Deriving utility: consumers' diligence under externalities and technical progress

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Abstract

If we present sales items or trade units, cars and apartments, in the units of consumption, miles and nights, like it takes place in the sharing&rental economy, we can get the model of the optimal consumption-leisure choice, where the efforts on pre-purchase search and after-purchase care produce non-monetary costs before the use of a trade unit. The paper argues that the productivity of these efforts differs from the efficiency of consumers' efforts on the workplace. The consumer searches diligently the quantity to be purchased, he spends money earned by his labor or high-productive industry on the purchase and, following his willingness to take care of the purchased item, he takes low-productive diligent efforts in order to finally enjoy it. While the purchase price of the trade unit is equal to consumer's willingness to pay, the total costs of his industry and diligence become equal to his willingness to accept or to sell the trade unit, the car and the apartment, where his marginal and average costs become equal to the equilibrium price of the unit of consumption, a mile or a night, and total costs become equal to the equilibrium price of the trade unit. The consumers' productivity function really gets the S-shape, which slows the growth of monetary costs and accelerates the growth of non-monetary costs.

While the consumers' diligence derives the utility from the trade item at the equilibrium level, it enlarges also the spectrum of solutions for the Coase theorem, because the consumers' diligence copies also with externalities. The trade-off between quantity of consumption units to be purchased and non-monetary efforts for its' efficient use appears. The assets are redistributed for its more efficient use, from slight to great diligence, or from low to high willingness to take care of the trade unit just in accordance with the Black's Law Encyclopedia where the great or high diligence is defined as the diligence that a very prudent person exercises in handling his or her own property like that at issue.

The model demonstrates that the labor augmenting technical progress decreases the marginal monetary costs of consumers' industry and increases non-monetary costs of consumers' diligence at the equilibrium level that can be explained by the loss in the quality of trade units, cars and apartments.

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The outcome of the service augmenting technical progress is ambiguous. While it raises the equilibrium price, the consumption falls. But the fall in consumption reduces consumers' diligence and results in the development of the sharing&rental economy. However, if the production and services are gross complements, the consumption growths and Veblen effect is to be expected where consumption becomes "bad" with respect to leisure and the consumers' diligence becomes excessive.

Key words: search, diligence, willingness to take care, Coase theorem, externalities, technical progress

JEL classification: D11, D83.

Introduction

The paper continues to develop the concept of the optimal consumption-leisure choice under the pre-purchase search and the after-purchase willingness to take care of an item, presented at the 68th AFSE Congress (Malakhov 2019). The model challenges the traditional theories of home production (Becker 1965, Gronau 1977) and returns to the theory of attributes (Lancaster 1966). While the cleaning of an apartment represents efforts before its following use and it appears as an option either to buy a service or to clean the apartment up oneself, the cleaning in particular and the care of the item in general works like a pre-purchase search with the same option – either to pay high price for the search and delivery or to cut expenses and to search oneself. So, we start with the search model but under strict limits of the classical labor-leisure choice. The need to describe the search model as the derivative from the classical labor-leisure choice, where the equivalence of the marginal utility of both consumption and leisure should be confirmed, explains the choice of the static optics.

Then we proceed from the search to the care of the purchased item. Although there the dynamic optics becomes more urgent, we stay on the static base because we assume that at the moment of purchase a consumer takes into account some expected quantity of consumption units, mileage and nights, to be purchased and used after the purchase. Other words, the consumer esteems the time horizon of his choice and the intensity of consumption of the trade units, the car and the apartment. Here, the static optics makes the presentation of the diligence as a natural way to copy with externalities more transparent and discovers the classical static cost curves, now with regard to consumer's pre-purchase search and after-purchase care of trade units.

Allocation of time for search and the consumption-leisure utility function

If we presuppose that the search S displaces the labor L and the leisure H from the time horizon until the next purchase like an ice squeezes out whiskey and soda from the glass, we get the general rule of the allocation of time and the value of the propensity to search $\partial L/\partial S<0$:

$$L+S+H=T; \quad (1.1)$$

$$(-\partial L/\partial S)+(-\partial H/\partial S)=1; \quad (1.2)$$

$$dH(S)=dS\frac{\partial H}{\partial S}=-dS\frac{H}{T};\rightarrow\frac{\partial H}{\partial S}=-\frac{H}{T}; \quad (1.3)$$

$$\frac{\partial L}{\partial S}=\frac{H-T}{T}=-\frac{L+S}{T} \quad (1.4)$$

$$\frac{L+S}{T}+\frac{H}{T}=1 \quad (1.5)$$

If we multiply the propensity to search $\partial L/\partial S$ by the wage rate w, we get the value of the marginal loss of monetary labor income during the search $w\partial L/\partial S$. According to the famous George Stigler's rule we can equalize it with the marginal benefit of the search $Q\partial P/\partial S$, where quantity demanded Q is given and the price of purchase depends on search $P(S)$. This behavioral explicit rule can be used as the constraint to some utility function $U(Q,H)$, where the quantity to be purchased Q becomes the variable value and the value of the marginal benefit per unit of purchase $\partial P/\partial S<0$ is given by the place of purchase. Indeed, at the optimum level this implicit solution should match the explicit behavioral constraint:

$$\max U(Q,H) \; subject\,to \; w\frac{\partial L}{\partial S}=Q\frac{\partial P}{\partial S} \quad (2.1)$$

$$\Lambda=U(Q,H)+\lambda(w-\partial P/\partial S\frac{Q}{\partial L/\partial S}) \quad (2.2)$$

$$\frac{\partial U}{\partial Q}=\lambda\frac{\partial P/\partial S}{\partial L/\partial S} \quad (2.3)$$

$$\frac{\partial U}{\partial H}=-Q\frac{\partial P/\partial S}{(\partial L/\partial S)^2}\partial^2L/\partial S\partial H=-\frac{w}{\partial L/\partial S}\partial^2L/\partial S\partial H \quad (2.4)$$

$$MRS(H\,for\,Q)=-\frac{w}{\partial P/\partial S}\partial^2L/\partial S\partial H \quad (2.5)$$

$$\partial^2L/\partial S\partial H=\frac{\partial(H-T/T)}{\partial H}=1/T \quad (2.6)$$

$$MRS(H\,for\,Q)=-\frac{w}{T\partial P/\partial S}=-\frac{Q}{T\partial L/\partial S}=\frac{QT}{T(L+S)}=\frac{Q}{L+S} \quad (2.7)$$

$$MRS(H\,for\,Q)=\frac{Q}{L+S}\frac{H/T}{H/T}=\frac{Q}{H}\frac{(-\partial H/\partial S)}{(-\partial L/\partial S)} \quad (2.8)$$

$$U(Q,H)=Q^{-\partial L/\partial S}H^{-\partial H/\partial S} \quad (2.9)$$

We can suppose that the consumption-leisure relationship is described by the utility function $U(Q,H)=Q^{-\partial L/\partial S}H^{\partial H/\partial S}=Q^{(L+S)/T}H^{H/T}|_{(L+S)T+H/T=1}$ and its curve is tangent at the point of the optimal choice $(Q^*;H^*)$ to the budget constraint line (Equations 2.5-2.9 and Figure 1):

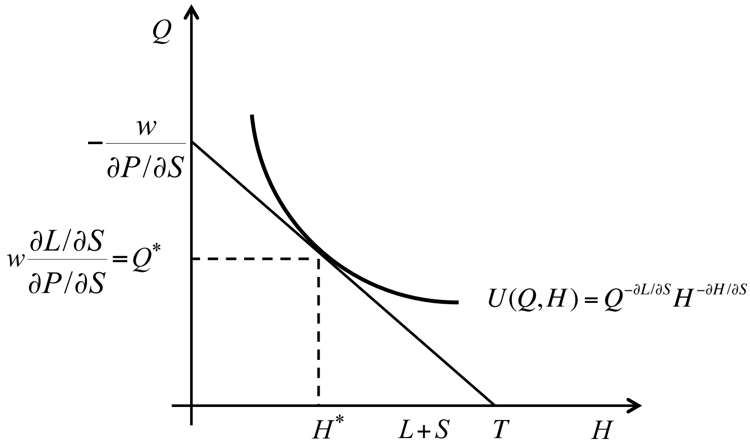

Fig.1. Implicit consumption-leisure choice under the search

Now we can simplify step by step the unusual values, do not forget that *∂P/∂S<0;∂L/∂S<0*, in order to confirm their correspondence to the classical labor-leisure choice. First, we present the behavioral choice of the fixed quantity demanded *Q* and the variable price of purchase $P_P(S)$ (Figure 2):

