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The Revenue Maximization Oligopoly Model: Comment

Since Professor Baumol's substitution of constrained revenue maximization for profit maximization in the objective function attributed to rational oligopolists, the nature of the relationships between advertising outlay, price, output, and production cost in the oligopoly equilibrium solution has been radically altered [1]. However, because these relationships are clearly articulated neither graphically nor in the mathematical appendix of Baumol's book (even the revised edition), their appreciation has been limited. Moreover, a recent article integrating both the price and advertising outlay variables and touted as *the* general statement of the Baumol model is, as will be shown, but a special case of the Baumol model [2]. In this commentary, we shall present a general model of the sales maximization hypothesis, articulating the above relationships and remedying the shortfalls in generality.

In the text of *Business Behavior, Value and Growth*, Baumol presents his admittedly "partial" sales maximization argument together with its equilibrium advertising outlay-price-output solution. The equilibrium output is defined with production costs and price as variables but with advertising outlay excluded. The equilibrium level of advertising depends on advertising outlay with production costs and presumably output held constant. Total revenue is a standard parabolic function of output, marginal production cost is a positive and increasing function of output, and marginal revenue is a positive but decreasing function of advertising outlay. Given a minimum profit constraint, Baumol demonstrates the impossibility of any combination of advertising outlay and output yielding both a profit above the constraint and maximum sales revenue [1, p. 60].

In the mathematical note, this problem is stated as one of maximizing a total revenue function (price times quantity) subject to a minimum profit constraint which includes both advertising and production outlays. Here, marginal production cost is taken to be positive and relevant to the determination of the equilibrium price and output [1, p. 69].

In an article by R. L. Sandmeyer [2], Baumol is criticized for failing to integrate the sales maximizer's price policy with his advertising policy in describing the equilibrium. This situation is then "remedied" through a graphical model which "allows price, advertising outlay, and product output to change" [2, p. 1078]. Beginning in a no-advertising situation in which total revenue is maximized at an output unconstrained by the profit minimum, Sandmeyer demonstrates that profit in excess of the minimum will be devoted to advertising outlay, generating a new and higher total revenue function, a new price and a new excess profit level which in turn is devoted to advertising outlay, etc., until eventually equilibrium output, price, and advertising outlay are determined. At this equilibrium, the level of profit equals the minimum acceptable.

Two things should be noted with respect to this model. First, it is a "partial" model in that all production and marketing costs are fixed and, consequently, the nonadvertising marginal cost of increasing sales is zero

over the relevant range. Second, because of the implicit zero marginal cost, the equilibrium defined by Sandmeyer occurs at the peak of one of the total revenue functions, i.e., where marginal revenue is zero—a solution which is inconsistent with the basic equilibrium conditions of the model as stated by Baumol [1, pp. 61, 69].

In the following model, the equilibrium for a revenue maximizer will be shown when price, cost, output, and advertising outlay are all free to vary. This model will be referred to as the general Baumol model of which both Baumol's "partial" presentation and Sandmeyer's model are special cases.

Define, as does Baumol,

$D(x, a)$ —demand function representing the price which will be paid when x units are offered after a dollars of advertising outlay;

π —minimum acceptable profits;

$C(x)$ —total cost of production function;

$A(a)$ —total cost of advertising function;

$x \cdot D(x, a)$ —total revenue function representing the maximum total revenue saved from selling x units after a units of advertising.

Given these definitions, we maximize the total revenue function, subject to a profit constraint:

$$(1) \quad x \cdot D(x, a) - \pi - C(x) - A(a) \geq 0.$$

Using the Lagrange multiplier technique (and Kuhn-Tucker conditions), we have:

$$(2) \quad \frac{\partial}{\partial x} \{x \cdot D(x, a) - \lambda[x \cdot D(x, a) - \pi - C(x) - A(a)]\} \leq 0,$$

$$(3) \quad \frac{\partial}{\partial a} \{x \cdot D(x, a) - \lambda[x \cdot D(x, a) - \pi - C(x) - A(a)]\} \leq 0.$$

Differentiating (2) and (3), solving for λ , and substituting, we have, for interior points,

$$(4) \quad x \frac{\partial D}{\partial a} \cdot \frac{dC}{dx} = D \frac{dA}{da} + x \frac{\partial D}{\partial x} \cdot \frac{dA}{da},$$

which is the statement of the equilibrium condition. Letting $TR(x, a) = x \cdot D(x, a)$, this equilibrium condition (4) can be written as:¹

$$(5) \quad \frac{\partial TR}{\partial a} \left(\frac{dC}{dx} \right) = \frac{\partial TR}{\partial x} \left(\frac{dA}{da} \right).$$

Note that, by assumption, the Baumol formulation requires the following conditions,

$$\frac{\partial TR}{\partial a} > 0, \quad \frac{dC}{dx} > 0, \quad \text{and} \quad x \frac{dA}{da} = 1,$$

¹ This condition can easily be generalized to allow for more than one output and advertising outlay. See [1, pp. 68–69].

