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Welfare-improving cooperation with a consumer-friendly multiproduct corporation

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Abstract

This paper considers a multiproduct corporation that adopts consumer-friendly activities and cooperates with single plant firms for improving welfare. Comparing competition and (full or partial) cooperation, we show that full cooperation is socially beneficial if products are strong complements, whereas partial cooperation with a higher consumer-friendly activities is beneficial if products are substitutes. We also examine a sequential choice game of endogenous cooperation in which the multiproduct corporation can induce cooperation and show that our findings are supportive at equilibrium. We finally compare different ownerships of the single plant firms and find that foreign ownership decreases the benefits of cooperation. Therefore, a cooperation with multiproduct corporation should be accompanied with active governmental guideline for promoting consumer-friendly activities.

Keywords: consumer-friendly activities; full cooperation; partial cooperation; multiproduct corporation; welfare-improving cooperation;

1. Introduction

As the society’s demands for corporate social responsibility (CSR) are expanding and the firm’s engagement of CSR activities has become a global business practice, particularly large companies, recent research on the effects of CSR activities and its profits and welfare consequences has been also becoming increasingly prominent.\textsuperscript{1}

One of challenging policy issues is the assessment of cooperation among competing firms in the name of CSR. Traditionally, it is believed that firms’ cooperation may induce anti-competitive effect of cartel in production-market and thus it raises serious skepticism among economists and policy makers. For instance, Salant et al. (1983) and Farrell and Shapiro (1990) expressed that in the absence of synergic benefits, the

\textsuperscript{1}CSR embraces a wide range of behaviors, such as being consumer-friendly, employee-friendly, environment-friendly, mindful of ethics, respectful of communities where the firm is located, of which call for duty extends beyond the firm’s business realm. The global phenomenon of CSR trend has been surveyed by various surveys such as KPMG (2013) and CASS (2017).
firms' gains from cooperation come at the expense of the consumers. However, this view generally ignores CSR activities of the firms, such as public investments in R&D, green product design, and the consumer-oriented expansion of multi-markets, which create other channels through firms' cooperation to provide positive effects on the consumers.\(^2\)

On the other hand, as Eckel and Neary (2010) and Crifo and Forget (2015) point out that typical characteristic of current economies is the presence of multiproduct firms, many corporations comprise firms that produce various types of goods at various production plants in the real world.\(^3\) Examples include digital companies such as Google and Apple, as well as business markets for processed materials, car manufacturers, and various types of industrial machinery. Furthermore, multiproduct firms involve mergers and adopt CSR as well.\(^4\)

In the theoretical model of multimarket configuration,\(^5\) we consider a multiproduct corporation that adopts consumer-friendly activities and cooperates with single plant profit-maximizing firms, and provide a new beneficial effect of product-market cooperation on the consumers. We find that cooperation does not always hurt consumer or social welfare but its beneficial effect depends on the level of CSR by a multiproduct corporation and products substitutability.\(^6\) In specific, we examine an industry comprised of a multiproduct corporation that is a consumer-friendly firm (hereafter, CSR-corp.) and owns two production plants, each of which produces differentiated products and competes with a single plant firm. We show that full cooperation with two single plant firms is socially beneficial if both products are strong complements, whereas partial cooperation with a single plant firm is socially beneficial if consumer-friendly activities are high when products are substitutes.

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\(^2\)Several leading companies in developed countries performed eco-design project of developing proactive consumer-oriented products such as automobiles and electronic appliances, with cooperation with customers and companies. For example, Yim (2007) provides some industrial applications of eco-design process within cooperation projects organized by research centers and companies in Austria and Germany. For more case studies, see also Liczmaniska-Kopcewicz and Zastempowski (2020). In recent research, Symeonidis (2008), Mukherjee (2010), Mukherjee and Sinha (2019), and Cho et al. (2019) showed that product-market cooperation may benefit the consumers under certain situations, such as input market imperfection or strategic trade policies.

\(^3\)In plenty of empirical works, multiproduct firms are regarded as canonical real identities in the listed firms data. Regarding recent studies on multiproduct competition without CSR, for example, see Armstrong and Vickers (2018) and Johnson and Rhodes (2019).

\(^4\)Empirical studies explored the link between CSR and firm’s profitability in mergers. See Deng et al. (2013), Bereskin et al. (2018) and Gomes and Marsat (2018).

\(^5\)Recent theoretical study on the multiproduct firms with CSR concerns has just caught attention of research scholars. For examples, see Bárcena-Ruiz and Garzón (2017), Fanti and Buccella (2017, 2019), Dong et al. (2018), Dong and Wang (2019) and Garcia et al. (2020).

\(^6\)Instead of multiproduct markets with product substitutability, if we use a model of homogeneous product market where one CSR-firm competes with two profit-maximizing firms, we can show that welfare-improving full cooperation cannot be supportive in equilibrium because cooperation can increase the profits of rival firms but does not improve consumer surplus with perfect substitutes, which makes the CSR-firm with a higher concern on CSR deviate from full cooperation. A detailed analysis can be provided by authors upon request.
Under full cooperation, total outputs in each market can be higher than that under competition when products are strong complements but it does not happen when products are substitutes. However, under partial cooperation between the CSR-corp. and one single plant firm it can improve consumer surplus if CSR activities are sufficiently high even when products are substitutes. This is because even though cooperation between two competing firms could reduce the output of CSR-corp., it can also increase the output of other competitive single plant firms when products are substitutes, which works for inducing cooperation with profit-maximizing firms. Thus, there exists an internalization effect of substitutability. Due to this output trade-off in the interaction between the substitutes, the level of CSR activities is crucial to determine total outputs effects. In particular, total output-decreasing effect from the cooperation might be lessened with a higher CSR activities even when products are strong substitutes. As a result, there exists a beneficial effect of product-market cooperation on the consumers and welfare in the presence of firm’s CSR. Furthermore, in a sequential choice game of endogenous cooperation in which the CSR-corp. can induce either full or partial cooperation, we can show that our findings are supportive at equilibrium. Therefore, our findings suggest useful policy implications on fair competition law that anti-competitive welfare effect of cooperation should be re-explored when the CSR-corp. induces cooperation, but the cooperation should be accompanied with active governmental guideline for promoting consumer-friendly activities.

