Endogenous Timing of RD Decisions and Privatization Policy with Research Spillovers

Lee, Sang-Ho and Muminov, Timur

Chonnam National University, Chonnam National University

February 2020

Endogenous Timing of R&D Decisions and Privatization Policy with Research Spillovers

Sang-Ho Lee* and Timur K. Muminov**

Abstract

This study investigates an endogenous R&D timing game between duopoly firms which undertake cost-reducing R&D investments and then play Cournot output competition. We examine equilibrium outcomes in private and mixed markets and find that spillovers rate critically affects contrasting results. We show that a simultaneous-move appears in a private duopoly only if the spillovers rate is low while a sequential-move appears in a mixed duopoly irrespective of spillovers. We also show that public leadership is the only equilibrium if the spillovers rate is intermediate and its resulting welfare is the highest. Finally, we show that the implementation of privatization policy transforms a public leader to a private competitor, but this can decrease the social welfare.

Running Head: Endogenous R&D Timing and Privatization

JEL Classifications: L13; L32; H21

Keywords: private duopoly; mixed duopoly; R&D spillovers; endogenous R&D timing game.

1. Introduction

Previous literature of an endogenous timing game in a Cournot oligopoly has heavily used an observable delay game since it is formulated by Hamilton and Slutsky (1990). It is shown that in private duopoly profit-maximizing firms decide simultaneously when competing in quantities and sequentially when competing in prices. However, in a mixed duopoly where a profit-maximizing private firm

* Sang-Ho Lee: Department of Economics, Chonnam National University, 300 Yongbong-ro, Bukgu, Gwangju 61186 South Korea. E-mail: sangho@chonnam.ac.kr

** Timur Muminov: Department of Economics, Chonnam National University, 300 Yongbong-ro, Bukgu, Gwangju 61186 South Korea. E-mail: smtk0171@gmail.com

competes with a welfare-maximizing public firm, the results are reversed and both firms decide sequentially when competing in quantities and simultaneously when competing in prices.[1]

Besides understanding these conflicting results in production market competition, as innovations have intensified R&D competition among firms, governments have recognized the importance of R&D policies and emphasized the facilitation of innovation in society.[2] Recent policy concerns over innovation suggest the need for further examination of what allows for endogenous R&D decisions for the possibility of considering public policies such as regulating public institutions and organizations or privatization of the public firms.[3]

Due to its cost-reducing features associated with the existence of R&D spillovers (knowledge sharing) and its implications on innovation and competition activities, the works on R&D policies are currently gaining importance and have become highly popular. There have been considerable researches devoted to investigate main determinants which lead firms to internalize R&D performances and spillovers. For instance, Katsoulacos and Ulph (1998), Poyago-Theotoky (1995, 1999) and Baranes and Tropeano (2003) constructed a theoretical model of endogenous R&D spillovers. Gil Molto et al. (2011, 2018), Kesavayuth and Zikos (2013), Lee and Tomaru (2017), Lee et al. (2017) and Haruna and Goel (2017, 2018) investigated the role of R&D policies in mixed oligopolies where firms compete in R&D investments and examined the relationship between R&D activity and public policy.

---

[1] Ono (1978), Dowrick (1986), Robson (1990) and Matsumura (1999) discussed endogenous timing in private markets. Since the work by Pal (1998), Bárcena-Ruiz (2007). Lu (2006), Lu and Poddar (2009) and Heywood and Ye (2009) extended the analysis into the mixed markets. For recent analysis, see Bárcena-Ruiz and Garzon (2010), Tomaru and Kiyono (2010), Balogh and Tasnadi (2012), Amir and Feo (2014), Matsumura and Ogawa (2010, 2014, 2017), Naya (2015), Din and Sun (2016) and Lee and Xu (2018) among others.

[2] EU institutions have reaffirmed their commitment to R&D policies and consequently, the budgets of the research Framework Programs (FPs) have grown from EUR 3.3 billion in the first FP, launched in 1984, to EUR 80 billion of Horizon 2020. For more details, see Miyagiwa and Ohno (2002), Marinucci (2012) and Lee and Muminov (2017).

[3] In the last generation, there have been considerable theoretical works on R&D and policy implications for innovation under imperfect competition along with different structures of ownership. In the literature of mixed oligopolies, the works by Delbono and Denicolo (1993) and Nett (1994) have motivated the analysis of R&D decision. For recent studies, see Ishibashi and Matsumura (2006), Heywood and Ye (2009), Gil-Molto et al. (2011), Kesavayuth and Zikos (2013), Lee et al. (2017) and Lee and Tomaru (2017) among others.

Previous analyses have heavily investigated the fixed timing of firms' decision on R&D where mostly simultaneous-move game in R&D investment is exogenously given. However, firms may choose their R&D level sequentially (not only what action to take, but when to take them) given conditions for knowledge sharing in a certain market-place. We incorporate this important aspect into the model with an endogenous timing game and analyze the strategic movement of R&D investment.

This paper is the first to investigate an endogenous timing game on strategic timing choice of cost-reducing R&D investment with R&D spillovers in private and mixed duopolies. We extend an observable delay game into the two-period duopoly game that firms first determine their cost reducing R&D investments either simultaneously or sequentially given the rate of spillovers, and then play Cournot output competition. That is, we consider a Cournot duopoly in output competition but allow for an endogenous timing in R&D investments. This structural enhancement of the model allows us to anticipate when the firms are likely to play either a leader or a follower in making their R&D investments decision.

We highlight that the rate of spillovers is crucial in determining equilibrium of endogenous R&D choices in both private and mixed markets. We also indicate that welfare consequences are contrast between the two markets, depending on the timing of R&D choices. Therefore, in the process of privatization policy, not only the rate of spillovers but also the endogenous leadership position in R&D timing are important factors in determining welfare consequences.

Our main findings are as follows: First, a leader's R&D and output are higher than those of a follower for any rate of spillover under the sequential-move game in a private market while public firm's R&D and output are higher than those of the private firm for any market structure in a mixed duopoly. Hence, the presence of public firm enhances not only total R&D expenditures but also total market outputs.

