Dynamic Asymmetry and Fiscal Policy

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Abstract

We introduce a new time series model for public consumption expenditure, tax revenues and real income that is capable to incorporate oscillations characterized by asymmetric phase and duration (or dynamic asymmetry). A specific-to-general econometric strategy is implemented in order to exclude the null hypotheses that these variable are linear or symmetric and, consequently, to ensure that these can be parsimoniously modelled. The U.S. postwar data suggest that the dynamic asymmetry – either in cycle, either in trend – is effectively a reasonable hypothesis for government expenditure and tax revenue, but also that a simple vector model unifying the (different) nonlinearities of each single series is unfeasible. Such an "Occam-razor" failure hinders econometricians in building impulse responses for calculation of fiscal multiplier and is here circumvented via empirical indexes.

Keywords: Nonlinearities, Spending, Modelling, Multiplier, Testing, Selection.

JEL: C12, C22, E32, E62.

1 Introduction

The dynamics of the U.S. fiscal variables is one of the key topics in the political and economic debate. In this framework, the main object of interest in literature is the fiscal multiplier, which, according to the Keynesian perspective, indicates the marginal effect of a shock in the Government spending on the aggregated income. Because of the impactive policy consequences, the estimation of this object is one of the main objectives of the modern macroeconometrics. The difficulties in the pursue of such an objective have been exacerbated during the Sovereign Debt Crisis of 2011-13, when many Countries was facing a so high debt and deficit spending to lead International Monetary Fund to suggest severe cuts in the public balances that subsequently revealed unnecessary; see, among others, Blanchard and Leigh (2013).

This has forced economic literature to modify the econometric modelling in favor of a nonlinear parametrization of the system under investigation; see Auerbach and Gorodnichenko (2012, here adopted as benchmark for our analysis), and Caggiano, Castelnuovo, Colombo, and Nodari (2015); Ramey and Zubairy (2018), among others. This new strand of literature assumes that the macroeconomic system moves according to a Smooth Transition (Vector) Autoregression (ST(V)AR, henceforth). This family of models assumes that the observables are (at least partially) represented by a nonlinear function (called *transition function*) depending on at least two parameters – slope and location – and a *transition variable*, which links the nonlinear dynamics of the observables to other variables or lags of the same observables. Asymmetric transitions are commonly associated to a logistic curve because the sigmoid characterizing that function ensures that the phases of expansions are steeper than the following phase of recessions; such "steepness" is the most classical definition of asymmetry in the literature on business cycle and, to our knowledge, the only one adopted in literature on fiscal policy.

Other two definitions of asymmetry are equally fundamental, and namely: the "deepness", occurring if the series undergoes contraction at an accelerating pace until a minimum, after which it starts to recover with quickly decreasing acceleration until it smoothly recovers the peak; and the "sharpness", occurring when the probability that the transitions to and from the expansion and contraction phases are not identical. Dynamic asymmetry occurs when *all* these definitions of asymmetry holds jointly. Zanetti Chini (2018) shows that traditional STAR-type models can only incorporate steepness but not deepness, hence introduces a generalized version of the STAR model (GSTAR, henceforth) capable to model dynamic asymmetry in the cycle.

Are U.S. fiscal variables dynamically asymmetric? And does dynamic asymmetry affect the fiscal multiplier? This paper shows that the affirmative answer to these questions has important consequences in terms of measurement. The next section 2 summarizes the model and introduce a set of indexes to deal with this methodological problem; the results are described in Section 3; Section 4 concludes; finally, a separated Supplement includes econometric details and further results.

2 Methods

2.1 Data and Econometric Modelling

Let consider the triple of economic variables

$$\{G,\ T,\ Y\}, \tag{1}$$

where G is the aggregate public consumption, T the total public revenues and Y is the real output. Such a triple of variable coincides with the one adopted by Auerbach and Gorodnichenko (2012). The corresponding data (quarterly growth rates from 1947:Q1 to 2008:Q1) are plotted in Figure 1, jointly with the corresponding estimated models that we are going to discuss.

We model dynamic asymmetry in time series $\{y_t^i\}_t^T$, where $i = \{G, T, Y\}$, by using a

GSTAR(p) model here described:

$$
\begin{aligned}
y_t^i &= \boldsymbol{\phi}'\mathbf{z}_t + \boldsymbol{\theta}'\mathbf{z}_t G(\boldsymbol{\xi}) + \epsilon_t, \quad \epsilon_t \sim I.I.D.(0,\sigma^2), \quad E[\epsilon_t|\Omega_{t-1}] = 0, \\
F(\boldsymbol{\xi}) &= \left(1 + \exp\left\{ -\prod_{k=1}^{K} h\big(c_k, s_t\big)\right\}\right)^{-1}, \\
h(\eta_t) &\doteq \begin{cases} \gamma_1^{-1}\exp(\gamma_1|\eta_t| - 1) & \text{if } \gamma_1 > 0, \\ 0 & \text{if } \gamma_1 = 0, \\ -\gamma_1^{-1}\log(1-\gamma_1|\eta_t|) & \text{if } \gamma_1 < 0, \end{cases} \quad \text{for } \eta_t \geq 0 \text{ and} \\
h(\eta_t) &\doteq \begin{cases} -\gamma_2^{-1}\exp(\gamma_2|\eta_t| - 1) & \text{if } \gamma_2 > 0, \\ 0 & \text{if } \gamma_2 = 0, \\ \gamma_2^{-1}\log(1-\gamma_2|\eta_t|) & \text{if } \gamma_2 < 0, \end{cases} \quad \text{for } \eta_t < 0.
\end{aligned} \tag{2}
$$

