Towards a theory of horizontal mergers

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forms. Indeed space constraints are such that the full variety of price discrimination has only been hinted at above. Many other applications could have been considered.

A further important point to emerge is that price discrimination need not be harmful to buyers, even in monopoly markets. Such a pricing policy may be necessary to ensure that particular markets are served. It has also been shown that non-linear prices can Pareto-dominate non-discriminatory prices.

In a more general sense, price discrimination can facilitate market entry and market interpenetration by competing sellers, with buyers being offered a greater variety of goods than would otherwise be available.

NOTES

1. Not all of the examples presented in this section will be 'explained' in the subsequent analysis. Where no explanation is offered, the interested reader should refer to the additional readings noted in section 1.2.
2. With non-linear costs, this is marginal cost of aggregate output.
3. An exception to this would be if the proportion of group 2 consumers were 'high' or if θ_2 were 'significantly greater' than θ_1.
4. Some analysis of second-degree price discrimination is now available, but this is technically very complicated: see Gal-Or (1988); Oren, Smith and Wilson (1983).
5. This case would arise if, for example, $p^*_0 - t_{02} = p^*_0 - 2t = (a_0 - 5t)/3 < 0$, in which case the seller located at 0 could not sell at a profit in market 2.
6. Problems of non-existence of equilibria have plagued spatial price models in the *absence* of price discrimination (Gabszewicz and Thisse, 1986). These problems tend to be resolved when price discrimination is allowed.

6. Towards a Theory of Horizontal Mergers

Gérard Gaudet and Stephen W. Salant

INTRODUCTION

The purpose of this chapter is to explain and illustrate some recent developments in the theory of horizontal mergers. In Section 2, we derive the key property of oligopoly models – analysed in detail in Gaudet and Salant (1991a) – on which our subsequent merger analysis is based. This property has widespread implications, not only for the theory of mergers but also for international trade theory, the relationship of Stackelberg to Cournot equilibrium and other topics. We briefly discuss these implications before focusing on mergers.

To clarify its implications for the theory of mergers, we examine two examples in Section 3. In the first example, we consider the merger of firms engaged in Cournot competition and producing perfect substitutes – such as oil extractors. In the second example, we consider the merger of firms engaged in Bertrand competition and producing complementary inputs – such as different railroads whose tracks adjoin 'end-to-end' and whose services are therefore *jointly* needed to ship a good from one end of the line to the other.

In each case we assume provisionally that the firms merge and focus on two issues: (1) how the merger would affect the *private profitability* of the merging firms and, as a separate matter, (2) how it would affect *social welfare*. Paradoxically – as was first proved in Salant, Switzer and Reynolds (1983) – some exogenous mergers would induce losses for the merging firms.

Section 4 analyses a two-stage game – first formulated by Kamien and Zang (1990) – in which the merger decision is endogenized. We discuss the basic characteristics of the equilibrium and prove that none of the unprofitable mergers previously noted would occur endogenously under *laissez-faire*. Indeed, in the acquisition game we consider, some exogenous mergers which would *increase* the profits of the merging firms do not occur. We briefly explain why and then use the two examples from the previous section to illustrate that socially undesirable mergers may occur in equilibrium and socially desirable mergers may fail to occur. This suggests two types of situations where government intervention might be contemplated. Section 5 discusses extensions and concludes the chapter.

Our goal here is a modest one: to integrate selectively a few recent contributions to the theory of horizontal mergers, to point out fundamental differences between the merger of firms producing substitutes and firms producing complements and to suggest, by means of illustrations, a few results of importance to policy. References have been included to guide readers interested in particular merger models to the relevant technical literature.

2 A COMPARATIVE-STATIC PROPERTY OF OLIGOPOLY MODELS

Merger theory builds on oligopoly theory.[1] Our task in this section is to clarify one aspect of oligopoly models on which our future analysis depends. The remaining sections will apply this property to mergers. In the remainder of this section we discuss a few of its other important applications.

As an opening teaser, consider an industry in which three identical firms produce a homogeneous good at constant marginal cost and engage in Cournot (quantity) competition. For simplicity, assume the demand curve is linear.

1. If two of the firms are located in the 'home country' while the third firm is located in another country, is the optimal trade policy for the home government an export subsidy? For simplicity, assume the government of the 'other country' (where the other firm is located) is pre-committed to *laissez-faire*; furthermore assume that no home residents consume the good so the entire output is exported.
2. Assume that a 'Stackelberg leader' gains control of two firms and can pre-commit to outputs at each before the third firm responds. Will the leader strictly increase their outputs relative to the levels under Cournot competition?

One might well answer both questions in the affirmative. It might *seem* that an export subsidy is the optimal trade policy since that result has been shown by Brander and Spencer (1983) to hold where there are *two* firms in total – one in the home country and one in the other country.

It might *seem* that a Stackelberg leader would wish to increase the

output at each of the firms, since, in the textbook case where the Stackelberg leader controls only one firm, it is profitable for him to produce more than in Cournot equilibrium. However compelling each of these answers may seem, each is incorrect. The correct answer to each question is 'no!'

Consider an industry composed of n firms with identical cost functions in a symmetric Cournot equilibrium. Suppose the equilibrium is displaced by an exogenously-induced *marginal* contraction of the output of a subset of s of these firms. To resolve each of the foregoing teasers, we must identify the circumstances in which the profits of each firm in the subset would increase as a result of the exogenous output contraction.

First note that, since each of the s firms is optimizing, a marginal contraction in its own output in a neighbourhood of equilibrium would have no effect on its profits were it not for the induced changes in the output of the $n-1$ other firms. Assuming goods are substitutes in demand, the profit of each contracting firm will increase if and only if the aggregate output of the other $n-1$ firms decreases.

By assumption, the output of $s-1$ of these firms decreases since they are in the designated subset. Assuming downward-sloping reaction functions (strategic substitutes), the output of the other $n-s$ firms will increase. To determine which effect dominates and under what circumstances requires some algebra.

Let $P(\cdot)$ denote the inverse demand function, $C(\cdot)$ the identical total cost function of each firm and Q_{-i} the aggregate output of every firm other than i. Denote the output of each of the $n-s$ firms as q and let $\overline{q}$ denote the output of each of the s firms whose output will be contracted exogenously. Since a symmetric equilibrium is being displaced, $q = \overline{q}$ at the initial equilibrium. Nonetheless it is important to distinguish the two variables in the notation since one variable is displaced exogenously while the other adjusts endogenously in response. If firm i is one of the firms whose output is being contracted, then, given these definitions:

$$Q_{-i} = (n - s)q + (s - 1)\overline{q}. \tag{6.1}$$

Let π denote the profit of this firm. It can be expressed as a function of two variables:

$$\pi(\overline{q}, Q_{-i}) = \overline{q}P(\overline{q} + Q_{-i}) - C(\overline{q}). \tag{6.2}$$

Under standard assumptions, if $q = \overline{q}$, a unique, symmetric Nash equilibrium in pure strategies will exist[2]. At the equilibrium,

$$\pi_1(\overline{q}, Q_{-i}) = 0, \tag{6.3}$$

where the subscript on π denotes partial differentiation with respect to the subscripted argument.

