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Abstract:
Advances in gravity literature have presented econometric approaches for the theoretically consistent estimation of structural gravity. When estimating the impact of policy-shocks on trade values however, researchers are confronted with two problems. Once multilateral resistances are taken into account, through time-varying importer and exporter fixed effects, they absorb the effect of policy-shock indicator variables. Hence, we cannot obtain a coefficient for the impact of policy. The second problem is rooted in the necessary panel data dimensions in structural gravity that requires multiple-exporters and multiple-importers. The (at least) three dimensional panel implies that any coefficients/impacts that are estimated apply to the whole set of exporters rather than the country related to the scope of the research. I propose a method to approach these two problems, estimate the impact that policy-shock variables have on trade and differentiate the results for the country/countries related to the scope of the research. A short application on the impact that the Global Financial Crisis had on trade values is presented.

Keywords: Trade, Structural Gravity, PPML, Poisson Pseudo Maximum Likelihood, Global Financial Crisis

JEL Codes: F10; F14; C10; C20; C23

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Abstract:
Advances in gravity literature have presented econometric approaches for the theoretically consistent estimation of structural gravity. When estimating the impact of policy-shocks on trade values however, researchers are confronted with two problems. Once multilateral resistances are taken into account, through time-varying importer and exporter fixed effects, they absorb the effect of policy-shock indicator variables. Hence, we cannot obtain a coefficient for the impact of policy. The second problem is rooted in the necessary panel data dimensions in structural gravity that requires multiple-exporters and multiple-importers. The (at least) three dimensional panel implies that any coefficients/impacts that are estimated apply to the whole set of exporters rather than the country related to the scope of the research. I propose a method to approach these two problems, estimate the impact that policy-shock variables have on trade and differentiate the results for the country/countries related to the scope of the research. A short application on the impact that the Global Financial Crisis had on trade values is presented.

1. Introduction

The empirical estimation of structural gravity has undergone many changes over the last two decades. A "naïve" approach that was employed until Anderson and van Wincoop (2003) (AvW) presented the structural gravity model, examined the value of exports as a function of the economic mass of partner countries, costs and a number of augmenting variables. After AvW (2003), theory-rooted methods of estimation that concentrate on the proper inclusion of all relative prices and bilateral relations gradually evolved.

Santos Silva and Tenreyro (2006) proposed the Poisson Pseudo Maximum Likelihood (PPML) estimator towards the proper, theoretically-grounded, empirical estimation of structural gravity that has since become the most commonly employed method in the literature. While the application of PPML on structural gravity usually employs three-dimensional panel data with multiple exporters to properly account for multilateral resistances, application is also possible in cross-sections of bilateral trade data. When panel data are employed, however, time-varying importer and exporter fixed effects are usually included in the specification to take into account the effect of multilateral

resistances in a theoretically consistent manner[1] (Olivero and Yotov, 2012; Fally, 2015). At the same time, country-pair fixed are also usually employed to alleviate endogeneity problems encountered with variables that capture regional trade agreements. The necessary inclusion of fixed effects in the econometric specification, does not allow researchers to obtain coefficients for any variables that exhibit either within or between variation but not both. This limits the applicability of structural gravity models as the impact from policy shock indicator variables cannot be estimated. Hence researchers cannot quantify exporter-specific, importer-specific and time-specific variables. This problem is also mentioned in Head and Mayer (2014) and Yotov et.a. (2016)[2].

A second difficulty originates from the three-dimensional nature of the panel data that includes (at least) multiple exporters, multiple importers and time. The estimated coefficients apply to all the exporters included in the study. Hence, researchers need a way to differentiate results that apply to one country (or any sub-groups of countries) related to the scope of the research, from results that apply to the whole set of exporters included in the panel.

In this article I propose a rather unconventional approach in order to estimate the impact from importer/exporter- specific policy shock variables in structural gravity PPML, which are otherwise absorbed by the inclusion of time-varying fixed effects. I also offer a discussion on how to differentiate the results to those that apply to the whole group of exporters from those that apply to the country related to the scope of the research.

The next section of this article presents the modifications to the empirical structural gravity model that allow us to estimate policy impacts using PPML as well as the method we employ to differentiate the results for (groups of) exporters (section 2). Section 3 presents a short application to the Global Financial Crisis and Section 4 the conclusions and extensions of this research.