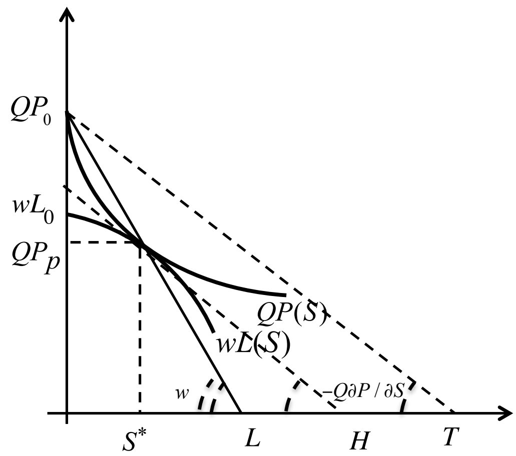

Fig.2.Explicit choice of the pre-determined quantity to be purchased

Here we take the $Q\partial^2P/\partial S^2>0$ - shape of the *QP(S)* curve with regard to the assumption of the diminishing marginal efficiency of the search and the $w\partial^2L/\partial S^2<0$ – shape of the *wL(S)* curve can be easily drawn from the Equation 2.1 for the values of the propensity to search under the Archimedes' "whiskey-soda-use" rule as $-1<\partial L/\partial S<0$.[1] We see that $QP(S)$ and $wL(S)$ curves becomes tangent at the moment of purchase QP_P with the $Q\partial P/\partial S$ slope according to the behavioral constraint (2.1). It gives us the price per unit of consumption $P_0=-T\partial P/\partial S$ and the price of the trade unit QP_0 at the zero search level. This is the price paid by *shoppers*, consumers with zero search costs (Dahl 1989).

$$w\frac{\partial L}{\partial S}=Q\frac{\partial P}{\partial S}=-w\frac{L+S}{P} \quad (3.1)$$

$$w(L+S)=-QT\partial P/\partial S=QP_0 \quad (3.2)$$

$$MRS(H\ for Q)=-\frac{w}{\partial P/\partial S}\partial^2 L/\partial S\partial H=-\frac{w}{T\partial P/\partial S}=\frac{w}{P_0} \quad (3.3)$$

This is the equilibrium price because it equalizes consumer's marginal costs on purchase with his average costs:

$$MRS(H\ for Q)=\frac{Q}{L+S}=\frac{w}{P_0}\Rightarrow P_0=\frac{w(L+S)}{Q}=AC \quad (4.1)$$

$$MC=\frac{\partial w(L+S)}{\partial Q}=\frac{\partial QP_0}{\partial Q}=P_0 \quad (4.2)$$

$$P_0=AC=MC=P_e \quad (4.3)$$

While under the behavioral constraint $Q\neq Q(S)$, both the $Q(S)$ and $S(Q)$ exist under the allocation of time in the utility function itself because the $\partial Q/\partial H$ relationship presumes the existence of $\partial L/\partial Q$ and $\partial S/\partial Q$ relationships at the implicit utility level.

Now we can prove the identity of marginal utility of both consumption and leisure under the classical labor-leisure choice and the choice on imperfect market under the search with the help of the methodology for the analysis of the Lagrangian multiplier, proposed once by American mathematicians J.V.Baxley and J.C.Moorhouse (Baxley and Moorhouse 1984, Malakhov 2015):

[1] The value $\partial L/\partial S<-1$ goes beyond the time horizon and produces «the leisure model» of behavior $(\partial Q/\partial H>0)$, that will be presented by the analysis of the service augmenting technical progress (S.M.).

$$Classical\ labor-leisure\ choice:$$

$$\lambda=\frac{\partial U^{*}/\partial w}{T-H} \quad (5.1);$$

$$MU_{Q}=\lambda P_{e}=P_{e}\frac{\partial U^{*}/\partial w}{T-H} \quad (5.2);$$

$$MU_{H}=\lambda w=w\frac{\partial U^{*}/\partial w}{T-H} \quad (5.3);$$

$$Choice\ under\ the\ search:$$

$$\lambda=\partial U^{*}/\partial w;$$

$$MU_{Q}=\lambda\frac{\partial P/\partial S}{\partial L/\partial S}=-\lambda\frac{T\partial P/\partial S}{L+S}=P_{e}\frac{\partial U^{*}/\partial w}{T-H} \quad (5.4);$$

$$MU_{H}=-\lambda\frac{w}{\partial L/\partial S}\partial^{2}L/\partial S\partial H=\lambda\frac{wT}{T(L+S)}=\lambda\frac{w}{T-H}=w\frac{\partial U^{*}/\partial w}{T-H} \quad (5.5)$$

It means that in the static world the search and care don't change marginal utility of both consumption and leisure. But we see that the purchase price is not related directly to the marginal utility of consumption. The same happens with the purchase price of leisure, which is equal to the value $(-w\partial L/\partial S=w(L+S)/T)$, that corresponds to the "price" of leisure got by field studies in the economics of transportation and the economics of tourism as $P_{H}\approx 1/4-1/2w$ (Cesario 1976).

However, when the time horizon is divided between labor, search, and leisure ***the search represents any activity, which decreases the purchase price.*** Thus, the marginal benefit of search $Q\partial P/\partial S$ becomes equal to the marginal benefit of ***home production*** with regard to the corresponding market services (Aguiar and Hurst 2007a).

The pre-purchase search and the after-purchase care don't change the marginal utility of both consumption and leisure with respect to the classical labor-leisure choice. However, the need to take care after the purchase in order to derive utility from the trade items changes definitely the optics on the quantity demanded. At the equilibrium level consumers become *shoppers* and they bear neither pre-purchase, nor after-purchase costs they also don't make efforts to derive the utility. Here we need the optics of the sharing&rental economy, where consumers buy miles and nights. It means that at the equilibrium level where transaction costs equal to zero, the equilibrium price for vehicles is equal to ***the price of a mile in taxi*** and the equilibrium price for real estate is equal to ***the price for a night in the hotel***. However, this is not the unique solution, because the sharing economy, like it takes place in rent-a-car or real estate business, offers the options of miles-days and nights-square meters. It is clear that at the equilibrium level the utility of miles equals to the utility of days of driving as well as the utility of nights in the apartment equals to the utility of it's square meters.

However, on the level of trade units, where cars and apartments need some efforts to be kept in use, buyers become *searchers*, i.e. consumers with positive search costs (Dahl 1989), and the ***driving like the house maintenance becomes a specific form of home production***, where the option to produce or to buy the corresponding driving or maintenance market service always exists.

Productivity of industry and diligence

We see that the equilibrium price collects search&care costs of *searchers*, consumers with positive transaction costs, it equalizes ***the willingness to pay*** (*WTP*) of *shoppers* with ***the willingness to accept*** (*WTA*) of *searchers*. It means that a *searcher* gets an opportunity to re-sell the purchased item to a *searcher* like any new owner of a car gets a chance to sell miles as illegal taxi driver. However, the *WTA* doesn't mean that the searcher certainly sells an item. But if he does it, the *searcher* sells his property to the *shopper* at the equilibrium price of the trade unit QP_e. While *shoppers* have different quantity demanded, the equilibrium price dispersion appears, like it take place on the market of used cars, where good cars offers greater expected mileage Q_g with regard to the expected mileage of bad cars Q_b. There, good cars are sold at the purchase price Q_gP_{Pg}, and bad cars at the purchase price Q_bP_{Pb}. But any mile either in good or in a bad car has the same equilibrium price P_e, which is equal to the price of a mile in taxi. This price determines the equilibrium prices for trade units, i.e., a good car Q_gP_e and a bad car Q_bP_e. These prices appear implicitly like as the cars' owner decides to become taxi driver and to sell miles to *shoppers*. And the home production of cars' owner, i.e., the search in the given model, includes driving itself and handling – fueling, maintenance, washing and cleaning.

