Consumer Asset Pricing Model Based on Heterogeneous Consumers and the Mystery of Equity Premium

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abstract

As one of the core models of finance, the consumer capital asset pricing model (CCAPM) has produced the puzzle of equity premium. In order to explain this problem and get a more realistic pricing formula, this paper uses constant absolute risk aversion coefficient (Cara) utility function and introduces heterogeneous consumers to improve the original model, and finally gets a more effective form and there is no original puzzle in this form. At the end of the article, the American data are used to verify the results. The regression results support this model very well.

1 Introduction

1.1 Foreword

In the research process of modern finance, how to establish a reasonable asset pricing model has been a core problem that has not been completely solved. Capital asset pricing model (CAPM) is a classical model which is widely recognized and questioned, but it focuses more on the variance of return rate and asset value volatility, which is not completely compatible with the traditional economic theory. There are many proven schemes of CAPM through the classical economic framework in succession, but no matter from the macro level or from the macro level There are many defects in microcosmic. As an asset pricing model, consumption based capital asset pricing model (CCAPM) is also a macro-economic model widely concerned. The derivation method is completely from the classical economic framework. The assumption is clear and only two kinds of assets, market assets and risk-free assets, being considered. The conclusion is very simple and beautiful. After the classic CCAPM was put forward, a classic problem came into being, that is, the puzzle of equity premium. That is to say, the expected value of equity return can be explained by the variance of risk-free return and consumption fluctuation after being solved. However, the required risk aversion coefficient is much higher than the coefficient obtained in microeconomic experiment when the parameter is introduced. The emergence of this problem has caused extensive discussion. A natural thought is whether there is deviation in the basic setting of the model.

In fact, there are some problems in CCAPM's assumption based on CRRA utility function

The first point is that the relative risk aversion coefficient of CRRA utility function at any point is a constant, which is a strong assumption.

From the reality, it can be seen that people's risk aversion in the face of different costs of investment is not the same. For example, if a young man works in a front-line City, if his investment decision wealth is the remaining tens of dollar per month, his investment decision is often to buy several lottery tickets to see if he can get excess return, but if his investment decision-making is the remaining several thousand dollar, he may choose assets with certain risks, such as stocks or funds, in order to change his life. And if the wealth of decision-making is larger, such as inheriting a multi-billion dollar legacy, most ordinary people will choose to put the property in the bank for saving or buying national debt. Then they can get some interest every month and live a luxurious life.

The second point is that the financial market is complete and representative consumer is used, which is obviously not suitable in reality.

Thirdly, in the traditional model, representative consumer is used, and the utility function hypothesis of representative consumers is highly technical. In order to take care of the applicability of total consumption, when using representative consumer, we are often unable to choose the utility function freely, and have to use CRRA utility function, which will leads to the first problem.

And one of these problems is simply reflected in the micro experiment with relatively obvious results

Ask the same person the following two questions:

Question a: Be sure to get 100 yuan. The probability of 30 $\%$is 10000 yuan, and the probability of 70 $\%$is 0 yuan.

Question b: We must get 1 billion yuan. The probability of 30 $\%$is 100 billion yuan, and the probability of 70 $\%$is 0 yuan.

It is believed that most people will choose the latter in question a, which is determined by the form of utility function, and most people are risk averse. But for the second question, it is easy to see most people who are not super rich will choose to get a billion yuan. This simple small experiment shows that when people face different amounts of money, they will show different risk preferences. Generally, it is speculated that risk aversion will increase with the increase of money.

In the classical model, it is always assumed that the relative risk aversion coefficient is constant, that is, (where $u$represents utility function, Crepresents consumption, and $gamma$represents relative risk aversion coefficient):

$$-c\frac{u'(c)}{u''(c)} = \gamma;$$

The CRRA utility function satisfies this property:

$$u(c) = \frac{c^{(\gamma-1)} - 1}{\gamma - 1}.$$

In reality, CRRA utility function shows that consumers at different asset levels have the same expectation and variance of return, which is obviously inconsistent with the above simple micro experimental results.

In order to deal with the addition problem of non representative consumers, the main conclusion of this paper is based on the fact that the relative risk aversion coefficient is increasing and the utility function is set as follows:

$$-c\frac{u'(c)}{u''(c)} = c\alpha;$$

The CARA utility function satisfies this relation:

$$u(c) = -e^{-\alpha c}.$$

What the CARA utility function shows is that consumers of different asset levels have the same expectation and variance for the same small consumption amount (in the neighborhood where Taylor formula performs well), that is, consumers of CARA utility function type should have the same response to gambling games of several cents regardless of their income, but their attitude towards the rate of return affecting the overall asset is not very good same.