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[1] Other methods are available in the literature such as iterative nonlinear least squares (AvW, 2003), remoteness indexes (Wei, 1996; Baier and Bergstrand, 2009) and elimination methods through appropriate transformations (Head and Ries, 2001; Head et. al., 2010; Novy, 2013).

[2] Recent attempts employed intra-national trade flows together with international flows in order to quantify the impact of country specific variables.

2. The Model

We start with the structural gravity model from AvW (2003):

$$X_{ijt} = \frac{Y_{it}E_{jt}}{Y_t^W}\left(\frac{t_{ijt}}{\Pi_{it}P_{jt}}\right)^{1-\sigma} \quad (1)$$

where Y_{it}, E_{jt} represent the economic mass of exporting country i and importing country j respectively, at time t. The term $\left(\frac{t_{ijt}}{\Pi_{it}P_{jt}}\right)^{1-\sigma}$ reflects the effect that trade costs have on exports with t_{ijt} being the bilateral trade cost factor between countries i and j, σ the elasticity of substitution and Π_{it}, P_{jt} the outward and inward Multilateral Resistance Terms. In structural gravity these are estimated as:

$$\Pi_{i,t}^{1-\sigma} = \sum_j \frac{t_{ijt}^{1-\sigma}}{P_{jt}^{1-\sigma}}\frac{E_{jt}}{Y_t^W} \quad (2)$$

$$P_{jt}^{1-\sigma} = \sum_i \left(\frac{t_{ijt}}{\Pi_{it}}\right)^{1-\sigma}\frac{Y_{it}}{Y_t^W} \quad (3)$$

We now consider a policy shock common for all exporters. We name this variable $Policy_t$ and we denote it only with a time subscript. Our transformation will require a second variable we name DS_{ijt} that varies across all three dimensions. Next we multiply both sides of equation 1 by $Po_{ijt} = e^{\frac{Policy_t}{DS_{ijt}}}$ and obtain a transformed model

$$X_{ijt} = X_{ijt}Po_{ijt} = X_{ijt}\left(e^{\frac{Policy_t}{DS_{ijt}}}\right) = \frac{Y_{it}E_{jt}}{Y}\left(\frac{t_{ijt}}{\Pi_{it}P_{jt}}\right)^{1-\sigma}e^{\frac{Policy_t}{DS_{ijt}}} \quad (4)$$

The purpose of this multiplication is twofold. First, the new variable $\left(Po_{ijt}\right)$ varies across all dimensions and can be estimated when time-varying importer and exporter fixed effects

are included in the specification of the modified model. Second, the formulation of the right hand side variables will allow us to extract a post-estimation impact of the policy shock on the original dependent variable $\left(X_{ijt}\right)$ of exports. We note the difference between the trade flows $\left(X_{ijt}\right)$ and the transformed model $\left(X_{ijt}\right)$ flows which hold for $Po_{ijt}=1$.

The usual proxy for trade costs employs standard observable variables:

$$\left(1-\sigma\right)\ln t_{ijt}=\beta_1\ln Dist_{ij}+\beta_2 CNTG_{ij}+\beta_3 LANG_{ij}+\beta_4 CLNY_{ij}+\beta_5 FTA_{ijt} \qquad (5)$$

Where $Dist_{ij}$ is the distance between trading partners i and j, and variables $CNTG_{ij}, LANG_{ij}, CLNY_{ij}$ and FTA_{ijt} represent indicator variables that capture contiguous borders, common language, colonial ties and regional trade agreements.

Substituting equation 5 in 4 and considering that time varying fixed effects and country pair fixed effects absorb all the coefficients that exhibit only within or between variation we obtain our econometric specification[3]

$$X_{ijt}=X_{ijt}\left(e^{\frac{Policy_t}{DS_{ijt}}}\right)=e^{\beta_5 FTA_{ijt}+\beta_6\frac{Policy_t}{DS_{ijt}}-\pi_{it}-\chi_{jt}-k_{ij}}\varepsilon_{ijt} \qquad (6)$$

