RD incentives with uncertain probability of success

Chatterjee, Rittwik and Chattopadhyay, Srobonti and Kabiraj, Tarun

Indian Statistical Institute, Vidyasagar College for Women, Indian Statistical Institute

31 January 2020

R&D incentives with uncertain probability of success

Rittwik Chatterjee*
Srobonti Chattopadhyay†
Tarun Kabiraj‡§¶

Abstract

A firm's decision to invest in R&D depends on a number of factors like availability of funds, extent of R&D spillovers, market structure, and success probability. However, probability of success depends, to a large extent, on factors endogenous to a firm. This means, success probability can be known to the firm undertaking R&D investment, not to the rivals, hence there is incomplete information about probability of success in R&D. There are also uncertainties about rival's R&D decision and R&D status. In a duopoly we show that there is a non-monotone relation between R&D incentives and the level of information.

Keywords R&D incentives, Duopoly, Incomplete information, Type distribution

JEL Classification D43, D82, L13, O31

*Economic Research Unit, Indian Statistical Institute, **Email:** rittwik@gmail.com
†Vidyasagar College for Women, Kolkata, **Email:** srobonti@gmail.com
‡Indian Statistical Institute, Kolkata, **Email:** tarun@isical.ac.in
§Correspondence to: Srobonti Chattopadhyay, Vidyasagar College for Women, 39, Sankar Ghosh Lane, Kolkata - 7000006, INDIA.
¶Authors would like to thank Prabal Roy Choudhury and Krishnendu Ghosh Dastidar for their helpful comments and suggestions. Any errors are responsibility of the authors alone.

1 Introduction

Research and Development (R&D) activity is considered important from the viewpoint of both society and individual firms in oligopolistic industries. From the perspective of firms, R&D is considered valuable, primarily because of the reason that R&D results either in lower costs of production or development of new products. For both these cases, the firm enjoys an edge over its rivals, by being able to produce at a lower cost or enhance its market. Due to lowering of production cost, and consequently lower selling price, even the consumers benefit, which is refelcted in the form of higher consumer surplus. Consumers also tend to gain from increased product differentiation when R&D results in innovating new and related products. Also, R&D activities push out the technological frontier and thus take the economy to a higher technological plane. R&D may even lead to generation of environment-friendly technologies (e.g. developing less polluting production techniques or reusing wastes from industries or households). For all these reasons, quite often governments also provide incentives for investing in R&D.

However, at the firm level, the intensity of R&D activity depends on various factors including the availability of funds for conducting R&D, possibility of spillovers of R&D outcomes and the probability of success in R&D. In addition, the level of information that a firm possesses about its rivals in an oligopolistic industry may act as a decisive factor. Whether a firm has perfect or imperfect information, or complete or incomplete information about its rivals' various features (like marginal cost of production, R&D status, prospective spillover benefits etc.), may affect a firm's decision to invest in R&D.

Role of incomplete information as a decisive factor behind R&D investment is a relatively less explored area in R&D literature. Some of the known works in this direction include Brocas (2004), Grishagin, Sergeyev and Silipo (2001), Bacchiega and Garella (2008), Conti (2013), Kabiraj and Chattopadhyay (2015) and Chattopadhyay and Kabiraj (2015). Among these, Brocas (2004), Kabiraj and Chattopadhyay (2015) and Chattopadhyay and Kabiraj (2015) discuss the role of incomplete information in choosing the R&D organisation, that is, whether to perform R&D collaboratively or single-handedly. Conti (2013) discusses the same issue in the presence of spillovers. Bacchiega and Garella (2008) talk about a firm's choice between witholding as opposed to disclosing R&D information. Grishagin, Sergeyev and Silipo (2001), on the other hand, elaborate on a firm's choice regarding investment in R&D, but the setup deals more with imperfect information, rather than incomplete information, since the paper discusses a context where firms involve in a patent race, and do not know each other's relative position in terms of R&D - a frim decides to perform R&D only if it starts in the same position as its rival initially. However, the main research question of this paper relates to the issue of rent dissipation in the context of patent race.

Then two recent papers by Chatterjee, Chattopadhyay and Kabiraj (2018, 2019) have addressed directly the issue of how incomplete information may affect the R&D decision. Both these papers consider a Cournot duopoly where the firms decide whether to invest in R&D prior to competition in the product market. The firms always perform R&D non-cooperatively and there is no uncertainty in R&D outcomes. In Chatterjee, Chattopadhyay and Kabiraj (2019) incomplete information arises due to unobservability in the reduction of marginal cost of the rival. Moreover, a firm may or may not know whether the rival has invested in R&D or not. There is, however, no spillover. Chatterjee, Chattopadhyay and Kabiraj (2018) have, on the other hand, analyzed the problem where spillovers are present, but the extent to which a firm can benefit from spillover of rival's R&D outcomes is private information. Both these papers conclude that incomplete information may enhance, under some parametric situations, the incentive to invest in R&D activities compared to the case of complete information.

The present paper seeks to extend the above analysis of incomplete information to another direction. Now, we consider that not only the R&D outcome is uncertain, hence probabilistic, but the firms have incomplete information about the probability of the success of the rival. Therefore, the probability of success in R&D is private information to the concerned firm, that means, the probability of success of R&D for a particular firm constitutes its type.

So, if $p_i \in [0, 1]$ be the probability of success of R&D, then p_i has a probability distribution function $F(.)$ and density function $f(.)$ with full support. Under incomplete information the innovating firm knows p_i, but the rival knows that p_i is drawn from $F(\cdot)$. Firms first simultaneously decide whether to do R&D or not, and if any firm does, it spends an amount $M > 0$. We restrict to the scenario where innovation, if successful, is non-drastic, therefore in the post-innovation situation all firms will operate at a positive output level. Thus the purpose of the paper is to study how R&D incentives of firms get affected when the probability of success is private information. This is not yet studied in R&D literature.

One interesting result, we have derived in this paper, is non-monotonicity of R&D incentive with respect to availability of information about the R&D characteristics of the rival firm. We have rival's type unknown. Then we compare it to the case where both the rival's R&D investment decision and status of R&D result are known (Level I incomplete information); R&D incentive of a firm goes up as R&D status becomes unknown (Level II Incomplete Information). Now if, in addition, R&D investment decision also becomes incomplete information (Level III incomplete information), R&D incentive of a firm goes down. This happens because of the signaling effect (to be discussed in an appropriate place later in this paper) in case of Level II incomplete information as distinct from other cases.

The organisation of the paper is as follows: section 2 specifies the model setup, section 3 describes the benchmark case of complete information framework, section 4 elaborates on the three alternative incomplete information structures, viz. Level I, Level II and Level III. Then section 5 compares the different information structures and derives the important results. Finally, section 6 concludes the paper.