where the $T \times 1$ vector y_t is a dependent variable; $\mathbf{z}_t = (1, y_{t-1}, \ldots, y_{t-p})'$, $\boldsymbol{\phi} = (\phi_0, \phi_1, \ldots, \phi_p)'$, $\boldsymbol{\theta} = (\theta_0, \theta_1, \ldots, \theta_p)'$ are autoregressive parameter vectors; $\Omega_{t-1} = [y_{t-1}, \ldots, y_{t-p}]$ is the set of histories up to time $t-1$, $G(\boldsymbol{\xi}) \doteq F(\boldsymbol{\gamma}, h(c_k, s_t))$ is a transition function – continuous in $\boldsymbol{\gamma}$ – of the vector of nonlinear parameters $\boldsymbol{\xi} = [\boldsymbol{\gamma}, h(c_k, s_t)]$, which is formed by the vector $\boldsymbol{\gamma} = (\gamma_1, \gamma_2)$ and a function of the K location parameter c_k; the transition variable $s_t = y_{t-d}$, with $d > 0$ denoting the delay, and defining $\eta_t \equiv (s_t - c)$ for ease of notation. We consider three types of transition variables:

$$
\begin{aligned}
&S1 : s_t = y_{t-d},\ d > 0 \\
&S2 : s_t = x_t,\ x_t \sim MA(7); \\
&S3 : s_t = t/T.
\end{aligned} \tag{3}
$$

The first transition variable, S1, is a classical autoregressive transition variable with delay parameter determined according to some criterion like the minimal p-value of the test for dynamic asymmetry. The exogenous transition S2 mimics the one adopted by Auerbach and Gorodnichenko (2012) and has been adopted for comparison with the literature. S3 postulates the transition is a linear trend, and thus implies that the model is dynamically asymmetric around the trend.

The two equations for $h(\eta_t)$ model the higher and the lower tail of the transition function, respectively; so they allow for the dynamically asymmetric behavior introduced by the slope parameters γ_1 and γ_2, which control the velocity of the transition in each half of the same function. When $\gamma_1, \gamma_2 > 0$ ($\gamma_1, \gamma_2 < 0$), $h(\eta_t)$ is an exponential (logarithmic) rescaling that increases more quickly (more slowly) than a standard logistic function does. In turn, this logarithmic (exponential) rescaling in the levels is directly connected to the form of the density function: when $\gamma_1 = \gamma_2 = \gamma$, the model nests a symmetric STAR(p) and the sample density tends to concentrate in the two extremes; when $\gamma \rightarrow +\infty$, $F(\cdot)$ nests an indicator function $\mathrm{I}_{(s_t>c)}$, so that GSTAR(p) nests a threshold autoregression and the distribution of the process degenerates a rectangle-'U'; finally, when $\gamma = 0$, $F(\cdot)$ nests a straight line around 1/2 for each transition variable and the distribution becomes symmetric, so that the model nests a linear AR(p). The requirement that $h(\cdot) = 0$ when the slope parameter is zero is necessary for ease of exposition and allows us to build a test for the null of linearity against of dynamic asymmetry. In our empirical application, we adopt the Specific-to-General modelling strategy summarized in Supplement.

2.2 Measuring Fiscal Efficiency

Once $\hat{y}_t^i$ has been obtained from (2) – (10) we estimate two Static/Dynamic Fiscal Pseudo-Multiplier Indexes (SFPMI and DFPMI, henceforth):

$$\begin{aligned} SFPMI_t^{(1)} = \hat{Y}_t/\hat{G}_t, \quad SFPMI_t^{(2)} = \hat{Y}_t/(\hat{T}_t\hat{G}_t) \\ DFPMI_t^{(1)} = \Delta\hat{Y}_t/\Delta\hat{G}_t, \quad DFPMI_t^{(2)} = \Delta\hat{Y}_t/(\Delta\hat{T}_t + \Delta\hat{G}_t) \end{aligned} \tag{4}$$

where term "pseudo-" underlines the merely empirical nature of the index, which differs from the with from the theoretical coefficient adopted in the New-Keynesian literature; see Woodford (2011). The indexes $SFPMI^{(2)}$ and $DFPMI^{(2)}$ serves to clean the indicator from fiscal revenues, thus giving an idea of the effective amount of the financial efficiency of public expenditure for investments. Moreover, we compute also their cumulated (average) version:

$$\begin{aligned} CSFPMI_t^{(1)} &= 1/T \sum_{t=1}^{T} \hat{Y}_t/\hat{G}_t, \quad CSFPMI_t^{(2)} = 1/T \sum_{t=1}^{T} \hat{Y}_t/\big(\hat{T}_t + \hat{G}_t\big) \\ CDFPMI_t^{(1)} &= 1/T \sum_{t=1}^{T} \Delta\hat{Y}_t/\Delta\hat{G}_t, \quad CDFPMI_t^{(2)} = 1/T \sum_{t=1}^{T} \Delta\hat{Y}_t/\big(\Delta\hat{T}_t + \Delta\hat{G}_t\big) \end{aligned} \tag{5}$$

The use of these indexes is necessary in light of the difficulties in modelling all the three variables of interest with the same parametrization and specification via a unique vector representation, as the next Section is going to explain.

3 Results

We start our empirical investigation by applying the two tests for the null hypotheses of linearity and dynamic symmetry against the alternative of dynamic asymmetry (see the Supplement for details) using all the three different transition variables S1, S2 and S3 and three different delays. According to Table 1, two variables (namely, G and T) are nonlinear and, in most of the cases, dynamically asymmetric. On the opposite side, Y is linear. Interestingly, in each test, there are important spreads between different transition variable: for example, T is nonlinear according to S1, while the nonlinearity cannot be rejected for S2 if considering one quarter delay. Moreover, there is not perfect correlation among nonlinearity and asymmetry. This seems us an (demanding, but feasible) issue for the specification of the econometric model, since our experience lead us to conclude that – limitatly to G and T – a proper choice of the autoregressive order p and number of transition regimes K delivers good estimates also if the sample is suspected to be linear according to the linearity test. On the other side – and *a fortiori* in light of the above mentioned differences in nonlinearity testing – the choice of the transition variable is a economic issue and cannot be delegated to the econometric mechanics.