At the same time, since we set the utility function like this, it is obviously not appropriate to use the representative consumer when considering the risk, so we can only use the heterogeneous consumers to describe this problem.

It is found that the equity premium, that is, the logarithmic return of risk assets minus the logarithmic return of risk-free assets, is a linear expression of the average consumption of the current period (in the formula, Rtrepresents the gross return of risk assets, RF, trepresents the gross return of risk-free assets, $hatC_t$represents the average consumption of the T period, αrepresents the absolute risk aversion in the utility function Coefficient, $sigma$is the standard deviation of total consumption growth rate):

$$\ln E_t R_t - \ln E_t R_{f,t} = \hat{c}_t \alpha \sigma^2$$

The classic CCAPM just ignored the influence of this aspect.

At the end of the theoretical model, we use the Standard Poor's and one-year treasury bond interest rate regression, and find that the regression results support our conclusion to some extent.

However, from the perspective of model robustness, our formal assumption of utility function is a little too strict, which is likely to have a huge impact on our final results. A very intuitive idea is that even if average consumption really has an impact on equity premium, it may not be a simple linear impact, but there may be various impact ways, and the impact ways may depend on different utility functions of different people. Therefore, from the empirical point of view, this paper uses the support vector regression machine, a nonlinear regression machine learning algorithm, to compare the previous linear regression, and finds that at least we have reason to think that the impact of average consumption on equity premium should be positive.

1.2 Literature Review

Mehra and Prescott (1985) pointed out in [10] that there is a very serious problem in the formula of risk asset premium, that is, to explain the part of the return on risk asset which is higher than the risk-free return, the relative risk aversion coefficient of investors is much higher than the micro experimental results, which is called the puzzle of equity premium.

The puzzle in the consumption based asset pricing model has aroused a lot of scholars' research interest. Campbell (2002) [3] uses the data of securities markets in the United States, the United Kingdom, Australia and other countries for nearly 30 years to test the classic CRRA utility function. It is found that the puzzle holds in different degrees in these countries, that is, there are higher excess returns, which means that there are higher excess returns. In other words, the estimated value of the relative risk aversion coefficient is far beyond the reasonable range, which shows that the puzzle of equity premium is a widespread problem in the global market and a market anomaly.

To solve this problem, scholars from different countries have made explanations through different schemes. Among them, the most concerned and accepted one is that the scheme is still based on rational investors as the basic assumption, using the way of changing market preference or changing market friction to explain this problem, such as [12] and [1]. This scheme can reduce the required level of investor risk aversion from the perspective of model structure.

Other scholars describe this problem through some new financial research theories, for example, irrational expectation, seeing [4]; behavioral finance, seeing [14], incomplete market, seeing [6]; heterogeneous consumer, seeing [5]. These papers consider the situation of non-traditional consumers, and use some special schemes in utility function, expectation, etc., which can relax the requirements of risk aversion.

From the empirical point of view, there are also many scholars who think that the puzzle of equity premium may arise because there is a certain bias in the selection of basic data. The empirical data in the literature ([12]) deduct quasi durable goods such as medicine, and the literature ([13]) only uses the consumption data of people who have invested in stocks. The literature ([11]) defines housing as an asset and consumer goods The literature ([8]) focuses on the high-income population, which makes the risk aversion coefficient of model demand decrease.

2 Model and Solution

2.1 Classical Model and Conclusion

In the classic CCAPM, we solve the following representative consumer problem (Crepresents consumption, $q$represents the price of risk assets, $x$represents the dividend of risk assets, Zrepresents the holding amount of risk assets, $p$represents the price of risk-free assets, Brepresents the holding amount of risk-free assets):

$$\max E_0 \sum_{t-0}^{\infty} \beta^t u(c_t)$$
$$s.t.\ c_t + q_t z_t + p_t b_t \leqslant (q_t + x_t) z_{t-1} + b_{t-1}.$$

Let

$$u(c) = \frac{c^{(\gamma-1)} - 1}{\gamma - 1}.$$

Combined with some classical assumptions, we can get the following results:

$$\ln E_t R_t - \ln E_t R_{f,t} = \gamma\sigma^2$$

2.2 Hypothesis of This Model

1. There will be a total of infinite trading periods from phase 0.

2. There are limited consumers.

3. Complete information. Consumers know the distribution of all assets in each period and the conditional distribution information in each period.