Second, in a private duopoly, a simultaneous-move is equilibrium if the spillovers rate is low while a sequential-move is otherwise. This implies that the simultaneous-move game in R&D decisions is plausible in the case of no spillovers, but if there exist R&D spillovers, it might be problematic. We also show that a sequential-move in R&D choices yields a higher total R&D investments and total market outputs. Thus, a higher rate of spillovers can work for changing the equilibrium of an endogenous R&D timing game to a sequential-move game and enhance the welfare. This result complements previous findings in Hamilton and Slutsky (1990) who showed that the equilibrium of an endogenous timing game is that both firms choose outputs (under the same R&D investments in the absence of spillovers) simultaneously. However, when the spillovers rate is high, both firms prefer a sequential-move in R&D choices even though they prefer a simultaneous-move in output choices.

Third, in a mixed duopoly, a simultaneous-move game cannot be equilibrium. Thus, in the existence of public firm, the sequential game in R&D decisions is more plausible irrespective of the rate of spillovers. Further, public leadership is the only equilibrium if the spillovers rate is intermediate and its welfare is always higher than that under the other two cases. This result is in contrast to the previous studies in Pal (1998), Tomaru and Kiyono (2010) and Matsumura and Ogawa (2010, 2017) who showed that private leadership is more robust in a mixed market with quantity competition. Our findings highlight the key role of R&D spillovers which can change the equilibrium outcome between private and public leadership of the endogenous R&D timing game in a mixed market.

Finally, we demonstrate that implementation of privatization policy by government may transform public leader to a private leader which in turn decrease the social welfare for any rate of spillovers. This is because privatization policy decrease total industry R&D investments and outputs for any rate of spillovers. Our analysis reveals that the rate of spillovers does not affect welfare consequences under privatization policy. This is consistent with the result of Gil-Moltó et al. (2011), who showed that privatization is not desirable, regardless of whether the government provides R&D subsidies to private and public firms.

The remainder of the paper is structured as follows. In section 2, we present a basic duopoly model of R&D investment with spillovers. In section 3 and 4, we analyze an endogenous R&D timing game in private and mixed duopolies, respectively. We then compare the equilibrium outcomes and examine the welfare effect of privatization policy in section 5. Finally, section 6 concludes the paper.

2. The Model

We consider a duopoly with homogeneous goods in an R&D and then quantity-setting game. Let the inverse demand function be $P(Q) = a - Q$, where $P(\mathrm{Q})$ is market price, $Q = q_0 + q_1$ is market output, and q_i is output of firm $i = 0{,}1$, respectively. The consumer surplus is calculated as $CS = Q^2/2$.

We assume a (convex) production cost function of the firm where each firm has an increasing cost function in production and R&D investment. In specific, we assume the following cost function, which is used in the literature of the cost-reducing innovation[4]:

$$C(q_i, x_i) = \left(c - x_i - \beta x_j\right)q_i + q_i^2 \text{ and } \Gamma(x_i) = x_i^2, \quad for\ i,j = 0{,}1\ and\ i \neq j. \tag{1}$$

where $a > c > 0$ and x_i denotes the amount of R&D investment for firm i, which exhibits decreasing returns to scale, i.e., the firm has to spend x_i^2 to implement cost-reducing R&D, x_i. Note that the initial cost c is reduced not only by each firm's R&D investment, x_i, but by the rival's R&D investment, βx_j, where $\beta \in [0,1]$ denotes the R&D spillovers rate. Thus, R&D investment can reduce a firm's own cost by x_i and the rival firm's cost by βx_i per unit of output, depending on the spillovers rate.

The profit function of the firm is as follows:

$$\pi_i = \left(a - q_i - q_j\right)q_i - (c - x_i - \beta x_j)q_i - q_i^2 - x_i^2, \quad for\ i,j = 0{,}1\ and\ i \neq j. \tag{2}$$

We define social welfare as the sum of consumer surplus and both firms' profits:

$$W = CS + \pi_0 + \pi_1. \tag{3}$$

[4] The model with linear demand and quadratic cost functions is a standard formulation and popularly used in the literature since D'Aspremont and Jacquemin (1988). In the literature of mixed oligopolies, see Gil Molto et al. (2011, 2018), Kesavayuth and Zikos (2013), Lee et al. (2017) and Leal et al. (2018, 2019). It can rule out an uninteresting corner solution case of a public monopoly. Matsumura and Okamura (2015) and Kim et al. (2019) also provided the economic rationale behind this formulation. The production cost shows that a firm's R&D investment shifts its marginal cost function downwards, $\partial C/\partial q_i = c - x_i - \beta x_j + 2q_i$, but does not alter its slope.

In the followings, we examine two models of private and mixed duopolies in which one of two firms (say firm 0) is either a private firm or a public firm while the other firm (say firm 1) is a private firm in both cases. We assume that private firm maximizes its profit in (2) while the public firm is fully owned by a benevolent government which maximizes the social welfare in (3).

The setting is a multi-stage game with an observable delay period formulated by Hamilton and Slutsky (199). In the first stage, each firm simultaneously chooses whether to move early or late in determining its R&D investment. The following game played in the second stage is either a simultaneous R&D if both firms choose the same period or a sequential R&D otherwise. In the last stage, both firms compete with outputs simultaneously. We solve the subgame perfect Nash equilibrium of these games by backward induction.

3. Private duopoly

In this section, we first consider a fixed-timing game in R&D decisions in a private duopoly, and then examine the first-stage in an endogenous timing game.

3.1. Output decisions

In the last stage of output choice, the first-order conditions of private firms are as follows:[5]

$$\frac{\partial \pi_i}{\partial q_i} = a - q_j - 4q_i - (c - x_i - \beta x_j) = 0, \quad i,j = 0,1 \; and \; i \neq j. \tag{4}$$

Equation (4) yields the following response functions of each firm:

$$q_i = \frac{a - q_j - (c - x_i - \beta x_j)}{4}, \quad i,j = 0,1 \; and \; i \neq j. \tag{5}$$

As usual, outputs are strategic substitutes for both firms. Solving these reaction functions provide the following equilibrium output levels of each firm in the last stage:

[5] Note that all the second-order conditions of the equilibrium outcomes in this study are satisfied.

$$q_i(x_i, x_j) = \frac{3(a-c) + (4-\beta)x_i - (1-4\beta)x_j}{15}, \quad i, j = 0,1 \; and \; i \neq j. \tag{6}$$

The relationship between outputs and R&D choices are as follows:

$$\frac{\partial q_i(x_i,x_j)}{\partial x_i} = \frac{4-\beta}{15} > 0 \;\; and \;\; \frac{\partial q_i(x_i,x_j)}{\partial x_j} = -\frac{1-4\beta}{15} \gtreqless 0 \text{ if } \beta \gtreqless 1/4, \quad i, j = 0,1 \; and \; i \neq j.$$

It shows that an increase in R&D by the firm increases its output, but increases (decreases) its rival's output if the spillovers rate is high (low).

