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Abstract

Emulating consumer price inflation with an increasing path of consumption taxes when the nominal interest rate binds and monetary policy becomes ineffective, as proposed by Correia et al. [1] in the Standard New Keynesian model, may not neutralize a liquidity trap of very long duration. Instead this paper presents a wealth redistributive tax policy, in an OLG model with credit constraints, able to prevent or counteract a liquidity trap caused by a credit shock. The tax prescription is opposite to the one proposed by Correia et al. [1]

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1 Introduction

Short term monetary policy is ineffective to counteract a recession when inflation is low and the zero bound for nominal interest rates prevents the real rate to reach a negative *natural rate of interest*[1]. Standard fiscal policy prescriptions to circumvent a liquidity trap[2] usually depend on increasing public spending to stimulate demand. Recently, however, Correia et al. [1] proposed an alternative approach based on the use of distortionary taxes, with *"no need to use inefficient policies such as wasteful public spending or future commitments to low interest rates"*. To allow a first-best allocation when the zero bound is binding their fiscal prescription consists on emulating consumer price inflation, the ones that matter for intertemporal decisions, via an increasing path of consumption taxes. They use a standard single agent New Keynesian model where a slump is by construction not permanent.

This paper also inspects how tax policy may circumvent a liquidity trap, but when a recession may become permanent, in an economy where credit is constrained, as well as public spending and debt. The economic framework used in this paper is based on the recent *Secular Stagnation* work of Eggertsson and Mehrotra [3]. A distortionary tax extension is coupled to a general version of their three periods OLG model where the natural rate of interest can be persistently negative. In contrast with an *inflation emulation* tax policy type as proposed by Correia et al. [1], which increases gross consumption prices by increasing consumption taxes to allow an implementable first-best allocation, the proposed tax policy prescription to neutralize a permanent slump in a credit constrained environment is based on a wealth redistributive mechanism from an unconstrained middle age generation, to younger credit constrained households and to the old that dissave to consume: An increase of labor taxes on the working middle generation would reduce their net income, and thus consumption and savings, leading to an increase of the *natural rate of interest*

[1] The natural rate of interest has been defined in the literature as the equilibrium full-employment real interest rate.

[2] Besides other non-fiscal approaches, namely the one proposed by Eggertsson and Woodford [4] where the central bank commits to keep interest rates at a lower level even after a recession resulting from a liquidity trap is over.

to a higher stable level[3]. The proceeds from this tax increase on the unconstrained agents are used to reduce the consumption tax, such that the credit constrained younger households, as well as the old, would be able to consume more.

Curiously, although this paper and the one of Correia et al. [1] both propose to circumvent the potential damages of a liquidity trap with no need to increase public spending, but by using tax policy to ensure that first best full-employment solutions are implementable, their prescriptions are reversed. Correia et al. [1] propose to neutralize the effects of the zero bound by generating inflation in consumer prices, the ones that matter for intertemporal decisions, with an increasing path of consumption taxes during the slump. Simultaneously labor taxes are reduced such that producer price inflation remains at the zero level. Their results hold in a standard single agent New Keynesian framework without credit constraints, where recessions are by construction temporary, but do not seem to hold in an OLG framework with credit constrained an unconstrained agents, where a permanent slump is possible.

In fact, a policy that neutralizes a persistent recession must have an also persistent impact in time, and thus cannot rely on a permanent increasing path of consumption taxes in order to permanently sustain the natural rate of interest in a set of first best full employment implementable solutions. A sustained increase of the natural rate of interest can instead be implemented with a contraction of loan supply by increasing labor income taxation on unconstrained middle generation; the tax redistribution in the benefit of the young constrained households and the old who dissave to consume is done via reducing consumption taxes. This redistributive tax policy could also be implemented with lump-sum taxes and transfers, from middle age to young and old agents. Lump-sum taxation is then effective in this heterogeneous agents environment with borrowing constraints, but not in a standard single agent New Keynesian framework.

With respect to recent economic literature, this paper explores the benefits of

[3] assuming a standard context where the derivative of excess savings with respect to the real interest rate is positive. Excess savings being defined as the difference between supply and demand for loans.

a redistributive tax policy at the zero bound to counteract persistent recessions in a credit constrained economy with heterogeneous agents, and with no need to increase public spending and debt. Moreover, this paper can also be read as a tax extension of the *Secular Stagnation* work of Eggertsson and Mehrotra [3], or as a complement for credit constrained economies of the *Unconventional Fiscal Policy* paper of Correia et al. [1].

Going forward, section 2 describes a three period OLG model with credit constraints, nominal prices and downward rigid wages, where households pay taxes on labor income, consumption and capital income[4], and firms pay taxes on hired labor. Section 3 analyzes and compares two alternative tax policies to avoid a credit shock zero bound recession. The first one being the *Unconventional Tax Policy at the Zero Bound* by Correia et al. [1], and the second one a redistributive fiscal policy based on an opposite tax prescription. It is shown that the first one cannot neutralize a slump in a credit constrained heterogeneous agents environment. Section 4 shows that the redistributive tax policy is also effective in ensuring the transition from a permanent recession to a full-employment steady state. Reminding the *Paradox of Toil*[2], an aggregate supply contraction caused by an increase of labor taxes on firms when wages are rigid, would boost inflation and force a transition from a stable deflation recession to a positive inflation full-employment steady state. A capital income tax is introduced later in the paper.

2 A Simple OLG Model with Distortionary Taxes

This section describes a general version with distortionary taxes of the 3 periods overlapping generations model proposed by Eggertsson and Mehrotra [3], where households interact with firms and a Government, real interest rates can be persistently negative and recessions long lasting.