4. Every consumer is rational.

5. Consumers use the expected utility function with discount based on consumption.

6. The income of consumers only comes from the initial endowment and investment income.

7. Dividends shall be distributed before trading in each period.

8. The consumer is the receiver of the price.

9. All consumers have the same utility function, which is the constant absolute risk aversion coefficient utility function of CARA.

10. There is only one kind of risk assets and one kind of risk-free asset in the market.

2.3 Model Solution

At first, we consider a two phases model:

$$\begin{aligned}
&\max\ u_i(c_0^i) + E_0\beta_i u_i(c_1^i) \\
&s.t.\ c_1^i \leqslant R_f(w - qz^i + c_0^i) + x_1 z^i
\end{aligned}$$

and

$$u(c) = -e^{-\alpha c}.$$

x_1 is the total consumption of phase 1 and x_0 is the total consumption of phase 1, and x_1 is uncertanty.The results are as follows:

$$\begin{aligned}
&E(\beta_i e^{-\alpha(c_1^i - c_0^i)} R_f) = 1 \\
&E(\beta_i e^{-\alpha(c_1^i - c_0^i)} \frac{x_1}{q}) = 1 \\
&E(\frac{x_1}{q} - R_f) = \frac{n}{\sum E(\beta_i e^{-\alpha(c_1^i - c_0^i)})} \alpha \hat{c_0} \frac{x_0}{q} Var(\frac{x_1}{x_0})
\end{aligned}$$

$\frac{x_0}{q}$ is close to consumption-asset-ratio and this equation means that the relationship between equity premium and average consumption is linear.

$$
\begin{aligned}
&\max\ E_0 \sum_{t-0}^{\infty} (\beta^i)^t u_i(c_t) \\
&s.t.\ c_t + q_t z_t + p_t b_t \leqslant (q_t + x_t) z_{t-1} + b_{t-1}
\end{aligned}
$$

After the solution, we can get the following conclusions

$$
E_t(\beta^i \frac{u_i'(c_{t+1})}{u_i'(c_t)} \frac{q_{t+1} + x_{t+1}}{q_t}) = 1 \tag{2.1}
$$

$$
E_t(\beta^i \frac{u_i'(c_{t+1})}{u_i'(c_t)} \frac{1}{p_t}) = 1 \tag{2.2}
$$

It is assumed that the utility function is CARA utility function

$$
u(c) = -e^{-\alpha c}.
$$

After that, we can get a pricing formula:

$$
R_{f,t} = \frac{1}{\beta^i E_t(e^{\alpha(c_{t+1}^i - c_t^i)})}. \tag{2.3}
$$

Here we assume that total consumption and individual consumption follow different random walks:

$$
\lambda_t := \frac{c_{t+1} - c_t}{c_t} \sim N(\mu, \sigma^2). \tag{2.4}
$$

$$
\frac{c_{t+1}^i - c_t^i}{c_t^i} \sim N(\mu^i, (\sigma^i)^2). \tag{2.5}
$$

From the above assumptions(2.5)and(2.3)，we can get

$$
\ln E_t(R_{f,t}) = -\ln(\beta^i) + \mu^i \alpha c_t^i - \frac{1}{2}(\sigma^i)^2 \alpha^2 (c_t^i)^2 \tag{2.6}
$$

Next, consider risk assets. Let's first assume $\frac{x_t}{q_t} = w$, and then verify that $w$is a constant. Bring in fixed price(2.1), we have

$$
w = \frac{1 - E_t(\beta^i e^{-\alpha(c_{t+1}^i - c_t^i)} \lambda_t)}{E_t(\beta^i e^{-\alpha(c_{t+1}^i - c_t^i)} \lambda_t)}
$$

To solute the model, **we assume w is a constant**. Next, let's use this formula to take back to the pricing formula(2.1)to get the result:

$$
E_t(R_t) = \frac{E_t(\lambda_t)}{E_t(\beta^i e^{-\alpha(c_{t+1}^i - c_t^i)} \lambda_t)};
$$

then with approximate replacement $\lambda_t \approx e^{(\lambda_t)}$

$$\ln E_t(R_t) = -\ln(\beta^i) + \mu^i \alpha c_t^i - \frac{1}{2}(\sigma^i)^2 \alpha^2 (c_t^i)^2 + COV(c_t^i \alpha \mu_i, \lambda_t); \tag{2.7}$$

From this formula, we can see some interesting conclusions (the individual is the recipient of the yield):

1. Because $\mu$is what people like, and $\sigma$is what people hate. If people can choose $beta$, the bigger $beta$, the better the growth.

