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A Mechanism of Recession that Accompanies Persistent Pareto Inefficiency

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February 2020

Abstract

Recessions are generated by various shocks. In particular, if fundamental shocks change the steady state, severe recessions will be generated. In this paper, I show that when such a shock occurs, it is possible for households to rationally select a Nash equilibrium consisting of a Pareto inefficient transition path to the new steady state in an economy in which households behave according to a procedure that is not based on the expected utilities discounted by the rate of time preference. They select this path because they are non-cooperative and risk averse and want to reach what I call the "maximum degree of comfortability" or MDC. The MDC mechanism behind choosing a Pareto inefficient path is basically the same as that in an economy in which households behave according to the usually assumed procedure based on the rational expectations hypothesis.

JEL Classification code: E00, E10, E32

Keywords: Economic fluctuation; MDC-based procedure; Pareto inefficiency; Rational expectations hypothesis; Recession

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1 INTRODUCTION

Recessions are generated by various shocks (e.g., Rebelo, 2005; Blanchard, 2009; Ireland, 2011; Schmitt-Grohé and Uribe, 2012; McGrattan and Prescott, 2014; Hall, 2016). In particular, if a fundamental shock or shocks cause the steady state to move downwards, a severe recession will be generated. When the steady state is changed downwards by such a shock, households must change their consumption path to the one on which consumption diminishes towards the lower level corresponding to the posterior steady state. This means that the growth rate becomes negative; that is, a recession begins. However, this description of recession raises a question. Pareto efficiency can be maintained only if households largely and immediately (i.e., discontinuously) increase their consumption from that of the prior steady state to a point on the posterior saddle path at the time when the shock occurs, even if they know that the level of consumption at the posterior steady state is lower than before. Do households actually increase consumption largely and abruptly to maintain Pareto efficiency even though they well know that they must soon begin to decrease it?

Harashima (2004, 2009, 2012a, 2018a, 2019b) showed a mechanism by which households do not suddenly increase ("jump" up) their consumption when these shocks occur because they are intrinsically risk averse and non-cooperative. In a strategic situation after such a shock occurs, the possibility exists that risk averse and non-cooperative households choose a Pareto inefficient path (a "Nash equilibrium of a Pareto inefficient path").

However, Harashima's model is constructed on the basis of the rational expectations hypothesis, which has been predominant in economics since it was popularized by Lucas (1972) and Sargent et al. (1973), both of whose papers were based on that of Muth (1961). However, the idea of rational expectations has been criticized for imposing substantial demands on economic agents. To generate rational expectations, households are assumed to do something equivalent to computing complex large-scale non-linear dynamic macro-econometric models. Can a household routinely do such a thing in normal daily life? Evans and Honkapohja (2001) argued that this problem can be solved by introducing a learning mechanism (see also, e.g., Marcet and Sargent, 1989; Ellison and Pearlman, 2011), but this solution is not necessarily regarded as being sufficiently successful because arbitrary learning rules have to be assumed.

Harashima (2018b, 2019a) presented an alternative procedure for a household to reach steady state (the maximum degree of comfortability [MDC]-based procedure), which does not necessitate using the rational expectations hypothesis. This alternative procedure is very simple. A household only has to subjectively estimate its self-assessed value of the combination of its earned (labor) income and wealth (capital) as measured by the wage-to-capital ratio (CWR). It then adjusts its consumption to the point at which it feels most comfortable (its MDC). A household is not required to do anything equivalent to computing a complex model to generate rational expectations, and furthermore, it is not even required to be aware of any sort of economic model. Moreover, households naturally reach a steady state that can be interpreted as the same steady state reached by the conventionally assumed procedure that relies on generating rational expectations based on the rate of time preference (RTP), which I refer to as the "RTP-based procedure" in this paper.

The purpose of this paper is to examine the mechanism of recession in an economy in which households behave according to the MDC-based procedure. I show that the mechanism under the MDC-based procedure works basically in the same manner as that under the RTP-based procedure shown in Harashima (2004, 2009, 2012a, 2018a, 2019b). When a shock on MDC occurs, each household has to adjust its capital (wealth) to satisfy its new MDC. However, because households are risk averse and non-cooperative and therefore behave strategically, they make their decisions based on strategic considerations. As a result, a Pareto inefficient process of capital adjustment after the shock can be chosen by households.

Although the mechanisms under the MDC- and RTP-based procedures work basically in the same manner, there is an important difference between them. The expected utilities need to be discounted by RTP under the RTP-based procedure, but they need not be under the MDC-based procedure.

2 MDC-BASED PROCEDURE

The MDC-based procedure is explained in brief in this section on the basis of Harashima (2019a).

2.1 "Comfortability" of CWR

Let k_t and w_t be the capital and wage (labor income) per capita, respectively, in period t. Under the MDC-based procedure, a household should first subjectively evaluate the value of $\frac{\breve{w}_t}{\breve{k}_t}$, where $\breve{w}_t$ and $\breve{k}_t$ are w_t and k_t of the household, respectively (i.e., how much labor income it earns and how much capital (wealth) it possesses). Let Γ be the subjective valuation of $\frac{\breve{w}_t}{\breve{k}_t}$ by a household and Γ_i be the value of $\frac{\breve{w}_t}{\breve{k}_t}$ of household i ($i = 1, 2, 3, \ldots, M$). The household should next assess whether it feels comfortable with its current Γ, that is, its combination of income and capital. "Comfortable" in this context means "at ease," "not anxious," and other related feelings.