(i) Households

Households go through three stages of life, young, middle aged and old, and use

[4]In appendix.

their income net of taxes, borrow and save, to maximize an objective function of consumption given by:

$$\max_{C_t^y, C_{t+1}^m C_{t+2}^o} E_t\{U(C_t^y) + \beta U(C_{t+1}^m) + \beta^2 U(C_{t+2}^o)\} \tag{1}$$

$$\text{s.t. } P_t(1+\tau_t^c)C_t^y = P_t B_t^y \tag{2}$$

$$P_{t+1}(1+\tau_{t+1}^c)C_{t+1}^m = Z_{t+1} + W_{t+1}L_{t+1}(1-\tau_{t+1}^l) - (1+i_t)P_t B_t^y + P_{t+1}B_{t+1}^m \tag{3}$$

$$P_{t+2}(1+\tau_{t+2}^c)C_{t+2}^o = -(1+i_{t+1})P_{t+1}B_{t+1}^m \tag{4}$$

$$(1+i_t)P_t B_t^y \leqslant P_{t+1}D_t, \text{ an exogenous borrowing limit.} \tag{5}$$

The young generation borrows $P_t B_t^y$ from middle-aged to consume. Middle-aged save $-P_t B_t^m$ by lending to the young and to the government, and pay back their loans to the previous generation. The old receive back with interest i_t what they have lent to the young and the Government when middle aged, and dissave to consume. It is assumed that each of the three stages of life has a duration of 20 years. $U(C)$ is a constant elasticity of inter-temporal substitution utility function expressed by $U(C) = \frac{C^{1-\sigma}}{1-\sigma}$. P_t is the aggregate price level. Consumption is taxed at a rate τ_t^c. Only the middle earn income in the form of firms profits $Z_t = P_t z_t$, and labor net income $(1-\tau_t^l)L_t W_t$, where nominal wages $W_t = P_t w_t$ are taxed at a rate τ_t^l. L_t is labor endowment supplied inelastically by middle generation at $\bar{L}$, but firms may hire only part of it so that $L_t \leq \bar{L}$. It is also assumed that the standard Fisher equation holds:

$$1 + i_t = (1+r_t)\Pi_t \tag{6}$$

where $\Pi_t = \frac{P_{t+1}}{P_t}$ is the growth rate of price level. Borrowing is constrained by an exogenous binding debt upper limit D_t faced by the young, or $B_t^y = \frac{D_t}{1+r_t}$, so that the amount borrowed by the young does not directly depend on any fiscal instrument but just on the real interest rate and the borrowing constraint. Instead, consumption of the young inversely depends on the consumption tax, and is given by:

$$(1+\tau_t^c)C_t^y = B_t^y = \frac{D_t}{1+r_t} \tag{7}$$

So, unless an increase of consumption tax goes along with a sufficient reduction

of equilibrium real interest rates, it would result in a contraction of consumption of younger agents. The same could be said about the old, but in this case with no relation with current equilibrium interest rate, since their consumption depends on savings and real rate from previous period:

$$(1+\tau_t^c)C_t^o = -(1+r_{t-1})B_{t-1}^m \tag{8}$$

Consequently an increase of consumption taxation, as proposed by Correia et al. [1], would lead in this environment to a contraction of consumption of young and old generations In turn, unconstrained middle age consumption is determined by the intertemporal condition given by:

$$1+i_t = \frac{1}{\beta}\Pi_t \frac{1+\tau_{t+1}^c}{1+\tau_t^c}\mathbb{E}_t \frac{U_c(C_t^m)}{U_c(C_{t+1}^o)} \Leftrightarrow 1+r_t = \frac{1}{\beta}\frac{1+\tau_{t+1}^c}{1+\tau_t^c}\mathbb{E}_t \frac{U_c(C_t^m)}{U_c(C_{t+1}^o)} \tag{9}$$

This Euler equation is similar to the one derived by Correia et al. [1], and suggests that increasing consumption taxes could be an alternative to increasing price level in order to sustain the equilibrium nominal interest rate above the binding zero bound, thus ensuring that a first best full-employment equilibrium is implementable when monetary policy becomes ineffective. Next it is shown why this is not necessarily the case in this credit constrained economy where persistent recessions are possible.

(ii) Firms

Firms are perfectly competitive, hire labor to maximize profits on a period by period basis, pay taxes on labor hired at a rate τ_t^w. The firm problem is given by:

$$Z_t = \max_{L_t} P_t Y_t - W_t L_t (1+\tau_t^w) \tag{10}$$

$$\text{s.t. } Y_t = L_t^\alpha \tag{11}$$

Firms' labor demand condition is given by:

$$W_t = \frac{\alpha}{1+\tau_t^w}\frac{P_t Y_t}{L_t} = \frac{\alpha}{1+\tau_t^w} P_t L_t^{\alpha-1} \tag{12}$$

so that $Z_t = (1-\alpha)P_t Y_t$ in equilibrium. A wage downward rigidity parametrized

by γ is considered such that households will not accept working for a wage lower than a nominal wage norm $\tilde{W}$ which depends on the nominal wage of the previous period adjusted by expected inflation of current period given by the Central Bank target inflation Π^*. The wage norm expression is given by:

$$\tilde{W}_t = (\Pi^* W_{t-1})^{\gamma} \left(W_t^{flex}\right)^{1-\gamma}, \text{ where } W_t^{flex} = \frac{\alpha}{1+\tau_t^w} P_t \bar{L}^{\alpha-1} \tag{13}$$

whereas the nominal wage will always be greater or equal than the flexible labor full-employment nominal wage:

$$W_t = \max\{\tilde{W}_t, W_t^{flex}\} \tag{14}$$

If firms need to adjust the nominal wage (12) downwards, namely due to a decline of aggregate price level or an increase of labor taxes on firms, but are constrained by the rigidity, then labor market may clear at a level lower than full-employment $\bar{L}$. From equation (14) labor demand can be expressed by:

$$L_t = \bar{L} \min\left\{1, \left(\frac{\Pi_t}{\Pi^*} \frac{1+\tau_{t-1}^w}{1+\tau_t^w}\right)^{\frac{\gamma}{1-\alpha}} \left(\frac{L_{t-1}}{\bar{L}}\right)^{\gamma}\right\} \leq \bar{L} \tag{15}$$