As a closely related work, García et al. (2020) considered a CSR-concerned multiproduct corp. by adopting a shareholder’s viewpoint in a strategic managerial delegation framework and examined the profit-oriented incentive to form a cooperative group. They showed that full cooperation with two single plant firms can be an equilibrium for lower substitutability but partial cooperation between the consumer-friendly multiproduct firm and one single plant firm can be an equilibrium for higher substitutability. However, the welfare effect when the CSR-concerned multiproduct corp. decides on the cooperation with single plant firms are not included in their analysis. Therefore, the anti-competitive effect of cooperation is limited in their analysis, which is one of significant policy issues in merger decisions. 7

This paper extends their analysis into a few important directions. First, we adopt a stakeholder’s viewpoint in which the level of ethical CSR in a multiproduct corp. is given to satisfy social philanthropy, rather than the shareholder’s viewpoint where the strategic CSR activities can be manipulated as an instrument of the firm’s choice variables. 8

Second, we further compare consumer surplus and welfare effects between competition and cooperation

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7In the recent literature on merger simulation and anti-competitive effects of mergers, Nevo (2000), Farrell and Shapiro (2010), Nocke and Whinston (2010, 2013) and Jaffe and Weyl (2013) have heavily influenced antitrust practice.

8In the academic literature, research debate on the motives pushing firms to engage in CSR activities has been becoming increasingly prominent. Baron (2001) described two polar definitions between shareholder theory and stakeholder theory. Benabou and Tirole (2010), Schreck (2011), Kitzmueller and Shimshack (2012), Crifo and Forget (2015), Planer-Friedrich and Sahn (2018) and Kim et al. (2019) also provided recent developments in business economics and explained the social benefits and costs of CSR.
in different regimes of ownerships, and then show that either full or partial cooperation with consumer-friendly activities is beneficial to the society. We also show that foreign ownership of the single plant firms decreases the benefits of cooperation. These findings provide policy implications that in the presence of foreign firms, a cooperation with multiproduct corporation should be encouraged by active governmental guideline for promoting consumer-friendly activities.

Finally, we include the case that products are complements and show that our findings are quite robust but complementarity effect between the products increases the benefits of full cooperation. Therefore, CSR activities of the multiproduct firms create more positive effects to the society through full cooperation under certain situation, especially when the products are complements or less substitutes.

The remainder of this paper is organized as follows: Section 2 presents the basic model of multiproducts CSR-corp. Section 3 analyzes the equilibrium outcomes under competition and cooperation among the firms. We then compare outputs, consumer surplus and welfare consequences under different regimes of ownership of single plant firms, respectively, in Section 4 and 5. In section 6, we examine a sequential choice of endogenous cooperation in which the multiproduct corporation can induce full or partial cooperation. Finally, conclusion is provided in Section 7.

2. Model

We consider two markets with differentiated goods, denoted by \( i = 1, 2 \), which may be substitutes or complements. In each market, there are a CSR-corp. (denoted by firm A) and a single plant for-profit (FP) firm (denoted by firm F). The CSR-corp. A has two plants in each market where one plant produces good 1 and the other good 2 in each market \( i \), denoted by \( 1A \) and \( 2A \), respectively. The FP firm has a single plant in each market where one firm produces good 1 and the other good 2 in each market \( i \), denoted by \( 1F \) and \( 2F \), respectively.

On the demand side, there is a continuum of consumers of the same type. The representative consumer has a utility function \( U(q_1, q_2) \), which is quadratic, strictly concave and symmetric in \( q_1 \) and \( q_2 \): \( U(q_1, q_2) = (q_1 + q_2) - ((q_1^2 + 2\delta q_1 q_2 + q_2^2))/2 \), where \( \delta \in (-1, 1) \) and \( q_1 = q_{1A} + q_{1F} \) and \( q_2 = q_{2A} + q_{2F} \) are the quantity of good 1 and 2 respectively. Note that \( q_{ik} \) is the output produced by firm or plant \( ik \), \( i = 1, 2; \ k = A, F \).

Then, the consumer maximizes \( U(q_1, q_2) - p_1 q_1 - p_2 q_2 \), where \( p_i \) is the price of good \( i \). The inverse demand functions are linear and given by:

\[
p_i = 1 - (q_{iA} + q_{iF}) - \delta(q_{jA} + q_{jF}), \quad i \neq j, \quad i, j = 1, 2; \quad -1 < \delta < 1
\]

where goods are substitutes if \( \delta > 0 \), complements if \( \delta < 0 \), and independent in demand if \( \delta = 0 \).

On the supply side, we assume that firms have identical technologies represented by the following quadratic cost function: \( c(q_{ik}) = q_{ik}^2 \), \( i = 1, 2; \ k = A, F \). Thus, the profit function of a plant or firm \( ik \) is:

\[
\pi_{ik} = p_i q_{ik} - q_{ik}^2
\]

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and the profit of the multiplant CSR-corp. is:

$$\pi_A = \pi_{1A} + \pi_{2A}$$  \hspace{1cm} (3)$$

We assume that the CSR-corp. maximizes the joint profits of its two plants plus a fraction ($\theta$) of consumer surplus ($CS$) in production. Thus, the payoff that the multiplant CSR-corp. maximizes in output decision process is as follows:

$$V = \pi_A + \theta CS$$  \hspace{1cm} (4)$$

where $CS = ((q_{1A} + q_{1F})^2 + 2\delta(q_{1A} + q_{1F})(q_{2A} + q_{2F}) + (q_{2A} + q_{2F})^2)/2$. Note that the parameter $\theta \in [0, 1]$ measures the degree of concern on consumer surplus when the corporation adopts CSR activities.\(^9\) Then, as $\theta$ increases, the CSR-corp. puts more weights on its outputs, given the degree of product differentiation.

In the below, we analyzes three different scenarios in output production among three firms: (i) competition, (ii) cooperation between CSR-corp. and one FP firm and (iii) full-cooperation among CSR-corp. and two FP firms.

3. Analysis

3.1. Competition

As a benchmark, we consider a competition case: CSR-corp. $A$ chooses the outputs $q_{1A}$ and $q_{2A}$ that maximise eqn. (4) while FP firm $iF$ chooses the output $q_{iF}$ that maximises its profit given by eqn. (2). Solving these problems, we obtain the following:

$$q_{iA}^B = \frac{3 + \theta(1 + \delta)}{15 + 10\delta + \delta^2 - 3(1 + \delta)\theta}, \hspace{1cm} q_{iF}^B = \frac{3 + \delta - \theta(1 + \delta)}{15 + 10\delta + \delta^2 - 3(1 + \delta)\theta}, \hspace{1cm} i = 1, 2$$  \hspace{1cm} (5)$$

where superscript $B$ stands for "benchmark". Note from eqn. (5) that:

a) $q_{iA}^B < q_{iF}^B$ if $0 < \delta < 1$ and $\theta < \frac{\delta}{2 + 2\delta}$, whereas $q_{iA}^B > q_{iF}^B$ otherwise. That is, when products are substitutes and CSR effort is low, the output in market $i$ produced by FP firm $i$ is higher than that produced by CSR-corporation $iA$. Otherwise, the output produced by CSR-corporation $iA$ is higher than that from its FP counterpart. It implies that only when the CSR-corp. puts a higher weight on output productions to take care of consumers, it induces rivals to reduce their outputs under strategic substitutes relationship.

b) $q_{1A}^B = q_{2A}^B$ and $q_{1F}^B = q_{2F}^B$. Therefore, $q_1^B = q_2^B$, where $q_i^B = q_{iA}^B + q_{iF}^B$.

c) $\frac{\partial q_{iA}^B}{\partial \theta} > 0$, $\frac{\partial q_{iF}^B}{\partial \theta} < 0$ and $\frac{\partial q_i^B}{\partial \theta} > 0$. That is, the market output always increases with CSR effort.

