Shadow effect from Laffer tax allergy: New tax policy tool to fight tax evasion

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Abstract

This study is inspired by the Laffer curve to develop and formalize a concept around optimal tax policy considering asymmetric information. This is the "Shadow effect". This theory states that when the tax burden is high, producers tend to inflate their fictitious expenses in order to reduce their declared profit (in order to avoid paying a high tax). The theoretical developments show that the propensity of producers to the Shadow effect is positively related to the square of tax rate. The relationship is non-linear. They also show that there is an inverse and non-linear relationship between the tax rate and the level of production. In addition, producers' sensitivity to the Shadow Effect can be influenced by fluctuating the tax burden. This study provides to governments a new fiscal policy tool. For instance, a numerical application has shown that if the Cameroonian government wants to encourage production in such a way that it could reach 50% more, it should reduce the corporate tax rate down *ceteris paribus,* to 16.19%.

Keywords: Tax evasion; Tax burden; Laffer Curve; Shadow effect.
JEL Codes: H21; H26.

1.1 Introduction

"We have a system that increasingly taxes work and subsidizes nonwork." – Milton Friedman

Can markets regulate without state intervention? There is not a lot of differences between this question and the following: Should we pay the tax? Indeed, tax is one of the main government's source of funding. Without the tax that gives to the State the means of implementing its politics (Fauvelle-Aymar, 1999), the latter wouldn't have any power on markets, or even conversely (Quinn & Shapiro, 1991; Best, 1976). This brings tax at the centre of the liberalism – interventionism debate. But, even if the major school of thought agree on the necessity of the presence of the State as an institution in the economy (at least for the Friedrich Hayek's Rule of law and property right), the problem of the '' degree '' of intervention will still persists (Friedman & Friedman, 1998). This issue carries with him that of the management of the tax: what to tax? who to tax? where to tax? and above all, how to tax?

The washouts of capitalism during 1929 and 2008's crises (Stiglitz J. , 2010), the failure of the Marxist's socialism and communism in Ex-URSS have shown the limits of economic radicalism. By extrapolating, it's obvious that not taxing is as dangerous as overtaxing. It is therefore necessary to find the right balance. But, economic literature around that issue is abundant and even proposes some solutions. One of them is the theory of optimal taxation. Indeed, the optimal tax theory aims to design and implement a tax that maximises a social welfare function that is subjected to economic constraints (Mankiw & Weinzierl, 2009). From this theory, the majority of taxes distort individual behaviour and consequently reduces the individual incentive to the taxed activity (Keane, 2011). Although useful, optimal tax theory is often criticized to not considering administrative costs of tax systems (Burgess & Stern, 1993). And the practice of ignoring the full set of tax instruments under uncertainty leads to misleading results (Dhami & al-Nowaihi, 2006). Moreover, in its current state, optimal tax theory is incomplete as a guide to action for critical issues in tax policy. It is incomplete because it has not yet come to terms with taxation as a system of coercively collecting revenues from individuals who will tend to resist (Slemrod, 1990).

Another attempt to build appropriate tax policy lays in the famous Laffer Curve. Much closer to a political concept than an economic theory, the Laffer curve also aims to provide solution to the optimal taxation issues. Indeed, an anecdote reported by Jude Wanniski in *The Public Interest* says that during a dinner, Arthur Laffer grabbed his napkin and a pen and sketched a curve on the napkin illustrating the trade-off between tax rates and tax revenues (Wanniski, 1978; Laffer, 2004).

Laffer's curve is particularly interesting and seductive as it is simple to explain. About that Wanniski (1978) summarises it into that Laffer's

statement: "There are always two tax rates that yield the same revenues." And the famous Curve has the following shape:

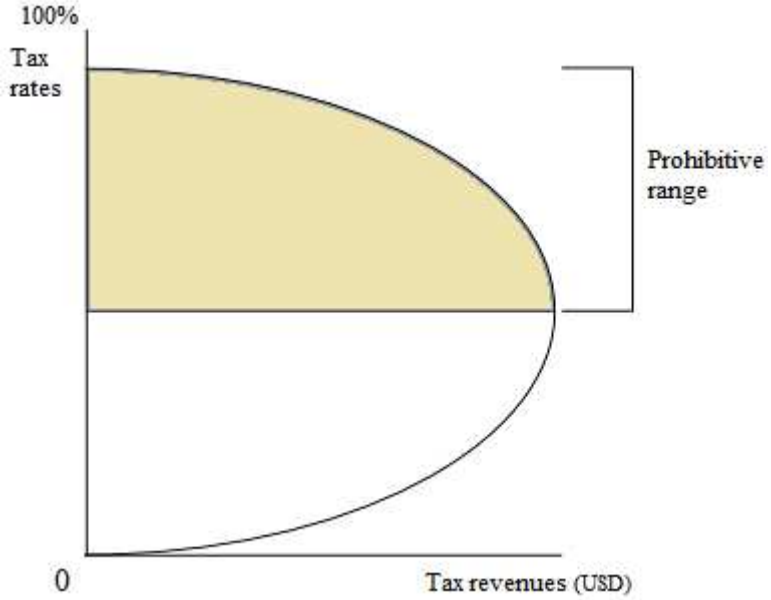

Figure 1: The Laffer Curve

Source: (Laffer, 2004)

As exposed on the figure, Laffer curve is a hump-shaped curve showing tax revenue as a function of the tax rate. Revenue initially increases with the tax rate but then it can decrease (prohibitive range) if taxpayers reduce market labour supply and investments, if they switch compensation into non-taxable forms, and engage in tax evasion (Fullerton D. , 2008). From the Curve, the revenue-maximizing tax rate can be calculated from an estimate of the elasticity of taxable income with respect to the after-tax share. Fullerton D. (2008) explains that the mid-range for this elasticity is around 0.4, with a revenue peak around 70%.

According to Laffer (2004), lower tax rates change economic behaviour and stimulate growth, which causes tax revenues to exceed static estimates. Furthermore, Wanniski (1978) argue that when the tax rate is 100%, all production ceases in the money economy (as distinct from the barter economy); Indeed, people will not work in the money economy if all the fruits of their labours are confiscated by the government, leading government revenues to zero (Wanniski, 1978).

The particularity of this study is that it focuses on the tax evasion alternative, meaning that instead of ceasing production when tax burden is high, producers increase their informal activities. This behaviour is considered as the "Shadow effect". Indeed, Mirowski (1982) was already arguing that the derivation of the Laffer Curve has nothing to do with tax evasion. Thus, Mirowski (1982) place a particular emphasis on this study "Shadow effect". Moving in the same

direction, the optimal tax problem is a game of imperfect information between taxpayers and the social planner (Mirrlees, 1971). Indeed, imperfect information is due to asymmetric information that enables producers to produce fraudulent financial statements by inflating their expenses. Their goal being to avoid high tax burden. An issue that has not been formally considered by the optimal tax theory (Slemrod, 1990) and the so-called Laffer Curve (Mirowski, 1982).

However, this did not taint the Laffer Curve glamor. Maybe because Laffer only reinvented the wheel. Indeed, Laffer himself said that the first studies on the relationship between the tax rate and the economic growth date back to the fourteenth century with the writings of Ibn Khaldoun[1] (Laffer, 2004). Moreover, in the *Wealth of Nations*, Adam Smith yet observed that "…the economic incomes of private people are of three main types: rent, profit and wages. Ordinary taxpayers will ultimately pay their taxes from at least one of these revenue sources". Also, Jean-Baptiste Say in *Treaty of Political Economy* concluded that an excessive tax destroys the basis which carries it. In fact, the curve named the Laffer curve was formally presented by French economist Jules Dupuit as early as the 1840s (Giertz, 2008).

The tax is so important that it seems as far back as the man stated to live in community, it was there. In this connection, Marshall Sahlins (1976) explained that primitive societies already used their surplus of production as offerings to deities for their protection. A little latter, the tax will rather serve to constitute the mode of social organisation of the sedentary population (Sahlins, 1976). Nowadays, in addition to ensuring the state's sovereign functions, taxes are supposed to contribute to the reduction of inequality and poverty by redistributing the wealth created (Lambert, 1993). Goals that are very far from being achieved (Leigh, 2008) and whose can even have negative consequences on financial markets[2].

Indeed, the tax has not always been considered by taxpayers as a contribution or a participation. They consider it more like a penalty, a punishment; Although, a famous quote from an unknown author says, "A fine is a tax for doing something wrong. A tax is a fine for doing something right." Indeed, too much tax kills tax; and this can lead to tax evasion. Facing high tax burden, entrepreneurs will choose to inflate their fictitious expenses in order to declare a low profit and therefore maintain their living condition. This is what this study considers as being the "Shadow effect". In such a situation, tax authorities may need to have clear information on how low their have to reduce

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[1] Ibn Kahaldun wrote in the 14th century that: at the beginning of the dynasty, taxation yields a large revenue from small assessments. At the end of the dynasty, taxation yields a small revenue from large assessments (Giertz, 2008).

[2] Financing after bailout is costly because increased taxation reduces the non-financial sector's incentives to invest (Acharya, Drechsler, & Schnabl, 2014).

the tax rates. Such policy, as an incentive may encourage production and therefore economic development.
This is the framework in which this study fits. The objective of this paper is to develop and formalise the Shadow effect theory. Then, the study will provide policy maker with an effective and reliable tool for deriving tax policy considering tax evasion.
As recounted by Mirowski (1982), when Laffer was called before US Congress to testify on the then- proposed Kemp-Roth tax cut. Senator Packwood questioned Laffer point-blank on his method of empirically determining the peak of the Laffer Curve; Laffer answers: "I cannot measure it frankly, but I can describe to you what the characteristics of it are (…)".
This study aims to measure shadow effect propensity in addition to describing its characteristics.