These considerations forces us to estimate one dynamically asymmetric and one symmetric model for each economic variable and for each of the three transition variables, for a total of 18 models. The estimated models are reported in Table 3 of Supplement. Several facts arises: first, and coherently with the previously mentioned difficulty of the sample to pass the classical linearity test, many of the parameter of the STAR models are non-significant, while the majority of GSTAR ones are. Second, the asymmetric parameterizations is more parsimonious in terms of autoregressive order (3 for STAR, up to 5 for STAR); in two cases on three (G and Y) the STAR models is characterized by three regimes (thus, two transition functions, corresponding to labels "G1", "G2", "Y1", "Y2", respectively), making the number of parameter to estimate blowing-up; such an over-parametrization of the STAR model reflects to only slightly higher R^2 with respect to the GSTAR; in the peculiar case of STAR model for T with specification S3, we were not able to find a sufficiently performing model, also if augmenting the possible lags up to 7. Third, and most importantly, the form of the estimated transition functions $F(\cdot)$ plotted in Figure 2 differs significantly among the two main parameterizations as well as among the three S-specifications: the GSTAR model suggests a common dynamic symmetry for only Y and G, where an immediate bust during recessions is followed by a rapid acceleration in expansions; instead, T is a bit a more gradual either in the accelerating recovery, either in the smooth peak. Differently, the $F(\cdot)$ resulting from STARs are characterized by extremes oscillations in Y (which was suspected to be linear), while G and T transitions are extremely smooth; for further details, see Figure 3 of Supplement.

This impossibility to summarize all the three economic variable via unique nonlinear transition motivates our skepticism in adoption of a vector-type structural modelling: in fact, an improper transition function would reflect in non-credible impulse-response functions – which in turn are the final object of interest in the literature. Our indexes serves to circumvent this methodological gap. Ideally, these indexes are supposed to be positive (that is, evidencing a clear positive multiplier effect of public expenditure on economic growth) and, similarly to Woodford (2011), small in magnitude. Our estimates reported in Table 2 are almost surprising: although the evidence of a positive cumulated effect of G (or G-and-T) on Y is sufficient if looking at the overall exercise (in facts, 2/10 static and 4/10 dynamic indexes are negative), there is an important difference among symmetric and asymmetric indexes: an half of the former is negative, while this results reduces to 1/10 for the asymmetric ones. Similarly the asymmetric specifications are different also in terms of magnitude, albeit mostly in the dynamic version. The temporal evolution of the indexes plotted in Figures 4 – 6 of Supplement indicates that all the indexes are stationary processes with outliers. These last varies according to the parametrization as well as the S-specification: the dynamically asymmetric models are globally more stable than their symmetric counterpart – specially in the static version, whose few outliers therein are relatively small-sized; on the opposite side, the dynamic indexes are characterized by ultra-high sized outliers in either symmetric and asymmetric specifications, in particular with S2.

4 Conclusions

The U.S. public consumption expenditure and tax revenues are characterized by dynamically asymmetry. An application of the Generalized Logistic STAR model lead us to conclude that there is a non-negligible spread among the estimated transition function of each fiscal variable, and, noticeably, among fiscal variables and real income. Secondly, that the efficiency of public consumption expenditure measured by cumulative indexes computed by classical symmetric STAR model is ambiguous, and such an ambiguity is due to the fact that this parametrization is prone to over-evaluate the duration as well as the magnitudo of recession phases. Our dynamic asymmetric specification allows for non-ambiguous results. As a consequence, any vector model aiming to summarize the nonlinear behavior of fiscal variables and understand the business cycle phase-effect on fiscal multiplier is required to take in account for the presence of several independent, selective transition functions. Finally, that the selection of the transition variable is a non-postponable issue. In fact, while the new generation of large dataset publicly available for macroeconometric analysis is formed by hundreds of variables, repeating the current analysis is not recommendable. Hence, we invoke further research to arrive to a unifying structural model.

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Table 1: Linearity and Dynamic Symmetry Tests

Data	G		T		Y	
H0:	Dyn. Symmetry	Linearity	Dyn. Symmetry	Linearity	Dyn. Symmetry	Linearity
d=1						
S1	0.0005	0.2187	0.0076	0.0136	0.5188	0.9699
S2	0.0236	0.7008	0.6991	0.8288	0.7176	0.6085
d=2						
S1	0.0871	0.1085	1.15e-04	0.1162	0.9201	0.6700
S2	0.1918	0.0783	0.0376	0.0953	0.9703	0.3490
d=4						
S1	0.0021	0.0984	0.0011	0.0648	0.1776	0.2825
S2	6.3e-06	0.0431	0.2211	0.2055	0.9969	0.6877
S3	0.0236	0.0.5480	2.7e-06	0.2785	0.4319	0.6085

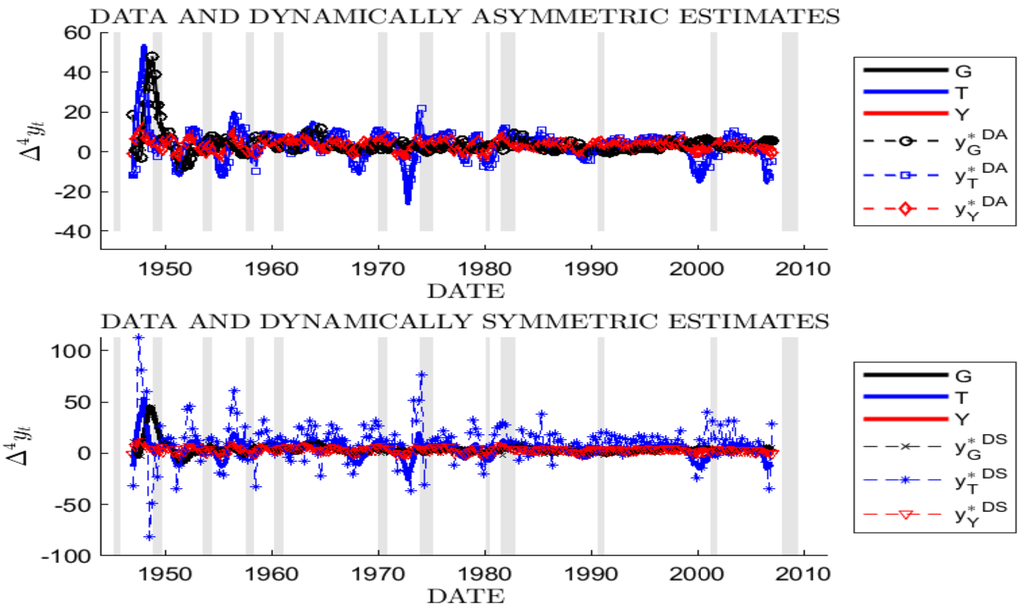
NOTES: This table reports the *p*-values of LM statistic (9) of Supplement for all different types of transition variable and four delay parameters. All the variables are in quarterly growth rates. The specification of transition variable S3 is a linear trend and, thus, cannot be used in lag determinations.

