Banks' capital structures and capital regulations

Okahara, Naoto

Graduate School of Economics, Kyoto University

27 August 2018

Banks' capital structure and capital regulations

Naoto Okahara*†

Abstract

This study proposes a model that describes banks' decisions about their capital structures and analyzes the effects of regulating banks' capital adequacy ratios (CAR); that is, the ratio of equity financing to risky assets. This study investigates whether bank lending decreases when the banks need to raise their CAR to satisfy the regulation. We analyze a model in which households have bargaining power regarding deposits and a bank must adjust its capital structure indirectly through the households' decision-making, and compare the results which that obtained in a model in which the bank has the bargaining power. In either case, the bank can suffer a loss when it raises its CAR. However, changes in the amount of lending in the two models differ. When the bank has the bargaining power, it always chooses to just use equity financing more, and thus there is no probability that bank lending decreases. When the households has the bargaining power, contrariwise, this model shows that the more risk-averse households are, the more likely the amount of lending is to decrease. These results can explain why banks' reaction to the CAR regulation are different from each other. Moreover, the results indicate a positive probability that regulating banks' capital structures has a negative effect on the economy.

Keywords: Bank capital, Bank lending, Capital regulation,

JEL classification: E10, G18, G21

*Graduate School of Economics, Kyoto University, Yoshida-Honmachi, Sakyo-ku, Kyoto 606-8501, Japan, E-mail address: okahara.naoto.73u@st.kyoto-u.ac.jp

†This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

1 Introduction

After the financial crisis of 2007–2008, the danger of negative externalities that highly indebted financial institutions face gained attention and some argue that financial institutions should have more equity so that they can absorb shock. The new regulations on banks, Basel Ⅲ, require banks to have enough equity to satisfy the required capital adequacy ratio (CAR). Many studies analyze the relationship between banks' capital structure and financial stability, and most find that increasing banks' CAR contributes more or less to financial stability. However, there is no consensus on the cost of the regulation; that is, whether or not the regulation decreases banks' lending is still disputed.

This is because the models that prior studies use are not sufficient to investigate the regulations on banks' CAR. In order to analyze effects of the regulation, research must investigate the optimal strategies for banks on not only its lending but also its financing. In other words, we need investigate how banks mix deposit and equity financing and how the regulation affects banks' decisions. However, most models in the literature are not suited for such an investigation.

Recent analyses of financial policies use dynamic stochastic general equilibrium (DSGE) models. Before the crisis, there was little attention on financial intermediaries, and few analyses adopted DSGE-type models that include the financial intermediary sector. Then, after the crisis, new types of DSGE models emerged in which banks (financial intermediaries) play the role of enhancing a shock. However, these models do not consider banks' equity financing and suppose that they accumulate their retained earnings as their capital for tractability. Thus, it is not possible to investigate how banks mix deposit and equity financing with these models.

Other models focus on analysis of banks' decision making. Before the crisis, most of these analyses were based on the traditional idea that deposit financing was optimal for banks. Then, the crisis raised doubts about this idea and many studies constructed models to analyze merits of equity financing. However, the problem in these analyses is that they do not consider the advantages of the other financing means; in other words, banks can use both deposit and equity financing, but only one of them is always superior. Then, without some restrictions, models in which deposit financing is always optimal show that the optimal CAR for banks without regulations is 0, whereas models in which equity financing is always optimal show that the optimal CAR is 1. Moreover, it is difficult to compare these models in order to obtain general implications on the effects of regulations on banks' CAR.

On the other hand, there are two important results in the existing literature. First, Lindquist (2004) and Aiyar et al. (2015) point out that some banks held enough capital to satisfy the levels in Basel Ⅲ before the crisis. In addition, Berger et al. (2008) points out that the capital in these banks is not retained earnings, but obtained by issuing new shares in the U.S. These results imply that banks' optimal CAR is neither 0 nor 1, and affected by the properties of the banks and the economy. Hence, we need to investigate not only how banks determine their capital structures, but also how they adjust them. Second, Kanngiesser et al. (2017) point out that some banks in the EU decrease lending in order to increase their CARs. Ben Naceur et al. (2018) analyze data on bank holding companies in the United States and Europe, and they show that capital regulations have more significant and negative impacts on European banks than they do on U.S. banks. Therefore, banks' reactions to a CAR regulation are not uniformly determined, and it is possible that banks decrease their lending under the regulation, although those who support regulations on banks' CARs argue[1] that banks can increase their CARs by issuing new shares and the regulations do not decrease banks' lending. Thus, we need to investigate what causes some banks to decrease their lending to satisfy the regulation whereas others issue new shares. Recently, it is pointed out that the effect of capital regulations is nonlinear with respect to their capitalizations (Olszak et al., 2016), but we analyze a nonlinear effect regarding to depositors' properties.

In conclusion, this study investigates what leads banks to decrease their lending to raise their CARs when they can issue new share for that purpose. We therefore analyze how a bank determines and adjusts its capital structure using models in which it has an incentive to use both deposit and equity financing; that is, the optimal CAR is likely to be neither 1 nor 0. In addition, we investigate banks' decisions, taking into account the interactions between banks and households (banks' depositors).

In our theoretical models, we suppose that the bank's objective is to maximize its expected return on equity (ROE),[2] [3] but it must keep the probability of bankruptcy under a fixed level in order to attract depositors.[4] Based on these suppositions, we investigate two types of models. The bank has bargaining power regarding deposits in the first model, and the households (the bank's depositors) has the bargaining power in the second one.

The first model shows that the optimal CAR for the bank can be either an interior solution or 0. When the lower limit of the reserve ratio is sufficiently small or the parameter of the depositors' demand for the bank's soundness is sufficiently large, the bank uses both deposit and equity financing and its CAR is an interior solution, otherwise the bank use only equity financing and its CAR is 0. With regard to regulating CARs, we have two results using this model. First, there is a probability that banks with a small CAR relative to the level of the new regulation will suffer a decrease in their expected ROE. Second, there is no probability that these unsound banks decrease their lending to satisfy the regulation, and thus regulating banks' CARs never harms the economy.

In the second model, the households determine the amount of deposit, and thus the bank must

[1] Admati et al. (2014) summarizes supporters' arguments for regulations on banks' CARs and responses to the opponents' assertions.

[2] Bhattacharyya and Purnanandam (2011) points out that many banks determined rewards for their managers based on their ROE before the crisis, and that this payment structure led to the banks' risk-taking and made the financial system more unstable.

[3] With regard to why banks are reluctant to issue new shares, some argue that debt financing (deposit financing) is superior to equity financing in terms of reducing tax payments.

[4] As these suppositions indicate, the ratio of deposit financing to equity financing plays an important role in this analysis, and the amount of these financing sources and total size of the bank's balance sheet are indeterminate. Although the heterogeneity of banks in response to the size of their balance sheets is an important issue, it is beyond the scope of this study and we do not treat the issue.

indirectly lead them to hold a higher share. Then, we obtain two new results when the households are risk-averse. First, the optimal CAR is more like to be an interior solution than it is in the first model, because they want hold deposits rather than shares. This result indicated that households' demand is one of the factors that determine the amount of banks' deposit financing, and this is similar to the result of Van den Heuvel (2008). Second, there is a probability that unsound banks decrease their lending to satisfy the regulation; that is, a CAR regulation can work as to slow down economic activities when we consider the household's decisions. These results imply that we should pay more attention to the interaction between banks' decisions and households' demand and their adjustment process.

The remainder of this paper is organized as follows. Section 2 reviews literature. Section 3 presents the model in which the bank has bargaining power regarding deposits, and examines the effects of CAR regulation. Section 4 presents the model in which the households have bargaining power regarding deposits, and examines effects of the regulation. Section 5 presents our conclusion. Proofs of the all lemmas, propositions, and corollaries are presented in the Appendix.

2 Review of literature

As Thakor (2014), theories about the effect of bank capital (equity financing) on the bank's behaviors are divided into three groups. The first group argues that banks should rely on deposit financing and that high leverage is optimal for banks. It supposes that deposit financing is superior to equity financing because banks can obtain rent via deposit financing. Thus, CAR regulation is not always desirable for banks because it leads them to decrease their deposit financing. Various factors can be the source of rent. For example, DeAngelo and Stulz (2015) argued that banks' deposit financing works as a provision of liquidity to households who cannot access the capital market and earns rent. Krishnamurthy and Vissing-Jorgensen (2012) empirically showed that bank deposit have social values as safe and liquid assets.

Theories in the second group also argue that banks should rely on deposit financing, though for a different reason. Though theories in the first group are based on the merit of bank deposits, theories in the second group emphasize that the bank deposit financing works as discipline on banks. When a bank relies heavily on deposit financing, it is exposed to a high risk of bankruptcy. Thus, the bank avoids taking too much risk in order to prevent depositors from withdrawing en masse. Based on this idea, CAR regulation is not always desirable for either banks or the entire economy because it could decrease banks' deposit financing and leads banks to aggressive risk taking. This idea is based on such studies as those by Diamond and Dybvig (1983) and Calomiris and Kahn (1991), and theories in this group represent a large part of the literature on banks' behavior from the 1980's to the early half of the 2000's.

Theories in the third group argue that banks should rely on equity financing. These theories can be divided further into three subgroups according to why equity financing is desirable. Theories in the first subgroup regard equity financing as a means to make the bank's objectives consistent with those of stockholders, and restrains the bank's asset substitution; that is, excessive risk-taking (Jensen and Meckling, 1976; Keeley, 1990). Theories in the second subgroup emphasize that banks' reliance on equity financing improves their ability to absorb shocks (Repullo, 2004; Coval and Thakor, 2005). Theories in the third subgroup argue that equity financing rather than deposit financing forces discipline on banks because bank managers pay attention to their stockholders' profit.

As the above shows, theories in these three groups shore some criteria for judging bank financing methods. However, as Thakor (2014) pointed out, most of the models these theories use are inconsistent with each other. On the one hand, models in theories supporting deposit financing do not consider any merits of equity financing, while models in theories supporting equity financing show that deposit financing leads only to an increasing risk of bankruptcy. Thus, the optimal CAR in these models is always either 1 or 0, unless there is a limitation on the amount of deposit or equity. Because discussions on costs of CAR regulation after the crisis are based mainly on these models, there is no consensus on the cost yet.

Recently, however, some models in which both deposit and equity financing have merits appeared, and the models in this study belong to this group. For example, Acharya et al. (2016) supposed that using deposit financing gives banks an incentive to monitor their borrowers, whereas using equity financing prevents banks from taking excessive risk. Acharya and Thakor (2016) found that both deposit and equity financing discipline banks, though how they do so differs. Chen (2016) argued that banks can reduce the risk of bankruptcy by relying on equity financing, but the cost of equity financing depends on the severity of the competition in the credit market, and thus pointed out that deposit financing could be relatively desirable.

With respect to the effects of capital regulations, Carlson et al. (2013) test the hypothesis that the association between capital regulations and growth of banks' lending is nonlinear and provides strong empirical support for that. Olszak et al. (2016) also shows that lending of poorly capitalized banks is more affected by capital regulation than lending of well-capitalized banks. However, in the theoretical literature, how changes in CAR restrictions affect banks' capital structure, or whether increasing a bank's CAR leads it to decrease its lending is still not clear. Moreover, most prior studies analyze banks' behaviors based on relationship between banks and their borrowers (firms); that is, from the perspective of bank lending. Thus, we here focus on the relationship between banks and their depositors and investigate how the CAR affects banks' capital structures and their lending.

3 Baseline Model

In this section, we develop a simple one-period ($T = 0, 1$) model with a bank and households, and the bank determines the amount of deposit. The settings of the bank are based on Thakor (2014), and we introduce the bank's bankrupt as Allen et al. (2015). We then investigate how the bank determines its capital structure and how regulations on its CAR affect its decisions using the model.

3.1 Settings

Suppose that the economy has one bank and households. At $T = 0$, the bank collects fund by receiving deposits from the households (deposit financing) and issuing shares (equity financing). The total amount of funds that the bank can collect from households is fixed and denoted as G. In addition, we denote the amount of funds collected using deposit financing as D_{B} and that collected using equity financing as E_{B} (subscript B indicates that it the bank's choice). Thus, $G = D_{\mathrm{B}} + E_{\mathrm{B}}$ is always satisfied.

After collecting G, the bank invests the funds in a risky project, and at $T = 1$, it receives the return. We suppose that the bank holds some parts of G as reserves and does not invest it, and that the amount of the reserves is determined by the amount of deposits D_{B}. We denote the reserve as sD_{B} with the bank's reserve ratio s. Thus, the bank's investment amount, L_{B}, is defined as

$$L_{\mathrm{B}} \equiv D_{\mathrm{B}} + E_{\mathrm{B}} - sD_{\mathrm{B}} = G - sD_{\mathrm{B}}.$$

The profit ratio r of the bank's investment is a random variable and uniformly distributed on the support $[0,\, R]$ such that $R > 2r_d$ is satisfied. We denote the density function and the cumulative distribution function of r as $f(r)$ and $F(r)$, respectively. In addition, we suppose that the profit ratio of reserves is 1 and there exists a lower limit on s s, and denote it as $\underline{s} \in (0, 1)$.

At $T = 1$, the bank receives the return on its investment, and first repays to the depositors, and then pays dividends to stockholders.We assume that the repayment to the depositors is the sum of the principal and the interest and denote the repayment ratio as r_d. Then, thus, the amount of repayment to depositors is $r_d D_{\mathrm{B}}$. The amount of funds that the bank can use for payment is $sD_{\mathrm{B}} + rL_{\mathrm{B}}$, and thus, there is no dividend unless $sD_{\mathrm{B}} + rL_{\mathrm{B}} > r_d D_{\mathrm{B}}$ is satisfied. In other words, when $sD_{\mathrm{B}} + rL_{\mathrm{B}} < r_d D_{\mathrm{B}}$ is satisfied, the return on the investment is not sufficient to even repay depositors, and consequently, the bank goes bankrupt and its funds are distributed among the depositors equally.

There is a threshold rate $\underline{r}$ below which the bank goes bankrupt, and it is defined as

$$\underline{r} = \frac{(r_d - s)D_{\mathrm{B}}}{L_{\mathrm{B}}}.$$

The probability that the bank goes bankrupt is $F(\underline{r})$. We assume that the bank must keep the probability not more than $\gamma \in [0, 1]$ in order to receive deposits from households. We interpret this parameter γ as the depositors' demand for the bank's soundness. In addition, we suppose that $R\gamma < 1$ is satisfied, meaning that the threshold profit rate $\underline{r}$ is less than 1 when the bank is sound enough to satisfy the constraint $F(\underline{r}) \leq \gamma,\ \gamma \in [0, 1)$.[5]

We suppose that the bank tries to maximize its expected ROE, and define ROE as the dividend per amount of equity financing. Based on the above suppositions, the bank has an incentive to be reluctant to issue shares to keep its expected ROE high on the one hand, and it will be reluctant to rely only on deposit financing to keep its probability of going bankrupt small on the other hand.

[5]The probability that a bank goes bankrupt is $F(\underline{r}) = \underline{r}/R$; thus, we denote the constraint on the probability as $\underline{r}/R \leq \gamma$. By rewriting the expression, we have $\underline{r} \leq R\gamma$, and thus have $\underline{r} < 1$ when $R\gamma < 1$ is satisfied.

3.2 Analysis of Baseline Model: Optimal CAR

The maximization problem for the bank is

$$\max_{D_{\mathrm{B}},\ E_{\mathrm{B}},\ s} \ \mathrm{E}[ROE] \equiv \int_{\underline{r}}^{R} \frac{1}{E_{\mathrm{B}}}\Big[(D_{\mathrm{B}} + E_{\mathrm{B}} - sD_{\mathrm{B}})r + sD_{\mathrm{B}} - r_d D_{\mathrm{B}}\Big] f(r)dr,$$
$$s.t. \qquad F(\underline{r}) \leq \gamma,$$
$$0 < \underline{s} \leq s \leq 1.$$

We denote the ratio of deposit financing to total funding as $d = D_{\mathrm{B}}/G$, and denote the expected ROE as $\mathcal{R}_{\mathrm{B}}(d, s)$, a function of d and s. Because $F(r)$ is defined as r/R, we can rewrite the problem above as follows.

$$\max_{d,\ s} \qquad \mathcal{R}_{\mathrm{B}}(d, s) \equiv \frac{\big[R(1 - sd) - (r_d - s)d\big]^2}{2R(1 - d)(1 - sd)} \tag{3.1}$$

$$s.t. \qquad d \ \leq \ \frac{R\gamma}{(r_d - s) + R\gamma \cdot s} \ \equiv d_\gamma(s) \tag{3.2}$$

$$\underline{s} \leq s \leq 1$$

Considering that the constraint $F(\underline{r}) \leq \gamma$, $d = 0$ must be satisfied when $\gamma = 0$, we suppose $\gamma \in (0, 1]$. In addition, from equation (3.1), we assume that $d \neq 1$ and $sd \neq 1$ are satisfied.