Note that if labor tax on firms is constant, and inflation is lower than its target, wages cannot adjust to its flexible value and labor demand must be lower than its full-employment level:

$$\Pi_t < \Pi^* \frac{1+\tau_t^w}{1+\tau_{t-1}^w} \left(\frac{\bar{L}}{L_{t-1}}\right) \overset{\tau_t^w = \tau_{t-1}^w}{\leq} \Pi^* \Rightarrow L_t < \bar{L} \tag{16}$$

(iii) Government

The Government budget constraint is given by:

$$B_t^g = G_t - T_t + \frac{1+r_{t-1}}{1+g_{t-1}} B_{t-1}^g, \text{ where } T_t = \tau_t^c C_t + (\tau_t^l + \tau_t^w) w_t L_t \tag{17}$$

B_t^g is Government borrowing per middle age household, and $g_t = N_t/N_{t-1} - 1$ is

young generation growth rate from $t-1$ to t. N_t is the size of the young generation at time t, and N_{t-1} the size of middle generation also at time t. Assuming that the market of goods clears, $C_t = Y_t - G_t$, and substituting out the wage equilibrium condition (12), an alternative expression for the budget constraint relating the fiscal instruments is obtained:

$$\frac{G_t}{Y_t} = 1 - \frac{1-\alpha\tau_t^{lw}}{1+\tau_t^c} + \frac{1}{Y_t}\left[B_t^g - \frac{1+r_{t-1}}{1+g_{t-1}}B_{t-1}^g\right] \tag{18}$$

where $\tau_t^{lw} = \frac{\tau_t^l + \tau_t^w}{1+\tau_t^w}$ is a labor tax index combining labor taxes on the working middle generation τ_t^l and firms τ_t^w.

(iv) Central Bank

Suppose that the central bank sets the nominal interest rate according to a standard Taylor rule:

$$1 + i_t = \max\left\{1, (1+i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right\} \Rightarrow i_t \geq 0 \tag{19}$$

where i^* and Π^* are respectively the Central Bank targets for interest rate and inflation, and $\phi_\pi > 1$ is the Taylor parameter. Note that the nominal interest rate zero bound is binding when inflation is lower than a threshold level $\Pi^{kink} = \frac{\Pi^*}{(1+i^*)^{\frac{1}{\phi_\pi}}} \leq \Pi^*$, so that a corresponding lower bound for the real interest rate is given by:

$$1 + r_t = \frac{1+i_t}{\Pi_t} \geq \frac{(1+i^*)^{\frac{1}{\phi_\pi}}}{\Pi^*} = 1 + r^{kink} \tag{20}$$

An implementable first-best allocation in this economy requires the *natural rate of interest*[5], r_t^n, to be greater than the threshold r^{kink}.

(v) Equilibria

Equilibrium in the loan market[6] implies that aggregate borrowing of the young

[5]Defined as the real interest rate required for full-employment in unrestricted zero lower bound conditions

[6]Note that equilibrium in the loan market implies that the market of goods are in equilibrium too.

and government, $B_t^{yg} = N_t B_t^y + N_{t-1} B_t^g$, equals savings of the middle age $-N_{t-1} B_t^m$, or:

$$(1+g_t)B_t^y + B_t^g = -B_t^m \tag{21}$$

An equilibrium is defined as a set of processes $\{C_t^y, C_t^m, C_t^o, B_t^y, B_t^m, L_t, Y_t, Z_t\}$ and prices $\{P_t, W_t, r_t, i_t\}$ solving (2), (6), (7), (8), (9), (10), (11), (12), (43), (17), (21). This model is able to generate persistently negative real rates (see Eggertsson and Mehrotra [3]).

Loan market equilibrium condition (21) is used to analyze how equilibrium real interest rates are affected by exogenous factors, namely credit shocks or tax changes. Equilibrium in the loan market requires that equality (21) is satisfied, where the right-hand side is loan demand $L_t^d = (1+g_t)B_t^y + B_t^g$, and the left is loan supply $L_t^s = -B_t^m$. *Excess savings* is defined as the difference between loan supply and demand, and is zero in equilibrium. Using the the *Implicit Function Theorem*[7] the derivative of equilibrium real interest rate r_t with respect to an exogenous parameter x can be expressed by:

$$S_t = L_t^s - L_t^d = 0 \Rightarrow \frac{dr_t}{dx} = -\frac{\frac{\partial S_t}{\partial x}}{\frac{\partial S_t}{\partial r_t}} \tag{22}$$

Loan demand can be expressed in terms of the borrowing limit D_t substituting out for the borrowing of the young (7):

$$L_t^d = B_t^{yg} = \frac{N_t}{N_{t-1}} B_t^y + B_t^g = \frac{1+g_t}{1+r_t} D_t + B_t^g \tag{23}$$

Loan demand is a decreasing function of the real interest rate, and contracts if the Government needs to deleverage, or if a credit shock reduces the binding borrowing limit D_t. Note also that loan demand is not directly affected by any tax instrument.

Loan supply can be derived by combining the intertemporal condition (9), con-

[7] Assuming *excess savings* is continuous and differentiable.

sumption of the old (8), young (7), and middle age (3):

$$L_t^s = -B_t^m = \frac{(1-\alpha\tau_t^{lw})Y_t - D_{t-1}}{1+\frac{1}{\beta^{\frac{1}{\sigma}}}\left[(1+r_t)\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right]^{1-\frac{1}{\sigma}}} \tag{24}$$

