Environmental Regulations in Private and Mixed Duopolies: Emission Taxes versus Green RD Subsidies

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Environmental Regulations in Private and Mixed Duopolies:

Emission Taxes versus Green R&D Subsidies

Sang-Ho Lee* and Chul-Hi Park**

Abstract

In the presence of R&D spillovers, we compare environmental regulations between emission taxes and green R&D subsidies in private and mixed duopoly markets. We show that the green R&D subsidy is better (worse) than the emission tax when the green R&D cost is low (high) irrespective of the R&D spillovers, whereas the existence of a public firm encourages the government to adopt the subsidy policy. We then show that the optimal policy choice depends on the level of the R&D cost and the degree of R&D spillovers. In particular, when the R&D cost is high and the spillover rate is (not) weak, the government should choose the emission tax and (not) privatize the public firm. However, when the R&D cost is low, such a privatization policy is not desirable to society irrespective of the R&D spillovers.

Keywords: emission tax; green R&D subsidy; R&D spillovers; privatization policy

JEL Classification: L13; L21; M14

I. Introduction

Over the past generation, environmental policies have been implemented in polluting industries across the world given the global concern about climate change. As part of the Paris COP21 agreement an important number of countries in the world submitted independent nationally determined contributions along with environmental policies, including market-oriented mechanisms and stricter emission standards. For example, many progressive countries have already adopted market-based environmental regulation by using emission taxes, cap-and-trade, and pollution abatement (green R&D) subsidies.1

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1 For example, the USA implemented permits trading systems such as the sulfur dioxide (SO2) trading program, in which allowances were freely allocated under the Clean Air Act, and California's CO2 cap-and-trade program. Also, many countries including the EU and China introduced this program gradually during the last decade. Nowadays, it becomes a successful international experiment for controlling a large amount of greenhouse gases in
In the academic literature, researchers have devoted considerable efforts to evaluate various initiatives by using a range of theoretical and empirical approaches. Since 1990s, many economists explored the effect of emission taxes or tradeable permit systems in polluting markets. They showed that governments can promote social welfare by implementing market allocation of tradable permits (quantity regulation) or emission taxes (price regulation) since those mechanisms can minimize abatement costs when there are differences with respect to the abatement technologies among regulated firms. Further, a public policy with a green R&D subsidy has long been proposed to accelerate the adoption and diffusion of cleaner technologies and help support environment-friendly products.

On the other hand, from the administrative perspective of the ownership of the firms, technological and managerial improvements in public domains have shown the possible benefits of public ownership in polluting industries. Research encouraging the development of cleaner technologies is being paid more attention by governments with mixed markets where private firms compete against public firms which care for environment. As a first attempt, Beladi and Chao (2006) examined the environmental effect of public ownership while Bárcena-Ruiz and Garzon (2006) and Ohori (2006) investigated the effect of privatization policy on environmental regulation. Since then, a series of research have explored the interaction between privatization policy and environmental regulation, but relatively few economic analyses have addressed the comparative welfare effects of different policy instruments in polluting industries.

In fact, comparing emission taxes with green R&D subsidies in oligopolies has clear policy importance for developing a sustainable system. There have been considerable empirical and

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the world. Stavins (1998), Kato (2006), Burtraw and McCormack (2017) and Garcia et al. (2018) introduced some useful real-world discussions on the tradable emission permits as climate change policy instrument in the UN and the USA.


4 Mixed markets now exist in a broad range of industries that emit pollutants in the production process, such as oil, gas, automobile, steel, chemical, electricity, power generation, and healthcare. In transition economies, many state-owned industries are reliant on highly polluting technologies.

5 However, environmental concerns in a mixed market have been analyzed by several researchers relatively recently. For example, Kato (2013), Bárcena-Ruiz and Garzón (2014), Pal and Saha (2015), Xu and Lee (2015), Xu et al. (2016), Haruna and Goel (2017, 2018), Lee and Xu (2018) and Ouattara (2019) have explored the effect of product differentiation, partial privatization and tariff on the environmental policy.
theoretical works on R&D incentives and policy implications. A significant number of research has concluded that both the R&D spillovers and the effectiveness of the R&D cost are critical to assess the welfare effect of governmental intervention. For example, Poyago-Theotoky (2003) investigated the effect of emission taxes and green R&D subsidies in a differentiated goods duopoly, and compared the market conduct between Cournot and Bertrand in order to find optimal environmental policy. Youssef and Dinar (2011) analyzed the optimal combination of emission taxes and R&D subsidies, and showed that R&D investment is taxed when the marginal damage cost of pollution is sufficiently high. Poyago-Theotoky and Yong (2019) examined managerial compensation contract in a Cournot duopoly and showed that the level of green R&D cost is crucial to the effect of an emission tax on the green R&D decisions. However, the parametric properties of green R&D technologies such as R&D spillovers and the effectiveness of R&D cost and the progressive role of government ownership on the environmental policy are not sufficiently discussed.

In the recent development on green R&D policy in mixed markets, several works have provided a significant relationship between the government ownership and green R&D technologies. For example, Haruna and Goel (2019) examined the effect of emission taxes on green R&D investment in a mixed market and showed that privatization can lead to reductions in R&D and output. Further, Ouattara (2019) studies the impact of partial privatization on firm’s R&D investment under the emission taxes. However, policy comparisons between emission taxes and green R&D subsidies have attracted insufficient attention in contrast to the key role of government policy in facilitating environmental innovation.

In this study, we examine policy comparisons between emission taxes and green R&D subsidies in both private and mixed markets, respectively, and investigate policy interactions with R&D spillovers and the level of R&D cost. We show that the green R&D subsidy is better for society than the emission tax when the R&D cost is low irrespective of the R&D spillovers. This is because a green R&D subsidy is more effective to firms’ R&D investments, which can directly reduce the environmental damages with the lower emission abatement cost. However, the reverse occurs when the R&D cost is high. In that case, emission tax can reduce the output productions directly, which can indirectly reduce the environmental damages under the higher

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emission abatement cost. We also show that public firm will be more aggressive in R&D investments and output productions only when the R&D cost is low. Therefore, the existence of a public firm will encourage the government to adopt the green R&D subsidy policy rather than emission taxes.