Table 2: Cumulated Fiscal Pseudo-Multiplier Indeces

Index Type	Static				Dynamic			
Model	Dyn. Sym.		Dyn. Asym.		Dyn. Sym.		Dyn. Asym.	
Index spec.	I_1	I_2	I_1	I_2	I_1	I_2	I_1	I_2
S1	-1.5865	-1.4418	0.6317	0.2642	-1.4139	1.0653	-7.4466	0.0713
S2	2.1453	1.4530	1.7042	0.9828	-0.1108	0.7714	12.5302	5.6378
S3		1.3862		0.8456		-0.3723		0.0962

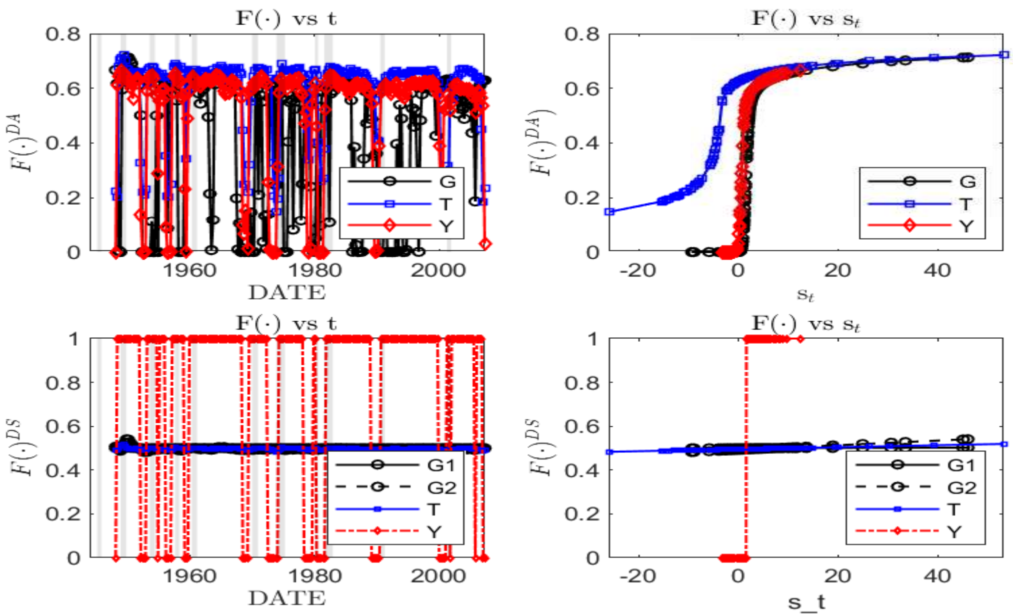
NOTES: This table reports the estimated CFPMI indexes in (5) for the case that $\hat{y}^i$ is estimated via dynamically symmetric , and asymmetric model. I_1 and I_2 label the specification of the cumulative index without and with T, respectively.

Figure 1: Data and Fitted models



NOTE: The upper panel plots the GSTARs; the lower panel plots (symmetric) STARs. Colored bands represent NBER recession dates.

Figure 2: Comparsion of Dynamically Symmetric and Asymmetric Models



NOTE:The upper panels plot GSTAR models; the lower panels plots STAR models; the left-hand panels plot the nonlinear function versus time; the right-hand panels plot the same function versus transition variable S1. Colored bands represent NBER recession dates.

A Supplement (for online publication only)

This Supplement, partially taken from Zanetti Chini (2018), provides additional contents in terms of methodology adopted in the Main Text as well as additional results. Namely, Subsection A.1 illustrates the test for the null hypothesis of dynamic symmetry and linearity; Subsection A.2 describes the estimation; Subsection A.3 describes the modelling strategy; finally Section B gives additional results.

A.1 Testing

According to the definition of GSTAR model given in Section 2 of Main Document, the dynamic asymmetry of the series is modelled by parameters γ_1 and γ_2. Hence, a test for the presence of dynamic asymmetry in the process y_t requires the following hypothesis system:

$$\begin{aligned} &H_0 : \gamma_1 = 0 \;\text{ and }\; \gamma_1 = 0 \;\text{ in }\; (2) \text{ of Main Document,} \\ &H_1 : \gamma_1 \neq 0 \;\text{ and }\; \gamma_1 \neq 0 \;\text{ in }\; (2) \text{ of Main Document.} \end{aligned} \tag{6}$$

On a slightly different perspective, testing for the null hypothesis of dynamic symmetry requires substituting $h(\eta_t) = \eta_t$ in equation (2) of Main Document. In both cases, the alternative hypothesis remains dynamic asymmetry. In both the cases the idea of this test is the same as that of Luukkonen, Saikkonen, and Teräsvirta (1988): the GLSTAR(p) model is linearized via Taylor expansion in order to build an artificial regression whose coefficients incorporate these two slopes. This linearization leads to the following auxiliary regression:

$$\hat{u}_t = \hat{\mathbf{z}}'_{\mathbf{1t}}\tilde{\boldsymbol{\beta}}_{\mathbf{1}} + \sum_{j=1}^{p} \beta_{2j} y_{t-j} y_{t-d} + \sum_{j=1}^{p} \beta_{3j} y_{t-j} y_{t-d}^2 + \sum_{j=1}^{p} \beta_{4j} y_{t-j} y_{t-d}^3 + v_t \;, \tag{7}$$