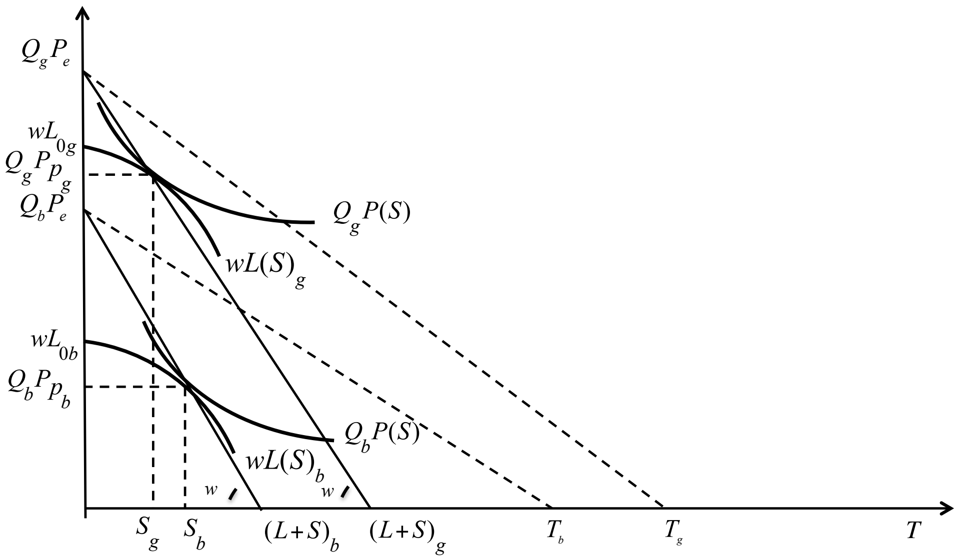

Fig.3. Equilibrium price dispersion

However, handling the property also faces the option “to produce or to buy”. Even the taxi driver can purchase a corresponding service on the market, or do cleaning and maintenance himself.

At the equilibrium level, the consumer does nothing and becomes a *shopper*. In taxi he leaves driving to the cabman and in the hotel the hostess cleans the room and makes the bed. The static search model accumulates all monetary costs as labor costs like it is done by the price of a mile in taxi and the price of a night in the hotel, while all non-monetary costs become search costs in their general sense, like any physical costs, time and efforts, that decrease monetary expenses.

If we try to compare the productivity of monetary and non-monetary efforts, we come to the well-known S-shaped productivity function. We know that on imperfect market the purchase price P_P depends on quantity demanded, or $\partial P_p/\partial Q<0$. However, the equilibrium price of a trade unit QP_e is equal to the willingness to pay of consumers with zero transaction costs and its stays constant for any dispersion $[Q_g>Q_b]$ of purchase prices for a trade unit QP_P. And the constant equilibrium price highlights the dynamics of labor and search costs.

Although the purchase price P_P falls with the increase in consumption units Q, or $\partial P_p/\partial Q<0$, the purchase price of a trade unit growths, or $\partial QP_P/\partial Q>0$, but it rises slowly, or $\partial^2 QP_p/\partial Q^2<0$. However, while the purchase price of a trade unit is equal to labor costs, or $QP_P=wL$, the constant equilibrium price per consumption unit slows down the growth of labor costs ($\partial wL/\partial Q>0$; $\partial^2 wL/\partial Q^2<0$ but accelerates the increase in search costs ($\partial wS/\partial Q>0$; $\partial^2 wS/\partial Q^2>0)$. And both the $\partial^2 wL/\partial Q^2<0$ and $\partial^2 wS/\partial Q^2>0$ values results in the corresponding inverse productivity relationships.

All these considerations produce the traditional S-shaped productivity curve $Q=Q(L;S)$ ($\partial Q/\partial L>0;\ \partial^2 Q/\partial L^2>0;\ \partial Q/\partial S>0;\ \partial^2 Q/\partial S^2<0$) and the traditional cubic total costs curve ($\partial wL/\partial Q>0$; $\partial^2 wL/\partial Q^2<0;\ \partial wS/\partial Q>0;\ \partial^2 wS/\partial Q^2>0$). But before we start to examine total costs we should pay particular attention to the productivity itself (Figure 4):

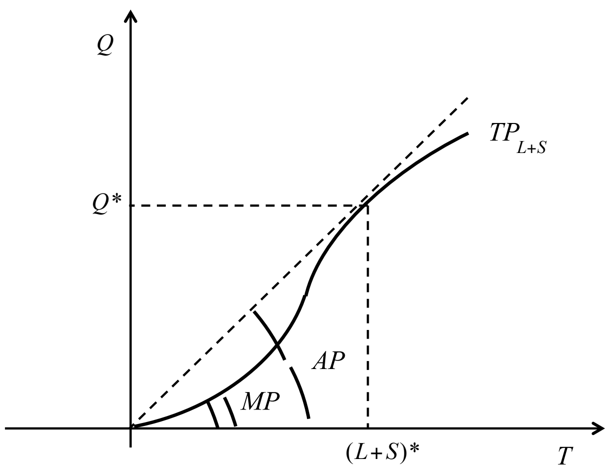

Fig.4.The productivity of consumer's efforts

We can divide all consumer's efforts on pre-purchase search, the purchase itself, and after-purchase care of the trade unit between ***high-productive industry*** ($\partial^2 Q/\partial L^2 > 0$) and ***low-productive diligence*** ($\partial^2 Q/\partial S^2 < 0$). Here we suppose that just the diligence supports the willingness to accept or to sell the trade item as it is presented in the common law where the great or high diligence means the "*diligence that a very prudent person exercises in handling his or her own property like that at issue"* (Black's Law Encyplopedia). The great or high diligence results in very thorough treatment of the trade item according to consumer's ***willingness to take care*** of it (Malakhov 2019). With regard to the trade item lifecycle it looks like the consumer earns industrially money to buy an item, searches it thoroughly, spend labor income on it, and handles it carefully after the purchase. Other words ***the consumer uses his industry to buy an item and his diligence to derive correctly the utility from it***.

Production possibility frontier with regard to consumer's diligence

However, the care as the specific form of the search decreases consumer's leisure time but it can be bought on the market in order to save leisure time. We can suppose that the same producer, who sells the item, proposes also its after-purchase maintenance and the costs of this maintenance raise the purchase price. By this the producer sells not only some consumption units but also he "supplies" leisure time to consumers. But the producer's resources are limited and he always has an option either to produce more consumption units or more services. And it happens not only with durables. Even the baker can either leave his son to work in the bakery or to send him with warm bread to customers. It means that we can present some sort of his production possibility frontier, which demonstrates the trade-off between the production of consumption units Q and the creation of consumers' leisure H. Of course, there is some relevant range. In point A his son works in bakery, where he reduces consumer's leisure because now they should go themselves to the bakery, and in point B his son takes a bicycle and delivers warm bread to customers.

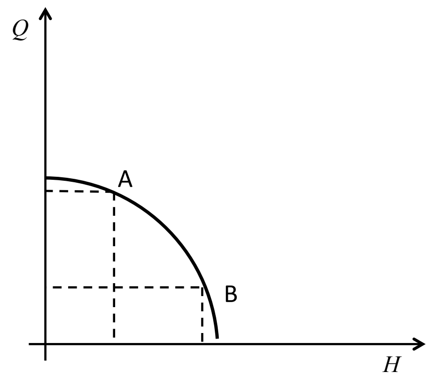

Fig.5.Consumption-leisure production possibility frontier

We take another example, now of a gardener who also has option either to leave his son to work on the plantation or to send him to the customers to trim hedges, to spray fertilizers, and to mown lawns. There are *shoppers* in the community who buy all gardening services at the equilibrium price level. However, there are also *searchers* who make the gardening themselves. And their willingness to accept looks reliable because the prudent customer practices usually the gardening like his house with the garden at an issue.