Let the "degree of comfortability" (DOC) represent how comfortable a household feels with its Γ. The higher the value of DOC, the more a household feels comfortable with its Γ. For each household, there will be a most comfortable CWR value because the household will feel less comfortable if CWR is either too high or too low. Therefore, for each household, a maximum DOC exists. Let $\tilde{s}$ be a household's state at which its DOC is the maximum (MDC), and let $\Gamma(\tilde{s})$ be a household's Γ when it is at $\tilde{s}$. $\Gamma(\tilde{s})$ therefore indicates the Γ that gives a household its MDC, and $\Gamma(\tilde{s}_i)$ is household i's Γ_i when it is at $\tilde{s}_i$.

2.2 A homogeneous population

I first examine the behavior of households in a homogeneous population, that is, the case where all households are identical.

2.2.1 Rules

Household i should act according to the following rules:

Rule 1-1: If household i feels that the current Γ_i is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption for any i.
Rule 1-2: If household i feels that the current Γ_i is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption until it feels that Γ_i is equal to $\Gamma(\tilde{s}_i)$ for any i.

2.2.2 Steady state

Households can reach a steady state even if they behave only according to Rules 1-1 and 1-2. Let S_t be the state of the entire economy in period t, and $\Gamma(S_t)$ be the value of $\frac{w_t}{k_t}$ of the entire economy at S_t (i.e., the economy's average CWR). In addition, let $\tilde{S}_{MDC}$ be the steady state at which MDC is achieved and kept constant by all households, and $\Gamma(\tilde{S}_{MDC})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC}$. Let also $\tilde{S}_{RTP}$ be the steady state in a Ramsey-type growth model in which households discount utilities by θ where θ (> 0) is the RTP of a household, and $\Gamma(\tilde{S}_{RTP})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP}$.

Proposition 1: If households behave according to Rules 1-1 and 1-2, and if the value of θ that is calculated from the values of variables at $\tilde{S}_{MDC}$ is used as the value of θ in the Ramsey-type growth model, then $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{S}_{RTP})$.
Proof: See Harashima (2018b, 2019a).

Proposition 1 indicates that we can interpret that $\tilde{S}_{MDC}$ is equivalent to $\tilde{S}_{RTP}$. This means that the MDC-based and RTP-based procedures can function equivalently and that MDC is substitutable for RTP as a guide for household behavior.

2.3 A heterogeneous population

In actuality, households are not identical—they are heterogeneous—and if heterogeneous households behave unilaterally, there is no guarantee that a steady state other than corner solutions exists (Becker, 1980; Harashima, 2010, 2012b, 2017). However, Harashima (2010, 2012b, 2017) showed that a sustainable heterogeneity (SH), at which all optimality conditions of all heterogeneous households are simultaneously satisfied, exists under the RTP-based procedure. Moreover, Harashima (2018b, 2019a) showed that SH also exists under the MDC-based procedure, but Rules 1-1 and 1-2 have to be revised and a rule for the government should be added in a heterogeneous population.

Suppose that households are heterogeneous in MDC and for simplicity that they are identical except for MDC. Let $\tilde{S}_{MDC,SH}$ be the steady state at which MDC is achieved and kept constant by any household (i.e., SH in a heterogeneous population under the MDC-based procedure), and let $\Gamma(\tilde{S}_{MDC,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{MDC,SH}$. In addition, let Γ_R be a household's numerically adjusted value of Γ for SH based on the information it has about its estimated values of $\Gamma(\tilde{S}_{MDC,SH})$. Specifically, let $\Gamma_{R,i}$ be Γ_R of household i, and let T be the net transfer that a household receives from the government with regard to SH. Specifically, let T_i be the net transfer that household i receives (i = 1,2,3, …, M).

2.3.1 Revised and additional rules

Household i should act according to the following rules in a heterogeneous population:

Rule 2-1: If household i feels that the current $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$, it maintains the same level of consumption as before for any i.
Rule 2-2: If household i feels that the current $\Gamma_{R,i}$ is not equal to $\Gamma(\tilde{s}_i)$, it adjusts its level of consumption or revises its estimated value of $\Gamma(\tilde{S}_{MDC,SH})$ so that it perceives that $\Gamma_{R,i}$ is equal to $\Gamma(\tilde{s}_i)$ for any i.

At the same time, a government should act according to the following rule:

Rule 3: The government adjusts T_i for some i if necessary so as to make the number of votes cast in response to increases in the level of economic inequality equivalent to that cast in response to decreases in economic inequality in elections.

2.3.2 Steady state

Even if households and the government behave according to Rules 2-1, 2-2, and 3, there is no guarantee that the economy can reach $\tilde{S}_{MDC,SH}$. However, thanks to the government's intervention, SH can be approximately achieved. Let $\tilde{S}_{MDC,SH,ap}$ be the state at which $\tilde{S}_{MDC,SH}$ is approximately achieved (see Harashima, 2018b, 2019a) and $\Gamma(\tilde{S}_{MDC,SH,ap})$ be $\Gamma(S_t)$ at $\tilde{S}_{MDC,SH,ap}$ on average. Here, let $\tilde{S}_{RTP,SH}$ be the steady state that satisfies SH under the RTP-based procedure when households are identical except for their RTPs, and let $\Gamma(\tilde{S}_{RTP,SH})$ be $\Gamma(S_t)$ for $S_t = \tilde{S}_{RTP,SH}$.

Proposition 2: If households are identical except for their values of $\Gamma(\tilde{s})$ and behave unilaterally according to Rules 2-1 and 2-2, if the government behaves according to Rule 3, and if the value of θ_i that is calculated back from the values of variables at $\tilde{S}_{MDC,SH,ap}$ is used as the value of θ_i for any i under the RTP-based procedure in which households are identical except for their RTPs, then $\Gamma(\tilde{S}_{MDC,SH,ap}) = \Gamma(\tilde{S}_{RTP,SH})$.
Proof: See Harashima (2018b, 2019a).