1.2 Background of the study

There is a plethora of work on the taxation. These works cover many aspects; they are related to the nature of taxation as well as to its purpose. In this regard, Ramsey's contribution has had a significant impact on tax theory as well as other fields such as public goods pricing and regulation. Indeed, Ramsey (1927) proposes to tax only goods and services so that goods with the most inelastic demand are the most heavily taxed. The latter explains that, when taxes focus on goods whose demand varies little with price, it is more likely that the consumer will not change his consumption behaviour too much (Ramsey, 1927). Kaldor (1965) connected taxation issue with economic development from two points of view: the point of view of incentives and the point of view of resources. According to him, improving the tax system from an incentive point of view is made thru the granting of additional concessions of various kind, with less regard to the unfavourable effects on the public revenue. On the other side, additional taxation is made at the expense of worsening its disincentive effects (Kaldor, 1965). Afterwards, Mirrlees's work shed new light on optimal taxation. Indeed, the author focused on the management of inequality at the centre of taxation by suggesting a way to formalize the planner's problem that deals explicitly with unobserved heterogeneity among taxpayers. According to him, if the planner taxes income in an attempt to tax those of high ability, individuals will be discouraged from exerting as much effort to earn that income (Mirrlees, 1971). A pioneering thought close to Laffer curve. Moreover, his thought already considers the asymmetry of information. Indeed, the optimal tax problem turns out to be a game of imperfect information between taxpayers and the social planner.
It will not be long before the concept of '' Laffer Curve '' emerges. Wanniski was one of the first economist to speak of it in a scientific paper. According to

him, all around the Laffer curve, there is this simple but powerful statement by Arthur Laffer: "There are always two tax rates that yield the same revenues.". Indeed, when the tax rate is 100 percent, all production stops in the money economy. Wanniski (1978) from Laffer curve, argues that people will not work in the money economy if all the fruits of their labours are confiscated by the government. Thus, as production ceases, there is nothing for the 100 % rate to confiscate, leading government revenues to zero. On another side, when the tax rate is zero, people can keep 100 % of what they produce in the money economy (Wanniski, 1978). There could therefore be a critical point or an equilibrium from which the tax becomes heavy. Any further increase in the tax rate from this point would result in a reduction in tax revenues. But, at political equilibrium, both governmental decision and makers and tax payers, as a group find themselves in a dilemma (Buchanan & Lee, 1982). In such a situation, Buchanan & Lee (1982) explains that both would be better off if rates could be reduced and revenues increased. But this will not be the end of the dilemma because taxpayers will not respond to the reduction in rates as they predict a return to the equilibrium rate. Therefore, government cannot increase tax revenues by moving down the long-run Laffer curve unless it can convince taxpayers that the rate cuts are permanents (Buchanan & Lee, 1982).

When Arthur Laffer plotted total tax revenue as a function of a particular tax rate, he drew an upward-sloping segment called the normal range, followed by a downward-sloping segment called the prohibitive range (Fullerton D. , 1982). From this, Fullerton (1982) indicates that tax rates on the prohibitive range in theoretical and empirical models have been caused by high tax rates, high elasticity parameters, or both.

But the Laffer Curve does not make unanimity among researchers. Indeed, into ''What's wrong with the Laffer Curve?'', Mirowski (1982) raises some of the main criticisms of the Laffer Curve. The author groups them in 4 points: the first one lays in questions about the magnitudes of elasticities of incentives that are not formally determined; the second lays into the problems of empiricism; the third one is the omission of some potentially relevant variables; the last one is the subsidiary controversy about the size of the underground economy. The most acute criticism is that the procedure of empirical attempts to formalize the Laffer curve lacks both theoretical and statistical rationale (Mirowski, 1982). For a scientific theory, it is undeniably an important issue.

But yet, critics have not reduced economists' craze for the Laffer Curve. In the same wake, Feige & McGee (1983) focus on the public finance implications of Laffer curve. They developed a simple macro-model from which it is possible to derive a Laffer curve. Their model reveals that the shape and position of Laffer curve depend upon the strength of supply side effects, the progressivity of the tax system and the size of the unobserved economy (Feige & McGee, 1983). But in a general equilibrium model with one private good, one public

good, labour and an income tax, Malcomson (1986) explains that certain widely-assumed properties of the Laffer curve do not necessarily hold. Indeed, for well-behaved functional forms it may not be continuous and may not have an interior maximum (Malcomson, 1986). But, one of the major contributions of Malcomson (1986) lies in the fact that the slope of Laffer curve depends on technology as well as on the tax elasticity of labour supply.

Stiglitz (1987) comes back on asymmetry of information and the inequality issues around taxation. According to him, the new Welfare Economics is distinguished by two features: first, it does not assume that the government has at its disposal the information required to make lump-sum redistributions and second, it identifies who is able to pay higher taxes (Stiglitz J. E., 1987). But there exist so many factors influencing tax revenue. About that, the existence of a negatively-sloped section on the tax revenue – tax rate relationship is shown to crucially depend on the nature of government expenditures (Gahvari, 1989).

According to Slemrod (1990), the optimal tax theory, also named the theory of optimal taxation is the study of designing and implementing a tax that reduces inefficiency and distortion in the market under given economic constraints. But in its current state, optimal tax theory is incomplete as a guide to action for serious issues in tax policy. It is incomplete because it has not yet come to terms with taxation as a system of coercively collecting revenues from individuals who will tend to resist (Slemrod, 1990). This contribution allows to consider wilful tax evasion. According to the latter, the differences in the ease of administering various taxes are critical determinant of appropriate tax policy. But appropriate tax policy may be reach thru tax equilibrium. About that, Guesnerie & Jerison (1991) investigate the form of the tax equilibrium set in simple Diamond-Mirrlees models and characterizes the corresponding Laffer curves. They argue that the curves need not ever slope downward and can have multiple local maxima. Thus, local information about them is thus not sufficient to place restrictions on optimal choice among tax systems (Guesnerie & Jerison, 1991).

Some empirical studies exist around the taxpayers' sensitivity of taxable income to changes in tax rates. This is the case of Feldstein (1995). The latter argues that changes in marginal tax rates induce taxpayers to alter their behaviour in ways that affect taxable income and therefore tax revenue. The magnitude of this response is of critical importance in the formulation of appropriate tax and budget policies (Feldstein, 1995). Theses empirical studies also focuses on Laffer curves. This is the case of Hsing (1996) whose examines the Laffer curve for the U.S., based on time-series data during 1959–1991. His results show that the bell-shaped Laffer curve is statistically significant and that the revenue-maximizing tax rate is between 32.67% and 35.21% (Hsing, 1996). But all economists do not agree with the critical level of the maximum tax rate, maybe because of the existence of tax evasion. Indeed, in the presence

of differing abilities to evade taxes, markets select producers for their evasive skills and their abilities to keep costs of production low (Palda, 1998). According to Palda (1998), when the least efficient firms are the best tax evaders, adverse selection is severe and output comes entirely from the high cost end of the supply curve. Inefficiency can have different causes. About that, Goolsbee, Hall, & Katz (1999) argue that basic theory suggests that high marginal rates cause an inefficiency that rises with the square of the tax rate. The greater the behavioural response, the less revenue is raised by the higher rates (Goolsbee, Hall, & Katz, 1999). In the same vein of empirical work, the elasticity of taxable income has received much attention. About that, Gruber & Saez (2002) show that the overall elasticity of taxable income is approximately 0.4, and the elasticity of real income, is much lower (Gruber & Saez, 2002).

Since income can have different sources, the same is true for tax revenues. Indeed, Mirrlees pioneer contribution paves the way for several works around optimal taxation (Salanie, 2003; Kaplow, 2008). These studies suggest that taxation of income can be based on capital, environmental, credits for low-income families, and consumption tax (Salanie, 2003). Thus, much progress has been made in this area. Progress that has shadowed the Laffer curve drowned in its criticisms.

The premises of response to critics on of the Laffer curve's lacks in theoretical and statistical rationale lies into Laffer (2004). In his paper, Laffer (2004) argues that lower tax rates change economic behaviour and stimulate growth, which causes tax revenues to exceed static estimates. Because tax cuts create an incentive to increase output, employment, and production, they help balance the budget by reducing means-tested government expenditures (Laffer, 2004). This contribution by Laffer brings some theoretical elements, but still suffers from the same empiricism and theoretical demonstration limits. Indeed, the custom is to speculate on the shape of the curve, with or without empirical elements. This is the case of Fullerton (2008). According to him, Laffer curve is a hump-shaped curve showing tax revenue as a function of the tax rate. Revenue initially increases with the tax rate but then can decrease if taxpayers reduce market labour supply and investments, switch compensation into non-taxable forms, and engage in tax evasion (Fullerton D. , 2008).

Meanwhile, the theory of the optimal tax has made significant progress; especially through the social aspect. Indeed, from the theory of optimal taxation, a tax system should be chosen in order to maximize a social welfare function subject to a set of constraints (Mankiw & Weinzierl, 2009). Here, the social planner is considered as a utilitarian; meaning that the social welfare function is based on the utilities of individuals in the society. According to Mankiw & Weinzierl (2009), that welfare function is a nonlinear function of individual utilities. Summarizing the theory of the optimal tax, Mankiw & Weinzierl (2009) highlight eight general lessons suggested by optimal tax

theory. The first one suggests that the optimal marginal tax rate schedules depend on the distribution of ability; the second is that the optimal marginal tax schedule could decline at high incomes; the third would like that a flat tax, with an universal lump-sum transfer could be close to optimal; the fourth is that optimal extent of redistribution rises with wage inequality; the fifth explains that taxes should depend on personal characteristics as well as income; the sixth lesson is that only final goods ought to be taxed, and typically they ought to be taxed uniformly; the seventh is that capital income ought to be untaxed, at least in expectation; and the last one is that, in stochastic dynamic economies, optimal tax policy requires increased complexity (Mankiw & Weinzierl, 2009).