\(^9\)Note that we consider consumer surplus as the proxy of CSR, which has been widely utilized in related literature. For more discussion in recent works, see Lambertini and Tampieri (2015), Leal et al. (2018, 2019), Garcia et al. (2019) and Dong and Wang (2019) among others.
The equilibrium profits, consumer surplus and CSR-corp. payoff are, respectively:

\[
\begin{align*}
\pi_A^H &= \frac{2(3 + \theta((1 + \delta))(3(2 + \delta) - 4\theta(1 + \delta)))}{(15 + 10\delta + \delta^2 - 3(1 + \delta)\theta^2)}, \\
\pi_{iF}^H &= \frac{2(3 + \delta - \theta((1 + \delta)))^2}{(15 + 10\delta + \delta^2 - 3\theta - 3\delta\theta)^2}, \quad i = 1, 2 \\
CS^H &= \frac{(1 + \delta)(6 + \delta)^2}{(15 + 10\delta + \delta^2 - 3\theta - 3\delta\theta)^2}, \\
V^H &= \frac{1 + (\delta)(6 + \delta)^2\theta + 2(3 + \theta + \delta\theta)(3 + (1 + \delta)(3 - 4\theta))}{(15 + 10\delta + \delta^2 - 3\theta - 3\delta\theta)^2},
\end{align*}
\]  

(6)

3.2. Partial cooperation between CSR-corp. and FP firm i

Under this scenario, the CSR-corp. A and one FP firm coordinate their production to maximize the joint payoffs: \( V + \pi_{iF}, i = 1, 2 \); while FP firm \(-i\) chooses the output \( q_{-iF} \) that maximizes its profits. Solving these problems, we obtain the following:

\[
q_{-iA}^{pc} = \frac{9 - 7\delta + \delta^2 + (3 - 2\delta)\delta\theta + (-1 + \delta^2)\theta^2}{45 - 16\delta^2 - 3(8 - 5\delta^2)\theta + 3(1 - \delta^2)\theta^2}, \quad q_{-iF}^{pc} = \frac{9 - 2\delta - 2\delta^2 - 3(2 - \delta^2)\theta + (1 - \delta^2)\theta^2}{45 - 16\delta^2 - 3(8 - 5\delta^2)\theta + 3(1 - \delta^2)\theta^2}, \\
q_{ik}^{pc} = \frac{15 - 9\delta + (-3 + 5\delta)\theta}{90 - 32\delta^2 - 6(8 - 5\delta^2)\theta + 3(1 - \delta^2)\theta^2}, \quad k = A, F
\]  

(7)

For expositional convenience, we assume that CSR-corp. cooperates with FP firm 1. Let us define the functions \( \bar{\theta}_1(\delta) = \frac{6 + 3\delta - 5\delta^2 - \sqrt{36 - 44\delta - 27\delta^2 + 103\delta^3 + 3\delta^4}}{4(1 - \delta^2)}, \) and \( \bar{\theta}_2(\delta) = \frac{9 + 5\delta - 6\delta^2 - \sqrt{57 + 503\delta - 27\delta^2 - 203\delta^3 + 43\delta^4}}{4(1 - \delta^2)}. \) Now, from eqn. (7) we have that:

a) \( q_{1A}^{pc} = q_{1F}^{pc} \), and \( q_{2A}^{pc} \geq q_{2F}^{pc} \) if \( \bar{\theta}_1 \leq \bar{\theta}_1 \). Note that \( \bar{\theta}_1 \geq 0 \) only if \( \delta \geq 0 \) and \( \lim_{\delta \to 1} \bar{\theta}_1(\delta) = \frac{1}{2} \). That is, when products are substitutes and CSR effort is low, the output in market 2 produced by firm \( 2F \) is higher than that produced by CSR-corp. \( 2A \). Otherwise, the output from CSR-corp. \( 2A \) is higher than that from its FP counterpart.

b) \( q_{1A}^{pc} < q_{1F}^{pc} \) for any \(-1 < \delta < 1 \) and \( 0 \leq \theta \leq 1 \); and \( q_{1F}^{pc} \geq q_{1F}^{pc} \) if \( \theta \geq \bar{\theta}_2 \). Note that \( \theta_2 \geq 0 \) only if \( \delta \geq \frac{1}{2} \left( 5 - \sqrt{73} \right) \) and \( \max_{\delta} \left( \bar{\theta}_2(\delta) \right) = 0.504 \) when \( \delta \to 0.836 \). That is, regardless of whether products are substitutes or complements, when CSR effort is relatively low, the output produced by firm \( 1F \) is lower than that from firm \( 2F \). Then \( q_{1A}^{pc} \geq q_{1F}^{pc} \) if \( \theta \geq \frac{\delta - \delta^2}{2(1 - \delta^2)} \). That is, when products are substitutes and CSR effort is sufficiently low, the total output in market 2 is higher than that in market 1.

c) \( \frac{\partial q_{1A}^{pc}}{\partial \theta} > 0 \), \( \frac{\partial q_{1F}^{pc}}{\partial \theta} < 0 \) and \( \frac{\partial q_{1F}^{pc}}{\partial \theta} > 0 \). Further, \( \frac{\partial q_{1A}^{pc}}{\partial \delta} > 0 \), \( k = A, F \), unless products are sufficiently strong complements.

6
The equilibrium profits, consumer surplus and CSR-corp. payoff are, respectively:

\[
\pi_{pc}^A = \frac{\Omega_1}{4(45 - 16\delta^2 - 3(8 - 5\delta^2)\theta + 3(1 - \delta^2)\theta^2)^2},
\]

\[
\pi_{pc}^1_F = \frac{\Omega_2}{4(45 - 16\delta^2 - 3(8 - 5\delta^2)\theta + 3(1 - \delta^2)\theta^2)^2},
\]

\[
\pi_{pc}^2_F = \frac{2(9 - 2\delta - 2\delta^2 - 3(2 - \delta^2)\theta + (1 - \delta^2)\theta^2)^2}{(45 - 16\delta^2 - 3(8 - 5\delta^2)\theta + 3(1 - \delta^2)\theta^2)^2},
\]

\[
CS_{pc} = \frac{\Omega_3}{2(45 - 16\delta^2 - 3(8 - 5\delta^2)\theta + 3(1 - \delta^2)\theta^2)^2},
\]

\[
V_{pc} = \frac{\Omega_4}{4(45 - 16\delta^2 - 3(8 - 5\delta^2)\theta + 3(1 - \delta^2)\theta^2)^2}
\]

where \(\Omega_j j = 1, 2, ..., 8\) are as presented in Appendix A.