Proposition 2 indicates that we can interpret that $\tilde{S}_{MDC,SH,ap}$ is equivalent to $\tilde{S}_{RTP,SH}$. No matter what values of T, Γ_R, and $\Gamma(\tilde{S}_{MDC,SH})$ are severally estimated by households, any $\tilde{S}_{MDC,SH,ap}$ can be interpreted as the objectively correct and true steady state. In addition, a government need not necessarily provide the objectively correct T_i for $\tilde{S}_{MDC,SH,ap}$ even though the $\tilde{S}_{MDC,SH,ap}$ is interpreted as objectively correct and true.

2.4 "Undiscounted" utility

Although households do not use RTP (more precisely, they cannot know their "correct" and "true" values of RTP) under the MDC-based procedure, they still behave considering the utilities they obtain from consumption. A household's utility v is a function of c, which measures the level of current or future consumption estimated by a household. It is important to note that c is simply an estimated value, and it is not discounted by RTP. For simplicity, a typical constant relative risk aversion (CRRA) utility function is assumed such that

$$v = \frac{c^{1-\gamma}}{1-\gamma} \qquad \text{if } \gamma \neq 1$$
$$v = \ln c \qquad \text{if } \gamma = 1 \qquad (1)$$

and thereby

$$\gamma = -\frac{c\frac{d^2v}{dc^2}}{\frac{dv}{dc}} (> 0) .$$

3 MDC SHOCK

3.1 *Shock on the estimated* $\Gamma(\tilde{S}_{MDC,SH})$

3.1.1 Vulnerability of $\tilde{S}_{MDC,SH,ap}$

$\tilde{S}_{MDC,SH,ap}$ crucially depends on the estimated values of a few variables, in particular, $\Gamma(\tilde{S}_{MDC,SH})$, T, and Γ_R. Because these values are generally estimated with incomplete information, $\tilde{S}_{MDC,SH,ap}$ is vulnerable to various shocks and can occasionally fluctuate widely. Vulnerabilities will emerge because of the following factors.

- Limited information: A household can access only limited information about various aspects of the economy to estimate these values through its own direct experiences. In addition, publicly disseminated information will not necessarily be comprehensive, and more importantly, it may not necessarily be correct and may even be purposefully misleading or incorrect.
- Permanent capital and income: The value of CWR should be modified by removing any temporal elements, but this modification may not be easy.
- Capital or wealth: Conceptually, CWR should be the ratio of labor income to capital, not wealth. However, it seems likely that many households would use wealth as a substitute for capital, but the prices of various kinds of wealth fluctuate more widely and frequently than both the general price level and the price of capital.

3.1.2 Revision of the estimated values of $\Gamma(\tilde{S}_{MDC,SH})$

Because of these vulnerabilities, households will occasionally revise their estimated values of $\Gamma(\tilde{S}_{MDC,SH})$, T, and Γ_R when new pieces of information arrive or some kinds of shocks are recognized. In some cases, the estimated values of $\Gamma(\tilde{S}_{MDC,SH})$ of many households may be simultaneously revised. I call this kind of phenomenon a "MDC shock."

Suppose, for example, that because of some new information that becomes simultaneously known to all households, the estimated values of $\Gamma(\tilde{S}_{MDC,SH})$ of all households are simultaneously revised upward, and all households perceive this simultaneous revision. Because of the increases in the estimated values of $\Gamma(\tilde{S}_{MDC,SH})$, households begin to estimate that w_t and the net transfers they receive in the future from the government will become smaller than they had previously estimated. In other words, they begin to feel poorer. Because they begin to estimate smaller incomes while the amounts of capital remain unchanged, households begin to feel that their current values of Γ_R are smaller than previously estimated. Therefore, they begin to adjust the values of Γ_R upward to make them equal to their values of $\Gamma(\tilde{s})$ by adjusting consumption according to Rule 2-2.

Adjusting the value of Γ_R upward means that part of a household's accumulated capital has become excessive and must be reduced, but how can it be reduced? One possibility is that a household temporarily increases its consumption by a large amount to reduce its accumulated capital (following Rule 2-2). However, it seems unlikely that a household would do so because it now feels poorer than it did before lowering its estimated future income. In addition, an individual household may act differently from other households because it estimates a different new $\Gamma(\tilde{S}_{MDC,SH})$ and thereby also a different lower future income. Because households are non-cooperative with regard to consumption, they will behave strategically after an MDC shock, considering the actions taken by other households. Because the overall process of capital reduction in the economy will differ depending on how each household acts, the impact of simultaneous revisions on a household will differ depending not only on its own action but also on the actions of other households.

3.2 Strategic behaviors and options

For simplicity, I examine the effect of a MDC shock in a homogeneous population; that is, I assume that all households are identical and they behave according to Rules 1-1 and 1-2, not Rules 2-1 and 2-2. In a homogeneous population, a MDC shock is equivalent to a shock on $\Gamma(\tilde{s})$ because $\Gamma(\tilde{S}_{MDC}) = \Gamma(\tilde{s})$ and is identical for all households. Note, however, that even if the effect of a MDC shock is examined in the framework of a heterogeneous population, the result is basically the same as that in the framework of a homogeneous population because each household is "linked" via $\tilde{S}_{MDC,SH,ap}$ in a heterogeneous population. Harashima (2018b, 2019a) showed that, because of these links, all heterogeneous households respond to a shock in the same manner, and that the structure of the links at the posterior $\tilde{S}_{MDC,SH,ap}$ is almost same as that at the previous $\tilde{S}_{MDC,SH,ap}$. Therefore, the effect of a MDC shock is basically the same whether a homogeneous or heterogeneous population is assumed.