3.2 R&D decisions

In the second stage, each firm chooses R&D investment to maximize its profits, depending on the timing of movement chosen in the first stage. We analyze two scenarios: simultaneous and sequential choices of R&D investment.

3.2.1. Simultaneous R&D

Putting (6) into the profit function in (2) and taking the first-order conditions provide the following response functions of each firm:

$$x_i(x_j) = \frac{2(4-\beta)\big(3(a-c) - (1-4\beta)x_j\big)}{193 + 2(8-\beta)\beta}, \quad i, j = 0,1 \; and \; i \neq j. \tag{7}$$

Note that R&D decisions are strategic substitutes when the rate of spillovers is low but become strategic complements as the rate of spillovers increases. That is, $\frac{x_i(x_j)}{\partial x_j} \gtreqless 0$ if $\beta \gtreqless 1/4$. Solving the reaction functions provides the equilibrium R&D investments:

$$x_i^{CP} = \frac{2(a-c)(4-\beta)}{67 - 2(3-\beta)\beta}. \tag{8}$$

where the superscript *CP* denotes the equilibrium under the simultaneous-move **C**ournot game in a **P**rivate market. It shows that firm's R&D investment is positive and decreasing in the rate of spillovers, that is, $\frac{\partial x_i^{CP}}{\partial \beta} < 0$.

Then, putting (8) into (6), we have the equilibrium outputs of the last stage:

$$q_i^{CP} = \frac{15(a-c)}{67 - 2(3-\beta)\beta}. \tag{9}$$

Note that both firms increase their outputs as the rate of spillovers increases, that is, $\frac{\partial q_i^{CP}}{\partial \beta} > 0$.

The resulting profit of the private firm and social welfare are as follows, respectively:

$$\pi_i^{CP} = \frac{2(a-c)^2(193 + 2(8-\beta)\beta)}{(67 - 2(3-\beta)\beta)^2},$$

$$W^{CP} = \frac{2(a-c)^2(611 + 4(8-\beta)\beta)}{(67 - 2(3-\beta)\beta)^2}. \tag{10}$$

Then, firm's profits and social welfare increase as the spillovers rate increases.

3.2.2. Sequential R&D leadership

In this case, we further assume that firm 0 plays a leading position in a private market. Using backward induction, a follower firm 1 chooses its R&D investment after observing the R&D investment by the firm 0. The first-order condition of the firm 1 provides the response function in (7). Inserting $x_1(x_0)$ into the profit function of firm 0 and using the first-order condition, we get the followings:

$$x_0^{LP} = \frac{2(a-c)(4-\beta)(13 + 2\beta^2)(37 + 10\beta - 2\beta^2)}{31841 + 8880\beta - 2518\beta^2 + 768\beta^3 - 228\beta^4 + 64\beta^5 - 8\beta^6},$$

$$x_1^{LP} = \frac{2(a-c)(4-\beta)(475 + 178\beta - 48\beta^2 + 20\beta^3 - 4\beta^4)}{31841 + 8880\beta - 2518\beta^2 + 768\beta^3 - 228\beta^4 + 64\beta^5 - 8\beta^6}. \tag{11}$$

where the superscript *LP* denotes the equilibrium under the sequential-move game with firm's **L**eadership in a **P**rivate market. Note that a leading firm always undertakes larger investment than that of a following firm, i.e., $x_0^{LP} \geq x_1^{LP}$. However, as the spillovers rate increases, both firms decrease their R&D investments, i.e., $\frac{\partial x_i^{LP}}{\partial \beta} < 0$,

Then, putting (11) into (6), we have the equilibrium outputs of the last stage:

$$q_0^{LP} = \frac{(a-c)(193+2(8-\beta)\beta)(37+2(5-\beta)\beta)}{31841+8880\beta-2518\beta^2+768\beta^3-228\beta^4+64\beta^5-8\beta^6},$$

$$q_1^{LP} = \frac{15(a-c)(475+178\beta-48\beta^2+20\beta^3-4\beta^4)}{31841+8880\beta-2518\beta^2+768\beta^3-228\beta^4+64\beta^5-8\beta^6}. \quad (12)$$

Note that a leader always produces larger output than that of a follower, i.e., $q_0^{LP} \geq q_1^{LP}$. Thus, private leader is more aggressive in both R&D investment and output production for any rate of spillovers under the sequential-move R&D in a private market. Note also that as the spillovers rate increases, both firms increase their outputs, i.e., $\frac{\partial q_i^{LP}}{\partial \beta} > 0$.

The resulting profits of private firms and social welfare are as follows, respectively:

$$\pi_0^{LP} = \frac{2(a-c)^2(37+10\beta-2\beta^2)^2}{31841+8880\beta-2518\beta^2+768\beta^3-228\beta^4+64\beta^5-8\beta^6},$$

$$\pi_1^{LP} = \frac{2(a-c)^2(193+16\beta-2\beta^2)(475+178\beta-48\beta^2+20\beta^3-4\beta^4)^2}{(31841+8880\beta-2518\beta^2+768\beta^3-228\beta^4+64\beta^5-8\beta^6)^2}. \quad (13)$$

$$W^{LP} = \frac{2(a-c)^2(138015643+108999896\beta+627808\beta^2-3582584\beta^3+1797532\beta^4-812816\beta^5+137056\beta^6-15392\beta^7-3344\beta^8+1152\beta^9-64\beta^{10})}{(31841+8880\beta-2518\beta^2+768\beta^3-228\beta^4+64\beta^5-8\beta^6)^2}. \quad (14)$$

Then, profits of both private firms and social welfare increase as the spillovers rate increases.