To determine the effect on firm i's equilibrium profits of a change in $\bar{q}$, we totally differentiate the expression for its equilibrium profits:

$$\frac{d\pi}{d\bar{q}} = \underset{0}{\pi_1} + \underset{-}{\pi_2}\,\underset{?}{\frac{dQ_{-i}}{d\bar{q}}}. \tag{6.4}$$

From equation (6.1):

$$\frac{dQ_{-i}}{d\bar{q}} = (n - s)\frac{dq}{d\bar{q}} + (s - 1). \tag{6.5}$$

Clearly, in the special case where $s = n$, $dQ_{-i}/d\bar{q} > 0$ and a marginal contraction in the output of the s firms will cause Q_{-i} to fall and hence the profits of firm i to increase. This explains why it is always profitable for a monopolist who takes over an industry of n independent firms to contract the production of each. If, at the other extreme, $s = 1$, then $dQ_{-i}/d\bar{q} < 0$ and a marginal contraction in the output of this firm will have the effect of increasing the output of the other firms and hence reducing the profit of the contracting firm. This explains why a Stackelberg leader who takes over one firm in a duopoly will *expand* its production.

To understand what happens in intermediate cases, we must evaluate $dq/d\bar{q}$. This is done by differentiating the first-order condition of the typical firm which is not being contracted. After some simplification, we conclude.[3]

$$\frac{d\pi}{d\bar{q}} \gtreqless 0 \Leftrightarrow s - \alpha(n - s) \lesseqgtr 1, \tag{6.6}$$

where

$$\alpha = \frac{P' + qP''}{P' - C''}. \tag{6.7}$$

To interpret this condition, consider first the benchmark case of linear demand and constant marginal costs. Then $\alpha = 1$ and the condition says that a marginal contraction raises the profits of the firms in the subset if and only if the number of firms in the subset, s, exceeds the number of other firms, $n - s$, by strictly more than one. In the general case, if $P' + qP'' < 0$, reaction functions will be downward-sloping. If in addition $P' - C'' < 0$, which is a common stability condition in oligopoly models, α will be positive.[4] It is helpful to regard α as a multiplicative adjustment in the number of outside firms. The adjustment factor differs from unity in predictable ways when non-linearities are introduced.

In the linear cases outlined at the beginning of the section, $s = 2$, $n = 3$, and $\alpha = 1$. Since $s - \alpha(n - s) = 1$, a marginal change (contraction or expansion) will have no effect on the profits of firms in the subset. This explains each of the cases referred to at the outset. With two firms in the home country and one outside, the optimal trade policy in the home country is free trade rather than a subsidy or tariff. A tariff would be the optimal policy if we dropped the linearity assumption in a way which decreased[5] α or, alternatively, if we maintained the linearity assumption but appropriately increased the number of home firms relative to foreign firms (for example, $s = 3$, $n = 4$).

A Stackelberg leader would not alter the outputs of the two firms relative to their Cournot level. If he controlled only one firm in a two-firm industry, he would want to *expand* its production to maximize profits. This is the duopoly case which dominates most textbook treatments.[6] But if instead he controlled three of four firms, then in the linear case he would want to *contract* their production relative to their Cournot levels.

So far, we have considered only the case of quantity competition. But similar results hold when firms compete in price, provided reaction functions in price are downward-sloping. This is not the 'standard' case textbooks examine when discussing price competition but, as we shall see, it is of practical importance in analysing the merger of providers of complementary inputs.

In such cases, whether a marginal reduction in the price of a subset of s firms will increase or reduce their profits will depend on the size of the subset relative to the number of the outside firms. Rather than derive the results formally, we simply outline here how our previous argument can be modified for the case of price competition.

A forced marginal reduction in the price of any firm in the subset would have no effect on its profit if the prices of the other firms did not change, since it is already optimizing with respect to its own price. But by assumption, $s - 1$ of those other prices will decrease and $n - s$ of those prices will increase. Assume the goods are complements in the sense that higher prices on the part of the other firms reduce the amount one's own firm is asked to supply. If $s = n$, all of the prices will decrease and this reduction will unambiguously raise profits. If, on the other hand, $s = 1 < n$ then all of the other prices will rise and this will unambiguously reduce profits. For intermediate values of s a marginal reduction in the price of firms in the subset could increase or decrease profits.

This explains Cournot's discovery in comparing monopoly and duopoly provision of perfect complements – in his case zinc and copper producers selling to bronze makers. In a pricing game among sellers of complements,[7] he proved that the monopolist ($n = s$) would find it optimal to charge less in total than the sum of the prices charged by the oligopolists! This seemingly odd result occurs because the monopolist would internalize the effect that a higher price would have on reducing demand at the other firms.

The above analysis has relied on the assumption of downward-sloping reaction functions in both the quantity-competition case and the price-competition case. But what if we were dealing in either case with *strategic* complements (upward-sloping reaction functions)? The analysis is then quite straightforward: an exogenous marginal change in the strategic variable of a subset of s firms will induce a marginal change in the same direction in the strategic variable of the $n - s$ other firms and therefore the effect on profit will not depend on the relative size of s.[8] Thus, in the case of quantity competition with substitutes in demand, an exogenous marginal contraction in output will always be profitable to the s firms in the subset. Similarly, if the firms compete in price with the goods complements in demand, an exogenous marginal reduction in price will always be profitable to the s firms in the subset.[9]

3 LOSSES FROM EXOGENOUS MERGER

In this section we first discuss the implications of the comparative-static property of the previous section for exogenous mergers.[10] We then analyse two distinct kinds of mergers involving strategic substitutes. In both cases, since reaction functions are downward-sloping, the profitability of the exogenous merger depends on the size of the merged entity relative to the size of the industry. In the first example we consider a merger of oil extractors producing perfect substitutes and engaged in quantity competition. In the second example we consider a merger of railroads producing perfect complements (transport on adjoining tracks) and engaged in price competition.