Households are risk averse as described by equation (1), behave non-cooperatively, and determine their actions strategically after a MDC shock in a similar manner to that described by Harashima (2004, 2009, 2012a, 2018a, 2019b) for the case of a RTP shock under the RTP-based procedure. Suppose that an upward MDC shock occurs; that is, the $\Gamma(\tilde{s})$ of all households is simultaneously changed upward. It is assumed for simplicity that there are two options for each household with regard to consumption just after the shock. The first is a jump option (**J**), in which a household immediately increases its consumption largely to reduce its capital. If most households choose option **J**, the amount of capital in the economy steadily decreases to the level consistent with the new $\Gamma(\tilde{S}_{MDC})$. In this case, no resources are wasted or destroyed thanks to the increased consumption. The second option is non-jump (**NJ**), in which a household does not increase its consumption but instead decreases it directly to the level consistent with the new $\Gamma(\tilde{S}_{MDC})$. If all households choose the **NJ** option, large amounts of resources are simply destroyed to reduce capital. The difference of the level of consumption during the transition period to the new $\Gamma(\tilde{S}_{MDC})$ between the **J** and **NJ** options is measured by b (> 0), and therefore the level of consumption in the case of **J** is higher than that of **NJ** by b during the transition.

Although all households are identical, the choice a household makes after the shock can be different from that of other households because they are non-cooperative and behave strategically. Let **Jalone** indicate that a household chooses the **J** option but the other households choose the **NJ** option, **NJalone** indicate that the household chooses the **NJ** option but the other households choose the **J** option, **Jtogether** indicate that all households choose the **J** option, and **NJtogether** indicate that all households choose the **NJ** option. It is assumed that, when a household chooses a different option from that of the other households, the difference in the accumulation of financial assets resulting from the difference in consumption during the transition period to the new $\Gamma(\tilde{S}_{MDC})$ is reflected in consumption after reaching the new $\Gamma(\tilde{S}_{MDC})$. That is, the difference in the return on financial assets is added to (or subtracted from) the household's consumption after reaching the new $\Gamma(\tilde{S}_{MDC})$.

4 RATIONAL CHOICE OF A PARETO INEFFICIENT PATH

4.1 Undiscounted utility after an upward MDC shock

As noted in Section 2.4, future utility is not discounted by RTP under the MDC-based procedure, and households estimate only "undiscounted" future utilities.

4.1.1 Undiscounted utility

4.1.1.1 Undiscounted utility in the case of NJtogether

Let $\bar{c}$ be the level of consumption for a household after an upward MDC shock in the case of **NJtogether**. In this case, the undiscounted utility that a household estimates is described as $\tilde{v}(\bar{c})$ both during the transition period to the new $\tilde{S}_{MDC}$ and after reaching the new $\tilde{S}_{MDC}$.

4.1.1.2 Undiscounted utility during the transition in the case of NJalone

The undiscounted utility during the transition in the case of **NJalone** is also $\tilde{v}(\bar{c})$ because the level of consumption ($\bar{c}$) during the transition period is the same as that in the case of **NJtogether**.

4.1.1.3 Undiscounted utility during the transition in the case of Jtogether and Jalone

Choosing option **J** (**Jtogether** or **Jalone**) indicates that a household experiences a higher level of consumption during the transition period by b (> 0) than it does in the case of choosing option **NJ** as discussed in Section 3.2. Hence, the undiscounted utilities in the cases of **Jtogether** and **Jalone** during the transition period are both estimated to be $\tilde{v}(\bar{c} + b)$.

4.1.1.4 Undiscounted utility after reaching the new $\tilde{S}_{MDC}$ in the case of Jtogether

The undiscounted utility after reaching the new $\tilde{S}_{MDC}$ in the case of **Jtogether** is $\tilde{v}(\bar{c})$ because no household has extra capital after reaching the new $\tilde{S}_{MDC}$ (as is also the case with **NJtogether**). Therefore, the level of consumption after a household reaches the new $\tilde{S}_{MDC}$ is the same $\bar{c}$ as in the case of **NJtogether**.

4.1.1.5 Undiscounted utility after reaching the new $\tilde{S}_{MDC}$ in the case of Jalone

If a household chooses **Jalone**, it accumulates fewer financial assets because of its greater consumption as compared with the **NJ** households. As a result, it reduces consumption by $\bar{a}$ from $\bar{c}$ after reaching the new $\tilde{S}_{MDC}$ because it has a smaller amount of financial assets, as assumed in Section 3.2. Hence, the undiscounted utility after reaching the new $\tilde{S}_{MDC}$ in the case of **NJalone** is estimated to be $\tilde{v}(\bar{c} - \bar{a})$.

4.1.1.6 Undiscounted utility after reaching the new $\tilde{S}_{MDC}$ in the case of NJalone

If a household chooses **NJalone**, it accumulates more financial assets because of the difference in consumption as compared to **J** households. As a result, it consumes $\bar{a}$ in addition to $\bar{c}$ using the extra accumulated financial assets after reaching the new $\tilde{S}_{MDC}$, as assumed in Section 3.2. Hence, the undiscounted utility in the case of **NJalone** after the household reaches the new $\tilde{S}_{MDC}$ is estimated to be $\tilde{v}(\bar{c} + \bar{a})$.

For this discussion, I assume that $0 < \bar{a} < b$ and therefore $\tilde{v}(\bar{c} + b) > \tilde{v}(\bar{c} + \bar{a})$. Note, however, that even if the assumption is $0 < b < \bar{a}$, the main conclusion of this paper does not change (see Section 4.3.2).