3.3 Endogenous R&D timing

We examine the modified format of the observable delay game in Hamilton and Slutsky (1990) where both private firms choose its timing to move between "early" (t_i =1) and "late" (t_i=2) in determining their R&D choices. If both firms choose the same period, it yields the equilibrium of a simultaneous-move game. Otherwise, the equilibrium under a sequential-move game with leadership emerges.

Table 1 provides the payoff matrix of the observable delay game in a private market.[6]

Table 1: Payoff Matrix in a Private Market

Firm 0 \ Firm 1	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	π_0^{CP}, π_1^{CP}	π_0^{LP}, π_1^{LP}
$t_0 = 2$	π_1^{LP}, π_0^{LP}	π_0^{CP}, π_1^{CP}

Lemma 1. $\pi_1^{LP} < \pi_i^{CP} < \pi_0^{LP}$ if $\beta < 0.25$ while $\pi_i^{CP} < \pi_0^{LP} < \pi_1^{LP}$ if $\beta > 0.25$.

It states that a leader's profit is higher (lower) than that of follower when the spillovers rate is low (high), i.e., $\pi_0^{LP} \gtreqless \pi_1^{LP}$ if $\beta \lesseqgtr 0.25$. This is because private leader is more aggressive in both R&D investment and output production under the sequential-move game, which might cause lower profit to the leader (but higher than that under simultaneous-move game) when the spillovers rate is high and thus free-riding effect is strong. Using this lemma, we have the following result:

Proposition 1. The equilibrium of endogenous R&D timing game in a private market is as follows:

(i) If $\beta \in [0, 0.25]$, then a simultaneous-move is the only equilibrium; $(t_0, t_1) = (1,1)$;

(ii) If $\beta \in (0.25, 1]$, a sequential-move is an equilibrium; $(t_0, t_1) = (1,2)$ or $(t_0, t_1) = (2,1)$.

Proposition 1 states that the rate of spillovers is crucial in determining the equilibrium of an endogenous R&D timing game in a private market. In special, the only equilibrium is a simultaneous-move game if spillovers rate is low while one of sequential-move games appear otherwise. Thus, simultaneous-move game cannot be an equilibrium for a higher rate of spillovers. It implies that the sequential game in R&D

[6] The proofs of lemmas and propositions are provided in Appendix.

decisions is more plausible in a private market for higher rate of spillovers. This result complements the findings in the quantity setting game that in the absence of spillovers, private firms decide the timing of movement simultaneously (Hamilton and Slutsky, 1990). However, when the spillovers rate is high, both firms prefer a sequential-move in R&D choices even though they prefer a simultaneous-move in output choices.

Finally, we can show that social welfare under the sequential-move in R&D is always higher than that under the simultaneous-move game in a private market for any rate of spillovers, i.e., $W^{CP} \leq W^{LP}$ where the equality satisfies if $\beta = 0.250$. Thus, we have the following result.

Proposition 2. When the spillovers rate is not so low, the equilibrium with a sequential-move R&D leadership in a private market is always socially beneficial.

4. Mixed duopoly

In this section, we assume that firm 0 is a fully nationalized public firm that maximizes the welfare in (3). We first consider a fixed-timing game in R&D decisions in a mixed duopoly, and then examine the first-stage in an endogenous timing game.

4.1. Output decisions

In the last stage of output choice, the first-order conditions of the public and private firms are as follows, respectively:

$$\frac{\partial W}{\partial q_0} = a - 3q_0 - q_1 - (c - x_0 - \beta x_1) = 0,$$

$$\frac{\partial \pi_1}{\partial q_1} = a - q_0 - 4q_1 - (c - x_1 - \beta x_0) = 0. \qquad (15)$$

Rearranging these two equations yields the following response functions of firms:

$$q_0 = \frac{a - q_1 - (c - x_0 - \beta x_1)}{3},$$

$$q_1 = \frac{a - q_0 - (c - x_1 - \beta x_0)}{4}. \quad (16)$$

Again, outputs are strategic substitutes for both firms. Solving these reaction functions provide the following equilibrium output levels of both firms in the last stage, respectively:

$$q_0(x_0, x_1) = \frac{3(a - c) + (4 - \beta)x_0 - (1 - 4\beta)x_1}{11},$$

$$q_1(x_0, x_1) = \frac{2(a - c) - (1 - 3\beta)x_0 + (3 - \beta)x_1}{11},$$

$$Q(x_0, x_1) = \frac{5(a - c) + (3 + 2\beta)x_0 + (2 + 3\beta)x_1}{11}. \quad (17)$$

The relationship between outputs and R&D are as follows:

$$\frac{\partial q_0(x_0, x_1)}{\partial x_0} = \frac{4-\beta}{11} > 0, \qquad \frac{\partial q_0(x_0, x_1)}{\partial x_1} = -\frac{1-4\beta}{11} \gtreqless 0 \text{ if } \beta \gtreqless 1/4 \text{ and}$$

$$\frac{\partial q_1(x_0, x_1)}{\partial x_1} = \frac{3-\beta}{11} > 0, \qquad \frac{\partial q_1(x_0, x_1)}{\partial x_0} = -\frac{1-3\beta}{11} \gtreqless 0 \text{ if } \beta \gtreqless 1/3$$

It shows that an increase in R&D by the firm increases the outputs of the firms, but that of the public firm is higher than that of the private firm, that is, $\frac{\partial q_0(x_0, x_1)}{\partial x_0} > \frac{\partial q_1(x_0, x_1)}{\partial x_1} > 0$. It also shows that firm's R&D increases (decreases) the output of the rival firm if the rate of spillovers is relatively high (low).

4.2 R&D decisions

In the second stage, both firms choose R&D investments to maximize their objective functions, respectively, depending on the timing of movement chosen in the first stage. We analyze three scenarios: one simultaneous and two sequential choices with public firm leadership and private firm leadership, respectively, in R&D investment.