This expression shows that an increase of labor taxation on middle generation permanently contracts loan supply. At full-employment, when $Y_t = Y_f \Leftrightarrow L_t = \bar{L}$, the slope of loan supply with respect to the real interest rate is positive if the elasticity of intertemporal substitution EIS $= \frac{1}{\sigma}$ is greater or equal than one. In this situation the slope of excess savings with respect to r is positive. Otherwise, if EIS < 1, then a sufficient condition for a positive slope of excess savings S_t with respect to r_t is that the ratio of government borrowing over total borrowing in the economy $\frac{B_t^g}{L_t^g}$ does not exceed $\frac{1}{\sigma}$ (Proof[8]). This is a reasonable assumption in developed economies, given the standard estimations of EIS usually around and above 0.5 (see Havranek et al. [6]), in particular if EIS is equal or close to 1, or if the ratio of government debt B^g to total loan demand is low enough. It is then assume that inequality $\frac{B_t^g}{L_t^d} < \frac{1}{\sigma}$ is satisfied throughout the paper. This implies that the sign of the partial derivative of the natural rate of interest with respect to an exogenous parameter is opposite to the sign of the partial derivative of loans supply with respect to that same parameter:

$$\frac{B_t^g}{L_t^d} < \frac{1}{\sigma} \Rightarrow S_{r_t} \equiv \frac{\partial S_t}{\partial r_t} > 0 \Rightarrow \frac{dr_t}{dx} \text{ and } \frac{\partial S_t}{\partial x} \text{ have opposite signs.} \tag{25}$$

3 Preventing a permanent recession during a credit shock

A credit shock is characterized by a reduction of the borrowing limit D_t, having a negative impact on the natural rate of interest in two consecutive periods. $\frac{dr_t}{dD_t}$ and

[8]Proof: $\frac{\partial S_t}{\partial r_t} > 0 \Leftrightarrow -L_t^d > -L_t^s \Leftrightarrow \frac{L_t^d(1-B_t^g/L_t^d)}{1+r_t} > \left(1-\frac{1}{\sigma}\right)\frac{L_t^s}{1+r_t}\frac{1}{1+\beta^{\frac{1}{\sigma}}\left(\frac{1}{1+r_t}\frac{1+\tau_{t+1}^c}{1+\tau_t^c}\right)^{\frac{\sigma-1}{\sigma}}} \Leftarrow \frac{B_t^g}{L_t^d} < \frac{1}{\sigma}$.

$\frac{dr_{t+1}}{dD_t}$ can directly be expressed from (25):

$$\frac{dr_t}{dD_t} = \frac{1}{S_{r_t}}\frac{\partial S_t}{\partial D_t} = -\frac{1}{S_{r_t}}\frac{\partial L_t^d}{\partial D_t} = -\frac{1}{S_{r_t}}\frac{1+g_t}{1+r_t} < 0 \tag{26}$$

$$\frac{dr_{t+1}}{dD_t} = \frac{1}{S_{r_{t+1}}}\frac{\partial S_{t+1}}{\partial D_t} = \frac{1}{S_{r_{t+1}}}\frac{\partial L_{t+1}^s}{\partial D_t} < 0 \tag{27}$$

In the first period, a reduction of the binding borrowing limit D_t contracts loan demand, thus lowering the real rate. And in the second period, middle age households that have borrowed less while young can now save more, loan supply expands which further lowers the real rate eventually below the threshold r^{kink}, so that a first-best full-employment allocation becomes unimplementable. An effective fiscal policy to avoid a persistent recession and sustain full-employment during a credit shock should then prevent the natural rate of interest to fall below the threshold r^{kink}. Two alternative fiscal policies are next analyzed and compared.

3.1 Generating consumer price inflation with consumption tax

In their recent paper *"Unconventional Fiscal Policy at the Zero Bound", Correia et al. [1]* propose to neutralize the negative effects of the zero bound in a standard single agent New Keynesian model where a slump is by construction transitory, by generating consumer price inflation through a temporary increasing path of consumption taxes such that the intertemporal condition (9) is satisfied while real rates ought to be negative. At the same time, they propose to keep producer price inflation at zero by decreasing labor taxes.

In this model where the duration of each period correspond to 20 years, a reaction to a credit shock could depend on emulating the same type of consumer price inflation by a Government commitment at time t to increase consumption taxes from τ_t^c at time t to τ_{t+1}^c at time $t+1$. The effects of that policy on the natural rate of interest are next analyzed during the current period t, the next period $t+1$, and in steady state.

(i) Period $0 \equiv t-1$*:* The economy is at a full-employment steady state and the

natural rate of interest $r^n_{t-1} > r^{kink}$.

(ii) Period $1 \equiv t$*:* A credit shock contracts the borrowing limit from D_{t-1} to $D_t < D_{t-1}$, and r^n_t ought to be lower than r^{kink}. Since loan demand is not affected by tax instruments, the decision during period t to increase consumption tax in the next period $t+1$ only affects real interest rate through loan supply at time t, and depends on the coefficient of relative risk aversion σ as shown by the following expressions[9], derived from (22) and (24):

$$\frac{dr_t}{d\tau^c_{t+1}} = -\frac{\frac{\partial S_t}{\partial \tau^c_{t+1}}}{\frac{\partial S_t}{\partial r_t}} = -\frac{\overset{(\sigma>1?)}{\partial L^s_t}}{\partial \tau^c_{t+1}} \Big/ \frac{\overset{(+)}{\partial S_t}}{\partial r_t} \tag{28}$$

$$\text{where } \frac{\partial L^s_t}{\partial \tau^c_{t+1}} = \left(\frac{\sigma-1}{\sigma}\right) \left(L^s_t \frac{(1+\tau^c_{t+1})^{\frac{1}{\sigma}} \left[(1+r_t)(1+\tau^c_t)\right]^{-(1+\frac{1}{\sigma})}}{1+\frac{1}{\beta^{\frac{1}{\sigma}}} \left[(1+r_t)\frac{1+\tau^c_t}{1+\tau^c_{t+1}}\right]^{1-\frac{1}{\sigma}}} \right) \tag{29}$$

If $\sigma > 1 \Rightarrow \frac{dr_t}{d\tau^c_{t+1}} < 0$ then households are rather risk averse, and an income effect related to an expected increase of consumption taxation in the next period would increase savings propensity in the current period to sustain future consumption. This effect would prevail over the substitution effect resulting from the fact that consumption in the future would become more *costly* having an opposite impact on consumption and savings in current period. Thus, if $\sigma > 1$, the net effect of a Government commitment to increase consumption taxation in the next period would lead to a contraction of middle age consumption in current period, expanding loan supply and oughting to drag further down the real rate in period t.