2. Interestingly, there is a negative correlation between c_t^i and μ^i.

3. The choice between expectation and variance is linear.

Combining (2.6) and (2.7), we can get

$$\ln E_t(R_t) - \ln E_t(R_{f,t}) = COV(c_t^i \alpha \mu_i, \lambda_t); \tag{2.8}$$

Use an obvious property: $\sum c_t^i \mu_i = c_t \mu$ and sum (2.8)， we can get the final result

$$\ln E_t(R_t) - \ln E_t(R_{f,t}) = \hat{c}_t \alpha \sigma^2; \tag{2.9}$$

The difference between the above formula and the classic CCAPM is that $\gamma$is replaced by average consumption multiplied by α, which is a good explanation for the puzzle of equity premium. This formula is fully verified in the following measurement.

3 Data Introduction and Empirical Results

3.1 Data Introduction

The data source of this paper is CEIC database.

表 a:

	Index: SP: 500	Medium term treasury bond yield: fixed term: nominal: Ma: 10 years	Population: residents and armed forces overseas	Personal consumption expenditure (PCE): seasonally adjusted by annual rate	GDP
region	U.S.A	U.S.A	U.S.A	U.S.A	U.S.A
time country					
frequency	month	month	month	month	quarter
unit	1941-1943=10	annual interest rate%	Thousands of people	billion dollars	billion dollars
data sources	Standard & Poor's	Federal Reserve Board	US Census Bureau	Bureau of Economic Analysis	Bureau of Economic Analysis
status	continue	continue	continue	continue	continue
mnemonic					
function information					
first observation date	01/1976	04/1953	01/1952	01/1959	03/1947
last observation date	03/2019	03/2019	03/2019	01/2019	06/2019
series re-					
marks					
can					
average	884.88	5.80769	242396	4715.95	1522.26
variance	509468	8.12112	2.5E+09	1.7E+07	2389844
standard deviation	713.77	2.84976	49715.8	4152.32	1545.91
skewness	0.78908	0.89721	0.10259	0.68023	0.86979
kurtosis	-0.11216	0.5186	-1.14833	-0.85499	-0.5485
coefficient of variation	0.80663	0.49069	0.2051	0.88048	1.01554
minimum	87.04	1.504	156309	306.091	58.497
maximum	2913.98	15.3238	328743	14238.9	5360.92
median	825.88	5.23835	238679	3483.75	884.735
number of observation	519	792	807	721	290

3.2 Basic Regression

This paper focuses on the empirical results of the conclusions in model 2, that is, the expected return of risk assets in the current period minus the risk-free return is linear with the current social

表 b:

	Depository institution: legal reserve ratio	Deposit institutions: monetary base: currency in circulation	balance of trade	Report on business: Purchasing Manager Index	Report on business: PMI: price index
region	U.S.A	U.S.A	U.S.A	U.S.A	U.S.A
time country					
frequency	month	month	month	month	month
unit	million dollar	million dollar	million dollar	NA	NA
data sources	Federal Reserve Board	Federal Reserve Board	US Census Bureau	Institute for Supply Management	Institute for Supply Management
status	continue	continue	continue	continue	continue
mnemonic			US.TBAL.VA.USD-MN-M		
function information					
first observation date	02/1984	01/1959	01/1989	01/1948	01/1948
last observation date	09/2019	09/2019	08/2019	09/2019	09/2019
series re-marks					
can					
mean	35157.9	448288	-41217.8	52.9137	62.0732
variance	1.2E+09	2.2E+11	5.8E+08	52.7509	222.698
standard deviation	34984.9	463684	24171.2	7.26298	14.9231
skewness	1.5169	1.16561	0.12143	-0.30383	-0.06319
kurtosis	1.16269	0.3496	-1.41693	0.76585	-0.49652
coefficient of variation	0.99508	1.03434	-0.58642	0.13726	0.24041
minimum	3151	31234	-89097	29.4	10.6
maximum	135103	1762924	-1890	77.5	100
median	23350	245627	-44491.5	53.5	62.5
number of observation	428	729	368	861	861