2 Model Setup

Consider two firms, A and B. They compete in quantities in the product market. The market price of the product is given by $P = \max\{0, a - Q\}$, where $a > 0$ is the demand parameter and Q is the aggregate output produced in the market. The marginal cost of each firm is c. The firms can invest in a cost reducing R&D. The cost of research, $M > 0$, is same for both the firms. If a firm invests in R&D and is successful, its marginal cost will be $(c - D)$; $c > D > 0$. The probability of success in R&D for firm i is $p_i \in [0, 1]$. We assume that p_i is private information and is distributed according to distribution function $F(\cdot)$ and density function $f(\cdot)$ with full support. For technical reason we assume $a > c + 3D$.[1]

Consider the following notations: $K := a - c$, $q(x) := \frac{K+x}{3}$, $\Pi(x) := q^2(x)$ and $\Theta(x) := \int_x^1 y \frac{dF(y)}{1-F(x)}$.[2] It is easy to check that $q' > 0$, $\Pi' = \frac{2}{3}q(x) > 0$, $\Theta'(x) > 0$ when $x \in (0, 1)$ and $\lim_{x \to 1} \Theta(x) = 1$.[3]

We further denote 'doing research' by R and 'not doing research' by N, 'success' by S and 'failure' by F. When firm A chooses to invest in R&D and succeeds and firm B does not invest in R&D, we denote profit (or expected profit) of firm A by $\Pi_A^{[SN]}$ ($E\Pi_A^{[SN]}$) and that of firm B by $\Pi_B^{[SN]}$ ($E\Pi_B^{[SN]}$). Similar notations will be used for other cases.

We consider three pieces of information that will affect the decision to invest in R&D of a firm:

1. Type of the rival, that is, whether rival's type is known to the firm.
2. The decision of the rival to invest, that is, whether it is observable whether the rival has invested or not.

[1] This states that the demand is sufficiently high.
[2] $\Theta(x)$ gives the average value of y given that y lies between x and 1.
[3] The intuition is that $\Theta(x)$ must lie between x and 1.

3. The status of the rival's research, that is, whether the rival has come up with a success or failure in R&D when it has invested in R&D.

Our objective is to find out how the decision to perform R&D depends on the type of a firm and the level of information available to it. We construct a two stage game. In the first stage all firms decide simultaneously whether to invest in research or not, and in the second stage they compete in the product market a la Cournot. To facilitate our analysis we first provide the result for complete information model.

3 Complete Information: A Benchmark Case

We assume in this section that everything is common knowledge, including the types of the firms. Since we are considering duopoly, at equilibrium several cases can happen: SS, SF, SN, FS, FF, FN, NS, NF and NN.[4] The lemma below summarizes the payoffs of a firm under different equilibrium situations.

Lemma 1. *The following holds for firm A*

$$\Pi_A^{[NN]} = \Pi_A^{[NF]} = \Pi(0), \qquad \Pi_A^{[FF]} = \Pi_A^{[FN]} = \Pi(0) - M, \qquad \Pi_A^{[SS]} = \Pi(D) - M,$$
$$\Pi_A^{[SN]} = \Pi_A^{[SF]} = \Pi(2D) - M, \qquad \Pi_A^{[FS]} = \Pi(-D) - M, \qquad \Pi_A^{[NS]} = \Pi(-D).$$

Similarly for firm B.

Note that, when a firm is going to invest in R&D, it does not know whether it will succeed or not.

When firm i does not invest in R&D but its rival does then its expected profit is $p_j\Pi(-D) + (1-p_j)\Pi(0)$. When neither of them invests, its expected profit is $\Pi(0)$.

Similarly, the expected profit of firm i when both firms invest is

$$p_j\left[p_i\Pi(D) + (1-p_i)\Pi(-D)\right] + (1-p_j)\left[p_i\Pi(2D) + (1-p_i)\Pi(0)\right] - M.$$

But if firm i alone invests, its expected profit is $p_i\Pi(2D) + (1-p_i)\Pi(0) - M$.

Therefore when the rival is not doing R&D, the expected "gain" from doing R&D of firm i is $p_i\left[\Pi(2D) - \Pi(0)\right] - M$ and when the rival is doing research, the expected "gain" of doing R&D for firm i is $p_i\left[p_j\left(\Pi(D) - \Pi(-D)\right) + (1-p_j)\left(\Pi(2D) - \Pi(0)\right)\right] - M$.

[4] SS denotes the situation that both firms are successful in R&D; SF (or FS) means one firm is successful and the other firm has failed in R&D; SN (NS) implies that one firm has invested and succeeded in R&D while the other firm has not invested in R&D; FN (or NF) implies that one firm has not invested in R&D and the other firm has invested and failed in R&D; FF indicates that both the firms have invested in R&D but failed; and finally NN denotes the case where no firm invests in R&D.

When a firm does not invest in R&D, its type is taken as zero. Define $U(p_i, p_j)$ for all $i, j = \{A, B\}$ and $i \neq j$, as

$$U(p_i, p_j) := p_i \left[p_j \Pi(D) + (1 - p_j) \Pi(2D)\right] + (1 - p_i) \left[p_j \Pi(-D) + (1 - p_j) \Pi(0)\right]$$

So the firms have the following payoff matrix for doing and not doing R&D

Firm A \ Firm B	R	NR
R	$U(p_A, p_B) - M$, $U(p_B, p_A) - M$	$U(p_A, 0) - M$, $U(0, p_A)$
NR	$U(0, p_B)$, $U(p_B, 0) - M$	$U(0,0)$, $U(0,0)$

Table 1: Payoff matrix under complete information

If we define gross strategic incentive (GSI) as the difference in gross payoffs between performing and not performing R&D when the rival firm is performing R&D, and gross non-strategic incentive (GNSI) as the difference in gross payoffs between performing and not performing R&D when the rival firm is not performing R&D, then

- $GSI(p_i, p_j) := p_i \left[p_j \left[\Pi(D) - \Pi(-D)\right] + (1 - p_j)\left[\Pi(2D) - \Pi(0)\right]\right]$,
- $GNSI(p_i) := p_i \left[\Pi(2D) - \Pi(0)\right]$.

Then we must have $GNSI(p_i) \geq GSI(p_i, p_j)$, because $\Pi(\cdot)$ is strictly increasing and strictly convex, hence, $[\Pi(2D) - \Pi(0)] > [\Pi(D) - \Pi(-D)]$.

Now define

$$V(x) := \frac{M}{x(\Pi(D) - \Pi(-D)) + (1 - x)(\Pi(2D) - \Pi(0))}.$$

Then $V'(x) > 0$.

Proposition 1. *Under complete information,*

(a) if $p_i < V(0)$, then firm i will never invest in research;

(b) if $p_i > V(p_j)$, then firm i will always invest in research, and

(c) if $V(p_j) > p_i > V(0)$, then firm i will invest in research provided firm j does not invest in research.

The results underlying Proposition 1 are shown in Figure 1, which is similar to the one we have used in an earlier paper (see Chatterjee, Chattopadhyay and Kabiraj (2019)). Here $V(p_A)$ and $V(p_B)$ are increasing functions. Given the types of the firms, both the

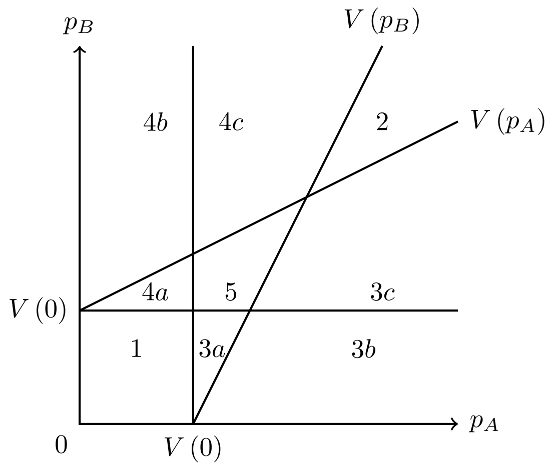

Figure 1: Research Possibilities under Complete Information

firms will invest in region 2, only firm A will invest in region $3(\cdot)$, and only firm B will invest in region $4(\cdot)$. In contrast in region 5, either of the firms (but not both) will invest.