3.3. Full cooperation

Under this scenario, the objective function becomes \(V + \pi_{1F} + \pi_{2F}\), and firms decide outputs cooperatively. Solving this problem, we obtain the following:

\[
q_{ful}^{ik} = \frac{1}{6 + 4\delta - 2(1 + \delta)\theta}, \ i = 1, 2; \ k = A, F
\]

From eqn. (9) we have that:

a) \(q_{ful}^{1A} = q_{ful}^{1F}\) and \(q_{ful}^{2A} = q_{ful}^{2F}\). Due to the symmetry, the output in market \(i\) is evenly distributed between the firms and total outputs in market 1 are tantamount to the market 2.

b) \(\frac{\partial q_{ful}^{ik}}{\partial \theta} > 0\). That is, market output always increases with CSR effort.

4. Outputs Comparisons

We will compare equilibrium outputs between competition and partial or full cooperation, respectively. We then provide the beneficial effects of output changes to consumer surplus in the two comparisons.
4.1. Competition vs. Partial cooperation

1) \( q_{1A}^B > q_{1A}^{pc} \) for any \( \delta \in (-1, 1) \) and \( \theta \in [0, 1] \), and \( q_{2A}^B > q_{2A}^{pc} \) if \( \delta \in (0, 1) \) and \( \theta \in [0, 1] \). It implies that the output that is produced by the cooperation will always decrease regardless of whether products are substitutes or complements, whereas the output that is produced by the CSR-corp. in the market competing with rival single plant firm will decrease only when products are substitutes. But, CSR-corp.’s output under competition is higher than that from partial cooperation, that is, \( q_{1A}^B + q_{2A}^B > q_{1A}^{pc} + q_{2A}^{pc} \) for any \( \delta \in (-1, 1) \) and \( \theta \in [0, 1] \).

2) Regarding FP firm i’s output,

(a) \( q_{2F}^B > q_{2F}^{pc} \) if \( \delta \in (-1, 0) \) and \( \theta \in [0, 1] \). Otherwise, \( q_{2F}^B \leq q_{2F}^{pc} \). That is, when products are complements (substitutes), the output of firm 2F resulting from competition is higher (lower) than that resulting from cooperation between the CSR-corp. and the FP firm 1. Thus, partial cooperation induces rival firm to decrease (increase) their outputs with complements (substitutes) relationship.

(b) \( q_{1F}^B > q_{1F}^{pc} \) if \( \theta \geq \bar{\theta}_1(\delta) \), where \( \bar{\theta}_1 \) satisfies \( q_{1F}^B = q_{1F}^{pc} \). That is, regardless of whether products are substitutes or complements, when CSR effort is relatively high, the output of FP firm 1 resulting from its cooperation with CSR-corp. is higher than that resulting from competition.

3) Regarding total outputs in market i, let us define \( \bar{\theta}_4(\delta) = \frac{18+215-\delta^2-\delta^3-\sqrt{7224+15408+26135+14851-1745+25\delta+5^2}}{12(1+\delta)} \)

and \( \bar{\theta}_5(\delta) = \frac{54+55\delta-16\delta^2-10\delta^3-\sqrt{1296+296+16803-8045^2-4045^2-16\delta^2+168^2}}{8(1+\delta)} \).

(a) \( q_2^B > q_2^{pc} \) only if \( \theta > \bar{\theta}_4(\delta) \). Note that \( \bar{\theta}_4 \geq 0 \) only if \( \delta \geq 0 \) and \( \max_{\delta}\{\bar{\theta}_4(\delta)\} = 0.148 \) when \( \delta \to 0.7805 \). Thus, market 2’s total outputs resulting from partial cooperation can be higher than that under competition when products are substitutes and CSR efforts are sufficiently low.

(b) \( q_1^B > q_1^{pc} \) only if \( \theta < \bar{\theta}_5(\delta) \). Note that \( \bar{\theta}_5 > 0 \) for any \( \delta \in (-1, 1) \) and \( \min_{\delta}\{\bar{\theta}_5(\delta)\} = 0.661 \) when \( \delta \to 1 \). Thus, when CSR activities are sufficiently high, market 1’s total outputs resulting from partial cooperation can be higher than that under competition.

4.2. Competition vs. Full cooperation

1) \( q_{1A}^B > q_{1A}^{ful} \) \( i = 1, 2 \) for any \( \delta \in (-1, 1) \) and \( \theta \in [0, 1] \). That is, CSR-corp.’s output under competition is always higher than that under full cooperation. Then, \( q_{1A}^B + q_{2A}^B > q_{1A}^{ful} + q_{2A}^{ful} \) for any \( \delta \in (-1, 1) \) and \( \theta \in [0, 1] \).

2) Regarding FP firm i’ output, let us define \( \bar{\theta}_6(\delta) = \frac{3(3+2\delta)-\sqrt{37+14\delta+12\delta^2}}{4(1+\delta)} \). Then, \( q_{1F}^{ful} \geq q_{1F}^B \) if \( \theta \geq \bar{\theta}_6(\delta) \). Note that \( \bar{\theta}_6 \geq 0 \) only if \( \delta \geq \frac{1}{3} (-4 + \sqrt{7}) \) and \( \lim_{\delta \to 1} \bar{\theta}_6(\delta) = 0.54 \). Thus, regardless of whether products are substitutes or complements, when CSR effort is high, FP firm i’ output resulting from full cooperation is higher than that under competition.

3) Regarding total outputs in market i, \( q_i^B \geq q_i^{ful} \) if \( \theta \leq \frac{3+5\delta+\delta^2}{3+4\delta+3\delta^2} \). Thus, market i’s total outputs resulting from full cooperation can be higher than that under competition when products are strong complements and CSR efforts are high.
4.3. Partial cooperation vs Full cooperation

1) \( q_{pc}^{1A} > q_{ful}^{1A} \) if \( \delta \in (0, 1) \) and \( \theta \in [0, 1] \), and \( q_{pc}^{2A} > q_{ful}^{2A} \) for any \( \delta \in (-1, 1) \) and \( \theta \in [0, 1] \). It implies that the output that is produced by the cooperation between all firms will decrease only when products are substitutes, whereas the output that is produced by the CSR-corp. in the market competing with rival single plant firm will always decrease regardless of whether products are substitutes or complements. Then, \( q_{pc}^{1A} + q_{pc}^{2A} > q_{ful}^{1A} + q_{ful}^{2A} \) for any \( \delta \in (-1, 1) \) and \( \theta \in [0, 1] \).