表 c:

	Report on business: PMI: employment index	retail of consumer goods	Index: Shanghai Stock Exchange: Composite Index	Deposit interest rate: Deposit: fixed term: 1 year	population
region	U.S.A	China	China	China	China
time country					
frequncy	month	month	month	month	year
unit	NA	billion yuan	1990.12.19=100	annual interest rate%	million persons
data sources	Institute for Supply Management	National Bureau of Statistics	Shanghai Stock Exchange	People's Bank of China	National Bureau of Statistics
status	continue	continue	continue	continue	continue
mnemonic					
function information					
first observation date	01/1948	01/1990	12/1990	09/1988	12/1949
last observation date	09/2019	10/2019	11/2019	11/2019	12/2018
series re-marks					
can					
mean	50.0301	943.219	1948.85	4.53992	1012.57
variance	57.6939	1008562	1130286	11.0843	76149.2
standard deviation	7.59565	1004.27	1063.15	3.32931	275.951
skewness	-0.39827	1.2034	0.60404	0.97441	-0.22925
kurtosis	0.54762	0.19161	0.37823	-0.59696	-1.36718
coefficient of variation	0.15182	1.06473	0.54553	0.73334	0.27253
minimum	27.2	55.43	113.94	1.5	541.67
maximum	73.7	3810.43	5954.77	11.34	1395.38
median	51.1	436.825	1810.52	3	1036.83
number of observation	861	342	348	375	70

average consumption. The return on risk assets is the standard Poor's 500 index, and the risk-free yield is the US 10-year Treasury bond interest rate. In particular, the average consumption has been converted to US dollars per person at this time.

The first regression is to use the simplest OLS. In the second regression, the general method of macro research is to logarithm the independent variable and the dependent variable, but because there is only one independent variable in our model, if the conclusion of the model is established, the coefficient of logarithm and regression should be 1, so this regression also tests the establishment of the model, such as If the coefficient of regression is not 1, then there are two possibilities: one is that there are some problems in model setting itself, and the relationship may not be linear; the other is that there may be missing variables in the process of regression, which is also the problem of model setting.

In addition to the above two kinds of regression, the time series often has its own heteroscedasticity. After the above two regression, the stationarity test of the time series is carried out after the regression. But the test result is not 100% reliable, so we use FGLS to recalculate the above two regression.

表 d: ols and gls

	Yield difference	Logarithm of yield difference (logarithm of independent variable)	Yield difference（fgls）	Logarithm of yield difference (logarithm of independent variable, fgls)
average consumption	2.6296E-7***	0.0561***	1.1025E-7***	0.0315***
	4.8763E-08	9.6695E-03	1.9349E-07	3.9181E-02
constant	-1.9367E-02	-2.5979E-01	0.0165***	3.9726E-02
	1.3696E-02	1.1861E-01	5.5482E-02	4.8038E-01
R-squared	0.0642	0.0734	0.0642	0.0734
adftest	1（stable）	1	1	1

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

In the model, we can see that the coefficient of risk return of risk assets that we are most concerned about is indeed positive with respect to the average value of consumption, which shows that the formula we get has certain significance. For the OLS regression model without FGLS, we can also see that the time sequence test of residual is indeed stable, and it is more satisfactory that the constant term does not reject the hypothesis of 0, which is more suitable for our model Type is a very favorable support, but the coefficient of constant term is significant after considering FGLS, which makes us still have certain uncertainty.

However, it can be seen that for log regression, the coefficients are far less than 1, 1 is far beyond the confidence interval, that is to say, there are probably nonlinear factors in the model. Now, the possible problems are listed as follows:

First, list the possible data problems:

1. SP 500 can't well represent all the risk assets in the whole market, as well as a large number of bonds, real estate and other stocks.

2. It is assumed in the model that the consumers in this period are homogeneous, but in the century of the 20th century, many profound changes have taken place.

From the perspective of politics and international relations, in this century, there have been two world wars, which have experienced the opposition and impact between socialism and capitalism. After the 1950s, the cold war between the United States and the Soviet Union has also greatly affected the society. In war time and non war time, people's view of risk is very different.