These theoretical developments in the field of the optimal tax will be followed by empirical contributions attempting to make the Laffer curve useful. For instance, revisiting Laffer curve, Trabandt & Uhlig (2011) show that the United States can maximally increase tax revenues by 30% with labour taxes and 6% with capital taxes. They obtain 8% and 1% for the European Union. According to them, the consumption tax Laffer curve does not peak. Moreover, endogenous growth and human capital accumulation affect the taxes quantitatively (Trabandt & Uhlig, 2011). Similarly, other macroeconomic aggregates are also tied to the tax. Indeed, economic theory suggests that inconsistent tax handling of investments distorts investment decisions and drives disfavoured investments high at the expense of good investments (Simkovic, 2015). According to Simkovic (2015), differences in the tax treatment of higher education relative to other forms of investment could create an undersupply of educated labour relative to physical or financial capital. Therefore, such distortions would reduce economic growth and social welfare.

1.3 THE SHADOW EFFECT'S THEORICAL PRESENTATION

1.3.1 Theoretical assumptions and definitions

The shadow effect is the propensity of producers to inflate their fictitious expenses. They do it in order to avoid the tax burden.

The study will define the other terms as they are introduced into the theoretical developments.

The theoretical assumptions are:

A1: The study assumes that entrepreneurs (producers) are sensitive to the Shadow effect;

A2: The fictitious expenses are determined ex-post based on the real value of the tax rate. The study name those fictitious expenses "input-tax", as they are used as input in the profit declaration to tax authorities[3];
A3: The production function is a Coob-Douglas with 3 production factors and constant returns to scale;
A4: The real profit of entrepreneurs is strictly non-negative.

Indeed, the profit is as: $\pi \geq 0$; moreover, if $\pi = 0$ the entrepreneurs have no reason to create fictitious expenses. This is because the tax authorities do not tax zero profits.
The non-negative condition is also because the official corporate tax rate can't be negative: $t_r \geq 0$.
In addition to the assumptions made by Cobb and Douglas, the study stated that:
1. If either tax-input, labour or capital vanishes, then so will production.
2. The marginal productivity of tax-input is proportional to the amount of production per unit of
tax-input.

1.3.2 Model description

As mentioned above, this study considers a Coob-Douglas production function (Cobb & Douglas, 1928). In fact, in most analyses on tax issues, authors generally use a social welfare function that is a nonlinear function of individual utilities (Mankiw & Weinzierl, 2009).
Therefore, the production function without fictitious cost is as follows:

$$\mathbf{Y = A.K^{\alpha}.L^{\beta}} \qquad (1.1)$$

With $\alpha + \beta = 1$;
Y is the level of production;
K is the capital input;
L is the labour input;
A is the level of the total factor productivity;
α and β are respectively, the output elasticities of labour and capital. These values are constants and they are determined by available technology used by producer.

The study assumes that the entrepreneur inflates his fictitious expenses in order to reduce the amount of the corporate tax he will have to pay. This is

[3] Producers create fictitious costs and maintain their level of production; this practice allows them to reduce the profit declared to the tax authorities. Thus, they increase their informal income.

the "Shadow effect". There are several reasons for such a behaviour; one of them is to avoid the high tax burden.
In order to measure the propensity of producers to blow up their expenses, the study will first estimate the maximum amount of fictitious expenses that entrepreneurs can add to their real expenses. This fictitious amount is named: the input-tax ($\overline{T}$).
$\overline{T}$ is as: $TC_{fictitious} = K + L + \overline{T}$,
With $TC_{fictitious}$ the Total fictitious Costs (the amount entrepreneurs will declare to tax authorities).
This is consistent with the Mirrlees framework. The latter states that the optimal tax problem becomes a game of imperfect information between taxpayers and the social planner (Mirrlees, 1971).
Indeed, the real expenses (Total Cost) are: $TC_{real} = K + L$, with $\overline{T} > 0$.
As $\overline{T} > 0$, $\Rightarrow TC_{fictitious} > TC_{real}$; $\Rightarrow \pi_{fictitious} > \pi_{real}$
When tax burden increase, entrepreneurs do not reduce their production output (in this case, because they have the possibility to produce fraudulent financial statements), they prefer to increase their fictitious expenses. This is why in theoretical developments, (for the first economic problem) the level of production is a constant.
From this, the production function become:

$$\mathbf{Y = A.K^{\propto}.L^{\beta}.\overline{T}^{\gamma}} \qquad (1.2)$$

1.3.2.1 Ex-post determination of the input-tax

Since the input tax is determined ex-post, the entrepreneurs calculate it so as to maintain their level of production. The formula is as follows:

$$\overline{T} = t_r.\pi = t_r.(Y - TC_{real}) = t_r.(Y - K - L)$$

The value of the input-tax is computed ex-post using the value of K and L from the real production function that is (1.1).
Once the value of $\overline{T}$ is obtained, the Total Cost becomes $TC = K + L + \overline{T}$.
From this, the production function becomes as it follows: $Y = A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma}$ (1.2)

1.3.2.2 The entrepreneur's economic problem

The standard theory of optimal taxation posits that a tax system should be chosen to maximize a social welfare function subject to a set of constraints (Mankiw & Weinzierl, 2009). But this study considers to minimise the cost function of entrepreneurs.
Since total production remains constant, but not total cost, the economic problem of the producer is to minimize the costs subjected to a constant amount of production.

In order words:

$$\begin{cases} Min\ K + L + \overline{T} \\ Subject\ to \\ A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma} = Y \end{cases} \qquad (1.3)$$

The Lagrangian is written as follows:

$$\mathcal{L}(K, L, \overline{T}, \lambda) = K + L + \overline{T} - \lambda(A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma} - Y) \qquad (1.4)$$

The first-order conditions are written as follow:

a) $\frac{\partial \mathcal{L}}{\partial K} = 0 \Rightarrow 1 = \lambda.\alpha.A.K^{\alpha-1}.L^{\beta}.\overline{T}^{\gamma}$

b) $\frac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow 1 = \lambda.\beta.A.K^{\alpha}.L^{\beta-1}.\overline{T}^{\gamma}$

c) $\frac{\partial \mathcal{L}}{\partial \overline{T}} = 0 \Rightarrow 1 = \lambda.\gamma.A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma-1}$

d) $\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow Y = A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma}$

The resolution of the above system of equations will give the values of K and L which enable to keep the level of production constant while inflating the input-cost costs at $\overline{T}$.

$\frac{(a)}{(b)}$: $\Rightarrow \frac{\alpha}{\beta}.K^{\alpha-1-\alpha}.L^{\beta-\beta+1} = 1 \Rightarrow L = \frac{\beta}{\alpha}.K$

$\frac{(a)}{(c)}$: $\Rightarrow \frac{\alpha}{\gamma}.K^{\alpha-1-\alpha}.\overline{T}^{\gamma-\gamma+1} = 1 \Rightarrow K = \frac{\beta}{\alpha}.\overline{T}$

$\frac{(a)}{(c)}$: $\Rightarrow \frac{\beta}{\alpha}.L^{\beta-1-\beta}.\overline{T}^{\gamma-\gamma+1} = 1 \Rightarrow L = \frac{\beta}{\gamma}.\overline{T}$

By replacing K and L by their values from $\frac{(a)}{(c)}$ and $\frac{(a)}{(c)}$ into d), we have:

d) $\quad Y = A.(\frac{\beta}{\alpha}.\overline{T})^{\alpha}.(\frac{\beta}{\gamma}.\overline{T})^{\beta}.\overline{T}^{\gamma}$

The logarithmic transformation thru the natural logarithm gives:

(d) $\Rightarrow \ ln\frac{Y}{A} = \alpha.ln\frac{\alpha}{\gamma} + \alpha.ln\overline{T} + \beta.ln\frac{\beta}{\gamma} + \beta.ln\overline{T} + \gamma.ln\overline{T}$

$$\Rightarrow \qquad \overline{\mathbf{T}} = \frac{\mathbf{Y}}{\mathbf{A}}.\frac{\boldsymbol{\gamma}^{\boldsymbol{\alpha}+\boldsymbol{\beta}}}{\boldsymbol{\alpha}^{\boldsymbol{\alpha}}.\boldsymbol{\beta}^{\boldsymbol{\beta}}} \qquad (1.5)$$

(1.5) gives the value of the optimal tax-input. The value of K and L are the following:

$\frac{(a)}{(c)}$: $\Rightarrow \qquad K = \frac{\beta}{\alpha}.\frac{Y}{A}.\frac{\gamma^{\alpha+\beta}}{\alpha^{\alpha}.\beta^{\beta}} \qquad \Rightarrow \qquad K = \frac{Y}{A}.\frac{\gamma^{\alpha+\beta-1}}{\alpha^{\alpha}.\beta^{\beta-1}}$

$\frac{(a)}{(c)}$: $\Rightarrow \qquad L = \frac{\beta}{\gamma}.\frac{Y}{A}.\frac{\gamma^{\alpha+\beta}}{\alpha^{\alpha}.\beta^{\beta}} \qquad \Rightarrow \qquad L = \frac{Y}{A}.\frac{\gamma^{\alpha+\beta-1}}{\alpha^{\alpha}.\beta^{\beta-1}}$

1.3.2.3 The Shadow effect propensity

As shown above, (1.5) gives the amount of the optimal input-tax that allows the producer to reduce the taxable profit.

The study focuses on the value of γ: the responsiveness of the output to a change in levels of the input-tax factor. From the point of view of tax officials, γ is interpreted as the propensity of entrepreneurs to Shadow effect.

Therefore, the objective of the tax authorities (or government) is to bring γ closer to 0. This is opposite to the producer's goal the tax burden increase. This statement means that there is a positive link between the propensity of entrepreneurs to Shadow effect (γ) and the level of tax rate (t_r).

From the value of the input-tax into (1.5), the value of γ can be derived:

(1.5) $\Rightarrow \overline{T} = \frac{Y}{A}.\frac{\gamma^{\alpha+\beta}}{\alpha^{\alpha}.\beta^{\beta}}$; $\Rightarrow \gamma = \exp\left[\frac{1}{\alpha+\beta}\ln\left(\frac{\alpha^{\alpha}.\beta^{\beta}}{Y}A.\overline{T}\right)\right]$

(1.6)

From there, the study can express the tax rate (t_r) as a function of the shadow effect propensity (γ).