Definition 1. *Define the threshold value of d as*

$$\hat{d}(s) \equiv \frac{2(r_d - s) - R(1 - s)}{(R - 1)s^2 - (R - r_d + 1)s + r_d}\ .$$

Then, we have the following lemma on the $\mathcal{R}_{\mathrm{B}}(d, s)$ defined in equation (3.1).

Lemma 1. *1. The following relationship between $\mathcal{R}_B(d, s)$ and s is satisfied.*

- *(a) When $d > 0$, $\mathcal{R}_B(d, s)$ is monotonically decreasing in s.*
- *(b) When $d = 0$, $\mathcal{R}_B(d, s)$ does not depends on s.*

2. The following relationship between $\mathcal{R}_B(d, s)$ and d holds, where $\hat{d}(s)$ is the threshold value defined as Definition 1 .

- *(a) When $s < \dfrac{R - 2r_d}{R - 2}$, $\mathcal{R}_B(d, s)$ is monotonically increasing in d.*
- *(b) When $\dfrac{R - 2r_d}{R - 2} \leq s \leq \dfrac{R - r_d}{R - 1}$,*
  - *i. When $d > \hat{d}(s)$, $\mathcal{R}_B(d, s)$ is monotonically increasing in d.*
  - *ii. When $d = \hat{d}(s)$, $\mathcal{R}_B(d, s)$ depends on only s.*

iii. When $d < \hat{d}(s)$, $\mathcal{R}_B(d,s)$ is monotonically decreasing in d.

(c) When $\dfrac{R-r_d}{R-1} < s$, $\mathcal{R}_B(d,s)$ is monotonically decreasing in d.

For the threshold value $\hat{d}(s)$, we can rewrite equation $\hat{d}(s) = 1$ as

$$(R-1)(1-s)\left(\frac{R-r_d}{R-1} - s\right) = 0.$$

Thus, the curve $d = \hat{d}(s)$ intersects with the line $d = 1$ at $s = 1$ and $s = (R-r_d)/(R-1)$. In addition, it intersects with the line $d = 0$ at $s = (R-2r_d)/(R-2)$. Then, the relationship between s and d is shown in Figure 1, with some $\underline{s} < (R-2r_d)/(R-2)$

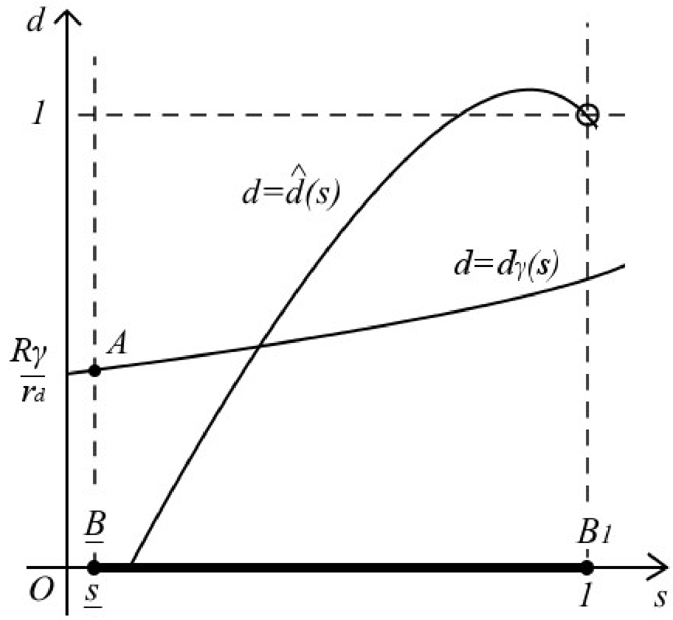


Figure 1: the relationship between s and d

As equation (3.2) shows, $d = d_\gamma(s)$ is monotonically increasing in s, and $0 < d_\gamma(0) < d_\gamma(1) < 1$ holds. From Lemma 1.1 , the bank has an incentive to decrease s unless $d = 0$. In addition, from Lemma 1.2 , a larger d is optimal when $d > \hat{d}(s)$, whereas a smaller d is optimal when $d < \hat{d}(s)$. Thus, in Figure 1, the possible equilibria of the maximization problem is point A and the points on segment $\underline{B}B_1$.

To investigate the equilibrium, we have the following definitions.

Definition 2. *Define a bank's CAR as*

$$\tau_B(s,d) \equiv \frac{E_B}{L_B} = \frac{1-d}{1-sd}.$$

Definition 3. *Define the function $\tilde{\gamma}(s)$ as*

$$\tilde{\gamma}(s) \equiv \frac{2(r_d - s) - R(1-s)}{r_d - s}.$$

The function $\tilde{\gamma}(s)$ is the function to calculate the threshold value of γ, on which the bank's optimal choice of (d, s) depends.

We thus summarize the equilibrium of the maximization problem as follows.

Proposition 1. *Denote the equilibrium of the maximization problem as (d^*, s^*), and denote the bank's expected ROE and CAR as $\mathcal{R}_B^* \equiv \mathcal{R}_B(d^*, s^*)$ and $\tau_B^* \equiv \tau_B(d^*, s^*)$, respectively. In addition, define $\tilde{\gamma}(s)$ as Definition 3. Then, (d^*, s^*), $\mathcal{R}_B^*$ and τ_B^* is determined as follows.*

1. *When $\underline{s} < \dfrac{R - 2r_d}{R-2}$, $\gamma > \tilde{\gamma}(\underline{s})$ always holds. Then, the equilibrium is $(d_\gamma(\underline{s}), \underline{s})$; that is, point A in Figure 1. In this case, express (d^*, s^*), $\mathcal{R}_B^*$, and τ_B^* as*

$$d^* = \frac{R\gamma}{(r_d - \underline{s}) + R\gamma \cdot \underline{s}} \ , \ s^* = \underline{s} \ , \ \mathcal{R}_B^* = \frac{R}{2} \cdot \frac{(r_d - \underline{s})(1-\gamma)^2}{(r_d - \underline{s}) - R\gamma(1-\underline{s})} \ , \ \tau_B^* = \frac{(r_d - \underline{s}) - R\gamma(1-\underline{s})}{r_d - \underline{s}} \ .$$

2. *When $\dfrac{R-2r_d}{R-2} \le \underline{s} \le \dfrac{R - r_d}{R-1}$,*

   (a) *When $\gamma > \tilde{\gamma}(\underline{s})$, the equilibrium is $(d_\gamma(\underline{s}), \underline{s})$, and (d^*, s^*); $\mathcal{R}_B^*$ and τ_B^* are the same as the first case in this proposition.*

   (b) *When $\gamma \le \tilde{\gamma}(\underline{s})$, the equilibrium is $(0, \underline{s})$, and $\mathcal{R}_B^*$ and τ_B^* are the same as the third case in this proposition.*

3. *When $\dfrac{R - r_d}{R-1} < \underline{s}$, $\gamma \le \tilde{\gamma}(\underline{s})$ always holds. Then, the equilibrium is $(0, \underline{s})$; that is, as any point on segment $\underline{B}B_1$. In this case, express (d^*, s^*), $\mathcal{R}_B^*$, and τ_B^* as*

$$d^* = 0 \ , \quad \forall\, s^* \in [\underline{s}, 1], \quad \mathcal{R}_B^* = \frac{R}{2}, \quad \tau_B^* = 1 \ .$$

Thus, when the lower limit of the reserve ratio $\underline{s}$ is sufficiently small or the parameter of the depositors' demand for the bank's soundness, γ, is sufficiently large, the bank uses both deposit and equity financing, and the equilibrium is thus $(d_\gamma(\underline{s}), \underline{s})$ and the optimal CAR is an interior solution. On the other hand, when $\underline{s}$ is sufficiently large or γ is sufficiently small, it uses only equity financing, and its CAR is thus 1.

3.3 Analysis of Baseline Model: CAR Regulation

In this subsection, we analyze how regulating the bank's CAR affects its capital structure. Suppose that a new regulation requires the bank to keep its CAR below the mandatory level of $\underline{\tau} \in (0, 1)$. In other words, the bank must satisfy the new constraint on its CAR, $\tau_{\mathrm{B}}(d, s) \geq \underline{\tau}$. By using Definition 2, we can rewrite the constraint as

$$d \leq \frac{1-\underline{\tau}}{1-s\underline{\tau}} \equiv d_{\tau}(s) \ . \tag{3.3}$$

As equation (3.3) shows, $d = d_{\tau}(s)$ is monotonically increasing in s, and $0 < d_{\tau}(0) < d_{\tau}(1) = 1$ holds. Thus, the function $d = d_{\tau}(s)$ is as in Figure 2 with some $\underline{\tau}$, and the larger $\underline{\tau}$ is, the lower the curve $d = d_{\tau}(s)$ is. As Figure 2 shows, the two curves $d = d_{\gamma}(s)$ and $d = d_{\tau}(s)$ have one intersection point within $s < 1$ as long as $d_{\gamma}(0) > d_{\tau}(0)$. When $d_{\gamma}(0) \leq d_{\tau}(0)$, all of the equilibria described in Proposition 1 satisfy the regulation. Therefore, we here suppose that the regulation level $\underline{\tau}$ satisfies $d_{\gamma}(0) > d_{\tau}(0)$.

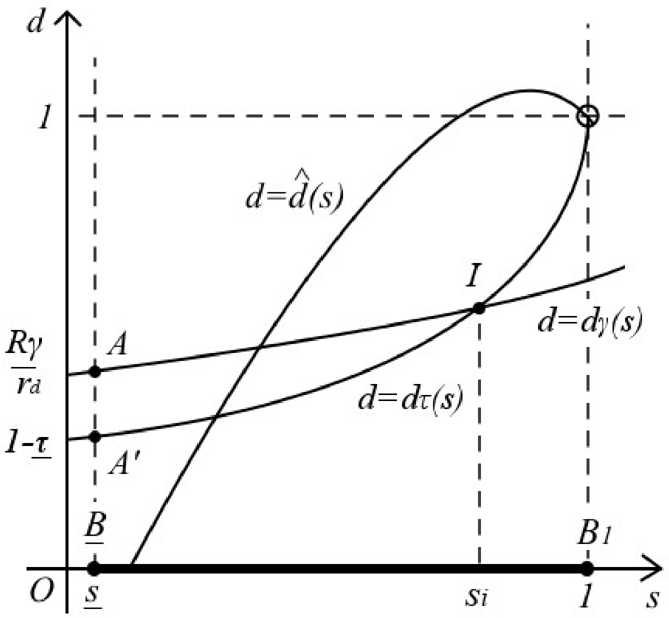


Figure 2: the relationships between d, s, and CAR regulation

When $d = 0$ holds at the equilibrium, the bank's CAR is 1, as in Proposition 1. Thus, the new CAR regulation affects only the bank that chooses $(\hat{d}(\underline{s}), \underline{s})$; that is, point A, as optimal choice without the regulation. In addition, Lemma 1 still holds when there is the new constraint $d \leq d_{\tau}(s)$, and the possible equilibria under the new regulation are point A' and the points on segment $\underline{B}B_1$ in Figure 2.

In order to investigate the equilibrium with a regulation on the bank's CAR, we have the following definitions.

Definition 4. *Define the functions* $\tilde{\tau}(s)$ *as*

$$\tilde{\tau}(s) \equiv \left[\frac{R(1-s)-(r_d-s)}{r_d-s}\right]^2 . \tag{3.4}$$

Definition 5. *Define* s_i*, which satisfies* $d_\gamma(s_i) = d_\tau(s_i)$*, as*

$$s_i \equiv \frac{R\gamma-(1-\underline{\tau})r_d}{R\gamma-(1-\underline{\tau})} .$$

s_i *is positive and less than 1 as long as* $R\gamma/r_d > 1-\underline{\tau}$*; in other words,* $\tau_B(d_\gamma(\underline{s}), \underline{s}) < \underline{\tau}$*.*

We summarize the equilibrium of the maximization problem with a new regulation on the bank's CAR as follows.

Proposition 2. *Suppose that a new regulation on banks' CAR,* $\tau(d, s) \geq \underline{\tau}$*, is put into force and* $\tau(d_\gamma(\underline{s}, \underline{s})) < \underline{\tau}$*.*

Denote the equilibrium of the maximization problem under the new regulation as (d_τ^*, s_τ^*)*, and denote the bank's expected ROE and CAR under the regulation as* $\mathcal{R}_{B,\tau}^* \equiv \mathcal{R}_B(d_\tau^*, s_\tau^*)$ *and* $\tau_{B,\tau}^* \equiv \tau_B(d_\tau^*, s_\tau^*)$*, respectively. In addition, define* s_i *as Definition 5, and suppose that* $\underline{s} \leq s_i$*.*

Then, express (d_τ^*, s_τ^*)*,* $\mathcal{R}_{B,\tau}^*$*, and* $\tau_{B,\tau}^*$ *as follows, where* $\tilde{\tau}(s)$ *is defined as the expression (3.4) in Definition 4.*

1. *When* $\underline{s} < \dfrac{R-2r_d}{R-2}$*,* $\underline{\tau} < \tilde{\tau}(\underline{s})$ *always holds. Then, the equilibrium is* $(d_\tau(\underline{s}), \underline{s})$*,*
   , and is point A' in Figure 2. In this case, express (d_τ^*, s_τ^*)*,* $\mathcal{R}_{B,\tau}^*$*, and* $\tau_{B,\tau}^*$ *as*

$$d_\tau^* = \frac{1-\underline{\tau}}{1-\underline{s\tau}}, \qquad s_\tau^* = \underline{s}, \quad \mathcal{R}_{B,\tau}^* = \frac{1}{2R\underline{\tau}}\left[R-(1-\underline{\tau})\frac{r_d-\underline{s}}{1-\underline{s}}\right]^2, \quad \tau_{B,\tau}^* = \underline{\tau} .$$

2. *When* $\dfrac{R-2r_d}{R-2} \leq \underline{s} \leq s_i$*,*

   (a) *When* $\underline{\tau} < \tilde{\tau}(\underline{s})$*, the equilibrium is* $(d_\gamma(\underline{s}), \underline{s})$*, and* (d_τ^*, s_τ^*)*, and* $\mathcal{R}_{B,\tau}^*$*, and* $\tau_{B,\tau}^*$ *are the same as in the first case in this proposition.*

   (b) *When* $\underline{\tau} \geq \tilde{\tau}(\underline{s})$*, the equilibrium is* $(0, \underline{s})$*, and is any point on segment* $\underline{B}B_1$*. In this case, express* (d_τ^*, s_τ^*)*,* $\mathcal{R}_{B,\tau}^*$*, and* $\tau_{B,\tau}^*$ *as*

$$d^* = 0, \quad \forall\, s^* \in [\underline{s}, 1], \quad \mathcal{R}_B^* = \frac{R}{2}, \quad \tau_B^* = 1 .$$

When $\underline{s}$ *satisfies* $s_i < \underline{s} \leq 1$*, the equilibrium is determined as in Proposition 1.*

In addition, we have the following lemma on the expected ROE $\mathcal{R}_{\mathrm{B}}(d_\tau(s), s)$.

Lemma 2. *The expected ROE* $\mathcal{R}_B(d_\tau(s), s)$ *has the following properties.*

1. *When* $\underline{\tau}^2 < \tilde{\tau}(s)$, $\mathcal{R}_B(d_\tau(s), s)$ *is monotonically decreasing in* $\underline{\tau}$.
2. *When* $\tilde{\tau}(s) < \underline{\tau}^2$, $\mathcal{R}_B(d_\tau(s), s)$ *is monotonically increasing in* $\underline{\tau}$.

Then, we have following result with regard to how regulating the bank's CAR affects its capital structure.

Corollary 1. *Suppose that a bank chooses* $(d_\gamma(\underline{s}), \underline{s})$*, but needs to increase its CAR to satisfy the regulation* $\tau_B \geq \underline{\tau}$. *In addition, suppose that* $\underline{\tau} < 1$. *Then, the following results hold.*

1. *When* $\underline{s} \leq \dfrac{R - 2r_d}{R - 2}$ *and* $d_\tau^* \neq 0$, $\mathcal{R}_{B,\tau}^* < \mathcal{R}_B^*$ *hold.*
2. *In order to increase its CAR, the bank always chooses to increase the amount of its shares and does not decrease its lending.*

Corollary 1 implies two important results from the model in terms of regulating CARs. First, when the regulation exists and the lower limit of s is sufficiently small, banks with a small CAR relative to the level of the new regulation suffer a decrease in their expected ROE. Second, there is no probability that these unsound banks decrease their lending to satisfy the regulation. Thus, under the suppositions in the model in this section, regulating banks' CARs never harms the economy.