2) Regarding FP firm \( i \)'s output,

(a) \( q_{pc}^{2F} > q_{ful}^{2F} \) if \( \theta < \bar{\theta}_7(\delta) \), where \( \bar{\theta}_7 \) satisfies \( q_{pc}^{2F} = q_{ful}^{2F} \). That is, regardless of whether products are substitutes or complements, when CSR effort is relatively high, the output of single plant firm 2F resulting from full cooperation is higher than that resulting from cooperation between CSR-corp. and the single plant firm 1F.

(b) \( q_{pc}^{1F} > q_{ful}^{1F} \) if \( \delta \in (0, 1) \) and \( \theta \in [0, 1] \). Otherwise, \( q_{pc}^{1F} \leq q_{ful}^{1F} \). That is, when products are complements, the output of single plant firm 1F resulting from full cooperation is higher than that resulting from its cooperation with CSR-corp.

3) Regarding total outputs in market \( i \),

(a) \( q_{pc}^{2} > q_{ful}^{2} \) if \( \theta < \bar{\theta}_8(\delta) \), where \( \bar{\theta}_8 \) satisfies \( q_{pc}^{2} = q_{ful}^{2} \). Thus, market 2’s total outputs resulting from full cooperation can be higher than that under partial cooperation either when products are strong complements or when products are substitutes and CSR efforts are high.

(b) \( q_{1}^{pc} > q_{1}^{ful} \) if \( \delta \in (0, 1) \) and \( \theta \in [0, 1] \). Otherwise, \( q_{1}^{pc} \leq q_{1}^{ful} \). Thus, when products are complements, market 1’s total outputs resulting from full cooperation can be higher than that under partial cooperation.

4.4. Consumer Surplus

We now compare consumer surplus in the three cases and find the regime that generates the highest consumer surplus. Fig. 1 graphically depicts Proposition 1.

**Proposition 1.**

a) Full cooperation can generate the highest consumer surplus when products are complements and CSR activities are relatively high. (Fig. 1, yellow area)

b) Partial cooperation can generate the highest consumer surplus either when products are strong substitutes and CSR activities are sufficiently high or when products are sufficiently strong complements. (Fig. 1, red area)

c) Competition can generate the highest consumer surplus when CSR activities are not so high and products are either substitutes or weak complements. (Fig. 1, green area)

**Proof.** See Appendix B.1
Proposition 1 represents the conditions that cooperation with CSR-corp. can be beneficial to consumers especially when cooperation can induce to increase total outputs, as shown in the previous output comparisons. On the one hand, if products are complements, total outputs under cooperation are higher than those under competition and thus cooperation increases consumer surplus unless CSR activities are relatively low. Further, when the complementarity is strong, it is always beneficial to the consumers regardless of CSR activities.

On the other hand, if products are substitutes, total outputs under cooperation might be higher than those under competition when CSR activities are sufficiently high. This is because even though cooperation between two competing firms could reduce the outputs of CSR-corp. by internalization of substitutability, it can increase the outputs of other competitive single plant firms. Due to the output trade-off in the intra-interaction and inter-interaction between the substitutes, the level of CSR activities are crucial to determine total output effects. Thus, the output-decreasing effects from the cooperation might be lessen with higher CSR activities when products are strong substitutes.

Therefore, there exists a beneficial effect of product-market cooperation on the consumers in the presence of firm’s CSR. It also indicates that cooperation will increase welfare mostly when products are complements and might increase with substitutable products if CSR efforts are high enough.

5. Welfare Effects of Cooperation

In this section, we will consider the ownership of the single plant firm and analyze the welfare effects of cooperation. We then compare the welfare resulting from three cases: (1) both single plant FP firms are domestic (2) one FP firm is foreign and (3) both FP firms are foreign.

5.1. Both FP firms are domestic

Because both single plant FP firms are domestic, the producer surplus is given by \( PS = \pi_A + \pi_{1F} + \pi_{2F} \). Then, the welfare level under competition and cooperation regimes are as follows:

\[
W^B = \frac{108 + 90\delta + 17\delta^2 + \delta^3 - 2(1 + \delta)(18 + \delta)\theta - 4(1 + \delta)^2\theta^2}{(15 + 10\delta + \delta^2 - 3(1 + \delta)\theta)^2}
\]

\[
W^{pc} = \frac{2(45 - 16\delta^2 - 3(8 - 5\delta^2)\theta + 3(1 - \delta^2)\theta^2)}{\Omega_5}
\]

\[
W^{ful} = \frac{4 + 3\delta - 2(1 + \delta)\theta}{(3 + 2\delta - (1 + \delta)\theta)^2}
\]

We now determine which regime is the most relevant with regard to aggregate surplus. Fig. 2 graphically depicts Proposition 2.

**Proposition 2.**

a) Full cooperation can generate the highest welfare when either products are strong complements or CSR activities are high. (Fig. 2, yellow area)
b) Partial cooperation can generate the highest welfare when products are substitutes and CSR activities are intermediate. (Fig. 2, red area)

c) Competition can generate the highest welfare when CSR activities are not so high and products are not so strong complements. (Fig. 2, green area).

Proof. See Appendix B.2

Proposition 2 represents that cooperation between CSR-corp. and domestic firms can be beneficial to the society when products are strong complements or CSR activities are not low. Since cooperation increases the profits of cooperative group and might increase consumer surplus, the welfare consequences critically depends on the level of CSR activities. (Compare Fig 1 and Fig. 2) This implies that the active role of societal guideline for promoting CSR, rather than considering it on a voluntary basis, is necessary from the viewpoint of society.

Further, government guideline can provide a policy and institutional framework that stimulates CSR-corp. to raise its CSR performance beyond minimum legal standards. In the guideline, government can encourage and incentivize the adoption of CSR by the corporation through best practice and appropriate regulation with fiscal incentives.10

5.2. One FP firm is foreign-owned

In this case, we have two scenarios: a) FP firm 2 is the foreign firm and b) FP firm 1 is the foreign one.