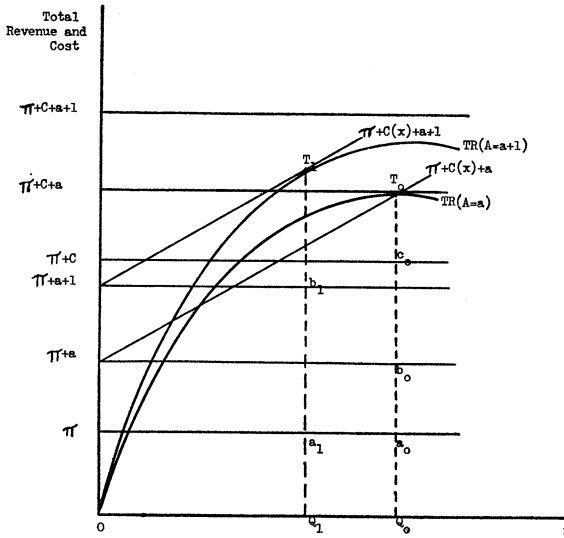


FIGURE 1

[1, pp. 54, 59] which, in turn, necessitate $\partial TR/\partial x > 0$. This is to be compared with the $\partial TR/\partial x = 0$ claimed by Sandmeyer as a characteristic of the equilibrium solution.

Graphically, Sandmeyer's equilibrium position is shown as T_0 in Figure 1. With the TR functions representing the relationship of total revenue to output at different levels of advertising outlay, the minimum profit constraint represented by π , and the production cost function implied by the horizontal curve $\pi + C$, this solution gives an output of OQ_0 , a price of T_0Q_0/OQ_0 , and advertising outlay of $c_0T_0 = a_0b_0$. Clearly, this solution implies $\partial TR/\partial x = dC/dx = 0$, which, as noted above, is inconsistent with the requirements of the generalized Baumol model.

The solution of the general Baumol model is also shown in Figure 1. Again, the TR functions represent a family of total revenue functions at different levels of advertising outlay and the level of minimum profit is given by π . In this case the production cost function is implied by $\pi + C(x) + a$. For each level of advertising outlay, a new total cost function (including π) is observed. At output OQ_0 , the advertising outlay is $a_0b_0 = c_0T_0$, total production cost is equal to b_0T_0 , and total revenue equals total cost including the minimum profit constraint. These conditions, then are true of both the Sandmeyer model and the general Baumol model at output Q_0 . However, while the sales maximizer is in equilibrium at Q_0 in the Sandmeyer model, he is *not* in equilibrium in the general Baumol model. The next unit of advertising will be purchased, output will be cut back from OQ_0 to OQ_1 , the price of the output will be raised from T_0Q_0/OQ_0 to T_1Q_1/OQ_1 , and total production cost will decrease from T_0b_0 to T_1b_1 . The change will be undertaken because total sales revenue is increased from T_0Q_0 to T_1Q_1 .

In general, then, the sales maximizer in the general Baumol model will

demonstrate a smaller output, a greater advertising outlay, a higher price, and a greater sales volume than will the sales maximizer in the Sandmeyer version of the Baumol model. With $dC/dx > 0$, the oligopolist will find it in his interest to allocate profit surplus from production to advertising in the process of generating sales revenue. Moreover, in the general Baumol model the tendency toward high product price, excessive advertising outlay, and more intense output restriction will be positively related to the elasticity of marginal cost.

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1. W. J. BAUMOL, *Business Behavior, Value and Growth*, rev. ed. New York 1967.
2. R. L. SANDMEYER, "Baumol's Sales-Maximization Model: Comment," *Am. Econ. Rev.*, Dec. 1964, 54, 1073-1081.

Incidence of the Corporation Income Tax in U.S. Manufacturing: Comment

In responding to previous critiques of our incidence study [4], we have conceded that our initial effort should eventually come to be replaced by a more complex approach, involving a structural model in which price, wage, and shifting behavior are specified, and all equations are identified. While the required data are exceedingly difficult to obtain, such a reformulation may eventually be forthcoming. Such a model may not only provide better information on total shifting, but also show how shifting comes about and what "direction" it takes. It is with great anticipation, therefore, that one follows beyond the opening pages of a recent article by R. J. Gordon, where such an approach appears to be taken.¹ But as the paper proceeds, it becomes apparent that no such improvement is offered.

Tax-less Model

Gordon begins with a model for a tax-less world. He defines profits as sales minus costs, with costs a function of quantity, wage rates, and material prices. Sales equals product price times quantity, where price is determined by a behavioral equation involving mark-up prices. This assumption is worth testing, although other behavior hypotheses might have been used as well. In equation (7), Gordon attempts to present the system in reduced form. Some of the variables in (7) being unavailable, he proceeds to make certain substitutions, thus arriving at equation (12),

¹ See [2]. To simplify and to permit reference to our model, we consider the Gordon model only in its rate of return formulation.