Indeed, $\overline{T} = t_r.\pi$ and $\pi = Y - TC = Y - K - L$

$\Rightarrow \quad \overline{T} = t_r.\pi = t_r.(Y - K - L)$

(1.6) becomes:

$$\gamma = \exp\left[\frac{1}{\alpha+\beta}\ln\left(\frac{\alpha^{\alpha}.\beta^{\beta}}{Y}A.t_r.(Y - K - L)\right]\right.$$

$$\Rightarrow \quad \ln t_r = (\alpha+\beta).\ln\gamma - \ln\left[\left(1 - \frac{K+L}{Y}\right)\alpha^{\alpha}.\beta^{\beta}.A\right]$$

$\Rightarrow$

The tax rate is as:

$$t_r(\gamma) = \frac{\gamma^{\alpha+\beta}}{\left(1-\frac{K+L}{Y}\right)\alpha^{\alpha}.\beta^{\beta}.A} \qquad (1.7)$$

With $0 \leq t_r(\gamma) \leq 1$;

When the tax function reaches his minimum, $t_r(\gamma = 0) = 0$; This is consistent with the study's assumptions.

Proof:

According to the above function, the only way to get γ to equal zero is to set t_r to zero. Indeed, the function $t_r(\gamma)$ exists if and only if: $\left(1 - \frac{K+L}{Y}\right)\alpha^{\alpha}.\beta^{\beta}.A \neq 0$

Since $\alpha^{\alpha}.\beta^{\beta}.A \neq 0$; $\Rightarrow 1 \neq \frac{K+L}{Y}$;

Indeed, $\alpha + \beta \neq 0$ because $\alpha + \beta = 1 - \gamma$ (due to the constant return to scale assumption -A3-).

Now, $K + L = TC$ cannot be greater than Y because of assumption A4. Indeed, A4 states that the real profit of entrepreneurs is strictly non-negative. Meaning that $\frac{K+L}{Y} \geq 1$.

But, since $1 \neq \frac{K+L}{Y}$,

$t_r(\gamma)$ exists if and only if $\frac{K+L}{Y} > 1$.

Meaning that the only way to bring γ to zero is to set t_r to zero.

The relationship between the shadow effect propensity and the tax rate can also be presented in the other direction. That is the expression of γ as a function of t_r.

From (1.7), $\gamma^{\alpha+\beta} = \left(1 - \frac{K+L}{Y}\right)\alpha^{\alpha}.\beta^{\beta}.A.t_r$

$\Rightarrow \quad \ln\gamma = \frac{1}{\alpha+\beta}.\ln\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}.t_r\right]$

$\Rightarrow$

$$\boldsymbol{\gamma(t_r) = \left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}.t_r^{\frac{1}{\alpha+\beta}}} \qquad (1.8)$$

This function (1.8) summarizes the relationship between the shadow effect and the tax rate. This function summarizes the relationship between the shadow effect and the tax rate. From here, raises the first instrument to influence entrepreneurs' propensity to shadow effect. This is the tax rate.

1.3.2.4 Slope of the curve and some properties

The slope of γ enables to represent the graph of the function and determine the elasticities during variations.

- The extremums of the curve are:

$\gamma(t_r = 0) = 0$;

$\gamma(t_r = 1) = \left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}$;

$\Rightarrow \gamma(t_r = 1) \neq 1$ if $\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}} \neq 1$

$\Rightarrow \gamma(t_r = 1) = 1$ if $\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}} = 1$

The curve on the graph below in shape (a) gives the shape of the function for $\gamma(t_r = 1) = 1$. In such a situation:

$ln\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}} = ln1 = 0$; $\Rightarrow \frac{1}{\alpha+\beta}\ln\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right] = 0$;

$\Rightarrow \left(1 - \frac{K+L}{Y}\right) = \frac{1}{\alpha^{\alpha}.\beta^{\beta}.A}$.

- The slope of the curve is: $\frac{\partial\gamma(t_r)}{\partial t_r} = \frac{\partial\gamma}{\partial t_r}\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}.t_r^{\frac{1}{\alpha+\beta}}$

$\Rightarrow \quad \frac{\partial\gamma(t_r)}{\partial t_r} = \frac{1}{\alpha+\beta}.\left[A.\left(1 - \frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}.t_r^{\frac{1-\alpha-\beta}{\alpha+\beta}}$;

$\Rightarrow \quad \frac{\partial\gamma(t_r)}{\partial t_r} > 0$, meaning that the curve has a positive slope.

- From $t_r^{\frac{1}{\alpha+\beta}}$, as $\frac{1}{\alpha+\beta} \neq 1$, the curve is non-linear.
- Another property of the function is already given above as: $\gamma(t_r)$ exists if and only if $\frac{K+L}{Y} > 1$. Meaning that the only way to bring γ to zero is to set t_r to zero.

The curve of shadow effect propensity as a function of tax rate will therefore have the following graphical representation:

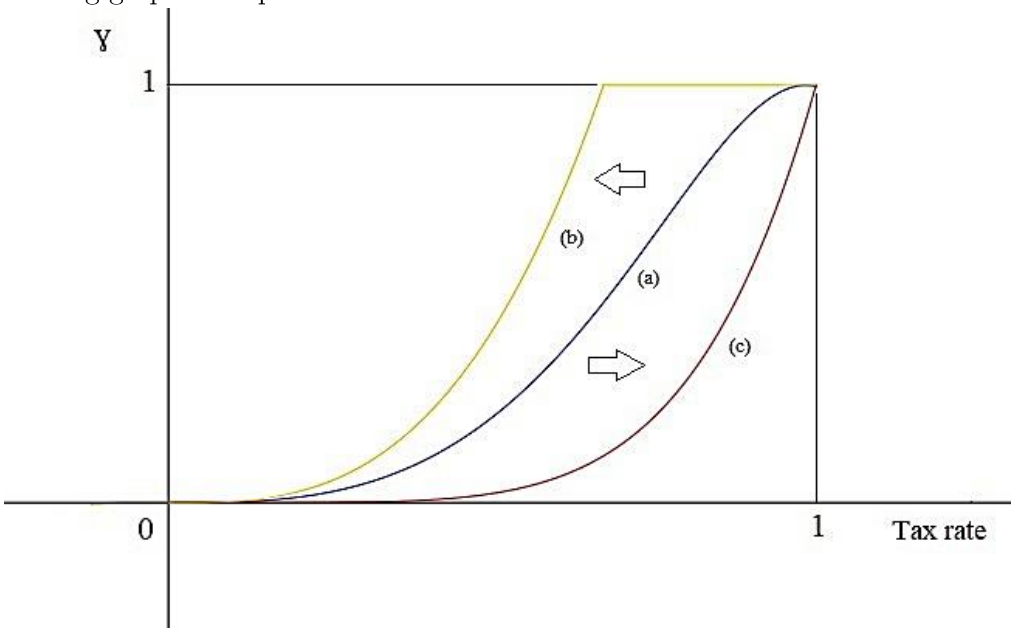

Figure 2:Shadow effect propensity as a function of tax rate

Source: Author

All along the line of each curve, we have the different combinations of γ and t_r. There is a positive but non-linear relationship between the entrepreneur propensity to shadow effect and the level of tax rate.

The curve will shift to the left from position (a) to position (b) due, *ceteris paribus*, to a decrease into the value of the slope.

From $\frac{\partial\gamma(t_r)}{\partial t_r}$, when $\alpha + \beta$ increase, the slope tends to decrease. Therefore, the curve can shift to the right, from (a) to (c).

This is an important finding. It suggests that the technology used to produce goods has a negative influence on the producer's sensitivity to shadow effect (the slope of the curve).

Indeed,

- $\lim_{\alpha+\beta\to 1}\left[\frac{\partial\gamma(t_r)}{\partial t_r}\right] = A.\left(1 - \frac{K+L}{Y}\right) > 0$;
- $\lim_{\alpha+\beta\to 0}\left[\frac{\partial\gamma(t_r)}{\partial t_r}\right] = 0$.

1.3.3 The use of tax rate leverage on shadow effect propensity

From the objective of this paper, the study aims to provide tools to reduce the shadow effect. The first instrument the study proposes is the corporate tax rate. Theoretical development suggests that an increase in the level of tax rate will lead to an increase in the producers' propensity to shadow effect. But with different proportions.

As shown above, the slope of $\gamma(t_r)$ is not constant and depends on α and β. That slope is considered as the sensitivity of producers to the shadow effect when there is a change in the tax rate level. Meaning that the curve of $\gamma(t_r)$ can have a lot of shapes.

Indeed, when the shape of the curve (slope of the curve that also is the sensitivity of producers to the shadow effect) is known, government can move along the $\gamma(t_r)$ curve by changing the value of the tax rate.