4.1.2 The undiscounted utility in each case

The undiscounted utilities a household estimates are summarized below:

	During the transition	After reaching the new $\tilde{S}_{MDC}$
Jtogether:	$\tilde{v}(\bar{c}+b)$	$\tilde{v}(\bar{c})$
NJtogether:	$\tilde{v}(\bar{c})$	$\tilde{v}(\bar{c})$
NJalone:	$\tilde{v}(\bar{c})$	$\tilde{v}(\bar{c}+\bar{a})$
Jalone:	$\tilde{v}(\bar{c}+b)$	$\tilde{v}(\bar{c}-\bar{a})$

Let $E(Jalone)$, $E(NJalone)$, $E(Jtogether)$, and $E(NJtogether)$ be the undiscounted utilities of **Jalone**, **NJalone**, **Jtogether**, and **NJtogether**, respectively, that are estimated by a household. Based on the results shown above, their values can be defined as:

$$E(Jtogether) = \tilde{v}(\bar{c}+b)+\tilde{v}(\bar{c}) \tag{2}$$
$$E(NJtogether) = \tilde{v}(\bar{c})+\tilde{v}(\bar{c}) \tag{3}$$
$$E(NJalone) = \tilde{v}(\bar{c})+\tilde{v}(\bar{c}+\bar{a}) \tag{4}$$
$$E(Jalone) = \tilde{v}(\bar{c}+b)+\tilde{v}(\bar{c}-\bar{a})\ . \tag{5}$$

4.1.3 Undiscounted utilities for the J and NJ options

Let $E(J)$ and $E(NJ)$ be the undiscounted utilities estimated by a household if it chooses the **J** and **NJ** options, respectively. Let also $p(0 \leq p \leq 1)$ be the subjective probability of a household that all the other households choose option **J** (e.g., $p = 0$ indicates that all the other households choose option **NJ**). With p, the undiscounted utility estimated by a household choosing option **J** is

$$E(J) = pE(Jtogether) + (1-p)E(Jalone)\ , \tag{6}$$

and the undiscounted utility estimated by a household choosing option **NJ** is

$$E(NJ) = pE(NJalone) + (1-p)E(NJtogether)\ . \tag{7}$$

4.2 Rational choice of the NJ option

4.2.1 Choice between $E(J)$ and $E(NJ)$

A household's decision on whether to choose option **J** or **NJ** is determined based on the values of $E(J)$ and $E(NJ)$. If $E(J) - E(NJ) > 0$, the **J** option will be chosen, but if $E(J) - E(NJ) < 0$, the **NJ** option will be chosen. By equations (6) and (7),

$$\begin{aligned} &E(J) - E(NJ) \\ &\quad = p[E(Jtogether) - E(NJalone)] + (1-p)[E(Jalone) - E(NJtogether)]. \end{aligned} \tag{8}$$

Here, because $b > \bar{a}$, by equations (2) and (4),

$$E(Jtogether) - E(NJalone) = \tilde{v}(\bar{c} + b) - \tilde{v}(\bar{c} + \bar{a}) > 0 \, .$$

That is, always

$$E(Jtogether) - E(NJalone) > 0 \, . \tag{9}$$

On the other hand, by equations (3) and (5),

$$E(Jalone) - E(NJtogether) = \tilde{v}(\bar{c} + b) - \tilde{v}(\bar{c}) + \tilde{v}(\bar{c} - \bar{a}) - \tilde{v}(\bar{c}) \, . \tag{10}$$

Because

$$\begin{aligned} &\tilde{v}(\bar{c} + b) - \tilde{v}(\bar{c}) > 0 \text{ and} \\ &\tilde{v}(\bar{c} - \bar{a}) - \tilde{v}(\bar{c}) < 0 \end{aligned}$$

for any $\gamma(> 0)$ by equation (1), the sign of $E(Jalone) - E(NJtogether)$ will differ depending on the value of parameter γ. As a result, by equation (8), the sign of $E(J) - E(NJ)$ will also differ depending on the value of γ.

4.2.2 Positive probability of choosing option NJ

Here,

$$\lim_{\gamma\to\infty} \left\{ \frac{1-\gamma}{\bar{c}^{1-\gamma}} [\tilde{v}(\bar{c} + b) - \tilde{v}(\bar{c})] \right\} = \lim_{\gamma\to\infty} \left(\frac{\bar{c}}{\bar{c} + b} \right)^{\gamma - 1} - 1 = -1$$

and

$$\lim_{\gamma\to\infty} \left\{ \frac{1-\gamma}{\bar{c}^{\ 1-\gamma}} [\tilde{v}(\bar{c} - \bar{a}) - \tilde{v}(\bar{c})] \right\} = \lim_{\gamma\to\infty} \left(\frac{\bar{c}}{\bar{c} - \bar{a}} \right)^{\gamma - 1} - 1 = +\infty \, .$$

Therefore,

$$\lim_{\gamma\to\infty}\frac{1-\gamma}{\bar{c}^{\;1-\gamma}}[\tilde{v}(\bar{c}+b)-\tilde{v}(\bar{c})+\tilde{v}(\bar{c}-\bar{a})-\tilde{v}(\bar{c})]=+\infty\ .$$

Because $\lim_{\gamma\to\infty}\frac{1-\gamma}{\bar{c}^{1-\gamma}} < 0$,

$$\lim_{\gamma\to\infty}[\tilde{v}(\bar{c}+b)-\tilde{v}(\bar{c})+\tilde{v}(\bar{c}-\bar{a})-\tilde{v}(\bar{c})]<0\ , \tag{11}$$

and thereby, by equation (10) and inequality (11),

$$\lim_{\gamma\to\infty}[E(Jalone)-E(NJtogether)]<0\ .$$

On the other hand, because $b>\bar{a}$,

$$\lim_{\gamma\to 0}[\tilde{v}(\bar{c}+b)-\tilde{v}(\bar{c})+\tilde{v}(\bar{c}-\bar{a})-\tilde{v}(\bar{c})]=b-\bar{a}>0 \tag{12}$$

and thereby, by equation (10) and inequality (12),

$$\lim_{\gamma\to 0}[E(Jalone)-E(NJtogether)]>0\ .$$

Lemma 1: If γ is sufficiently small, $E(Jalone)-E(NJtogether)>0$.
Proof: Because $\lim_{\gamma\to 0}\{\tilde{v}(\bar{c}+b)-\tilde{v}(\bar{c})+\tilde{v}(\bar{c}-\bar{a})-\tilde{v}(\bar{c})\}>0$ as indicated by inequality (12), then by equation (10), if $\gamma(>0)$ is sufficiently small, $E(Jalone)-E(NJtogether)>0$. ■