$\sigma = 1 \Rightarrow \frac{dr_t}{d\tau^c_{t+1}} = 0$: Income and substitution effect cancel-out each-other so that an expected change of consumption taxes during next period would have no impact on current real interest rates.

If $\sigma < 1 \Rightarrow \frac{dr_t}{d\tau^c_{t+1}} > 0$ the substitution effect would prevail over the income effect. Households would rather increase their consumption in the current period if expecting an increase of consumption taxes in the future, so that loan supply would

[9]Note that expression (29) has the sign of $\sigma - 1$ and is equal to zero for $\sigma = 1$, since the right parenthesis is positive.

contract and real interest rates would rise back in period t.

(iii) Period $2 \equiv t+1$*:* In the following period consumption taxes increase and labor taxes must decrease to balance Government budget[10], as also suggested by Correia et al. [1]. Then, according to the next equation, the real rate in period $t+1$ would fall to a level lower than before the Government commitment to increase consumption taxes. Let's analyze how and why:

$$\frac{dr_{t+1}}{d\tau^c_{t+1}} = -\frac{\frac{\partial S_{t+1}}{\partial \tau^c_{t+1}}}{\frac{\partial S_{t+1}}{\partial r_{t+1}}} = -\left(\overset{(=0)}{\frac{\partial L^s_{t+1}}{\partial \tau^c_{t+1}}} + \overset{(-)}{\frac{\partial L^s_{t+1}}{\partial \tau^{lw}_{t+1}}} \overset{(-)}{\frac{\partial \tau^{lw}_{t+1}}{\partial \tau^c_{t+1}}} \right) / \overset{(+)}{\frac{\partial S_{t+1}}{\partial r_{t+1}}} < 0 \tag{30}$$

$$\text{where } \frac{\partial L^s_t}{\partial \tau^{lw}_t} = \frac{-\alpha Y^f_t}{1 + \frac{1}{\beta^{\frac{1}{\sigma}}} \left[(1+r_t) \frac{1+\tau^c_t}{1+\tau^c_{t+1}} \right]^{1-\frac{1}{\sigma}}} < 0 \tag{31}$$

It is assumed that consumption tax adjusts only once (in this case during a 20 year period, as it could not grow forever), such that from time $t+1$ onwards it stays constant. Since consumption taxes affect loan supply only through the intertemporal condition, then when constant they have no direct partial effect on real interest rate level, (or $\frac{\partial L^s_{t+1}}{\partial \tau^c_{t+1}} = 0$), but only an indirect effect through the Government budget constraint (18). An increase of consumption tax in $t+1$ would then require a reduction of public debt, or a decrease of the labor taxes, assuming that government spending is prevented to increase, which would drag down the real rate independently of σ. In fact, a decline of public debt would contract loan demand leading to a decline of the real interest rate. And a decrease of labor taxes would increase middle generation net income, which in turn would increase their consumption and savings, expand supply of loans and drag down real rate.

(iv) Steady state, at $t+1$*:* The final natural rate of interest would be even lower and unimplementable than the one caused by the credit shock, and previous to the tax policy implementation decision. Note that the equilibrium equation (21) only depends on taxes of same period, as they are assumed constant in steady state, so

[10] it is assumed that government spending is prevented to increase, and public debt per middle age household remains constant (although this second assumption is not required for the results to hold).

that consumption taxes do not affect real rates in steady state. Since the derivative of steady state real rate with respect to labor tax index is positive, the natural rate of interest falls even further with the proposed reduction of labor taxes:

$$\frac{dr}{d\tau^{lw}} = -\frac{\overset{(-)}{\partial L^s}}{\partial \tau^{lw}} \Big/ \frac{\overset{(+)}{\partial S}}{\partial r} > 0, \text{ where } \frac{\partial L^s}{\partial \tau^{lw}} = \frac{-\alpha Y^f}{1+\frac{1}{\beta}\frac{1}{\bar{\sigma}}(1+r)^{1-\frac{1}{\sigma}}} < 0 \qquad (32)$$

So, the *Unconventional Fiscal Policy at the Zero Bound* proposed by Correia et al. [1] to neutralize the negative effect of a liquidity trap is not effective in this economy, with credit constraints and heterogeneous households. First, because the real rate can become persistently negative so that no policy depending on an increasing (or decreasing) path of taxes would be sustainable. And second, because the required decrease of labor taxes when consumption taxes are increased would reduce even further steady state real rate, as consumption taxes are constant. Moreover, by increasing consumption taxation this tax policy would penalize consumption of the credit constrained households as well as the old who dissave to consume in favor of the unconstrained middle generation.

3.2 Taxing the unconstrained, and relieving the constrained

To sustain a first-best allocation during a credit shock, the tax prescription next analyzed is opposite to the previous one in terms of signs of tax changes. It is based on increasing labor taxes on unconstrained middle generation or firms, and redistribute the proceeds among all households via a consumption tax reduction.

Labor and consumption taxes change in period t, and are expected to remain constant thereafter. So that the intertemporal condition is independent of consumption taxes, and equilibrium in the loan market given by (21) only depends on current period taxes. Then, without loss of generality, a log utility of consumption ($\sigma = 1$) is assumed going forward so that previous algebraic expressions are simplified, and

the natural rate of interest gets a closed-form expression given by:

$$1+r_t^n = \frac{(1+g_t)D_t}{\frac{\beta}{1+\beta}\left[(1-\alpha\tau_t^{lw})Y_t^f - D_{t-1}\right] - B_t^g} \tag{33}$$

