Munich Personal RePEc Archive

Strategic Issues in College Admissions with Early Decision

Mumcu, Ayse and Saglam, Ismail

Bogazici University, TOBB University of Economics and Technology

11 February 2020

Online at https://mpra.ub.uni-muenchen.de/98587/
MPRA Paper No. 98587, posted 14 Feb 2020 16:26 UTC

Strategic Issues in College Admissions with Early Decision*

Ayşe Mumcu[a] and Ismail Saglam[b,†]

[a] *Department of Economics, Bogazici University, Bebek, 34342, Istanbul, Turkey*

[b] *Department of Economics, TOBB University of Economics and Technology, Sogutozu Cad. No:43, Sogutozu, 06560, Ankara, Turkey*

In this paper, we consider college admissions with early decision using a many-to-one matching model with two periods. As in reality, each student commits to only one college in the early decision period and agrees to enroll if admitted. Under responsive and consistent preferences for both colleges and students, we show that there exists no stable matching system, consisting of early and regular decision matching rules, which is nonmanipulable via early decision quotas by colleges or early decision preferences by colleges or students.

Keywords: College admissions; early decision; manipulability; many-to-one matching.

JEL Classification Numbers: C71; C78.

*The authors have no conflicts of interests to declare. An early draft of this paper was written while the second author received financial support from Turkish Academy of Sciences, in the framework of Distinguished Young Scientist Award Program (TUBA-GEBIP). The usual disclaimer applies.

†Corresponding author. E-mail: isaglam@etu.edu.tr.

1 Introduction

The competition among colleges to attract high quality students has led to the adoption of early admission programs over the last five decades, and turned the college admission process into a complicated 'admission game' in the United States.[1] Today the most prominent colleges offer a choice over a variety of admission programs: 'early action', 'restrictive early action', 'single-choice early action', 'early decision (I and II)', and 'regular decision'.

According to the 2019 State of College Admissions Report of the National Association for College Admission Counseling (NACAC), twenty-five percent of the respondents taking the Admission Trends Survey[2] offered early decision (ED) plan while about thirty-eight percent offered early action (EA) plan. These figures are much higher for private institutions, and for selective colleges.[3] While EA presents a student a chance to gain an admission decision in advance of the Regular Decision (RD) date without a commitment to attend, ED requires the student to apply early to only one college and matriculate if admitted. EA and ED programs usually require high school seniors to apply near November with a decision by late December. RD offers a later application deadline (January 1) and time to decide whether to matriculate until May 1.[4]

The literature on college admissions offers various arguments to explain why colleges use ED programs and students prefer to apply to it. Colleges,

[1] *The Early Admissions Game* (Avery, et al., 2003) is the seminal empirical study of the effects of early admission policies in the US.

[2] The survey response rate was 35 percent.

[3] While only 5 percent of public institutions offered ED programs, this ratio goes up to 37 percent for private colleges. Fifty-six percent of the selective colleges (those accepting fewer than 50 percent of applicants) offered an ED plan. Forty-six percent of colleges with yield rates lower than 30 percent used EA plan. (See 'State of College Admission 2019 at http://www.nacacnet.org.)

[4] Refer to the 2018-2019 Admission Trends Survey of NACAC, available at http://www.nacacnet.org, for a detailed description of each type of early admission program.

in particular lower-ranked ones, may use ED to attract the desired students in restricted application pools in order to mitigate the competition in RD and manage the enrollment uncertainty (Chen et al., 2018).[5] ED programs may provide the student to signal her enthusiasm about a particular college, (Avery and Levine, 2010). Colleges may use ED programs as a screening device to avoid the winner's curse (Lee, 2009; Kim, 2010; Chapman and Dickert-Conlin, 2012).

2018-2019 Admission Trends Survey of NACAC also reports that between Fall 2017 and Fall 2018, there was an average increase of 11 percent in the number of ED applicants. Students prefer to apply early if the chance to be admitted is higher at ED than RD. Lee (2009) shows that ED results in lower admission standards than in RD, and that ED may increase allocative efficiency. Using the data from two liberal arts school, Chapman and Dickert-Conlin (2012) finds the evidence that applying ED raises the probability of acceptance by 40 percentage points. Considering need-blind colleges, Kim (2010) finds that while ED programs benefit the lower ability full-pay and higher ability financial aid students, they are detrimental to lower ability financial aid students. In contrast, Murra-Anton (2019) shows that low-income students are strictly better off when early admissions are allowed.

Despite its advantages, colleges have had an unsteady engagement with ED, which can be seen as corroborating evidence of the so called early admission game. In April of 2002, following the announcement of Yale University's president Richard Levine to drop their ED policy, the University of North Carolina at Chapel Hill became the first major selective college to abandon ED admissions. By November 2002, Yale and Stanford switched from ED to EA programs. In 2007-2008, Harvard and Princeton had eliminated the early admissions programs entirely. Nevertheless, after 2011, Harvard, Princeton, Stanford and Yale resumed single-choice EA.