where v_t is a $N.I.D.(0, \sigma^2)$ process, $\tilde{\boldsymbol{\beta}}_{\mathbf{1}} = (\beta_{10}, \boldsymbol{\beta}'_{\mathbf{1}})'$, $\beta_{10} = \phi_0 - (c/4)\theta_0$, $\boldsymbol{\beta}_{\mathbf{1}} = \boldsymbol{\phi} - (c/4)\boldsymbol{\theta} + (1/4)\theta_0 \mathbf{e}_d$, $\mathbf{e}_d = (0, 0, \ldots, 0, 1, 0, \ldots, 0)'$ with the d-th element equal to unit and $T_3(G) = f_1 G + f_3 G^3$ is the third-order Taylor expansion of $G(\Xi)$ at $\boldsymbol{\gamma} = \mathbf{0}$, $f_1 = \partial G(\Xi)/\partial \Xi\big|_{\boldsymbol{\gamma}=\mathbf{0}}$

and $f_3 = (1/6)\partial^3 G(\Xi)/\partial\Xi\big|_{\boldsymbol{\gamma}=\mathbf{0}}$, $G(\Xi)$ being defined in previous section[1]. The null hypothesis is

$$H_0' : \beta_{2j} = \beta_{3j} = \beta_{4j} = 0 \;\; j = 1, \dots, p, \tag{8}$$

The test statistic:

$$LM = (SSR_0 - SSR)/\hat{\sigma_v}^2 \;, \tag{9}$$

with SSR_0 and SSR denoting the sum of the squared estimated residuals from the estimated auxiliary regression (7) and under the null and alternative, respectively, and $\sigma_v^2 = (1/T)SSR$ has an asymptoticχ^2_{3p} distribution under H_0'. A similar argument with different definitions of $\hat{u}_t$, $\tilde{\boldsymbol{\beta}}_\mathbf{1}$, $\boldsymbol{\beta}_\mathbf{1}$, H_0', holds for the other types of $F(\cdot)$.

A.2 Estimation

Estimation of model (2) in Main Document is done by concentrating the Sum of Square Residuals function with respect to $\boldsymbol{\theta}$ and $\boldsymbol{\phi}$, that is minimizing:

$$SSR = \sum_{t=1}^{T} \left(y_t - \hat{\boldsymbol{\psi}}' \tilde{\xi}_\mathbf{t}' \right)^2, \tag{10}$$

where $\hat{\boldsymbol{\psi}} = [\hat{\boldsymbol{\phi}}, \hat{\boldsymbol{\theta}}] = \left(\sum_{t=1}^{T} \tilde{\xi}_\mathbf{t}'(\boldsymbol{\gamma}, \mathbf{c}) \tilde{\xi}_\mathbf{t}(\boldsymbol{\gamma}, \mathbf{c}) \right)^{-1} \left(\sum_{t=1}^{T} \tilde{\xi}_\mathbf{t}'(\boldsymbol{\gamma}, \mathbf{c}) y_t \right)$ and $\tilde{\xi}_\mathbf{t}(\hat{\boldsymbol{\gamma}}, \hat{\mathbf{c}}) = \left[\mathbf{z}, \mathbf{z}_\mathbf{t}' G(\hat{\boldsymbol{\gamma}}, h(\hat{\mathbf{c}}, s_t) \right]$. Both γ_1 and γ_2 have been chosen between a minimum value of -10 and a maximum of 10 with rate 0.25 in the first three examples the grid for parameter $c_1$is the set of values computed between the 10^{th} and 90^{th} percentile of s_t with rate computed as the difference of the two and divided for an arbitrarily high number (here, 200).

A.3 Econometric Modelling

In this section we discuss the modelling strategy for a GSTAR model.

The modelling strategy is based on the implicit assumptions that a linear process is

[1]Notice the difference from similar expressions in Teräsvirta (1994): here $\boldsymbol{\tau}_\mathbf{2}$ is a vector and $\hat{\mathbf{z}}_\mathbf{2t}$ is the double (it was $-\frac{1}{4}\{\cdots\}$). The LM statistic and the terms in the auxiliary regression remain unchanged.

a peculiar case of a nonlinear one and, in turn, that the dynamic asymmetry is the most general case of nonlinearity. Thus, we use a specific-to-general modelling strategy, consisting in the following 7 steps:

1. Specify a linear autoregressive model.
2. Test linearity for different values of d, and if rejected, determining d in (2).
3. Choose between LSTAR, LSTAR2 or ESTAR by the Teräsvirta's rule.
4. Test the symmetry of the tails transition function according to the result in Step 3.
5. If the hypothesis of symmetry is rejected, estimate the GSTAR model with the most appropriate transition function given by step 3.
6. Evaluate the new parametrization by some diagnostic tests.
7. Use the estimated GSTAR model for forecasting aims.

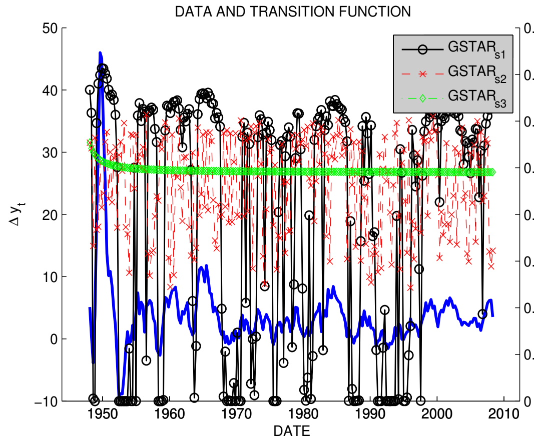
In our examples the autoregressive order p is selected according to Bayesian Information Criterion, which is combined with the result with a portmanteau test for serial correlation in order to avoid a wrong rejection of symmetry hypothesis. This is due to the fact that the GSTAR model requires a lower autoregressive order with respect to its symmetric counterpart.