In scenario a) the producer surplus is given by $PS = \pi_A + \pi_1F$; whereas in b) the producer surplus is given by $PS = \pi_A + \pi_2F$. Then, the welfare level under competition and cooperation regimes are as follows:

\[
W^B = \frac{90 + 78\delta + 15\delta^2 + \delta^3 + 2(-12 + \delta)(1 + \delta)\theta - 6(1 + \delta)^2\theta^2}{(15 + 10\delta + \delta^2 - 3(1 + \delta)\theta)^2}
\]

\[
W^{apc} = \frac{90 + 78\delta + 15\delta^2 + \delta^3 + 2(-12 + \delta)(1 + \delta)\theta - 6(1 + \delta)^2\theta^2}{(15 + 10\delta + \delta^2 - 3(1 + \delta)\theta)^2}
\]

\[
W^{bpc} = \frac{90 + 78\delta + 15\delta^2 + \delta^3 + 2(-12 + \delta)(1 + \delta)\theta - 6(1 + \delta)^2\theta^2}{(15 + 10\delta + \delta^2 - 3(1 + \delta)\theta)^2}
\]

\[
W^{full} = \frac{13 + 10\delta - 6(1 + \delta)\theta}{4(3 + 2\delta - (1 + \delta)\theta)^2}
\]

We now determine which regime is the most relevant with regard to aggregate surplus. Then, Fig. 3 and Fig. 4 graphically depict Propositions 3 and 4, respectively.

**Proposition 3. Under the case where FP firm 1 is a domestic firm**

10 The promotion of CSR has become a top priority in the policy agenda for sustainable development in many countries and international organization. For example, further encouragement of CSR became a central policy objective in the United States and European Union. See, for example, the UK government website (http://www.csr.gov.uk/policy.shtml), Steurer (2010) and Xu and Lee (2019).
a) Full cooperation can generate the highest welfare either when products are strong complements or when CSR activities are sufficiently high. (Fig. 3, yellow area)

b) Partial cooperation can generate the highest welfare either when CSR activities are intermediate. (Fig. 3, red area)

c) Competition can generate the highest welfare when CSR activities are not so high and products are not so strong complements. (Fig. 3, green area)

Proof. See Appendix B.3

Proposition 3 represents that in the presence of foreign rival firm cooperation between CSR-corp. and domestic firm can be beneficial to the society when products are strong complements or CSR activities are not so low. This result is quite similar to Proposition 2 where both single plant firms are domestic. However, it is noteworthy that partial cooperation is better than full cooperation in the intermediate ranges for CSR activities. (Compare Fig. 2 and Fig. 3) Therefore, the role of societal guideline for promoting CSR can be minimal when products are complements under partial cooperation with domestic firm.

Proposition 4. Under the case where FP firm 1 is a foreign firm

a) Full cooperation can generate the highest welfare either when products are strong complements or when CSR activities are sufficiently high. (cf. Fig. 4, yellow area)

b) Partial cooperation can generate the highest welfare when products are strong substitutes and CSR activities are sufficiently high. (cf. Fig. 4, red area)

c) Competition can generate the highest welfare when CSR activities are not so high and products are not strong complements. (cf. Fig. 4, green area)

Proof. See Appendix B.4

Proposition 4 represents that cooperation between CSR-corp. and foreign firm in the presence of domestic rival firm can be beneficial to the society when products are strong complements or CSR activities are sufficiently high. This result is somewhat different to Proposition 3 where the CSR-corp. cooperates with domestic firm in the presence of foreign rival firm. (Compare Fig. 3 and Fig. 4) In particular, partial cooperation is worse than not only full cooperation but competition when the CSR-corp. cooperates with foreign firm. Therefore, the active role of government for formulating societal guideline for promoting CSR should be stronger to improve the social welfare in the case of partial cooperation with foreign firm.

5.3. Both FP firms are foreign-owned

When both single plant FP firms are foreign-owned, the producer surplus is given by $PS = \pi_A$. Then, the welfare level under competition and cooperation regimes are as follows:
\[ W^B = \frac{72 + 66\delta + 13\delta^2 + 6(-2 + \delta)(1 + \delta)\theta - 8(1 + \delta)^2\theta^2}{(15 + 10\delta + \delta^2 - 3(1 + \delta)\theta)^2} \]
\[ W^{pc} = \frac{\Omega_8}{4(45 - 16\delta^2 - 3(8 - 5\delta^2)\theta + 3(1 - \delta^2)\theta^2)^2} \]
\[ W^{ful} = \frac{5 + 4\delta - 2(1 + \delta)\theta}{2(3 + 2\delta - (1 + \delta)\theta)^2} \]

We now determine which regime is the most relevant with regard to aggregate surplus. Fig. 5 graphically depicts Proposition 5.

**Proposition 5.** Under the case where both FP firms are foreign-owned

a) Full cooperation can generate the highest welfare either when products are strong complements or when CSR activities are sufficiently high and products are either complements or weak substitutes. (Fig. 5, yellow area)

b) Partial cooperation can generate the highest welfare only when products are substitutes and CSR activities are sufficiently high. (Fig. 5, red area)

c) Competition can generate the highest welfare when CSR activities are not so high and products are not strong complements.

**Proof.** See Appendix B.5

Proposition 5 represents that if the competing firms are all foreign, cooperation by CSR-corp. can be beneficial to the society only when products are strong complements or CSR activities are sufficiently high. Compared with proposition 4, however, it requires stronger complements or higher CSR activities for improving welfare from cooperation. (Compare Fig. 4 and Fig. 5) Further, competition can generate the highest social welfare unless CSR activities are sufficiently high or complementarity is sufficiently strong. Therefore, when domestic market is more open to foreign firms, cooperation with CSR-corp. should accompany with the active adoption of higher CSR to improve social welfare.

6. Endogenous cooperation induced by a CSR-corp.

In this section, we will endogeneize the firm’s choices on cooperation group by considering a sequential case that CSR-corp. initiates partial or full cooperation and then two FP firms decide whether to cooperate.\(^{11}\)

Then, we can find an equilibrium in a sequential game of endogenous cooperation in the following process.

\(^{11}\)Unlike Garcia et al. (2020) where the firms decide endogenous cooperation group simultaneously, we will restrict our analysis into a sequential case that CSR-corp. plays a key role of the first-mover in formulating cooperation group in order to identify how and when consumer-friendly CSR activities can enhance the welfare under cooperation. Thus, our analysis of the cooperation induced by a CSR-corp. does not include a partial cooperation between the FP firms only.
First, we consider the incentive of the CSR-corp. by comparing its objective function values when it initiates a cooperation with single plant firms.