The merged entity is assumed to choose the levels of the strategic variables (quantities in Cournot competition or prices in Bertrand competition) of the merged firms in order to maximize their joint profits, given the choices of the firms outside the merger. If the goods are substitutes (complements) in demand and the firms compete in quantities (prices), it will want to reduce the outputs (the prices) of the subset of firms which is being merged in an attempt to increase its profits. If in fact none of the outside firms were to change its output (its price), this would result in greater joint profits for the merged entity. But, with downward-sloping reaction functions, the best reply of each of the outside firms will be an increase in output (price). As a result, the net effect on the profit of the merged firms is unclear and the possibility exists of losses from an exogenous merger.

This is clearly a consequence of the comparative-static property discussed in the previous section and the results derived there can be applied. Consider the case of quantity competition. If s is the number of firms being merged, a contraction of outputs *in a neighbourhood of the initial equilibrium* will result in losses to the merged group whenever s exceeds the adjusted number of outside firms, $\alpha(n - s)$, by less than one. Let us assume that equilibrium profits of each individual firm in the merged subset is a concave function of its own output. It then follows that, if $s - \alpha(n - s) < 1$, any reduction of output – even a non-local one – will reduce profits and any exogenous merger is unprofitable to the merged firms.

If the merger is to be profitable, the number of firms involved must therefore exceed the adjusted number of outsiders by more than one. In that case a *marginal* reduction of output is profitable, as was shown in the previous section. But since a merger involves a *non-marginal* reduction in output, this condition is not sufficient. It must instead be verified that:

$$\pi(q^M, Q^M_{-i}) \geq \pi(q^*, Q^*_{-i}), \tag{6.8}$$

where q^* is the output of firm i (and Q^*_{-i} is the aggregate output of the other $n - 1$ firms) in the pre-merger equilibrium (that is, $\bar{q} = q = q^*$); similarly, q^M is the output of firm i (and Q^M_{-i} is the aggregate output of the other $n - 1$ firms) in the post-merger equilibrium. Let $q(x)$ denote the equilibrium output of an outsider if each merged firm produces x. Consider the roots defined implicitly by the following equation:

$$P(s\gamma + (n - s)q(\gamma))\gamma - C(\gamma) = P(nq^*)q^* - C(q^*). \tag{6.9}$$

Assuming the left-hand side is strictly concave in γ, there will be two real roots – one trivial and one non-trivial. The trivial root will be: $\gamma_1 = q^*$. The non-trivial second root will be smaller than q^* if and only if $s - \alpha(n - s) > 1$. Profitability of an exogenous merger therefore requires both $s - \alpha(n - s) > 1$ and $q^* > q^M > \gamma_2$.

Exactly the same type of argument holds for price competition with complements in demand. Even though a marginal reduction in the price of each merging firm would be profitable, there will exist a critical price (denoted below as γ_2^p) such that, if each insider charges less in the post-merger equilibrium, the merger is unprofitable.

The following examples will help clarify the implications of exogenous mergers in those two cases.

3.1 Merging under Quantity Competition with Perfect Substitutes

Consider the case of an industry composed of n identical firms engaged in the activity of oil extraction. The oil extracted by any given firm is viewed by consumers as a perfect substitute for the oil produced by any of the $n - 1$ other firms. The individual firm outputs can therefore be summed to give the total industry output of oil. Assume, for simplicity, that the inverse demand curve for oil is linear. By proper choice of units, it can always be written $P = \beta - Q = \beta - (q + Q_{-i})$, where, as before, q is the output of individual firm i and Q_{-i} is the aggregate output of the $n - 1$ other firms, Q being the total industry output. Assume also that each firm can extract oil at no cost.[11]

If one of the firms were to increase its output of oil, the best reply on the part of each of the other firms is a reduction in output. We thus have strategic substitutes (downward-sloping reaction functions).

Now suppose a subset of s of those n firms is forced to merge. The question arises as to whether this exogenously-imposed merger will increase the equilibrium profit per firm of those s oil producers. As has already been pointed out in the previous section, with a linear demand and constant marginal costs, we have $\alpha = 1$ in equation (6.6). Therefore, if the merger is to raise profits, the merged subset must outnumber the unmerged subset by more than one; that is, $s > n - s + 1$. This is the local profitability condition, which guarantees that an exogenous marginal contraction of output is profitable. But, since the merger will result in a non-marginal reduction of output, this condition is not sufficient to assure that the post-merger equilibrium profit will in fact exceed the pre-merger equilibrium profit of the merged oil producers. In addition, it must be verified that q^M, the post-merger output per merged firm, exceeds γ_2, the critical level output defined by equation (6.9). This may be referred to as the global profitability condition.

It is straightforward to verify that the pre-merger symmetric Nash-equilibrium output per firm in an n-firm industry, when the inverse demand is linear and marginal cost is zero, will be $q^* = \beta/(n+1)$. Each firm's equilibrium profit is then $P(nq^*)q^* = \beta^2/(n+1)^2$. If we substitute in (6.9), we find that γ must satisfy:

$$\left(\beta - s\gamma - (n - s)\frac{\beta - s\gamma}{n - s + 1}\right)\gamma = \frac{\beta^2}{(n + 1)^2}, \tag{6.10}$$

where $(\beta - s\gamma)/(n - s + 1) = q(\gamma)$, the equilibrium output of each outside firm when each merged firm produces γ.

Equation (6.10) is a quadratic in γ with two real roots. The first root is trivial: $\gamma_1 = \beta/(n + 1)$. The second root is the one we seek:

$$\gamma_2 = \frac{n - s + 1}{s}\frac{\beta}{n + 1}.$$

It is straightforward to verify that $\gamma_2 < q^*$ if and only if $s > n - s + 1$. Now the post-merger output per merged firm will be $q^M = \beta/s(n - s + 2)$.[12] The global necessary condition for profitability of the merger ($q^* > q^M > \gamma_2$) yields:

$$\frac{\beta}{s(n - s + 2)} > \frac{n - s + 1}{s}\frac{\beta}{n + 1} \tag{6.11}$$

or, equivalently:

$$(n - s + 1)^2 - s < 0. \tag{6.12}$$

Salant, Switzer and Reynolds (1983, p. 193) show that (6.12) requires $s > 0.8n$.[13] In other words, with a linear demand curve and constant marginal costs, the exogenous merger will be profitable only if it includes at least 80 per cent of the firms in the industry.[14]

Thus, in this example, if $s \leq (n + 1)/2$, no reduction of output, even marginal, from its pre-merger Cournot-equilibrium level can increase the profits of the s affected firms and a non-marginal reduction will definitely decrease it. Such mergers will cause losses to the merged firms. If $(n + 1)/2 < s < 0.8n$, the merged producers would benefit from a joint *marginal* contraction of their outputs from the pre-merger level. But the actual reduction of output which a merger would induce is so large that the merger would result in losses for the merging firms.