In the next section we develop the analysis under incomplete information. We consider the following three situations:

(i) Level I incomplete information - here a firm knows the rival's R&D and its R&D status, but does not know the type of its rival.

(ii) Level II incomplete information - here a firm knows its rival's R&D decision, but does not know the type and the status of R&D of its rival.

(iii) Level III incomplete information - here the rival's R&D decision, its R&D status as well as its type are unknown to a firm.

4 Incomplete Information

In this section we discuss R&D incentives of firms under various levels of incomplete information.

4.1 Level I Incomplete Information: Type unknown, decision and status known

We assume incomplete information about the rival's type. Thus a firm does not know the probability of success of the rival if the rival is doing R&D. But the firm knows, just before production takes place, whether the rival has invested in R&D or not, and whether the rival is successful or not if it has invested in R&D. However, at the time of investment in R&D the firm does not know the type, decision and status (i.e. whether success or failure) of its rival firm. Let τ_1 be the threshold type of a firm such that if its type is greater than this threshold, it will invest in R&D. So, our objective will be to find this threshold value, given M.

Note that the final profit of a firm does not depend on the type of its rival after it observes the decision and the status of R&D of its rival. So, the profit levels are exactly identical to those in the complete information section. However, while deciding whether to invest in R&D, no firm knows the decision of its rival and therefore does not know the status of its rival's R&D. A firm just knows that the probability that its rival will invest in R&D is $(1 - F(\tau_1))$, and if its rival invests in R&D then the expected probability of success is $\Theta(\tau_1)$.

The expected profit of the firm when it does not invest in R&D is

$$F(\tau_1)\Pi(0) + (1 - F(\tau_1))\left[\Theta(\tau_1)\Pi(-D) + (1 - \Theta(\tau_1))\Pi(0)\right].$$

and the expected profit of the firm when it does invest in R&D is

$$\begin{aligned} &p_i\left[F(\tau_1)\Pi(2D) + (1 - F(\tau_1))\left[\Theta(\tau_1)\Pi(D) + (1 - \Theta(\tau_1))\Pi(2D)\right]\right] \\ &+ (1 - p_i)\left[F(\tau_1)\Pi(0) + (1 - F(\tau_1))\left[\Theta(\tau_1)\Pi(-D) + (1 - \Theta(\tau_1))\Pi(0)\right]\right] - M. \end{aligned}$$

Define $T_1(x;\tau_1)$ as gross opportunity "gain" of doing R&D, when the type of the firm is x. Here

$$\begin{aligned} T_1(x;\tau_1) := {} & x\left[F(\tau_1)\Pi(2D) + (1 - F(\tau_1))\left[\Theta(\tau_1)\Pi(D) + (1 - \Theta(\tau_1))\Pi(2D)\right]\right] \\ & + (1 - x)\left[F(\tau_1)\Pi(0) + (1 - F(\tau_1))\left[\Theta(\tau_1)\Pi(-D) + (1 - \Theta(\tau_1))\Pi(0)\right]\right] \\ & - \left[F(\tau_1)\Pi(0) + (1 - F(\tau_1))\left[\Theta(\tau_1)\Pi(-D) + (1 - \Theta(\tau_1))\Pi(0)\right]\right] \\ = {} & x\left[\left[\Pi(2D) - \Pi(0)\right] - (1 - F(\tau_1))\Theta(\tau_1)\left[\Pi(2D) - \Pi(0) - \Pi(D) + \Pi(-D)\right]\right] \end{aligned}$$

$T_1(x;\tau_1)$ is increasing in x. Also a firm whose type is τ_1 must be indifferent between investing and not investing in R&D. Therefore, the threshold value τ_1 will be solved from the equation $T_1(\tau_1;\tau_1) = M$. We have the following result.

Proposition 2. *Given any $M > 0$, suppose $M < T_1(1;1)$. Then there exists a unique τ_1 such that a firm will invest in research if and only if its type is greater than or equal to τ_1, when τ_1 is solved from $T_1(\tau_1;\tau_1) = M$. If $M > T_1(1;1)$, then no firm will invest in research.*

Proof. We have $T_1(0;0) = 0$. With slight abuse of notation, $T_1(1;1) = \lim_{\tau_1 \to 1} T_1(\tau_1;\tau_1) = \Pi(2D) - \Pi(0) > 0$. Further, $T_1(x;x)$ is increasing in $x \in (0,1)$. To see this,

$$T_1(x;x) = x\left\{\left[\Pi(2D) - \Pi(0)\right] - (1 - F(x))\Theta(x)\left[\Pi(2D) - \Pi(0) - \Pi(D) + \Pi(-D)\right]\right\}$$

Hence,

$$T_1'(x;x) = \{[\Pi(2D) - \Pi(0)] - (1 - F(x))\Theta(x)[\Pi(2D) - \Pi(0) - \Pi(D) + \Pi(-D)]\}$$
$$+ x^2 f(x)[\Pi(2D) - \Pi(0) - \Pi(D) + \Pi(-D)] > 0$$

given $\Pi(x)$ strictly increasing and convex in x so that $\Pi(2D) - \Pi(0) > \Pi(D) - \Pi(-D)$. Hence the result. □

Given $T_1(x;x)$ is an increasing function, the critical τ_1 will increase as M increases. This is because as R&D becomes more costly, a firm requires a higher probability of success in order to have incentive for investment in R&D. The following example shows the result.

Example 1. Let us assume: $a = 10$, $c = 2$, $M = 2$, $D = 1$ and p_i is distributed uniformly for all $i \in \{A, B\}$. Therefore, $F(p_i) = p_i$, $K = 8$ and $\Theta(p_i) = \frac{p_i+1}{2}$. Then τ_1 must solve the equation, $\tau_1 [17 + \tau_1^2] = 9$. The above equation solves uniquely to $\tau_1 = 0.5211$. If research cost is too high (i.e. more than 4), then no firm will do the research. On the other hand, if there is no research cost, then both of them will invest in research, irrespective of their types.

4.2 Level II Incomplete Information: Type unknown, investment decision known but status unknown

In this section we assume that a firm does not know the type of its rival. Also, if its rival has taken up R&D investment, the firm does not know whether the rival will succeed or fail. However, the firm can observe whether the rival does R&D or not. Let τ_2 be the threshold value such that if the type of a firm is greater than or equal to τ_2, then it will invest in R&D. Our objective in this case is to find out such a τ_2, given M.

The expected payoffs of the firms under level II incomplete information are given in the following lemma.

Lemma 2. *The following results hold*

(a) If none of the firms invests in R&D, the profit of firm i is $\Pi_i^{[NN]} = \Pi(0)$.