Any point in the relevant range of the production possibility frontier corresponds to some price offer with regard to complementary services. At point *A* the price is net of services, while at point *B* it includes all services. But the equilibrium price for the unit of consumption stays the same along the production possibility frontier:

$$w(L+S) = QP_e;$$
$$wL_A / Q_A = P_{pA} : wL_B / Q_B = P_{pB} \quad (6.1);$$
$$\frac{w(L_A + S_A)}{Q_A} = \frac{w(L_B + S_B)}{Q_B} = P_e \quad (6.2)$$

Here the equilibrium price looks like the monopoly price, like Peter Diamond explained it, and the producer discriminates customers with respect to their wage rate. While the equilibrium price is constant the efficient allocation *MRS (H for Q)= w/*P_e depends only on the wage rate. At point *A* all low-income consumers are *searchers* and they mowing their lawns themselves. At point *B* all consumers are *shoppers* and the son of the gardener mows their lawns (figure 6):

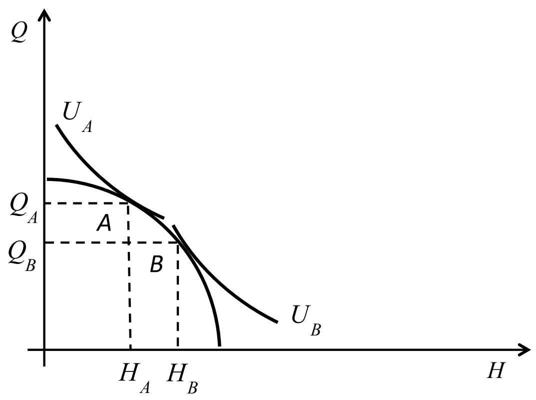

Fig.6.Multiple equilibria under wage rate differential

If the equilibrium price for a square meter of the lawn or the meter high of trees is constant, the community's optimum will be determined by the wage rate along the production possibility frontier.

But it is not the monopoly price. When the gardener raises the price for the square meter of the lawn mowing, the *shopper* immediately addresses to his neighbor, the *searcher*, and pays to his son the fair price for lawn mowing.

By this way the *searcher* starts to play the first chair in the equilibrium price dispersion orchestra. The *shopper* needs the *searchers* because they protect him from unfair offers. However, the producer also needs *searchers* because he can leave for them some inefficient work. It means that a trade-off between quantities of consumption units demanded with and without services should exist in some narrow margin because finally it should result in some trade-off between the production of consumption units and the "supply" of leisure, or *dQ/dH*. This is the way that enlarges the field for the Coase theorem.

The Coase theorem and consumers' diligence

Let's take a developer who constructs a residence and sells well-isolated apartments. But sometime it happens when consumers start to visit the construction when it is not finished yet and some apartments are waiting works on isolation.

The consumer starts to examine the residence, the developer asks the usual question about the budget and makes an offer *B* of well-isolated small apartment with Q_B square meters. The buyer to his turn makes another question about the price per meter without isolation. The seller gives an answer and the buyer tells to him that at this price he is ready to buy a greater apartment without isolation because he can make it himself under his personal guaranties.[2]

[2] While this practice is forbidden now in France, it still exists in other countries (S.M.)

If the trade-of of square meters and isolation exists within some narrow range of the *PPF*, the developer accepts this proposal and sells the greater apartment without guarantees on isolation.

From the point of view of the Coase theorem, by this way the seller produces the negative externality of both heat and cold, localized by the space of the apartment. But the nature of the problem of externality is really reciprocal here. Making the isolation himself, the buyer avoids the harm of heat and cold but he inflicts harm on the seller because he cuts his revenues on isolation. Then, he makes a step toward the seller and offsets the negative income effect, buying more square meters. As a result, the seller moves along some indifference curve along the narrow range of his *PPF*, searching for the new trade-off between consumption and leisure to be supplied, and the buyer, although he cuts his leisure time by more labor time L for the purchase of the greater apartment and by some working time S on isolation, comes to the upper level of his utility function, from U_0 to U^* (Figure 7):

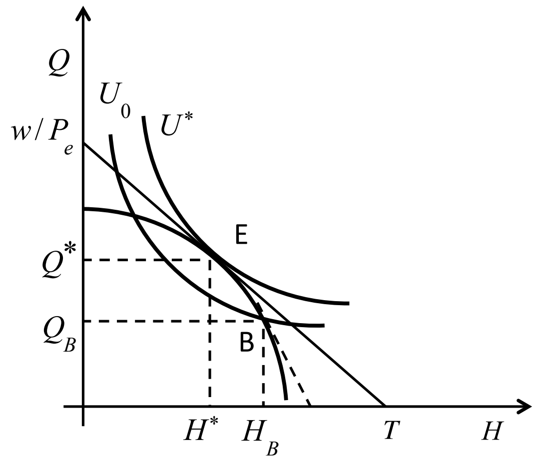

Fig.7.The reciprocal optimization under consumer's diligence

This logic proves both hypothesis of the Coase theorem (Medema and Zerba 2000, *Encyclopedia of Law&Economics*, p.838). The ***efficiency hypothesis*** is confirmed by the fact that the asset, here the apartment, is distributed for its more efficient use because if the purchaser is not diligent, the developer will keep the apartment for its sale. The ***invariance hypothesis*** is confirmed here by the fact that the new apartment appears on the market at its equilibrium price regardless alternative assignment of property rights because the new owner ***handles his property like that at issue***, i.e., when his diligence creates his *WTA*.

Here it is very important to follow step by step his decision-making. But before we start to analyze the buyer's decision-making, we should pay attention to the general outcome of the model. We see that it simply reproduces the logic of the Edgeworth box. Copying with the externality, one part raises its utility level while another part moves along its indifference curve. The solution is very simple – to restore the equilibrium price level of the damaged part and to keep the equilibrium price level of another part.

This approach can be applied to the classical case Sturges v. Bridgman, presented by Ronald Coase, where the building of a separate wall, which could deaden the noise and vibration, had been examined by the court. The confectioner's machinery had cut the equilibrium price level of the doctor's practice. However, the mortars also produced the negative effect to the confectionery itself. The court had documented the fact that the garden wall had been subjected to vibration. Nobody knew whether that vibration would destroy the garden wall or not, but it is evident that if confectioner decided once to sell his business, its prudent purchaser would certainly deduct costs of constructing a new wall from the market value of the confectionery. It means that the equilibrium price had to take into account the costs of construction of the new wall. If there was no risk that the garden wall, if it was destroyed by the vibration, damaged also the new consulting room, the confectioner could construct a new wall to deaden the noise and vibration or to ask the doctor for the permission to work in night time when there were no patients that could be equal to the cost of constructing of a new wall. So, the equilibrium price of the confectionery would stay at the same level, i.e., at its market value less the construction costs, moving the confectioner along the indifference curve from the construction of a new wall to the working at night. However, if the risk to damage the new consulting room existed, it would cut more seriously the equilibrium price of the confectionery. And the cost of the restoration of the consulting room could be equal to the replacement of the machinery. Thus, the confectioner would go down to the lower indifference curve where he moves from the restoration of the consulting room to the replacement of the machinery. We see that it was better for both parts to bargain before the building of the new consulting room, when the option to lease the end of the garden existed, if the confectionery was more efficient than the medical practice, because the doctor re-established the market value of his practice in any way but the confectioner could stay on his upper indifference curve.