The estimated coefficient β_6 allows us to discuss only the impact of the transformed policy variable on the transformed dependent variable. However, it also allows us to extract a semi-elasticity of the effect the policy variable has on export values. The semi-elasticity that we directly obtain from the econometric estimation of equation 6 is $\hat{\sigma}=\frac{X_{ijt,Policy=1}-X_{ijt,Policy=0}}{X_{ijt,Policy=0}}$. Substituting the transformed variable into this expression we find

[3] See Appendix A1

$$\hat{\sigma} = \frac{X_{ij,t,D=1} e^{\frac{1}{DS_{ijt}}} - X_{ij,t,D=0} e^{\frac{0}{DS_{ijt}}}}{X_{ij,t,D=0} e^{\frac{0}{DS_{ijt}}}} = \frac{e^{\beta_6 \frac{1}{DS_{ijt}}} \varepsilon_{ijt}}{e^{\beta_6 \frac{0}{DS_{ijt}}} \varepsilon_{ijt}} - 1 \qquad (7)$$

We note two things in equation 7. First, we can eliminate some of the exponential terms on the left and right hand sides and simplify the expression to obtain

$$\frac{e^{\frac{1}{DS_{ijt}}} X_{ij,t,D=1} - X_{ij,t,D=0}}{X_{ij,t,D=0}} = e^{\beta_6 \frac{1}{DS_{ijt}}} - 1 \qquad (8)$$

which leaves us with the additional term $e^{\frac{1}{DS_{ijt}}}$ on the left hand side. We eliminate the additional term to obtain an expression for our semi-elasticity, such that the impact of the policy shock variable on trade is[4]

$$\hat{\sigma}_{ijt} = \frac{X_{ij,t,D=1} - X_{ij,t,D=0}}{X_{ij,t,D=0}} = \frac{\left(e^{\frac{\beta_6}{DS_{ijt}}} - 1 \right) + 1 - e^{\frac{1}{DS_{ijt}}}}{e^{\frac{1}{DS_{ijt}}}} = e^{\frac{\beta_6 - 1}{DS_{ijt}}} - 1 \qquad (9)$$

Expression 9 estimates the desired semi-elasticity of the policy change variable, in other words the percentage change in trade values due to the change in the policy variable. This is the impact of the policy variable on the original $\left(X_{ijt} \right)$, rather than the transformed $\left(X_{ijt} \right)$, dependent variable.

The second thing to note is that the semi-elasticity in 9 now depends on i, j, t. We do not obtain a single value that applies to all observations in our data but rather an estimate for each observation. We can average out the impact to get an effect for the whole sample or we can obtain an average per year or per country, depending on the scope of the research. The impact can be estimated either for one (or more) years. For example, to obtain the

[4] See Appendix A2

impact for all exporters (i) towards all destinations (j) in the study for the years 2007 and 2008 we average

$$\tilde{\sigma}_{All\ Exporters,2007-2008} = \frac{\sum_{t=2007}^{T=2008}\sum_{i=1}^{I}\sum_{j=1}^{J}\hat{\sigma}_{ijt}}{(T-t+1)\cdot I\cdot J} \quad (10)$$

or for one country (or a set of countries)

$$\tilde{\sigma}_{EU\,28,2007-2008} = \frac{\sum_{t=2007}^{T=2008}\sum_{i=1}^{28}\sum_{j=1}^{J}\hat{\sigma}_{ijt}}{(T-t+1)\cdot 28\cdot J} \quad (11)$$

This will yield a single value for the semi-elasticity that pertains to the scope of the research.

3. Application: The Effect of the 2007-2008 Global Financial Crisis on Trade

Our application incorporates a total of 211 exporters and 232 importers for the years 2003-2017. The data for exports is in nominal terms, it comes from the UN Comtrade database and the Organization of Economic Complexity. It was downloaded using 6-digit data and it was aggregated to annual, country-level observations. Data for GDP is in nominal terms and it was extracted from the World Development indicators of the World Bank in US dollars. Free Trade Agreements were also extracted from the World Bank and include a "catch-all" measure coded as one if a country has any type of agreement with a partner country in year t. The $Policy_t$ variable was coded as one for the years 2007 and 2008, the years of the Global Financial Crisis. The weighing variable DS_{ijt} was estimated as the average GDP of the trading partners at time t.