Lemma 2: If γ is sufficiently large, $E(Jalone)-E(NJtogether)<0$.
Proof: Because $\lim_{\gamma\to\infty}[\tilde{v}(\bar{c}+b)-\tilde{v}(\bar{c})+\tilde{v}(\bar{c}-\bar{a})-\tilde{v}(\bar{c})]<0$ as indicated by inequality (11), then by equation (10), if γ is sufficiently large, $E(Jalone)-E(NJtogether)<0$. ■

Lemmas 1 and 2 indicate that if γ is larger than a critical value, $E(Jalone)-E(NJtogether)<0$.

Lemma 3: There is a $\gamma^*(>0)$ such that if $\gamma^*<\gamma$, $E(Jalone)-E(NJtogether)<0$.
Proof: If $\gamma(>0)$ is sufficiently small, then $E(Jalone)-E(NJtogether)>0$ by

Lemma 1, and if γ is sufficiently large, then $E(Jalone) - E(NJtogether) < 0$ by Lemma 2. Hence, there is a certain $\gamma^*(> 0)$ such that, if $\gamma^* < \gamma$, $E(Jalone) - E(NJtogether) < 0$. ■

Lemma 3 and Equation (8) indicate that, for some values of p, the inequality $E(J) - E(NJ) < 0$ will be true.

Proposition 1: If $\gamma^* < \gamma$, then there is a p^* ($0 \leq p^* \leq 1$) such that if $p = p^*$, $E(J) - E(NJ) = 0$, and if $p < p^*$, $E(J) - E(NJ) < 0$.

Proof: By equation (8), $E(J) - E(NJ) = p[E(Jtogether) - E(NJalone)] + (1 - p)[E(Jalone) - E(NJtogether)]$. Here, $E(Jtogether) - E(NJalone) > 0$ always holds as inequality (9) indicates. On the other hand, by Lemma 3, if $\gamma^* < \gamma$, $E(Jalone) - E(NJtogether) < 0$. Therefore, if $\gamma^* < \gamma$, $\lim_{p \to 0} E(J) - \mathrm{E}(NJ) < 0$ and $\lim_{p \to 1} E(J) - E(NJ) > 0$. Hence, by the intermediate value theorem, there is a $p^*(0 \leq p^* \leq 1)$ such that if $p = p^*$, $E(J) - \mathrm{E}(NJ) = 0$, and if $p < p^*$, $E(J) - \mathrm{E}(NJ) < 0$.

■

If p is sufficiently small for a household (i.e., if the household estimates that the other households will most likely not choose option **J**), the household does not choose option **J** (i.e., jump its consumption).

Proposition 1 is the same as the case of an upward RTP shock under the RTP-based procedure of Harashima (2004, 2009, 2012a, 2018a, 2019b).

4.3 Selection of Nash equilibrium

4.3.1 Nash equilibria

A household strategically determines whether to choose the **J** or **NJ** option considering other households' choices. Particularly, the choice depends on how it estimates the value of p.

Because all households are identical, the best response of each household is also identical. In addition, all households know that each of them uses the same process to make a choice. Suppose that there are $H(\in N)$ households in the economy, where H is sufficiently large. Let $q_\eta(0 \leq q_\eta \leq 1)$ be the probability that household $\eta(\in H)$ chooses option **J**. Because H is sufficiently large, the average undiscounted utility of the other households almost equals that of all households, and therefore the average undiscounted utility of the other households that choose the **J** and **NJ** options are $E(Jtogether)$ and $E(NJtogether)$. Hence, the payoff matrix of the H-dimensional symmetric mixed-strategy game can be described as shown in Table 1.

Table 1: The payoff matrix for the mixed-strategy game when choosing a consumption option (J or NJ)

		Any other household	
		J	**NJ**
A household	**J**	$E(Jtogether),\ E(Jtogether)$	$E(Jalone),\ E(NJtogether)$
	NJ	$E(NJalone),\ E(Jtogether)$	$E(NJtogether),\ E(NJtogether)$

Each household determines its behavior on the basis of this payoff matrix. In this mixed-strategy game, strategy profiles

$$(q_1,q_2,\ldots,q_H) = \{(1,1,\ldots,1), (p^*, p^*,\ldots, p^*), (0,0,\ldots,0)\}$$

are Nash equilibria for the following reason. By Proposition 1, the best response of household η is **J** (i.e., $q_\eta = 1$) if $p > p^*$, it is indifferent between **J** and **NJ** (i.e., any $q_\eta \in [0,1]$) if $p = p^*$, and it chooses **NJ** (i.e., $q_\eta = 0$) if $p < p^*$. Because all households are identical, the best-response correspondence of each household is identical such that $q_\eta = \{1\}$ if $p > p^*$, $[0,1]$ if $p = p^*$, and $\{0\}$ if $p < p^*$ for any household $\eta \in H$. Hence, the mixed-strategy profiles $(1, 1,\ldots,1)$, $(p^*, p^*,\ldots, p^*)$, and $(0,0,\ldots,0)$ are the intersections of the graph of the best-response correspondences of all households. **Jtogether** $(1,1,\ldots,1)$ and **NJtogether** $(0,0,\ldots,0)$ are pure strategy Nash equilibria, and the strategy profile $(p^*, p^*,\ldots, p^*)$ is a mixed-strategy Nash equilibrium.