Now we can come back to the logic of the buyer of the apartment. At point B he spends zero personal efforts S_B but he is ready to make them thoroughly with all his diligence because his low-productive diligence $\partial^2 Q/\partial S^2<0$ results in accelerated growth of costs of his physical efforts $\partial^2 wS/\partial Q^2>0$ (Eq.7.1) If the buyer accepts the offer B, his physical $MRS\ (H\ for\ Q)=Q_B/(L_B+S_B)$ (the dotted tangent line) will be greater than monetary $MRS\ (H\ for\ Q)=w/P_e$. Moreover, this offer doesn't equalize marginal loss with marginal benefit on purchase and the total costs are less than the equilibrium price of the trade unit, here the apartment Q_BP_e (Eq.7.2-3).

$$S_B = 0; \frac{\partial wS_B}{\partial Q} > 0; \frac{\partial^2 wS_B}{\partial Q^2} > 0 \quad (7.1)$$

$$\frac{w}{P_e} < \frac{Q_B}{L_B + S_B} \Rightarrow w\frac{L_B + S_B}{T} < -Q_B \frac{\partial P}{\partial S}; -w\frac{L_B + S_B}{T} > Q_B \frac{\partial P}{\partial S} \quad (7.2)$$

$$\frac{w}{P_e} < \frac{Q_B}{L_B + S_B} \Rightarrow w(L_B + S_B) < Q_B P_e \quad (7.3)$$

$$\frac{\partial wL_B}{\partial Q} + \frac{\partial wS_B}{\partial Q} > 0; \frac{\partial Q_B P_e}{\partial Q} = P_e > 0 \quad (7.4)$$

$$\frac{\partial Q_B P_e}{\partial Q} = P_e \Rightarrow \frac{\partial^2 Q_B P_e}{\partial Q^2} = 0 \quad (7.5)$$

$$\frac{\partial^2 wL}{\partial Q^2} < 0; \left.\frac{\partial^2 wL_{B+q}}{\partial Q^2}\right|_{Pp\,const} = 0; \frac{\partial^2 wS}{\partial Q^2} > 0; \frac{\partial^2 wL_{B+q}}{\partial Q^2} + \frac{\partial^2 wS_{B+q}}{\partial Q^2} > 0 \; (7.6)$$

$$Q_{B+q} = Q^* > Q_B; -w\frac{L^* + S^*}{T} = Q^* \frac{\partial P}{\partial S}; \frac{w}{P_e} = \frac{Q^*}{L^* + S^*}; w(L^* + S^*) = Q^* P_e \quad (7.7)$$

Both total costs and the equilibrium price per trade unit are rising with square meters (Eq.7.4.). The equilibrium price for the apartment is rising linearly (the equilibrium price per square meter P_e is constant), as well as the purchase price for the trade unit $QP_P(Q)=wL(Q)$ because the seller keeps the same price P_P for the square meter without isolation for some interval $B+q$ but non-monetary efforts of the buyer continue to rise (Eq.7.5-6). It means that the total suboptimal costs are rising faster than the equilibrium price per trade unit and once the increase in square meters re-establishes the marginal rate of substitution and the equality of marginal loss on self-made isolation with its marginal benefit at the equilibrium price level of the upper utility level (Eq.7.7).

All these considerations reproduce the well-known total cost curve, here the TC_{L+S} curve (Figure 8):

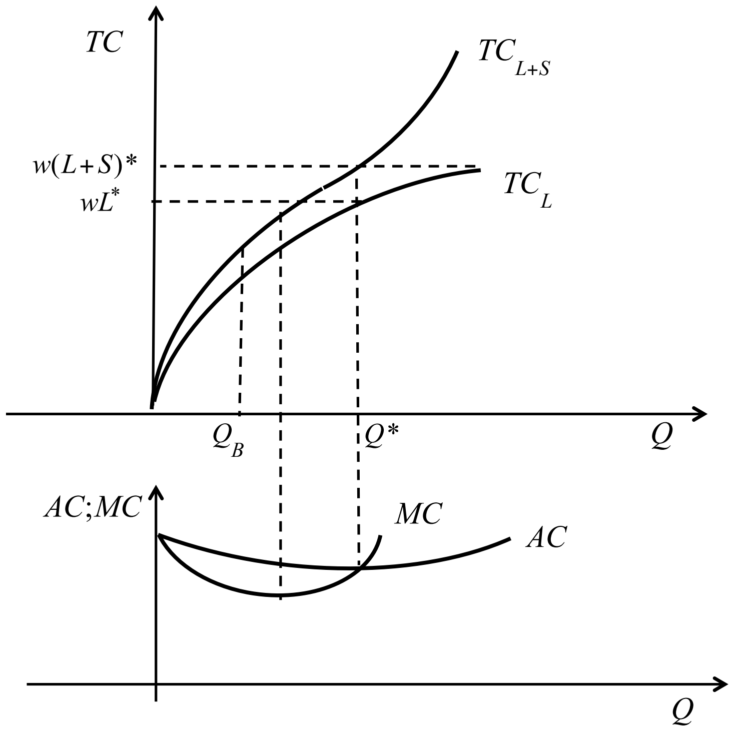

Fig.8. Total, average, and marginal consumer's costs

The buyer pays $wL^*(Q)=Q^*P_P$ for the apartment without isolation and starts to work on it. And his diligence changes the shape of the total costs curve. While once in the interval $B+q$ labor costs become linear, the diligence rate $\partial^2 wS/\partial Q^2$ certainly becomes equal to the industry rate $\partial^2 wL/\partial Q^2$ and marginal costs come to its optimal level $\partial MC/\partial Q=0$. However, the average costs' inertia continues to decline them until the meeting point with marginal costs where they come together to the equilibrium price level of the isolated square meter, or $MC=AC=P_e$.

While theoretically the externality cannot be eliminated totally, we can take into account its residual effect, here in the form of isolation materials, sold by the developer to the buyer. But it doesn't change the logic of the model. Here we see how the static optics reproduces the traditional cubic total costs curve. But now it depends not only on the productivity itself. It is created by the ***co-existence of the imperfect market*** with $\partial P_P/\partial Q<0$ of *searchers* and ***the perfect market*** of *shoppers* $\partial P_P/\partial Q=0$ with the constant equilibrium price per unit of consumption. In some sense, the productivity itself becomes the outcome of the state of the market. The Figure 8 gives an intuitive argument that the state of market changes the shape of the total costs curve. When the market is close to its perfect state, the shape of labor costs curve becomes close to linear and it doesn't need much diligence from buyers. However, when the market stays imperfect, even the increase in productivity, either of consumption units or services cannot eliminate consumers' diligence, other words the home production. This intuitive argument can be verified by the analysis of the labor augmenting and service augmenting technical progress.

Prices and allocation of time under labor augmenting technical progress

When workers learn to do their job better, the productivity of labor is augmented over some time. This technical progress increases the output of Q and lowers marginal costs of production MC_Q. The production possibility frontier becomes steeper and the marginal rate of substitution of leisure for consumption $MRS\ (H\ for\ Q)=w/P_e=MC_H/MC_Q$ rises. We can expect both equilibrium price and purchase price to fall. And the prices' dynamics gives an answer to the question whether the technical progress reduces the consumers' diligence or not.

To understand this, we should determine first of all the dynamics of purchase price. We understand that now we cannot take the time horizon as the constant value due to the increase in both output and consumption. However, the idea that the time horizon rises proportionally to the consumption, or $e_{T,Q}=1$ doesn't look reliable.

It is quite reasonable to assume that the absolute value of marginal savings on purchase $|\partial P/\partial S|$ follows the purchase price, or $e_{|\partial P/\partial S|,\ Q}=e_{Pp,Q}$. Tis assumption limits the elasticity of the time horizon. To keep the producers' inflow positive we need $e_{Q|\partial P/\partial S|,Q}=e_{QPp,Q}>0$.

The increase in the $MRS\ (H\ for\ Q)=w/P_e= -w/T\partial P/\partial S =w/T|\partial P/\partial S|$ means the fall in the equilibrium price and in the value $(-T\partial P/\partial S)$. If $e_{T,Q}=1$, we can expect $e_{T|\partial P/\partial S|,Q}= e_{Q|\partial P/\partial S|,Q} > 0$. But when $e_{Pe,Q}<0$, we get $e_{T|\partial P/\partial S|,Q} <0$. It means that the assumption of the unit consumption elasticity of the time horizon $e_{T,Q}=1$ is wrong. As a result, we get the inelastic time horizon with regard to consumption, or $e_{T,Q}<1$.