Three models were estimated. Model 1 of Table 1 presents the usual PPML estimation results where the dependent variable is in levels and the right hand side includes exporter-time, importer-time and country-pair fixed effects. The time-varying fixed effects absorb the impact from GDP and the Policy variable whereas the inclusion of country-pair fixed effects absorbs the impact from variables that vary bilaterally such as distance, contiguity,

colonial history etc. The only variable that we can estimate and interpret is the coefficient in FTA which is statistically significant and implies that FTA's have had a positive impact on export values equal to $e^{0.05} - 1 = 0.051$.

Table 1. PPML Results

	(1) X_{ijt}	(2) $X_{ijt} \exp(Policy_t / DS_{ijt})$	(3) X_{ijt}
GDP exporter			0.00***
			(0.00)
GDP importer			0.00***
			(0.00)
FTA	0.05***	0.05***	0.15***
	(0.01)	(0.01)	(0.01)
GFC		376344979.97***	0.08***
		(142771127.75)	(0.00)
Constant	22.13***	22.12***	21.63***
	(0.00)	(0.00)	(0.01)
Observations	435,698	367,850	367,850
Effects	ET, IT, CP	ET, IT, CP	CP
Impact	-		
2007		0.016	
2008		0.027	
Total		0.0214	0.089
Total 2007-8 US		0.00005	
Total 2007-8 EU 28		0.004	

Note: Author Estimates. *** p<0.01, ** p<0.05, * p<0.1, ET refers to Exporter-Time fixed effects, IT to Importer-Time and CP to Country-Pair

The second model present the results of equation 6 on the transformed dependent variable. The impact of the FTA can be interpreted either in terms of the transformed variable or the original variable[5]. The estimate does not differ from model 1 which can be considered as a test for the robustness of the model[6]. Our coefficient on the GFC variable

[5] See Appendix A3.

[6] Note that this result differs from the case where we would multiply both sides with an existing independent variable where the transformed model coefficients would follow from the initial. If for example we had made the following

is positive and statistically significant. This coefficient however, cannot be interpreted until we transform the equations. Estimating $e^{\frac{\beta_6 - 1}{DS_{ijt}}} - 1$ for each observation, we can then obtain an average impact for the years 2007-2008 equal to 0.0214 (equation 10). Hence, during the GFC export values were 2.14% higher holding everything else constant (Table 1). This breaks down to a 1.6% increase for 2007 and 2.7% increase for 2008. For both the US and EU (equation 11) the impact is very small with lower sized countries absorbing most of the effect of the GFC.

The third model is a simple PPML estimation that does not account for multilateral resistances. The purpose of this model is to compare the impact of the GFC variable. This is estimated as $e^{\beta_6} - 1 = 0.088$ which can be compared to our total estimate of the impact with model 2 (2.14% increase). The result is different, however, model three diverges from the theoretically informed model. Differences in the estimated coefficients are not only found with respect to the GFC variable but also with respect to FTA which shows a coefficient of 0.15.

4. Conclusions

We modified the basic structural gravity specification in order to extract a coefficient for the impact of policy-shocks in structural gravity. The impact cannot otherwise be estimated as exporter-time and importer-time fixed effects that need to be included in the specification absorb the coefficient of the policy variable. The methodology employed further allowed us to extract the impact on a per-country or an annual basis or both. While we employed annual, aggregated data in our analysis, higher level data should be employed when possible, allowing for a more reliable averaging of the impact on a per-country/per-year basis.

The proposed method presents the only alternative researchers have at the time when structural gravity estimation concentrates around policy shocks and they need to

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transformation in a simplified model $X_{ijt} = X_{ijt} \exp\left(CNTG_{ij}\right) = \exp\left(\beta_0 + \beta_1 \ln Dist_{ij} + \beta_2 CNTG_{ij}\right)\exp\left(CNTG_{ij}\right)\varepsilon_{ijt}$ -therefore multiplying both sides with $\exp\left(CNTG_{ij}\right)$- then the result would be $X_{ijt} = \exp\left(\beta_0 + \beta_1 \ln Dist_{ij} + \left(\beta_2 + 1\right) CNTG_{ij}\right)\varepsilon_{ijt}$ which would imply that the coefficient would be the same in the original and the transformed model.

differentiate the results that apply for the whole set of exporters from those that apply to one or a few countries.