4 Model 2: Households have the bargaining power on deposits

In this section, we develop a model similar to the one in Section 2, except that the households determine the amount of deposits rather than the bank in this case.

4.1 Settings

Suppose that the economy has one bank and households, and the total size of the households is normalized to 1. Most of the settings here are the same as those in Section 2. Each household i is homogeneous and is uniformly distributed on the support $[0, 1]$. At $T = 0$, the bank collects funds D and E from the households using deposit financing and equity financing, respectively. Then, it keeps sD as reserves and invests the remaining amount. Define the bank's total funds as G, and the bank's investment is then $L \equiv G - sD$. At $T = 1$, the bank receives the return on investment, and thereafter repays $r_d D$ to depositors, and uses the remaining profits, if any, to pay dividends to stockholders. The bank's objective is to maximize its expected ROE.

In this section, we have new important assumptions on the bank's financing. We suppose that the households have all of the bargaining power as to the amount of deposits, and the bank cannot determine how much funding it receives as deposits. Therefore, households' demand for deposits determines the amount of the bank's deposit financing, D. Then, the constraint on the probability of the bank's bankruptcy affects not only the bank's decision, but also those of the households.

4.2 Households

We suppose that the households' objective is to maximize sum of the expected utilities of two term of consumption. The household receives fixed wage income w at $T = 0$, and there is no additional earning at $T = 1$. Thus, they need to invest some part of w as bank deposits and/or shares at $T = 0$ in order to gain the return at $T = 1$ for the consumption at $T = 1$.

Denote household i's consumption at time T as $C_{i,T}$ $(T = 0, 1)$. In addition, denote the amount of household i's earning used for the investment in the assets as Q_i, and denote the total profit ratio of the investment as μ_i. We express the households' utilities as the logarithm of the utility function. Then, household i's utility maximization problem is defined as follows, where ρ_i expresses the time preference rate of household i.

$$\begin{aligned} \max_{c_{i,0},\ c_{i,1}} \quad & \ln(c_{i,0}) + \rho_i \ln(c_{i,1}) \\ s.t. \quad & c_{i,0} = w - Q_i \\ & c_{i,i} = Q_i \mu_i \end{aligned}$$

From first order differentiation, we have

$$Q_i = \frac{W}{1 + \rho_i} \ .$$

Although the amount of household i's investment Q_i is determined, the composition of the investment, that is, the amounts of deposit and shares are not determined. Taking into account that household i's utility is increasing in the total profit ratio of the investment μ_i, we assume that it choose the amounts of deposit and shares that maximize μ_i. Because the value of Q_i does not depend on μ_i, we can consider the maximization problem for μ_i separately from the above one.

Denote the ratio of household i's expenditure for the bank deposit to the total investment; that is, household i's deposits-to-total-assets ratio, as d_i. Because the households are homogeneous, all will choose the same ratio, and we thus treat all households as one household and omit the subscript i. Denote the total amounts of the household's deposits and investments as D_{H} and Q_{H} and the total profit ratio of the investment as $\mu_{\mathrm{H}}(= \mu_i)$, respectively. Then, the household's deposits-to-total-assets ratio d_{H} is defined as $d_{\mathrm{H}} \equiv D_{\mathrm{H}}/Q_{\mathrm{H}}$.

As it is mentioned above, the household determines the amount of deposit D in the model. In other words, the amount of bank's deposit financing D is always equal to D_{H}. In addition, households' investment determine the total amount of the bank's funding; that is, G is always equal to Q_{H}. Thus, we can rewrite the ratio of the bank's deposit financing to total funds, D/G as d_{H}.

Suppose that the household chooses d_{H} to maximize the profit ratio of the total investment, μ_{H}, taking the risk of the bank's shares into account. Then, in order to define μ_{H}, we need to calculate the expected profit rates of bank deposits and shares. We denote the expected profit rate of deposits as π_d, and then define π_d as

$$\pi_d \equiv \frac{1}{D_{\mathrm{H}}}\left[\int_{\underline{r}}^{R}(r_d D_{\mathrm{H}})f(r)dr + \int_{0}^{\underline{r}}(rL + sD_{\mathrm{H}})f(r)dr\right] .$$

Considering that we can rewrite L as $Q_{\mathrm{H}} - sD_{\mathrm{H}}$, we can rewrite the expression above as

$$\pi_d \equiv r_d - \frac{(r_d - s)^2}{2R} \cdot \frac{d_{\mathrm{H}}}{1 - sd_{\mathrm{H}}} . \tag{4.1}$$

Next, denote the expected profit rate as π_{E}. Because π_{E} is $E[ROE]$, we express π_{E} as

$$\pi_{\mathrm{E}} \equiv \frac{\left[R(1 - sd_{\mathrm{H}}) - (r_d - s)d_{\mathrm{H}}\right]^2}{2R(1 - d_{\mathrm{H}})(1 - sd_{\mathrm{H}})} . \tag{4.2}$$

Using π_d and π_{E}, we define the profit ratio μ_{H} as $d_{\mathrm{H}}\pi_d + (1 - d_{\mathrm{H}})\pi_{\mathrm{E}}$. Then, as it is mentioned above, we suppose that the household tries to maximize the profit ratio of its portfolio considering the risk of the bank's shares. Then, the objective function is

$$\left[d_{\mathrm{H}}\pi_d + (1 - d_{\mathrm{H}})\pi_{\mathrm{E}}\right] - \frac{1}{2}\lambda_{\mathrm{H}}\sigma_E^2(1 - d_{\mathrm{H}})^2,$$

where λ_{H} is the parameter of the household's risk aversion, and σ_E^2 is the variance of the dividend on the bank's shares.

As it is mentioned before, the household takes into account equation (3.2) as the constraint. Thus, the household's portfolio optimization problem is defined as follows.

$$\begin{aligned} \max_{d_{\mathrm{H}}} \quad & d_{\mathrm{H}}\pi_d + (1 - d_{\mathrm{H}})\pi_{\mathrm{E}} - \frac{1}{2}\lambda_{\mathrm{H}}\sigma_E^2(1 - d_{\mathrm{H}})^2 \\ s.t. \quad & d_{\mathrm{H}} \leq d_\gamma(s) \\ & 0 \leq d_{\mathrm{H}} \leq 1 \end{aligned}$$

In addition, by substituting equation (4.1) and (4.2) into the objective function, we can rewrite the above problem as

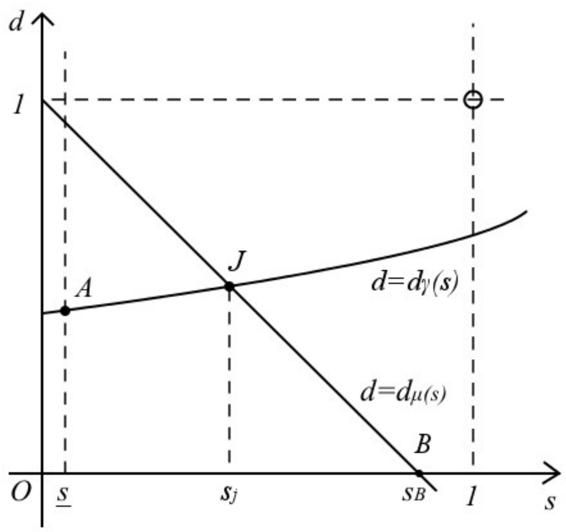
$$\max_{d_{\mathrm{H}}} \quad \frac{1}{2}\left[R - (R - 2)sd_{\mathrm{H}} - \lambda_{\mathrm{H}}\sigma_E^2(1 - d_{\mathrm{H}})^2\right] \tag{4.3}$$

$$\begin{aligned} s.t. \quad & d_{\mathrm{H}} \leq d_\gamma(s) \\ & 0 \leq d_{\mathrm{H}} \leq 1 \end{aligned}$$

By differentiating equation (4.3) by d_{H}, we have

$$\frac{\partial \mu}{\partial d_{\mathrm{H}}} \geq 0 \quad \Leftrightarrow \quad 1 - \frac{(R-2)s}{2\lambda_{\mathrm{H}}\sigma_E^2} \equiv d_\mu(s) \geq d_{\mathrm{H}}. \tag{4.4}$$

Thus, based on equations (3.2) and (4.4), the relationship between d_{H} and s is as in Figure 3 and Figure 4. Denote the intersection point of the two curves $d = d_\gamma(s)$ and $d = d_\tau(s)$ as point J, and denote the value of the s-coordinate at point J as s_j. In addition, denote the intersection point of line $d = d_\mu(s)$ and s-axis as point B, and denote the value of the s-coordinate at point B as s_{B}.



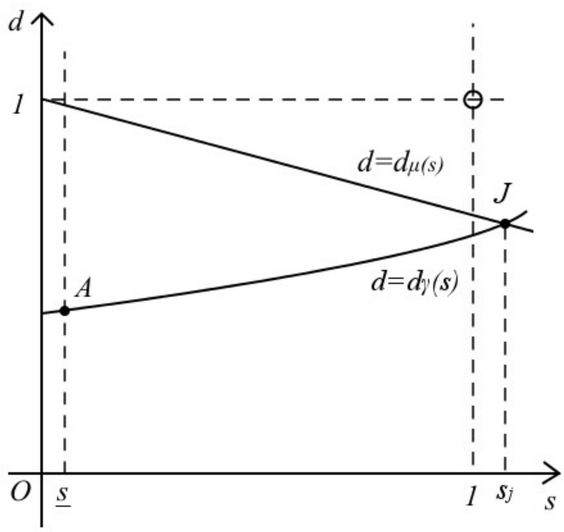


Figure 3: the relationship between s and d ($s_j < 1$) Figure 4: the relationship between s and d ($1 \leq s_j$)

From equation (4.4), with some s, the optimal value of d is fixed at $d_\mu(s)$. When $s_j < 1$ holds, as in Figure 3, $d_{\mathrm{H}} = d_\gamma(s)$ is satisfied with $\forall s \in [\underline{s}, s_j]$, whereas $d_{\mathrm{H}} = d_\mu(s)$ is satisfied with $\forall s \in [s_j, \min\{s_{\mathrm{B}}, 1\}]$, and $d = 0$ is satisfied with $[s_{\mathrm{B}}, 1]$ when $s_{\mathrm{B}} < 1$. On the other hand, when s_j satisfies $1 \leq s_j$ as in Figure 4, $d_{\mathrm{H}} = d_\gamma(s)$ always holds.

4.3 Bank

In the model in this section, the bank tries to maximize its expected ROE. Although the objective function is the same as in equation (3.1), the ratio of the bank's deposit financing d is always equal to the household's deposits-to-total-assets ratio d_{H}, and thus the bank chooses only its reserve ratio s.

4.4 Analysis of Model 2: Optimal CAR

Denote the intersection point of $d = d_\mu(s)$ and $d = d_\gamma(s)$ as Point J and its value of s-coordinate as s_j. In this study, we suppose that $\underline{s}$ is so small that $\underline{s} < s_j$ always holds.

First, suppose that $1 \leq s_j$ holds, as in Figure 4, and $d_{\mathrm{H}} = d_\gamma(s)$ always holds. Considering that $\mathcal{R}_{\mathrm{B}}(d, s)$ is decreasing in s as long as $d \neq 0$, the possible equilibrium is $(d_\gamma(\underline{s}), \underline{s})$.

Next, suppose that $s_j < 1$ holds, as in Figure 3. Then, the bank's maximization problem is

$$
\begin{aligned}
\max_{s} \quad & \mathcal{R}_{\mathrm{B}}(d_{\mathrm{H}}, s) \equiv \frac{\left[R(1 - s d_{\mathrm{H}}) - (r_d - s) d_{\mathrm{H}}\right]^2}{2R(1 - d_{\mathrm{H}})(1 - s d_{\mathrm{H}})} \\
s.t. \quad & d_{\mathrm{H}} = \frac{R\gamma}{(r_d - s) + R\gamma \cdot s} \equiv d_\gamma(s) \quad \text{(when } s \leq s_j \text{ is satisfied)} \\
& d_{\mathrm{H}} = 1 - \frac{(R-2)s}{2\lambda_{\mathrm{H}} \sigma_E^2} \equiv d_\mu(s) \quad \text{(when } s > s_j \text{ is satisfied)} \\
& \underline{s} \leq s \leq 1
\end{aligned}
\tag{4.5}
$$

Because the expected ROE as defined in equation (4.5) is the same as in equation (3.1), the properties of $\partial \mathcal{R}_{\mathrm{B}}(d_{\mathrm{H}}, s)/\partial d_{\mathrm{H}}$, and $\partial \mathcal{R}_{\mathrm{B}}(d_{\mathrm{H}}, s)/\partial s$ are the same as in Lemma 1. Then, the above maximization problem is described as in Figure 5 and Figure 6. In Figure 6, point C expresses the intersection of the two lines $d = d_\mu(s)$ and $s = 1$.

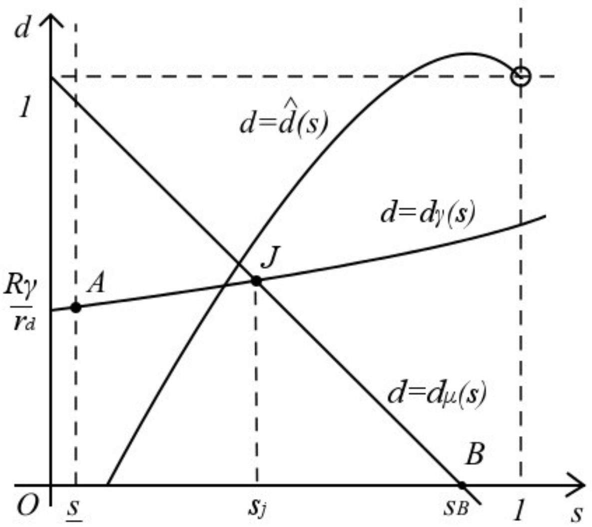


Figure 5: the relationship between d and s ($s_B < 1$)

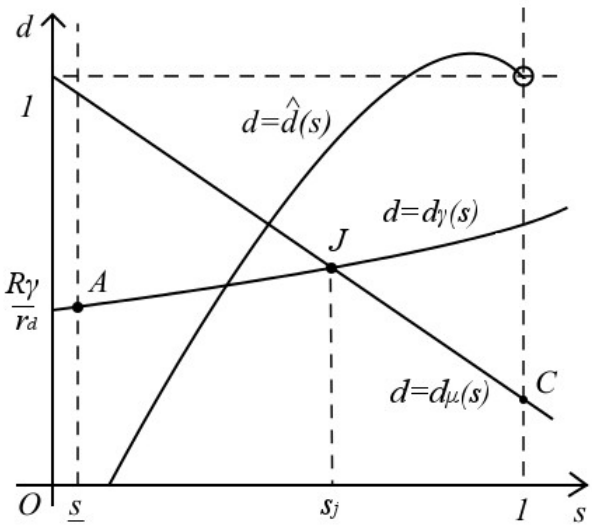


Figure 6: the relationship between d and s ($s_B \geq 1$)

As it is mentioned above, the household's choice of d is either $d_\gamma(s)$ or $d_\mu(s)$, or 0 when $d_\mu(s)$ is negative. Thus, there are four possible equilibria: $(d_\mu(\underline{s}), \underline{s})$, $(d_\gamma(\underline{s}), \underline{s})$, $(0, s_{\mathrm{B}})$, and $(d_\mu(1), 1)$. The latter three are points A, B, and C in Figure 5 and Figure 6, respectively.[6]

In order to summarize the properties of the equilibrium, we have the following two definitions.

[6] Point $(d_\mu(\underline{s}), \underline{s})$ exists when $\underline{s}$ is sufficiently large and point J is located in the upper left of the curve $d = \hat{d}(s)$. In other words, $(d_\mu(\underline{s}), \underline{s})$ can be the equilibrium when $d_\gamma(\underline{s})$ cannot be chosen because $d_\mu(\underline{s}) < d_\gamma(\underline{s})$ holds.

Definition 6. *Define the value* $\ddot{\gamma}_{\underline{s}}$ *as*

$$\ddot{\gamma}_{\underline{s}} \equiv \frac{(r_d - \underline{s})\big[2\lambda_H \sigma_E^2 - (R-1)\underline{s}\,\big]}{R\big[2(1-\underline{s})\lambda_H\sigma_E^2 + (R-1)\underline{s}^2\,\big]}\ ,$$

where $R\ddot{\gamma}_{\underline{s}}/\big[(r_d - \underline{s}) - R\ddot{\gamma}_{\underline{s}}\underline{s}\big] = 1 - (R-1)\underline{s}/\big[2\lambda_H\sigma_E\big]$ *holds.*

Definition 7. *Define the solutions to the equation below as* $\underline{\gamma}_1$ *and* $\underline{\gamma}_2$ $(\underline{\gamma}_1 \leq \underline{\gamma}_2)$*, respectively.*

$$\gamma^2 + \frac{R(1-\underline{s})\Gamma - 2(r_d - \underline{s})}{r_d - \underline{s}}\gamma + (1-\Gamma) = 0$$

$$\text{where} \quad \Gamma \equiv \left\{1 - \frac{(r_d - 1)\big[2\lambda_H\sigma_E^2 - (R-1)\big]}{R(R-1)}\right\}^2$$

We have following definitions as the possible equilibria.