The choice of the type of nonlinear function has not been investigated, being the literature homogeneously logistic-supportive. For what concerns Step 4, the linear as well as dynamic symmetry hypotheses are tested by the LM-type test discussed in Section A.1. The choice of the delay parameter d and the choice of the transition function can be done with the same procedure adopted in Teräsvirta (1994). In this paper, results for step 6 are not shown for economy of space, but they are available under request. Finally step 7 is not part of the aims of this paper.

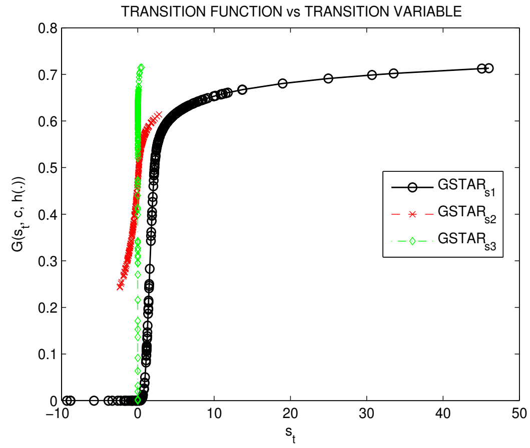
B Additional Figures and Tables

Figure 3: Data and Nonlinear functions: a comparison

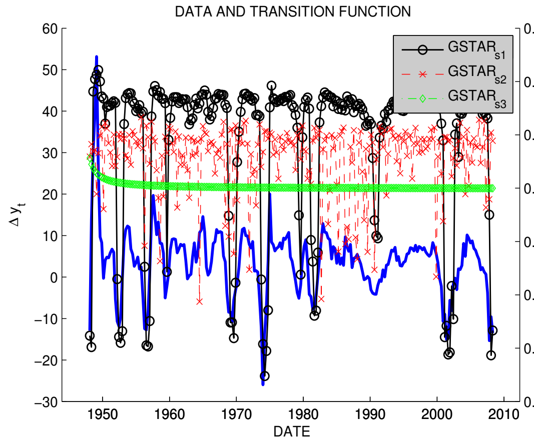
(a) $\hat{F}^G$ versus time



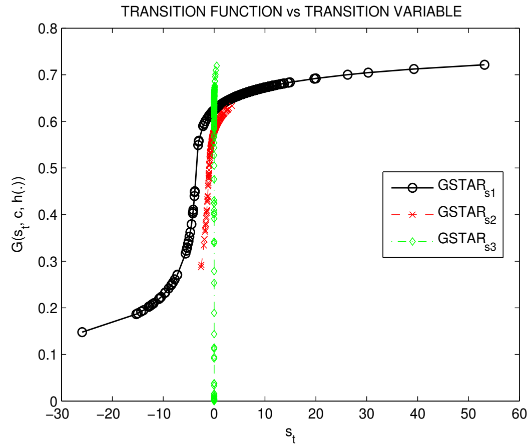
(b) $\hat{F}^G$ versus s_t



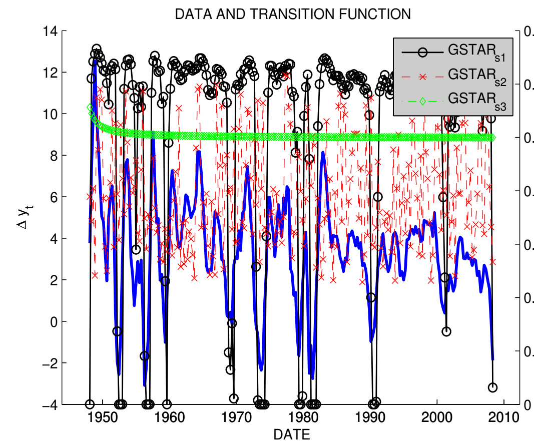