Updates to this article will first examine the selection of the scaling variable $\left(DS_{ijt}\right)$. In this article the weighing variable was chosen as the average GDP of the trading partners which may cause problems of endogeneity. The variable choice must be justified, the impact on the results must be identified and the robustness of the results to different choices of the weighing variable must be examined. Second, the functional form for the interaction of the country specific (or only time-varying) policy variable with the weighting variable $\left(DS_{ijt}\right)$ that varies over i, j, t has to be chosen. Finally, an adjustment for the standard errors must be presented to obtain the significance of country-specific (or year-specific) effects.

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6. Appendix

A 1.

We start with the basic model where we multiply both sides with our transformed variable:

$$X_{ijt}\left(e^{\frac{Policy_t}{DS_{ijt}}}\right) = \frac{Y_{it}E_{jt}}{Y_t^W}\left(\frac{t_{ijt}}{\Pi_{it}P_{jt}}\right)^{1-\sigma} e^{\frac{Policy_t}{DS_{ijt}}}$$

Rename the left hand side transformed variable:

$$X_{ijt} = \frac{Y_{it}E_{jt}}{Y_t^W}\left(\frac{t_{ijt}}{\Pi_{it}P_{jt}}\right)^{1-\sigma} e^{\frac{Policy_t}{DS_{ijt}}}$$

Write the model in the form necessary for PPML estimation

$$X_{ijt} = \mathrm{e}^{\ln\left(\frac{Y_{it}E_{jt}}{Y}\left(\frac{t_{ijt}}{\Pi_{it}P_{jt}}\right)^{1-\sigma} e^{\frac{Policy_t}{DS_{ijt}}}\right)}\varepsilon_{ijt}$$

Expand the logarithm on the right hand side

$$X_{ijt} = \mathrm{e}^{\ln Y_{it}+\ln E_{jt}+(1-\sigma)\ln t_{ijt}-(1-\sigma)\ln\Pi_{it}-(1-\sigma)\ln P_{jt}+\frac{Policy_t}{DS_{ijt}}}\varepsilon_{ijt}$$

Substitute equation 5 for the costs and add the fixed effects

$$X_{ijt} = \mathrm{e}^{\alpha_0\ln Y_{it}+\alpha_1\ln E_{jt}+\beta_1\ln Dist_{ij}+\beta_2 CNTG_{ij}+\beta_3 LANG_{ij}+\beta_4 CLNY_{ij}+\beta_5 FTA_{ijt}-\pi_{it}-\chi_{jt}-k_{ij}+\beta_6\frac{Policy_t}{DS_{ijt}}}\varepsilon_{ijt}$$

Eliminate the terms that are absorbed by the time varying fixed effects and country pair fixed effects. More specifically the importer-time and exporter-time fixed effects (π_{it}, χ_{jt}) absorb $\ln Y_{it}, \ln E_{jt}$. The country pair fixed effects (k_{ij}) absorb $\ln Dist_{ij}$, $CNTG_{ij}, LANG_{ij}, CLNY_{ij}$ and we are left with

$$X_{ijt} = \mathrm{e}^{\cancel{\alpha_0\ln Y_{it}}+\cancel{\alpha_1\ln E_{jt}}+\cancel{\beta_1\ln Dist_{ij}}+\cancel{\beta_2 CNTG_{ij}}+\cancel{\beta_3 LANG_{ij}}+\cancel{\beta_4 CLNY_{ij}}+\beta_5 FTA_{ijt}-\pi_{it}-\chi_{jt}-k_{ij}+\beta_6\frac{Policy_t}{DS_{ijt}}}\varepsilon_{ijt}$$

$$X_{ijt} = e^{\beta_5 FTA_{ijt}+\beta_6\frac{Policy_t}{DS_{ijt}}-\pi_{it}-\chi_{jt}-k_{ij}}\varepsilon_{ijt}$$

A 2.