We then examine the welfare effect of privatization policy and analyze its policy relation with environmental regulations. We show that when the R&D cost is low, the privatization policy does not improve social welfare irrespective of the R&D spillovers. Since public firm is more aggressive in the market activities when the R&D cost is low, it can achieve cost efficiency from the R&D investments and allocation efficiency from output production. However, when the R&D cost is high, the government should choose the emission tax whereas the privatization policy depends crucially on the degree of R&D spillovers. In particular, the public firm should be privatized if the spillover effect is weak, while it should not be privatized otherwise. This is because the public firm can increase allocative efficiency from higher output productions only when the spillover effect is strong. Otherwise, however, privatization policy is beneficial to the society because there exists a trade-off between cost efficiency and allocation efficiency in the presence of a public firm.

The remainder of this paper is organized as follows. In section 2, we present the basic model of environmental regulations. In sections 3 and 4, we provide the equilibrium results in private and mixed markets, respectively. We then compare the welfare effects of two policies in section 5. Finally, we conclude our analysis in section 6.

II. The Model

We consider a duopoly with homogeneous products, firm 0 and firm 1, facing a linear inverse demand function, \( P = A - Q \), where \( A \) is market size, \( q_i \) is each firm’s output and \( Q = q_0 + q_1 \) is total market output. For comparison purpose, we assume that each firm has a quadratic production cost of output,\(^7\) \( c(q_i) = \frac{1}{2} q_i^2 \), for firm \( i = 0, 1 \). The output of each firm emits pollution, \( e_i \). For analytic simplicity, we normalize the emission efficiency and we assume that each unit of output generates one unit of pollution, \( e_i = q_i \). We additionally adopt a green R&D investment, \( z_i \), which has the spillover effect, \( \beta \in [0,1] \). Then, the emission

\(^7\) The model with linear demand and quadratic cost functions is a standard formulation and popularly used in the literature on mixed oligopolies in order to rule out the uninteresting case of a public monopoly.
function affected by green R&D investment is modified as: \( e_i(q_i, z_i, z_j, \beta) = q_i - z_i - \beta z_j \).
That is, the emission emitted by firm \( i \) can be reduced by not only its own R&D but also the other firm’s R&D with the R&D spillover effect, \( \beta \). We also assume that each firm has a quadratic R&D cost: \( \frac{\gamma}{2} z_i^2 \) where \( \gamma > 0 \) is the level of green R&D cost and is assumed to satisfy that\(^8\) \( \gamma > \gamma \). Thus, higher (lower) \( \gamma \) represents the high (low) level of the green R&D cost. Then, the profit function of the firm without any environmental regulation becomes:
\[
\pi_i = (A - Q)q_i - \frac{1}{2} q_i^2 - \frac{\gamma}{2} z_i^2. \tag{1}
\]
It is easy to see that the firm has no incentive to invest in green R&D, \( z_i = 0 \).

Finally, environmental damage function is given as: \( D(E) = E^2 \) where \( E = e_0 + e_i \) denotes total emissions. Then, marginal environmental damage becomes \( MDE = 2E \). Social welfare can be calculated as the aggregated sum of the consumer and producer surpluses less environmental damage:
\[
W = \int_0^Q (A - u) \, du - \frac{1}{2} \sum_{i=0}^1 (q_i^2 + \gamma z_i^2) - E^2 \tag{2}
\]
As a benchmark, it is worthy to note that the first-best (FB) outcomes that directly maximizes welfare in (2) is as follows:
\[
Q_{FB}^{FB} = \frac{2A(4(1+\beta)^2+\gamma)}{12(1+\beta)^2+7\gamma} \quad \text{and} \quad Z_{FB} = \frac{8A(1+\beta)}{12(1+\beta)^2+7\gamma} \tag{3}
\]

In the following analysis, we consider two typical environmental regulations, an emission tax and a green R&D subsidy, in both private and mixed markets. We assume that each private firm maximizes its regulated profits in the private market, while the public firm maximizes social welfare in the mixed market where the private and public firms coexist.

The timing of the game is as follows. In the first stage, the government sets environmental regulation to maximize social welfare. In the second stage, each firm chooses its R&D cost to maximize its own objectives individually and simultaneously. In the third stage, both firms compete in outputs to maximize their own objectives. We will solve all games by backward induction and find subgame perfect Nash equilibria.

\(^8\) In Appendix A, we define \( \gamma > 0 \). This assumption ensures that all the equilibrium outcomes are positive.
III. Private Market

In a private market, both firms are profit-oriented entities without concerning the environment but each private firm maximizes its regulated profits under environmental regulations. We will examine an emission tax and a green R&D subsidy regulation, respectively, and compare the welfare consequences.

3. 1. Emission Tax

Under the emission tax, the objective function of the private firm of which unregulated profit is (1) becomes as follows:

\[ T_i = \pi_i - t(q_i - z_i - \beta z_j) \]  \hspace{1cm} (4)

In the third stage of output competition, we have the following equilibrium quantities:

\[ q_i = \frac{A - t}{4} \]  \hspace{1cm} (5)

Equilibrium output decreases as the emission tax increases, as expected. This implies that the emission tax directly affects the output decision of the firms.

In the second stage of R&D competition, we have the following equilibrium green R&D:

\[ z_i = \frac{t}{\gamma} \]  \hspace{1cm} (6)

There is a positive relationship between green R&D and the emission tax, but its effect is disproportional to the level of R&D cost.