4.3.2 Selecting NJtogether equilibrium

Determining which Nash equilibrium, either **NJtogether** $(0,0,\ldots,0)$ or **Jtogether** $(1,1,\ldots,1)$, is dominant requires a refinement of the Nash equilibrium, which necessitates additional criteria. Here, suppose that households have a worst-case aversion preference in the sense that they avoid options that include the worst-case scenario when its probability is not known.

By equations (4) and (5),

$$E(Jalone) - E(NJalone) = \tilde{v}(\bar{c} + b) - \tilde{v}(\bar{c} + \bar{a}) - \tilde{v}(\bar{c}) + \tilde{v}(\bar{c} - \bar{a}) \ , \quad (13)$$

where $\tilde{v}(\bar{c}+b)-\tilde{v}(\bar{c}+\bar{a})>0$ and $-\tilde{v}(\bar{c})+\tilde{v}(\bar{c}-\bar{a})<0$.

By Lemma 3, if $\gamma^{*}<\gamma$,

$$\tilde{v}(\bar{c}+b)-\tilde{v}(\bar{c})<\tilde{v}(\bar{c})-\tilde{v}(\bar{c}-\bar{a}) \ . \tag{14}$$

Because

$$\tilde{v}(\bar{c}+b)-\tilde{v}(\bar{c}+\bar{a})<\tilde{v}(\bar{c}+b)-\tilde{v}(\bar{c}) \ ,$$

then, by inequality (14), if $\gamma^{*}<\gamma$,

$$\tilde{v}(\bar{c}+b)-\tilde{v}(\bar{c}+\bar{a})<\tilde{v}(\bar{c})-\tilde{v}(\bar{c}-\bar{a}). \tag{15}$$

Hence, by equation (13) and inequality (15), if $\gamma^{*}<\gamma$,

$$E(Jalone)-E(NJalone)<0 \ . \tag{16}$$

On the other hand, by equations (3) and (4),

$$E(NJalone)-E(NJtogether)=\tilde{v}(\bar{c}+\bar{a})-\tilde{v}(\bar{c})>0 \ . \tag{17}$$

In addition, by Lemma 3, if $\gamma^{*}<\gamma$,

$$E(Jalone)-E(NJtogether)<0 \ . \tag{18}$$

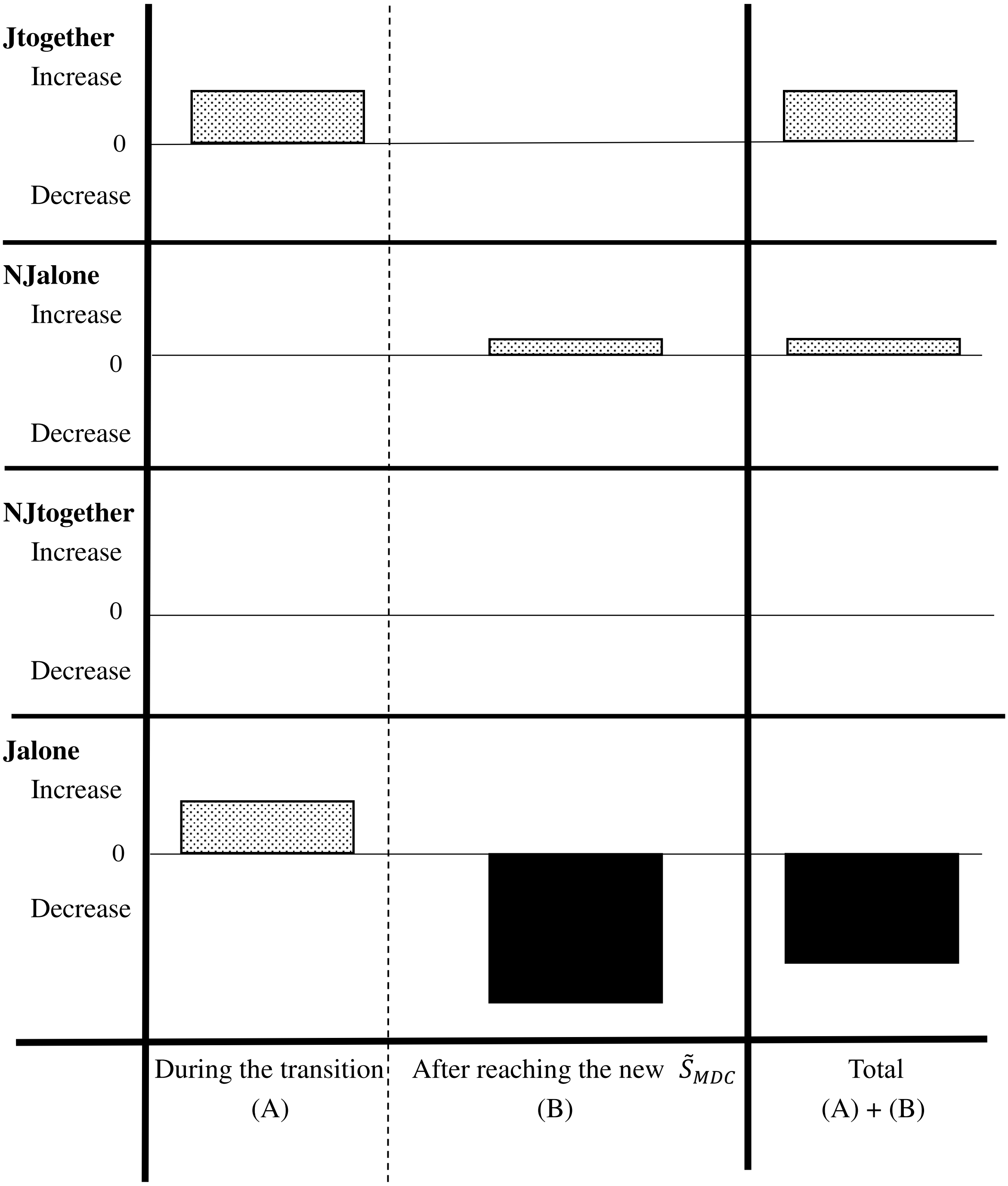
In sum, if $\gamma^{*}<\gamma$, by inequalities (9), (16), (17), and (18),

