and the value of L is determined by $L = \frac{Ll}{l(Ll)}$. Ll can not be larger than $L_f l(L_f)$. However, it may be strictly smaller than $L_f l(L_f)$. Then, there exists *involuntary umemployment*, that is, $L < L_f$ because Ll is strictly increasing in L.

If we consider the following budget constraint for the government with a lump-sum tax T on the younger generation consumers,

$$G = T,$$

the aggregate demand and the aggregate supply are

$$\alpha(WLl + \Pi - G) + G + M = \alpha(P^1 Lly(Ll) - G) + G + M = P^1 Lly(Ll).$$

Then, we get[4]

$$Ll = \frac{1}{(1-\alpha)y(Ll)}[(1-\alpha)g + m],$$

If labor supply of each individual is small, there exists no unemployment. If it is not so small, however, it is likely that there exists involuntary unemployment without sufficiently large value of $g + m$.

If

$$F'\left(1 - \frac{1}{\eta}\right)(1+\zeta)\, y\,(Ll) > \varphi(1,\rho)G'\,(l) \text{ for any } 0 < l < 1, \text{given } L,$$

individuals choose $l = 1$, and then the labor supply is indivisible.

On the other hand, if

$$F'\left(1 - \frac{1}{\eta}\right)\lim_{Ll\to 0}(1+\zeta)y(Ll) < \varphi(1,\rho)G'(0),$$

individuals choose $l = 0$. However, if $G'(0)$ is sufficiently small, $l > 0$.

[3] $\frac{1}{1-\alpha}$ is a multiplier.

[4] This equation means that the balanced budget multiplier is 1.

Summary of discussions

1. The real aggregate demand and "employment $\times$ labor supply" (Ll) are determined by the value of $g + m$ according to (15).
2. Labor supply of each individual is determined by Ll according to (12).
3. The employment L is determined by

$$L = \frac{Ll}{l(Ll)}.$$

   The employment may be smaller than the population of labor, then there exists involuntary unemployment.
4. The real wage rate is determined by Ll according to (11).

There exists no mechanism to reduce involuntary unemployment unless $g + m$ is increased.

Comment on the nominal wage rate The reduction of the nominal wage rate induces a proportionate reduction of the price even when there exists involuntary unemployment, and it does not rescue involuntary unemployment (please see Chapter 2 of Otaki (2016)[5].

In the model of this section no mechanism determines the nominal wage rate. When the nominal value of $G + M$ increases, the nominal aggregate demand and supply increase. If the nominal wage rate rises, the price also rises. If the rate of an increase in the nominal wage rate is smaller than the rate of an increase in $G + M$, the real aggregate supply and the employment increase. Partition of the effects by an increase in $G + M$ into a rise in the nominal wage rate (and the price) and an increase in the employment may be determined by bargaining between labor and firm[6].

Full-employment case If $L = L_f$, full-employment is realized. Then, (15) is written as

$$L_f l(L_f) y(L_f l(L_f)) = \frac{1}{1-\alpha}(g + m). \tag{16}$$

$l(L_f)$ is obtained from

$$(1 + \zeta) y(L_f l) = \varphi(1, \rho) G'(l).$$

$L_f l(L_f) > Ll(L)$ for any $L < L_f$ because $Ll(L)$ is strictly increasing in L. Since $L_f l(L_f)$ is constant, (16) is an identity not an equation. On the other hand, (15) is an equation not an identity. (16) should be written as

$$L_f l(L_f) y(L_f l(L_f)) \equiv \frac{1}{1-\alpha}(g + m). \tag{17}$$

This defines the value of $g + m$ which realizes the full-employment state.

From (17) we have

$$P^1 = \frac{1}{(1-\alpha) L_f l(L_f) y(L_f l(L_f))}(G + M),$$

where

$$g = \frac{G}{P^1},\ m = \frac{M}{P^1}.$$

[5] However, there is room for improvement of employment if the nominal values of government expenditure and consumption by older generation are maintained when the nominal wage rate is reduced.

[6] Otaki (2009) has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow (1981). The arguments of this paper, however, do not depend on bargaining.

Therefore, the price level P^1 is determined by $G+M$, which is the sum of nominal values of government expenditure and consumption by older generation. Also the nominal wage rate is determined by

$$W=(1+\zeta)y(L_f l(L_f))P^1.$$

Steady state At the steady state $\rho=1$. If $g+m$ is constant, the employment is constant.