In the first stage, the government decides the following optimal emission tax: \( t^{TP} \)

\[ t^{TP} = \frac{A\gamma(16(1 + \beta) + 3\gamma)}{4(4(1 + \beta) + \gamma)^2 + \gamma(16 + 3\gamma)} \]  \hspace{1cm} (7)

\[ \frac{\partial Q}{\partial t} \]  \hspace{1cm} (8)

9 Solving this problem gives the following first-order condition:

\[ (A - Q(t)) \frac{dQ}{dt} - \sum_{i=0}^{1} q_i(t) \frac{dq_i}{dt} - \gamma \sum_{i=0}^{1} z_i(t) \frac{dz_i}{dt} - MED(t\frac{dQ}{dt} - (1 + \beta) \frac{dz_i}{dt}) = 0. \]

The first term on the left-hand side measures the loss from decreasing the utility of the representative consumer, the second term measures the benefit from decreasing the output production cost, the third term measures the loss from increasing the R&D cost, and the last term measures the benefit from the reduction in environmental damage, all caused by an increase in the emission tax.
where the superscript TP denotes the optimal emission tax in private market. Note that the optimal emission tax increases as the R&D cost parameter increases. However, it decreases (increases) as the R&D spillover effect increases the R&D cost is low (high). Then, we obtain the equilibrium outcomes under the emission tax in the private market:

\[ q_i^{TP} = \frac{A(16\beta^2 + (4 + \gamma)^2 + 4\beta(8 + \gamma))}{64(1 + \beta)^2 + 16(3 + 2\beta)\gamma + 7\gamma^2} \]  
\[ z_i^{TP} = \frac{A(16 + 16\beta + 3\gamma)}{4(4 + 4\beta + \gamma)^2 + \gamma(16 + 3\gamma)} \]  
\[ e_i^{TP} = \frac{A\gamma(5 + \beta + \gamma)}{64(1 + \beta)^2 + 16(3 + 2\beta)\gamma + 7\gamma^2} \]

Comparing the first-best outcomes, we have \( Q^{FB} > Q^{TP} \) and \( Z^{FB} > Z^{TP} \). This shows that emission tax policy is insufficient not only in green R&D investment but the output production. Hence, there are both under-investment and under-production under emission tax regulation, which results in cost inefficiency from the R&D investments and allocation inefficiency from output production.

Finally, social welfare under the emission tax in the private market is as follows:

\[ W^{TP} = \frac{A^2(20(1 + \beta)^2 + (9 + 8\beta)\gamma + \gamma^2)}{64(1 + \beta)^2 + 16(3 + 2\beta)\gamma + 7\gamma^2} \]

We have the following lemma.\(^{12}\)

**Lemma 1.** \( t^{TP} \leq MED^{TP} \iff \gamma \geq 4(3\beta - 1) \)

It indicates that the emission tax depends on the level of green R&D cost and spillover effect. In particular, if spillover effect is low, \( \beta \leq \frac{1}{3} \), then the emission tax is always lower than marginal environmental damage. This result is consistent to the traditional tax rule that the emission tax decreases (increases) as \( \beta \) increases.

\(^{10}\) That is, \( \frac{\partial t^{TP}}{\partial \gamma} = \frac{16(64(1 + \beta)^3 + 24(1 + \beta)^2 \gamma + (2 - \beta)\gamma^2)}{(64(1 + \beta)^2 + 16(3 + 2\beta)\gamma + 7\gamma^2)^2} > 0 \).

\(^{11}\) That is, \( \frac{\partial t^{TP}}{\partial \beta} \geq 0 \iff \gamma \leq 4(1 + 3\beta + \sqrt{5 + \beta(14 + 13\beta)}) \). Regarding the R&D cost is low (high), the optimal emission tax decreases (increases) as \( \beta \) increases.

\(^{12}\) All the proofs of the lemmas and propositions are provided in Appendix B.
optimal emission rate in imperfectly competitive markets falls short of the Pigouvian rule, i.e.,
marginal environmental damage. This is because the lower emission tax can directly
encourage the output decision of the firms. However, if spillover effect is high, the emission
tax with the low (high) R&D cost can be higher (lower) than the marginal environmental
damage in order to internalize the externalities.

3. 2. Green R&D Subsidy

Under the green R&D subsidy, the objective function of the private firm is as follows:

\[ T_i = \pi_i + sz_i \] (12)

In the third stage of output competition, we have the following equilibrium quantities:

\[ q_i = \frac{A}{4} \] (13)

It is noteworthy that a green R&D does not affect the output decision of the firms.

In the second stage of R&D competition, we have the following equilibrium green R&D:

\[ z_i = \frac{s}{\gamma} \] (14)

There is also a positive relationship between green R&D investment and the R&D subsidy, but
its effect is disproportional to the level of green R&D cost.

In the first stage, the government decides the following optimal green R&D subsidy: \( s_{SP} \)

\[ s_{SP} = \frac{A(1 + \beta)\gamma}{4(1 + \beta)^2 + \gamma} \] (15)

where the superscript SP denotes the equilibrium under the green R&D subsidy in the private
market. Note that the green R&D subsidy increases as the level of green R&D cost increases.\(^{15}\)

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\(^{13}\) See, for example, Barnett (1980) for a monopoly and Levin (1985) for an oligopoly.

\(^{14}\) Solving this problem gives the following first-order condition:

\[ MED((1 + \beta) \frac{dz}{ds} - \gamma \sum_{i=0}^{1} z_i(s) \frac{dz_i}{dz}) = 0. \]

The first term on the left-hand side measures the loss from increasing the R&D cost and the second term measures
the benefit from the reduction in environmental damage, all caused by an increase in the subsidy.

\(^{15}\) That is, \( \frac{\partial s_{SP}}{\partial \gamma} = \frac{4A(1+\beta)^2}{(4(1+\beta)^2+\gamma)^2} > 0 \)
However, it decreases (increases) as the R&D spillover effect increases under the low (high) R&D cost.\footnote{That is, }$
abla s^{SP} = Ay(\gamma - 4(1 + \beta)^2) < 0 \iff \gamma < 4(1 + \beta)^2$. Regarding the R&D cost is low (high), the optimal emission tax decreases (increases) as $\beta$ increases.