Definition 8. *Denote the equilibrium as* (d_H^*, s_H^*)*, and the bank's expected ROE and CAR at the equilibrium as* (d_H^*, s_H^*)*,* $\mathcal{R}_H^* \equiv \mathcal{R}_B(d_H^*, s_H^*)$ *and* $\tau_H^* \equiv \tau_B(d_H^*, s_H^*)$*, respectively, where* $\mathcal{R}_B(d,s)$ *and* $\tau_B(d,s)$ *are defined as in expressions* (4.5) *and Definition* 2*.*

Then, we have the following definitions.

1. *Equilibrium* A_1 *is* $(d_H^*, s_H^*) = (d_\gamma(\underline{s}), \underline{s})$*. Then, Equilibrium* A_1*, the bank's expected ROE and CAR in this case are expressed as*

$$d_H^* = \frac{R\gamma}{(r_d - \underline{s}) + R\gamma\cdot\underline{s}}\,,\qquad s_H^* = \underline{s},\qquad \mathcal{R}_H^* = \frac{R}{2}\cdot\frac{(r_d-\underline{s})(1-\gamma)^2}{(r_d-\underline{s}) - R\gamma(1-\underline{s})},\qquad \tau_H^* = \frac{(r_d - \underline{s}) - R\gamma(1-\underline{s})}{r_d - \underline{s}}\ .$$

2. *Equilibrium* A_2 *is* $(d_H^*, s_H^*) = (d_\mu(\underline{s}), \underline{s})$*. Then, Equilibrium* A_2*, the bank's expected ROE and CAR in this case are expressed as*

$$d_H^* = 1 - \frac{R-2}{2\lambda_H\sigma_E^2}\underline{s}\,,\qquad s_H^* = \underline{s},\qquad \mathcal{R}_H^* = \frac{R}{2}\cdot\frac{(r_d-\underline{s})(1-\ddot{\gamma}_{\underline{s}})^2}{(r_d-\underline{s}) - R\ddot{\gamma}_{\underline{s}}(1-\underline{s})},\qquad \tau_H^* = \frac{(r_d - \underline{s}) - R\ddot{\gamma}_{\underline{s}}(1-\underline{s})}{r_d - \underline{s}}\ ,$$

$$\text{where}\quad \ddot{\gamma}_{\underline{s}} \equiv \frac{(r_d - \underline{s})\big[2\lambda_H\sigma_E^2 - (R-1)\underline{s}\,\big]}{R\big[2(1-\underline{s})\lambda_H\sigma_E^2 + (R-1)\underline{s}^2\big]}\ .$$

3. *Equilibrium* B *is* $(d_H^*, s_H^*) = (0, s_B)$*. Then, Equilibrium* B*, the bank's expected ROE and CAR in this case are expressed as*

$$d_H^* = 0,\qquad s_H^* = \frac{2\lambda_H\sigma_E^2}{R-2},\qquad \mathcal{R}_H^* = \frac{R}{2},\qquad \tau_H^* = 1.$$

4. *Equilibrium* C *is* $(d_H^*, s_H^*) = (d_\mu(1), 1)$*. Then, Equilibrium* C*, the bank's expected ROE and CAR in this case are expressed as*

$$d_H^* = 1 - \frac{R-2}{2\lambda_H\sigma_E^2}\,,\qquad s_H^* = 1,\qquad \mathcal{R}_H^* = \frac{1}{2R}\left[R - (r_d - 1)\frac{2\lambda_H\sigma_E^2 - (R-2)}{R-2}\right]^2,\qquad \tau_H^* = 1\ .$$

Then, we summarize the properties of the equilibrium of the bank's maximization problem as follows.

Proposition 3. *Define $\tilde{\gamma}(s)$ as Definition 3, $\ddot{\gamma}_{\underline{s}}$ as Definition 6, $\underline{\gamma}_1$ and $\underline{\gamma}_2$ as Definition 7, and Equilibrium A_1, Equilibrium A_2, Equilibrium B, and Equilibrium C as Definition 8, respectively. In addition, suppose that $\underline{s} < s_j < 1$ holds.*[7]

Then, the equilibrium of the bank's maximization problem, and the bank's expected ROE and CAR at the equilibrium, are determined as follows.

1. *When $\lambda_H \leq (R-2)/(2\sigma_E^2)$, and*
   (a) *When $\tilde{\gamma}(\underline{s}) < \gamma \leq \ddot{\gamma}(\underline{s})$ holds, the equilibrium is Equilibrium A_1, defined as in Definition 8.1.*
   (b) *When $\tilde{\gamma}(\underline{s}) < \ddot{\gamma}(\underline{s}) < \gamma$ holds, the equilibrium is Equilibrium A_2, defined as in Definition 8.2.*
   (c) *when $\min\{\gamma, \ddot{\gamma}(\underline{s})\} \leq \tilde{\gamma}(\underline{s})$ holds, the equilibrium is Equilibrium B, defined as in Definition 8.3.*
2. *When $\lambda_H > (R-2)/(2\sigma_E^2)$, and*
   (a) *when $\underline{s} > \dfrac{R-r_d}{R-1}$, $0 < \underline{\gamma}_1 < 1 < \underline{\gamma}_2$ holds. Then,*
      i. *when both $\gamma \leq \ddot{\gamma}(\underline{s})$ and $\gamma < \underline{\gamma}_1$ hold, the equilibrium is Equilibrium A_1, defined as in Definition 8.1.*
      ii. *when both $\ddot{\gamma}(\underline{s}) < \gamma$ and $\ddot{\gamma}(\underline{s}) < \underline{\gamma}_1$ hold, the equilibrium is Equilibrium A_2, defined as in Definition 8.2.*
      iii. *when $\underline{\gamma}_1 < \min\{\gamma, \ddot{\gamma}(\underline{s})\}$ holds, the equilibrium is Equilibrium C, defined as in Definition 8.4.*
   (b) *when $\underline{s} \leq \dfrac{R-r_d}{R-1}$ holds and $\underline{\gamma}_1$ and $\underline{\gamma}_2$ satisfy $0 \leq \underline{\gamma}_1 < \underline{\gamma}_2 \leq 1$, then,*
      i. *when $\gamma \leq \ddot{\gamma}(\underline{s})$ holds and γ satisfies either $0 \leq \gamma < \underline{\gamma}_1$ or $\underline{\gamma}_2 < \gamma \leq 1$, the equilibrium is Equilibrium A_1, defined as in Definition 8.1.*
      ii. *when $\ddot{\gamma}(\underline{s}) < \gamma$ holds and $\ddot{\gamma}(\underline{s})$ satisfies either $0 \leq \ddot{\gamma}(\underline{s}) < \underline{\gamma}_1$ or $\underline{\gamma}_2 < \ddot{\gamma}(\underline{s}) \leq 1$, the equilibrium is Equilibrium A_2, defined as in Definition 8.2.*
      iii. *when $\underline{\gamma}_1 < \min\{\gamma, \ddot{\gamma}(\underline{s})\} < \underline{\gamma}_2$ holds, the equilibrium is Equilibrium C, defined as in Definition 8.4.*
   (c) *when $\underline{s} \leq \dfrac{R-r_d}{R-1}$ holds and $\underline{\gamma}_1$ and $\underline{\gamma}_2$ does not exist as real solutions,*
      i. *when $\gamma \leq \ddot{\gamma}(\underline{s})$, the equilibrium is Equilibrium A_1, defined as in Definition 8.1.*

[7] This supposition guarantees that $d_\mu(s) < d_\gamma(s)$ holds and that point B or C can be the equilibrium.

ii. when $\ddot{\gamma}(\underline{s}) < \gamma$, the equilibrium is Equilibrium A_2, defined as in Definition 8.2.

In this section, d is determined by both the bank and the household, and thus, in order to obtain the optimal level of d, the bank must adjust s and lead the household to choose the level. Then, the response of d_{H} to the change in s is mostly determined by λ_{H}; that is, the slope of line $d = d_\mu(s)$, and thus the level of λ_{H} affects how the results differ from those obtained in the model in the previous section.

When λ_{H} is sufficiently small; that is, the slope of the line $d = d_\mu(s)$ is sufficiently large, there is a probability that the household will choose not $d_\gamma(\underline{s})$, but $d_\mu(\underline{s})$. Then, the possible equilibrium is $(d_\mu(\underline{s}), \underline{s})$ and the value at which the bank compares to $\tilde{\gamma}(\underline{s})$ is not γ, but $\ddot{\gamma}(\underline{s})$. Therefore, with the parameters at which the equilibrium is $d^* \neq 0$ in the model in the previous section, $d^* = 0$ can be satisfied in the equilibrium in this section.

When λ_{H} is sufficiently large, there are two important changes. First, there can be a new possible equilibrium $(d_\mu(1), 1)$; that is, point C, where the bank uses both deposit and equity financing, and its CAR is 1.

Second, when point C exists with $d_\mu(\underline{s}) > 0$, the bank's CAR at the equilibrium is more likely to satisfy $0 < \tau^*_{\mathrm{H}} < 1$. In other words, the equilibrium $(d^*_{\mathrm{H}}, s^*_{\mathrm{H}})$ is more likely to satisfy $d^*_{\mathrm{H}} > 0$ and $s^*_{\mathrm{H}} = \underline{s}$. As we showed, when λ_{H} is sufficiently small, the threshold that determines whether $d^*_{\mathrm{H}} > 0$ holds or not is $\tilde{\gamma}(s)$, and thus the process of determination of the equilibrium is not so largely different from that in the model in the previous section. On the other hand, when λ_{H} is sufficiently large and point C exists and $d_\mu(1) > 0$, $d^*_{\mathrm{H}} > 0$ holds, not only with sufficiently large γ and $\ddot{\gamma}(s)$, but also sufficiently small ones. When point C exists with $d_\mu(1) > 0$, the household is so heavily risk-averse that it still demands the bank's deposit with $s = 1$. In other words, the bank always obtains $d > 0$ with $\forall s \in [\underline{s}, 1]$. Then, when γ is sufficiently small, the bank can obtain a small d by choosing $\underline{s}$. Because it cannot obtain $d = 0$, and with some $d \neq 0$, the smaller s is, the more optimal it is for the bank. Thus, when λ_{H} is sufficiently large and point C exists with $d_\mu(1) > 0$, $s^*_{\mathrm{H}} = \underline{s}$ with sufficiently small γ and $\ddot{\gamma}(\underline{s})$, the bank's CAR at the equilibrium is more likely to satisfy $0 < \tau^*_{\mathrm{H}} < 1$.

4.5 Analysis of Model 2: CAR Regulation

In this subsection, we analyze how regulating the bank's CAR affects its capital structure. Again, suppose that a new regulation is put into force and the bank' CAR, τ_{B}, needs to satisfy $\tau_{\mathrm{B}} \geq \underline{\tau}$. Thus, as in the previous section, we add equation (3.3) as a constraint to the bank's maximization problem. Then, we describe the relationship between d and s as in Figure 7 and Figure 8 when $\underline{\tau}$ is sufficiently small. In these figures, points A" and J express the intersections of the curve $d = d_\tau(s)$ with line $d = d_\mu(s)$ and with curve $d = d_\gamma(s)$, respectively, and the values of the s-coordinates at these points are s_μ and s_i, respectively. In addition, point I expresses the intersection of the two curves $d = d_\tau(s)$ and $d = d_\gamma(s)$, and s_i denotes the value of the s-coordinate at point I. Moreover, the new regulation affects only the bank that chooses $(d_\gamma(\underline{s}), \underline{s}))$ or $(d_\mu(\underline{s}), \underline{s})$; that is, Equilibrium A_1 or Equilibrium A_2, defined as in Definition 8.

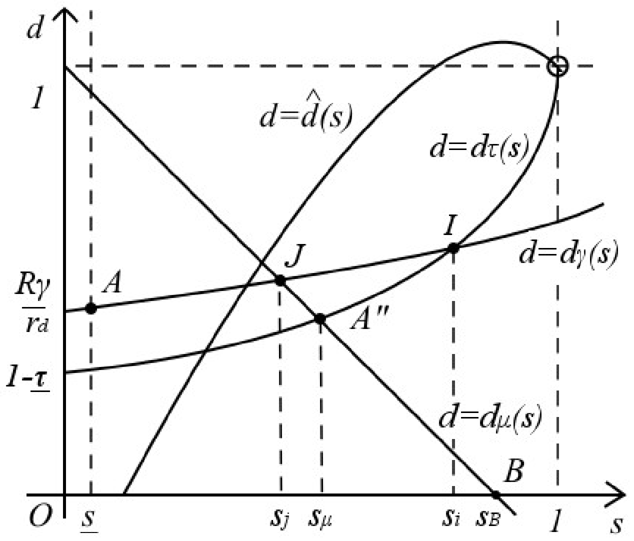


Figure 7: the relationships between d and s ($s_B < 1$)

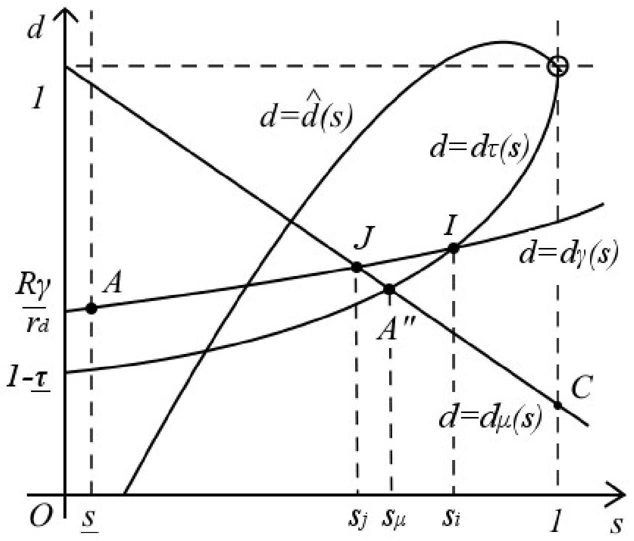


Figure 8: the relationships between d and s ($1 \leq s_B$)

The CAR regulation does not affect the household, and thus their choice of d is either $d_($$\gamma s)$, $d_\mu(s)$, or 0 when $d_\mu(s)$ is negative. Thus, there are four possible equilibria: $(d_\mu(s_\mu), s_\mu)$, $(d_\gamma(s_i), s_i)$, $(0, s_\mathrm{B})$, and $(d_\mu(1), 1)$, which we describe as points A'', J, B, and C in Figure 7 and Figure 8.

To summarize the properties of the equilibrium, we have the following definition.

Definition 9. *Define the solutions to the equation below as $\underline{\tau}_1(s)$ and $\underline{\tau}_2(s)$ ($\underline{\tau}_1(s) \leq \underline{\tau}_2(s)$), respectively.*

$$(1-\underline{\tau})^2 + \frac{R(1-s)\big[R(1-s)\Gamma - 2(r_d - s)\big]}{(r_d - s)^2}(1-\underline{\tau}) + \frac{R^2(1-s)^2(1-\Gamma)}{(r_d - s)^2} = 0$$

$$\textit{where} \quad \Gamma \equiv \left\{1 - \frac{(r_d - 1)\big[2\lambda_H \sigma_E^2 - (R-1)\big]}{R(R-1)}\right\}^2$$

In addition, we have the following definitions as the new possible equilibria.

Definition 10. *Denote the equilibrium under the regulation $\tau_B(d, s) \geq \underline{\tau}$ as $(d^*_{H,\tau}, s^*_{H,\tau})$, and define the bank's expected ROE and CAR at the equilibrium as $\mathcal{R}^*_{H,\tau} \equiv \mathcal{R}_B(d^*_{H,\tau}, s^*_{H,\tau})$ and $\tau^*_{H,\tau} \equiv \tau_B(d^*_{H,\tau}, s^*_{H,\tau})$, respectively.*

Then, we have following definitions.