[5]According to the 2018-2019 Admission Trends Survey of NACAC, colleges with lower total yield rates tended to admit a greater percentage of their ED applicants compared to those with higher yield rates.

In this paper we study strategic issues in regard to ED in a centralized college admissions model, by extending the one-period many-to-one matching model of Gale and Shapley (1962) to a two-period model with early and regular decision markets. Our model involves two finite and disjoint sets of individuals, colleges and students. Each college has a finite capacity that limits the number of students that it can accept in the two periods, and each student can enroll to at most one school during the whole matching process.[6] In the RD period, each college has a preference relation over the possible subsets of students which is responsive to its preference over the set of students and each student has a preference relation over the set of colleges and being unmatched. The capacities of colleges together with the preference profiles of colleges and students in the RD period constitute a regular RD market.

In the ED period, each college announces out of its total capacity an ED quota, which it aims to fill with respect to its ED preference ordering. This ordering is responsive to some restriction of its RD preference ordering on a subset of students. On the other side of the market, each student has an ED preference ordering which is a restriction of his or her RD preference ordering on a singleton subset of colleges. The quotas of colleges together with the preference profiles of colleges and students in the ED period define an ED market. Clearly, for each RD market, there is a set of induced ED markets.

An allocation in the ED market is a many-to-one ED matching where no college is assigned more students than its ED quota and no student is assigned more than one college. Given a binding ED matching, an allocation in the RD market is a many-to-one RD matching where all the assignments realized in the ED market are preserved, no college is assigned more students than its overall capacity and no student is assigned more than one college. We assume that any student rejected from a college in the ED market can

[6] Many colleges and universities have priority categories for athletes, alumni children, and minorities. We assume that the capacity of each college in our model is net of its priority quota.

still apply to the same college in the RD market.[7]

A matching in the ED market is stable if no student prefers remaining unassigned to his or her assignment, no college prefers having a student slot vacant rather than filling it with one of its assignments, and there exists no unmatched college-student pair such that the college prefers the student to one of its assignments or keeping a vacant slot (if any) or the student prefers the college to his or her assignment. Given a matching realized in the ED market, a matching in the RD market is stable if no student having a regular assignment prefers remaining unassigned to his or her assignment, no college prefers having a regularly assigned student slot vacant rather than filling it with one of its regular assignments, and there exists no unmatched college-student pair such that the college prefers the student to one of its regular assignments or keeping a vacant slot (if any) or the student prefers the college to his or her regular assignment.

An ED matching rule selects a matching for every ED market, and is stable if it always selects a stable matching. A RD matching rule selects a matching for every RD market, given any matching in any ED market induced by the associated RD market. A RD matching rule is stable at an ED matching rule if it always selects a stable matching, given any realization of the ED matching rule applied to any ED market that is induced by the RD market.

An ED matching rule and an RD matching rule as an ordered pair form a matching system. A matching system is stable if it involves a stable ED matching rule at which the RD matching rule within the system is also stable.

We study manipulation of a matching system via ED quotas and preferences, and show that there is no matching system that is stable and nonmanipulable by colleges or students.

Our results can be related to those in the literature dealing with manipula-

[7] As pointed out by Avery et.al. (2003), "...historically most colleges rejected 5 percent or fewer of their early applicants in December. Some, such as Cornell, Georgetown, MIT, and Tufts, have automatically deferred to the regular pool all early applicants who are not admitted in December."

tion of preferences under two-sided matching in a single-period. Roth (1982) shows that there is no stable matching rule which is immune to preference manipulation. Mongell and Roth (1991) report a high percentage of truncated preference profiles (single alternative preference) submitted in sorority rush. Roth and Vande Vate (1991) show that in a decentralized one-to-one matching with random matching process, for any strategies of the other players, each player will always have a truncation strategy as a best response. Roth and Rothblum (1999) introduce the truncation of the true preferences as a potentially profitable strategic behavior, instead of changing the order of true preferences, in a low information environment in one-to-one matchings. Sönmez (1999) shows that there is no stable matching rule in hospital-intern markets which is immune to manipulation via early contracting (unraveling) between a hospital and a single intern.[8] The paper most related to our study is Mumcu and Saglam (2009), studying a similar problem between hospitals and interns, though with some significant differences. Like ours, their model considers two periods of matching, involving a regular market followed by an aftermarket. Although the regular market can be treated as the ED period in our model, students are not restricted to apply to one college (or to any number of colleges for that matter) like in our model with ED. Therefore, the negative result in Mumcu and Saglam (2009) about the nonmanipulability of the matching systems by colleges through their quotas does not imply ours. Moreover, the focus of Mumcu and Saglam (2009) is only manipulation (and strategic games) in quotas while our paper also considers manipulation in preferences.

The organization of the paper is as follows: Section 2 introduces the model. Section 3 gives results on manipulability of matching systems. Finally Section 4 concludes.

[8]Unraveling was previously studied by Roth and Xing (1994) showing that the instability of matchings realized at the final date of transactions are neither necessary or sufficient for the unraveling to occur. The two potential causes of unraveling are evolving uncertainty and the exercise of market power.