Then, we obtain the equilibrium outcomes under the green R&D subsidy in the private market:

\begin{equation}
q_i^{SP} = \frac{A}{4} \tag{16}
\end{equation}

\begin{equation}
z_i^{SP} = \frac{A(1 + \beta)}{4(1 + \beta)^2 + \gamma} \tag{17}
\end{equation}

\begin{equation}
e_i^{SP} = \frac{Ay}{4(1 + \beta)^2 + \gamma} \tag{18}
\end{equation}

Comparing the first-best outcomes, we have $Q^{FB} < Q^{SP}$ and $Z^{FB} < Z^{SP}$, depending on the levels of green R&D cost and spillover effect. This shows that the welfare consequences of the green R&D subsidy depends crucially on the cost and spillovers of green R&D investment. However, comparing emission tax outcomes, we have $Q^{SP} > Q^{TP}$ and $Z^{SP} > Z^{TP}$. It implies that the green R&D subsidy induces more output production and more R&D investment than the emission tax in a private market. Therefore, depending on the level of green R&D cost and spillover effect, the green R&D subsidy can increase not only cost efficiency from the R&D investments and allocation efficiency from output production.

Finally, social welfare under the green R&D subsidy in the private market is as follows:

\begin{equation}
W^{SP} = \frac{A^2(20(1 + \beta)^2 + \gamma)}{16(1 + \beta)^2 + \gamma} \tag{19}
\end{equation}

**Lemma 2.** $s^{SP} > MED^{SP}$

It states that the optimal green R&D subsidy is always higher than marginal environmental damage irrespective of the level of R&D cost and spillovers. This is because the R&D subsidy
can encourage not only R&D investments directly but total emission amounts indirectly, compared to emission tax, and thus the government will set higher R&D subsidy rate than marginal environmental damage in order to increase more output production and more R&D investment.

3.3. Comparisons

We compare social welfare between under the emission tax and under the green R&D subsidy in the private market.

Proposition 1.

\[ W^{SP} < W^{TP} \iff \gamma > \frac{2(1 + \beta)}{9} \left( \sqrt{313 + \beta(98 + 361\beta)} \right) - (5 - 19\beta) \equiv \gamma^P \]

It shows that the superiority of the regulatory instruments between the emission tax and the green R&D subsidy in a private market depends not only on the spillover effect but also on the level of the R&D cost. In particular, social welfare under the emission tax can be higher (lower) than those under the green R&D subsidy with the higher (lower) R&D cost. Note that welfare thresholds, \( \gamma^P \), is increasing in the spillovers, i.e., \( \frac{\partial \gamma}{\partial \beta} > 0 \).

IV. Mixed Duopoly

In a mixed market where the private and public firms coexist, it is assumed that the private firm maximizes its regulated profits while the public firm maximizes social welfare. We will also examine an emission tax and a green R&D subsidy regulation, respectively, and compare the welfare consequences.

4.1. Emission Tax

Under the emission tax, the public firm maximizes social welfare in (2), while the private firm maximizes its own objective function in (4). In the third stage of output competition, we have the following equilibrium quantities:

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17 If we put the equilibrium outcomes in (13) and (14) into \( e_i = q_i - z_i - \beta z_j \), we have \( e_i^{SP}(s) = \frac{Ay - 4s(1 + \beta)}{4\gamma} \).

Then, we can show that \( \frac{\partial e_i^{SP}(s)}{\partial s} = -\frac{1 + \beta}{\gamma} < 0 \).
\[ q_0 = \frac{3t + 6(1 + \beta)(z_0 + z_1)}{9} \]  
\[ q_1 = \frac{3A - 4t - 2(1 + \beta)(z_0 + z_1)}{9} \]  
\[ Q = \frac{3A - t + 4(1 + \beta)(z_0 + z_1)}{9} \]

Then, the equilibrium output of the public (private) firm increases (decreases) as either the emission tax or the R&D cost increases. Thus, emission tax can encourage the public firm to be more aggressive in output production but, because the outputs are strategic substitutes, discourage the private firm to be more passive. As the cost of R&D investment is low, the public firm which already produces too much output, will be less aggressive and encourage the private firm to produce more in order to save social production cost. That is, there is an output substitution effect that is increases as the cost of R&D investment increases. However, total market output decreases (increases) as the emission tax (R&D investment) increases.

In the second stage of R&D competition, we have the following equilibrium green R&D:

\[ z_0 = \frac{2(1 + \beta)(22A(1 + \beta)^2 + 90A\gamma - t(199 + \beta (239 + 40\beta) + 48\gamma))}{3\gamma(94(1 + \beta)^2 + 81\gamma)} \]  
\[ z_1 = \frac{2t(1 + \beta)^2(199 + 40\beta) - 44A(1 + \beta)^3 - 54A(1 + \beta)\gamma + 9t(35 + 8\beta)\gamma}{3\gamma(94(1 + \beta)^2 + 81\gamma)} \]  
\[ Z = \frac{t(73 - 8\beta) + 42A(1 + \beta)}{94(1 + \beta)^2 + 81\gamma} \]

Note that the public firm’s R&D decreases as the emission tax increases, whereas the private firm’s R&D increases. This implies that from the viewpoint of government policy, the decisions on the emission tax and the public firm’s R&D investment are strategically substitutable policy instruments.