The natural rate of interest declines with the contraction of aggregate borrowing, either because public debt B_t^g is reduced, or credit constraints are worsen by a lower binding borrowing limit D_t. But a sufficient contraction of loan supply through an increase of labor taxes τ_t^{lw} can neutralize the negative impact of a credit shock on r_t^n. The required increase of the labor index τ_t^{lw} to offset the negative impact of a credit shock on r_t^n, respectively caused by a contraction of public or private borrowing[11], is given by the following equations:

$$\partial\tau_t^{lw} \geq -\frac{1}{\alpha}\left(\frac{1+\beta}{\beta}\right)\frac{\partial B_t^g}{Y_t^f} \tag{34}$$

$$\partial\tau^{lw} \geq -\frac{1}{\alpha}\left[1+\left(\frac{1+\beta}{\beta}\right)\frac{1+g}{1+r^n}\right]\frac{\partial D}{Y^f} \tag{35}$$

Equilibrium in the market of goods[12] requires that aggregate demand Y_t is the sum of aggregate consumption C_t and Government spending G_t per middle age household. If the government is prevented to spend more than an exogenous upper limit G, then consumption C_t cannot be lower than $C^f = Y^f - G$ at full-employment. By combining the budget constraints of the young (7) and the old (8), the Euler equation (9), and loan market equilibrium (23), aggregate consumption is expressed by:

$$N_{t-1}C_t = N_tC_t^y + N_{t-1}C_t^m + N_{t-2}C_t^o \Leftrightarrow \tag{36}$$

$$C_t = \frac{1}{1+\tau_t^c}\left\{\left[\left(\frac{1+\beta}{\beta}\right)\frac{1+g_t}{1+r_t}D_t + D_{t-1}\right] + \left[\frac{B_t^g}{\beta} + \frac{1+r_{t-1}}{1+g_{t-1}}B_{t-1}^g\right]\right\} \tag{37}$$

In this credit constrained economy consumption is unsurprisingly a function of the public and private credit limits, respectively D and B^g. In fact, if households' credit limit is reduced, then the young generation has to lower consumption if everything

[11] The private borrowing inequality uses the steady state version of the natural rate of interest (33), since the transition effect of a households credit limit contraction lasts for two consecutive periods.

[12] No capital in the model.

else remains unchanged. The middle generation lowers lending and thus saves less, assuming the Government is also prevented to increase public debt. Then the old would also need to reduce consumption in the following period since their savings were reduced. Moreover, for the intertemporal condition to hold when real interest rate is kept unchanged, middle age consumption would also be reduced in current period. This would lead to a contraction of aggregate consumption and demand[13].

A sufficient reduction of consumption tax τ_t^c can offset the negative effect on aggregate consumption and demand of a contraction of the credit limit D, when the real rate remains unchanged and government spending and debt cannot increase. So that the constrained young would increase consumption, as well as the old who dissave to consume, and thus the middle generation via the intertemporal condition. The Government budget constraint is satisfied if the increase of labor taxes τ_t^{lw} required to sustain the real rate at the level prior to the credit shock is combined with a decrease of consumption taxes τ_t^c required to sustain full employment[14], or $C_t = C_t^f$, for which steady state expressions are given by:

$$\partial \tau^c = \left[\frac{1}{\beta} + \frac{1+r_s}{1+g}\right] \frac{\partial B^g}{Y^f - G} \tag{38}$$

$$\partial \tau^c = \left[1 + \left(\frac{1+\beta}{\beta}\right) \frac{1+g}{1+r}\right] \frac{\partial D}{Y^f - G} \tag{39}$$

In this credit constrained environment, a redistributive policy based on taxing the unconstrained to relieve the constrained is effective in sustaining full-employment. Curiously, this tax prescription is opposite to the *Unconventional Fiscal Policy at the Zero Bound* proposed by Correia et al. [1].

4 Transition from a zero bound recession to full-employment

The same redistributive fiscal prescription described previously is also effective at transitioning out of a persistent zero bound recession. Making reference to the

[13] We have assumed that Government is prevented to increase public spending G_t

[14] Note that equilibrium in the loan market $(L_t^s - L_t^d = 0)$, implies equilibrium in the goods market $(Y_t = C_t + G_t)$.

Secular Stagnation paper of Eggertsson and Mehrotra [3], a full employment equilibrium may coexist with a stable recession in the set of implementable allocations of this model, with a graphical representation of aggregate demand and supply curves in Figure 1. Aggregate demand is expressed by:

$$\Pi_t > \Pi^{kink}\text{: } Y_t^d = \frac{1}{1+\tau_t^c}\left[\frac{(1+\beta)(1+g_t)D_t}{\beta}\frac{\Pi_{kink}^{\phi_\pi}}{\Pi_t^{\phi_\pi-1}} \qquad +D_{t-1}\right] + G_t \tag{40}$$

$$\Pi_t \leq \Pi^{kink}\text{: } Y_t^d = \frac{1}{1+\tau_t^c}\left[\frac{(1+\beta)(1+g_t)D_t}{\beta}\Pi_t \qquad +D_{t-1}\right] + G_t \tag{41}$$

where $\Pi^{kink} \equiv \frac{\Pi^*}{(1+i^*)^{\frac{1}{\phi\pi}}} \leq \Pi^*$, and $Y^{kink} \equiv Y(\Pi^{kink})$ is aggregate demand upper bound, $Y_t^d < Y^{kink}$. If the natural rate of interest is implementable ($r_t^n \geq r^{kink}$), then a full-employment first-best allocation is also implementable ($Y_t^f \leq Y^{kink}$) although not ensured. Moreover, if inflation is greater than Π^{kink} then aggregate demand Y_t^d is a negative function of inflation and is affected by monetary policy. Otherwise aggregate demand is a positive function of inflation, and a negative function of real interest rate which is now greater than r^{kink} and the natural rate of interest. Furthermore, the lower segment of aggregate demand is not affected by monetary policy.

Aggregate supply is, in turn, given by:

$$Y_t = \min\left\{Y^f, Y^f\left(\frac{\Pi_t}{\Pi^*}\frac{1+\tau_{t-1}^w}{1+\tau_t^w}\right)^{\gamma\frac{\alpha}{1-\alpha}}\left(\frac{Y_{t-1}}{Y^f}\right)^{\gamma}\right\} \tag{42}$$

For algebraic simplicity it is assumed going forward that $\Pi^* = 1$ and $i^* = 0$ so that $\Pi^{kink} = 1$ and $r^{kink} = 0$.