2 Model

We consider a college admission problem involving an early decision (ED) market and a regular decision (RD) market. Formally, this problem is denoted by the list (C, S, q, q^E, P^R, P^E) while the pairs (q^E, P^E) and (q, P^R) denote the ED market and the RD market, respectively. The first two components of a college admission problem are non-empty, finite and disjoint sets of colleges C and students S. The third component is a list of positive natural numbers $q = (q_c)_{c \in C}$, where q_c is the total capacity of college c. The fourth component is a list of nonnegative natural numbers $q^E = (q_c^E)_{c \in C}$, where $q_c^E \leq q_c$ denotes the quota of college c in the ED market. We define for all $q \in N_+^n$, the sets $\mathcal{Q}_c^E(q) = \{0, 1, ..., q_c\}$ and $\mathcal{Q}^E(q) = \times_{c \in C} \mathcal{Q}_c^E(q)$. Let $\mathcal{Q}^E = \cup_q \mathcal{Q}^E(q)$. The fifth component of a college admission problem is a list of strict preference relations $P^R = (P_i^R)_{i \in C \cup S}$ where P_i^R denotes the strict preference relation of individual i in the RD market. Finally, the last component P^E denotes a list of strict preference relations for colleges and students in the ED market.

For any $c \in C$, P_c^R is a linear order on $\Sigma_c^R = 2^S$ and P_c^E is a linear order on some $\Sigma_c^E \subseteq \Sigma_c^R$ such that $\emptyset \in \Sigma_c^E$. Also, for any $s \in S$, P_s^R is a linear order on $\Sigma_s^R = \{\{c_1\}, \{c_2\}, \ldots, \{c_m\}, \emptyset\}$ and P_s^E is a linear order on some $\Sigma_s^E \subseteq \Sigma_s^R$ such that $\emptyset \in \Sigma_s^E$ and $|\Sigma_s^E \backslash \{\emptyset\}| \leq 1$; i.e., each student can apply to at most one college in the ED market.

Given any college c with a strict preference relation P_c^R, we can derive its weak preference relation R_c^R, where $s \, R_c^R \, s'$ for any $s, s' \in S$ if and only if $s \, P_c^R \, s'$ or $s = s'$. Analogously, given any student s with a strict preference relation P_s^R, we can derive his or her weak preference relation. We introduce the notations $(\succ_c^R, \succeq_c^R, \succ_s^R, \succeq_s^R)$ and $(\succ_c^E, \succeq_c^E, \succ_s^E, \succeq_s^E)$ associated with $(P_c^R, R_c^R, P_s^R, R_s^R)$ and $(P_c^E, R_c^E, P_s^E, R_s^E)$ to represent the strict and weak preference of college c and student s over any two alternatives in the ED and RD markets.

We assume that the ED preference P_c^E of any college c is consistent with its RD preference P_c^R; i.e., for any $T, T' \in \Sigma_c^R$, we have $T \succ_c^E T'$ only if

$T \succ_c^R T'$.[9] Likewise, we assume that for any $s \in S$, P_s^E is consistent with P_s^R, i.e., for any $c \in C$ we have $c \succ_s^E \emptyset$ only if $c \succ_s^R \emptyset$ and for any $c, c' \in C$ we have $c \succ_s^E c'$ only if $c \succ_s^R c'$.

We also assume that P_c^R is responsive as in Roth (1985). That is, for all $S' \subset S$ it is true that

i) for all $s \in S \backslash S'$, $S' \cup \{s\} \succ_c^R S'$ if and only if $\{s\} \succ_c^R \emptyset$,

ii) for all $s, s' \in S \backslash S'$ such that $s \neq s'$, $S' \cup \{s\} \succ_c^R S' \cup \{s'\}$ if and only if $\{s\} \succ_c^R \{s'\}$.

Obviously, preferences of students over individual colleges are responsive. Also note that preferences of both colleges and students in the ED market become automatically responsive if their preferences are responsive in the RD market, due to our assumption that the ED preferences must be consistent with the RD preferences.

Let $\mathcal{P}_c^R$ and $\mathcal{P}_s^R$ respectively denote the set of all responsive preference relations for college c and for student s in the RD market. Define $\mathcal{P}^R = \times_{k \in C \cup S} \mathcal{P}_k^R$. Also, given any $P_c^R \in \mathcal{P}_c^R$, let $\mathcal{P}_c^E(P_c^R)$ denote for college c the set of all responsive preference relations, in the ED market, which are consistent with P_c^R. Similarly, given any $P_s^R \in \mathcal{P}_s^R$, let $\mathcal{P}_s^E(P_s^R)$ denote for student s the set of all responsive preference relations, in the ED market, which are consistent with P_s^R. For any $P^R \in \mathcal{P}^R$ define $\mathcal{P}^E(P^R) = \times_{k \in C \cup S} \mathcal{P}_k^E(P_k^R)$ and $\mathcal{P}^E = \times_{P^R \in \mathcal{P}^R} \mathcal{P}^E(P^R)$.