From the perspective of science and technology, the 20th century is also a century of changeable situation. At the beginning of the 20th century, it was just entering the era of electrification, but by the end of the 20th century, it had entered the era of Internet information. From the perspective of the early twentieth century, classical physics tends to achieve great success, and social change is relatively stable and slow. At the end of the twentieth century, human beings have made tremendous changes in science and technology in the past 100 years, and scientific and technological innovation is emerging in an endless stream, and the speed of scientific and technological progress is far faster than that of the past era. Moore's law of computer, the computing speed of computer will double every few years, this kind of progress speed is far unimaginable in the past, which will also lead to the profound change of human utility function.

All of these indicate that the regression coefficient in our model is time-varying and can not be obtained by simple estimation.

3. There may be a problem of non-linear term, that is, the relationship between risk return and

average consumption is not linear, so we consider the possible error of regression method, we use support vector regression to regression the model.

3.3 Mechanism Analysis and Test

In this paper, we mainly choose the stock index data as the representative of risk assets to replace the risk asset market portfolio in the model. In reality, the price is not as discrete as we set in the model (in fact, in the matching of the model and the real data, the model needs to change the price once a year, which still has a big gap with the reality). In fact, such setting also leads to many problems in the regression. After all, generally speaking, for risky assets such as stocks, the rate of return is higher than the risk-free rate of return (unlike the risk-free rate of return such as insurance, the stock market is generally Pro cyclical with respect to the economic situation), and the daily fluctuations will make the data we collect appear not small biases. We can often see that the return of the stock index in a certain year is lower than the risk-free rate of return in that year, in fact, this situation is not rare.

From the micro point of view, everyone's consumption actually happens. Even when it happens, there is no clear plan. Therefore, it is the allocation and transaction of assets. People will update their views on assets anytime and anywhere, and then change their asset allocation. Especially in today's popularity of smart phones, transactions can be carried out anytime and anywhere

Therefore, we need an analysis basis to make the model look more reasonable and make it more convenient for us to carry out mechanism analysis. In order to better adapt to the data, we assume that an economic entity determines the expectation of the dividend and price of the stock in the next year after the annual report is issued, and plans its own risk-free return and risk-free return after the expectation is formed. The proportion of assets in consumption is obviously not completely accurate in reality, but for ordinary people, even if they don't plan their own asset distribution at the beginning of a year, they will form a rough plan, which will be implemented to a certain extent according to this plan, and the expectation of assets also depends on the annual report to a large extent.

On the basis of this discussion, we can make some interpretations of this influence mechanism according to the way people choose. For the sake of understanding, we might as well think that all people sell all their own assets before buying according to the new plan when they make a transaction, which will undoubtedly help us to analyze.

1. The most direct way is to see from the reality that the total income of residents is certain. When they choose to consume more, the total amount of investment will be smaller. Due to the effect of clearing, the price of assets in the current period will fall. When the price and dividend in the next period are fixed, the rate of return of risk assets will naturally rise, while the rate of return of risk-free assets will rise But the difference between the two will naturally increase under the same rising proportion. Although there is no reason to think that the rising proportion of the two is the same in reality, it can also be explained to some extent.

2. If people expect that the future rate of return will be relatively high, they will be affected by two functions. On the one hand, due to the high return, investment may be increased to obtain a higher return in the future, which will reduce people's current consumption. On the other hand, due

to more people buying risk assets, the gap between risk-free assets and risk-free assets will become smaller. On the other hand, in order to smooth consumption, There will be more consumption and lower investment, at this time, the return of risk assets will further increase. In the analysis of these two aspects, both reflect the positive relationship between average consumption and return on assets.

3. Considering the relationship between the rate of return on consumption and investment and the economic environment, in the period of economic prosperity, people's income will increase, their output will also increase, and their consumption will increase. From the perspective of market, strong demand will stimulate production, and make transactions more frequent. The products produced by factories will be sold quickly, and the natural dividend will increase. In the period of prosperity, people tend to More optimistic expectations for future prices will also increase, and the return in the next period will naturally rise. In the economic prosperity, the risk-free interest rate will be greatly reduced, that is to say, in this analysis path, the average consumption will increase, the return on risk assets will increase, and the risk-free return will decrease. Of course, in the economic depression, it turns into a decrease in average consumption, a decrease in the rate of return on risky assets and an increase in the rate of return on risk-free assets.

4. From the perspective of consumption in the past, the fluctuation of consumption growth rate is relatively low, so consumption in the last period and this period together affect the level of yield.