However, this value might be either positive ($0<e_{T,Q}<1$), or negative ($e_{T,Q}<0$). If we take the positive elasticity of time horizon with regard to consumption, we see that when elasticity of both equilibrium price and purchase price with regard to consumption is negative, the fall of purchase price is deeper, or $e_{Pp,Q} < e_{Pe,Q}$ (Equation 8.6).

$$P_e = -T / \partial S = T\,|\partial P / \partial S| \quad (8.1)$$

$$e_{Pe,Q} = e_{T,Q} + e_{|\partial P/\partial S|,Q} \quad (8.2)$$

$$if\ e_{|\partial P/\partial S|,Q} = e_{Pp,Q} \Rightarrow e_{Pe,Q} = e_{T,Q} + e_{Pp,Q} \quad (8.3)$$

$$e_{Pe,Q} < 0; e_{Pp,Q} < 0 \quad (8.4)$$

$$if\ 0 < e_{T,Q} < 1 \quad (8.5)$$

$$e_{Pp,Q} < e_{Pe,Q} \quad (8.6)$$

While the difference between the purchase price of a trade unit $QP_p=wL$ and the equilibrium price of a trade unit $QP_e=w(L+S)$ is equal to the value of non-monetary costs wS, the conclusion (8.6) become evident. But we can precise this result:

$$\frac{\partial P_e}{\partial Q} - \frac{\partial P_P}{\partial Q} = \frac{1}{Q^2}\left(Q\frac{\partial S}{\partial Q} - S\right) = \frac{S}{Q^2}(e_{S,Q} - 1) \quad (9)$$

We see when purchase price is more sensitive to the technical progress and the following growth of both supply and demand than the equilibrium price because the non-monetary costs per unit of consumption wS are increasing ($e_{S,Q}>1$). While we understands that the concept of non-monetary costs describes physical efforts that consumers need to derive a utility from the trade item, the result (9) means that the labor augmenting technical progress increases consumers' efforts for the recovery of the utility from the trade item. We have no formal grounds to talk here about the quality because it is not measurable, but at the commonsense level we understand that this conclusion means ***the fall in the quality of trade items under the labor augmenting technical progress***. There, workers become more industrious but less diligent. So, these are consumers who should become more diligent under the labor augmenting technical progress.

However, we can come to absolutely opposite conclusions, if we subsequently change signs in Equations (8-9). Here we can see that the reduction of physical efforts per consumption unit results in the cut of the time horizon, or $e_{T,Q}<0$. It happens when the fall in purchase price is less than in the equilibrium price. At the margin consumers can buy for the same price high-quality items.

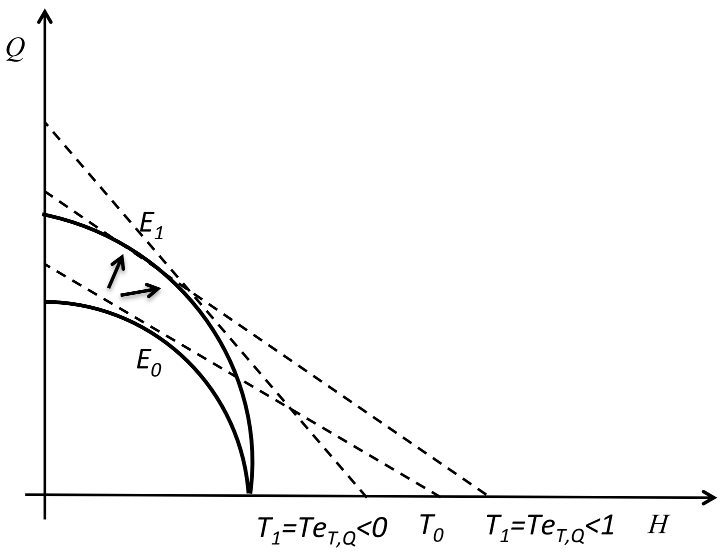

Fig.9.Labor augmenting technical progress

It looks reasonable for necessities, primarily for food. The reduction in the time horizon means the increase in shopping frequency, and it happens with the income growth (Kunst 2019). And the fall in prices means the income growth. Sometimes, it happens also with the big-ticket items. For example, the age of the car in recession might be greater (Statista 2019). However, the increase in shopping frequency for durables adapts well to the phenomenon of ***the planned obsolescence***.

The cut in the time horizon means the reduction of consumers' diligence and their willingness to take care of a big-ticket item because now producers become more diligent. However, the development of the customer relationship management (CRM) means the accounting of low-productive diligence as labor costs. It results in the growth of quality but reduces the productivity. The productivity function becomes almost linear, the purchase price becomes less sensitive to changes in quantity supplied and the demand becomes very elastic. However, the increase in the services' productivity can drastically change the situation.

Service augmenting technical progress: from negative productivity to "bad" consumption

The service augmenting technical progress means the increase in the productivity of services. By this way producers "supply" more leisure to consumers because services cut home production and consumers get more leisure time. However, the increase in the productivity of services can result in ambiguous outcomes with respect to the substitution effect of consumption units for services. If it is strong, the production of consumption units falls and the equilibrium price rises that makes the budget constraint line flatter under the low $MRS\ (H\ for\ Q)=w/P_e$ (Figure 10). Here, the increase of the time horizon is produced by the growth of both labor and leisure time because consumers really cut their search and care time due to the value $e_{S,Q}>1$ (Equation 9.2).

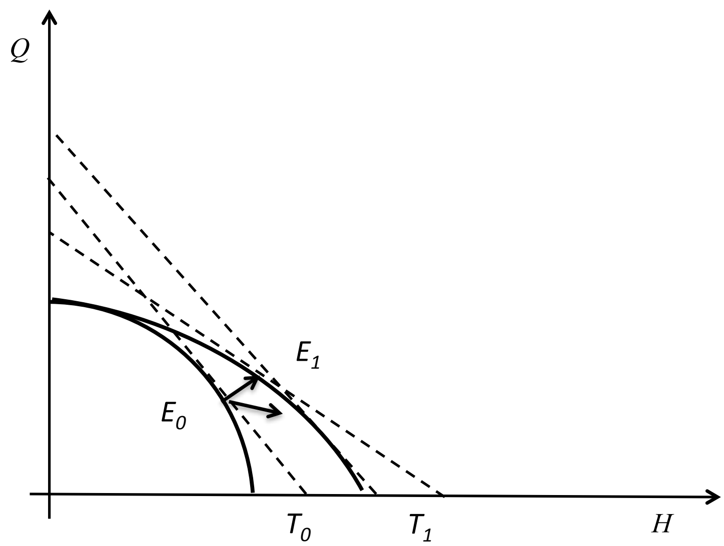

Fig.10. The service augmenting technical progress

However, the increase in services' costs rises consumers' spending. And when total consumers' labor costs wL are rising, the productivity itself falls with respect to consumption units, or $\partial Q/\partial L<0$. Here, the phenomenon of the negative productivity appears.

On the other hand, the increasing service productivity can result in more phenomenal outcome. In the substitution effect is small and services and consumption units are gross complements, the decrease in the marginal costs of "the production of leisure" is so important that it gives a possibility for producers to increase the production of consumption units (Figure 10). However, under the constant wage rate it looks like the equilibrium needs the increase in the equilibrium price $RPT=MC_H/MC_Q=MRS\ (H\ for\ Q)=w/P_e$ with the fall of MC_H value At a glance, this result doesn't seem paradoxical because it discovers the price growth, now with the costs of services. The technical progress gives for producers an opportunity to add more services to the price of the consumption unit.

At the beginning producers also add services to the price of consumption units when they sell trade units to *shoppers* but there the equilibrium price stays constant because there producers move along the *PPF* and discharge *shoppers* from search&care costs that rise the marginal costs of services, i.e., of the "production" of leisure MC_H. Other words, they substitute production for services. The MC_H growth meets the high wage rate and the high willingness to pay of *shoppers* that keeps the equilibrium price of the consumption unit constant. The service augmenting technical progress shifts the *PPF* and cuts the value MC_H while the wage rate stays constant. Other words, producers add services to any level of consumption.

We see that the equilibrium price elasticity of consumption becomes positive, or $e_{Q,Pe}>0$. While this result is produced by the increase in leisure time with the fall of its marginal costs MC_H, the analysis of the budget constraint (2.1) with regard to the leisure time can explain the positive $e_{Q,Pe}$ elasticity.