Now, we describe matching problems. Let $\mathcal{E}^R = N_+^n \times \left(\times_{k \in C \cup S} \mathcal{P}_k^R \right)$ denote the class of all matching problems in the RD market. For any $(q, P^R) \in \mathcal{E}^R$ and $q^E \in \mathcal{Q}^E(q)$, let us also define $\mathcal{E}^E(q, P^R, q^E) = \{q^E\} \times \mathcal{P}^E(P^R)$, denoting the class of all matching problems in the ED market. Let $\mathcal{E}^E = \cup_{(q,P^R)} \cup_{q^E \in \mathcal{Q}^E(q)} \mathcal{E}^E(q, P^R, q^E)$.

[9] Since Σ_c^E can be a proper subset of Σ_c^R, the consistency assumption does not prevent college c from compromising in the ED market. For example, given a college admission problem where $S = \{s_1, s_2\}$, $C = \{c_1\}$, $P_{c_1}^R = s_1, s_2, \emptyset$, and $P_{c_1}^E = s_2, \emptyset$, we should observe that $P_{c_1}^E$ is consistent with $P_{c_1}^R$ even though c_1 compromises in the ED market by not accepting its top-ranked student s_1 with respect to $P_{c_1}^R$.

A matching μ^E in the ED market (simply an ED matching) with the quota profile q^E is a function from the set $C \cup S$ into $2^{C \cup S}$ such that

i) for all $s \in S$, $|\mu^E(s)| \leq 1$ and $\mu^E(s) \subseteq C$;

ii) for all $c \in C$, $|\mu^E(c)| \leq q^E_c$ and $\mu^E(c) \subseteq S$;

iii) for all $(c, s) \in C \times S$, $\mu^E(s) = \{c\}$ if and only if $s \in \mu^E(c)$.

We denote the set of all ED matchings at q^E by $\mathcal{M}^E(q^E)$. Let $\mathcal{M}^E = \cup_{q^E} \mathcal{M}^E(q^E)$.

Given any (q^E, P^E) and any two ED matchings $\mu^E_1, \mu^E_2 \in \mathcal{M}^E(q^E)$, we say that student s strictly prefers μ^E_1 to μ^E_2 if and only if $\mu^E_1(s) \succ^E_s \mu^E_2(s)$ and weakly prefers μ^E_1 to μ^E_2 if and only if $\mu^E_1(s) \succeq^E_s \mu^E_2(s)$. We do the same for each college.

Given any ED matching μ^E and any capacity profile q, we define an RD matching μ^R as a function from the set $C \cup S$ into $2^{C \cup S}$ such that

i) for all $s \in S$, $|\mu^R(s)| \leq 1$, and $\mu^E(s) \subseteq \mu^R(s) \subseteq C$;

ii) for all $c \in C$, $|\mu^R(c)| \leq q_c$, and $\mu^E(c) \subseteq \mu^R(c) \subseteq S$;

iii) for all $(c, s) \in C \times S$, $\mu^R(s) = \{c\}$ if and only if $s \in \mu^R(c)$.

The function μ^R preserves the early matchings achieved under μ^E, i.e. early decisions are binding. Given (q, μ^E), we denote the set of all RD matchings by $\mathcal{M}^R(q, \mu^E)$. Let $\mathcal{M}^R = \cup_{(q,\mu^E)} \mathcal{M}^R(q, \mu^E)$.

Given any two RD matchings μ^R_1 and μ^R_2, we say that student s strictly prefers μ^R_1 to μ^R_2 if and only if $\mu^R_1(s) \succ^R_s \mu^R_2(s)$ and weakly prefers μ^R_1 to μ^R_2 if and only if $\mu^R_1(s) \succeq^R_s \mu^R_2(s)$. We do the same for each college. For any $P \in \mathcal{P}^E \cup \mathcal{P}^R$, we let $A(P_c)$ denote the set of all acceptable students for college c at P_c, i.e., $A(P_c) = \{s \in S : s \succ_c \emptyset\}$. Similarly, we let $A(P_s)$ denote the set of all acceptable colleges for student s at P_s, i.e., $A(P_s) = \{c \in C : c \succ_s \emptyset\}$.

The choice of a college c from a group of students $T \subseteq S$ in the ED market (q^E, P^E) is defined as

$$Ch^E_c(P^E_c, q^E_c, T) = \{T' \subseteq T \cap A(P^E_c) : |T'| \leq q^E_c \text{ and } T' \succ^E_c T''$$

$$\text{for all } T'' \subseteq T \cap A(P_c^E) \text{ such that } T'' \neq T' \text{ and } |T''| \leq q_c^E\}.$$

Similarly, given any ED matching μ^E, the choice of a college c from a group of students $T \subseteq S \backslash \mu^E(c)$ available for matching in the RD market (q, P^R) is defined as

$$Ch_c^R(P_c^R, q_c, \mu^E, T) = \{T' \subseteq T \cap A(P_c^R) : |T'| \leq q_c - |\mu^E(c)| \text{ and}$$
$$T' \cup \mu^E(c) \succ_c^R T'' \cup \mu^E(c) \text{ for all } T'' \subseteq T \cap A(P_c^R)$$
$$\text{such that } T'' \neq T' \text{ and } |T''| \leq q_c - |\mu^E(c)|\}.$$