(b) If firm A does R&D but firm B does not, then the respective expected profits are $E\Pi_A^{[SN]} = \Pi\left(\frac{(3+\Theta(\tau_2))D}{2}\right)$, $E\Pi_A^{[FN]} = \Pi\left(\frac{\Theta(\tau_2)D}{2}\right)$ and $E\Pi_B^{[RN]} = \Pi(-\Theta(\tau_2)D)$.

(c) If both of them are investing in R&D, then expected profits of firm A are given by $E\Pi_A^{[SR]} = \Pi\left(\frac{(3-\Theta(\tau_2))D}{2}\right)$ and $E\Pi_A^{[FR]} = \Pi\left(\frac{-\Theta(\tau_2)D}{2}\right)$.

Similar expected profit expressions can be derived for firm B routinely.

Derivation of the payoffs underlying Lemma 2(b) and 2(c) are given in Appendix 1.

From Lemma 2, the expected "gain" of investing in R&D for firm i when the rival is not doing R&D is given by

$$p_i \Pi\left(\frac{(3+\Theta(\tau_2))D}{2}\right) + (1-p_i)\Pi\left(\frac{\Theta(\tau_2)D}{2}\right) - \Pi(0) - M.$$

Similarly, the expected "gain" of investing in R&D for firm i when the rival is doing R&D, is given by

$$p_i \Pi\left(\frac{(3-\Theta(\tau_2))D}{2}\right) + (1-p_i)\Pi\left(\frac{-\Theta(\tau_2)D}{2}\right) - \Pi(-\Theta(\tau_2)D) - M.$$

Therefore the gross expected "gain" of doing research for the firm whose type is x, is given by

$$\begin{aligned} T_2(x;\tau_2) := F(\tau_2) & \left[x\Pi\left(\frac{(3+\Theta(\tau_2))D}{2}\right) + (1-x)\Pi\left(\frac{\Theta(\tau_2)D}{2}\right) - \Pi(0)\right] \\ & + (1-F(\tau_2))\left[x\Pi\left(\frac{(3-\Theta(\tau_2))D}{2}\right) + (1-x)\Pi\left(\frac{-\Theta(\tau_2)D}{2}\right) - \Pi(-\Theta(\tau_2)D)\right] \end{aligned}$$

$T_2(x;\tau_2)$ is increasing in x. Also a firm whose type is τ_2 must be indifferent between investing and not investing in R&D; therefore, τ_2 will be solved from $T_2(\tau_2;\tau_2) = M$. We have

$$\begin{aligned} T_2(x;x) = & \left[x\Pi\left(\frac{(3-\Theta(x))D}{2}\right) + (1-x)\Pi\left(\frac{-\Theta(x)D}{2}\right) - \Pi(-\Theta(x)D)\right] \\ & + F(x)x\left[\Pi\left(\frac{(3+\Theta(x))D}{2}\right) - \Pi\left(\frac{(3-\Theta(x))D}{2}\right)\right] \\ & - F(x)x\left[\Pi\left(\frac{\Theta(x)D}{2}\right) - \Pi\left(\frac{-\Theta(x)D}{2}\right)\right] \\ & + F(x)\left[\Pi\left(\frac{\Theta(x)D}{2}\right) - \Pi\left(\frac{-\Theta(x)D}{2}\right) - \Pi(0) + \Pi(-\Theta(x)D)\right] \end{aligned}$$

Hence, $T_2(0;0) = \Pi\left(-\frac{\Theta(0)D}{2}\right) - \Pi(-\Theta(0)D) > 0$, and with slight abuse of notation, $T_2(1;1) := \lim_{\tau_2 \to 1} T_2(\tau_2,\tau_2) = \Pi(2D) - \Pi(0)$.

Lemma 3. *If $K > 3D$, then $T_2(x;x)$ is strictly increasing.*

Proof of Lemma 3 is given in Appendix 2. We can now write the following proposition.

Proposition 3. *Given the functions $T_2(x;\tau_2)$ and M, the following must hold:*

(a) if $M < T_2(0;0)$, then both the firms will invest in R&D,

(b) if $M > T_2(1;1)$, then no firm will invest in R&D, and

(c) when $T_2(0;0) < M < T_2(1;1)$, there exists a unique τ_2 such that a firm will invest

in R&D if and only if its type is greater than or equal to τ_2 where τ_2 is solved from the equality $T_2(\tau_2; \tau_2) = M$; then τ_2 is increasing in M.

Since in the second stage firms know the R&D decision of their respective rivals, this information can act as a signal. So, it is important now to check the incentive compatibility. We have claimed that a firm will invest in R&D if and only if the type of the firm is greater than or equal to τ_2. Suppose firm A follows this strategy and belief that firm B also follows the same strategy. Further, firm B knows firms A's strategy and belief.

Let

$$NR_{II}(x; \tau_2) := F(\tau_2)\Pi(0) + (1 - F(\tau_2))\Pi(-\Theta(\tau_2)D)$$

be the expected profit of a firm when it does not invest in R&D. Similarly, let

$$\begin{aligned} R_{II}(x; \tau_2) := x \left[F(\tau_2)\Pi\left(\frac{(3 + \Theta(\tau_2))\, D}{2}\right)\right] + (1 - F(\tau_2))\Pi\left(\frac{(3 - \Theta(\tau_2))\, D}{2}\right) \\ +(1 - x)\left[F(\tau_2)\Pi\left(\frac{\Theta(\tau_2)D}{2}\right) + (1 - F(\tau_2))\Pi\left(-\frac{\Theta(\tau_2)D}{2}\right)\right] \end{aligned}$$

be the gross expected profit of a firm which does invest in R&D. Note that $R_{II}(x; \tau_2)$ is an increasing function of x. Also $T_2(x; \tau_2) = R_{II}(x; \tau_2) - NR_{II}(x; \tau_2)$. Then we can make the following two observations.

First, suppose firm B's type is less than τ_2 but it decides to invest in R&D. Here, from the second stage onwards, firm A believes that the type of firm B is greater than τ_2. So, firm A will produce accordingly. Note here the signaling aspect. Since the R&D status of a firm can never be observed by its rival, but its R&D decision is fully observable, the concerned firm has a scope for signaling its type the way it wishes, irrespective of its true type. If it wants to make its rival believe that its type is greater than τ_2, it will conduct R&D, irrespective of its true type. It will do so if by inducing such a belief to the rival, it experiences a higher expected profit compared to what it could earn by acting according to its true type. The possibility of such signaling is unique in the case of Level II incomplete information.

So the expected profit of firm B under R&D investment is $R_{II}(p_B; \tau_2)$. However, if it had not invested, then its expected profit would have been $NR_{II}(p_B; \tau_2)$. Given the definition of τ_2, and that $T_2(x; \tau_2)$ is strictly increasing in x, the following must hold for all $p_B < \tau_2$:

$$NR_{II}(p_B; \tau_2) > R_{II}(p_B; \tau_2)$$

So, if firm B's type is less than τ_2, then given firm A's strategy and belief, it will never invest in R&D.

Secondly, suppose firm B's type is greater than τ_2, but it decides not to invest in R&D.