4.2.1. Simultaneous R&D

Putting (7) into the objective function of each firm in (2) for a private firm and (3) for a public firm and taking the first-order conditions provide the following response functions of each firm:

$$x_0(x_1) = \frac{(a-c)(31+28\beta) - \big(14-\beta(87-14\beta)\big)x_1}{197+14\beta(2-3\beta)}. \tag{18}$$

$$x_1(x_0) = \frac{2(3-\beta)(2(a-c)-(1-3\beta)x_0)}{103+2(6-\beta)\beta}. \tag{19}$$

It shows that R&D decisions are strategic substitutes when the rate of spillovers is relatively low but become strategic complements as the rate increases. That is, $\frac{x_0(x_1)}{\partial x_1} \gtreqless 0$ if $\beta \gtreqless 0.165$ and $\frac{x_1(x_0)}{\partial x_0} \gtreqless 0$ if $\beta \gtreqless 0.333$. Note also that if the spillovers rate is intermediate, $0.165 < \beta < 0.333$, the public firm's R&D increases as the private firm's R&D increases, i.e., strategic complements for a public firm, while the private firm's R&D decreases as the public firm's R&D increases, i.e., strategic substitutes for a private firm. Solving the reaction functions provides the equilibrium R&D investments:

$$x_0^{CM} = \frac{(a-c)(25+2(18-\beta)\beta)}{167+2(25-\beta)(1-\beta)\beta},$$

$$x_1^{CM} = \frac{2(a-c)(9-\beta^2)}{167+2(25-\beta)(1-\beta)\beta}. \tag{20}$$

where the superscript *CM* denotes the equilibrium under the simultaneous-move **C**ournot game in a **M**ixed market. Note that the public firm always undertakes larger investment than that of a private firm, i.e., $x_0^{CM} > x_1^{CM}$. As the spillovers rate increases, the public firm's R&D increases, i.e., $\frac{\partial x_0^{CM}}{\partial \beta} > 0$, while that of the private firm depends on the rate, i.e., $\frac{\partial x_1^{CM}}{\partial \beta} \gtreqless 0$ if $\beta \gtreqless 0.879$. However, the total R&D increases as spillovers rate increase, i.e., $\frac{\partial (x_0^{CM}+x_1^{CM})}{\partial \beta} > 0$.

Then, putting (20) into (17), we have the equilibrium outputs of the last stage:

$$q_0^{CM} = \frac{(a-c)(53+\beta(31-18\beta))}{167+2(25-\beta)(1-\beta)\beta},$$

$$q_1^{CM} = \frac{11(a-c)(3+\beta)}{167+2(25-\beta)(1-\beta)\beta}. \quad (21)$$

Note that $q_0^{CM} > q_1^{CM}$. It states that the public firm's R&D and output are higher than those of the private firm for any rate of spillovers. Thus, the public firm is more aggressive in both R&D investment and output production under simultaneous-move game in a mixed market. Note also that both firms' outputs increase as the spillovers rate increases, i.e., $\frac{\partial q_i^{CM}}{\partial \beta} > 0$.

The resulting profit of the private firm and social welfare are as follows, respectively:

$$\pi_1^{CM} = \frac{2(a-c)^2(3+\beta)^2(103+2(6-\beta)\beta)}{(167+2(25-\beta)(1-\beta)\beta)^2},$$

$$W^{CM} = \frac{(a-c)^2(7736+\beta(6550-\beta(2495+2\beta(864-239\beta))))}{(167+2(25-\beta)(1-\beta)\beta)^2}. \quad (22)$$

Then, firm's profits and social welfare increase as the spillovers rate increases.

4.2.2. Sequential R&D with public leadership

We further address a Stackelberg leadership situation, in which the public firm plays a leading position and the private firm acts as a follower in R&D choices. Using backward induction, the private firm chooses its R&D investment after observing the R&D investment by the public firm. The first-order condition of the private follower provides the response function in (18). Inserting $x_1(x_0)$ into the welfare function and using the first-order condition for the public firm's R&D, we get the followings:

$$x_0^{LM} = \frac{(a-c)(2551+3890\beta+764\beta^2-122\beta^3-84\beta^4+12\beta^5)}{17189+7440\beta-5922\beta^2-236\beta^3-4\beta^4+72\beta^5-12\beta^6},$$

$$x_1^{LM} = \frac{2(a-c)(927+126\beta-205\beta^2+92\beta^3-42\beta^4+6\beta^5)}{17189+7440\beta-5922\beta^2-236\beta^3-4\beta^4+72\beta^5-12\beta^6}. \quad (23)$$

where the superscript *LM* denotes the equilibrium under the sequential-move public **L**eadership in a **M**ixed market. Note that a public leader always undertakes larger investment than that of a private follower, i.e., $x_0^{LM} > x_1^{LM}$. As the spillovers rate increases, the public leader's R&D increases, i.e., $\frac{\partial x_0^{LM}}{\partial \beta} > 0$ while that of private follower depends on the spillovers rate, i.e., $\frac{\partial x_1^{LM}}{\partial \beta} \gtrless 0$ if $\beta \gtrless 0.759$. However, the total R&D increases as the spillovers rate increase, i.e., $\frac{\partial (x_0^{LM}+x_1^{LM})}{\partial \beta} > 0$.

Then, putting (23) into (17), we have the equilibrium outputs of the last stage:

$$q_0^{LM} = \frac{(a-c)(5447 + 3863\beta - 1562\beta^2 - 344\beta^3 + 54\beta^4)}{17189 + 7440\beta - 5922\beta^2 - 236\beta^3 - 4\beta^4 + 72\beta^5 - 12\beta^6},$$

$$q_1^{LM} = \frac{11(a-c)(309 + 145\beta - 20\beta^2 + 24\beta^3 - 6\beta^4)}{17189 + 7440\beta - 5922\beta^2 - 236\beta^3 - 4\beta^4 + 72\beta^5 - 12\beta^6}. \quad (24)$$

Note that $q_0^{LM} > q_1^{LM}$ and $\frac{\partial q_i^{LM}}{\partial \beta} > 0$. It states that the public leader's output is higher than that of the private follower. It also implies that the public leader is more aggressive in both R&D investment and output production for any spillover rate under the sequential-move with public leadership in a mixed market.