$$E(Jtogether)>E(NJalone)>E(NJtogether)>E(Jalone) \ . \tag{19}$$

This means that **Jalone** is the worst choice, followed by **NJtogether** and **NJalone**, and that **Jtogether** is the best.

The reason why inequality (19) holds is illustrated in Figure 1. As compared with the undiscounted utility in the case of **NJtogether** ($\tilde{v}(\bar{c})+\tilde{v}(\bar{c})$), the utility in the case of **Jtogether** ($\tilde{v}(\bar{c}+b)+\tilde{v}(\bar{c})$) is larger. It is also larger than that in the case of **NJalone** $\tilde{v}(\bar{c})+\tilde{v}(\bar{c}+\bar{a})$), but not by quite as much. In the case of **Jalone** ($\tilde{v}(\bar{c}+b)+\tilde{v}(\bar{c}-\bar{a})$), the undiscounted utility is much smaller, particularly after reaching the new $\tilde{S}_{MDC}$, although it is larger during the transition period.

Figure 1: Increases and decreases in undiscounted utilities from $\tilde{v}(\bar{c})$ when γ is sufficiently large



If households have a worst-case aversion preference as assumed above and avoid the option that includes the worst-case scenario when they have no information on its probability, a household will prefer the **NJ** option, in avoidance of the possible worst-case situation (i.e., **Jalone**). Since all households are identical and know the order of preference shown by inequality (19), all households will equally suppose that they all prefer the **NJ** option; therefore, all of them will suppose a very low p, particularly $p = 0$, and select the **NJtogether** (0,0,…,0) equilibrium. This result is the same as that in the case of an upward RTP shock under the RTP-based procedure of Harashima (2004, 2009, 2012a, 2018a, 2019b).

Note that if $0 < b < \bar{a}$ (not $0 < \bar{a} < b$), then if $\gamma^* < \gamma$,

$$E(NJalone) > E(Jtogether) > E(NJtogether) > E(Jalone) .$$

Clearly, **Jalone** is still the worst choice, and all households will choose the **NJ** option even if $0 < b < \bar{a}$.

4.3.3 Persistently high unemployment rates

NJtogether indicates that large amounts of resources (i.e., b) are simply destroyed during the transition period by leaving them unused, discarding them, or preemptively not producing them. Hence, Pareto efficiency is not satisfied during the transition. Nevertheless, households still take the **NJtogether** path because it is the best path. As a result, the economy falls into a recession or even depression, which is accompanied by persistently high unemployment rates (Harashima, 2012a, 2019b).

4.4 Downward MDC shock

What will happen in the case of a downward MDC shock, that is, if $\Gamma(\tilde{s})$ of all households is simultaneously changed downward? Suppose again that all households are identical and therefore $\Gamma(\tilde{s})$ is identical among them. Because of the decrease in $\Gamma(\tilde{s})$, households begin to estimate that w_t will become larger in the future than previously estimated. In other words, they begin to feel richer and therefore to feel that the current value of Γ is larger than previously estimated. Hence, they begin to adjust the value of Γ downward to make it again equal to the value of $\Gamma(\tilde{s})$ by adjusting consumption. Adjusting the value of Γ downward means that capital must be increased. Households may decrease their consumption temporarily to increase their capital, but it seems unlikely that households decrease their consumption if they feel richer. Therefore, risk averse and non-cooperative households will instead begin to increase consumption after strategically considering the consequences of their choices. As a result, an economic boom will begin and capital and labor will begin to be overused.

5 CONCLUDING REMARKS

Recessions are generated by various shocks. Particularly, if a fundamental shock changes the steady state, a severe recession will be generated. To keep Pareto efficiency after such a shock, households have to discontinuously change their consumption. However, such discontinuous large changes in consumption seem to be very unlikely because households are intrinsically risk averse. Harashima (2004, 2009, 2012a, 2018a, 2019b) showed a mechanism by which risk averse and non-cooperative households do not change their consumption discontinuously when the steady state is changed as a result of strategic considerations. That is, in some strategic situations, risk averse and non-cooperative households strategically choose a "Nash equilibrium of a Pareto inefficient path."

Harashima's model is, however, constructed on the basis of the rational expectations hypothesis (i.e., the RTP-based procedure), but this hypothesis has been criticized for imposing substantial demands on economic agents. As an alternative, Harashima (2018b, 2019a) presented the MDC-based procedure, which also shows how households reach the steady state. In this paper, I showed a mechanism by which a Nash equilibrium of a Pareto inefficient path is selected by households under the MDC-based procedure. This mechanism works basically in the same manner as that when households behave according to the RTP-based procedure. When a shock on CWR occurs, each household has to adjust its capital (wealth) to satisfy its MDC. However, because households are risk averse and non-cooperative and behave strategically, they make decisions based on strategic considerations. As a result, a Pareto inefficient process of capital adjustment after the shock can be chosen by households.

Although the mechanisms under the MDC- and RTP-based procedures are basically the same, there is an important difference between them. Household expected utilities have to be discounted by RTP under the RTP-based procedure, but they do not need to be discounted under the MDC-based procedure.

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