It is easy to show that the shift from E_0 to E_1 results in the following equations:

$$w\frac{\partial L}{\partial S}=Q\frac{\partial P}{\partial S};$$

$$e_{w\partial L/\partial S,H}=e_{Q\partial P/\partial S,H}=e_{Q,H}+\frac{-T\partial(\partial P/\partial S)}{H}\frac{H}{-T\partial P/\partial S}=e_{Q,H}+e_{Pe,H}=e_{QPe,H} \quad (10.1);$$

$$\frac{w\partial(\partial L/\partial S)}{\partial H}\frac{H}{w\partial L/\partial S}=e_{QPe,H} \quad (10.2)$$

Commonly, the value $\partial^2 L/\partial S\partial H$ is positive. We have seen that under Archimedes' principle ($-1<\partial L/\partial S<0$), the increase in leisure time reduces the absolute value of the propensity to search $\partial L/\partial S=|\partial L/\partial S|$, i.e., it rises its real value ($-(L+S)/T$). While the value of the propensity to search is strictly negative ($\partial L/\partial S<0$) and it is followed by the positive $\partial^2 L/\partial S\partial H$ value, it is expected that the leisure elasticity of equilibrium price for a trade unit, cars and apartments,

$e_{QPe,H}$ to becomes negative. Indeed, the value $e_{Pe,H}$ is negative because leisure reduces total costs *(L+S)* and the consumption-leisure relationship $e_{Q,H}$ is also negative. However, if we get the positive leisure elasticity $e_{QPe,H}$, like it takes place under the complementary service augmenting technical progress, it means that the value $\partial^2 L/\partial S\partial H$ changes its sign. ***It becomes negative.***

But if we come back to the value of the marginal utility of leisure in the consumption-leisure choice, we can see that under negative $\partial^2 L/\partial S\partial H$ value the marginal utility of leisure also becomes negative (2.4). The increase in leisure time makes the fall of the propensity to search $\partial L/\partial S<0$ deeper. It happens when the propensity to search becomes very strong, or $\partial L/\partial S<-1$. There, the reduction of labor time under the search and care is so important, that it raises not only search and care but also leisure:

$$dL(S) = dS\frac{\partial L}{\partial S}; \frac{dL}{dS} = \frac{\partial L}{\partial S} < -1 \quad (11.1)$$

$$-dL(S) > dS; \partial L/\partial H < 0; \partial H/\partial S > 0 \quad (11.2)$$

When the marginal utility of leisure become negative, it doesn't mean that we get here the excess leisure. Contrarily, the strong propensity to search creates a deficit of leisure. It seems that leisure becomes "negative" within the "negative" time horizon $\partial^2 L/\partial S\partial H = -1/T$ (Figure 11, Equation 12.2):

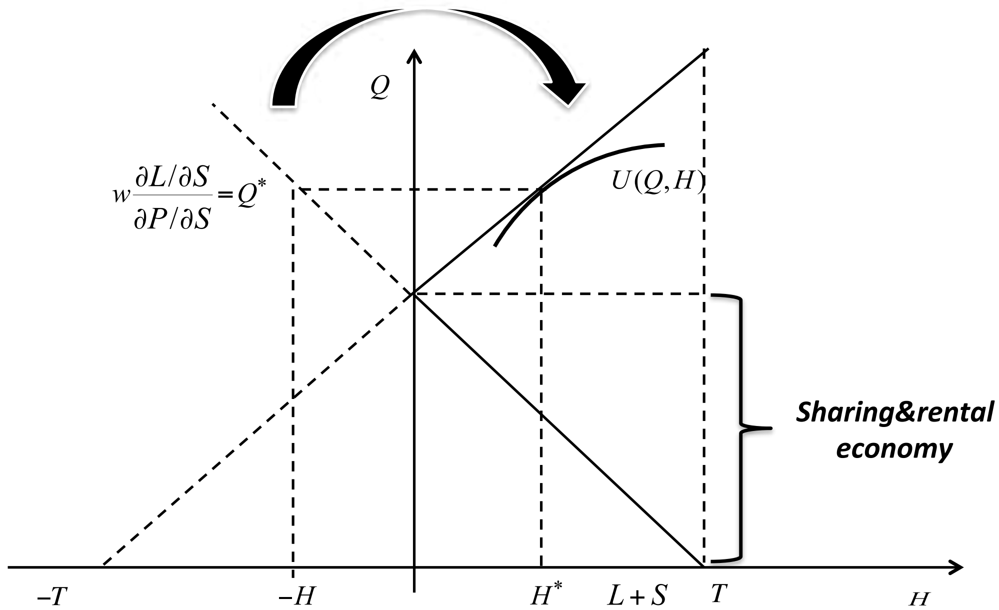

Figure 11. The deficit of leisure and the "bad" consumption

But we can see that the optimal quantity to be purchased Q^* doesn't depend on the virtual negative values of both time horizon and leisure. So, the following set of Equations (12) is true in any case and the change in signs of time horizon and leisure doesn't matter:

$$Q^* = Q_{\partial L/\partial S=-1} + dQ(H) = -\frac{w}{\partial P/\partial S} - H\frac{\partial Q}{\partial H} =$$
$$= -\frac{w}{\partial P/\partial S} - H\frac{w}{T\partial P/\partial S} = -\frac{w}{\partial P/\partial S}(1+H/T);$$
$$Q^*\frac{\partial P}{\partial S} = -w(1+H/T) \Rightarrow \partial L/\partial S = -1 - H/T \quad (12.1)$$
$$\partial^2 L/\partial S^2 = -\frac{\partial H/\partial S}{T} = -\frac{-1-\partial L/\partial S}{T} = \frac{\partial L/\partial S+1}{T};$$
$$\partial L/\partial S < -1 \Rightarrow \partial^2 L/\partial S^2 < 0;$$
$$\partial^2 L/\partial S\partial H = -1/T \quad (12.2)$$

We see that when the propensity to search becomes strong, or $\partial L/\partial S< -1$, it changes its value from $\partial L/\partial S= -(L+S)/T$ to $\partial L/\partial S= -(T+H)/T$ (Equation 12.1). However, both "negative" leisure and time horizon values are virtual because leisure is increasing with the value $\partial^2 L/\partial S\partial H<0$. Now it doesn't look like the deficit of leisure but like leisure becomes excessive under its negative marginal utility. However, this assumption is also invalid. The negative marginal utility of leisure radically changes the *MRS (H for Q)*:

$$MRS(H\ for Q) = -\frac{dQ}{dH} = \frac{MU_H}{MU_Q};$$
$$MU_H < 0 \Rightarrow dQ/dH > 0 \quad (13)$$

Now we understand that the positive leisure elasticity $e_{QPe,H} >0$ in (10.1) is true because the consumption-leisure relationship $e_{Q,H}$ at w_{const}, $\partial P/\partial S_{const}$ becomes definitely positive. But it is not the end of the story. The value $\partial L/\partial S<-1$ tells us that the leisure-search relationship also becomes positive, or $\partial H/\partial S>0$. The Archimedes' principle stops working.

It looks strange because the end of the service augmenting technical progress is to cut search and care time in favor of leisure. But both search and care are increasing here under the pressing of price growth. While the increase in the wage rate makes search and care less attractive with regard to the existing price reductions $\partial P/\partial S$, the price growth creates attractive high price reductions $\partial P/\partial S$ that motivate consumers to search and to care. However, after the price growth both search and care take place in upper price niches. For example, the insurance moves cars to the upper price niche and motivates buyers to search for cheap options.

As a result we get the following logical chain:

$$\partial S/\partial P > 0; \partial H/\partial S > 0; \partial Q/\partial H > 0 \Rightarrow \partial Q/\partial P > 0 \quad (14)$$

When the price growth under the service augmenting technical progress stimulates search and care, the Veblen effect $\partial Q/\partial P>0$ is to be expected.

This result explains the nature of the price growth under service augmenting technical progress, or $e_{Q,Pe}>0$. But it doesn't explain all changes produced by the re-allocation of time under the deficit of leisure, presented at Figure 11.

The price growth creates attractive price reductions and makes labor at a given wage rate unattractive. It becomes better to search and to care than to work. It looks like the labor decreases the utility. The shift of the constraint to the north-east occurs only when leisure recovers its positive marginal utility. And it can take place only when the marginal utility of labor income $MU_w= \lambda$ becomes negative. But the negative marginal utility of labor income makes consumption "bad" (Equation 2.3). The increase in consumption reduces the utility and consumers should accept it because they are interesting in leisure. For example, a music lover needs to buy or to rent an uncomfortable suit in order to go to opera. It looks like ***a particular price bundling of negative consumption with positive leisure takes place***. And sometimes this price bundling results in the ***sunk-costs sensitivity***, for example, in skiing, when consumers prefer not to rent ski but to buy the equipment and to depreciate it thoroughly by the increase in leisure time.