According to the producer's sensitivity to shadow effect (the slope of the curve), the curve can take the following shapes:

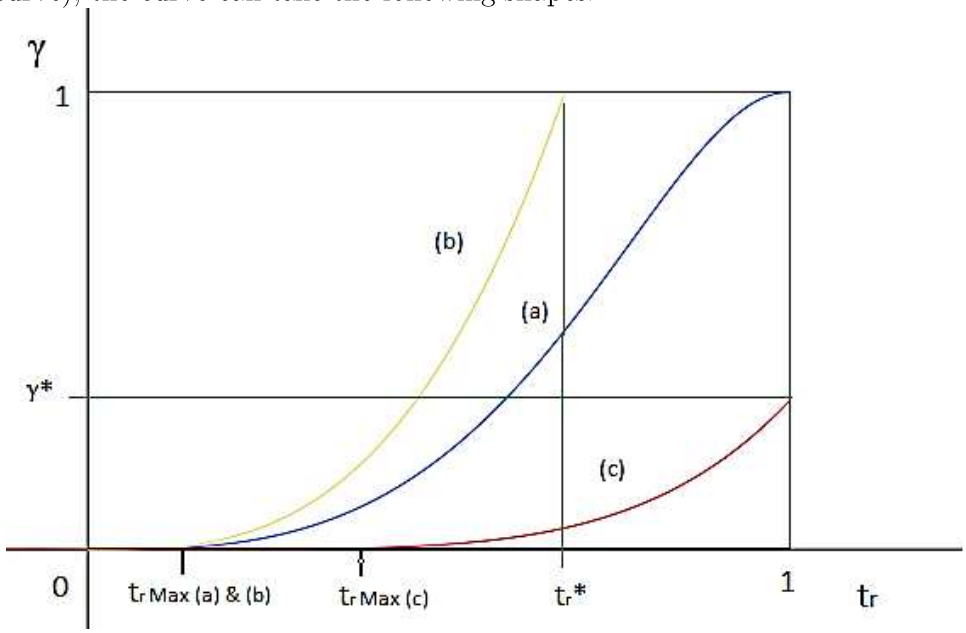

Figure 3: Movements of the curve due to changes in shadow effect sensitivity

Source: Author

Let's consider the 3 following shapes of the curve:

- Entrepreneurs shadow propensity is highly sensitives to tax burden (b). This situation is closed to the case of developing countries, informal enterprises, very small and small businesses;
- Entrepreneurs shadow propensity is proportionally sensitive to tax burden (a). This situation is closed to the case of emerging countries, medium enterprises;
- Entrepreneurs shadow propensity is poorly sensitives to tax burden (c). This situation is closed to the case of developed countries, large enterprises.

Explanations:

The same increase in the level of tax rate will lead to a higher increase in the producer's propensity to shadow effect in (b) and to a lower increase in (c). This means that government in situation (c) can have a maximum optimal tax rate ($t_{r\ Max}$) greater than the one of government in situation (b). The maximum

optimal tax rate is the maximum tax rate above with producers start creating fictitious expenses.

When producers are highly sensitives to shadow effect (b), up to a certain level of tax burden (t_r*), they declare zero profit because $\gamma(t_r) = 1$. Because the burden of the tax is too high for them, they increase their fictitious expenses to their maximum level. In such a situation (t_r*), any additional increase in tax rate will result to the same level of shadow effect propensity: $\gamma(t_r) = 1$. An explanation is due the fact that entrepreneurship is subsistence entrepreneurship (informal entrepreneurship in sub-Saharan countries for instance). In such a situation there is not industrial production and small business doesn't have legal form, but they pay their tax to the town hall. Another explanation leads in the fact that institutions are weak (high level of corruption for example).

When producers are poorly sensitive to shadow effect (c), it may happen that they never reach the maximum level of shadow effect. This can be due because of their scale or the technology they use. Another explanation can be the strength of institutions (tax authorities, credits registries, shareholders, stock markets, etc.) that constraint the producers to provide real information about the health of the company.

1.3.4 Reduce tax rate to improve tax revenue

The study still considers that entrepreneurs inflate their fictitious expenses in order to avoid tax burden. Above, the study has shown that shadow effect propensity increases with tax rate. This link enables to build up another useful tool: the level of production.

Indeed, the government's main objective, instead of reducing shadow effect can be: increase the country's Gross domestic products. In such a situation, government will put in place policies in order to increase the country's production, therefore, the entrepreneurs' output. One of these policies can be the reduction of corporate tax rate. Once entrepreneurs' production will increase, the government tax revenue will also increase. Indeed, the real profit of entrepreneurs will increase because tax burden has decreased (shadow effect).

The economic problem will therefore be stated as (Mankiw & Weinzierl, 2009):

$$\begin{cases} Max\ \mathrm{A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma}} \\ \quad Subject\ to \\ \mathrm{K + L + \overline{T} = TC} \end{cases} \qquad (2.1)$$

The Lagrangian is written as follows:

$$\mathcal{L}(\mathrm{K, L, \overline{T}}) = \mathrm{A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma}} - \lambda(\mathrm{K + L + \overline{T} - TC}) \qquad (2.2)$$

The first-order conditions are written as follow:

a) $\frac{\partial \mathcal{L}}{\partial K} = 0 \Rightarrow \alpha.\mathrm{A.K^{\alpha-1}.L^{\beta}.\overline{T}^{\gamma}} = \lambda$

b) $\frac{\partial \mathcal{L}}{\partial L} = 0 \Rightarrow \beta.A.K^{\alpha}.L^{\beta-1}.\overline{T}^{\gamma} = \lambda$

c) $\frac{\partial \mathcal{L}}{\partial \overline{T}} = 0 \Rightarrow \gamma.A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma-1} = \lambda$

d) $\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \Rightarrow K + L + \overline{T} = TC$

By solving the above system of equations, the following results are found:

- $\overline{T} = TC.\gamma$ (2.3)
- $K = TC.\alpha$ (2.4)
- $L = TC.\beta$ (2.5)

In this situation, the study considers that government doesn't consider the reduction of shadow effect as a priority, but the increase of tax revenues. Therefore, the propensity of shadow effect will be influenced indirectly from government policies.

(2.3) gives: $\overline{T} = TC.\gamma \quad \Rightarrow (Y - TC).t_r = TC.\gamma$

$\Rightarrow Y = \frac{TC.\gamma}{t_r} + TC$

$$\Rightarrow \quad \mathbf{Y(t_r) = (1+\gamma).TC.t_r^{-1}} \qquad (2.6)$$

This suggest that there is an inverse relationship between the level of production and the tax rate.

Indeed, the slope of the curve of $Y(t_r)$ is negative:

$$\frac{\partial Y}{\partial t_r} = -\gamma.TC.t_r^{-2} < 0 \qquad (2.7)$$

Since $\frac{\partial^2 Y}{\partial t_r} = 2\gamma.TC.t_r^{-3}$ and $Y(t_r)$ is non-linear, the shape of the curve is as:

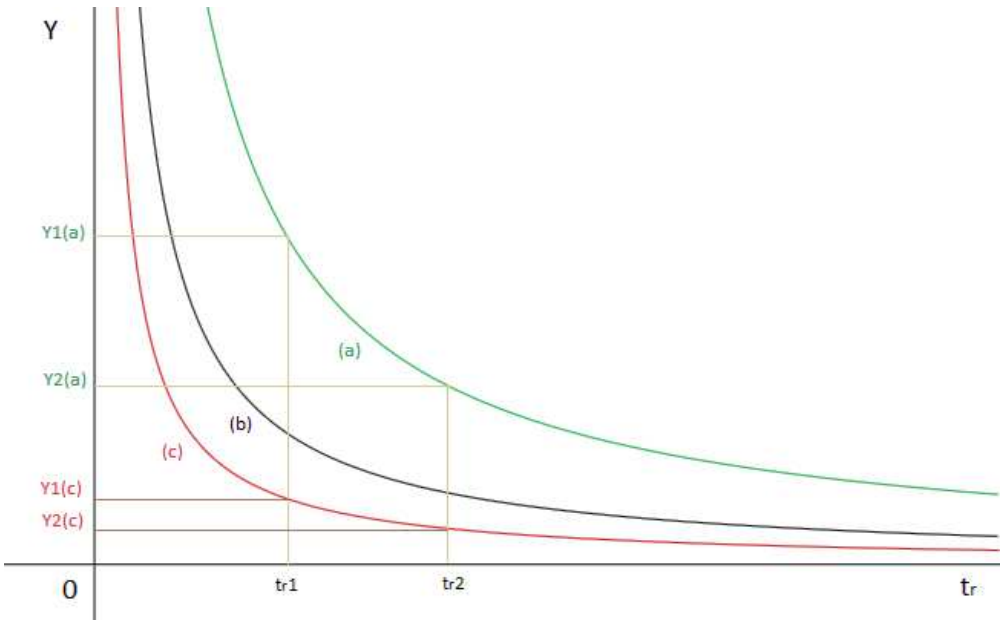

Figure 4: Changes in production due to tax rate changes

Source: Author

The above graph shows that from the same variation in tax rate (from t_r2 to t_r1), the variation in production changes depending on the curve slope and shape.

Indeed, the shape of the curve in (a) is explained by a higher level of γ. When γ increases, the curve shifts to the right; and when γ decreases, the curve shifts to the left (c). Meaning that γ in (a) is greater than γ in (c).
A higher level of shadow propensity leads to higher change in production when the tax rate changes. Indeed, from the graph, the same reduction in tax rate (from t_r2 to t_r1), leads to a higher level of increase in production (From $Y2(a)$ to $Y1(a)$ in (a) than (From $Y2(c)$ to $Y1(c)$ in (c).
From (2.7), the formula for computing the value of the change is the following:

$$\Delta \mathbf{t_r} = -\frac{\mathbf{t_r}^{-2}}{\gamma.\mathbf{TC}}.\Delta \mathbf{Y}(\mathbf{t_r}) \qquad (2.8)$$

This formula helps government to determine from how low their have to reduce their tax rate in order to increase their production.
For instance, if the government wants to increase production by 50%, it must reduce the level of taxation by: $\Delta t_r = -\frac{t_r^{-2}}{\gamma.TC}.50\%$.

1.4 Numerical application: the case of Cameroon

Numerical application of the above theoretical developments is made choosing the Cameroon's case. Cameroon is amongst the Sub-Saharan African countries with the higher tax burden levels. About that, the Paying Taxes 2018 report from PricewaterhouseCoopers (PwC) ranks the Cameroonian tax system 183rd at the world level. Indeed, from World Bank enterprise survey 2016, data shows that 76.45% of Cameroonian entrepreneurs point out tax rate as an obstacle for the growth of their activities. Moreover, 36.56% of those entrepreneurs even describes tax rate as a major and very severe obstacle. A worrying situation for the government. Besides, tax reforms (Institution of a single tax interlocutor, modernization of the Fiscal Investigations Brigade) in 2004 has contributed to the improvement of domestic tax revenues. Indeed, tax revenues from corporate tax rose to 1.6% of GDP in 2010 compared to 0.6% of GDP in 1995 (Assobo, 2011). But confronted to the needs of the state, this tax revenue increase is not enough. This is one of the reasons why the government has set itself the next challenges: to improve the level of tax revenues; and above all, to consider the possibility of lowering the corporate tax rate (Assobo, 2011).