In the first stage, the government decides the following optimal emission tax:

\[ t^{TM} = \frac{2A(88L^5(199 + 40\beta) + 6L^3(9883 + 1900\beta)\gamma + 27L(1517 + 464\beta)\gamma^2 + 4374\gamma^3)}{K} \]  
where the superscript TM denotes the optimal emission tax in mixed market, \( L = (1 + \beta) \) and \( K = 8L^4(199 + 40\beta)^2 + 6L^2(77701 + 8\beta(5920 + 1037\beta))\gamma + 9(21985 + 16\beta(1361 + 397\beta))\gamma^2 + 20412\gamma^3. \)
Then, we obtain the equilibrium outcomes under the emission tax in the mixed market:

\[ q_0^{TM} = \frac{6A(8L^4(13 + 3\beta)(199 + 40\beta) + 2L^2(11905 + \beta(8561 + 1516\beta))\gamma + 3L(2201 + 824\beta)\gamma^2 + 486\gamma^3)}{K} \]  
(27)

\[ q_1^{TM} = \frac{3A(88L^4(199+40\beta)+2L^2(11375+8\beta(278+61\beta))\gamma+3(2827+640\beta(2+\beta))\gamma^2+972\gamma^3)}{K} \]  
(28)

\[ z_0^{TM} = \frac{4A(2L^2(1+\beta)^2(11957+5686\beta+884\beta^2)+9(2501+2\beta(917+274\beta))\gamma+2916\gamma^2)}{K} \]  
(29)

\[ z_1^{TM} = \frac{4A(2L^2(9137+2\beta(869-122\beta))+3L(5413+8\beta(181-40\beta))\gamma+243(7-2\beta)\gamma^2)}{K} \]  
(30)

\[ e_0^{TM} = \frac{2A(4L^4(5+2\beta)(713+482\beta)+6L^2(4402+\beta(5149+1836\beta))\gamma+9(1553+\beta(1999+932\beta))\gamma^2+1458\gamma^3)}{K} \]  
(31)

\[ e_1^{TM} = \frac{A(6L^2(549-4\beta(1213+700\beta))\gamma+9(2071+8(25-82\beta)\beta)\gamma^2+2916\gamma^3-16L^4(5+2\beta)(257+221\beta))}{K} \]  
(32)

Comparing the first-best outcomes, we have \( Q^{FB} > Q^{TM} \) and \( Z^{FB} > Z^{TM} \). This shows that even in the presence of public firm, the emission tax policy is insufficient not only in green R&D investment but the output production. There are still both under-investment and under-production under emission tax regulation, which results in cost inefficiency from the R&D investments and allocation inefficiency from output production.

Finally, social welfare under the emission tax in the mixed market is as follows:

\[ W^{TM} = \frac{A^2(8L^4(12317+8\beta(575+49\beta))+2L^2(57605+8\beta(4265+688\beta))\gamma+9(3817+8\beta(529+143\beta))\gamma^2+2916\gamma^3)}{K} \]  
(33)

**Lemma 3.** \( t^{TM} \preceq MED^{TM} \Leftrightarrow \gamma \preceq \gamma^{TM} \) where \( \gamma^{TM} > \gamma \) and satisfies \( t^{TM} = MED^{TM} \).\(^{18}\)

It states that depending on the level of green R&D cost and spillover effect, the optimal emission tax in a mixed market can be higher (lower) than the marginal environmental damage under the low (high) green R&D cost. This result is similar with Lemma 1. Thus, the emission tax with the low (high) R&D cost can be higher (lower) than the marginal environmental damage in order to internalize the externalities.

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\(^{18}\) We can show that \( 0 < \gamma < \gamma^{TM} \) in Appendix B.
4. 2. Green R&D Subsidy

Under the green R&D subsidy, the public firm maximizes social welfare in (2), while the private firm maximizes its own objective function in (12). In the third stage of output competition, the first-order conditions yield the following equilibrium quantities:

\[ q_0 = \frac{2(1 + \beta)(z_0 + z_1)}{3} \]  \hspace{1cm} (34)
\[ q_1 = \frac{3A - 2(1 + \beta)(z_0 + z_1)}{9} \]  \hspace{1cm} (35)
\[ Q = \frac{3A + 4(1 + \beta)(z_0 + z_1)}{9} \]  \hspace{1cm} (36)

This shows that both the output of the public (private) firm and total market output increase (decrease) as green R&D increases. Thus, contrary to the private market, the presence of public firm change the output effect of a green R&D subsidy policy. In specific, due to the output substitution effect between the two public and private firms, a green R&D subsidy encourages the public firm’s output, which in return reduce the private firm’s output.

In the second stage of green R&D competition, we have the following equilibrium green R&D:

\[ z_0 = \frac{2(1 + \beta)(22A(1 + \beta)^2 + 90A\gamma - 159s(1 + \beta))}{3\gamma(94(1 + \beta)^2 + 81\gamma)} \]  \hspace{1cm} (37)
\[ z_1 = \frac{s(318(1 + \beta)^2 + 243\gamma) - (1 + \beta)A(44(1 + \beta)^2 + 54\gamma)}{3\gamma(94(1 + \beta)^2 + 81\gamma)} \]  \hspace{1cm} (38)
\[ Z = \frac{81s + 42A(1 + \beta)}{94(1 + \beta)^2 + 81\gamma} \]  \hspace{1cm} (39)

The public firm’s R&D decreases as the green R&D subsidy increases, whereas the private firm’s R&D and total R&D increase. This implies that from the viewpoint of government policy, the decisions on the green R&D subsidy and the public firm’s R&D investment are strategically substitutable policy instruments.

In the first stage, in which the government decides the optimal green R&D subsidy, the first-order condition yields the following:
where the superscript SM denotes the optimal green R&D subsidy in the mixed market. Then, we obtain the equilibrium outcomes under the green R&D subsidy in the mixed market:

\[ q_{o}^{SM} = \frac{80A(1 + \beta)^2}{212(1 + \beta)^2 + 81\gamma} \]  

(41)

\[ q_{1}^{SM} = \frac{44A(1 + \beta)^2 + 27A\gamma}{212(1 + \beta)^2 + 81\gamma} \]  

(42)

\[ z_{o}^{SM} = \frac{60A(1 + \beta)}{212(1 + \beta)^2 + 81\gamma} \]  

(43)

\[ z_{1}^{SM} = \frac{60A(1 + \beta)}{212(1 + \beta)^2 + 81\gamma} \]  

(44)

\[ e_{0}^{SM} = \frac{20A(1 + \beta)^2}{212(1 + \beta)^2 + 81\gamma} \]  