(c) $\hat{F}^T$ versus time



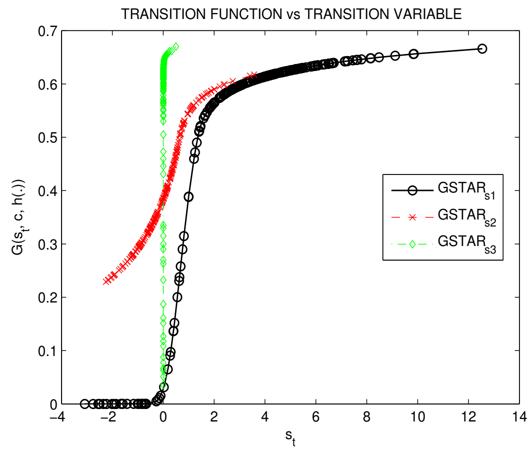
(d) $\hat{F}^T$ versus s_t



(e) $\hat{F}^Y$ versus time

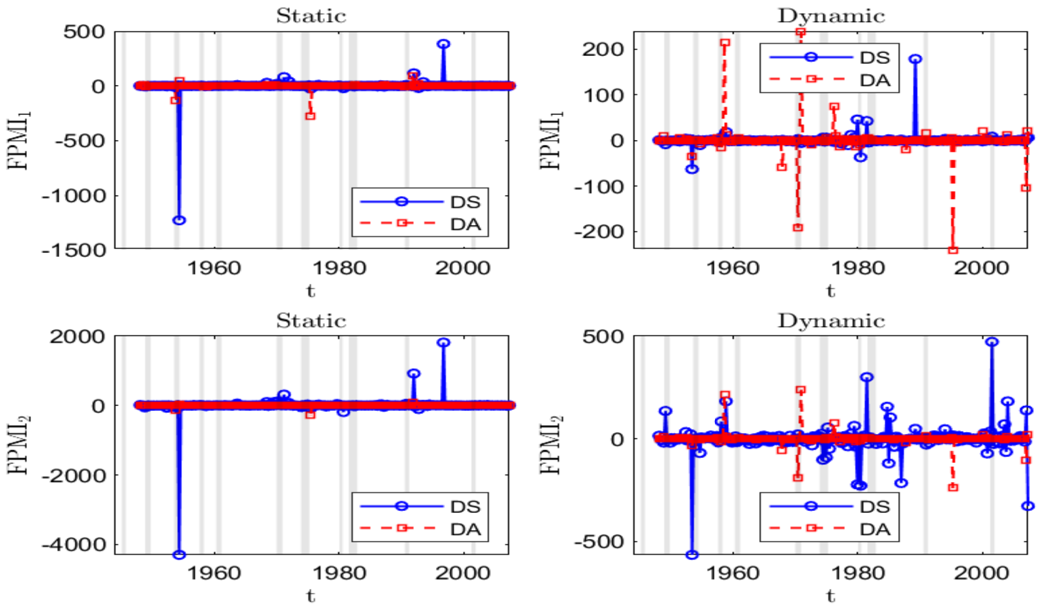


(f) $\hat{F}^Y$ versus s_t



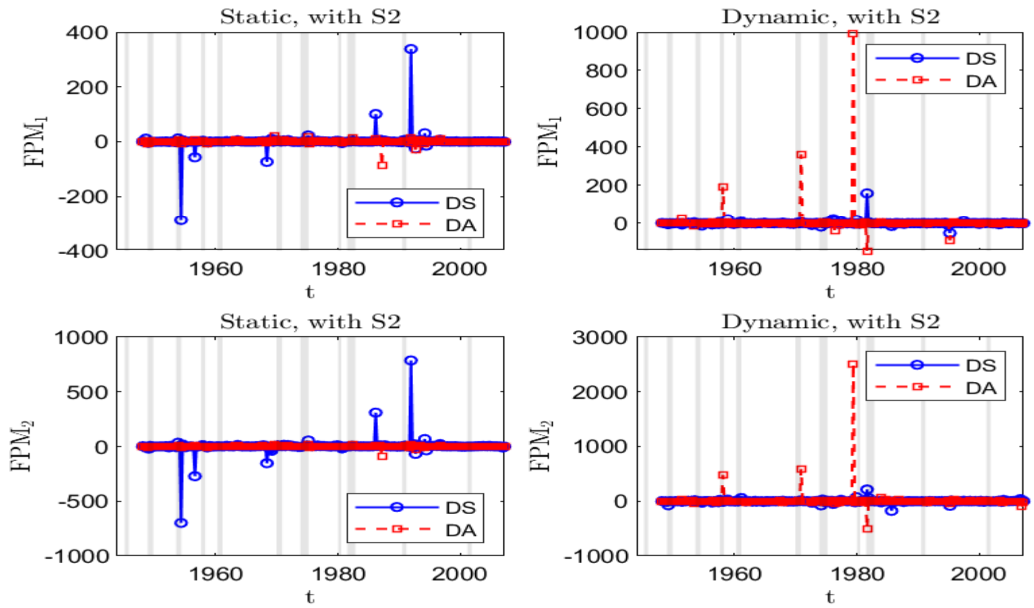
NOTE: This figure plots the estimated transition function of al the variables in consideration against time (left-hand side panels) and against transition variables S1– S3 (right-hand side panels)

Figure 4: Estimated FPMIs with transition variable S1



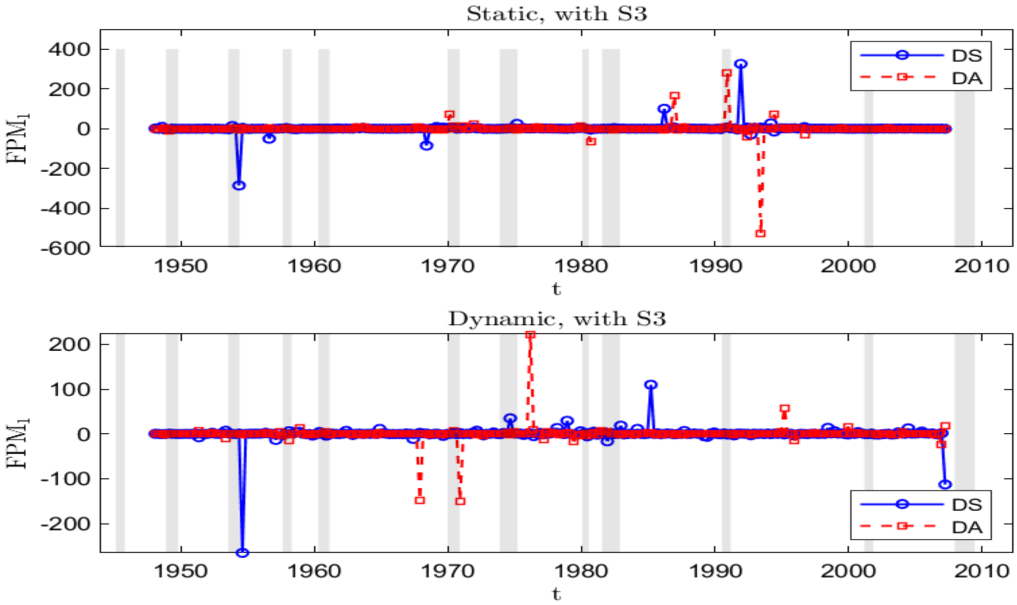
NOTE: Colored bands represent NBER recession dates.

Figure 5: Estimated FPMIs with transition variable S2



NOTE: Colored bands represent NBER recession dates.

Figure 6: Estimated FPMIs with transition variable S3



NOTE: Colored bands represent NBER recession dates.

Table 3: Estimation results of Dynamically Symmetric and Asymmetric models.