1.4.1 Data and determination of input-tax

The data used in the numerical application come from the world bank enterprise survey, last update 2016. The database provides information and opinions about the business environment in Cameroon.
The study uses the following variables from the database:

- n5a: Total annual expenditure for purchases of equipment in last fiscal year, the variable is labelled K;
- n2a: Total labour cost (including wages, salaries, bonuses, etc) in last fiscal year, the variable is labelled L;
- d2: Establishment's total annual sales in last fiscal year, the variable is labelled Y.

In order to estimate the Cobb-Douglas production function, the study uses the natural logarithm of the data. The description of the available and non-negative data is presented in the table below:

Table 1:Data description of Cameroonian producers

Variable	**Obs.**	**Mean**	**Std. Dev.**	**Min.**	**Max.**
lnY	144	17.83422	2.452546	13.12236	26.02158
lnK	144	15.01748	2.361146	9.903488	22.10956
lnL	144	15.7905	2.195856	11.69525	23.719

Source: Author

The sample for 2016 Cameroon Enterprise Survey is selected using stratified random sampling to obtain unbiased estimates for the whole population. Despite that the sampling consist of 363 firms, including small, medium and large firms, only 144 observation were complete in the database for the study selected variables.
From this database, the study derived the value of the input-tax as:

$$\overline{T} = t_r.\pi = t_r.(Y - TC_{real}) = t_r.(Y - K - L) = 35\%.(Y - K - L)$$

Indeed, the official corporate tax rate in Cameroon is 35%.
Using the ex-post value of the input-tax, the study can proceed to the estimation of the Cobb-Douglas production function of Cameroonian entrepreneurs.

1.4.2 Production function estimation

The second step is estimating the production function. The purpose is to find the elasticities of production factors. From the same above sample, the study uses Bootstrapped[4] Ordinary Least Square to estimate the output that is total sales. The results are presented in the table below:

[4] The bootstrapping consists of 10000 replications.

Table 2: OLS estimation result for production function

lnY	Observed Coef.	Bootstrap Std. Err.	z	P>z
lnK	0.1418157***	0.0232532	6.10	0.000
lnL	0.1564977***	0.0314389	4.98	0.000
lnT	0.719882***	0.0360549	19.97	0.000
_cons.	1.482066***	0.1327167	11.17	0.000

Number of Obs. =	144
Replications =	10000
Wald chi2(3) =	15190.58
Prob > chi2 =	0.0000
R-squared =	0.9902
Adj R-squared =	0.9900
Root MSE =	0.2452

Source: Author from Stata. With *** representing significance at 1% level.

From the above table, the study derives the Cameroonian entrepreneur's production function as:

$$\widehat{Y} = (4.40).\widehat{K}^{0.14}.\widehat{L}^{0.15}.\widehat{\overline{T}}^{0.71}$$

This is a Cobb-Douglas with constant returns to scale. Indeed, $\alpha + \beta + \gamma = 1$. And, all the coefficient values are significant at 1% level of significance. The exponential value of the constant coefficient is replaced into the Cobb-Douglas as the value of $A = 4.40$.

The next step is to determine the shadow effect propensity.

1.4.3 Shadow effect propensity derivation

As presented above, the economic problem of producer facing tax burden is to reduce its total costs subjected to its production. The problem to optimise can be stated as:

$$\begin{cases} Min\ K + L + \overline{T} \\ \quad Subject\ to \\ A.K^{\alpha}.L^{\beta}.\overline{T}^{\gamma} = Y \end{cases} \qquad (1.3)$$

The Cameroonian entrepreneurs' economic problem can be stated as:

$$\begin{cases} Min\ K + L + \overline{T} \\ Subject\ to \\ (4.40).\widehat{K}^{0.14}.\widehat{L}^{0.15}.\widehat{\overline{T}}^{0.71} = \widehat{Y} \end{cases}$$

The Lagrangean is written as:

$$\mathcal{L}(K, L, \overline{T}, \lambda) = K + L + \overline{T} - \lambda(4.40).\widehat{K}^{0.14}.\widehat{L}^{0.15}.\widehat{\overline{T}}^{0.71} = \widehat{Y})$$

In a concern for accuracy, the value of $\widehat{Y}$ is the value of the efficient producer. Therefore, efficiency scores are determined using DEA (Data Envelopment analysis). The DEA is output oriented and the study compute technical efficiency scores of Cameroonians producers.

When technical efficiency score is equal to zero, it means that producer (i) is not technical efficient. On the other hand, when technical efficiency score is equal to one, it means that firm (i) has reached the maximum technical efficiency (Coelli, 1996).

The DEA model is the following:

$$\begin{cases} Max\left(\dfrac{\beta_1.Y_i}{\sum(\alpha_1.K_i+\alpha_2.L_i+\alpha_3.\overline{T}_i)}\right) \\ Subject\ to: \dfrac{\beta_1.Y_i}{\sum(\alpha_1.K_i+\alpha_2.L_i+\alpha_3.\overline{T}_i)} \leq 1: \end{cases}$$

With $i \in [1;144]$ for the Cameroonian producers from World Bank Enterprise Survey with complete data availability into the database;

With $\sum_{k=1}^{K}\alpha_k = 1$: input weight and $\sum_{k=1}^{K}\beta_k = 1$: output weigh;

The results are presented in the annex[i].

From DEA results, the average efficient dmu (decision maker unit), the shadow propensity function is as follows:

$$\boldsymbol{\gamma(t_r) = \left[A.\left(1-\frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}.t_r^{\frac{1}{\alpha+\beta}} = 21,68.t_r^{3,45}}$$

The shadow effect propensity functions for the efficient Cameroonian entrepreneurs are presented into the following table:

Table 3: Shadow effect propensity functions for the efficient Cameroonian entrepreneurs

DMU	DMU Label	$\left[A.\left(1-\frac{K+L}{Y}\right).\alpha^{\alpha}.\beta^{\beta}\right]^{\frac{1}{\alpha+\beta}}$	$\gamma(t_r)$
339	A	11,8358908	$11,84.t_r^{3,45}$
189	B	12,7985868	$12,80.t_r^{3,45}$
319	C	16,3406329	$16,34.t_r^{3,45}$
256	D	12,2664263	$12,27.t_r^{3,45}$
238	E	13,6977065	$13,70.t_r^{3,45}$
17	F	16,4748069	$16,47.t_r^{3,45}$
114	G	20,9565672	$20,96.t_r^{3,45}$
346	H	17,9267581	$17,93.t_r^{3,45}$
314	I	18,8686879	$18,87.t_r^{3,45}$
Average DMU		21,6776137	$21,68.t_r^{3,45}$

Source: Author from Stata iterations

The study considers the average DMU data of Cameroonian producers in plotting the shadow effect propensity function.

The shape of the curve is shown in the figure below:

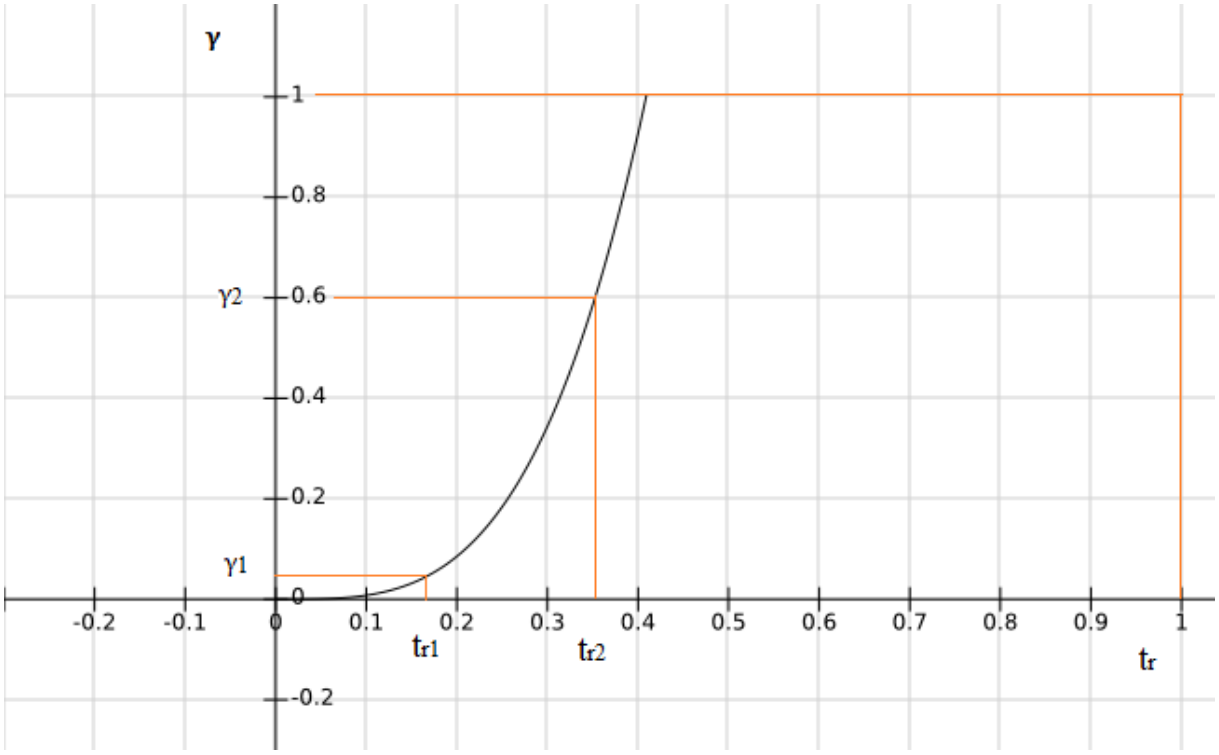

Figure 5:Shape of Shadow effect propensity for Cameroonian producers

Source: Author

The average DMU situation is closed to the overall Cameroonian producers' situation from the estimations. Indeed, the official corporate tax rate in Cameroon is 35%. That could be t_{r2} in the graph leading to γ_2.