The resulting profit of the private firm and social welfare are as follows, respectively:

$$\pi_1^{LM} = \frac{2(a-c)^2(103 + 12\beta - 2\beta^2)(309 + 145\beta - 20\beta^2 + 24\beta^3 - 6\beta^4)^2}{(17189 + 7440\beta - 5922\beta^2 - 236\beta^3 - 4\beta^4 + 72\beta^5 - 12\beta^6)^2},$$

$$W^{LM} = \frac{(a-c)^2(4768 + 3246\beta - 835\beta^2 - 192\beta^3 + 24\beta^4)}{17189 + 7440\beta - 5922\beta^2 - 236\beta^3 - 4\beta^4 + 72\beta^5 - 12\beta^6}. \quad (25)$$

Then, firm's profits and social welfare increase as the spillovers rate increases.

4.2.3. Sequential R&D with private leadership

We finally address the reversed Stackelberg leadership situation, in which the private firm plays a leading position while the public firm acts as a follower in a mixed market. Using backward induction, the public firm chooses its R&D investment after observing the R&D investment by the private firm. The first-order condition of the public follower provides the response function in (18). Inserting $x_0(x_1)$

into the profit function and using the first-order condition for the private firm's R&D, we get the followings:

$$x_0^{FM} = \frac{(a-c)(4897+2\beta(4039-\beta(622+467\beta)))}{32759+2\beta(7936-\beta(9576+\beta(692-761\beta)))},$$

$$x_1^{FM} = \frac{242(a-c)(3+\beta)(5-(2-\beta)\beta)}{32759+2\beta(7936-\beta(9576+\beta(692-761\beta)))}. \quad (26)$$

where the superscript *FM* denotes the equilibrium under the sequential-move private leadership (public **F**ollowership) in a **M**ixed market. Note that a private leader always undertakes smaller investment than that of a public follower, i.e., $x_0^{FM} > x_1^{FM}$. As the spillovers rate increases, the public follower's R&D increases, i.e., $\frac{\partial x_0^{FM}}{\partial \beta} > 0$ while that of the private leader depends on the spillovers rate, i.e., $\frac{\partial x_1^{FM}}{\partial \beta} \gtreqless 0$ if $\gtreqless 0.391$. However, the total R&D increases as the spillovers rate increase, i.e., $\frac{\partial (x_0^{FM}+x_1^{FM})}{\partial \beta} > 0$.

Then, putting (27) into (17), we have the equilibrium outputs of the last stage:

$$q_0^{FM} = \frac{(a-c)(10385+\beta(8163-2\beta(3260+\beta(269-294\beta))))}{32759+2\beta(7936-\beta(9576+\beta(692-761\beta)))},$$

$$q_1^{FM} = \frac{11(a-c)(3+\beta)(197+14\beta(2-3\beta))}{32759+2\beta(7936-\beta(9576+\beta(692-761\beta)))}. \quad (27)$$

Note that $q_0^{FM} > q_1^{FM}$ and $\frac{\partial q_0^{FM}}{\partial \beta} > 0$ but $\frac{\partial q_1^{FM}}{\partial \beta} \gtreqless 0$ if $\beta \gtreqless 0.011$. However, the total R&D and output increase as the spillovers rate increase, i.e., $\frac{\partial (q_0^{FM}+q_1^{FM})}{\partial \beta} > 0$. It states that the public follower's output is higher than that of the private leader. Thus, the public leader is more aggressive in both R&D investment and output production for any spillover rate under the sequential-move with private leadership in a mixed market.

The resulting profit of the private firm and social welfare are as follows, respectively:

$$\pi_1^{FM} = \frac{242(a-c)^2(3+\beta)^2}{32759+2\beta(7936-\beta(9576+\beta(692-761\beta)))},$$

$$W^{FM} = \frac{(a-c)^2 \left\{ \begin{array}{c} 297785216 + \beta(362599906 - \beta(197568983 + \\ 2\beta(108875022 - \beta(41846137 + 2\beta(7595446 - \beta \\ (2962433 + 882\beta(346 - 147\beta))))))) \end{array} \right\}}{(32759 + 2\beta(7936 - \beta(9576 + \beta(692 - 761\beta))))^2} \quad (28)$$

Then, firm's profits and social welfare increase as the spillovers rate increases.

4.3 Endogenous timing game

We examine an endogenous timing game in R&D choices in a mixed duopoly. If both firms choose the same period, it yields the equilibrium of a simultaneous-move game. Otherwise, the equilibrium under a sequential-move game with either public leadership or private leadership emerges.

Table 2 provides the payoff matrix of the observable delay game in a mixed market.

Table 2: Payoff Matrix in a Mixed Market

Firm 0 \ Firm 1	$t_1 = 1$	$t_1 = 2$
$t_0 = 1$	W^{CM}, π_1^{CM}	W^{LM}, π_1^{LM}
$t_0 = 2$	W^{FM}, π_1^{FM}	W^{CM}, π_1^{CM},

Lemma 2. $\pi_1^{CM} \leq \pi_1^{FM} \leq \pi_1^{LM}$ where the equality satisfies only if $\beta = 0.333$.

It states that irrespective of the spillovers rate the private firm's profit under its leadership is not higher than that under public leadership, but it is higher than that under the simultaneous-move game. This is because the public leader is the most aggressive in both R&D investment and output production, which increases production inefficiency of the public firm and causes the highest profits to the private follower.

Lemma 3.

(i) $W^{CM} < W^{LM} < W^{FM}$ if $0 < \beta < 0.163$

(ii) $W^{CM} < W^{FM} < W^{LM}$ if $0.163 < \beta < 0.165$

(iii) $W^{FM} < W^{CM} < W^{LM}$ if $0.165 < \beta < 0.333$

(iv) $W^{CM} < W^{LM} < W^{FM}$ if $0.333 < \beta < 1$

It shows that the spillovers rate is crucial in welfare rankings. The welfare under public leadership in R&D choices is always higher than that under the simultaneous-move game but is higher than that under the private leadership only when the spillovers rate is intermediate, i.e., $W^{FM} < W^{LM}$ if $0.163 < \beta < 0.333$. Using this lemma, we have the following result:

Proposition 3. The equilibrium of endogenous R&D timing game in a mixed market is as follows:

(i) If $\beta \in [0, 0.165)$ then either the public leadership or private leadership is an equilibrium; $(t_0, t_1) = (1,2)$ or $(t_0, t_1) = (2,1)$.