5 Concluding Remark

In this paper we have examined the existence of involuntary umemployment using a monopolistic competition model with increasing or constant returns to scale technology and homothetic preferences of consumers. It is a limited assumption that the goods are produced by only labor. The analysis of a case where the goods are produced by capital and labor is one of themes of future researches.

Appendix: Derivations of (5), (6), (7) and (8)

From (3) and (4)

$$\frac{\partial u}{\partial X^1}X^1\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\frac{\partial u}{\partial X^1}X^1=\frac{\lambda}{F'}\int_0^1 p^1(z)c^1(z)dz,$$

$$\frac{\partial u}{\partial X^2}X^2\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz=\frac{\partial u}{\partial X^2}X^2=\frac{\lambda}{F'}\int_0^1 p^2(z)c^2(z)dz.$$

Since $u(X^1,X^2)$ is homogeneous of degree one,

$$u(X^1,X^2)=\frac{\partial u}{\partial X^1}X^1+\frac{\partial u}{\partial X^2}X^2.$$

Thus, we obtain

$$\frac{\int_0^1 p^1(z)c^1(z)dz}{\int_0^1 p^2(z)c^2(z)dz}=\frac{\frac{\partial u}{\partial X^1}X^1}{\frac{\partial u}{\partial X^2}X^2},$$

and

$$u(X^1,X^2)=\frac{\lambda}{F'}\left[\int_0^1 p^1(z)c^1(z)dz+\int_0^1 p^2(z)c^2(z)dz\right]=\frac{\lambda}{F'}(Wl+\Pi). \tag{5}$$

From (1) and (2), we have

$$\left(\frac{\partial u}{\partial X^1}\right)^{1-\eta}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c^1(z)^{1-\frac{1}{\eta}}=\left(\frac{\lambda}{F'}\right)^{1-\eta}p^1(z)^{1-\eta},$$

and

$$\left(\frac{\partial u}{\partial X^2}\right)^{1-\eta}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c^2(z)^{1-\frac{1}{\eta}}=\left(\frac{\lambda}{F'}\right)^{1-\eta}p^2(z)^{1-\eta}.$$

They mean

$$\left(\frac{\partial u}{\partial X^1}\right)^{1-\eta}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\left(\frac{\lambda}{F'}\right)^{1-\eta}\int_0^1 p^1(z)^{1-\eta}dz,$$

and

$$\left(\frac{\partial u}{\partial X^2}\right)^{1-\eta}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz=\left(\frac{\lambda}{F'}\right)^{1-\eta}\int_0^1 p^2(z)^{1-\eta}dz.$$

Note that

$$F' = F'(u(X^1, X^2)),$$

and

$$X^1 = \left\{ \int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right\}^{\frac{1}{1-\frac{1}{\eta}}},\ X^2 = \left\{ \int_0^1 c^2(z)^{1-\frac{1}{\eta}} dz \right\}^{\frac{1}{1-\frac{1}{\eta}}}.$$

Then, we obtain

$$\frac{\partial u}{\partial X^1} = \left(\frac{\lambda}{F'}\right) \left(\int_0^1 p^1(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}} = \left(\frac{\lambda}{F'}\right) P^1,$$

and

$$\frac{\partial u}{\partial X^2} = \left(\frac{\lambda}{F'}\right) \left(\int_0^1 p^2(z)^{1-\eta} dz \right)^{\frac{1}{1-\eta}} = \left(\frac{\lambda}{F'}\right) P^2.$$

From them we get

$$u(X^1, X^2) = \left(\frac{\lambda}{F'}\right) (P^1 X^1 + P^2 X^2),$$

$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}}, \tag{6}$$

and

$$P^1 X^1 + P^2 X^2 = Wl + \Pi. \tag{7}$$

Since $u(X^1, X^2)$ is homogeneous of degree one, $\frac{\lambda}{F'}$ is a function of P^1 and P^2, and $\frac{F'}{\lambda}$ is homogeneous of degree one because proportional increases in P^1 and P^2 reduce X^1 and X^2 at the same rate given $Wl + \Pi$. We obtain the following indirect utility function.

$$V = F\left(\frac{Wl + \Pi}{\varphi(P^1, P^2)}\right) - G(l). \tag{8}$$

$\varphi(P^1, P^2)$ is a function which is homogenous of degree one.

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