The above graph shows that a reduction in corporate tax rate from 35% to approximatively 16% brings producers shadow effect propensity from approximatively 60% to approximatively 4%. The graph also shows that up to approximatively 41% of tax rate burden, Cameroonian entrepreneurs reaches they maximum shadow effect propensity.

1.4.4 Optimal tax rate reduction in Cameroon

The instrument built in this study enables to determine the optimal tax rate according to the needs of the government.

From (2.8), the necessary tax rate change is: $\mathbf{\Delta t_r = -\frac{t_r^{-2}}{\gamma.TC}.\Delta Y(t_r)}$

As shown above, this formula helps government to determine from how low their have to reduce their tax rate in order to increase their production. In the case of Cameroon, the considers a positive change of 50% in production. Indeed, "If SMEs were contributing to 50% of GDP, we would already be an emerging country." This statement is from Etoundi Ngoa, the Cameroonian Minister for Small and Medium-sized enterprises.

The table below gives the optimal reductions in corporate tax rate considering a 50% increase in efficient Cameroonian producers' outputs.

Table 4: Optimal reduction in corporate tax rate for Cameroonian producers

DMU	**DMU Label**	**Δt_r**	**New t_r**
339	A	-23,16	11,84%
189	B	-22,20	12,80%
319	C	-18,66	16,34%
256	D	-22,73	12,27%
238	E	-21,30	13,70%
17	F	-18,53	16,47%
114	G	-14,04	20,96%
346	H	-17,07	17,93%
314	I	-16,13	18,87%
Average DMU		-18,81	16,19%

Source: Author.

The results show that, considering shadow effect and, in order to reach 50% more in term of entrepreneur's production, the Cameroonian government should reduce the corporate tax rate from 35% to 16.19%. The study considers the value of the average DMU.

1.5 Conclusion

This study develops a new concept in order to explain the behaviour of entrepreneurs that is to create fictitious expenses in order to avoid tax burden. The study names that concept the: '' Shadow effect ''. The idea behind the concept is that, when the tax burden increases, entrepreneurs falsify their financial statements in order to reduce the tax base and maintain their profits. The idea is closed to the concept of Laffer Curve. But Laffer curves supposes that when tax burden is high, people reduce their work supply or increase it in order to maintain their living condition. This study adds another alternative linked to tax evasion: the shadow effect. In that alternative, people keep their real level of production constant, but inflate their expenses. In order words, their increase their informal activities in order to evade tax. Therefore, the objective of the paper is to provide to policy makers a tool to reduce the shadow effect. A tool to drive tax policy considering tax evasion. The instrument is based on the usage of corporate tax rate.

In order to measure the propensity of producers to inflate their expenses, the study first estimate the maximum amount of fictitious expenses that entrepreneurs can add to their real expenses. This fictitious amount is named: the input-tax ($\overline{T}$). The input-tax is determined ex-post of production. This is

due to the fact that the study supposes that producers inflate their expenses based on their real level of production.
The study made 4 theoretical assumptions:
A1: The study assumes that entrepreneurs (producers) are sensitive to the Shadow effect;
A2: The fictitious expenses are determined ex-post based on the real value of the tax rate;
A3: The production function is a Coob-Douglas with 3 production factors and constant returns to scale;
A4: The real profit of entrepreneurs is strictly non-negative.
From this, producer's economic problem is to reduce his expenses subjected to a constant level of production. A Lagrangian is derived from the producer economic problem and solved.
Working into this framework, the theoretical developments have shown that there is a positive non-linear relationship between entrepreneurs' sensitivity to the shadow effect and the evolution of the tax burden. Thus, an increase in the level of tax rate will lead to an increase in the producers' propensity to shadow effect. But with different proportions. Three interesting cases are highlighted:

- When entrepreneurs' shadow propensity is highly sensitives to tax burden (depending on the slope of the curve). This situation is closed to the case of developing countries, informal enterprises, very small and small businesses;
- When entrepreneurs' shadow propensity is proportionally sensitive to tax burden (depending on the slope of the curve). This situation is closed to the case of emerging countries, medium enterprises;
- When entrepreneurs' shadow propensity is poorly sensitives to tax burden (depending on the slope of the curve). This situation is closed to the case of developed countries, large enterprises.

Indeed, when producers are highly sensitives to shadow effect, up to a certain level of tax burden (tr*), they declare zero profit because the shadow effect propensity is $\gamma(t_r) = 1$. Because the burden of the tax is too high for them, they increase their fictitious expenses to their maximum level. In such a situation (tr*), any additional increase in tax rate will result to the same level of shadow effect propensity: $\gamma(t_r) = 1$.
In the other hand, when producers are poorly sensitive to shadow effect, it may happen that they never reach the maximum level of shadow effect. This can be due because of their scale or the technology they use. Another explanation can be the strength of institutions (tax authorities, credit bureau registries, shareholders, stock markets, etc.) that constraint the producers to provide real information about the health of the company.

The study also provides a simple instrument to government based on the

$$\Delta \mathbf{t_r} = -\frac{\mathbf{t_r}^{-2}}{\gamma.\mathbf{TC}}.\Delta \mathbf{Y}(\mathbf{t_r})$$

slope of the curve.

The above formula helps government to determine from how low their need to reduce their tax rate in order to increase their GDP. Indeed, when the government needs to spur economic growth while avoiding tax evasion (thru shadow effect), the above formula provides the needed change in tax burden. This tool is of a critical importance because if economic growth is not fairly distributed, poverty and inequality will increase.

An empirical application on the case of Cameroon (a high tax burden Sub-Saharan African country) has shown that, considering shadow effect and, in order to reach 50% more (as expected by the government) in term of entrepreneur's production, the Cameroonian government should reduce the corporate tax rate from 35% to 16.19%.

References

Acharya, V., Drechsler, I., & Schnabl, P. (2014). A pyrrhic victory? Bank bailouts and sovereign credit risk. *The Journal of Finance, 69*(6), 2689-2739.

Assobo, A. B. (2011). La réforme de l'administration fiscale au Cameroun. *Conférence sur la mobilisation des recettes fiscales dans les pays en développement.* Washington.

Best, M. (1976). Political power and tax revenues in Central America. *Journal of Development Economics, 3*(1), 49-82.

Buchanan, J. M., & Lee, D. R. (1982). Politics, Time, and the Laffer Curve. *Journal of Political Economy, 90*(4), 816-819.

Burgess, R., & Stern, N. (1993). Taxation and development. *Journal of Economic Literature*, 762-830.

Cobb, C. W., & Douglas, P. H. (1928). A Theory of Production. *The American Economic Review, 18*(1), 139-165.

Coelli, T. (1996). A Guide to DEAP Version 2.1: A Data Envelopment Analysis(Computer) Program. *CEPA Working Papers.*

Dhami, S., & al-Nowaihi, A. (2006). A simple model of optimal tax systems: taxation, measurement and uncertainty. *Manchester School, 74*(6), 645-669.

Fauvelle-Aymar, C. (1999). The political and tax capacity of government in developing countries. *International Review for Social Sciences*, 391-413.

Feige, E. L., & McGee, R. T. (1983). Sweden's Laffer Curve: Taxation and the Unobserved Economy. *Working Paper Series 95. Research Institute of Industrial Economics.*

Feldstein, M. (1995). The effect of marginal tax rates on taxable income: A panel study of the 1986 Tax Reform Act. *Journal of Political Economy*, 551–572.

Friedman, R., & Friedman, M. (1998). *Two Lucky People: Memoirs,*. University of Chicago Press.

Fullerton, D. (1982). On the possibility of an inverse relationship between tax rates and government revenues. *Journal of Public Economics*, 3–22.

Fullerton, D. (2008). Laffer Curve. *Palgrave Macmillan (eds) The New Palgrave Dictionary of Economics.*

Gahvari, F. (1989). The nature of government expenditures and the shape of the Laffer curve. *Journal of Public Economics, 40*(2), 251-260.

Giertz, S. A. (2008). How Does the Elasticity of Taxable Income Affect Economic Efficiency and Tax Revenues and what Implications Does this have for Tax Policy Moving Forward? *American Enterprise Institute for Public Policy Research*, 36–42.

Goolsbee, A., Hall, R. E., & Katz, L. F. (1999). Evidence on the High-Income Laffer Curve from Six Decades of Tax Reform. *Brookings Papers on Economic Activity*(2), 1-64.

Gruber, J., & Saez, E. (2002). The elasticity of taxable income: Evidence and implications. *Journal of Public Economics*, 1–32.

Guesnerie, R., & Jerison, M. (1991). Taxation as a social choice problem: The scope of the Laffer argument. *Journal of Public Economics, 44*(1), 37-63.

Hsing, Y. (1996). Estimating the Laffer Curve and Policy Implications. *Journal of Socio-Economics, 25*(3).

Kaldor, N. (1965). *The role of taxation in economic development.* Problems in Economic Development. International Economic Association Series.

Kaplow, L. (2008). The Theory of Taxation. *Princeton University Press.*

Keane, M. (2011). Labor supply and taxes: a survey. *Journal of Economic Literature, 49*(4), 961-1075.

Laffer, A. B. (2004). The Laffer Curve: Past, Present, and Future. *Backgrounder.*

Lambert, P. (1993). Inequality reduction through the income tax. *Economica, 60*(239), 357-365.

Leigh, A. (2008). Do redristributive state taxes reduce inequality? *National Tax Journal, 61*(1), 81-104.