4.1 Eliminating a steady state recession

Steady state expression of aggregate supply (42) given by the following expression does not depend on any fiscal instrument, assuming labor taxes are constant.

$$Y = Y^f \min\left\{1, \Pi^{\frac{\gamma}{1-\gamma}\frac{\alpha}{1-\alpha}}\right\} \tag{43}$$

Then the remotion of a secular stagnation steady state from the set of implementable allocations depends on sufficiently expanding aggregate demand, to trigger a sustainable transition from deflation to a positive inflation level. Graphically this would require that the intersection of the lower segment of aggregate demand moves from a positive to a negative inflation level (see Figure 1) to clear its intersection with the lower segment of aggregate supply. Achieving such an expansion of aggregate demand can be ensured by sufficiently reducing consumption tax[15]:

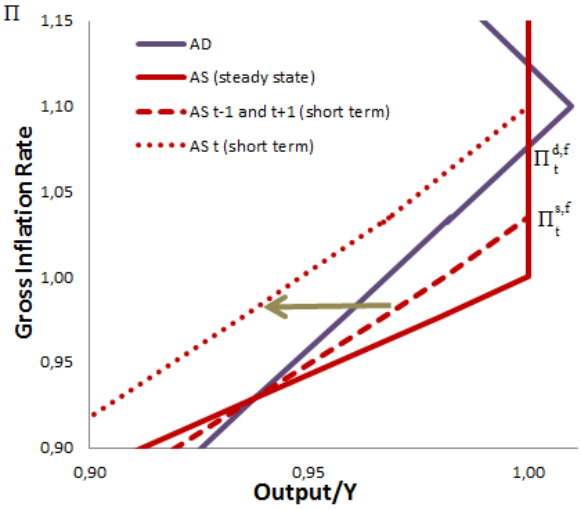
$$\partial\tau^c \leq (1+\tau^c_{ss})\left[\left(\frac{Y_{ss}-G}{Y^f-G}\right)\left(\frac{\Pi_{ss}+\frac{\beta}{(1+\beta)(1+g)}}{1+\frac{\beta}{(1+\beta)(1+g)}}\right)-1\right]<0 \tag{44}$$

where Y_{ss} and Π_{ss} are respectively the steady state recession levels of output and inflation. This would require that the natural rate of interest is implementable, $r^n \geq r^{kink} = 0$, where it has been assumed that $\Pi^* = 1+i^* = 1$. In fact, the lower segment of aggregate demand (41) intersects full employment at an inflation level $\Pi_t^{d,f}$ equal to the inverse of the gross natural rate of interest $(1+r_t^n)^{-1}$, so that a necessary condition for an implementable steady state equilibrium is a negative natural rate of interest, or $Y_t < Y^f \Rightarrow r_t^n < 0$, which is equivalent to state that $r_t^n > 0$ is a sufficient condition for full-employment.

But the decision to decrease consumption taxes during a recession may be difficult for a Government prevented to increase public debt. Then, to generate inflation, a contraction of aggregate supply could be an alternative to expanding aggregate demand.

[15] As intuitively shown by expression (41).

Figure 1: One period aggregate supply contraction shock



4.2 Supply contraction, and the *Paradox of Toil*[2]

In this economy where wages are downwardly rigid, deflation is a characteristic of a permanent recession since firms are prevented to fully decrease nominal wages, and thus need to adjust down labor demand. Then any effective policy in counteracting a recession needs to generate sufficient inflation, as is the case of an expansion of aggregate demand previously proposed, and a contraction of aggregate supply next analyzed.

However, and assuming that steady state taxes are constant, aggregate supply in steady state, given by expression (43), is unaffected by any tax instrument. Moreover, a stable recession may coexist with full-employment in the set of implementable allocations, so that the transition of this economy from the first to the second would at least require a temporary cancellation of the liquidity trap. This can be accomplished by a temporary contraction of aggregate supply through an increasing path of labor tax on firms as shown by expression (42). Although not sustainable in the long run, if this policy can generate sufficient inflation to temporarily offset the recession then the economy could move to full-employment. Moreover this transition could become permanent if the proposed increase of labor

tax on firms goes along with a reduction of consumption tax to fulfill Government budget constraint, which would also expand aggregate demand and possibly remove the steady state recession from the set of implementable allocations (see previous subsection).

Algebraically and graphically this can be explained as follows: A stable recession requires a negative natural rate of interest[16]. From Figure (1) this implies that short term aggregate supply (42) intercepts full employment at an inflation level $\Pi_t^{s,f}$ lower than the inflation level corresponding to aggregate demand determined at full employment $\Pi_t^{d,f}$, by construction equal to $\frac{1}{1+r_t^n}$:

$$\Pi_t^{s,f} = \left(\frac{1+\tau_t^w}{1+\tau_{t-1}^w}\right)\left(\frac{Y^f}{Y_{t-1}}\right)^{\frac{1-\alpha}{\alpha}} \overset{\tau_t^w=\tau_{t-1}^w}{=} \left(\frac{Y^f}{Y_{t-1}}\right)^{\frac{1-\alpha}{\alpha}} < \Pi_t^{d,f} = \frac{1}{1+r_t^n} \tag{45}$$

Then the transition from a recession to full-employment can be implemented by increasing labor tax on firms such that $\Pi_t^{s,f}$ becomes greater than $\Pi_t^{d,f}$, or from equation (45):

$$\partial\tau^w \geq (1+\tau_{ss}^w)\left[\frac{1}{1+r_{ss}^n}\left(\frac{Y_{ss}}{Y^f}\right)^{\frac{1-\alpha}{\alpha}} - 1\right] > 0 \tag{46}$$

Increasing labor tax on firms τ_t^w has also a positive effect on the natural rate of interest if labor tax on households τ_t^l is kept unchanged, so that the steady state recession can altogether be cleared. The Government budget constraint would require that consumption taxes are lowered as output approached full-employment, if government spending and debt are kept unchanged. Alternatively, the natural rate of interest could remain constant by combining increasing labor tax on firms with a reduction of labor tax on middle generation, so that budget balance would require a smaller decline of consumption tax.