1. *Equilibrium A" is* $(d^*_{H,\tau}, s^*_{H,\tau}) = (d_\mu(s_\mu), s_\mu)$. *The equilibrium at which the bank's expected ROE and CAR in this case are expressed as in*

$$d^*_{H,\tau} = \frac{1-\underline{\tau}}{1-s_\mu\underline{\tau}}, \quad s^*_{H,\tau} = s_\mu \equiv \frac{(R-2)+2\lambda_H\sigma_E^2\underline{\tau} - \sqrt{(R-2+2\lambda_H\sigma_E^2\underline{\tau})^2 - 8(R-2)\lambda_H\sigma_E^2\underline{\tau}^2}}{2(R-2)\underline{\tau}},$$

$$\mathcal{R}^*_{H,\tau} = \frac{1}{2R\underline{\tau}}\left[R-(1-\underline{\tau})\frac{r_d - s_\mu}{1-s_\mu}\right]^2, \quad \tau^*_{H,\tau} = \underline{\tau}.$$

2. *Equilibrium I is* $(d^*_{H,\tau}, s_\mu) = (d_\tau(s_i), s_i)$. *The equilibrium at which the bank's expected ROE and CAR in this case are expressed as in*

$$d^*_{H,\tau} = \frac{R\gamma-(1-\underline{\tau})}{R\gamma-(1-r_d\underline{\tau})}, \quad s^*_{H,\tau} = s_i \equiv \frac{R\gamma-(1-\underline{\tau})r_d}{R\gamma-(1-\underline{\tau})}, \quad \mathcal{R}^*_{H,\tau} = \frac{(1-\gamma)^2}{\underline{\tau}}, \quad \tau^*_{H,\tau} = \underline{\tau}.$$

We summarize the equilibrium under the new regulation as follows.

Proposition 4. *Suppose that a new regulation on banks' CARs,* $\tau(d,s) \geq \underline{\tau}$, *is put into force and* $\tau(d_\gamma(\underline{s},\underline{s})) < \underline{\tau}$ *holds. In addition, suppose that* $s_j < 1$. *Define* $\tilde{\tau}(s)$ *as in Definition 4,* $\underline{\tau}_1(s)$ *and* $\underline{\tau}_2(s)$ *as Definition 9, Equilibrium B and Equilibrium C as Definition 8, and Equilibrium A″ and Equilibrium I as Definition 10.*

Then, the equilibrium under the new regulation at which the bank's expected ROE and CAR in this case are expressed as follows.

1. *When* $\lambda_H \leq (R-2)/(2\sigma_E^2)$, *and*
   (a) *when both* $s_i < s_\mu$ *and* $\underline{\tau} < \tilde{\tau}(s_i)$ *hold, the equilibrium is Equilibrium I, defined as in Definition 10.2.*
   (b) *when both* $s_i \geq s_\mu$ *and* $\underline{\tau} < \tilde{\tau}(s_\mu)$ *hold, the equilibrium is Equilibrium A″, defined as in Definition 10.1.*
   (c) *when* $\underline{\tau} \geq \tilde{\tau}(\bar{s})$ *holds with* $\bar{s} \equiv \min\{s_i, s_\mu\}$, *the equilibrium is Equilibrium B, defined as in Definition 8.3.*
2. *When* $\lambda_H > (R-2)/(2\sigma_E^2)$ *holds, and*
   (a) *when* $\underline{\tau}_1(\bar{s})$ *and* $\underline{\tau}_2(\bar{s})$ *do not exist as real solutions with* $\bar{s} \equiv \min\{s_i, s_\mu\}$, *and*
      i. *when* $s_i < s_\mu$, *the equilibrium is Equilibrium I, defined as in Definition 10.2.*
      ii. *when* $s_i \geq s_\mu$, *the equilibrium is Equilibrium A″, defined as in Definition 10.1.*
   (b) *when* $\underline{\tau}_1(\bar{s})$ *and* $\underline{\tau}_2(\bar{s})$ *satisfy* $0 \leq \underline{\tau}_1(\bar{s}) < \underline{\tau}_2(\bar{s}) \leq 1$ *with* $\bar{s} \equiv \min\{s_i, s_\mu\}$, *and*
      i. *when* $s_i < s_\mu$, *and* $\underline{\tau}$ *satisfies either* $0 \leq \underline{\tau} < \underline{\tau}_1(s_i)$ *or* $\underline{\tau}_2(s_i) < \underline{\tau} \leq 1$, *the equilibrium is Equilibrium I, defined as in Definition 10.2.*

ii. *when $s_i \geq s_\mu$, and $\underline{\tau}$ satisfies either $0 \leq \underline{\tau} < \underline{\tau}_1(s_\mu)$ or $\underline{\tau}_2(s_\mu) < \underline{\tau} \leq 1$, the equilibrium is Equilibrium A″, defined as in Definition 10.1.*

iii. *when $\underline{\tau}$ satisfies $\underline{\tau}_1(\bar{s}) < \underline{\tau} < \underline{\tau}_2(\bar{s})$ with $\bar{s} \equiv \min\{s_i, s_\mu\}$, the equilibrium is Equilibrium C, defined as in Definition 8.4.*

In this section, d is determined by not the bank, but the household, and thus the bank must increase s when it adjusts its capital structure in order to increase its CAR. In other words, the bank cannot choose $\underline{s}$ under the regulation. Then, as in the previous subsection, the level of λ_{H} affects how the results differ from those obtained in the model in the previous section.

When λ_{H} is sufficiently small, the possible equilibrium $(d_\mu(s_{mu}), s_\mu)$ is relatively close to the point at which $(d_\tau(\underline{s}), \underline{s})$, and thus the process of determining the equilibrium is not so largely different from that in the model in the previous section.

When λ_{H} is sufficiently large and point C exists with $d_\mu(1) > 0$, on the other hand, both $d^*_{\mathrm{H},\tau} > 0$ and $s^*_{\mathrm{H},\tau} < 1$ hold at the equilibrium, not only with a sufficiently large $\underline{\tau}$, but also with a sufficiently small $\underline{\tau}$. In other words, the bank's CAR at the equilibrium is more likely to satisfy $0 < \tau^*_{\mathrm{H}} < 1$, as in the previous subsection.

In addition, the outcome of the regulation also changes due to the model's property that s increases when the bank adjusts its CAR. Then, we have following result as to how regulating banks' CARs affects their capital structures.

Corollary 2. *Suppose that the bank chooses $\underline{s}$ and the household chooses $d_\gamma(\underline{s})$, but the bank needs to increase its CAR to satisfy the new regulation $\tau_B \geq \underline{\tau}$. In addition, suppose that $\underline{\tau} < 1$ holds. Then, we obtain the following results.*

1. *When $\underline{s} \leq \dfrac{R - 2r_d}{R - 2}$ and $\min\{s_i, s_\mu\} < (R - r_d)/(R - 1)$, $\mathcal{R}^*_{H,\tau} < \mathcal{R}^*_B$ holds.*
2. *Suppose that $0 < d^*_\tau(s)$. Then, when the bank need to increase its CAR, there is a probability that the amount of its lending decreases, as long as at least one of the following conditions holds.*
   (a) $d_\gamma(\underline{s})$ is sufficiently small.
   (b) λ_H is sufficiently large.

The result of Corollary 2.1 implies that when the bank uses both deposit and equity financing under the regulation and its reserve ratio is not so large, its expected ROE decreases compared to the case with no regulation. In other words, there is still a probability that banks with low CARs relative to the mandatory level suffer a decrease in their expected ROEs, as in Corollary 1.

From Corollary 2.2, we derive two important implications. First, there is a probability that a CAR regulation reduces bank lending and works so as to slow down economic activities when we consider the household's decisions. Second, how a bank reacts to a CAR regulation depends on its depositors' properties. When the depositors are so risk-averse and they prefer holding deposits so much to holding shares, the bank's lending is likely to decrease. In other words, not only banks' capitalization ratios but also degrees of households' risk-aversion may cause differences in the capital regulation's effect among banks.

We derive the result of Corollary 2.2 as follows. As we described in the proof of Corollary 2, the bank's lending depends on the value of ds, and in order to keep the amount from decreasing, (d^*_τ, s^*_τ) must satisfy $d_\gamma(\underline{s})\underline{s} \le d^*_\tau s^*_\tau$.

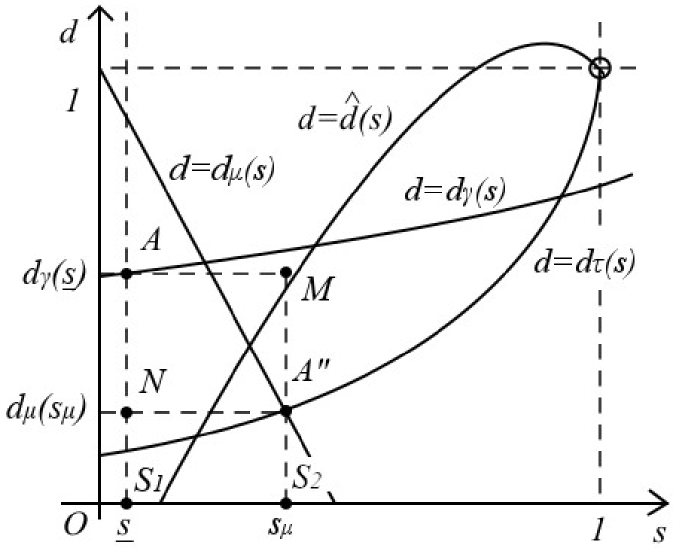


Figure 9: the case in which d_H increases under a regulation

Suppose that $(d^*_\tau, s^*_\tau) = (d_\mu(s_\mu), s_\mu)$ holds, as in Figure 9. Then, we can rewrite the inequality $d_\gamma(\underline{s})\underline{s} \le d^*_\tau s^*_\tau$ as

$$\begin{aligned} d_\gamma(\underline{s})\underline{s} \le d_\mu(s_\mu)s_\mu \quad &\Leftrightarrow \quad \frac{d_\gamma(\underline{s})}{s_\mu} \le \frac{d_\mu(s_\mu)}{\underline{s}} \ , \\ &\Leftrightarrow \quad \tan(\angle MOS_2) \le \tan(\angle NOS_1) \ . \end{aligned}$$

Then, with some $\underline{s}$ and $\underline{\tau}$, the smaller $d_\gamma(\underline{s})$ is, the smaller $\tan(\angle MOS_2)$ is, and the larger $\lambda_{\rm H}$ is, the larger $d_\mu(s_\mu)$ becomes, and the larger $\tan(\angle NOS_1)$ is. Thus, the condition $d_\gamma(\underline{s})\underline{s} \le d^*_\tau s^*_\tau$ can be likely to be satisfied. In other words, the bank's lending is likely to decrease under the regulation when $d_\gamma(\underline{s})$ is sufficiently small or $\lambda_{\rm H}$ is sufficiently large. Then, we have the result of Corollary 2.2, which implies that there is a probability that a CAR regulation works as to slow down economic activities when we consider the household's decisions.

5 Conclusion

We summarize the analytical results as follows. First, the analysis shows that the bank uses both deposit and equity financing, and the bank's CAR can be neither 1 nor 0 when it has an incentive to use deposit financing, but the constraint on the probability of the bank's bankruptcy prevents it from depending on it heavily. In addition, we show that the probability that its CAR is an interior solution is higher when the households have the bargaining power regarding deposits than that when the bank has the bargaining power.

Suppose that the households have a high demand for the bank to be sound enough not to go bankrupt. When the bank has the bargaining power, it decides not to use deposit financing and uses only equity financing, and its CAR is 1. On the other hand, when the households have the bargaining power and they are so heavily risk-averse that they always demand deposits, the bank can neither use deposit financing heavily nor stop using it. In this situation, there is a probability that the bank receives some deposits and holds some reserves and its CAR is neither 1 nor 0. In other words, when the households determine the amount of deposits and their demand for it is sufficiently high, the bank's CAR is more likely to be an interior solution.

Second, the analysis shows that banks' expected ROE can decrease under the CAR regulation. Suppose that a regulation on banks' CARs is put into force and a bank's CAR is neither 0 nor 1, both before and after the regulation enters into force. When the bank's reserve ratio is sufficiently small before the regulation and not so large after the regulation, the bank's expected ROE under the regulation is less than it obtained before the regulation, regardless of whether the households have the bargaining power on deposits. In other words, unsound banks may suffer a loss under a CAR regulation.

Third, the results show that regulating banks' CARs leads to a decrease in banks' lending when households' decision-making is taken into account. Thus, it implies that not only the banks' capitalization ratios but also their depositors' properties cause differences in regulation's effects among banks. When the households have the bargaining power on deposits, there is a probability that banks' lending will decrease under the regulation. Suppose that a regulation on banks' CARs is put into force and a bank needs to increase its CAR. When the bank has the bargaining power on deposits, it can adjust its capital structure directly to satisfy the regulation, and it thus chooses to use only more equity financing. Then, there is no probability that banks will decrease their lending under the regulation, as supporters of CAR regulation argue. When the households have the bargaining power on deposits, on the other hand, the bank must change its reserve ratio and indirectly lead households to hold a higher share, and thus the amount of the bank's reserve can increase under the regulation. In addition, an increase in the households' expenditure on the bank's shares means a decrease in the bank's deposits. Thus, when the households determine the amount of deposits and their demand for deposits is sufficiently high, the bank must increase its reserve ratio considerably in order to satisfy the regulation. Therefore, there is a probability that the bank's lending will decrease under the CAR regulation.

These results imply that we should pay more attention to the interaction between banks' decisions and households' demand and their adjustment process.

However, the results in this study depend on some specific suppositions, especially the model's feature that households' demand works as a constraint based on the assumption that the households' total expenditure on assets is always equal to the amount of the bank's funding. This supposition means that the only assets in the economy are offered by the bank, but this is not the case in the actual economy. An analysis that includes more assets, such as government bonds, is future research theme.

In addition, we consider that the merit of a bank's equity financing is to maintain a lower probability of bankruptcy. However, as we mentioned in Section 2, restraining the bank's excessive risk-taking is also a merit of equity financing, and recently there is increasing research in this direction. Thus, future studies should introduce firms into the model and analyze the effect of a CAR regulation on banks' capital structures in more detail.

6 Appendix

6.1 Proof of Lemma 1

First, we consider the bank's choice of s. When $d = 0$, we have $\mathcal{R}_{\mathrm{B}}(0, s) = R/2$ and the expected ROE does not depend on s, and thus we assume that $d > 0$.

By differentiating $\mathcal{R}_{\mathrm{B}}(0, s)$ defined in (3.1) by s, we have

$$\begin{aligned}
\frac{\partial \mathcal{R}_{\mathrm{B}}(d, s)}{\partial s} &= \frac{1}{2R(1-d)} \left\{ \frac{2\big[R(1-sd) - (r_d - s)d\big]}{1 - sd}(-Rd + d) + \frac{\big[R(1-sd) - (r_d - s)d\big]^2}{(1-sd)^2} d \right\}, \\
&= \frac{d\big[R(1-sd) - (r_d - s)d\big]}{2R(1-d)(1-sd)^2} \Big[-2(R-1)(1-sd) + R(1-sd) - (r_d - s)d \Big], \\
&= \frac{d\big[R(1-sd) - (r_d - s)d\big]}{2R(1-d)(1-sd)^2} \Big[-(R-2)(1-sd) - (r_d - s)d \Big].
\end{aligned}$$

Because $1 > sd$ and $r_d > s$ hold, the sign of the value in the large square braces is negative. Then, the sign of $\partial \mathcal{R}_{\mathrm{B}}(d, s)/\partial s$ is negative when the sign of $\big[R(1-sd) - (r_d - s)d\big]$ is positive.

Suppose that $[R(1 - \bar{s}d) - (r_d - \bar{s})d] \leq 0$ holds with some fixed value, $\bar{s} \in [\underline{s}, 1]$. Then, we have

$$\frac{R}{R\bar{s} + (r_d - \bar{s})} \leq d\,.$$

The left part of the inequality is decreasing in $\bar{s}$ and the smallest value is $R/(R + r_d - 1)$. Thus, when $\big[R(1-sd) - (r_d - s)d\big] \leq 0$ holds, d must satisfies $R/(R + r_d - 1) < d$. On the other hand, the bank must chooses d such that satisfies the constraint $d \leq d_\gamma(s) \equiv R\gamma/(R\gamma s + r_d - s)$. Because $R\gamma < 1$ holds by the assumption, $d_\gamma(s)$ is increasing in s and the largest value is $R\gamma/(R\gamma + r_d - 1)$ that is smaller than $R/(R + r_d - 1)$. Thus, the bank chooses d such that satisfies $d < R/(R + r_d - 1)$ and $\big[R(1-sd) - (r_d - s)d\big] > 0$ holds. Therefore, the sign of $\partial \mathcal{R}_{\mathrm{B}}(d, s)/\partial s$ is negative and the optimal value of s is the smallest one; that is, $\underline{s}$.