Given any q^E, a matching $\mu^E \in \mathcal{M}^E(q^E)$ is blocked by student $s \in S$ if $\emptyset \succ_s^E \mu^E(s)$, and blocked by college $c \in C$ if $\mu^E(c) \neq Ch_c^E(P_c^E, q_c^E, \mu^E(c))$. We say that μ^E is acceptable to a college, or to a student, that does not block it. Also, μ^E is blocked by a college-student pair $(c, s) \in C \times S$ if $\{c\} \succ_s^E \mu^E(s)$ and $\mu^E(c) \neq Ch_c^E(P_c^E, q_c^E, \mu^E(c) \cup \{s\})$. We say that μ^E is stable if it is not blocked by a student, a college, or a college-student pair. Given an ED market (q^E, P^E), we denote the set of all stable ED matchings by $\mathcal{S}^E(q^E, P^E)$.

Given an ED matching μ^E, an RD matching $\mu^R \in \mathcal{M}^R(q, \mu^E)$ is blocked by student $s \in S$ if $\emptyset \succ_s^R \mu^R(s) \backslash \mu^E(s)$ and blocked by college $c \in C$ if $\mu^R(c) \backslash \mu^E(c) \neq Ch_c^R(P_c^R, q_c, \mu^E, \mu^R(c) \backslash \mu^E(c))$. We say that μ^R is acceptable to a college, or to a student, that does not block it. Also, μ^R is blocked by a college-student pair $(c, s) \in C \times S$ if $\mu^E(s) = \emptyset$, $\{c\} \succ_s^R \mu^R(s)$ and $\mu^R(c) \backslash \mu^E(c) \neq Ch_c^R(P_c, q_c, \mu^E, \{s\} \cup \mu^R(c) \backslash \mu^E(c))$. We say that μ^R is stable if it is not blocked by a student, a college, or a college-student pair. Given an ED matching μ^E and an RD market (q, P^R), we denote the set of all stable RD matchings by $\mathcal{S}^R((q, P^R), \mu^E)$.

We say that college c and student s are achievable for one another in the ED market (q^E, P^E), if there is a stable ED matching in $\mathcal{S}^E(q^E, P^E)$ at which they are matched. Likewise, we define achievability in an RD market.

An ED matching rule is a function $\varphi^E : \mathcal{E}^E \rightarrow \mathcal{M}^E$ such that $\varphi^E(q^E, P^E) \in \mathcal{M}^E(q^E)$ for every $(q^E, P^E) \in \mathcal{E}^E$. Let $\bar{\varphi}^E$ denote the set of all ED matching rules. Similarly, an RD matching rule is a function $\varphi^R : \mathcal{E}^R \times \mathcal{M}^E \to \mathcal{M}^R$ such that $\varphi^R((q, P^R), \mu^E) \in \mathcal{M}^R(q, \mu^E)$ for every $(q, P^R) \in \mathcal{E}^R$, $q^E \in \mathcal{Q}^E(q)$, and $\mu^E \in \mathcal{M}^E(q^E)$. Let $\bar{\varphi}^R$ denote the set of all RD matching rules.

An ED matching rule φ^E is stable if $\varphi^E(q^E, P^E) \in \mathcal{S}^E(q^E, P^E)$ for every $(q^E, P^E) \in \mathcal{E}^E$. On the other hand, an RD matching rule φ^R is stable at an ED matching rule φ^E if $\varphi^R((q, P^R), \varphi^E(q^E, P^E)) \in \mathcal{S}^R((q, P^R), \varphi^E(q^E, P^E))$ for every $(q, P^R) \in \mathcal{E}^R$ and $(q^E, P^E) \in \mathcal{E}^E(q, P^R, q^E)$.

Given any ED matching rule $\varphi^E \in \bar{\varphi}^E$ and any RD matching rule $\varphi^R \in \bar{\varphi}^R$, the ordered pair (φ^E, φ^R) is called a matching system. A matching system (φ^E, φ^R) is stable if φ^E is stable and φ^R is stable at φ^E.

A matching system (φ^E, φ^R) cannot be manipulated by individual $k \in C \cup S$ via its ED preference if for all $(q, P^R) \in \mathcal{E}^R$, $(q^E, P^E) \in \mathcal{E}^E(q, P^R, q^E)$, and $\hat{P}_k^E \in \mathcal{P}_k^E(P_k^R)$ it is true that

$$\varphi^R((q, P^R), \varphi^E(q^E, P^E))(k) \ \ \succeq_k^R \ \ \varphi^R((q, P^R), \varphi^E(q^E, \hat{P}_k^E, P_{-k}^E))(k).$$

If the above holds for all colleges (students), then we say that the matching system (φ^E, φ^R) is nonmanipulable by colleges (students) via ED preferences.