Similarly to Correia et al. [1] a liquidity trap is resolved with an inflation boost, although with an opposite prescription in this credit constrained environment through

[16] and that for negative inflation levels, the slope of aggregate demand is positive and steeper than the slope of aggregate supply (see also Eggertsson and Mehrotra [3])

an increasing path of labor tax on firms combined with a reduction of consumption tax. Note also that the transition from a liquidity trap to full-employment is clearly welfare improving for all generations.

Capital income tax

Introducing capital, and a capital income tax in the model, does not qualitatively change the fiscal policy options derived so far to counteract persistent recessions. By having an impact similar to the consumption tax in expanding aggregate demand, reducing the tax on capital can be an effective alternative to a reduction of consumption taxes when further expanding consumption is not a desirable policy option. The model with capital is derived in appendix.

5 Final remarks

This paper proposes a redistributive tax policy to counteract zero bound recessions without the need to increase public spending or debt, in an economy where some households are credit constrained, real interest rates can be permanently negative, and an economic slump can last for a very long time. Ensuring that a first-best allocation is an implementable solution requires that the natural rate of interest clearing loan market at full-employment remains positive. Namely during a credit shock that permanently contracts the demand for loans. The proposed policy prevents the real rate from falling through a contraction of loan supply via increasing private and corporate taxes on labor. The proceeds of this labor tax increase, that alone would negatively affect consumption of unconstrained agents, are used to reduce consumption tax to the benefit of all households, and thus sustain consumption at its full-employment level. A tax policy based on annual lump-sum taxation of unconstrained households combined with transfers to the constrained could have equivalent results in this OLG framework, but would not be effective in reaching the same objective in the standard single agent New Keynesian model without borrowing constraints used by Correia et al. [1].

Their *Unconventional Fiscal Policy at the Zero Bound* is instead based on gen-

erating inflation in consumer prices, the ones that matter for intertemporal decisions, with an increasing path of consumption taxes during the transitory[17] slump. Producer price inflation is in turn kept at zero through a decreasing path of labor income taxes. Curiously, their tax prescription is opposite to the redistributive tax policy, and would worsen a recession in the OLG setting of this paper, as lower labor taxes would reduce even further the natural rate of interest in a permanent way, and increasing consumption tax would contract consumption further away from full-employment level.

To conclude, the most effective tax policies to counteract a zero bound recession without increasing public spending or debt can be based on opposite tax prescriptions depending on the model chosen to best fit a given economy.

[17] by construction.

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A Capital Income Tax

We introduce distortionary taxes including the tax on capital income in the budget constraints of the middle age and old, given by:

$$(1+\tau^c_{t+1})C^m_{t+1} = z_{t+1} + w_{t+1}L_{t+1}(1-\tau^l_{t+1}) + K_{t+1}[r^k_{t+1}(1-\tau^k_{t+1})-1] - (1+r_t)B^y_t + B^m_{t+1} \tag{47}$$

$$(1+\tau^c_{t+2})C^o_{t+2} = -(1+r_{t+1})B^m_{t+1} + K_{t+1}(1-\delta) \tag{48}$$

and the labor tax on firms is still considered in the firm problem, now given by:

$$Z_t = \max_{L_t,K_t}\{P_tY_t - W_tL_t(1+\tau^w_t) - P_tr^k_tK_t\} \text{ s.t. } Y_t = A_tL^\alpha_tK^{1-\alpha}_t \tag{49}$$

where, $w_t = \frac{W_t}{P_t} = \frac{\alpha L_t^{\alpha-1}}{1+\tau^w_t} = \frac{\alpha}{1+\tau^w}\frac{Y_t}{L_t}$ and $r^k_t = (1-\alpha)A_tL^\alpha_tK_t^{-\alpha} = (1-\alpha)\frac{Y_t}{K_t}$. From the return on capital expression we can directly observe that a reduction of the tax on capital income reduces the cost of capital:

$$r^k_t = \frac{1}{1-\tau^k_t}\left(1 - \frac{1-\delta}{1+r_t}\right) \tag{50}$$

Aggregate demand expands when if the tax on capital income decreases. This can be directly observed from the following expression of aggregate demand in real terms:

$$Y^d_t = \frac{1}{1-\alpha\tau^{lw}_t - (1-\alpha)B^l_t}\left[\frac{1+\beta}{\beta}\frac{(1+g_t)D_t}{1+r_t} - D_{t-1}\right] \tag{51}$$

Where $B^l_t = \frac{1}{r^k_t}\frac{1+\beta}{\beta}\frac{1-\delta}{r_t+\delta} = (1-\tau^k_t)\frac{1+\beta}{\beta}\frac{1-\delta}{r_t+\delta}$.

Regarding aggregate supply, its expressions is equal to the one derived previously and given by expression (43). But now full employment output is given by:

$$Y^f_t = A_t\bar{L}^\alpha K^{1-\alpha}_t = \bar{L}A^{\frac{1}{\alpha}}_t\left(\frac{1-\alpha}{r^k_t}\right)^{\frac{1-\alpha}{\alpha}} = \bar{L}A_t^{\frac{1-\alpha}{\alpha}}\left[\frac{(1-\alpha)(1-\tau^k_t)}{1-\frac{1-\delta}{1+r_t}}\right]^{\frac{1-\alpha}{\alpha}} \tag{52}$$

The difference lies on the aggregate supply expression for positive inflation levels

which is not constant, and expands when τ^k decreases, leading also to an aggregate supply expansion when inflation is negative. Although a reduction of the tax on capital income has an expanding impact on both aggregate supply and demand, the impact on demand prevails. The resulting impact on inflation, employment and the natural rate of interest are qualitatively similar to the ones derived for the consumption tax, being an available adequate alternative to this instrument in counteracting a persistent recession.