**Lemma 1.** CSR-corp. has the following preferences according to the objective function value, $V$, in Fig. 6:

- $V_{ful} > V_B > V_{pc}$ (zones I and VIII)
- $V_{ful} > V_{pc} > V_B$ (zones II and IX)
- $V_{pc} > V_{ful} > V_B$ (zones III and XI)
- $V_{pc} > V_B > V_{ful}$ (zones VI and X)
- $V_B > V_{pc} > V_{ful}$ (zone V)
- $V_B > V_{ful} > V_{pc}$ (zones IV and VII)

*Proof.* See Appendix B.6

It represents that the preferences of the CSR-corp. depends on the level of consumer-friendliness and products substitutability. From Fig. 6, we can summarize the incentive of the CSR-corp. as follows:

a) In the case of higher level of consumer-friendliness, full cooperation is the best to the CSR-corp. when the products are complements or weak substitutes. (Zone I, II) while partial cooperation is the best to the CSR-corp. when the products are not weak substitutes. (Zone III, VI)

b) In the case of intermediate level of consumer-friendliness, competition is always the best to the CSR-corp. (Zone IV, V, VII)

c) In the case of lower level of consumer-friendliness, full cooperation is the best to the CSR-corp. when the products are weak complements or substitutes. (Zone VIII, IX)

Second, given the preferences of the CSR-corp., we can check whether the two FP firms agree on the suggested cooperation in a sequential manner. Then, we can show the incentives of cooperation by the two FP firms as follows:

**Lemma 2.** When the CSR-corp. induces a full or partial cooperation, the two FP firms will agree on the suggested cooperation in each zone of Fig. 6:

a) Zone I and II: full cooperation will be agreed

b) Zone VIII: full cooperation will be agreed only when the products are weak substitutes.

c) Zone III: full cooperation will be agreed when the level of consumer-friendliness is very high and the products are strong substitutes, otherwise partial cooperation will be agreed.

d) Zone IX: full (partial) cooperation will be agreed when the level of consumer-friendliness is low and the products are (not so) weak substitutes.
e) Zone VI, X and XI: partial cooperation will be agreed.

f) Zone IV, V and VII: any cooperation will not be agreed.

Proof. See Appendix B.7 and figures 7 and 8.

Finally, using lemma 1 and 2 and our findings in the previous propositions on the welfare consequences of cooperation, we have the proposition in the equilibrium of a sequential game of endogenous cooperation.

**Proposition 6.** In a sequential game where the CSR-corp. can induce an endogenous cooperation, the equilibrium depends on the level of consumer-friendliness and products substitutability. In particular, only in the case of higher level of consumer-friendliness, the followings hold:

a) CSR-corp.-induced full cooperation can be endogenously chosen and it is welfare-improving when the products are complements or weak substitutes

b) CSR-corp.-induced partial cooperation can be endogenously chosen and it is welfare-improving when the products are not weak substitutes

This implies that our main findings on the socially beneficial cooperation with a consumer-friendly multiproduct corp. are consistent in the equilibrium of a sequential game of endogenous cooperation. Note also that these findings hold even in the presence of foreign firms.

7. Concluding Remarks

In this paper, we considered a multiproduct corporation that adopts consumer-friendly activities and cooperates with single plant firms. We found that cooperation between two competing firms could reduce the outputs of CSR-corp. but increase the outputs of other single plant firms when CSR activities are high. It could provide a beneficial effect of product-market cooperation on the consumers and welfare in the presence of firm’s CSR, especially when either products are strong complements or consumer-friendly activities are sufficiently high. Our findings are also consistent in a sequential game of endogenous cooperation.

We also compared different ownerships of the single plant firms and show that our findings are quite robust but foreign ownership decreases the benefits of cooperation. Therefore, in the presence of foreign firms, cooperation with multiproduct corporation should be encouraged by active governmental guideline for promoting consumer-friendly activities.

There remain some limitations of our analysis. If we endogeneize the CSR-initiatives and the incentives of cooperation together, we can further investigate the welfare effects of the instrumental use of CSR in a managerial delegation model, especially when there are multiple CSR activities among the competitive firms. Further, the firms can take strategic investments for adopting CSR and commit to credible CSR for encouraging cooperation in a signaling framework. The analytic consideration on the cost effect of CSR investment might be an important factor for choosing a strategic device to induce cooperation. These are future research directions for understanding real practice in CSR activities.
References


CASS, 2017. Csr report of china’s national brand.


Appendix A. The values of $\Omega_j$

\begin{align*}
\Omega_1 &= 1323 - 1008\delta - 337\delta^2 + 356\delta^3 - 46\delta^4 + 2 (-684 + 585\delta + 330\delta^2 - 367\delta^3 + 64\delta^4) \theta + (279 - 572\delta - 97\delta^2 + 452\delta^3 - 142\delta^4) \theta^2 + (54 + 90\delta - 130\delta^2 - 90\delta^3 + 76\delta^4) \theta^3 - 16 (-1 + \delta)^2 \theta^4 > 0; \\
\Omega_2 &= 45 - 9\delta - 14\delta^2 + 2\delta^3 + (-39 - 3\delta + 24\delta^2 - 25\delta^3) \theta + (6 - 6\delta^2) \theta^2 > 0, \\
\Omega_3 &= 549 - 54\delta - 468\delta^2 + 150\delta^3 + 19\delta^4 - 6 (51 - 22\delta - 56\delta^2 + 22\delta^3 + 5\delta^4) \theta + (45 - 30\delta - 56\delta^2 + 30\delta^3 + 11\delta^4) \theta^2 > 0; \\
\Omega_4 &= 1323 - 1008\delta - 337\delta^2 + 356\delta^3 - 46\delta^4 - (270 - 1062\delta + 276\delta^2 + 434\delta^3 - 166\delta^4) \theta - (333 + 308\delta - 575\delta^2 - 188\delta^3 + 202\delta^4) \theta^2 + (144 + 30\delta - 242\delta^2 - 30\delta^3 + 98\delta^4) \theta^3 - (16 - 32\delta^2 + 16\delta^4) \theta^4 > 0; \\
\Omega_5 &= 1872 + \delta(-972 + 3\delta(146 + \delta))) + (-1782 + 6\delta(182 + (195 - 118\delta)\delta))\theta + (504 - \delta(452 + \delta(425 + \delta(-362 + 13\delta))))\theta^2 + 2(-1 + \delta)(1 + \delta)(15 + \delta(-30 + 7\delta))\theta^3 - 4 (1 - \delta^2)^2 \theta^4 > 0; \\
\Omega_6 &= 1548 - \delta(828 + \delta(701 + \delta(-406 + 13\delta))) + (-1350 + 6\delta(166 + \delta(143 + 2\delta(-55 + 4\delta))))\theta + (288 - \delta(436 + \delta(193 + \delta(-346 + 65\delta))))\theta^2 + 2(-1 + \delta)(1 + \delta)(-9 + \delta(-30 + 19\delta))\theta^3 - 8 (1 - \delta^2)^2 \theta^4 > 0; \\
\Omega_7 &= 3069 + \delta(-1404 + \delta(-1529 + 24\delta(30 + \delta))) + (-2844 + 2\delta(813 + \delta(978 - \delta(547 + 14\delta))))\theta + (801 - \delta(664 + \delta(673 + 16(-34 + \delta)\delta)))\theta^2 + 2(-1 + \delta)(1 + \delta)(21 + \delta(-45 + 14\delta))\theta^3 - 8 (1 - \delta^2)^2 \theta^4 > 0; \\
\Omega_8 &= 2421 - \delta(1116 + \delta(1273 + 8(-82 + \delta)\delta)) + (-1980 + 2\delta(717 + \delta(666 + \delta(-499 + 34\delta)\delta)))\theta + (369 - \delta(632 + \delta(209 + 8\delta(-64 + 15\delta)\delta)))\theta^2 + 2(-1 + \delta)(1 + \delta)(-27 + \delta(-45 + 38\delta))\theta^3 - 16 (1 - \delta^2)^2 \theta^4 > 0.
\end{align*}
Appendix B. Proof of propositions