Once we estimate this model we can extract the semi elasticity. The percentage change in the transformed dependent variable as a result of the change in policy is

$$\hat{\sigma} = \frac{X_{ijt,Policy=1} - X_{ijt,Policy=0}}{X_{ijt,Policy=0}}$$

Substitute the transformed variable $\left(X_{ijt}\left(e^{\frac{Policy_t}{DS_{ijt}}}\right)\right)$ in the above expression

$$\hat{\sigma} = \frac{X_{ijt}\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right) - X_{ijt}\left(e^{\frac{Policy_{t,0}}{DS_{ijt}}}\right)}{X_{ijt}\left(e^{\frac{Policy_{t,0}}{DS_{ijt}}}\right)}$$

Substitute from the econometric model and add the values for the dummy variables:

$$\hat{\sigma} = \frac{X_{ijt}\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right) - X_{ijt}\left(e^{\frac{Policy_{t,0}}{DS_{ijt}}}\right)}{X_{ijt}\left(e^{\frac{Policy_{t,0}}{DS_{ijt}}}\right)} = \frac{e^{\beta_5 FTA_{ijt} + \beta_6 \frac{1}{DS_{ijt}} - \pi_{it} - \chi_{jt} - k_{ij}} \varepsilon_{ijt} - e^{\beta_5 FTA_{ijt} + \beta_6 \frac{0}{DS_{ijt}} - \pi_{it} - \chi_{jt} - k_{ij}} \varepsilon_{ijt}}{e^{\beta_5 FTA_{ijt} + \beta_6 \frac{0}{DS_{ijt}} - \pi_{it} - \chi_{jt} - k_{ij}} \varepsilon_{ijt}}$$

Eliminate some of the exponential terms where policy=0 and the exponential is equal to 1.

$$\hat{\sigma} = \frac{X_{ijt}\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right) - X_{ijt}}{X_{ijt}} = \frac{e^{\beta_5 FTA_{ijt} + \beta_6 \frac{1}{DS_{ijt}} - \pi_{it} - \chi_{jt} - k_{ij}} \varepsilon_{ijt}}{e^{\beta_5 FTA_{ijt} + \beta_6 \frac{0}{DS_{ijt}} - \pi_{it} - \chi_{jt} - k_{ij}} \varepsilon_{ijt}} - 1$$

Simplify the right hand side of this equation

$$\frac{X_{ijt}\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right) - X_{ijt}}{X_{ijt}} = e^{\beta_6 \frac{1}{DS_{ijt}}} - 1$$

The next step is to eliminate the extra term on the left hand side $\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right)$ so that we are left with the desired percentage change in trade values $\left(\frac{X_{ijt,P=1} - X_{ijt,P=0}}{X_{ijt,P=0}}\right)$. Multiply the right hand side of the previous expression with the denominator of the left side fraction $\left(X_{ijt}\right)$ and simplify

$$\frac{X_{ijt,P=1}\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right) - X_{ijt,P=0}}{X_{ijt,P=0}} = e^{\beta_6 \frac{1}{DS_{ijt}}} - 1$$

$$X_{ijt,P=1}\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right)-X_{ijt,P=0}=\left(e^{\beta_6\frac{1}{DS_{ijt}}}-1\right)X_{ijt,P=0}$$

$$X_{ijt,P=1}\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right)-X_{ijt,P=0}=\left(e^{\beta_6\frac{1}{DS_{ijt}}}X_{ijt,P=0}-X_{ijt,P=0}\right)$$

$$X_{ijt,P=1}\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right)=\left(e^{\beta_6\frac{1}{DS_{ijt}}}X_{ijt,P=0}-X_{ijt,P=0}\right)+X_{ijt,P=0}$$

Eliminate the redundant term from the left hand side

$$X_{ijt,P=1}=\frac{\left(e^{\beta_6\frac{1}{DS_{ijt}}}X_{ijt,P=0}-X_{ijt,P=0}\right)+X_{ijt,P=0}}{\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right)}$$

$$X_{ijt,P=1}=\frac{e^{\beta_6\frac{1}{DS_{ijt}}}X_{ijt,P=0}}{\left(e^{\frac{Policy_{t,1}}{DS_{ijt}}}\right)}$$

Subtract from both sides $X_{ijt,P=0}$ and then divide by $X_{ijt,P=0}$

$$\frac{X_{ijt,P=1}-X_{ijt,P=0}}{X_{ijt,P=0}}=\frac{\frac{e^{\beta_6\frac{1}{DS_{ijt}}}X_{ijt,P=0}}{\left(e^{\frac{1}{DS_{ijt}}}\right)}-X_{ijt,P=0}}{X_{ijt,P=0}}$$