(45)

\[ e_{1}^{SM} = \frac{A(27\gamma - 16(1 + \beta)^2)}{212(1 + \beta)^2 + 81\gamma} \]  

(46)

Comparing the first-best outcomes, we have \( Q^{FB} \gtrless Q^{SM} \) and \( Z^{FB} \lless Z^{SM} \), depending on the level of green R&D cost. This also shows that the welfare consequences of the green R&D subsidy depends crucially on the cost level and spillover effect of R&D investment. However, comparing emission tax outcomes, we have \( Q^{SM} > Q^{TM} \) and \( Z^{SM} > Z^{TM} \). It implies that the green R&D subsidy costs less than the emission tax in a mixed market. Therefore, depending on the level of green R&D cost and spillover effect, green R&D subsidy can increase not only cost efficiency from the R&D investments and allocation efficiency from output production. Finally, social welfare under the green R&D subsidy in the mixed market is as follows:

\[ W^{SM} = \frac{A^2(68(1 + \beta)^2 + 9\gamma)}{212(1 + \beta)^2 + 81\gamma} \]  

(47)

**Lemma 4.** \( s^{SM} > MED^{SM} \)

It states that even in the presence of public firm, the optimal green R&D subsidy is always
higher than marginal environmental damage irrespective of the level of green R&D cost and spillover effect. This result is similar with Lemma 2. Thus, compared to emission tax, the government will set higher R&D subsidy rate than marginal environmental damage in order to increase more output production and more R&D investment. ¹⁹ Note that the green R&D subsidy increases as either the green R&D cost or the spillover effect increases, i.e., \( \frac{\partial s_{SM}}{\partial \gamma} > 0 \) and \( \frac{\partial s_{SM}}{\partial \beta} > 0 \).

4.3. Comparisons

We compare social welfare between under the emission tax and under the green R&D subsidy in the mixed market.

**Proposition 2.**

\[ W^{SM} \leq W^{TM} \iff \gamma > \gamma^M \iff W^{SM} = W^{TM} \]

It shows that the superiority of the regulatory instruments between the emission tax and the green R&D subsidy in a mixed market depends not only on the spillover effect but also on the level of the green R&D cost. In particular, social welfare under the emission tax can be higher (lower) than those under the green R&D subsidy with the high (low) green R&D cost. Note that the welfare thresholds, \( \gamma^M \), is increasing in the spillovers, i.e., \( \frac{\partial \gamma^M}{\partial \beta} > 0 \).

V. Discussion: Privatization and Environmental Policy

We now examine the government’s optimal policy choices and discuss the welfare effect of privatization policy. We first compare the emission taxes and the green R&D subsidies in between private and mixed markets.

**Proposition 3.** \( t^{TM} > t^{TP} \) whereas \( s^{SM} \leq s^{SP} \iff \gamma \geq 44(1 + \beta)^2 \)

It states that privatization policy reduces the levels of the emission tax whereas reduces

¹⁹ If we put the equilibrium outcomes in (37) and (38) into (36) and (39), then from \( e_i = q_i - z_i - \beta z_j \), we have

\[ Q^{SM}(s) = \frac{A(50(1+\beta)^2+27\gamma)+36s(1+\beta)}{94(1+\beta)^2+81\gamma} \]

\[ Z^{SM}(s) = \frac{42A(1+\beta)+81s}{94(1+\beta)^2+81\gamma} \]

and

\[ E^{SM} = \frac{A(8+16\beta+8\beta^2+27\gamma)-45s(1+\beta)}{94+81\beta+94\beta^2+81\gamma} \]

Then, we can show that

\( \frac{\partial Q^{SM}(s)}{\partial s} > 0 \), \( \frac{\partial Z^{SM}(s)}{\partial s} > 0 \) and \( \frac{\partial E^{SM}(s)}{\partial s} > 0 \).
(increases) the level of green R&D subsidy with low (high) green R&D cost.

On the one hand, under the emission tax policy, both output productions and R&D investments are higher in a mixed market than those in a private market because there exists an aggressive public firm in a mixed market. But, the amounts of both production and R&D investments are still lower than those of the first-best. After privatization, therefore, a lower emission tax is required to increase total outputs but a higher emission tax is required to increase R&D investments in a private market. In total, the former effect on the welfare outweigh the latter effect and thus privatization policy reduces the level of emission taxes.  

On the other hand, under the green R&D subsidy policy, the level of output productions and R&D investments in mixed market is higher (lower) than those in private market with lower (higher) R&D cost. Also, the R&D subsidy induces more output productions and more R&D investments than the emission tax in both private and mixed markets. As a result, the amounts of both output productions and R&D investments might be higher than those of the first-best. Therefore, depending on the green R&D cost, a lower or a higher green R&D subsidy is required after privatization.

We now turn to the welfare consequences of privatization policy.

Lemma 5. $\gamma^M > \gamma^P$ for $\forall \beta$ and $\forall \gamma$

It shows that when comparing the welfare threshold between the emission tax and the green R&D subsidy in both private and mixed markets, the government requires a lower threshold of the green R&D cost for privatization. That is, the emergence of the public firm increases the threshold. It implies that the government might prefer to adopt the green R&D subsidy in the presence of public firm under certain conditions.

For visual comparisons, [Fig. 1] shows the thresholds between $\gamma^M$ and $\gamma^P$, which depend on the R&D spillovers, $\beta$. There exist three regions. The first is the lower region below $\gamma^P$ with a lower R&D cost. The second is the upper region above $\gamma^M$ with a higher R&D cost. The third is between $\gamma^M$ and $\gamma^P$ with the intermediate R&D cost. Hence, from Proposition 1

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20 This result is contrast to the results in Haruna and Goel (2019) who considered a reverse game structure where the firms decide their green R&D investments and then government chooses the emission taxes. In that case of time-inconsistency problem, the firms can choose more R&D investment opportunistically to induce the governments to reduce the tax level. Regarding time-inconsistency problem in environmental taxation, see also Leal, et al. (2018) and Garcia, et al. (2018).