A matching system (φ^E, φ^R) cannot be manipulated by college $c \in C$ via its ED quota if for all $(q, P^R) \in \mathcal{E}^R$, $(q^E, P^E) \in \mathcal{E}^E(q, P^R, q^E)$, and $\hat{q}_c^E \in \mathcal{Q}_c^E(q)$ it is true that

$$\varphi^R((q, P^R), \varphi^E(q^E, P^E))(c) \ \ \succeq_c^R \ \ \varphi^R((q, P^R), \varphi^E(\hat{q}_c^E, q_{-c}^E, P^E))(c).$$

If the above holds for all colleges, then we say that the matching system (φ^E, φ^R) is nonmanipulable via ED quotas.

3 Results

Proposition 1. *For any college admission problem with at least two colleges and one student, there exists no matching system that is stable and nonmanipulable via ED quotas.*

Proof. We first consider a college admission problem (C, S, q, q^E, P^R, P^E) with two colleges and one student; i.e., $C = \{c_1, c_2\}$ and $S = \{s_1\}$. Let $q_{c_1} = 1$, $q_{c_2} = 1$, $q^E_{c_1} = 0$, $q^E_{c_2} = 0$, $\hat{q}^E_{c_1} = 1$;

$$P^R_{c_1} = P^E_{c_1} = \{s_1\}, \emptyset;$$

$$P^R_{c_2} = P^E_{c_2} = \{s_1\}, \emptyset;$$

$$P^R_{s_1} = \{c_2\}, \{c_1\}, \emptyset;$$

$$P^E_{s_1} = \{c_1\}, \emptyset.$$

(Note that the strict preference relation of a student or a college is represented by an ordered list of acceptable mates.) We have $S^E(q^E, P^E) = \{\mu_1\}$, $S^R(q, P^R, \mu_1) = \{\mu_2\}$, $S^E(\hat{q}^E_{c_1}, q^E_{c_2}, P^E) = \{\mu_3\}$, and $S^R(q, P^R, \mu_3) = \{\mu_3\}$, where

$$\mu_1 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \emptyset \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \{s_1\} \end{pmatrix}, \quad \mu_3 = \begin{pmatrix} c_1 & c_2 \\ \{s_1\} & \emptyset \end{pmatrix}.$$

Consider any matching system (φ^E, φ^R) that is stable. Then, we must have $\varphi^E(q^E, P^E) = \mu_1$, $\varphi^R((q, P^R), \mu_1) = \mu_2$, $\varphi^E(\hat{q}^E_{c_1}, q^E_{c_2}, P^E) = \mu_3$, and $\varphi^R((q, P^R), \mu_3) = \mu_3$. Hence,

$$\varphi^R((q, P^R), \varphi^E(\hat{q}^E_{c_1}, q^E_{c_2}, P^E))(c_1) \succ^R_{c_1} \varphi^R((q, P^R), \varphi^E(q^E, P^E))(c_1).$$

So, college c_1 can manipulate the matching system (φ^E, φ^R) via its ED quota when $q^E_{c_1} = 0$. It can do so by announcing $\hat{q}^E_{c_1} = 1$ and accepting the unique student s_1 in the ED market. This completes the proof for the case of $|C| = 2$ and $|S| = 1$. In order to extend it to the general case of $|C| \geq 2$ and $|S| \geq 1$, we can include, to the above college admission problem, colleges whose top choice is admitting no student and students whose top choice is staying unmatched both in the ED market and in the RD market. ■

The proof of Proposition 1 suggests that a college may benefit from admitting students both in the ED market and in the RD market, when the rest of the colleges, or a sufficient number of them, consider admission only in the RD market. Naturally, Proposition 1 is not valid when there exists a unique college in the admission problem. In that case, a unique stable matching exists for the RD market (and for the ED market), and this stable matching is college-optimal (and also student-optimal), i.e., the unique college would be matched to the highest-ranked achievable students allowed by its quota. Thus, a college that faces no rivals cannot improve the quality of its matches in the RD market (which is already optimal), by changing its quota for the ED market (or by allocating/not allocating some of its total capacity to the ED market). The presence of an ED market would offer an unrivaled college only the opportunity to run and complete its admission process at an earlier time than planned for the RD market.

Next, we consider whether colleges have incentives to manipulate their ED preferences.

Proposition 2. *For any college admission problem with at least two colleges and one student, there exists no matching system that is stable and nonmanipulable by colleges via ED preferences.*

Proof. We first consider a college admission problem (C, S, q, q^E, P^R, P^E) with two colleges and one student; i.e., $C = \{c_1, c_2\}$ and $S = \{s_1\}$. Let $q_{c_1} = 1$, $q_{c_2} = 1$, $q^E_{c_1} = 1$, $q^E_{c_2} = 0$;

$$P^R_{s_1} = \{c_2\}, \{c_1\}, \emptyset; \qquad P^E_{s_1} = \{c_1\}, \emptyset;$$

$$P^R_{c_1} = \{s_1\}, \emptyset; \qquad P^E_{c_1} = \emptyset;$$

$$P^R_{c_2} = P^E_{c_2} = \{s_1\}, \emptyset;$$

$$\hat{P}^E_{c_1} = P^R_{c_1}.$$

Then, we have $S^E(q^E, P^E) = \{\mu_1\}$, $S^R(q, P^R, \mu_1) = \{\mu_2\}$, $S^E(q^E, \hat{P}^E_{c_1}, P^E_{-c_1}) = \{\mu_3\}$, $S^R(q, P^R, \mu_3) = \{\mu_3\}$, where

$$\mu_1 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \emptyset \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \{s_1\} \end{pmatrix}, \quad \mu_3 = \begin{pmatrix} c_1 & c_2 \\ \{s_1\} & \emptyset \end{pmatrix}.$$