We have so far restricted our attention to the *private* profitability of mergers. But what can be said about the *social* value of mergers in this example? Define social welfare as the sum of all the firms' profits and of consumer surplus.[15] With the firms producing at no cost, this is simply the area under the inverse demand curve up to equilibrium industry output. Since a merger of any number of firms will reduce aggregate industry output,[16] and thereby increase price, it is easy to see that it will always reduce social welfare.[17]

3.2 Merging under Price Competition with Perfect Complements

Picture a railroad track running from a coal mine to the market and assume that different segments of the track are owned by different companies. For simplicity, assume that initially each of n companies owns a single segment of track and that the companies set fares on their segments simultaneously.[18] Then a coal shipper would have to pay the fare on each of the segments to ship to market. For simplicity, we assume that the demand for coal is a linear function of the cost of transporting it: $Q = \beta - P = \beta - (p + P_{-i})$. Here P is the aggregate price paid by the shipper, p is the price charged on segment i and P_{-i} is the aggregate price charged on the $n - 1$ other segments. As in the previous example, we assume that both the fixed and the variable costs of operating each of the n segments of track are zero.

To the coal shipper, however, costs are not zero. An increase in the fare on the i^{th} segment would raise the shipper's costs and would shift leftward his demand for shipping on all of the other segments. In contrast to the usual case of substitutes, an increase in the fare of the other firms is therefore *bad* for i's business. The best reply to an increase in the sum of the rivals' prices is a reduction in one's own price. Hence perfect complements in demand give rise to downward-sloping reaction functions.

There is obviously a very close similarity between the mathematical structure of this example and that of the previous example. In fact, as shown in Sonnenschein (1968), there exists a dual relationship between the two formulations. In our context, this implies that the results concerning the private profitability of mergers under quantity competition with perfect substitutes can be applied to this example of merger under price competition with perfect complements simply by interchanging the role of prices and quantities. Hence the pre-merger equilibrium price per segment is $p^* = \beta/(n + 1)$ and the equilibrium profit of each railroad is $p^*Q(np^*) = \beta^2/(n + 1)^2$. Note that what we previously derived as the formula for the equilibrium quantity of an individual firm is in the current context the formula for a firm's equilibrium price; moreover the expression for the equilibrium profit of an individual firm is the same under price competition as it was under quantity competition. The post-merger price per segment after a forced merger of s railroads is $p^M = \beta/s(n - s + 2)$. Hence we get exactly the same expression as before for γ_2^p, where γ_2^p now represents the critical price level below which the non-marginal price reduction causes a loss.

Thus merging a subset of s railroads competing in prices will increase their profits only if they constitute more than 80 per cent of the firms in the pre-merger equilibrium. As in the previous example, if the merger includes less than 80 per cent of the railroads, it will inflict losses upon the merged firms, either because no price reduction would be beneficial ($s \leq (n + 1)/2$) or because the equilibrium price reduction induced by the merger is too large to be profitable ($(n + 1)/2 < s < 0.8n$).

Where this example differs markedly from the previous one, however, is with respect to the effect of a merger on social welfare. In the example of quantity competition with substitutes, any merger *reduces* welfare. But in our railroads example, where the goods are complements in demand and competition is in price, a merger will always reduce the aggregate price and, as a result, will always *increase* social welfare. Therefore some socially profitable mergers are privately unprofitable and would presumably not occur if the decision to merge were endogenized. We will examine this issue further in the next section.

Before moving on to consider a model of endogenous merger, a word should be said about merging under strategic complements. As noted in the last section, with strategic complements the equilibrating adjustment by outsiders will always reinforce an exogenous move by insiders. Hence the private profitability of mergers will be independent of the number of firms included in the merger. Since merger to monopoly is always profitable, smaller mergers must also be profitable. This explains the finding of Deneckere and Davidson (1985) that, given their assumptions, the merged firms always gain from merger when competition is in prices. Their result comes not from their assumption of price competition but instead from their assumption of upward-sloping reaction functions. As our railroad example shows, there exist important cases of price competition where the reaction functions in price space are downward-sloping. In such cases there can be losses from the exogenous merger of firms engaged in price competition.

4 A MERGER GAME

Recently Kamien and Zang (1990) have developed a two-stage merger game.[19] In the first stage, each of n players simultaneously selects an n-tuple: a bid for the other $n - 1$ firms and an asking price for its own firm. If the highest bid for a firm exceeds the asking price of that firm, it is sold to the highest bidder for the price he bids (if two or more bids tie, a tie-breaking rule determines the identity of the acquiring firm). The new market structure is then observed before the second-stage game begins.

In the second stage, some of the original n players have sold out and no longer control any technology. The remaining players may own one or more technologies. Kamien and Zang assume each of the remaining players selects the outputs of the firms under its control to maximize his joint profits – taking as given the outputs of the firms under the control of others.[20]

In the case of constant marginal costs and zero fixed costs, the cost of producing a given aggregate output is independent of the interfirm distribution of this output. It follows that whatever aggregate output a player controlling more than one firm chooses to produce can be provided at least cost by a single firm. Hence Kamien and Zang exploit the trick in Salant, Switzer and Reynolds and assume that a multi-firm player sets to zero the output of all but one firm.

Kamien and Zang's model endogenizes what we have so far treated as exogenous: the decision to merge. Using their model, we can now ask whether mergers which have been characterized as beneficial or harmful from either a social or a private point of view will in fact occur. In what follows we extend their analysis by considering in addition a price game played by sellers of perfect complements. As in the output game, an indeterminacy arises in the case of constant marginal costs and zero fixed costs. Hence, in the railroad example of the previous section, a player operating more than one segment of the track is indifferent as to vectors of fares set on the segments under its control as long as the components of each vector add to the same aggregate price. It will be convenient to assume in such cases that the player sets to zero all but one of the prices.

We also depart from Kamien and Zang by introducing fixed costs. There are really two sources of fixed costs in either oligopoly model: costs which can be avoided only by producing zero output (for example, costs of maintaining an oil pipeline or a railroad bed) and costs which can be avoided only by charging a zero price (for example, costs of collecting fares from buyers on a given segment). When analysing quantity competition, we typically ignore the latter type of fixed cost. This omission is inconsequential when analysing mergers under Cournot competition since the (common) price paid to each component of a merged entity is positive before and after the merger. Symmetrically, when discussing price competition, we will ignore the former type of fixed cost. This omission is inconsequential when analysing mergers under Bertrand competition since the (common) usage of each segment of track owned by the merged entity is positive before and after the merger. Henceforth, when we mention fixed costs in the context of quantity competition, we mean costs which can be avoided only at a zero output. When we mention fixed costs in the context of price competition, we mean costs which can be avoided only at a zero price.