(ii) If $\beta \in (0.165, 0.333)$, then the public leadership is the only equilibrium; $(t_0, t_1) = (1,2)$;

(iii) If $\beta \in (0.333, 1]$, then either the public leadership or private leadership is an equilibrium; $(t_0, t_1) = (1,2)$ or $(t_0, t_1) = (2,1)$.

Proposition 3 also states that the rate of spillovers is crucial in determining the equilibrium of endogenous R&D timing game in a mixed market. First, the simultaneous-move game cannot be an equilibrium, but either public or private leadership game in R&D choices will be an equilibrium. This result is in contrast to the results in a private market where simultaneous-move game emerges for a lower rate of spillovers. It also implies that the sequential game in R&D decisions is more plausible in a mixed market irrespective of the rate of spillovers. This finding is also in a sharp contrast to Pal (1998), Tomaru and Kiyono (2010) and Matsumura and Ogawa (2010, 2017) who showed that private leadership is more robust in a mixed market with quantity competition. Our findings highlight the key role of R&D factors which can change the equilibrium outcome between private and public leadership of the endogenous timing in a mixed market.

Finally, we can show that the public leadership is the only equilibrium if the spillovers rate is intermediate, i.e., $\beta \in (0.165, 0.333)$ and the welfare under the public leadership is always higher than that under the other two cases. Then, we have the following result.

Proposition 4. When the spillovers rate is intermediate, the equilibrium with a public leadership in R&D choices in a mixed market is always socially beneficial.

5. Welfare effect of privatization policy

In this section, we examine the welfare effect of privatization policy when the government implements privatization, which in return affects the firms' endogenous choice of R&D investments.

Lemma 4. $\mathrm{W^{CM}} > \mathrm{W^{CP}}$ and $\min\{\mathrm{W^{FM}}, \mathrm{W^{LM}}\} > \mathrm{W^{LP}} \geq \mathrm{W^{CP}}$.

Using this lemma and comparing the equilibrium in the endogenous R&D timing game in private and mixed duopolies, we obtain the following results:

Proposition 5 Privatization policy in an endogenous R&D timing game always lowers social welfare.

Proof: Comparing the welfare in Lemma 3 and 4, we obtain the following results: (i) $W^{CP} < W^{LM} < W^{FM}$ if $0 < \beta < 0.163$ (ii) $W^{CP} < W^{FM} < W^{LM}$ if $0.163 < \beta < 0.25$ (iii) $W^{LP} < W^{LM}$ if $0.25 < \beta < 0.333$ (iv) $W^{LP} < W^{LM} < W^{FM}$ if $0.333 < \beta < 1$.

This proposition resembles the results of Fjell and Heywood (2004) and Heywood and Ye (2009) who considered a mixed oligopoly with quantity competition and examined the welfare effect of privatization. They showed that privatization results in a public leader becoming a private leader and reduces welfare. Lee and Xu (2018) also examined an endogenous timing with externalities and demonstrated that privatization will always lower social welfare. Our findings confirm that privatization in an endogenous R&D timing game will always lower social welfare irrespective of the spillovers rate.

6. Concluding Remarks

This study examined an endogenous R&D timing game in private and mixed duopolies with R&D spillovers. We showed that the rate of spillovers is crucial in determining the equilibrium of endogenous R&D choices in both private and mixed markets. We also showed that its welfare consequences are contrast between the two markets, which is affected by the government's privatization policy.

In a private duopoly, we showed that a simultaneous-move is an equilibrium if the spillovers rate is low while a sequential-move is otherwise. We also show that a sequential-move in R&D choices yields a higher welfare to the society. Thus, a higher rate of spillovers can work for changing the equilibrium of an endogenous R&D timing game and enhance the welfare.

In a mixed duopoly, we showed that a simultaneous-move game cannot be an equilibrium. Thus, in the existence of public firm, the sequential game in R&D decisions is more plausible irrespective of the rate of spillovers. We also showed that the public leadership is the only equilibrium if the spillovers rate is intermediate and its welfare is always higher than that under the other two cases. Our findings highlight the key role of R&D spillovers which can change the equilibrium outcome between private and public leadership of the endogenous R&D timing in a mixed market.

Finally, we showed that the implementation of privatization policy transforms a public leader to a private competitor, but this can decrease the social welfare. Therefore, in the process of privatization policy, not only the rate of spillovers but the endogenous leadership position in R&D timing should be well-investigated.

However, limitations remain largely due to model-specific assumptions. We need to examine the robustness of the results when there are multiple domestic or foreign private firms under the general functional forms when the firms compete in prices or quantities sequentially or simultaneously. Further, it is also important to analyze the effects of endogenous entry in an oligopolistic market and investigate the effect of governmental intervention in terms of subsidization and partial privatization policies. These are promising and challenging topics for future research.

Reference

Amir, R. and De Feo, G. (2014), "Endogenous timing in a mixed duopoly," *International Journal of Game Theory* 43, pp. 629–658.

Balogh, T.L. and Tasnádi, A. (2012), "Does timing of decisions in a mixed duopoly matter?," *Journal of Economics* 106 (3), pp. 233–249.

Baranes, E. and Tropeano, J.-P. (2003), "Why are technological spillovers spatially bounded? A market orientated approach," *Regional Science and Urban Economics* 33(4), pp. 445–466.

Bárcena-Ruiz, J. (2007), "Endogenous timing in a mixed duopoly: price competition," *Journal of Economics*, vol. 91, pp. 263–272.

Bárcena-Ruiz, J. and Garzón, M. (2010), "Endogenous timing in a mixed oligopoly with semipublic firms," *Portuguese Economic Journal* 9, pp. 97–113.

D'Aspremont, C. and Jacquemin, A. (1988), "Cooperative and non cooperative R&D in duopoly with spillovers," *American Economic Review* 78, pp. 1133–1137.

Delbono, F. and Denicolo, V. (1993), "Regulating Innovative activity: the role of a public firm," *International Journal of Industrial Organization* 11 (1), pp. 35–48.

Din H. and Sun C. (2016), "Combining the endogenous choice of timing and competition version in a mixed duopoly," *Journal of Economics* 118, pp. 141–166.

Dowrick, S. (1986), "von Stackelberg and Cournot duopoly: choosing roles," *The RAND Journal of Economics* 17 (2), pp. 251–260.