To compare levels of CS under different regimes, let us define the functions: $\chi_r(\delta, \theta)$, and $\psi_B(\delta, \theta)$

$$
\begin{align*}
\chi_r(\delta, \theta) &= CS^{full}(\delta, \theta) - CS^r(\delta, \theta) \quad \text{with} \quad r = \{B, pc\} \\
\psi_B(\delta, \theta) &= CS^{pc}(\delta, \theta) - CS^B(\delta, \theta)
\end{align*}
$$

We plot the curves $\chi_r(\delta, \theta) = 0$, $\psi_B(\delta, \theta) = 0$ in $\delta$ and $\theta$ space and this pattern implies the ranking of CS into six zones. (Fig. 1) Then, based on Fig. 1, the consumer surplus ranking will be:

- $CS^{pc} > CS^{full} > CS^B$ (zone I)
- $CS^{full} > CS^{pc} > CS^B$ (zone II)
- $CS^{full} > CS^B > CS^{pc}$ (zone III)
- $CS^B > CS^{full} > CS^{pc}$ (zone IV)
- $CS^B > CS^{pc} > CS^{full}$ (zone V)
- $CS^{pc} > CS^B > CS^{full}$ (zone VI)

![Figure 1](image-url)
Appendix B.2. Proof of proposition 2.

Based on Fig. 2, the social welfare ranking will be:

- $W^{ful} > W^{B} > W^{pc}$ (zone I)
- $W^{ful} > W^{pc} > W^{B}$ (zone II)
- $W^{pc} > W^{ful} > W^{B}$ (zone III)
- $W^{pc} > W^{B} > W^{ful}$ (zone IV)
- $W^{B} > W^{pc} > W^{ful}$ (zone V)
- $W^{B} > W^{ful} > W^{pc}$ (zone VI)
Appendix B.3. Proof of propositions 3

Based on Fig. 3, the social welfare ranking will be:

- $W_{ful} > W_{apc} > W_B$ (zone I)
- $W_{apc} > W_{ful} > W_B$ (zone II)
- $W_{apc} > W_B > W_{ful}$ (zone III)
- $W_B > W_{apc} > W_{ful}$ (zone IV)
Appendix B.4. Proof of propositions 4

Based on Fig. 4, the social welfare ranking will be:

- \( W^{ful} > W^B > W^{bpc} \) (zone I)
- \( W^{ful} > W^{bpc} > W^B \) (zone II)
- \( W^{bpc} > W^{ful} > W^B \) (zone III)
- \( W^{bpc} > W^B > W^{ful} \) (zone IV)
- \( W^B > W^{bpc} > W^{ful} \) (zone V)
- \( W^B > W^{ful} > W^{bpc} \) (zone VI)
Appendix B.5. Proof of proposition 5.

Based on Fig. 5, the social welfare ranking will be:

- $W^{ful} > W^B > W^{pc}$ (zone I)
- $W^{ful} > W^{pc} > W^B$ (zone II)
- $W^{pc} > W^{ful} > W^B$ (zone III)
- $W^{pc} > W^B > W^{ful}$ (zone IV)
- $W^B > W^{pc} > W^{ful}$ (zone V)
- $W^B > W^{ful} > W^{pc}$ (zone VI)

Figure 5
Appendix B.6. Proofs of lemma 1

Figure 6 represent the regions where the following inequalities are satisfied.

- $V_{\text{ful}} > V^B$ in zones I,II,III, VIII, IX and XI, otherwise $V_{\text{ful}} \leq V^B$.
- $V_{\text{ful}} > V_{\text{pc}}$ in zones I,II,IV,VII,VIII and IX, otherwise $V_{\text{ful}} \leq V_{\text{pc}}$.
- $V_{\text{pc}} > V^B$ in zones II,III VI, IX,X and XI, otherwise $V_{\text{pc}} \leq V^B$.
Appendix B.7. Proofs of lemma 2

First, according to Figure 6, Zones I, II, VIII and IX represent the regions where full cooperation is the best for CSR-corp. and thus she will propose to cooperate to both FP firms. In Figure 7, are shown three areas inside the regions:

- **Gray area:** It represents that full cooperation will be agreed because $\pi_{iF}^{full} \geq \pi_{2F}^{pc}, i = 1, 2$ is satisfied. That is, none of FP firms would deviate from cooperation and thus both accept to cooperate.

- **Pink area:** It represents that partial cooperation will be agreed. If firms don’t accept full cooperation because $\pi_{iF}^{full} < \pi_{2F}^{pc}, i = 1, 2$, and given that $V^{pc} > V^B$ in zone IX, CSR corp. would suggest partial Cooperation to one FP firm, let say 1F. The latter would accept it if to cooperate is more profitable than to compete $\pi_{1F}^{pc} \geq \pi_{1F}^B$.

- **Yellow area:** It represents that competition would take place.

![Figure 7: $V^{full}$ is the highest](image)

Second, according to Figure 6, Zones III, VI, X and XX represent the regions where partial cooperation is the best for CSR-corp. and thus she will propose to cooperate to one FP firm, let say 1F. In Figure 8a, are shown two areas inside the regions:

- **Purple area:** It represents that partial cooperation will be agreed because $\pi_{1F}^{pc} \geq \pi_{1F}^B$ is satisfied and thus, FP firm 1F accepts to cooperate.
• Blue area: It represents that full cooperation will be agreed because $V_{ful} > V_B$ in zone III and $\pi_{iF}^{ful} \geq \pi_{iF}^{pc}$, $i = 1, 2$, are satisfied. That is, given that FP firm 1F wouldn’t accept partial cooperation, CSR corp. suggests full cooperation to both FP firms and both accept it.

Figure 8: $V_{pc}^*$ is the highest