In the case of output competition with constant marginal costs and positive fixed costs, setting to zero the output of all but one firm is the *unique* cost-minimizing solution to providing a given aggregate output.

Similarly, in the case of price-competition with perfect complements, setting to zero all but one of the prices is the unique least cost way of collecting a given aggregate price.

Merger behaviour in the first stage of Kamien and Zang's model depends on the profits which each player anticipates receiving in the Nash equilibrium of each second-stage subgame. The profits in each second-stage subgame are particularly easy to characterize when marginal costs are constant and equal. This is true whether the strategic variable in the second stage is quantity or price and the following discussion applies to both cases. If marginal costs are constant and equal, each player operating at least one firm receives an equilibrium profit before deduction of fixed costs which is a function of the number of such players. Denote this function $\hat{\pi}(z)$. Thus, if z players each control at least one firm at the end of the first stage, then each will receive $\hat{\pi}(z) - F$ in the second stage, where F denotes the fixed cost per firm.

Certain conditions must hold if, in a subgame perfect equilibrium of the two-stage game, one of the n players (denoted i) in the original industry acquires the firms of $s - 1$ of the other players, while the remaining players each operate one firm. First, each of the $s - 1$ players which sells its firm must receive at least $\hat{\pi}(n-s+2) - F$. If any did not receive that much, it would be more profitable to ask unilaterally an infinite price, operate solo, and earn $\hat{\pi}(n - s + 2) - F$. Hence, to acquire these $s - 1$ firms, player i must bid at least $\hat{\pi}(n - s + 2) - F$ for each of them.

Second, in an equilibrium player i must not pay so much for the $s - 1$ acquired firms that it would be strictly preferable for it to unilaterally drop some of those firms by bidding zero for them. Suppose player i considered dropping x firms and retaining the rest. Suppose it bid B_j $(j = 1,\ldots, s - 1 - x)$ for the firms it considers retaining. Since it must have bid at least $\hat{\pi}(n - s+2) - F$ for each of the firms it considers dropping, its net profit if it retains all of the $s - 1$ firms is no larger than $\hat{\pi}(n - s + 1) - F - x(\hat{\pi}(n - s + 2) - F) - \sum_{j=1}^{s-1-x} B_j$.

If instead player i bids zero for x of the $s - 1$ firms, its resulting profits will depend on which of two possibilities arises in the next stage: (1) the strategic variable of at least one of the dropped firms might be set to zero or (2) the strategic variable of each of the dropped firms might be set to the common non-zero level of the other firms. It is clear that the former possibility cannot arise in equilibrium. For, if the strategic variable of any firm player i acquires should be set to zero if it were dropped, then player i would not acquire that firm in the first place. It would not care whether it or some other player was responsible for setting the firm's strategic variable to zero and would prefer to avoid the cost of acquiring the firm. In equilibrium, therefore, player i knows that, if it were to drop some firms, their strategic variables would be set to the common non-zero level of the other firms in the market. The net profits it can anticipate after unilaterally deviating by dropping x firms are therefore $\hat{\pi}(n - s + x + 1) - F - \sum_{j=1}^{s-1-x} B_j$. It follows that, if the situation where player i acquires the $s - 1$ firms is an equilibrium, then, for all $x = 1, \ldots, s - 1$, the following inequality must hold:

$$\hat{\pi}(n - s + 1) - F - x(\hat{\pi}(n - s + 2) - F) - \sum_{j=1}^{s-1-x} B_j \geq \hat{\pi}(n - s + x + 1) - F - \sum_{j=1}^{s-1-x} B_j \tag{6.13}$$

or, equivalently, for all $x = 1, \ldots, s - 1$:

$$\hat{\pi}(n - s + 1) - x(\hat{\pi}(n - s + 2) - F) \geq \hat{\pi}(n - s + x + 1). \tag{6.14}$$

Otherwise player i would choose to drop the x firms.

It is a consequence of (6.14) that, if for some $x = 1, \ldots, s - 1$:

$$\hat{\pi}(n - s + 1) - x(\hat{\pi}(n - s + 2) - F) < \hat{\pi}(n - s + x + 1), \tag{6.15}$$

then one of the necessary conditions is violated and the acquisition of the $s - 1$ firms by player i cannot be an equilibrium outcome.

We can use this result to demonstrate that *none* of the unprofitable exogenous mergers identified in Salant, Switzer and Reynolds arises endogenously in Kamien and Zang's merger game. Recall that Salant, Switzer and Reynolds showed that, if in an n-firm oligopoly a subset of s firms is forced to merge ($s \geq 2$), then the collective profits of that subset may fall. This occurs if, for given n and s:

$$\hat{\pi}(n - s + 1) - F < s(\hat{\pi}(n) - F) \tag{6.16}$$

or, equivalently:

$$\hat{\pi}(n - s + 1) - (s - 1)(\hat{\pi}(n) - F) < \hat{\pi}(n). \tag{6.17}$$

But, since $\hat{\pi}(z)$ is a strictly decreasing function of z (Kamien and Zang, 1990; Gaudet and Salant, 1991c), we have $\hat{\pi}(n - s + 2) \geq \hat{\pi}(n)$ for $s \geq 2$. Therefore, if (6.17) holds, the following also holds:

$$\hat{\pi}(n - s + 1) - (s - 1)(\hat{\pi}(n - s + 2) - F) < \hat{\pi}(n), \tag{6.18}$$

which is simply (6.15) with $x = s - 1$. Since a condition necessary for the merger to arise endogenously is violated, no such merger would occur. Privately unprofitable mergers do not occur in this model; indeed, some privately *profitable* mergers do not occur. Unprofitable mergers do not occur because parties to the merger anticipate getting more if they unilaterally defect than they would get if the merger occurred. This same anticipation ensures that some profitable mergers do not occur either.[21].

Assume condition (6.14) holds for all $x = 1, \ldots, s - 1$. Then there must exist a subgame perfect equilibrium in which one player acquires the firms of $s - 1$ other players and each of the remaining $n - s$ players independently operates one firm. We can verify this claim by constructing such an equilibrium. Consider the following strategy combination in the first stage: player i bids $\hat{\pi}(n - s + 2) - F$ for each of the $s - 1$ firms it acquires and bids zero for each of the firms it does not acquire; every other player bids zero for all firms; finally, the $s - 1$ players which sell their firms each ask $\hat{\pi}(n - s + 2) - F$, while each of the $n - s + 1$ other players (including player i) asks infinity. Then, under the rules of the game, player i would acquire the firms of $s - 1$ other players. It is left to the reader to verify that no player could increase its profits by unilaterally deviating.