Specification	S1											
Variable	G				T				Y			
Model	Dyn. Symmetric		Dyn. Asymmetric		Dyn. Symmetric		Dyn. Asymmetric		Dyn. Symmetric		Dyn. Asymmetric	
Parameter	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE	Value	SE
ϕ_0	-3.8542	0.1195	0.9672	0.0671	-9.2104	12.2681	-10.3752	2.2592	1.0573	0.3616	-0.2290	0.2114
ϕ_1	-1.2978	65.8271	1.0011	0.0387	0.2032	6.9685	0.2601	0.1606	0.6354	0.2317	0.5252	0.1060
ϕ_2	-129.785	47.9899	-0.1348	0.0328	-0.0249	0.3649	-0.4066	0.0437	-0.3194	0.2081	-0.5224	0.0383
ϕ_3	-7.9140	46.8410			-0.2414	0.1803			0.0549	0.1999		
ϕ_4	4.7896	48.3562			-0.5268	0.1932			-0.4344	0.1575		
ϕ_5					0.2871	0.2744						
θ_0	13.2561	0.1024	-1.3346	0.4211	15.3575	20.5251	18.5734	6.2656	-0.1564	0.4198	2.2741	1.5560
θ_1	-118,639	8.5797	0.5307	0.1417	-3.4938	5.7683	1.1289	0.2075	0.5258	0.2454	0.9072	0.1811
θ_2	0.4296	13.0623	-0.5258	0.1323	1.1568	1.9298	0.0216	0.0707	0.0933	0.2359	0.0292	0.0597
θ_3	-0.1320	13.2070			-1.1274	2.3547			-0.2836	0.2296		
θ_4	0.4958	11.6285			1.9536	3.6901			0.4583	0.1737		
θ_5					-2.2385	3.9231						
γ_1	0.6520	0.4009	-6.1500	0.1932	2.7762	3.1240	-6.1500	0.1380	232.035	815.68	-3.1500	0.0942
γ_2	0.0476	0.0233	1.2500	0.2017	1.2194	2.0853	-2.2500	0.1615			-0.0823	0.1759
c_1	0.3282	4.8790	2.1862	0.1022	0.4132	0.8445	-3.5441	0.1775	0.2533	0.0217	1.0959	0.3704
c_2	1.1225	0.5212			1.3091	0.3503			0.3503	0.0217		
R^2	0.8276		0.8276		0.8144		0.7145		0.8001		0.7799	
LogLik.	-5.66e+04		-554.68		-1759.34		-3.258,86		-4.6164		-9.0876	
	S2											
ϕ_0	-9.0876	0.9199	2.0368	0.4612	1,3936	0.4701	-0.2376	0.1339	0.8348	0.1686	0.9898	0.2678
ϕ_1	0.0151	0.7059	-1.2747	0.6349	0.7763	0.0899	1.5433	0.0329	1.9264	0.1687	1.2820	0.1054
ϕ_2	0.2719	0.6249	1.5915	0.5040	0.2110	0.1347	-0.5674	0.0249	-0.4717	0.1071	-0.7724	0.2583
ϕ_3	0.5520	0.4005			0.0276	0.1146			-0.1249	0.0727		
ϕ_4	-0.3641	0.2832			-0.6677	0.1151			0.0588	0.0755		
ϕ_5					0.4200	0.0848						
θ_0	-1,2743	2.1568	-2.6258	0.5561	-1.9861	0.6500	3.4366	0.8424	0.8355	0.3390	-0.3083	0.3795
θ_1	4,5570	1.0620	4.6300	0.3262	0.6720	0.1286	-1.2635	0.2996	-0.5877	0.1417	-0.0284	0.1669
θ_2	-2,9702	1.3988	-3.7094	0.2785	-0.5747	0.1741	0.5985	0.1524	0.9471	0.2309	0.4487	0.2376
θ_3	-1,0830	1.1229			-0.1145	0.1832			-0.2797	0.2301		
θ_4	-0,2894	0.6779			0.2749	0.1724			-0.3156	0.1368		
θ_5					-0.0618	0.1229						
γ_1	3.2452	1.1147	-6.1500	0.0281	2,120,72	8.40e07	-6.1500	0.0859	1,363.63	2287.85	-3.1500	0.1349
γ_2	765.81	1.5e04	-1.1605	0.0239	11.2652	11.8550	1.2375	0.0971			-1.1189	0.1019
c_1	1.1225	0.1576	-0.7282	0.0798	-0.0461	0.0335	0.1785	0.0795	0.5640	0.0083	-1.0279	0.4187
c_2	1.0504	0.0311			1.3300	0.1077						
R^2	0.8657		0.8374		0.8044		0.7265		0.8066		0.7759	
LogLik.	-352.3147		-500.37		-1,900.80		-3,071.37		-3.4629		-10.1257	
	S3											
ϕ_0	-0.5980	2.7391	-44.6727	48.8091			-30.1383	74.6355	-2.9625	8,4927	-5.5562	16.3463
ϕ_1	0.7504	0.4195	4.8616	3.7639			-2.3279	7.8379	2.0589	2.3221	1.6230	1.0817
ϕ_2	0.1036	0.3733	-0.5768	0.3959			5.9505	14.4471	-2.0237	4.2199	-1.0634	1.3341
ϕ_3	0.4318	0.6372							-0.1220	1.5602		
ϕ_4	-0.4950	0.5246							-0.0028	1.1414		
ϕ_5												
θ_0	-0.5980	3.0097	91.2742	11.8626			61.5247	16.3987	111.1456	48.2376	12.6395	5.2861
θ_1	4.2967	0.4826	-7.0759	1.0369			6.6980	1.3029	-35.3972	26.1797	-0.7331	0.8347
θ_2	0.5635	0.4522	0.2354	0.7324			-12.3643	2.0998	75.7201	62.4688	1.11808	0.8132
θ_3	-0.8982	0.7196							3.3853	62.2005		
θ_4	0.7574	0.5903							-4.5736	42.9148		
θ_5												
γ_1	2.3982	2.2338	-6.1500	0.0279			-6.1500	0.0582	4.4504	1.8490	-3.1500	0.2112
γ_2			0.6187	0.0480			0.0413	0.0562	4.4264	1.7845	0.0413	0.2084
c_1	0.5246	0.0250	0.0413	0.0223			0.0046	0.7005	0.2310	0.3004	0.0046	1.3210
c_2									0.2410	0.2694		
R^2	0.8501		0.8342				0.7121		0.7767		0.8099	
LogLik.	-432.4223		-518.2096				-3,298.42		-2.9341		-9.9040	

NOTES: This table shows the estimates of the models here adopted. The Dynamically Symmetric model for T with S3 is not available. All the Dynamically Symmetric specifications have multiple transitions and only the first one is reported for economy of space.