5. It is expected that consumption will fall in the future, so defensive savings.

Because of the strong correlation between GDP and consumption, GDP is not put into the regression, mainly to test the impact of consumption in the next period and the previous period on the equity premium.

表 e: Mechanism test

	Yield difference	Logarithm of yield difference (logarithm of independent variable)	Yield difference (FGLS)	Logarithm of yield difference (logarithm of independent variable, FGLS)
Average consumption in the previous period	6.2925E-07	0.2664*	3.4952E-07	0.1964***
	5.1932E-07	0.1466	5.8204E-07	0.1685
Average consumption	2.9851E-06	0.7826***	-3.7395E-07***	-0.2061
	6.6181E-07	0.1949	5.8582E-07	0.1752
Average consumption in the next period	3.2615E-6***	0.7826***	2.5716E-07	0.0583
	4.9743E-07	0.1496	5.3365E-07	0.1676
Constant	0.01310***	0.2922**	-0.0092***	-0.1584
	1.4742E-02	0.1503	4.9329E-02	0.4466
R-squared	0.1569	0.1460	0.0377	0.0448
	1（stable）	1	0	0
		*** p¡0.01, ** p¡0.05, * p¡0.1		

The results of this regression are very interesting, On the contrary, the residual of FGLS fails to pass the stability test, so the results of OLS are very important.

Both regression results show that the current consumption and the average consumption together are very important. That is to say, the path of influence should be defensive savings and the 突tradeoff between current savings and consumption.

3.4 Robustness test

Because (2.9) is relatively simple, it is advisable to use OLS + time series stationarity test for regression. In order to be more convincing, this paper adds GLS regression.

However, recent studies have pointed out that financial pricing has a complex non-linear relationship, and the model in this paper can not completely shield this relationship. For a non-linear relationship, that is (consider a one variable non-linear relationship for simplicity):

$$y = g(x) + \epsilon$$

we care

$$\frac{1}{(b-a)} \int_a^b g'(x) dF(x)$$

That is, the value of the average slope, but can the linear regression we do really represent this value?

In our linear regression, the best estimate we can get is the solution of the following problem:

$$\min \int_a^b (g(x) - kx - b)^2 dF(x).$$

Unfortunately, the result of this solution is

$$k = \frac{\int_a^b xg(x)dF(x)}{\int_a^b x^2 dF(x)}.$$

This result obviously does not represent the result of our demand.

Therefore, this paper will adopt a non-linear fitting scheme: support vector regression machine, which is an extension of support vector machine. It ensures the minimization of structural risk while considering the empirical risk, which can solve the over fitting problem of high-dimensional small sample problem to a certain extent, while the kernel function method mapping to high-dimensional space solves the non-linear problem. Theoretically, any function can be approximated by RBF kernel function.

In order to calculate the p value, we use Chebyshev inequality, which makes the slope ka function of the independent variable x

$$pr(|k(x) - E(k(x))| \leq |E(k(x)) - 0|) \leq \frac{VAR(k(x))}{|E(k(x)) - 0|^2}$$

This P-value is significantly more conservative, and contains the probability of less than 0 and more than twice $mu(K(x))$.

表 f: svr regress

	Yield difference	Logarithm of yield difference (logarithm of independent variable)
Average consumption	2.793E-07（p:0.6542）	0.05329(p值0.59)
	2.259E-07	0.0410
R-squared	0.07788	0.0853
adftest	1（stable）	1

From the perspective of regression results, the coefficient is positive, which makes us very satisfied, and interestingly, the estimated results are very close to the results of OLS regression. Two-phase confirmation, we can think that this result is believable. And this number is very small, it can be said that there is no need to worry about the excessive risk aversion coefficient.

In order to test the universality of the model, we return the same data to China:

表 g: Chinese data

	Yield difference	Logarithm of yield difference (logarithm of independent variable)	Yield difference（FGLS）	Logarithm of yield difference (logarithm of independent variable,FGLS)
Average consumption (thousand yuan)	0.00039	0.0065*	0.0004	0.00065
	0.000348876	0.003430107	0.0003	0.0034
constant	-0.017***	0.25***	-0.017**	-0.017***
	0.004353672	0.006856241	0.0044	0.0069
R-squared	0.010751798	0.010751798	0.0039	0.0108
adftest	1（stable）	1	1	1

*** p¡0.01, ** p¡0.05, * p¡0.1

After the risk coefficient is regressed, other variables are introduced to test the robustness. The data period is from January 1976 to March 2019, and the data source is CEIC database. The data are: depository institutions: legal reserve ratio, trade balance: goods, report on business: purchasing manager index, report on business: PMI: price index, report on business: PMI: employment index.