Here, from the second stage onward, firm A believes that the type of firm B is less than τ_2. So, firm A will produce accordingly. Then the expected profit of firm B is $NR_{II}(p_B; \tau_2)$. However, if it had invested, its expected profit would have been $R_{II}(p_B; \tau_2)$. From the definition of τ_2, and since $T_2(x; \tau_2)$ is strictly increasing in x, we know that for all $p_B > \tau_2$ the following holds:

$$R_{II}(p_B; \tau_2) > NR_{II}(p_B; \tau_2)$$

So, if firm B's type is greater than τ_2, then given firm A's strategy and belief, it will always invest in research.

By optimal strategy under level II incomplete information we mean that the firm will invest in R&D if and only if its type is greater than or equal to τ_2 and it believes that the rival is following the same strategy. Thus, these two observations lead to the conclusion that given that the rival is following the optimal strategy mentioned above, it is always optimal for a firm to follow the same strategy. So, both the firms following this strategy is a perfect Bayesian Nash equilibrium.

The following example illustrates the result of Proposition 3.

Example 2. Assume that $a = 10$, $c = 2$, $M = 2$, $D = 1$ and p_i is distributed uniformly for all $i \in \{A, B\}$. Therefore, $F(p_i) = p_i$, $K = 8$ and $\Theta(p_i) = \frac{p_i+1}{2}$. Finally, τ_2 is solved from the following equation

$$\frac{28\tau_2^3 + 17\tau_2^2 + 470\tau_2 + 61}{144} = 2$$

This solves τ_2 uniquely to give $\tau_2 = 0.4689$. If the research cost is too high (i.e. more than 4), then no firm will do any research. On the other hand, if the cost is too low (i.e. less than 61/144), then both of them will invest in research.

4.3 Level III Incomplete Information: Type, decision and status unknown

In this subsection we assume that a firm does not know the type of its rival. Also, whether the rival does or does not invest in R&D is unobservable. Therefore, whether the rival has succeeded or failed in R&D is also unknown to the firm. As before, assume that a firm will invest in R&D if its type is greater than or equal to τ_3. Thus our objective in this section is to find out such a τ_3 given M. Let $H(x) := \frac{(1-F(x))\Theta(x)D}{2}$; then $H'(x) = -\frac{1}{2}xf(x)D < 0$.

The payoffs of a firm under various contingencies are given in Lemma 4 and are derived in Appendix 3.

Lemma 4. *The following results hold:*

(a) *if firm i invests in R&D and succeeds, then its expected profit is*

$$E\Pi_i^{[S]} = \Pi\left(\frac{3D}{2} - H(\tau_3)\right) - M,$$

(b) *if firm i invests in R&D and fails, then its expected profit is*

$$E\Pi_i^{[F]} = \Pi\left(-H(\tau_3)\right) - M,$$

and

(c) *if firm i does not invest in R&D, then its expected profit is $E\Pi_i^{[N]} = \Pi\left(-H(\tau_3)\right)$.*

Hence the gross expected "gain" from investing in R&D for firm i is

$$T_3(x;\tau_3) := x\left[\Pi\left(\frac{3D}{2} - H(\tau_3)\right) - \Pi\left(-H(\tau_3)\right)\right]$$

$T_3(x;\tau_3)$ is increasing in x. Define a firm of type τ_3 which is indifferent between investing and not investing in R&D. Then τ_3 is solved from $T_3(\tau_3;\tau_3) = M$. We have $T_3(0;0) = 0$, and (with slight abuse of notation) $T_3(1;1) := \lim_{\tau_3 \to 1} T_3(\tau_3,\tau_3) = \Pi(\frac{3D}{2}) - \Pi(0)$. Finally, $T_3(x;x)$ is strictly increasing in $(0,1)$.[5] Then we have the following results, as given in Proposition 4.

Proposition 4. *The following results hold:*

(a) *If $T_3(0;0) < M < T_3(1;1)$, there exists a unique τ_3 such that a firm will invest in research if and only if its type is greater than or equal to τ_3 when τ_3 can be obtained by solving the equality $T_3(\tau_3;\tau_3) = M$.*

(b) *If $M > T_3(1;1)$ then no firm will invest in R&D.*

The following example illustrates the above results.

Example 3. Assume that $a = 10$, $c = 2$, $M = 2$, $D = 1$ and p_i is distributed uniformly for all $i \in \{A,B\}$. Therefore, $F(p_i) = p_i$, $K = 8$ and $\Theta(p_i) = \frac{p_i+1}{2}$. Then τ_3 is solved from the following equation

$$\frac{\tau_3(\tau_3^2 + 34)}{12} = 2$$

This solves uniquely $\tau_3 = 0.696$. If research cost is too high (i.e. more than 35/12), no firm will perform research; on the other hand, if there is no R&D cost, then both of them will invest in research, irrespective of their types.

[5] Because, $T_3'(x;x) = \left[\Pi\left(\frac{3D}{2} - H(x)\right) - \Pi\left(-H(x)\right)\right] - \left[q\left(\frac{3D}{2} - H(x)\right) - q\left(-H(x)\right)\right]\frac{2}{3}H'(x)$. Both the parts of the RHS are strictly positive.

5 Comparison

In this section we compare the results under various information structures and examine the role of information.

5.1 Complete vs Incomplete Information

In this subsection we show that there are situations when incomplete information may enhance incentive for R&D investment compared to complete information case.

Proposition 5(A). *There always exists $M > 0$ and types of firms A and B, viz. p_A and p_B, such that both the firms will invest in R&D under Level II incomplete information, but will never invest in R&D under complete information.*

Proof. We know under complete information $GNSI\,(0) = 0$, and under Level II incomplete information $T_2(0; 0) > 0$. Since both $GNSI(\cdot)$ and $T(\cdot;\cdot)$ are continuous, there exists $b \in (0, 1)$ such that for all $x \in (0, b)$ we have $T_2(x; x) > GNSI(x)$. Then for all $M \in (0, GNSI(b))$ and for all $p_A, p_B \in (T_2^{-1}(M), GNSI^{-1}(M))$, both the firms will invest in R&D under Level II incomplete information, but will never invest in R&D under complete information. □

Proposition 5(B). *If both the firms are investing under Level I incomplete information, then at least one firm will definitely invest under complete information.*

Proof. Note that $T_1(0; 0) = GNSI(0)$ and $T_1(1; 1) = GNSI(1)$. But for all $x \in (0, 1)$ we have $GNSI(x) > T_1(x; x)$. Take any $M \in (0, GNSI(1))$, then $\tau_2 > GNSI^{-1}(M)$. Rest of the proof is trivial. □

Proposition 5(C). *If both the firms invest under Level III incomplete information, then they will definitely invest under complete information.*

Proof. Note that

$$9\left[\Pi\left(\frac{3D}{2} - H(x)\right) - \Pi(H(x))\right] = \left[2K + \frac{3D}{2} - 2H(x)\right]\frac{3D}{2}$$

Also

$$9\left[\Pi(D) - \Pi(-D)\right] = 4KD > \left[2K + \frac{3D}{2}\right]\frac{3D}{2} > 9\left[\Pi\left(\frac{3D}{2} - H(x)\right) - \Pi(H(x))\right]$$

So for all $x, y \in (0, 1)$, we have $GSI(x, y) > GSI(x, 1) > T_3(x; x)$. Rest of the proof is trivial. □

We can explain the results underling Proposition 5(·). As we move from complete to incomplete information, depending on the extents of uncertainty, there are three levels of incomplete information, where maximum uncertainty arises under Level III incomplete information. So under such a situation if for some (p_a, p_b) both firms do R&D then under complete information they will certainly do. In case of Level I incomplete information, uncertainty is lowest among these three cases of incomplete information. So for some (p_a, p_b) when at least one firm does invest in R&D under complete information, under Level I incomplete information both firms might do because one firm does not know the other's probability of succes with certainty. On the other hand, consideration of R&D investment under Level II incomplete information takes care of the signaling effect in the sense that if one firm invests in R&D, it may signal that its success is possibly high. Therefore, in the situation when no firm does R&D under complete information, both firms may invest because of the signaling effect. Thus our results show that there are situations when incomplete information may lead to a higher incentives for R&D investment compared to complete information.