Simplify the right hand side

$$\frac{X_{ijt,P=1}-X_{ijt,P=0}}{X_{ijt,P=0}}=\frac{\frac{e^{\beta_6\frac{1}{DS_{ijt}}}X_{ijt,P=0}-\left(e^{\frac{1}{DS_{ijt}}}\right)X_{ijt,P=0}}{\left(e^{\frac{1}{DS_{ijt}}}\right)}}{X_{ijt,P=0}}$$

$$\frac{X_{ijt,P=1}-X_{ijt,P=0}}{X_{ijt,P=0}}=\frac{e^{\beta_6\frac{1}{DS_{ijt}}}X_{ijt,P=0}-\left(e^{\frac{1}{DS_{ijt}}}\right)X_{ijt,P=0}}{X_{ijt,P=0}\left(e^{\frac{1}{DS_{ijt}}}\right)}$$

Eliminate all $X_{ijt,P=0}$ terms on the right hand side

$$\frac{X_{ijt,P=1}-X_{ijt,P=0}}{X_{ijt,P=0}}=\frac{e^{\beta_6\frac{1}{DS_{ijt}}}-\left(e^{\frac{1}{DS_{ijt}}}\right)}{\left(e^{\frac{1}{DS_{ijt}}}\right)}$$

Simplify

$$\frac{X_{ijt,P=1}-X_{ijt,P=0}}{X_{ijt,P=0}}=\frac{e^{\beta_6\frac{1}{DS_{ijt}}}}{\left(e^{\frac{1}{DS_{ijt}}}\right)}-1$$

$$\frac{X_{ijt,P=1}-X_{ijt,P=0}}{X_{ijt,P=0}}=e^{\frac{\beta_6}{DS_{ijt}}\left(-\frac{1}{DS_{ijt}}\right)}-1$$

$$\frac{X_{ijt,P=1}-X_{ijt,P=0}}{X_{ijt,P=0}}=e^{\frac{\beta_6-1}{DS_{ijt}}}-1$$

The last expression is equation 9 in the text.

We can also reach the same equation if we start from the desired estimation $\frac{X_{ijt,P=1}-X_{ijt,P=0}}{X_{ijt,P=0}}$ and then substitute the definition $X_{ijt}=X_{ijt}e^{\frac{Policy_t}{DS_{ijt}}}$ to obtain

$$\frac{\frac{X_{ijt}}{e^{\frac{Policy_t}{DS_{ijt}}}}-\frac{X_{ijt}}{e^{\frac{Policy_t}{DS_{ijt}}}}}{\frac{X_{ijt}}{e^{\frac{Policy_t}{DS_{ijt}}}}}$$

A 3.

Once we estimate this model we can now extract the semi elasticity. The change in the transformed dependent variable as a result of the "change" in FTA (countries that have an FTA signed) is

$$\hat{\sigma} = \frac{X_{ijt,FTA=1} - X_{ijt,FTA=0}}{X_{ijt,FTA=0}}$$

Substitute the transformed variable in the above expression

$$X_{ijt}\left(e^{\frac{Policy_t}{DS_{ijt}}}\right)$$

$$\hat{\sigma} = \frac{X_{ijt}\left(e^{\frac{Policy_{t,}}{DS_{ijt}}}\right) - X_{ijt}\left(e^{\frac{Policy_t}{DS_{ijt}}}\right)}{X_{ijt}\left(e^{\frac{Policy_t}{DS_{ijt}}}\right)}$$

Equivalently, perform the calculations by substituting from the econometric estimation while the common term $\left(e^{\frac{Policy_t}{DS_{ijt}}}\right)$ drops out:

$$\hat{\sigma} = \frac{X_{ijt}\left(e^{\frac{Policy_{t,}}{DS_{ijt}}}\right) - X_{ijt}\left(e^{\frac{Policy_t}{DS_{ijt}}}\right)}{X_{ijt}\left(e^{\frac{Policy_t}{DS_{ijt}}}\right)} = e^{\beta_5} - 1$$

$$\hat{\sigma} = \frac{X_{ijt} - X_{ijt}}{X_{ijt}} = e^{\beta_5} - 1$$