We are now in a position to address the two central questions in the theory of mergers. Can socially unprofitable mergers occur endogenously? Can socially profitable mergers fail to occur? To illuminate both questions, we endogenize the merger decision in the two examples of the previous section.

4.1 The Quantity Competition Example

Consider first the example of quantity competition with perfect substitutes developed in the previous section. If s oil extractors in our n-firm oil industry were forced to merge, then, by equation (6.16), the merger would increase the joint profits of the merged firms if and only if:

$$\frac{\beta^2}{(n - s + 2)^2} - F > s\left[\frac{\beta^2}{(n + 1)^2} - F\right]. \tag{6.19}$$

The threshold value of fixed costs for private profitability – that which equates both sides of (6.19) – is therefore:

$$F_p = \left[\frac{s}{(n + 1)^2} - \frac{1}{(n - s + 2)^2}\right]\frac{\beta^2}{s - 1}. \tag{6.20}$$

If, for given n and s, F is larger than F_p, then the merger is privately profitable. The merger is privately unprofitable if F is smaller than F_p.[22]

From the Kamien and Zang model, we know that such a merger can occur endogenously only if:

$$\frac{\beta^2}{(n-s+2)^2} - (s-1)\left[\frac{\beta^2}{(n-s+3)^2} - F\right] \geq \frac{\beta^2}{(n+1)^2} \tag{6.21}$$

Let F_0 denote that value of fixed costs for which the equality holds in (6.21), that is:

$$F_0 = \left[\frac{1}{(n+1)^2} + \frac{s-1}{(n-s+3)^2} - \frac{1}{(n-s+2)^2}\right]\frac{\beta^2}{s-1}. \tag{6.22}$$

Then if, for given n and s, F is smaller than F_0, the merger will fail to occur. If F is larger than F_0 there may exist an equilibrium in which one firm acquires $s-1$ other firms.[23]

Consider finally social surplus. In a z-firm quantity game with the linear inverse demand and zero marginal cost, aggregate profits net of fixed costs are $z(\beta/(z+1))^2 - zF$ and net consumer surplus is $0.5(z\beta/(z+1))^2$. Hence the sum of producer and consumer surplus is given by:

$$\frac{z(z+2)}{2(z+1)^2}\beta^2 - zF.$$

The merger of s firms will therefore increase social welfare if:

$$\frac{(n-s+1)(n-s+3)}{2(n-s+2)^2}\beta^2 - (n-s+1)F > \frac{n(n+2)}{2(n+1)^2}\beta^2 - nF \tag{6.23}$$

and the threshold value of fixed costs for social profitability of the merger is:

$$F_s = \left[\frac{n(n+2)}{2(n+1)^2} - \frac{(n-s+1)(n-s+3)}{2(n-s+2)^2}\right]\frac{\beta^2}{s-1}. \tag{6.24}$$

If, for given n and s, F is larger than F_s, then the merger is socially valuable. It will be socially harmful if F is smaller than F_s.

It can be verified that $F_p \leq F_0$ for all admissible n and s. This implies for the case of linear demand that, if a merger occurs endogenously, it must be privately profitable – a proposition we proved in another way for any demand curve at the beginning of this section.

It can also be verified that F_s is positive for all admissible n and s. This implies that, in the absence of fixed costs, all mergers in quantity competition are socially undesirable. Intuitively such mergers result in lower aggregate output and higher prices with no compensating change in costs. With fixed costs, however, there will be a social cost saving equal to the private cost saving of $(s-1)F$ from merging s firms. If F is large enough this may render the merger socially profitable, just as it may render privately profitable an otherwise privately unprofitable merger.

4.2 The Price Competition Example

Consider now the railroad example of price competition with perfect complements. With the costs and demand assumptions of the previous section, we will have $\hat{\pi}(z) = \beta^2/(z+1)^2$. This is the same expression for equilibrium profits as in the quantity competition example. Consequently the conditions remain the same for the non-occurrence (6.15) and the private unprofitability (6.17) of a merger; it follows that the expressions for F_0 and F_p given by (6.22) and (6.20) respectively apply as well to the example with price competition and complements.

However the expression for social welfare will now be different. In the quantity competition example, the goods are perfect substitutes and consumers pay an equilibrium price $\beta/(z+1)$ for an aggregate quantity $z\beta/(z+1)$; in the price competition example, the goods are perfect complements, so the consumers pay an aggregate price $z\beta/(z+1)$ for an equilibrium quantity $\beta/(z+1)$. In either case, producer surplus net of fixed costs will be $z(\beta/(z+1))^2 - zF$. However consumer surplus in the two cases will differ. In the case of Cournot competition, consumer surplus is

$$\frac{1}{2}\left[\beta - \frac{\beta}{(z+1)}\right]\frac{z\beta}{z+1}$$

whereas, in the case of Bertrand competition, consumer surplus is

$$\frac{1}{2}\left[\beta - \frac{z\beta}{z+1}\right]\frac{\beta}{z+1}.$$

Consequently, in the case of price competition, net social surplus in a z-firm industry will now be:

$$\frac{2z+1}{2(z+1)^2}\beta^2 - zF.$$

By equating pre- and post-merger equilibrium social welfare, we can solve for the threshold value of fixed costs for a merger to be socially beneficial in our example of price competition with perfect complements. This yields:

$$F_s^p = \left[\frac{2n+1}{2(n+1)^2} - \frac{2(n-s+1)+1}{2(n-s+2)^2}\right]\frac{\beta^2}{s-1}. \qquad (6.25)$$

If, for given n and s, F is larger than F_s^p, then the merger in the price game will increase social welfare. Social surplus will fall if F is smaller than F_s^p.

It can be verified that F_s^p is always negative for admissible values of n and s. This implies that merging is always socially beneficial in this example – even in the absence of fixed costs.[24] A merger will lower the aggregate price and will thereby increase demand. In the absence of fixed or variable costs, the induced output expansion must raise social surplus.

4.3 Policy Implications

We are now able to resolve the two questions posed at the outset. Can socially undesirable mergers occur endogenously? Can some socially desirable mergers fail to occur?[25]

Socially undesirable mergers can occur under quantity competition. A simple example is the merger of Cournot duopolists ($n = 2, s = 2$). In that case, it can be verified from (6.20), (6.22) and (6.24) that $F_p = F_0 < 0 < F_s$.[26] Hence, if the two firms had zero fixed costs, a merger to monopoly would be profitable enough to occur endogenously but would be socially harmful.