Consider any matching system (φ^E, φ^R) that is stable. Then, we must have $\varphi^E(q^E, P^E) = \mu_1$, $\varphi^R((q, P^R), \mu_1) = \mu_2$, $\varphi^E(q^E, \hat{P}^E_{c_1}, P^E_{-c_1}) = \mu_3$, and $\varphi^R((q, P^R), \mu_3) = \mu_3$. Hence,

$$\varphi^R((q, P^R), \varphi^E(q^E, \hat{P}^E_{c_1}, P^E_{-c_1}))(c_1) \succ^R_{c_1} \varphi^R((q, P^R), \varphi^E(q^E, P^E))(c_1).$$

So, college c_1 can manipulate the matching system (φ^E, φ^R) via its ED preference, completing the proof for the case of $|C| = 2$ and $|S| = 1$. In order to extend it to the general case of $|C| \geq 2$ and $|S| \geq 1$, we can include, to the above college admission problem, colleges whose top choice is admitting no student and students whose top choice is staying unmatched both in the ED market and in the RD market. ■

Below, we finally consider manipulation of matching systems by students.

Proposition 3. *For any college admission problem with at least two colleges and one student, there exists no matching system that is stable and nonmanipulable by students via ED preferences.*

Proof. We first consider a college admission problem (C, S, q, q^E, P^R, P^E) with two colleges and one student; i.e., $C = \{c_1, c_2\}$ and $S = \{s_1\}$. Let $q_{c_1} = 1$, $q_{c_2} = 1$, $q^E_{c_1} = 0$, $q^E_{c_2} = 1$;

$$P^R_{c_1} = P^E_{c_1} = \{s_1\}, \emptyset;$$

$$P^R_{c_2} = P^E_{c_2} = \{s_1\}, \emptyset;$$

$$P^R_{s_1} = \{c_1\}, \{c_2\}, \emptyset;$$

$$P^E_{s_1} = \{c_2\}, \emptyset;$$

$$\hat{P}^E_{s_1} = \{c_1\}, \emptyset.$$

We have $S^E(q^E, P^E) = \{\mu_1\}$, $S^R(q, P^R,\ \mu_1) = \{\mu_1\}$, $S^E(q^E, \hat{P}^E_{s_1}, P^E_{-s_1}) = \{\mu_2\}$, and $S^R(q, P^R, \mu_2) = \{\mu_3\}$, where

$$\mu_1 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \{s_1\} \end{pmatrix}, \quad \mu_2 = \begin{pmatrix} c_1 & c_2 \\ \emptyset & \emptyset \end{pmatrix}, \quad \mu_3 = \begin{pmatrix} c_1 & c_2 \\ \{s_1\} & \emptyset \end{pmatrix}.$$

Consider any matching system (φ^E, φ^R) that is stable. Then, we must have $\varphi^E(q^E, P^E) = \mu_1$, $\varphi^R((q, P^R), \mu_1) = \mu_1$, $\varphi^E(q^E, \hat{P}^E_{s_1}, P^E_{-s_1}) = \mu_2$, and $\varphi^R((q, P^R), \mu_2) = \mu_3$. Hence,

$$\varphi^R((q, P^R), \varphi^E(q^E, \hat{P}^P_{s_1}, P^E_{-s_1}))(s_1) \succ^R_{s_1} \varphi^R((q, P^R), \varphi^E(q^E, P^E))(s_1).$$

So, student s_1 can manipulate the matching system (φ^E, φ^R) via his or her ED preference, completing the proof for the case of $|C| = 2$ and $|S| = 1$. In order to extend it to the general case of $|C| \geq 2$ and $|S| \geq 1$, we can include, to the above college admission problem, colleges whose top choice is admitting no student and students whose top choice is staying unmatched both in the ED market and in the RD market. ■

Neither Proposition 2 nor Proposition 3 are valid when there exists a unique college in the admission problem (for the same reason as we stated after Proposition 1). In that case, a unique stable matching could exist in the RD market, and this matching would be both college-optimal and student-optimal, eliminating any incentives for manipulation by colleges or students.

4 Conclusions

Many colleges and universities in the United Stated may have strong incentives to continue their early decision programs as they can manipulate the admission and matriculation rate by means of these programs, which in turn determine the rankings of these institutions that students take into account when applying.[10] As a matter of fact, liberal arts colleges are argued to rely on early decision (and early action) programs much more than larger universities because small miscalculations about class size can have much more serious consequences than at larger institutions.[11] These arguments suggest that the intertemporal quota allocation may be an important reason behind

[10]See Avery et.al. (2004).