表 h:

	resuld	without GDP	logarithm	logarithm without GDP
legal reserve	0.68671942***	0.14793359*	0.00000045***	0.00000012***
	-0.12461	-0.06054	-9E-08	-3E-08
Trade balance	0.03569	0.16683	0.00026	0.00134
	-0.11142	-0.11111	-0.00099	-0.00097
Purchasing manager index	-0.11652923***	-0.11662984***	-0.00072441**	-0.00080027**
	-0.03225	-0.03313	-0.00025	-0.00026
Price index	0.0055	-0.13609	-0.00049	-0.00164
	-0.10636	-0.10516	-0.00102	-0.001
Employment index	-0.25845870***	-0.21237532***	-0.00011353***	-0.00009709***
	-0.02821	-0.02733	-1.3E-05	-1.3E-05
GDP	-2.82e+00***		-0.00008665***	
	-0.57525		-2.2E-05	
Average consumption	2.74e+00***	0.20628998*	0.00000280***	1.8E-07
	-0.52311	-0.09152	-6.6E-07	-1E-07
_cons	-6.65e+00***	-1.18e+00***	0.0016	0.04202
	-1.14	-0.23709	-0.02764	-0.0262
N	425	425	425	425
r2	0.28406	0.24272	0.25652	0.22784
F	2.36E+01	2.23E+01	2.06E+01	2.06E+01
p	0	0	0	0

Standard errors in parentheses

The regression results show that the coefficient in front of the average consumption is positive.

4 Summary

4.1 Article results

In order to overcome the inherent problems in the classical CCAPM model hypothesis and explain the puzzle of equity premium, this paper proposes an asset pricing model characterized by a heterogeneous constant absolute risk aversion coefficient. Surprisingly, this model can avoid the problem of equity premium puzzle. Fortunately, this model can also be added to a certain extent.

In the process of solving this model, we have found some interesting micro conclusions. The first point is that the more people have patience, the closer the β is close to 1, then the growth of people's consumption will increase at a faster rate. The second point is that the lower the level of consumption is, the faster the future will grow. These conclusions need micro data support.

At the end of the model, we find that the equity premium is a linear function of per capita consumption. In the second half of the paper, we test the relationship. The results of linear regression show that there is a positive linear relationship between the two variables, and the results are significant. At the same time, the results of the nonlinear support vector regression machine fitting and OLS regression are also highly consistent, which can be regarded as reliable. Most importantly, the regression coefficient shows that the puzzle of equity premium does not exist in this model.

In the mechanism test, regression shows that this relationship mainly comes from defensive savings for future consumption and the relationship with current consumption.

4.2 Expandable in the future

The model of this paper is relatively simple, the measurement is not very complex, and the data source is relatively single, so we can improve these three points in the later research.

For the simple problem of the model, we can make the following expansion:

1. In this paper, we only consider the heterogeneity of patience $beta$, and the heterogeneity of resources. In fact, the research of heterogeneous belief is more attractive in finance, but undoubtedly, adding heterogeneous belief will make aggregation more complex, and different risk aversion will make the model difficult to aggregate.

2. The model is a local equilibrium model without the introduction of producers, which can be introduced into the general equilibrium model. In this paper, the general equilibrium model is not used to better compare with the simplest classic CCAPM. In fact, the introduction of producers will not change the results.

3. It is assumed that the growth rate is normal distribution, but in reality, it is usually mixed normal distribution or stable distribution fitting better.

From the perspective of measurement, the following improvements can be made:

1. The control variables of regression are too simple, and simple methods are used in regression. In fact, this problem corresponds to the simple model setting, so the better solution is to introduce more macro variables into the model, and use panel or structural equation to regression, which can also better eliminate endogeneity.

2. Machine learning only uses one kind of support vector regression machine. More methods should be used and compared.

3. The robustness test is too simple and crude, so a more differentiated regression scheme should be used.

From the data point of view, the following extensions can be made:

1. The data of a wider range and a longer period of time should be used, i.e. for a longer period of time and more countries, and the differences between the coefficients of different countries should also be compared. The panel and time series can be compared at the same time.

2. More indexes should be used to synthesize risk return.

3. The best way is to get micro data to regress more equations.

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