5.2 Incomplete vs Incomplete Information

On comparing different cases of incomplete information we derive what we call non-monotonicity of R&D incentive with respect to different levels of incomplete information. In particular, we show that as less and less information becomes available, a firm's R&D incentive first increases, then it starts falling.

Let us first compare Level I and Level III incomplete information. It can be shown (see Appendix 4) that:

$$T_1(x; x) > T_3(x; x) \quad \forall x \in (0, 1)$$

Further, we have $T_1(0; 0) = T_3(0; 0)$, but $T_1(1; 1) > T_3(1; 1)$. Given that both T_1 and T_3 are increasing in x, this means we must have $\tau_1 < \tau_3$. This suggests that R&D incentive of a firm under Level I incomplete information is larger than that under Level III incomplete information. The reason is that compared to Level I incomplete information, there are many characteristics of the rival unknown to the firm.

Now we compare Level I and Level II incomplete information. However, it may be recalled that under Level II incomplete information, there is signaling effect through which a firm may get larger information about its rival. Now given that $T_1(0; 0) < T_2(0; 0)$, $T_1(1; 1) = T_2(1; 1)$, and both $T_1(\cdot; \cdot)$ and $T_2(\cdot; \cdot)$ are increasing functions of x, then if the signaling effect is strong enough, $T_2(\cdot; \cdot)$ will always be above $T_1(\cdot; \cdot), \forall x \in (0, 1)$. This is equivalent to restricting the $F(\cdot)$ function.

Assume that $F(\cdot)$ satisfies the following condition for all $x \in (0,1)$:

$$(\Theta(x)-x)\frac{K}{D}+\Theta(x)[(2.5-F(x))x+F(x)\Theta(x)] > 1.75x+0.75\Theta(x)^2 \qquad (1)$$

If the above condition holds, we can say that the signaling effect is strong, and under this condition we must have $T_2(x;x) > T_1(x;x), \forall x \in (0,1)$. The implication of this result is that,

$$\tau_2 < \tau_1,$$

i.e. R&D incentive of a firm is larger under Level II incomplete information than that under Level I incomplete information.

One may easily check that the condition (1) necessarily holds for uniform distribution function (a linear distribution function) and many other distribution functions.[6] Therefore, combining the above analysis we can write the following result:

Proposition 6. *Suppose the condition* (1) *holds. Then comparing all the threshold values we must have* $\tau_2 < \tau_1 < \tau_3$*, that is, if the signaling effect is strong enough, R&D incentive of a firm is larger under Level II incomplete information.*

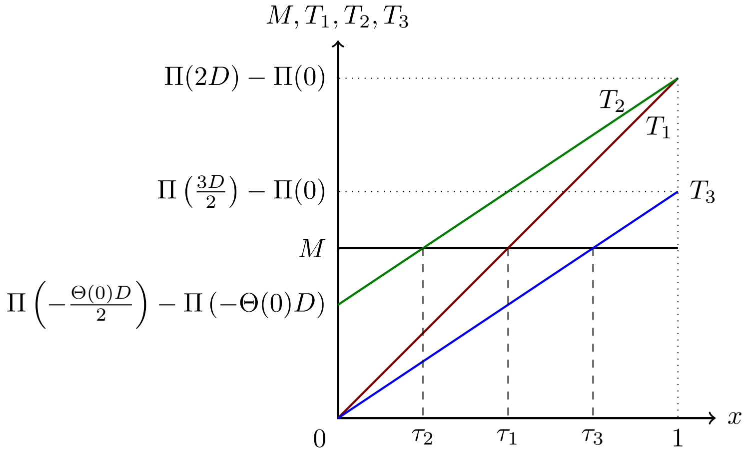

Figure 2: Non-monotonicity result when (1) holds

Figure 2 portrays the result. This shows that R&D incentive of a firm is non-monotone with respect to availability of information about rival's characteristics. Hence under incomplete information, as the level of information about the rival falls, R&D incentive rises initially. However, if the level of information falls further, R&D incentive falls. We illustrate the result with an example below.

Example 4. Let us assume $a = 10$, $c = 2$, $M = 2$, $D = 1$, and p_i is distributed uniformly for all $i \in \{A, B\}$. Our earlier examples show that in this case $\tau_1 = 0.5211$, $\tau_2 = 0.4689$ and $\tau_3 = 0.696$. So, here clearly we have, $\tau_2 < \tau_1 < \tau_3$.

[6] For example $F(x) = x^2$ (a convex distribution function) and $F(x) = \frac{2x}{1+x}$ (a concave distribution function) satisfy the above restriction.

Note that the restriction we impose on the distribution function is a sufficient condition for getting the non-monotone relationship; the condition is not necessary. So when the condition does not hold, suppose T_2 intersects T_1 at an intermediate probability, say τ_c.[7] Then given both T_2 and T_1 strictly increasing and $0 = T_1(0) < T_2(0) < T_2(\tau_c) = T_1(\tau_c)$, there exists $M \in (0, T_2(\tau_c))$ for which the desired non-monotonic relationship holds. This possibility is shown in Figure 3.

A firm has scope for signaling its type that may be different from its true type if it wants its rival firm to believe its signaled type under Level II incomplete information structure. This is, however, not possible under complete or other incomplete information structures. That is why a firm might invest in R&D even though it has a low type, just to convince its rival that it has a higher type under Level II incomplete information. This accounts for the observed non-monotonicity in the above result.

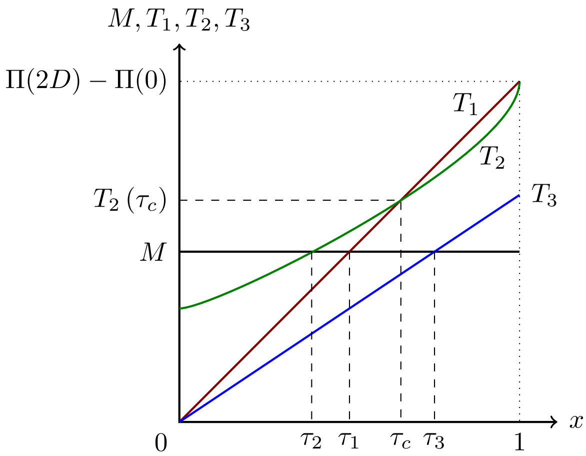

Figure 3: Non-monotonicity result when (*) does not hold

6 Conclusion

In the present paper we have discussed R&D incentives of a firm in a duopoly under various (incomplete) information structures hitherto not properly dealt with in the literature. First, that R&D outcome is uncertain is fully recognized in the literature - that is no guarantee that R&D investment will always yield a success. But more importantly, the probability of success in R&D depends to a large extend, on factors endogenous to a firm. Therefore, probability of success itself can be uncertain. It may be more known to the firm undertaking the R&D investment than to its rivals. Hence in the present paper we have assumed that a firm's probability of success in R&D is private information. This constitutes its type. Second, when a firm takes its decision regarding R&D investment,

[7]If there are multiple intersections then τ_c is the probability level for which we have the first intersection (i.e. p_t is minimum of all those probabilities for which we have intersections).

it may or may not know whether its rival is going to invest in R&D. Third, at time of production decision, even if a firm knows whether its rival has invested in R&D, still the firm under consideration is uncertain about the rival's R&D outcome, that is, even when it is known that the rival has invested in R&D, the firm may or may not know whether the rival has a success or failure.