Malcomson, J. (1986). Some analytics of the laffer curve. *Journal of Public Economics, 29*(3).

Mankiw, M., & Weinzierl, D. Y. (2009). Optimal Taxation in Theory and Practice. *The Journal of Economic Perspectives, 23*(4), 147-174.

Mirowski, P. (1982). What's Wrong with the Laffer Curve? *Journal of Economic Issues, 16*(3), 815-828.

Mirrlees, J. A. (1971). An Exploration in the Theory of Optimal Income Taxation. *Review of Economic Studies, 38*(114), 175–208.

Palda, F. (1998). Evasive Ability and the Efficiency Cost of the Underground Economy. *Canadian Journal of Economics, 31*(5), 1118–1138.

Quinn, D., & Shapiro, R. (1991). Business political power: the case of taxation. *American Political Science Review, 85*(3), 851-874.

Ramsey, F. (1927). A Contribution to the Theory of Taxation. *The Economic Journal, 37*(145), 47-61.

Sahlins, M. (1976). *Âge de pierre, âge d'abondance : Économie des sociétés primitives.* NRF Gallimard.

Salanie, B. (2003). The Economics of Taxation. *MIT Press.*

Simkovic, M. (2015). The Knowledge Tax. *University of Chicago Law Review.*

Slemrod, J. (1990). Optimal Taxation and Optimal Tax Systems. *Journal of Economic Perspectives, 4*(1), 157-178.

Stiglitz, J. (2010). *Le Triomphe de la cupidité.* Les Liens qui Libèrent.

Stiglitz, J. E. (1987). Pareto Efficient and Optimal Taxation and the New Welfare Economics. *Handbook on Public Economics*, 991–1042.

Trabandt, M., & Uhlig, H. (2011). The Laffer curve revisited. *Journal of Monetary Economics, 58*(4), 305-327.

Wanniski, J. (1978). Taxes, Revenues, and the 'Laffer Curve. *The Public Interest.*

Annex

[i] Data and efficiency scores from the World Bank Enterprise Survey 2016, Cameroon's database

dmu	lnK	lnL	lnY	lnT	Efficiency Scores	dmu	lnK	lnL	lnY	lnT	Efficiency Scores
339	9,90	14,91	16,12	14,71	1,00	53	14,91	16,30	17,03	15,07	0,93
359	10,13	14,51	15,61	14,15	0,99	84	14,91	14,29	17,07	15,83	0,94
174	10,31	12,61	13,12	10,99	1,00	133	14,91	16,81	17,22	14,71	0,96
278	10,82	13,59	14,91	13,53	0,98	176	14,91	14,51	17,37	16,17	0,94
189	10,92	14,98	17,82	16,71	1,00	225	14,91	15,42	17,62	16,38	0,94

129	11,00	14,00	15,42	14,08	0,98
265	11,51	13,12	14,91	13,64	0,97
290	11,51	13,46	14,91	13,55	0,96
327	11,51	14,22	16,12	14,89	0,97
167	11,92	13,12	14,91	13,62	0,96
244	11,92	13,59	15,20	13,88	0,96
194	12,21	12,39	14,51	13,21	0,96
259	12,21	13,82	16,12	14,94	0,96
322	12,21	13,82	14,91	13,35	0,95
342	12,21	14,22	15,89	14,61	0,96
232	12,43	13,82	14,51	12,48	0,96
92	12,55	17,50	19,52	18,33	0,99
70	12,61	13,18	15,61	14,41	0,96
99	12,61	14,29	15,42	13,90	0,94
279	12,61	13,82	15,76	14,51	0,95
281	12,61	14,22	15,52	14,07	0,95
286	12,61	13,82	15,07	13,55	0,94
319	12,61	18,20	20,72	19,59	1,00
354	12,61	14,73	15,76	14,20	0,95
179	12,77	14,51	16,12	14,80	0,95
234	12,79	13,12	15,07	13,74	0,95
191	12,90	11,92	15,20	14,00	0,98
43	13,12	14,00	16,35	15,15	0,95
69	13,12	14,91	15,89	14,27	0,94
74	13,12	12,61	16,52	15,42	0,98
101	13,12	15,07	15,76	13,86	0,95
130	13,12	14,29	15,61	14,13	0,94
243	13,12	15,76	16,52	14,78	0,95
258	13,12	13,82	15,42	14,02	0,94
71	13,30	14,40	15,42	13,72	0,94
117	13,30	15,61	17,55	16,33	0,95
147	13,30	16,38	18,08	16,82	0,96
222	13,46	14,40	15,42	13,68	0,93
266	13,46	14,69	16,52	15,24	0,94
313	13,46	14,51	16,12	14,75	0,94
344	13,46	14,22	14,91	12,54	0,95
152	13,59	15,10	16,81	15,51	0,94
256	13,59	11,70	17,22	16,14	1,00
34	13,82	16,30	18,60	17,44	0,96
140	13,82	15,94	18,06	16,87	0,95
148	13,82	14,51	17,73	16,62	0,96
163	13,82	14,91	16,52	15,16	0,93
220	13,82	17,50	19,67	18,50	0,97

302	14,91	14,91	16,30	14,56	0,93
316	14,91	15,95	16,76	14,79	0,93
238	15,07	11,92	16,07	14,53	1,00
16	15,20	14,51	17,20	15,92	0,94
105	15,20	16,91	17,91	16,29	0,93
236	15,20	14,91	16,45	14,71	0,93
274	15,20	14,69	16,21	14,29	0,93
143	15,42	16,81	19,19	18,01	0,95
182	15,42	14,40	16,81	15,35	0,93
198	15,42	14,22	16,52	14,91	0,93
307	15,42	16,81	18,83	17,59	0,94
310	15,42	14,22	16,52	14,91	0,93
323	15,42	14,85	16,45	14,59	0,93
332	15,42	15,96	17,11	15,37	0,93
173	15,61	15,32	17,03	15,44	0,92
226	15,76	16,12	16,81	13,86	0,95
18	15,89	15,68	17,37	15,79	0,92
26	15,89	18,83	19,64	17,96	0,95
28	15,89	14,91	17,73	16,43	0,94
29	15,89	14,73	16,52	14,27	0,94
30	15,89	14,09	16,81	15,13	0,94
48	15,89	14,40	17,22	15,77	0,94
184	15,89	15,15	18,20	16,99	0,94
178	16,12	16,52	19,28	18,11	0,95
262	16,12	18,83	20,72	19,50	0,96
336	16,12	16,12	20,72	19,65	0,98
347	16,12	15,42	17,91	16,57	0,93
135	16,21	13,82	17,03	15,33	0,94
201	16,24	16,00	17,37	15,45	0,93
11	16,30	16,81	17,62	15,33	0,94
17	16,30	14,73	16,52	12,07	1,00
31	16,30	16,76	19,40	18,22	0,95
273	16,38	18,42	20,50	19,30	0,95
353	16,45	16,01	18,34	17,00	0,93
86	16,52	17,15	20,08	18,94	0,95
102	16,52	16,30	18,17	16,70	0,93
116	16,52	19,81	22,11	20,95	0,97
299	16,76	17,22	18,13	16,02	0,93
249	16,81	16,71	19,34	18,12	0,94
338	16,81	18,70	20,66	19,43	0,95
109	17,03	15,52	17,71	15,68	0,93
224	17,03	18,42	20,72	19,54	0,95
39	17,22	17,15	19,81	18,60	0,94

268	13,82	12,21	15,42	14,10	0,97	40	17,22	15,79	18,89	17,58	0,94
289	13,82	13,82	14,91	12,77	0,94	114	17,22	23,72	26,02	24,87	1,00
295	13,82	14,51	15,42	13,46	0,93	137	17,22	15,42	17,73	15,47	0,94
326	13,82	15,42	16,93	15,57	0,94	346	17,22	16,45	22,64	21,58	1,00
68	14,00	15,83	17,37	16,04	0,94	89	17,50	19,67	22,11	20,96	0,96
205	14,00	15,25	16,43	14,88	0,93	126	17,73	18,47	19,92	18,45	0,94
23	14,22	16,08	18,20	17,00	0,95	231	17,94	17,50	18,62	15,76	0,97
64	14,22	15,15	16,43	14,88	0,93	352	17,97	20,22	22,39	21,21	0,96
73	14,22	16,30	17,22	15,57	0,94	107	18,06	20,03	21,64	20,33	0,96
110	14,22	15,91	17,91	16,68	0,95	146	18,13	18,06	20,42	19,15	0,94
190	14,22	15,89	17,22	15,79	0,94	112	18,20	17,62	20,03	18,69	0,94
141	14,51	15,20	16,81	15,40	0,93	260	18,42	14,51	18,86	16,74	0,96
151	14,51	15,42	16,71	15,16	0,93	207	18,83	20,72	23,03	21,85	0,97
165	14,51	15,42	16,52	14,85	0,93	122	19,19	17,71	19,86	17,82	0,96
175	14,51	15,14	16,52	14,99	0,93	6	19,52	18,83	21,13	19,72	0,96
237	14,51	14,40	17,50	16,35	0,95	103	19,52	21,82	24,64	23,52	0,99
197	14,63	15,66	16,95	15,43	0,93	351	19,52	20,56	23,43	22,30	0,98
283	14,69	13,12	17,73	16,62	0,97	12	19,54	19,08	24,31	23,24	0,99
21	14,73	15,42	16,81	15,29	0,93	59	19,67	19,11	20,44	18,06	0,97
214	14,73	15,20	16,52	14,91	0,93	7	19,82	20,30	22,67	21,46	0,97
227	14,73	15,42	16,12	13,68	0,94	136	20,03	22,33	23,36	21,80	0,98
14	14,91	19,34	21,75	20,61	0,98	216	20,72	19,61	21,82	20,19	0,97
25	14,91	15,42	17,03	15,60	0,93	314	21,42	14,22	23,03	21,75	1,00
52	14,91	16,52	17,34	15,54	0,93	56	22,11	19,81	23,72	22,42	0,99