The last consideration widens the understanding of the consumption model, presented at Figure 11. When the purchase of a trade unit can be substituted by the purchase of consumption units, like it take place in the sharing economy, the given wage rate keeps the consumption model in its "common" frames $(-1<\partial L/\partial S<0)$ for the given price level and respective marginal savings $\partial P/\partial S$. However, if there is some ***consumption threshold*** $Q_{H=0}$, the consumer should spend a time horizon to prepare himself to the purchase of a trade unit. Here we don't know how this negative time horizon is allocated between labor and search but we knows definitely that there is neither leisure, nor consumption itself. The missed consumption states the fact that ***the chosen trade unit doesn't represent the necessity***. Moreover, the consumption itself becomes a quasi-complement to the leisure, which becomes the end of consumption. The willingness to accept or to sell also leaves its economic grounds because now it depends on leisure, or $e_{QPe,H}=e_{WTA,H}>0$. This is not only the reason for the positive $e_{Q.Pe}$ elasticity. It is also an answer to the question why the equilibrium price rises under the service augmenting technical progress. While in the sharing&rental economy it goes up with the fall of production, or $e_{Q,Pe}<0$, here under the services&production complementarity, it rises due to the factor of leisure. Now the equilibrium price includes some leisure costs and while we speak about the willingness to sell, these costs represent ***leisure to be abandoned with the sale***. Indeed, the consumer leaves "***the common model***" $(-1<\partial L/\partial S<0;\partial H/\partial S<0)$ of behavior and comes to "***the leisure model***" $(\partial L/\partial S<-1;\partial H/\partial S>0)$.

The shift from the common model to the leisure model results in the kinked budget constraint where the consumer, trying to recover the deficit of leisure, produces a specific ***catastrophe*** at the consumption threshold $Q_{H=0}$, where the utility $U(Q,H)=Q^{-\partial L/\partial S}H^{-\partial H/\partial S}=Q^{(L+S)/T}H^{H/T}$ as well as its marginal value $MU_w=\lambda$ stays undefined due to $H=0$.[3] As a result, the *MRS (H for Q)* changes its sign in the leisure model. But it doesn't mean that the equilibrium price of consumption becomes negative. The equilibrium price keeps its positive value (Equation 15):

$$\partial L/\partial S < -1; \partial L/\partial S = -1 - H/T$$
$$w\frac{\partial L}{\partial S} = -w\frac{T+H}{T} = Q^*\frac{\partial P}{\partial S};$$
$$w(T+H) = -Q^*T\frac{\partial P}{\partial S} = Q^*P_e \quad (15)$$

Here we see that the equilibrium price really accounts not only labor and search costs of the negative time horizon but also leisure time of the current time horizon.

And the $MRS\ (H\ for\ Q) = MU_H>0/MU_Q<0=-w/P_e$ simply states the fact that the ***real wage becomes negative***. Indeed, when a trade unit doesn't represent the necessity, the quantity of consumption units demanded looks unnecessary for the current time horizon. ***The labor income is spent for something that produces the negative utility***.

If we take skiing as an example, we can see that the equilibrium price per unit of consumption, here, one downhill race, is formed by ski rentals, which complement this price by different price bundling. And the Figure 11 tells us that a person, who rents ski, can enjoy the same amount of leisure time in one day or in one season as a person who has bought the equipment before.

Of course, the purchase of the equipment can be depreciated in the next season or by more its intensive use in the same season. However, we can see that the change of the time horizon doesn't change the logic of the allocation of time because, as Equations (11.1-11.2) tell us, the strong propensity to search $\partial L/\partial S<-1$ results in the positive leisure-choice relationship $\partial H/\partial S>0$ for any given time horizon.

The logic of financial management can play here a trick. When we reject taxi and buy a car because we need many miles, we follow the financial logic of depreciation. The same happens with ski equipment. But there is an important difference between driving and skiing. The purchase of the car cut the price with respect to the equilibrium price, i.e., the mile in taxi. The purchase of ski also cut the price of one race with respect to the price of the ski rental. In all

[3] Zero to the power of zero, denoted by 0^0, is a mathematical expression with no agreed-upon value. The most common possibilities are 1 or leaving the expression undefined, with justifications existing for each, depending on context (Wikipedia).

cases it means the reduction of labor costs *wL* with respect to the equilibrium level. However, for the car the reduction in labor time is followed by the decrease in leisure time because both labor and leisure are squeezed out by the search, here the driving, because we assume that the search represents any activity, which reduces purchase price and labor time. In skiing the search takes a specific form of ski storage and ski maintenance, the grinding of the ski base and the polishing of the ski edges. If we pay for the ski maintenance at the resort, the maintenance fee raises the purchase price but it makes the ski maintenance costs irrelevant to the option to rent or to buy. So, the storage becomes the key factor for this option. But the ski storage is not driving and it doesn't reduce leisure time. Moreover, the increase in storage time can rise leisure time.[4] It happens when we buy ski not only for one but also for two-three seasons. From the financial point of view it looks very reasonable because the depreciation of the purchase becomes more evident. But if we take three-seasons time horizon, we see the reduction of labor time with respect to the equilibrium level of ski rental and the increase in both search and leisure time. And it means that the long-term efficient planning depreciates the positive purchase price of the negative marginal utility of ski.

If we take for this example the number of downhill races as the quantity demanded, the depreciation, as Figure 11 demonstrates, plays its nasty trick even in the first season, when the intensity of consumption of the purchased ski is much greater than the intensity of consumption of rented ski.

This illustration gives an idea that ***the sunk costs' sensitivity represents an attribute of the leisure model of behavior and results in the depreciation of the negative marginal utility of consumption***.

The analysis of the depreciation under the leisure model of behavior illustrates the commonsense idea that a durable item, for example, a washing machine, cannot stay idle. Once it is bought, it should work. If we represent the washing machine as a number of consumption units, i.e., clean clothes and household items, we should look for the equilibrium price in the price list of the laundry care nearby with free pick-up and delivery. And it doesn't worth field studies to confirm the assumption that the quantity of cleaned items will be greater in the case household cleaning.

This idea returns us to the substitution effect between consumption units and services under service augmenting technical progress, when the price growth results in the fall of the quantity demanded. The service augmenting technical progress under the common model of

[4] The same thing happens with the wine. We can either buy old luxury Bordeaux at the equilibrium level or to cut labor costs and to buy young wine in order to keep it. Keeping the wine means the increase in the time of care. The wine becomes better and when the bottle is finally open, it is consumed slowly. But it means that the care increases the time of enjoyment, i.e., leisure. And we get in total $\partial L/\partial S<-1$ and $\partial H/\partial S>0$.

behavior cuts the quantity demanded. As a result, it increases labor time in order to buy consumption units with more services but more services "produce" more leisure. Staying within the common model, the service augmenting technical progress decreases the time of search and care. By this way it reduces the consumers' diligence and develops the sharing&rental economy. But if consumers leave the common model and allocate their time under positive leisure-search&care relationship $\partial H/\partial S<0$, they stay diligent but their diligence becomes excessive.

Conclusion

The derivation of the equilibrium price of consumption unit seems not to be useful in the applied economic analysis but this is the only way to understand different consumers' efforts on search, purchase, and care. This difference becomes more evident if we take into account the wage rate growth, when the allocation of time changes under both income and consumption effect (Malakhov 2018). When the consumption effect is small, like it takes place with necessities, consumers reduce diligent efforts on search and care in favor of both labor and leisure. But if the consumption effect is strong, like it takes place with luxuries, the labor curve becomes backward-bending because consumers increase search and care efforts. Here the question is whether consumers stay within the "common model" of behavior with the decreasing leisure or they come to the "leisure model" with the increasing leisure, where their diligence becomes excessive.

The difference between the common and the leisure model behavior can explain the disparity of income growth and measures of happiness (Malakhov 2016). It also explains the major distinction between female and male models of the allocation of time discovered by field studies when women cuts the non-market work in favor of both market work and leisure, while men significantly cut the market work in favor of both non-market work and leisure (Aguiar and Hurst 2007b).

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