Given this kind of uncertainty, we have considered the following three information structures depending on the levels of uncertainty or incomplete information. Under Level I incomplete information, only rival's type is unknown, but the firm knows with certainty about the R&D investment decision and status of R&D outcome of the rival. Under Level II incomplete information, rival's type is unknown, investment decision is known, but the status of R&D is unknown. Finally, Level III incomplete information assumes that the rival's type, investment decision, as well as status of R&D, are all unknown to a firm.

In the paper we have first shown that given the level of information, there are situations when incomplete information enhances R&D incentive of a firm compared to complete information situation. This may happen because a firm under incomplete information structure may underestimate the R&D capability of its rival. Then we have derived an interesting result. We find that R&D incentive to invest in R&D is highest in the case of Level II incomplete information, but lowest under Level III incomplete information. Therefore, there is a monotone relation between the R&D incentive and the level of incomplete information. As information incompleteness goes up initially from Level I to Level II, R&D incentive goes up, but as further uncertainty about the rival's attributes increases, the firm's R&D incentive falls. R&D incentive under Level II is highest because of the presence of signaling effect in this case. Although a firm's type is unknown, but by means of its investment decision it can give a signal to its rival that it is of high type firm, irrespective of its true type. So the rival may believe that the firm's type is high, above the threshold value. Hence in the presence of signaling effect a firm's incentive for R&D is higher. But as further information available to a firm declines, the firm is discouraged to invest under increasing uncertainty.

Finally to mention, we have proved our results under a broad class of probability distribution functions of types. We have provided examples supporting our results.

References

[1] Bacchiega, E. and P.G. Garella (2008), "Disclosing versus withholding technology knowledge of a duopoly", *The Manchester School*, Vol. 76, No. 1, pp. 88-103.

[2] Brocas, I. (2004), "Optimal regulation of cooperative R&D under incomplete information", *The Journal of Industrial Economics*, Vol. LII, No. 1, pp. 81-119.

[3] Chattopadhyay, S. and T. Kabiraj (2015), " Incomplete information and R&D organisation", *Economics Bulletin*, Vol. 35, No. 1, pp. 14-20.

[4] Conti, C. (2013), "Asymmetric information in a duopoly with spillovers: New findings on the effects of RJVs (Working Paper No. 2013–04)". Retrieved from http://wp.demm.unimi.it/tl_files/wp/2013/DEMM-2013_04wp.pdf.

[5] Grishagin, V.A., Y.D. Sergeyev and D.B.Silipo (2001), "Firm's R&D decisions under incomplete information", *European Journal of Operational Research*, 129, pp. 414-433.

[6] Chatterjee, R., S. Chattopadhyay, and T. Kabiraj (2017), "R&D in a Duopoly under Incomplete Information." *International Journal of Economic Theory*.

[7] Chatterjee, R., S. Chattopadhyay and T. Kabiraj (2018), "Spillovers and R&D incentive under incomplete information", *Studies in Microeconomics*, Vol. 6, No. 1-2, pp.50-65.

[8] Kabiraj, T. and S. Chattopadhyay (2015), "Cooperative vs. non-cooperative R&D incentives under incomplete information", *Economics of Innovation and New Technology*, Vol. 24, No. 6, pp.624-632.

Appendices

Appendix 1 Proof of Lemma 2

Proof. (a) This is obvious.

(b) The expected profit function of firm A is

$$p_A\left[\left((K+D)-q_A^{[SN]}-q_B^{[RN]}\right)q_A^{[SN]}\right]+(1-p_A)\left[\left(K-q_A^{[FN]}-q_B^{[RN]}\right)q_A^{[FN]}\right]-M.$$

Similarly the expected profit of firm B is

$$\left(K-q_B^{[RN]}-\left[\Theta(\tau_2)q_A^{[SN]}+(1-\Theta(\tau_2))q_A^{[FN]}\right]\right)q_B^{[RN]}.$$

We have three reaction functions as given below

$$K+D=2q_A^{[SN]}+q_B^{[RN]},\qquad K=2q_A^{[FN]}+q_B^{[RN]},$$

and

$$K=2q_B^{[RN]}+\left[\Theta(\tau_2)q_A^{[SN]}+(1-\Theta(\tau_2))q_A^{[FN]}\right].$$

We also have three unknowns viz, $q_A^{[SN]}$, $q_A^{[FN]}$ and $q_B^{[RN]}$. Therefore by solving we get

$$q_A^{[SN]} = q\left(\frac{(3+\Theta(\tau_2))D}{2}\right),\ q_A^{[FN]} = q\left(\frac{\Theta(\tau_2)D}{2}\right) \text{ and } q_B^{[RN]} = q\left(-\Theta(\tau_2)D\right).$$

Hence the payoffs.

(c) The expected profit of firm A is given by

$$p_A\left[(K+D) - q_A^{[SR]} - \left[\Theta(\tau_2)q_B^{[RS]} + (1-\Theta(\tau_2))q_B^{[RF]}\right]\right] q_A^{[SR]}$$
$$+ (1-p_A)\left[K - q_A^{[FR]} - \left[\Theta(\tau_2)q_B^{[RS]} + (1-\Theta(\tau_2))q_B^{[RF]}\right]\right] q_A^{[FR]} - M.$$

So, firm A has two reaction functions

$$K + D = 2q_A^{[SR]} - \left[\Theta(\tau_2)q_B^{[RS]} + (1-\Theta(\tau_2))q_B^{[RF]}\right],$$

and

$$K = 2q_A^{[FR]} - \left[\Theta(\tau_2)q_B^{[RS]} + (1-\Theta(\tau_2))q_B^{[RF]}\right].$$

The expected profit of firm B is given by

$$p_B\left[(K+D) - q_B^{[RS]} - \left[\Theta(\tau_2)q_A^{[SR]} + (1-\Theta(\tau_2))q_A^{[FR]}\right]\right] q_B^{[RS]}$$
$$+ (1-p_B)\left[K - q_B^{[RF]} - \left[\Theta(\tau_2)q_A^{[SR]} + (1-\Theta(\tau_2))q_B^{[FR]}\right]\right] q_B^{[RF]} - M.$$