Fjell, K. and Heywood, J. (2004), "Mixed oligopoly, subsidization and the order of firm's moves: the relevance of privatization," *Economics Letters* 83(3), pp. 411–416.

Garcia, A., Leal, M. and Lee, S.H. (2019), "Endogenous timing with a socially responsible firm," *Korean Economic Review* 35 (2), pp. 345–370.

Gil-Molto, M., Poyago-Theotoky, J. and Zikos, V. (2011), "R&D subsidies, spillovers and privatization in mixed markets," *Southern Economic Journal* 78, pp. 233–255.

Gil-Molto, M., Poyago-Theotoky, J., Neto J. and Zikos, V. (2018), "Mixed oligopoly, privatization and R&D subsidization," DOI: http://dx.doi.org/10.2139/ssrn.3162021.

Hamilton, J. and Slutsky, S. (1990), "Endogenous timing in duopoly games: Stackelberg or Cournot equilibria," *Games and Economic Behavior* 2 (1), pp. 29–46.

Haruna S. and R.K. Goel (2017), "Output subsidies in mixed oligopoly with research spillovers," *Journal of Economics and Finance* 41, pp. 235–256.

Haruna, S. and Goel, R.K. (2018), "Optimal pollution control in a mixed oligopoly with research spillovers," *Australian Economic Papers* 58(1), pp. 21–40.

Heywood, J.S. and Ye, G. (2009), "Partial privatization in a mixed duopoly with an R&D rivalry," *Bulletin of Economic Research* 61, pp. 165–178.

Ishibashi, I. and Matsumura, T. (2006), "R&D competition between public and private sectors," *European Economic Review* 50, pp. 1347–1366.

Katsoulacos, Y. and Ulph, D. (1998), "Endogenous spillovers and the performance of research joint ventures," *The Journal of Industrial Economics* 46(3), pp. 333–357.

Kesavayuth, D. and Zikos, V. (2013), "R&D versus output subsidies in mixed markets," *Economics Letters* 118, pp. 293–296.

Kim, S-L., Lee, S.H. and Matsumura, T. (2019), "Corporate social responsibility and privatization policy in a mixed oligopoly," *Journal of Economics*, 128(1), pp. 67 – 89.

Leal, M. Garcia, A. and Lee, S.H. (2018). "The timing of environmental tax policy with a consumer-friendly firm," *Hitotsubashi Journal of Economics,* 59(1), pp. 25-43.

Leal, M., Garcia A. and Lee, S.H. (2019), "Sequencing R&D decisions with a consumer‐friendly firm and spillovers," The Japanese Economic Review, DOI: https://doi.org/10.1007/s42973-019-00028-5

Lee, S.H. and Muminov, T. (2017), "R&D Output Sharing in a Mixed Duopoly and Incentive Subsidy Policy," MPRA Paper 81732, University Library of Munich, Germany.

Lee, S.H., Muminov, T. and Tomaru, Y. (2017), "Partial privatization and subsidization in a mixed duopoly: R&D versus output subsidies," *"Hitotsubashi Journal of Economics"* 58(2), pp. 163–177.

Lee, S.H. and Tomaru, Y. (2017), "Output and R&D subsidies in mixed oligopoly," *Operations Research Letters* 45, pp. 238–241.

Lee, S.H. and Xu L. (2018), “Endogenous timing in private and mixed duopolies with emission taxes,” *Journal of Economics* 124, pp. 175–201.

Lu, Y. (2006), “Endogenous timing in a mixed oligopoly with foreign competitors: the linear demand case,” *Journal of Economics* 88, pp. 49–68.

Lu, Y. and Poddar, S. (2009), “Endogenous timing in a mixed duopoly and private duopoly -‘capacity-then-quantity’ game: the linear demand case,” *Australian Economic Papers* 48 (2), pp. 138–150.

Marinucci, M. (2012), “A primer on R&D cooperation among firms,” *Occasional Papers* No. 130*,* Bank of Italy.

Matsumura T. and Okamura M (2015), “Competition and privatization policies revisited: the payoff interdependence approach,” *Journal of Economics* 116 (2), pp. 137–150.

Matsumura, T. (1999), “Quantity-setting oligopoly with endogenous sequencing,” *International Journal of Industrial Organization* 17 (2), pp 289–296.

Matsumura, T. and Ogawa, A. (2010), “On the robustness of private leadership in mixed duopoly,” *Australian Economic Papers* 49 (2), pp. 149–160.

Matsumura, T. and Ogawa, A. (2014), “Corporate social responsibility or payoff asymmetry? A study of an endogenous timing game,” *Southern Economic Journal* 81 (2), pp. 457–473.

Matsumura, T. and Ogawa, A. (2017), “Endogenous Timing in Mixed Duopolies With Externality,” *Australian Economic Papers* 56 (4), pp. 304–327

Miyagiwa, K. and Ohno, Y. (2002), “Uncertainty, spillovers and cooperative R&D,” *International Journal of Industrial Organization* 20, pp. 855–876.

Naya, J. (2015), “Endogenous timing in a mixed duopoly model,” *Journal of Economics* 116, pp. 165–174.

Nett, L (1994), “Why private firms are more innovative than public firms,” *European Journal of Political Economy* 10(4), pp. 639–653.

Ono, Y. (1978), “The equilibrium of duopoly in a market of homogeneous goods,” *Economica* 45 (179), pp. 287–295.

Pal, D. (1998), “Endogenous timing in a mixed oligopoly,” *Economics Letters* 61 (2), pp. 181–185.

Poyago-Theotoky, J. (1995), “Equilibrium and optimal size of a research joint venture in an oligopoly with spillovers,” *Journal of Industrial Economics* 43, pp. 209–226.

Poyago-Theotoky, J. (1999), “A note on endogenous spillovers in a non-tournament R&D duopoly” *Review of Industrial Organization* 15, pp. 253–262.

Robson, A. J. (1990), “Duopoly with endogenous strategic timing: Stackelberg regained,” *International Economic Review* 31 (2), pp. 263–274.

Tomaru, Y. and Kiyono, K. (2010), “Endogenous timing in mixed duopoly with increasing marginal costs,” *Journal of Institutional and Theoretical Economics* 166 (4), pp. 591–613.