Next, we consider the bank's choice of d. Suppose that its reserve ratio is some fixed value, $\bar{s} \in [\underline{s}, 1]$. By differentiating $\mathcal{R}_{\mathrm{B}}(d, \bar{s})$ by d, we have

$$\begin{aligned}
\frac{\partial \mathcal{R}_{\mathrm{B}}(d, \bar{s})}{\partial d} &= \frac{2\big[R(1-\bar{s}d) - (r_d - \bar{s})d\big]}{2R(1-d)(1-\bar{s}d)} \big[-R\bar{s} - (r_d - \bar{s}) \big] \\
&\qquad - \frac{\big[R(1-\bar{s}d) - (r_d - \bar{s})d\big]^2}{2R[(1-d)(1-\bar{s}d)]^2} \big[2\bar{s}d - (1 + \bar{s}) \big], \\
&= \frac{R(1-\bar{s}d) - (r_d - \bar{s})d}{2R[(1-d)(1-\bar{s}d)]^2} \Big\{ -2(R\bar{s} + r_d - \bar{s})(1-d)(1-\bar{s}d) \\
&\qquad + \big[R(1-\bar{s}d) - (r_d - \bar{s})d\big] \big[(1-\bar{s}d) + \bar{s}(1-d)\big] \Big\}, \\
&= \frac{R(1-\bar{s}d) - (r_d - \bar{s})d}{2R[(1-d)(1-\bar{s}d)]^2} \Big\{ R(1-\bar{s}d)^2 - \bar{s}R(1-d)(1-\bar{s}d) - (r_d - \bar{s})\big[(1-d) + (1-\bar{s}d)\big] \Big\},
\end{aligned}$$

$$= \frac{R(1-\bar{s}d)-(r_d-\bar{s})d}{2R[(1-d)(1-\bar{s}d)]^2}\Big\{(1-\bar{s}d)\big[R(1-\bar{s})-(r_d-\bar{s})\big]-(r_d-\bar{s})(1-d)\Big\}.$$

As it is described above, $R(1-\bar{s}d)-(r_d-\bar{s})d>0$ holds when the constraint $d\le d_\gamma(\bar{s})$ holds, and thus the sign of $\partial\mathcal{R}_{\mathrm{B}}(d,\bar{s})/\partial d$ depends on the sign of the value in the large braces.

By rearranging $(1-\bar{s}d)\big[R(1-\bar{s})-(r_d-\bar{s})\big]-(r_d-\bar{s})(1-d)$, we define $\mathcal{J}(s)$ as

$$\mathcal{J}(d)\equiv(r_d-\bar{s})\left\{\left[\frac{R(1-\bar{s})}{r_d-\bar{s}}-1\right](1-\bar{s}d)-(1-d)\right\}.$$

First, suppose that $(R-r_d)/(R-1)<\bar{s}$ holds. Then, we have $R(1-\bar{s})/(r_d-\bar{s})<1$. Because both $(1-\bar{s}d)$ and $(1-d)$ are nonnegative with $\forall d\in[0,1]$, we have $\mathcal{J}(d)<0$. Next, suppose that $(R-2r_d)/(R-2)>\bar{s}$ holds. Then, we have $R(1-\bar{s})/(r_d-\bar{s})-1>1$. The both first and second terms in the large braces of $\mathcal{J}(d)$ are the linear functions of d, and $\mathcal{J}(d)>0$ holds with both $d=0$ and $d=1$. Thus, in this case, $\mathcal{J}(d)>0$ holds with $\forall d\in[0,1]$.

Lastly, suppose that $(R-2r_d)/(R-2)\le\bar{s}\le(R-r_d)/(R-1)$ holds. Then, the first term is always nonnegative but smaller than or equal to 1 with $d=0$. Thus, with some $\hat{d}(\bar{s})\in[0,1]$, $\mathcal{J}(\hat{d}(\bar{s}))=0$ is satisfied, and then $\mathcal{J}(d)<0$ holds with $d<\hat{d}(\bar{s})$, and $\mathcal{J}(d)>0$ holds with $d>\hat{d}(\bar{s})$, respectively. By rearranging $\mathcal{J}(\hat{d}(\bar{s}))=0$, $\hat{d}(s)$ is defied as

$$\hat{d}(s)\equiv\frac{2(r_d-s)-R(1-s)}{(R-1)s^2-(R-r_d+1)s+r_d}\ .$$

Then, because the sign of $\partial\mathcal{R}_{\mathrm{B}}(d,\bar{s})/\partial d$ is the same as that of $\mathcal{J}(d)$, we have the result of Lemma 1. □

6.2 Proof of Proposition 1

When the bank chooses $d=0$, the expected ROE is $\mathcal{R}_{\mathrm{B}}(0,s)=R/2$. Then the bank's choice of s does not affect the expected ROE, and thus, $\forall s\in[\underline{s},1]$ is optimal.

Suppose that the bank chooses $d\neq0$. Because $\mathcal{R}_{\mathrm{B}}(d,s)$ is decreasing in s as long as $d\neq0$ is satisfied, the bank chooses $\underline{s}$. When $\underline{s}$ satisfies $(R-2r_d)/(R-2)\le\underline{s}\le(R-r_d)/(R-1)$, $\mathcal{R}(d,s)$ can be either increasing or decreasing in d. With $\big(d_\gamma(\underline{s}),\underline{s}\big)$, the bank's expected ROE is calculated as

$$\mathcal{R}_{\mathrm{B}}(d_\gamma(\underline{s}),\underline{s})=\frac{R}{2}\cdot\frac{(r_d-\underline{s})(1-\gamma)^2}{(r_d-\underline{s})-R\gamma(1-\underline{s})}\ .$$

Because $\mathcal{R}(0,\underline{s})=R/2$, the inequality $\mathcal{R}(0,\underline{s})<\mathcal{R}_{\mathrm{B}}(d_\gamma(\underline{s}),\underline{s})$ can be rewritten as

$$\frac{R}{2}<\frac{R}{2}\cdot\frac{(r_d-\underline{s})(1-\gamma)^2}{(r_d-\underline{s})-R\gamma(1-\underline{s})}\quad\Leftrightarrow\quad0<(r_d-\underline{s})\gamma\left[\gamma-\frac{2(r_d-\underline{s})-R(1-\underline{s})}{r_d-\underline{s}}\right]\ .$$

Then, because $\underline{s}<1<r_d$ holds, the condition is summarized as

$$\frac{2(r_d-\underline{s})-R(1-\underline{s})}{r_d-\underline{s}}\equiv\tilde{\gamma}(\underline{s})<\gamma\ .$$

and thus $\tilde{\gamma}(\underline{s}) < \gamma$ holds when $(d_\gamma(\underline{s}), \underline{s})$ is the equilibrium, otherwise $(0, \bar{s}),\ \bar{s} \in [\underline{s}, 1]$ is the equilibrium.

By rewriting inequality $\tilde{\gamma}(s) < 0$, we have

$$2 - \frac{R(1-s)}{r_d - s} < 0 \quad \Leftrightarrow \quad s < \frac{R - 2r_d}{R - 2} \ ,$$

Thus, when $\underline{s} < (R - 2r_d)/(R-2)$ is satisfied, the condition $\tilde{\gamma}(\underline{s}) < \gamma$ holds and $(d_\gamma(\underline{s}), \underline{s})$ is the equilibrium.

By rewriting inequality $1 < \tilde{\gamma}(s)$, we have

$$1 < 2 - \frac{R(1-s)}{r_d - s} \quad \Leftrightarrow \quad \frac{R - r_d}{R - 1} < s \ .$$

Thus, when $(R - r_d)/(R-1) < \underline{s}$ is satisfied, the condition $\tilde{\gamma}(\underline{s}) < \gamma$ does not hold and $(0, \bar{s}),\ \bar{s} \in [\underline{s}, 1]$ is the equilibrium.

Then, we have the result of Proposition 1. □

6.3 Proof of Proposition 2

Suppose that there exists the new regulation $\tau_{\mathrm{B}} \geq \underline{\tau}$ and that $\tau_{\mathrm{B}}(d_\gamma(\underline{s}), \underline{s}) < \underline{\tau}$ holds.

When the bank chooses $d = 0$, its expected ROE is $\mathcal{R}_{\mathrm{B}}(0, s) = R/2$ and its CAR is 1 with $\forall s \in [\underline{s}, 1]$. Thus, it can choose any s in $[\underline{s}, 1]$ under the regulation. Moreover, its choice of s does not affect its expected ROE, and thus $\forall s \in [\underline{s}, 1]$ is optimal with $d = 0$.

Suppose that the bank chooses $d \neq 0$. When $\underline{s}$ satisfies $(R - 2r_d)/(R-2) \leq \underline{s} \leq (R - r_d)/(R-1)$, $\mathcal{R}(d, s)$ can be either increasing or decreasing in d. With $(d_\tau(\underline{s}), \underline{s})$, the bank's expected ROE

$$\mathcal{R}_{\mathrm{B}}(d_\tau(\underline{s}) = \frac{1}{2R\underline{\tau}} \left[R - (1 - \underline{\tau}) \frac{r_d - \underline{s}}{1 - \underline{s}} \right]^2 .$$

Then, because $\mathcal{R}_{\mathrm{B}}(0, \underline{s}) = R/2$ holds, the inequality $\mathcal{R}_{\mathrm{B}}(0, \underline{s}) < \mathcal{R}_{\mathrm{B}}(d_\tau(\underline{s}), \underline{s})$ can be rewritten as

$$\frac{R}{2} < \frac{1}{2R\underline{\tau}} \left[R - (1 - \underline{\tau}) \frac{r_d - \underline{s}}{1 - \underline{s}} \right]^2 \quad \Leftrightarrow \quad R^2 (1 - \underline{s})^2 \underline{\tau} < \left[R(1 - \underline{s}) - (1 - \underline{\tau})(r_d - \underline{s}) \right]^2 ,$$

$$\Leftrightarrow \quad 0 < (1 - \underline{\tau}) \left\{ \left[R(1 - \underline{s}) - (r_d - \underline{s}) \right]^2 - (r_d - \underline{s})^2 \underline{\tau} \right\} ,$$

and then, the condition is summarized as

$$\underline{\tau} < \left[\frac{R(1 - \underline{s}) - (r_d - \underline{s})}{r_d - \underline{s}} \right]^2 \equiv \tilde{\tau}(\underline{s}).$$

Then, when $\underline{\tau} < \tilde{\tau}(\underline{s})$ is holds, $(d_\tau(\underline{s}), \underline{s})$ is the equilibrium.

In addition, because the bank must satisfy the constraint $d \leq d_\gamma(s)$, it cannot chooses $d_\tau(\underline{s})$ when $d_\gamma(\underline{s}) < d_\tau(\underline{s})$ holds. We denote s such that equalizes $d_\gamma(s)$ to $d_\tau(s_i)$ as s_i. Then, when $s_i < \underline{s}$ holds, the bank chooses d based on not $d_\tau(s)$, but $d_\gamma(s)$. and thus the equilibrium is determined as it is in proof of Proposition 1.

Moreover, by rewriting inequality $1 < \tilde{\tau}(s)$, we have

$$(r_d - s)^2 < \big[R(1-s) - (r_d - s)\big]^2 \quad \Leftrightarrow \quad 0 < R(1-s)\big[R(1-s) - 2(r_d - s)\big] \ .$$

Then, when $\underline{s} < (R-2r_d)/(R-2)$ is satisfied, $\tilde{\tau}(\underline{s})$ is always larger than 1 and the condition $\underline{\tau} < \tilde{\tau}(\underline{s})$ always holds. In other words, when $\underline{s} < (R-2r_d)/(R-2)$ is satisfied, $(d_\tau(\underline{s}), \underline{s})$ is the equilibrium.

Then, we have the result of Proposition 2. □

6.4 Proof of Lemma 2

By differentiating $\mathcal{R}_{\mathrm{B}}(d_\tau(s), s)$ by $\underline{\tau}$, we have

$$\frac{\partial \mathcal{R}_{\mathrm{B}}(d_\tau(s), s)}{\partial \underline{\tau}} = \frac{1}{2R\underline{\tau}^2}\left[R - (1-\underline{\tau})\frac{r_d - s}{1-s}\right]\left[-R + (1+\underline{\tau})\frac{r_d - s}{1-s}\right] . \tag{6.1}$$

Denote $(r_d - s)/(1-s)$ as $\mathcal{K}(s)$. Then, equation (6.1) is quadratic function of $\mathcal{K}(s)$, and thus, we have $\partial\mathcal{R}_{\mathrm{B}}(d_\tau(s), s)/\partial\underline{\tau} > 0$ with $R/(1+\underline{\tau}) < \mathcal{K}(s) < R/(1-\underline{\tau})$, and $\partial\mathcal{R}_{\mathrm{B}}(d_\tau(s), s)/\partial\underline{\tau} < 0$ with $\mathcal{K}(s) < R/(1+\underline{\tau})$, or $R/(1-\underline{\tau}) < \mathcal{K}(s)$. The condition $R/(1+\underline{\tau}) < \mathcal{K}(s) < R/(1-\underline{\tau})$ can be rewritten as

$$\frac{R(1-s)}{r_d - s} - 1 < \underline{\tau} \quad \text{and} \quad 1 - \frac{R(1-s)}{r_d - s} < \underline{\tau} \ .$$

Thus, when $\partial\mathcal{R}_{\mathrm{B}}(d_\tau(s), s)/\partial\underline{\tau} > 0$ holds, $\underline{\tau}$ must satisfies

$$\left[1 - \frac{R(1-s)}{r_d - s}\right]^2 \equiv \tilde{\tau}(s) < \underline{\tau}^2 \ .$$

Then, we have the result of Lemma 2. □

6.5 Proof of Corollary 1

Denote the bank's choice of (d, s) when there exists no CAR regulation as (d^*, s^*), and the choice under the the regulation $\tau_{\mathrm{B}} \geq \underline{\tau}$, as (d^*_τ, s^*_τ), respectively. In addition, suppose that $(d^*, s^*) = (d_\gamma(\underline{s}), \underline{s})$ is satisfied; in other words, the bank uses both deposit and equity financing when there exists no CAR regulation.

The first result is derived as follows. As it is described in Lemma 2, when $\mathcal{R}_{\mathrm{B}}(d_\tau(s), s)$ is decreasing in $\underline{\tau}$, $\underline{\tau}$ satisfies $\tilde{\tau}(s) > \underline{\tau}^2$. Then, when $\tilde{\tau}(s) > 1$ holds, $\mathcal{R}_{\mathrm{B}}(d_\tau(s), s)$ is decreasing in $\underline{\tau}$ with $\forall \underline{\tau} \in [0, 1]$. The condition $\tilde{\tau}(s) > 1$ can be rewritten as

$$\left[1 - \frac{R(1-s)}{r_d - s}\right]^2 > 1 \quad \Leftrightarrow \quad \left[2 - \frac{R(1-s)}{r_d - s}\right]\left[-\frac{R(1-s)}{r_d - s}\right] > 0 \ .$$

Because $R(1-s)/(r_d - s)$ is nonnegative with $s \in [0,1]$, the condition can be reduced as

$$2 - \frac{R(1-s)}{r_d - s} < 0 \quad \Leftrightarrow \quad s < \frac{R - 2r_d}{R-2} \ .$$

Thus, when $\underline{s} \leq (R-2r_d)/(R-2)$ is satisfied, $\tilde{\tau}(\underline{s}) > \underline{\tau}^2$ is satisfied with $\underline{\tau} \in [0,1)$.

Then, because $(d_\gamma(\underline{s}), \underline{s})$ can be rewritten as $(d_\tau(\underline{s}), \underline{s})$ using some $\bar{\tau}$, $\mathcal{R}_{\mathrm{B}}(d^*, s^*)$ can be expressed as a function of $\bar{\tau}$. In addition, the assumption that the bank cannot choose (d^*, s^*) under the regulation means that $\bar{\tau} > \underline{\tau}$ is satisfied. Then, when both $\underline{s} \leq (R-2r_d)/(R-2)$ and $\underline{\tau} \neq 1$ hold, $\mathcal{R}_{\mathrm{B}}(d^*, s^*) > \mathcal{R}_{\mathrm{B}}(d^*_\tau, s^*_\tau)$ is satisfied. □

The second result is derived as follows. The bank's investment amount with some (d, s) is expressed as

$$L_{\mathrm{B}} = G - sD_{\mathrm{B}} = G(1 - sd) \ .$$

Then, because G is supposed to be fixed, the decrease of investment means increase of sd. In other words, when the amount of the bank's investment decreases after the regulation, sd must be satisfies $\underline{s}d_\gamma(\underline{s}) < s^*_\tau d^*_\tau$.