So, firm B has two reaction functions

$$K + D = 2q_B^{[RS]} - \left[\Theta(\tau_2)q_A^{[SR]} + (1-\Theta(\tau_2))q_A^{[FR]}\right],$$

and

$$K = 2q_B^{[RF]} - \left[\Theta(\tau_2)q_A^{[SR]} + (1-\Theta(\tau_2))q_A^{[FR]}\right].$$

So we have four reaction functions with four unknowns. By solving them we get for firm A

$$q_A^{[SR]} = q_B^{[RS]} = q\left(\frac{(3-\Theta(\tau_2))D}{2}\right) \quad \text{and} \quad q_A^{[FR]} = q_B^{[RF]} = q\left(\frac{-\Theta(\tau_2)D}{2}\right).$$

□

Appendix 2 Proof of Lemma 3

Proof. To prove Lemma 3, first consider the expression of $T_2(x;x)$. From the expression in the first brackets in $T_2(x;x)$,

$$\frac{d}{dx}\left[x\Pi\left(\frac{(3-\Theta(x))D}{2}\right) + (1-x)\Pi\left(\frac{-\Theta(x)D}{2}\right) - \Pi(-\Theta(x)D)\right]$$
$$= \left[\Pi\left(\frac{(3-\Theta(x))D}{2}\right) - \Pi\left(\frac{-\Theta(x)D}{2}\right)\right]$$

$$+\frac{2}{3}\left[2q(-\Theta(x)D) - xq\left(\frac{(3-\Theta(x))D}{2}\right) - (1-x)q\left(\frac{-\Theta(x)D}{2}\right)\right]\frac{\Theta'(x)D}{2}.$$

The first part of the RHS is clearly positive. For the second part, note that if $K > 3D$ then $2q(-\Theta(x)D) > q\left(\frac{(3-\Theta(x))D}{2}\right) > q\left(\frac{-\Theta(x)D}{2}\right)$ holds.

Similarly for the combined expression within the second and third brackets in $T_2(x;x)$,

$$\frac{d}{dx}\left[\Pi\left(\frac{(3+\Theta(x))D}{2}\right) - \Pi\left(\frac{(3-\Theta(x))D}{2}\right) - \Pi\left(\frac{\Theta(x)D}{2}\right) + \Pi\left(\frac{-\Theta(x)D}{2}\right)\right]$$
$$= \frac{2}{3}\left[q\left(\frac{(3+\Theta(x))D}{2}\right) + q\left(\frac{(3-\Theta(x))D}{2}\right) - q\left(\frac{\Theta(x)D}{2}\right) - \left(\frac{-\Theta(x)D}{2}\right)\right]\frac{\Theta'(x)D}{2}$$
$$> 0.$$

And that within the fourth brackets in $T_2(x;x)$

$$\frac{d}{dx}\left[\Pi\left(\frac{\Theta(x)D}{2}\right) - \Pi\left(\frac{-\Theta(x)D}{2}\right) - \Pi(0) + \Pi(-\Theta(x)D)\right]$$
$$= \frac{2}{3}\left[q\left(\frac{\Theta(x)D}{2}\right) + q\left(\frac{-\Theta(x)D}{2}\right) - 2q(-\Theta(x)D)\right]\frac{\Theta'(x)D}{2} > 0$$

Therefore all the three parts of the RHS of the $T_2(x;x)$ is strictly increasing. So, $T_2(x;x)$ is strictly increasing. □

Appendix 3 Proof of Lemma 4

Proof. Note that the expected profit of firm A if she invests in research and succeed is

$$(1-F(\tau_3))\left[(K+D) - q_A^{[S]} - \left[\Theta(\tau_3)q_B^{[S]} + (1-\Theta(\tau_3))q_B^{[F]}\right]\right]q_A^{[S]}$$
$$+ F(\tau_3)\left[(K+D) - q_A^{[S]} - q_B^{[N]}\right]q_A^{[S]} - M.$$

Similarly, the expected profit of firm A if she invests in research and failed is

$$(1-F(\tau_3))\left[K - q_A^{[F]} - \left[\Theta(\tau_3)q_B^{[S]} + (1-\Theta(\tau_3))q_B^{[F]}\right]\right]q_A^{[F]}$$
$$+ F(\tau_3)\left[K - q_A^{[F]} - q_B^{[N]}\right]q_A^{[F]} - M.$$

Finally, the expected profit of firm A, if she does not invest in research, is

$$(1-F(\tau_3))\left[K - q_A^{[N]} - \left[\Theta(\tau_3)q_B^{[S]} + (1-\Theta(\tau_3))q_B^{[F]}\right]\right]q_A^{[N]}$$
$$+ F(\tau_3)\left[K - q_A^{[N]} - q_B^{[N]}\right]q_A^{[N]}.$$

Looking at these expected profit functions it is clear that at equilibrium we have $q_A^{[F]} = q_A^{[N]}$; called it $q_A^{[N]}$.

So for firm A we have two reaction functions given below

$$K + D = 2q_A^{[S]} + \left[q_B^{[N]} + (1 - F(\tau_3))\Theta(\tau_3)\left(q_B^{[S]} - q_B^{[N]}\right)\right],$$

and

$$K = 2q_A^{[N]} + \left[q_B^{[N]} + (1 - F(\tau_3))\Theta(\tau_3)\left(q_B^{[S]} - q_B^{[N]}\right)\right].$$

Similarly for firm B we have two reaction functions given below

$$K + D = 2q_B^{[S]} + \left[q_A^{[N]} + (1 - F(\tau_3))\Theta(\tau_3)\left(q_A^{[S]} - q_A^{[N]}\right)\right],$$

and

$$K = 2q_B^{[N]} + \left[q_A^{[N]} + (1 - F(\tau_3))\Theta(\tau_3)\left(q_A^{[S]} - q_A^{[N]}\right)\right].$$

So we have four reaction functions and four unknowns, viz. $q_A^{[S]}$, $q_A^{[N]}$, $q_B^{[S]}$ and $q_B^{[N]}$. Solving them we get for firm A

$$q_A^{[S]} = q_B^{[S]} = q\left(\frac{3D}{2} - H(\tau_2)\right) \qquad \text{and} \qquad q_A^{[N]} = q_B^{[N]} = q\left(-H(\tau_3)\right).$$

The rest of the proof is trivial. □

Appendix 4 Comparing Threshold Values Under Level I and Level III Incomplete Information

Proof. Note that $\Pi(2D) - \Pi(0) > \Pi(D) - \Pi(-D) > \Pi\left(\frac{3D}{2} - H(x)\right) - \Pi(-H(x))$ for all $x \in (0,1)$, as $K > 3D$ and the $\Pi(\cdot)$ is strictly increasing and strictly convex to the origin. Let $G(x) := \Pi\left(\frac{3D}{2} - H(x)\right) - \Pi(-H(x))$.

Therefore,

$$\begin{aligned} &F(x)\left[\Pi(2D) - \Pi(0)\right] + (1 - F(x))\left[\Theta(x)\left(\Pi(D) - \Pi(-D)\right) + (1 - \Theta(x))\left(\Pi(2D) - \Pi(0)\right)\right] \\ &\quad > F(x)G(x) + (1 - F(x))\left[\Theta(x)G(x) + (1 - \Theta(x))G(x)\right] = G(x). \end{aligned}$$

□