Of course not all socially disadvantageous mergers need be of concern. We know mergers will not occur when they are unprofitable and, in linear examples, will be unprofitable if they involve less than 80 per cent of the industry. A merger of two firms in a three-firm industry ($n = 3, s = 2$) illustrates this case. It is easily verified from (6.20), (6.22) and (6.24) that $0 < F_p = F_o < F_s$.[27] Hence, if firms had zero fixed costs, the merger would not only be socially harmful but would harm the merging parties themselves and would not occur endogenously. The intuition for why the merger would not occur is as follows: the acquired firm would insist on at least $\beta^2/16$, since that is the profit it would expect by operating solo in a three-firm industry. But this exceeds the gain in profit ($\beta^2/9 - \beta^2/16$) which the acquiring firm would expect from operating in a two-firm rather than a three-firm industry.

In fact some socially disadvantageous mergers will not occur even though they are privately profitable. Consider a merger of all three firms in the same three-firm industry ($n = 3, s = 3$). It is easily verified in that case from (6.20), (6.22) and (6.24) that $F_p < 0 < F_o < F_s$.[28] Hence, if firms

had zero fixed costs, the merger to monopoly would be socially undesirable but – although profitable – would not be *sufficiently profitable* to arise endogenously. The merger would not occur for the following reason: each of the two acquired firms would insist on at least the duopoly profit – in aggregate at least $2\beta^2/9$ – since that is the profit each would expect if it operated solo in a two-firm industry. But the acquiring firm would not offer this amount since it exceeds the gain in profit ($\beta^2/4 - \beta^2/16$) which it would expect from operating in a one-firm rather than a three-firm industry.

Some socially desirable mergers will not occur endogenously without government intervention. In the context of quantity competition with fixed costs, Salant, Switzer and Reynolds (1983, p. 195) showed that some socially profitable mergers are privately unprofitable. To illustrate, consider a merger of two-firms in a four-firm industry ($n = 4, s = 2$). It is easy to calculate from (6.20), (6.22) and (6.24) that $0 < F_s < F_p = F_o$.[29] Hence, for any $F \in (F_s, F_p)$, such a merger is socially profitable but privately unprofitable and of course will not occur in the Kamien and Zang model.

In fact, even some socially and privately profitable mergers may not occur. Consider the case of a three-firm merger in the same four-firm industry ($n = 4, s = 3$). It can be verified that $0 < F_p < F_s < F_o$.[30] Therefore for $F \in (F_s, F_o)$ we have a merger which is both socially and privately profitable but will fail to occur.

Socially beneficial mergers may also fail to occur in the case of price competition among providers of complements. In fact in that case examples are particularly easy to construct since, as already pointed out, all mergers are then socially desirable, even in the absence of fixed costs ($F_s^p < 0$). To illustrate such failure to occur, consider the case of the merger of two railroads on a rail line with three independent companies ($n = 3, s = 2$). It can be verified that $F_s^p < 0 < F_p = F_o$.[31] If each firm had zero fixed costs, therefore, the merger would be socially beneficial but would be unprofitable and would not occur. Intuitively the merger would not occur since it involves less than 80 per cent of the industry and would therefore be unprofitable. But, since it would result in lower prices and increased output, it would be socially beneficial.

In fact, under price competition with perfect complements some mergers may fail to occur which are both socially and privately profitable. Consider a merger of all three firms in the same three-firm industry ($n = 3, s = 3$). It can be verified in this case that $F_s^p < F_p < 0 < F_o$.[32] If firms have zero fixed costs, therefore, such a merger is clearly both socially and privately profitable. But it will nonetheless fail to occur endogenously. To review the argument which by now should be familiar: each of the two acquired firms would insist on at least the duopoly profit ($2\beta^2/9$) since it expects that it would get that amount by unilaterally raising its asking price and operating solo in a two-firm industry. But the acquiring firm will not pay this since the amount exceeds the gain ($\beta^2/4 - \beta^2/16$) to it from turning the three-firm industry into a monopoly.

5 CONCLUDING REMARKS

In this chapter we have reviewed some recent contributions to the theory of horizontal mergers. After explaining a property of oligopoly models on which merger theory is based, we discussed the private and social implications of exogenous mergers. Finally we summarized a model of endogenous mergers.

The reader will surely have noticed (both from the dates on the papers we have cited and from the roughness of the ideas) that the theory of horizontal mergers is still in its infancy. Until Kamien and Zang's two-stage game, no one had built and analysed a model of endogenous mergers. Their paper constitutes an instructive beginning. Nonetheless, as they would surely be the first to admit, it omits important aspects of the merger decision (such as the behaviour of shareholders in determining whether a merger occurs). Future papers will presumably analyse more realistic merger games.

We have tried in this chapter to emphasize points which will remain important in future research. Thus future models will surely be based on the comparative statics discussed in Section 2. And, while they will presumably make different predictions from those of Kamien and Zang about which mergers will occur endogenously, attention will ultimately return to the two issues we have emphasized: the avoidance of socially injurious mergers which would occur endogenously and the promotion of socially beneficial ones which would not occur endogenously.

NOTES

1. This section is based on Gaudet and Salant (1991a).
2. See Gaudet and Salant (1991b) and note 4.
3. For details, see Gaudet and Salant (1991a, p. 660).
4. This pair of inequalities is closely related to the sufficiency conditions for existence and uniqueness of a pure strategy 'Cournot' equilibrium. See assumptions A. 4 and A. 5 of Gaudet and Salant (1991b).
5. For example, α would be smaller if we instead had a strictly convex inverse demand curve and constant marginal cost or, alternatively, a linear inverse demand curve and a strictly increasing marginal cost.
6. In the standard case, $n = 2$ and $s = 1$. But then $s - \alpha(n - s) = 1 - \alpha < 1$ for *any* $\alpha > 0$. Hence $d\pi/d\bar{q} > 0$. Thus, for this over-studied special case, the comparative-static results are independent of curvature assumptions.