21 Note that $Q^{FB} > Q^{SM} > Q^{SP}$ if $\gamma < \frac{4}{3}(1 + \beta)^2$ and $Q^{FB} < Q^{SM} < Q^{SP}$ if $\gamma > \frac{4}{3}(1 + \beta)^2$. 
and 2, the regions in which the emission tax (green R&D subsidy) is adopted by the government decreases (increases).

![Graph comparing $\gamma^M$ and $\gamma^P$](image)

**Fig. 1** Comparing $\gamma^M$ and $\gamma^P$

In the first region of the lower R&D cost, $\gamma < \gamma^P$, the green R&D subsidies in both private and mixed markets dominate the emission taxes, i.e., $W^SM > W^TM$ and $W^SP > W^TP$. Comparing $W^SM$ and $W^SP$, we have the following proposition.

**Proposition 4.** In the region of the lower R&D cost, we have $W^SM > W^SP$ for $\forall \beta$ and $\forall \gamma$.

It states that when the government adopts the green R&D subsidy policy, the mixed market always welfare-dominates the private market and thus privatization lowers social welfare under the lower R&D cost. As mentioned in Proposition 3, under the green R&D subsidy with the lower R&D cost, privatization policy induces less output productions and less R&D investments, which results in less social welfare.

In the second region of the higher R&D cost, $\gamma > \gamma^M$, the emission tax in both private and mixed markets dominates the green R&D, i.e., $W^SM < W^TM$ and $W^SP < W^TP$. Comparing $W^TM$ and $W^TP$, we have the following proposition.

**Proposition 5.** In the region of higher R&D cost, we have $W^TM \geq W^TP$ for $\beta \geq \beta^T$ where
\( \beta^T \) satisfies \( W^{TP} = W^{TM} \) and \( \beta^T > 0 \) for \( \gamma > \gamma^M \).

It states that when the government adopts the emission tax policy, the efficiency of the privatization policy depends on the levels of green R&D cost and spillover effect. In [Fig. 2], when the spillover effect is weak, the government should privatize the inefficient public firm that produces more output productions than the private firm. However, when the spillover effect is strong, the government should retain a public firm because the public firm invests more R&D, which increases externalities, and thus the spillover effect dominates the loss from the high green R&D cost.

![Fig. 2] Optimal Policy Choices

In the third region of the intermediate R&D cost, \( \gamma^P < \gamma < \gamma^M \), the efficiency between the emission tax and the green R&D subsidy depends on the existence of the public firm. In a mixed market, the green R&D subsidy welfare-dominates the emission tax, i.e., \( W^{SM} > W^{TM} \), while in a private market the emission tax welfare-dominates the green R&D subsidy, i.e., \( W^{TP} > W^{SP} \). Then, comparing \( W^{SM} \) and \( W^{TP} \), we have the following proposition.

**Proposition 6.** In the region of intermediate R&D cost, we have \( W^{TP} \gtrless W^{SM} \) for \( \gamma \gtrless \gamma^{TPSM} \) where \( \gamma^{TPSM} \) satisfies \( W^{TP} = W^{SM} \) and \( \gamma^{TPSM} > 0 \).
It states that the government policies on the environmental regulation and privatization depend on the relative R&D cost. In [Fig. 2], specifically, when the R&D cost is relatively high, the government should privatize the inefficient public firm and adopt the emission tax policy. However, when the R&D cost is relatively low, the government should retain a public firm and adopt the green R&D subsidy because with the lower R&D cost, privatization policy induces less output productions and less R&D investments while R&D subsidy welfare-dominates the emission tax when $\gamma^P < \gamma < \gamma^M$.

Finally, summarizing the findings in Proposition 4, 5 and 6, [Fig. 2] shows the optimal policy choices between the privatization policy and environmental regulation.

**Proposition 7.** Let $\gamma^* = \min\{\gamma^{TPSM}, \gamma^M\}$.

(i) When $\gamma < \gamma < \gamma^*$, then $W^{SM}$ is the highest welfare

(ii) When $\gamma > \gamma^*$, then $W^{TM}$ is the highest welfare if $\beta > \beta^T$ while $W^{TP}$ is otherwise.

It states that the optimal choices of government policy depends on the levels of green R&D cost and spillover effect. In specific, (i) the government should adopt the green R&D subsidy and keep the public firm under the lower R&D cost while (ii) adopt the emission tax and determine privatization policy depending on the R&D spillovers. The government should retain the public firm under the higher spillovers while privatize the public firm under the lower spillovers.

The economic explanation is as follows: When the R&D cost is low, the environmental improvement from the direct R&D subsidy outweighs the welfare loss from the R&D cost. Further, the R&D subsidy policy is more effective in the presence of the public firm. However, when the R&D cost is high, the welfare loss from R&D cost outweighs the environmental improvement from the direct R&D subsidy. In that case, the emission tax policy is superior, but the welfare effect of privatization depends on the R&D spillovers. When spillover effect is weak, the government should privatize the public firm in order to reduce higher R&D investments from the public firm. But, as the spillover effect increases, the government can maintain the public firm to encourage the output productions and R&D investments.

**VI. Conclusion**

In this study, we considered the green R&D cost and R&D spillovers, and compared environmental regulations between an emission taxes and green R&D subsidies in private and mixed duopoly markets. We then showed that the green R&D subsidy is better (worse) than the
emission tax when the green R&D cost is low (high) irrespective of the R&D spillovers, whereas the existence of a public firm encourages the government to adopt the subsidy policy. Finally, we examined the optimal policy choices between the environmental regulation and privatization policy. We showed that when the R&D cost is high and the spillover rate is (not) weak, the government should choose the emission tax and (not) privatize the public firm. However, when the R&D cost is low, such a privatization policy is not desirable to society irrespective of the R&D spillovers.