Then, when $d^*_\tau = 0$ holds, it is clear that the condition $\underline{s}d_\gamma(\underline{s}) < s^*_\tau d^*_\tau$ is not satisfied, and thus the bank's investment amount does not decrease. On the other hand, when $d^*_\tau \neq 0$ holds, $d^*_\tau = d_\tau(\underline{s}) < d_\gamma(\underline{s})$ and $s^*_\tau = \underline{s} = s^*$ are satisfied. Then, the condition $\underline{s}d_\gamma(\underline{s}) < s^*_\tau d^*_\tau$ does not satisfied and the bank's investment amount does not decrease. Thus, there is no probability that the bank decrease its investment to satisfy the regulation on its CAR.

Then, we have the results of Corollary 1. □

6.6 Proof of Proposition 3

Suppose that $\lambda_{\mathrm{H}} \leq (R-2)/2\sigma_{\mathrm{E}}$ holds and point B exists as Figure 5. Then, with some fixed value $\underline{s}$, the households' choice of d is $d_\gamma(\underline{s})$, $d_\mu(\underline{s})$ or 0. Suppose that the households choose $d_\mu(\underline{s})$. Then, $d_\mu(\underline{s}) \leq d_\gamma(\underline{s})$ must hold under the constraint $d_{\mathrm{H}} \leq d_\gamma(s)$. The inequality can be rewritten as

$$\begin{aligned} d_\mu(\underline{s}) \leq d_\gamma(\underline{s}) \ &\Leftrightarrow \ 1 - \frac{(R-1)\underline{s}}{2\lambda_{\mathrm{H}}\sigma^2_{\mathrm{E}}} \leq \frac{R\gamma}{(r_d - \underline{s}) + R\gamma\, \underline{s}} \\ &\Leftrightarrow \ \frac{(r_d - \underline{s})\big[2\lambda_{\mathrm{H}}\sigma^2_{\mathrm{E}} - (R-1)\underline{s}\,\big]}{R\big[2(1-\underline{s})\lambda_{\mathrm{H}}\sigma^2_{\mathrm{E}} + (R-1)\underline{s}^2\,\big]} \equiv \ddot{\gamma}(\underline{s}) \leq \gamma . \end{aligned}$$

Denote $R\ddot{\gamma}(s)/\big[\,(r_d - s) - R\ddot{\gamma}(s)s\,\big]$ as $\ddot{d}_\gamma(s)$. Then, when $\ddot{\gamma} = \gamma$ holds, we have $d_\mu(\underline{s}) = \ddot{d}_\gamma(\underline{s})$ and $\mathcal{R}_{\mathrm{B}}(d_\mu(\underline{s}), \underline{s})$ can be expressed as $\mathcal{R}_{\mathrm{B}}(\ddot{d}_\gamma(\underline{s}), \underline{s})$. In other words, we can treat the point $\big(d_\mu(\underline{s}), \underline{s}\big)$ as a point on the line $d = \ddot{d}_\gamma(s)$. Then, because $\gamma > \tilde{\gamma}(\underline{s})$ holds when $\mathcal{R}_{\mathrm{B}}(d_\gamma(\underline{s}), \underline{s}) > \mathcal{R}_{\mathrm{B}}(0, \underline{s})$ is satisfied, $\ddot{\gamma}(\underline{s}) > \tilde{\gamma}(\underline{s})$ must hold when $\mathcal{R}_{\mathrm{B}}(\ddot{d}_\gamma(\underline{s}), \underline{s}) > R/2$ is satisfied.

Suppose that $d \neq 0$ is optimal for the bank. Taking the households' decisions into account, the optimal value of d_{H} for the bank is $\min\{d_\mu(\underline{s}), d_\gamma(\underline{s})\}$. When $d_\mu(\underline{s}) < d_\gamma(\underline{s})$ holds, this inequality can be rewritten as $\ddot{\gamma}(\underline{s}) < \gamma$. The supposition that $d \neq 0$ is optimal means that $\mathcal{R}_{\mathrm{B}}(\ddot{d}_{\gamma}(\underline{s}), \underline{s}) > R/2$ is satisfied and $\ddot{\gamma}(\underline{s}) > \tilde{\gamma}(\underline{s})$ holds. Therefore, when $\tilde{\gamma}(\underline{s}) < \ddot{\gamma}(\underline{s}) < \gamma$ holds, the equilibrium is $(d_{\mu}(\underline{s}), \underline{s})$; contrariwise, when $\tilde{\gamma}(\underline{s}) < \gamma \leq \ddot{\gamma}(\underline{s})$ holds, the equilibrium is $(d_{\gamma}(\underline{s}), \underline{s})$. When $\min\{\gamma, \ddot{\gamma}(\underline{s})\} \leq \tilde{\gamma}(\underline{s})$ holds, the equilibrium is $(0, s_{\mathrm{B}})$.

Next, suppose that $\lambda_{\mathrm{H}} > (R-2)/2\sigma_{\mathrm{E}}$ is satisfied and point C exists as Figure 6. Then, with some $\underline{s}$, the households' possible choice of d is $d_{\gamma}(\underline{s})$, $d_{\mu}(\underline{s})$ or $d_{\mu}(1)$. As it is explained above, the bank's expected ROE with $(d_{\mu}(\underline{s}), \underline{s})$ can be rewritten as $\mathcal{R}_{\mathrm{B}}(\ddot{d}_{\gamma}(\underline{s}), \underline{s})$ using $\ddot{\gamma}(s)$ and $\ddot{d}_{\gamma}(s)$.

Suppose that $\mathcal{R}_{\mathrm{B}}(d_{\gamma}(\underline{s}), \underline{s}) < \mathcal{R}_{\mathrm{B}}(d_{\mu}(1), 1)$ is satisfied. Then, because $\mathcal{R}_{\mathrm{B}}(d_{\mu}(1), 1)$ is equal to $\mathcal{R}_{\mathrm{B}}(\ddot{d}_{\gamma}(1), 1)$, the above equation can be rewritten as

$$\frac{(r_d - \underline{s})(1-\gamma)^2}{(r_d - \underline{s}) - R\gamma(1-\underline{s})} < (1 - \ddot{\gamma}(1))^2 \ .$$

Denote $(1 - \ddot{\gamma}(1))^2$ as Γ, and then, the above equation can be rewritten as

$$\gamma^2 + \frac{R(1-\underline{s})\Gamma - 2(r_d - \underline{s})}{r_d - \underline{s}}\gamma + (1 - \Gamma) < 0 \ . \tag{6.2}$$

Denote the values of γ which equalize the both parts of inequality (6.2) as $\underline{\gamma}_1$ and $\underline{\gamma}_2$ ($\underline{\gamma}_1 \leq \underline{\gamma}_2$), respectively. When $\underline{\gamma}_1$ and $\underline{\gamma}_2$ exist as real solutions and $\underline{\gamma}_1 < \underline{\gamma}_2$ holds, the condition

$$\left[\frac{R(1-\underline{s})\Gamma - 2(r_d - \underline{s})}{r_d - \underline{s}}\right]^2 - 4(1-\Gamma) > 0$$

must be satisfied. The inequality can be rewritten as

$$\frac{\Gamma}{(r_d - \underline{s})^2}\left[R^2(1-\underline{s})^2\Gamma - 4R(1-\underline{s})(r_d - \underline{s}) + 4(r_d - \underline{s})\right] > 0 \ ,$$

$$\Leftrightarrow \quad \Gamma > \frac{4(r_d - \underline{s})\left[R(1-\underline{s}) - (r_d - \underline{s})\right]}{R^2(1-\underline{s})^2} \ . \tag{6.3}$$

The right part of the inequality (6.3) is maximized with $\underline{s} = (R - 2r_d)/(R-2)$ and the value is 1. Because $\Gamma \equiv (1 - \ddot{\gamma}(1))^2$ and $\ddot{\gamma}(1)$ is less than 1, $\Gamma \leq 1$ holds, and thus the inequality (6.3) cannot be satisfied with some $\underline{s}$. In other words, the two threshold values $\underline{\gamma}_1$, $\underline{\gamma}_2$ does not always exist as real solutions, and thus $\mathcal{R}_{\mathrm{B}}(d_{\gamma}(\underline{s}), \underline{s}) > \mathcal{R}_{\mathrm{B}}(d_{\mu}(1), 1)$ is satisfied with some $\underline{s}$.

Then, when the inequality (6.3) is satisfied and $\underline{\gamma}_1 < \gamma < \underline{\gamma}_2$ holds, the bank's expected ROE satisfies $\mathcal{R}_{\mathrm{B}}(d_{\gamma}(\underline{s}), \underline{s}) < \mathcal{R}_{\mathrm{B}}(d_{\mu}(1), 1)$; contrariwise, when $\gamma < \underline{\gamma}_1$ or $\underline{\gamma}_2 < \gamma$ holds or the inequality (6.3) is not satisfied, $\mathcal{R}_{\mathrm{B}}(d_{\gamma}(\underline{s}), \underline{s}) > \mathcal{R}_{\mathrm{B}}(d_{\mu}(1), 1)$ is satisfied.

It is clear that the left part of the inequality (6.2) is positive with $\gamma = 0$. Then, by substituting $\gamma = 1$, the value of the left part of inequality (6.2) is calculated as

$$1 + \frac{R(1-\underline{s})\Gamma}{r_d - \underline{s}} - 2 + (1 - \Gamma) = \Gamma\left[\frac{R(1-\underline{s})}{r_d - \underline{s}} - 1\right] \ .$$

Suppose that the inequality (6.3) is satisfied. Then, when $\underline{s} > (R-r_d)/(R-1)$ holds, the value in the square brackets is negative and $0 < \underline{\gamma}_1 < 1 < \underline{\gamma}_2$ is satisfied; contrariwise, when $\underline{s} \leq (R-r_d)/(R-1)$ holds and the inequality (6.3) is satisfied, the value is nonnegative and $0 < \underline{\gamma}_1 < \underline{\gamma}_2 \leq 1$ is satisfied.

The choice of d is determined as follows. First, when $\ddot{\gamma}(\underline{s}) < \gamma$ is satisfied, $d_\mu(\underline{s}) < d_\gamma(\underline{s})$ holds and $d_\mu(\underline{s})$ is chosen; contrariwise, when $\ddot{\gamma}(\underline{s}) \geq \gamma$ is satisfied, $d_\mu(\underline{s}) \geq d_\gamma(\underline{s})$ holds and $d_\mu(\underline{s})$ is chosen. Next, when $\underline{\gamma}_1 < \min\{\gamma, \ddot{\gamma}(\underline{s})\} < \underline{\gamma}_2$ is satisfied, $\mathcal{R}_{\mathrm{B}}(\ddot{d}_\gamma(1), 1)$ is larger than the expected ROE with the chosen d and $\ddot{d}_\gamma(1)$ is chosen; otherwise, the chosen d is the optimal choice. Because whether $\underline{\gamma}_1$, $\underline{\gamma}_2$ exist as real values or not depends partly on the value of $\underline{s}$, we consider the following two cases.

First, suppose that $\underline{s} \leq (R-r_d)/(R-1)$ is satisfied. When the inequality (6.3) is not satisfied, the equilibrium is determined based on the relationship between γ and $\ddot{\gamma}(\underline{s})$. When $\ddot{\gamma}(\underline{s}) < \gamma$ is satisfied, the equilibrium is $\big(d_\mu(\underline{s}), \underline{s}\big)$; otherwise the equilibrium is $\big(d_\gamma(\underline{s}), \underline{s}\big)$. When the inequality (6.3) is satisfied, $0 < \underline{\gamma}_1 < \underline{\gamma}_2 \leq 1$ holds. Therefore, when $\gamma \leq \ddot{\gamma}(\underline{s})$ is satisfied and γ does not satisfy $\underline{\gamma}_1 < \gamma < \underline{\gamma}_2$ the equilibrium is $(d_\gamma(\underline{s})$. When $\gamma > \ddot{\gamma}(\underline{s})$ is satisfied and $\ddot{\gamma}(\underline{s})$ does not satisfy $\underline{\gamma}_1 < \ddot{\gamma}(\underline{s}) < \underline{\gamma}_2$, the equilibrium is $(d_\mu(\underline{s}), \underline{s})$. Then, when $\underline{\gamma}_1 < \min\{\gamma, \ddot{\gamma}(\underline{s})\} < \underline{\gamma}_2$ is satisfied, the equilibrium in $(d_\mu(1), 1)$.

Second, suppose that $(R-r_d)/(R-1) < \underline{s}$ is satisfied. In this case, $0 < \underline{\gamma}_1 < 1 < \underline{\gamma}_2$ holds. Therefore, when $\gamma \leq \ddot{\gamma}(\underline{s})$ and $\gamma < \underline{\gamma}_1$ hold, the equilibrium is $(d_\gamma(\underline{s})$; contrariwise, when $\gamma > \ddot{\gamma}(\underline{s})$ and $\ddot{\gamma}(\underline{s}) < \underline{\gamma}_1$ hold, the equilibrium is $(d_\mu(\underline{s}), \underline{s})$. Then, when $\underline{\gamma}_1 < \min\{\gamma, \ddot{\gamma}(\underline{s})\} < \underline{\gamma}_2$ holds, the equilibrium in $(d_\mu(1), 1)$.

Then, we have the results of proposition Proposition 3. □

6.7 Proof of Proposition 4

Suppose that a new regulation $\tau_{\mathrm{B}} \geq \underline{\tau}$ is put into force and $\tau_{\mathrm{B}}(d_\gamma(\underline{s}), \underline{s}) < \underline{\tau}$ is satisfied.

Because the regulation does not affect the households' decisions directly, when the reserve ratio is s under the regulation, their choice of d is $\min\{d_\mu(s), d_\gamma(s)\}$ as it is without the regulation. Then, the bank adjusts the value of s to satisfy the constraint $d \leq d_\tau(s)$, and thus the points (d, s) that can be chosen are $(d_\mu(s_\mu), s_\mu)$; that is, the intersection point of $d = d_\tau(s)$ and $d = d_\mu(s)$, and $(d_\gamma(s_i), s_i)$; that is, the intersection point of $d = d_\tau(s)$ and $d = d_\gamma(s)$. In addition, $(0, s_{\mathrm{B}})$ and $(d_\mu(1), 1)$ can be chosen.

First, suppose that $\lambda_{\mathrm{H}} \leq (R-2)/2\sigma_{\mathrm{E}}$ is satisfied and point B exists with $s_{\mathrm{B}} \leq 1$. With some $\underline{\tau}$, the optimal value of d for the bank is $d_\gamma(s_i)$, $d_\mu(s_\mu)$, or 0, and the former two s is expressed as $d_\tau(s_i)$ and $d_\tau(s_\mu)$, respectively. Then, when $\mathcal{R}_{\mathrm{B}}(d_\tau(s), s) > \mathcal{R}_{\mathrm{B}}(0, s_{\mathrm{B}})$ is satisfied, the inequality

$$\left[\frac{R(1-s)-(r_d-s)}{r_d-s}\right]^2 \equiv \tilde{\tau}(s) > \underline{\tau}$$

holds. Therefore, when $\tilde{\tau}(s) < \underline{\tau}$ is satisfied with $s \in \{s_i, s_\mu\}$, $\mathcal{R}_{\mathrm{B}}(d_\tau(s), s) < \mathcal{R}_{\mathrm{B}}(0, s_{\mathrm{B}})$ holds.

Because the households choose $d_{\mathrm{H}} = d_\gamma(s)$ only with $s \in [\underline{s}, s_\mu]$, when $(d_\gamma(s_i), s_i)$ can be chosen, $\underline{s} \leq s_i \leq s_j$ must be satisfied. Then, because $d_\tau(s)$ is increasing in s, the intersection point of

$d = d_\mu(s)$ and $d = d_\tau(s)$ exists on the upper right-hand part of $(d_\gamma(s_i), s_i)$. It implies that $s_i < s_\mu$ and $d_\mu(s_\mu) > d_\gamma(s_\mu)$ hold. Thus, the condition $d_{\text{H}} \leq d_\gamma(s)$ is not satisfied with $(d_\mu(s_\mu), s_\mu)$ and it cannot be the equilibrium. Then, when $(d_\mu(s_\mu), s_\mu)$ can be chosen, $s_\mu < s_i$ holds because $d_\tau(s)$ is increasing in s. It implies that $\underline{s} \leq s_i \leq s_j$ is not satisfied and $(d_\gamma(s_i), s_i)$ cannot be the equilibrium. In other words, when $d \neq 0$ and $s \neq 1$ hold at the equilibrium, the value of d and s depend on the relationship between s_μ and s_i, wheres_i is defined as Definition 5, and s_μ are calculated as

$$s_\mu \equiv \frac{(R-2) + 2\lambda_{\text{H}}\sigma_E^2\,\underline{\tau} - \sqrt{(R-2+2\lambda_{\text{H}}\sigma_E^2\,\underline{\tau})^2 - 8(R-2)\lambda_{\text{H}}\sigma_E^2\,\underline{\tau}^2}}{2(R-2)\underline{\tau}} \; .$$

Then, when both $s_i \leq s_\mu$ and $\underline{\tau} < \tilde{\tau}(s_i)$ hold, the equilibrium is $(d_\gamma(s_i), s_i)$; contrariwise, when both $s_i > s_\mu$ and $\underline{\tau} < \tilde{\tau}(s_\mu)$ hold, the equilibrium is $(d_\mu(s_\mu), s_\mu)$. Then, when $\underline{\tau} \geq \tilde{\tau}(\bar{s})$ is satisfied with $\bar{s} \equiv \min\{s_i, s_\mu\}$, the equilibrium is $(0, s_{\text{B}})$.