[11]See page 274 of Avery et.al. (2003).

the adoption of ED programs. While the existing ED programs have been invented, to some extent, to strategically manipulate the outcome of the regular admission programs, even the ED programs, or the combinations of ED and RD programs, are prone to the manipulation of colleges (for example, via their quotas and preferences) and students (via their preferences), as shown by our results in this paper. Using a two period matching model with an ED market followed by an RD market, we have established that (i) there exists no stable matching system which is nonmanipulable via ED quotas by colleges (Proposition 1) and (ii) there exists no stable matching system which is nonmanipulable via ED preferences by colleges or students (Propositions 2 and 3, respectively). Interestingly, Proposition 1 suggests that it may not (always) be possible to eliminate strategic incentives of colleges to manipulate the existing college admission system by controlling or changing the (stable) matching rules followed in the ED and RD markets.

We should note here that we have modeled the college admission problem using a many-to-one matching setup in two periods, separating the early and regular decision markets in time dimension as in reality. An alternative, and much richer, model was very recently introduced by Yenmez (2018), who showed that college admissions with early and non-early decisions can be operated by a centralized clearinghouse that can deal with stable many-to-many matchings with contracts between colleges and students. We believe that one can fruitfully study whether our negative results as to the nonmanipulability of stable matching rules could also arise in the alternative model of Yenmez (2018).

Another important question that we leave for future research is why some colleges in the United States use only regular decision programs while others also offer at least one type of early admission program. Related to this question, we wonder whether the observed heterogeneity in the attitudes of colleges over the use, and the selection, of early admission programs can be sustained as an equilibrium behavior in a college admission game where colleges can strategically decide which early admission programs to offer given their beliefs about the choices of others. In case such an equilibrium exists and can be characterized in terms of the parameters of the college admission system, one can also study whether or how it could be improved by policy makers to the benefit of students and/or colleges.

References

Avery, C., Fairbanks, A. and Zeckhauser, R. *The Early Admission Game: Joining the Elite.* Cambridge: Harvard University Press, 2003.

Avery, C., Glickman, M., Metrick, A. and Hoxby, C.M. "A Revealed Preference Ranking of American Colleges and Universities," NBER Working Paper 10803, 2004.

Avery, C., and Levine, J. "Early Admissions at Selective Colleges,"*American Economic Review*, 2010, 100, 2125-2156.

Chapman, G. and Dickert-Conlin, S. "Applying Early Decision: Student and College Incentives and Outcomes," *Economics of Education Review*, 2012, 31,749-763.

Chen, W-C., Chen, Y-Y. and Kao, Y-C. "Limited Choice in College Admissions: An Experimental Study," *Game and Economic Behavior*, 2018, 295-316.

Gale, D. and Shapley, L.S. "College Admissions and the Stability of Marriage," *American Mathematical Monthly*, 1962, 69(1), 9-15.

Kim, M. "Early Decision and Financial Aid Competition among need-blind Colleges and Universities," *Journal of Public Economics*, 2010, 94, 410-420.

Lee, S-H. "Jumping the Curse: Early Contracting with Private Information in University Admissions," *International Economic Review*, 2009, 50(1), 1-38.

Mongell, S. and Roth, A.E. "Sorority Rush as a Two-Sided Matching Mechanism," *American Economic Review*, 1991, 81(3), 441-464.

Mumcu, A. and Saglam, I. "Games of Capacity Allocation in Many-to-One

Matching with an Aftermarket," *Social Choice and Welfare*, 2009, 33, 383-403.

Murra-Anton, Z. "College Early Admissions: Determinants and Welfare," 2019. Available at SSRN: https://ssrn.com/abstract=3451291 or http://dx.doi.org/10.2139/ssrn.3451291.

NACAC, 2018-2019 Admission Trends Survey, http://www.nacacnet.org.

Roth, A.E. "The Economics of Matching: Stability and Incentives," *Mathematics of Operations Research*, 1982, 7, 617-628.

Roth, A.E. "The College Admissions Problem is not Equivalent to the Marriage Problem," *Journal of Economic Theory*, 1985, 36(2), 277-288.

Roth, A.E. and Rothblum, U.G. "Truncation Strategies in Matching Markets in Search of Advice for Participants," *Econometrica*, 1999, 67(1), 21-43.

Roth, A.E. and Vande Vate, J.H. "Incentives in Two-Sided Matching with Random Stable Mechanism," *Economic Theory*, 1991, 1(1), 31-44.

Roth, A.E. and Xing, X. "Jumping the Gun: Imperfections and Institutions Related to the Timing of Market Transactions," *American Economic Review*, 1994, 84(4), 992-1044.

Sönmez, T. "Can Pre-Arranged Matches Be Avoided in Two-Sided Matching Markets?," *Journal of Economic Theory*, 1999, 86(1), 148-156.

Yenmez, M.B. "A College Admissions Clearinghouse," *Journal of Economic Theory*, 2018, 176, 859-885.