7. Cournot was dead by the time Bertrand erroneously criticized him for omitting discussion of price as a strategic variable.
8. Since $dq/d\bar{q} > 0$, equation (6.5) implies that $dQ_{-i}/d\bar{q}$ is always positive – regardless of the size of s. But equation (6.4) then implies that $d\pi/d\bar{q} < 0$, regardless of how many firms are in the subset whose output is contracted exogenously.
9. We restrict our attention in this chapter to cases of substitutes in demand when discussing quantity competition and complements in demand when discussing price competition. But the analysis can of course be extended to cover complements in quantity competition and substitutes in price competition. See Gaudet and Salant (1991a, Appendix) as well as Gaudet and Salant (1989).
10. This section is based on Salant, Switzer and Reynolds (1983).
11. Or, alternatively, that marginal cost is constant and P measures price net of this constant marginal cost. We of course neglect here all issues related to exhaustibility of the oil resource stock. To account for exhaustibility, the full cost should include the opportunity cost of extracting the oil today rather than leaving it in the ground for future extraction.
12. The aggregate output of the newly merged entity will be sq^M, which maximizes the joint profits of the s firms given the equilibrium output of the outsiders. Notice that this is the Cournot-equilibrium output of each firm in an $(n - s + 1)$-firm industry. Since marginal costs are constant (zero) and there are no fixed costs, the equilibrium distribution of the aggregate output of the merged entity between its s constituent firms is indeterminate: it will be indifferent, for example, between having each produce q^M or having one produce sq^M and the $s - 1$ others produce zero.
13. The left-hand side of (12) has two roots in s. One of them is inadmissible, since it requires $s > n$. The admissible root is $s = (2n + 3 - \sqrt{(4n + 5)})/2$, which attains its minimum value of 4 when $n = 5$.
14. Zachau (1987) has worked out a particularly striking example of losses from merger due to failure to meet the global condition. He shows that, with a quadratic cost function and a constant elasticity demand curve of the form $p = bQ^{-1}$, the only profitable merger is a merger to monopoly ($s = n$). The reason is that for $1 < s < n$, the merger results in a reduction of output so large that $q^M < \gamma$. This occurs even though, for $s > n - s + 1$, a marginal contraction of output in the neighbourhood of the initial equilibrium still results in a gain.
15. In a recent paper, Farrell and Shapiro (1989) show how the external effect of a merger on consumers and non-participant firms can be used to provide sufficient conditions for privately profitable mergers to raise social welfare.
16. The change in industry output will be $\Delta Q = (n - s + 1)sq^M - nq^* = -(s - 1)q^*/(n - s + 2)$, which is negative for $1 < s \leq n$.
17. This may change when there are positive fixed costs, which we will introduce in the next section.
18. For an investigation of four major end-to-end mergers see Vellturo (1988) and the references therein.
19. This section extends Kamien and Zang (1990).
20. They also consider a second model in which a subset of acquired firms is operated independently in a decentralized manner. For an analysis of this 'decentralized game', see Kamien and Zang (1990).
21. In contrast, Salant, Switzer and Reynolds (1983, p. 198) briefly sketch an endogenous merger game with different rules. In their game, a unilateral defection by any prospective party to the merger means that the merger would fail to occur and that each firm would collect its pre-merger profit. This alternative approach eliminates from the equilibrium set *only* the unprofitable mergers. It eliminates no profitable mergers.
22. We will restrict our attention, in our examples, to levels of fixed costs which leave initial equilibrium profits positive; that is, levels of F which satisfy $\beta^2/(n + 1)^2 > F$. It is easy to verify that $\beta^2/(n + 1)^2 > F_p$ for all n and s. Therefore there are values of F which simultaneously leave profits positive and exceed the threshold level F_p.
23. Condition (6.21) is condition (6.14) with $x = s - 1$. Although necessary, it is not by

itself sufficient for the endogenous merger of the s firms to constitute an equilibrium of the two-stage game. For this reason, F greater than F_o is not sufficient for the merger to be an equilibrium. A sufficient condition for the merger to be an equilibrium is that (6.14) holds for all $x = 1, \ldots, s - 1$. To illustrate, if $s = 2$ then the range of x degenerates to $x = 1$ and condition (6.21) is necessary and sufficient for existence of an equilibrium with endogenous merger.

24. The presence of fixed costs associated to charging a positive price makes mergers in price competition with complements more socially desirable and also more likely to occur.
25. As Vellturo (1988) discusses, the US government was unable for decades to induce railroads adjoining end-to-end to 'consolidate' despite the efficiency gains which such mergers would have created.
26. In particular, $F_p = F_o = -0.0278\beta^2$ and $F_s = 0.0694\beta^2$. In this case, $F > F_o$ is sufficient for the existence of an equilibrium with merger (see note 23).
27. In particular, $F_p = F_o = 0.0139\beta^2$ and $F_s = 0.0243\beta^2$.
28. In particular, $F_p = -0.0313\beta^2$, $F_o = 0.0174\beta^2$ and $F_s = 0.0469\beta^2$.
29. In particular, $F_s = 0.0113\beta^2$ and $F_p = F_o = 0.0175\beta^2$.
30. In particular, $F_p = 0.0044\beta^2$, $F_s = 0.0178\beta^2$ and $F_o = 0.0269\beta^2$.
31. In particular, $F^p_s = -0.0590\beta^2$ and $F_o = F_p = 0.0139\beta^2$.
32. In particular, $F^p_s = -0.0781\beta^2$, $F_o = 0.0174\beta^2$ and $F_p = -0.0313\beta^2$.

7. New Dimensions of the Patent System

Manfredi La Manna

1 INTRODUCTION

The last few years have witnessed a renewed and sustained interest in the economics of patents by theoretical economists. Unfortunately this interest has been due more to the opportunity to apply to patents concepts developed in other fields (notably game theory, and especially bidding games and multi-stage games) than to a genuine interest in the specific – and often intriguing – features of the patent system.

In this chapter we will not consider models of patent races, not because they have already been surveyed elsewhere,[1] but for the deeper reason that much of the literature has extended the basic patent race model in directions that ignore some of the very characteristics of patents. Partha Dasgupta, a leading contributor to the new economics of R&D, has admirably captured the underlying structure of patent races models, as follows:

> N players ($N \geqslant 2$) bid for an indivisible object valued by each at V (> 0). All bids are forfeited. The highest bidder wins the object. If there are K ($\leqslant N$) highest bidders each of these (K) players wins the object, with probability $1/K$. (Dasgupta, 1986, pp. 535–6)

Undoubtedly the analogy of patent races with bidding games is a powerful one and has furnished many interesting insights, but at a price – the neglect of some of the distinctive features of patents and their relationships with R&D.

The criticism advanced in this chapter is not the sterile one of 'unrealism': as Dasgupta has pointedly reminded economic theorists of R&D, a model should be judged, among other things, with respect to its intended use and, given a specific purpose, the more 'economical' the model, the better. The point is that many models of patents abstract from issues that ought to be of fundamental importance in designing a 'good' (if not optimal) patent system, such as: *What* should patents be awarded to? How