Our findings on the policy relation between market-oriented environmental regulations and privatization in oligopolies will provide clear policy importance for developing a sustainable green growth system. In particular, both the R&D spillovers and the effectiveness of the R&D cost are critical to assess the welfare effect of governmental policies. Therefore, the optimal combinations of green policy and privatization policy should be well-designed for the government in facilitating environmental innovation.

There remain future research. The limitations of the simple duopoly model should be generalized in the further extensions. First, we could consider different market structures such as Stackelberg and Bertrand competition in the markets with differentiated products. Second, we assumed that the environmental concerns of the players such as consumers and firms are exogenous irrespective of R&D activities. We could extend this study to examine the inclusive case with environmental awareness or environmental corporate social responsibility. For example, recent evidence shows that more consumers prefer to buy environment-friendly products and more firms are participating in environmental corporate social responsibility. This stimulates firms to carry out more environmental R&D investment in polluting industries. These research directions remain for future work.

22 For example, Lambertini and Tampieri (2015), Liu et al. (2015), Hirose et al. (2017), and Lee and Park (2019) explained how corporate environmentalism is desirable for both firms and society.
Appendix A. Lower boundary of the green R&D cost

Let $\gamma^{TM}$ satisfy equation (32) = 0 and $\gamma^{SM}$ satisfy equation (47) = 0. Then, we can show $\gamma^{TM} > \gamma^{SM}$ for $\forall \beta$ and $\forall \gamma$, in [Fig. A1]. Hence, it is assumed that $\gamma > \gamma = \gamma^{TM}$.

[Fig. A1] Lower boundary of the green R&D cost

Appendix B. Proofs

Lemma 1.

We can derive that $MED^{TP} = \frac{4Ay(5+\beta+\gamma)}{64(1+\beta)^2+16(3+2\beta)\gamma+7\gamma^2}$. Then, we can easily show that:

$t^{TP} - MED^{TP} = \frac{Ay(4(3\beta-1)-\gamma)}{64(1+\beta)^2+16(3+2\beta)\gamma+7\gamma^2} \leq 0 \iff \gamma \geq 4(3\beta - 1)$

Lemma 2.

We can derive that $MED^{SP} = \frac{Ay}{4(1+\beta)^2+\gamma}$. Then, we can easily show that:

$s^{SP} - MED^{SP} = \frac{A\beta\gamma}{4(1+\beta)^2+\gamma} > 0 \text{ for } \forall \beta \text{ and } \forall \gamma.$

Proposition 1.

We can derive that $W^{TP} - W^{SP} = \frac{A^2\gamma(9\gamma^2+4(1+\beta)(5-19\beta)\gamma-128(1+\beta)^3)}{16(4(1+\beta)^2+\gamma)(64(1+\beta)^2+16(3+2\beta)\gamma+7\gamma^2)}$. Then, we can show
that: $W_{TP} \leq W_{SP} \iff \gamma_{TP} > \gamma_{SP}$. 

**Lemma 3.**

We can derive that:

$$M_{ED}^{TM} = \frac{2A(8L^4(5+2\beta)(199+40\beta)+6L^2(9353+5446\beta+872\beta^2)\gamma+9(5177+2\beta(2099+604\beta))\gamma^2+5832\gamma^3)}{K}.$$ 

Then, we can show that:

$$t_{TM} - M_{ED}^{TM} = \frac{6A(8L^4(2+3\beta)(199+40\beta)+2L^2(530+\beta(6337+1028\beta))\gamma-3(626-\beta(1745+184\beta))\gamma^2-486\gamma^3)}{K}$$

Let $\gamma_{TM}$ satisfy $t_{TM} = M_{ED}^{TM}$. Note that $\gamma_{TM} > \gamma$ for $\forall \beta$ and $\forall \gamma$. Then, we can show that: $t_{TP} < M_{ED}^{TP} \iff \gamma_{TP} < \gamma_{TM}.$

**Lemma 4.**

We can derive that: $M_{ED}^{SM} = \frac{8A(1+\beta)^2+54A\gamma}{212(1+\beta)^2+81\gamma}$. Then, we can easily show that:

$$s_{SM} - M_{ED}^{SM} = \frac{8A(1+\beta)^2(8+11\beta)+18A(4+13\beta)\gamma}{636(1+\beta)^2+243\gamma} > 0 \text{ for } \forall \beta \text{ and } \forall \gamma.$$

**Proposition 2.**

Let $\gamma_{M}$ satisfy $W_{SM} = W_{TM}$. Then, we can show $\gamma_{M} > \gamma$ for $\forall \beta$ and $\forall \gamma$ in [Fig. 1].

**Proposition 3.**

We can easily show that $t_{TM} > t_{TP}$ and $s_{SM} - s_{SP} = \frac{A(1+\beta)(352(1+\beta)^4+388(1+\beta)^2\gamma-9\gamma^2)}{3(848(1+\beta)^4+536(1+\beta)^2\gamma+81\gamma^2)}$. Then, can derive that $s_{SM} < s_{SP} \iff \gamma_{SP} < 44(1+\beta)^2$ for $\forall \beta$ and $\forall \gamma$.

**Lemma 5.**

It is easy to show that $\gamma_{M} > \gamma_{P}$ for $\forall \beta$ and $\forall \gamma$ in [Fig. 1].

**Proposition 4.**

We can easily show that $W_{SM} - W_{SP} = \frac{7(4A(1+\beta)^2-3A\gamma)^2}{16(848(1+\beta)^4+536(1+\beta)^2\gamma+81\gamma^2)} > 0 \text{ for } \forall \beta \text{ and } \forall \gamma.$

**Proposition 5.**
Let $\beta^T$ satisfy $W^{TP} = W^{TM}$. Then, we can show $\beta^T > 0$ for $\gamma > \gamma^M$ in [Fig. 2].

**Proposition 6.**

Let $\gamma^{TPS_M}$ satisfy $W^{TP} = W^{SM}$. Then, we can show $\gamma^P < \gamma^{TPS_M}$ for $\forall \beta$ and $\forall \gamma$ in [Fig. 2].
References