Next, suppose that $\lambda_{\text{H}} > (R-2)/2\sigma_{\text{E}}$ is satisfied and point C exists with $d_\mu(1) > 0$. Then, with some $\underline{\tau}$, the optimal value of d for the bank is $d_\gamma(s_i)$, $d_\mu(s_j)$, or $d_\mu(1)$.

Suppose that $\mathcal{R}_{\text{B}}(d_\tau(s), s) < \mathcal{R}_{\text{B}}(d_\mu(1), 1)$ is satisfied. Then, as it is described in proof of Proposition 3, $\mathcal{R}_{\text{B}}(d_\mu(1), 1)$ is equal to $\mathcal{R}_{\text{B}}(\ddot{d}_\gamma(1), 1)$, and thus the above equation can be rewritten as

$$\frac{1}{2R\underline{\tau}}\left[R - (1-\underline{\tau})\frac{r_d - s}{1-s}\right]^2 < \frac{R}{2}(1-\ddot{\gamma}(1))^2 \; .$$

Denote $(1-\ddot{\gamma}(1))^2$ as Γ. Then, the inequality can be rewritten as

$$(1-\underline{\tau})^2 + \frac{R(1-s)\big[R(1-s)\Gamma - 2(r_d - s)\big]}{(r_d - s)^2}(1-\underline{\tau}) + \frac{R^2(1-s)^2(1-\Gamma)}{(r_d-s)^2} < 0 \; . \tag{6.4}$$

Denote the value of $\underline{\tau}$ which equalize the both part of inequality (6.4) as $\underline{\tau}_1(s)$ and $\underline{\tau}_2(s)$ ($\underline{\tau}_1(s) \leq \underline{\tau}_2(s)$), respectively. When $\underline{\tau}_1(s)$ and $\underline{\tau}_2(s)$ exist as real solutions and $\underline{\tau}_1(s) < \underline{\tau}_2(s)$ holds, the condition

$$\begin{aligned}
& \frac{R^2(1-s)^2}{(r_d-s)^2}\left\{\Big[R(1-s)\Gamma - 2(r_d-s)\Big]^2 - 4(1-\Gamma)(r_d-s)^2\right\} > 0 \; , \\
\Leftrightarrow \quad & \frac{R^2(1-s)^2}{(r_d-s)^2}\,\Gamma\left\{R^2(1-s)^2\Gamma - 4R(1-s)(r_d-s) + 4(r_d-s)^2\right\} > 0 \; , \\
\Leftrightarrow \quad & \Gamma > \frac{4(r_d-s)\big[R(1-s) - (r_d-s)\big]}{R^2(1-s)^2}
\end{aligned} \tag{6.5}$$

must be satisfied. The inequality (6.5) is same as inequality (6.3) except the notation of s, and thus, as it is described in proof of Proposition 3, with some $\bar{s}$ such that $\bar{s} = (R-2r_d)/(R-2)$, the inequality (6.5) cannot be satisfied. In other words, the threshold value $\underline{\tau}_1(\bar{s})$ and $\underline{\tau}_2(\bar{s})$ does not always exist as real solutions. Thus, $\mathcal{R}_{\text{B}}(d_\tau(\bar{s}), \bar{s}) > \mathcal{R}_{\text{B}}(d_\mu(1), 1)$ is satisfied with some $\bar{s}$.

Then, with some $\bar{s} \in [0,1]$, when $\underline{\tau}_1(\bar{s}) < 1 - \underline{\tau} < \underline{\tau}_2(\bar{s})$ is satisfied, $\mathcal{R}_{\text{B}}(d_\tau(\bar{s}), \bar{s}) < \mathcal{R}_{\text{B}}(d_\mu(1), 1)$ hold. With $\underline{\tau} = 1$, the left part of the inequality (6.4) is nonnegative. By substituting $\underline{\tau} = 0$ into the left part, we have

$$1+\frac{R^2(1-s)^2}{(r_d-s)^2}\Gamma-\frac{2R(1-s)}{(r_d-s)}+\frac{R^2(1-s)^2(1-\Gamma)}{(r_d-s)^2}=1-\frac{2R(1-s)}{(r_d-s)}+\frac{R^2(1-s)^2}{(r_d-s)^2}\ ,$$
$$=\left[1-\frac{R(1-s)}{r_d-s}\right]^2\geq 0\ .$$

Thus, when $\underline{\tau}_1(s)$ and $\underline{\tau}_2(s)$ exist as real solutions, $0\leq\underline{\tau}_1(s)\leq\underline{\tau}_2(s)\leq 1$ is satisfied.

As it is described above, when $d\neq 0$ and $s\neq 1$ hold at the equilibrium, the value of d and s depend on the relationship between s_μ and s_i. First, suppose that the inequality (6.5) is satisfied. Then, when $s_i<s_\mu$ holds and $\underline{\tau}$ does not satisfies $\underline{\tau}_1(s_\mu)<\underline{\tau}<\underline{\tau}_2(s_\mu)$, the equilibrium is $(d_\gamma(s_i),s_i)$; contrariwise, when $s_i\geq s_\mu$ holds and $\underline{\tau}$ does not satisfies $\underline{\tau}_1(s_i)<\underline{\tau}<\underline{\tau}_2(s_i)$, the equilibrium is $(d_\mu(s_\mu),s_\mu)$. When $\underline{\tau}$ satisfies $\underline{\tau}_1(\bar{s})<\underline{\tau}<\underline{\tau}_2(\bar{s})$ with $\bar{s}\equiv\min\{s_i,s_\mu\}$, the equilibrium is $(d_\mu(1),1)$.

Second, suppose that the inequality (6.5) is not satisfied. Then, when $s_i<s_\mu$ holds, the equilibrium is $(d_\gamma(s_i),s_i)$; contrariwise, when $s_i\geq s_\mu$ holds , the equilibrium is $(d_\mu(s_\mu),s_\mu)$.

Then, we have the results of proposition Proposition 4. □

6.8 Proof of corollary Corollary 2

Denote the bank's choice of (d,s) without regulation and that under the regulation $\tau_{\mathrm{B}}\geq\underline{\tau}$ as (d^*,s^*) and (d^*_τ,s^*_τ), respectively. In addition, suppose that the bank uses both deposit and equity financing when the regulation does not exits and $(d^*,s^*)=(d_\gamma(\underline{s}),\underline{s})$ holds.

The result of Corollary 2.1 is derived as follows. As it is described in proof of Corollary 1, when $\underline{s}\leq(R-2r_d)/(R-2)$ is satisfied, $\tilde{\tau}(\underline{s})>\underline{\tau}^2$ holds with $\underline{\tau}\in[0,1)$, and thus $\mathcal{R}_{\mathrm{B}}(d_\gamma(\underline{s}),\underline{s})>\mathcal{R}_{\mathrm{B}}(d_\tau(\underline{s}),\underline{s})$ is satisfied. Then, with some $\bar{s}$, $\mathcal{R}_{\mathrm{B}}(d_\gamma(\underline{s}),\underline{s})>\mathcal{R}_{\mathrm{B}}(d_\tau(\bar{s}),\bar{s})$ holds when $\mathcal{R}_{\mathrm{B}}(d_\tau(\underline{s}),\underline{s})>\mathcal{R}_{\mathrm{B}}(d_\tau(\bar{s}),\bar{s})$ is satisfied.

Suppose that $s\neq 1$ holds. Then, by differentiating $\mathcal{R}_{\mathrm{B}}(d_\tau(s),s)$ with s, we have

$$\frac{\partial}{\partial s}\left\{\frac{1}{2R\underline{\tau}}\left[R-(1-\underline{\tau})\left(\frac{r_d-s}{1-s}\right)\right]^2\right\}=\frac{1}{2R\underline{\tau}}\left[2(1-\underline{\tau})^2\left(\frac{r_d-s}{1-s}\right)-2R(1-\underline{\tau})\right]\frac{r_d-1}{(1-s)^2}$$
$$=\frac{1-\underline{\tau}}{R\underline{\tau}}\left[(1-\underline{\tau})\left(\frac{r_d-s}{1-s}\right)-R\right]\frac{r_d-1}{(1-s)^2}\ .$$

Thus, we have

$$\frac{\partial\mathcal{R}_{\mathrm{B}}(d_\tau(s),s)}{\partial s}<0\quad\Leftrightarrow\quad(1-\underline{\tau})\left(\frac{r_d-s}{1-s}\right)<R\ ,$$
$$\Leftrightarrow\quad s<\frac{R-(1-\underline{\tau})r_d}{R-(1-\underline{\tau})}\ .$$

Therefore, when $s<(R-r_d)/(R-1)$ holds, $\mathcal{R}_{\mathrm{B}}(d_\tau(s),s)$ is decreasing in s. In other words, when $\underline{s}<s<(R-r_d)/(R-1)$ holds, $\mathcal{R}_{\mathrm{B}}(d_\tau(s),s)<\mathcal{R}_{\mathrm{B}}(d_\tau(\underline{s}),\underline{s})$ is satisfied. In addition, because

$(R-2r_d)/(R-2) < (R-r_d)/(R-1)$ holds, there exists some s that satisfies $\underline{s} < s < (R-r_d)/(R-1)$ when $\underline{s} \leq (R-2r_d)/(R-2)$.

In conclusion, when both $\underline{s} \leq (R-2r_d)/(R-2)$ and $\min\{s_i, s_\mu\} \equiv \bar{s} < (R-r_d)/(R-1)$ are satisfied, the bank's expected ROE satisfies $\mathcal{R}_{\mathrm{B}}(d_\gamma(\underline{s}), \underline{s}) > \mathcal{R}_{\mathrm{B}}(d_\tau(\bar{s}), \bar{s})$; in other words, $\mathcal{R}_{\mathrm{B}}(d^*, s^*) > \mathcal{R}_{\mathrm{B}}(d^*_\tau, s^*_\tau)$ □

The result of Corollary 2.2 is derived as follows. As it is described in proof of Corollary 1, the amount of the bank's investment with some (d, s) is expressed as $G(1-sd)$. Then, when the amount of the bank's investment decreases under the regulation, the choice (d^*_τ, s^*_τ) satisfies $\underline{s}d_\gamma(\underline{s}) < s^*_\tau d^*_\tau$.

When $d^*_\tau = 0$ holds, it is clear that the above inequality is not satisfied. Then, suppose that $d^*_\tau \neq 0$ holds. In addition, suppose that $(d^*_\tau, s^*_\tau) = (d_\gamma(s_i), s_i)$ is satisfied. This supposition means that $s_i < s_\mu$ is satisfied and implies that the slope of line $d = d_\mu(s)$ is sufficiently gentle; in other words, λ_{H} is sufficiently large. Then, because $s_i > \underline{s}$ and $d_\gamma(s_i) > d_\gamma(\underline{s})$ hold, the inequality $\underline{s}d_\gamma(\underline{s}) < s^*_\tau d^*_\tau$ is satisfied. Therefore, the amount of the bank's investment decreases under the regulation.

Then, suppose that $(d^*_\tau, s^*_\tau) = (d_\mu(s_\mu), s_\mu)$ is satisfied. The inequality $\underline{s}d_\gamma(\underline{s}) < s^*_\tau d^*_\tau$ can be rewritten as

$$\underline{s}d_\gamma(\underline{s}) < \left[1 - \left(\frac{R-2}{2\lambda_{\mathrm{H}}\sigma_E^2}\right) s_\mu\right] s_\mu \ .$$

Then, the inequality is likely to be satisfied with some $\underline{s}, s_\mu$ when λ_{H} is sufficiently large and/or $d_\gamma(\underline{s})$ is sufficiently small. □

Acknowledgements

My heartfelt appreciation goes to Prof. Sasaki and Prof. Kurose who provided helpful comments and suggestions. I am also grateful to Prof. Shibata for his invaluable comments and warm encouragements. The responsibility for the final formulation, and any errors that it may concern, are entirely mine.

Declarations of interest: none.

References

Acharya, Viral and Anjan Thakor, 2016. The dark side of liquidity creation: Leverage and systemic risk. Journal of Financial Intermediation, 28, 4–21.

Acharya, Viral, Hamid Mehran, and Anjan Thakor, 2016. Caught between scylla and charybdis? Regulating bank leverage when there is rent seeking and risk shifting. Review of Corporate Finance Studies, 5 (1), 36–75.

Admati, Anat, Peter DeMarzo, Martin Hellwig, and Paul Pfleiderer, 2014. Fallacies and Irrelevant Facts in the Discussion on Capital Regulation. in: Goodhart, Charles, Daniela Gabor, Jakob Vestergaard, and Ismail Ertürk. (Eds.), Central Banking at a Crossroads: Europe and Beyond, London; New York, NY: Anthem Press, Chap. 3, 33–50.

Aiyar, Shekhar, Charles Calomiris, and Tomasz Wieladek, 2015. Bank capital regulation: Theory, empirics, and policy. IMF Economic Review, 63 (4), 955–983.

Allen, Franklin, Elena Carletti, and Robert Marquez, 2015. Deposits and bank capital structure. Journal of Financial Economics, 118 (3), 601–619.

Ben Naceur, Sami, Katherin Marton, and Caroline Roulet, 2018. Basel III and bank-lending: Evidence from the United States and Europe. Journal of Financial Stability, 39, 1–27.

Berger, Allen, Robert DeYoung, Mark Flannery, David Lee, and Özde Öztekin, 2008. How do large banking organizations manage their capital ratios?. Journal of Financial Services Research, 34 (2-3), :123–49.

Bhattacharyya, Sugato and Amiyatosh Purnanandam, 2011. Risk-taking by banks: What did we know and when did we know it?. AFA 2012 Chicago Meetings Paper.

Calomiris, Charles and Charles Kahn, 1991. The role of demandable debt in structuring optimal banking arrangements. American Economic Review, 81 (3), 497–513.

Carlson, Mark, Hui Shan, and Missaka Warusawitharana, 2013. Capital ratios and bank lending: A matched bank approach. Journal of Financial Intermediation, 22 (4), 663–687.

Chen, Yehning, 2016. Bank capital and credit market competition: Will competitive pressure lead to higher capital levels?. Journal of International Money and Finance, 69, 247–263.

Coval, Joshua and Anjan Thakor, 2005. Financial intermediation as a beliefs-bridge between optimists and pessimists. Journal of Financial Economics, 75, 535–569.

DeAngelo, Harry and René Stulz, 2015. Liquid-claim production, risk management, and bank capital structure: Why high leverage is optimal for banks. Journal of Financial Economics, 116 (2), 219–236.

Diamond, Douglas and Philip Dybvig, 1983. Bank runs, deposit insurance, and liquidity. Journal of Political Economy, 91 (3), 401–419.

Jensen, Michael and William Meckling, 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. Journal of Financial Economics, 3 (4), 305–360.

Kanngiesser, Derrick, Reiner Martin, Laurent Maurin, and Diego Moccero, 2017. Estimating the impact of shocks to bank capital in the euro area. ECB Working Paper (2077).

Keeley, Michael, 1990. Deposit insurance, risk, and market power in banking. American Economic Review, 80 (5), 1183–1200.

Krishnamurthy, Arvind and Annette Vissing-Jorgensen, 2012. The aggregate demand for treasury debt. Journal of Political Economy, 120 (2), 233–267.

Lindquist, Kjersti-Gro, 2004. Banks' buffer capital: How important is risk. Journal of International Money and Finance, 23 (3), 493–513.

Olszak, Małgorzata, Mateusz Pipień, and Sylwia Roszkowska, 2016. The impact of capital ratio on lending of EU banks – the role of bank specialization and capitalization. Equilibrium. Quarterly Journal of Economics and Economic Policy, 11 (1), p. 43.

Repullo, Rafael, 2004. Capital requirements, market power, and risk-taking in banking. Journal of Financial Intermediation, 13 (2), 156–182.

Thakor, Anjan, 2014. Bank capital and financial stability: An economic trade-off or a faustian bargain?. Annual Review of Financial Economics, 6, 185–223.

Van den Heuvel, Skander, 2008. The welfare cost of bank capital requirements. Journal of Monetary Economics, 55 (2), 298–320.