General Trade Equilibrium of Integrated World Economy

Guo, Baoping

Individual Researcher

October 2015

General Trade Equilibrium of Integrated World Economy

Baoping Guo[1]

Fairfax, VA, USA

Abstract – Dixit and Norman (1980) provided a remarkable result of integrated world equilibrium that the world prices remain the same when the allocation of factor endowments of two countries changes within the parallelogram formed by the rays of diversification cone. What structure are for the prices? This paper derived the equalized factor prices and general trade equilibrium embedded in the IWE diagram. The study demonstrated that the equalized factor prices are the function of world factor endowments. Moreover, the world prices make sure that countries participating in free trade gain from trade. This result is helpful for the studies of factor price non-equalizations when countries have different productivities.

Keywords:
Factor content of trade; factor price equalization; General equilibrium of trade; Integrated World Equilibrium; IWE

1. Introduction

Samuelson (1948) presented the famous theorem of factor price equalization. Immediately, he made a verbal argument that the equalized factor prices will not change when factors mobilized across countries (see Samuelson 1949). Thirty years later, Dixit and Norman (1980) provided the Integrated World Equilibrium (IWE) to illustrate the FPE and price-trade equilibrium, which fulfilled mobile factor analysis perfectly. Helpman and Krugman (1985) normalized the assumption of the integrated equilibrium. They derived two important analytical relationships for price-trade equilibrium by placing equal trade volume lines on the IWE diagram (see Helpman and Krugman, 1985, pp23-24). Their result can be used to derive price-trade equilibrium independently[2] and can serve as a reference to the correctness of equilibrium solution. It is the earliest study to try to solve the price-trade equilibrium. Deardorff (1994) derived the conditions of the FPE for many goods, many factors, and many countries by using the IWE approach. He discussed the FPE for all possible allocations of factor endowments within lenses identified.

Woodland (2013, pp39) described the importance of the general equilibrium, "General equilibrium has not only been important for a whole range of economics analyses, but especially so for the study of international trade". Deardorff (1984, pp685) said, "A trade equilibrium is somewhat more complicated". The Heckscher-Ohlin theories still do not achieve this primary goal, even for the simplest $2 \times 2 \times 2$ model.

---

[1] Former faculty member of The College of West Virginia (renamed as Mountain State University), author Email address: bxguo@yahoo.com.

[2] See their two math formulas in section 2 of this paper.

The one focus of studies on the general equilibrium for constant returns and perfect competition is by the social utility function and direct and indirect trade utility function (offer curve). It is not easy neither for those approachs to get a desired price-trade equilibrium. It did provide a framework for solutions of equilibriums from consumption.

This study found that behind the shared world prices and the equalized factor price within IWE, there is a clear relationship of price-trade equilibrium. It showed that the prices at equilibrium are the functions of the world factor endowments. The result is consistent with the price inference Dixit and Norman made four decades ago. The solution stands with price-trade relationships proposed by Helpman and Krugman. Moreover, the study derived the autarky prices and illustrated that the equalized factor prices ensure gains from trade for countries participating in the trade.

This study is divided into five sections. Section 2 introduces the solution of price–trade equilibrium by the IWE diagram. Section 3 provides a way to estimate autarky prices. The logic is that the autarky factor endowments determine autarky prices. Section 4 presents the equilibrium for cases of two factors, two commodities, and multiple countries. Section 5 is a brief discussion.

2. The Price-Trade Equilibrium by Geometric Analyses within the IWE

We take the following normal assumptions of the Heckscher-Ohlin model in this study: (1) identical technology across countries, (2) identical homothetic taste, (3) perfect competition in the commodities and factors markets, (4) no cost for international exchanges of commodities, (5) factors are completely immobile across countries but that can move costlessly between sectors within a country, (6) constant return of scale and no factor intensity reversals, and (7) full employment of factor resources. We denote the Heckscher-Ohlin model as follows. The production constraint of full employment of factor resources is

$$AX^h = V^h \qquad (h = H, F) \qquad (2\text{-}1)$$

where A is the 2×2 matrix of direct factor inputs, X^h is the 2×1 vector of commodities of country h, V^h is the 2×1 vector of factor endowments of country h. The elements of matrix A is $a_{ki}(w/r), k = K, L, i = 1,2$. We assume that A is not singular. The zero-profit unit cost condition is

$$A'W^h = P^h \qquad (h = H, F) \qquad (2\text{-}2)$$

where $W^h$is the 2×1 vector of factor prices, its elements are r rental for capital and w wage for labor, P^h is the 2×1 vector of commodity prices.

We provide three different ways to derive the same price-trade equilibrium solution, in this section.

Method 1

Figure 1 is a regular IWE diagram. The dimensions of the box represent world factor endowments. The origin of the home country is the lower-left corner, for the foreign country is the right-upper corner. ON and OM are the rays of the cone of factor diversifications. Any point within the parallelogram formed by ONO^*M is an available allocation of factor endowments of two countries. Suppose that an allocation of the factor endowments is at point E, where the home country is capital abundant. Point C represents the trade equilibrium point. It indicates the sizes of the consumption of the two countries.

We introduce two parameters, which are the shares of the home country's factor endowment to their world factor endowments respectively,

$$0 < \lambda_L < 1 \tag{2-3}$$

$$0 < \lambda_K < 1 \tag{2-4}$$

The factor endowments of the home country can be expressed as

$$L^H = \lambda_L L^W \tag{2-5}$$

$$K^H = \lambda_K K^W \tag{2-6}$$

where K^W is the world capital endowment, and L^W is the world labor endowment. The allocation of point E in Figure 1 is $E(\lambda_L L^W, \lambda_K K^W)$.

The factor contents of trade from HOV theorem are

$$F_K^H = K^H - s^H K^W = (\lambda_K - s^H)K^W \tag{2-7}$$

$$F_L^H = L^H - s^H L^W = (\lambda_L - s^H)L^W \tag{2-8}$$

where s^H the share of the GNP of country H to the world GNP.
Using the trade balance of factor contents yields

$$\frac{r^*}{w^*} = -\frac{F_L^H}{F_K^H} = \frac{(s^H - \lambda_L)L^W}{(\lambda_K - s^H)K^W} \tag{2-9}$$

where r^* is the equalized rental, w^* is the equalized wage. Introduce a constant q as

$$q = \frac{(s^H - \lambda_L)}{(\lambda_K - s^H)} \tag{2-10}$$

Substituting it into (2-9) yields

$$\frac{r^*}{w^*} = q\frac{L^W}{K^W} \tag{2-11}$$

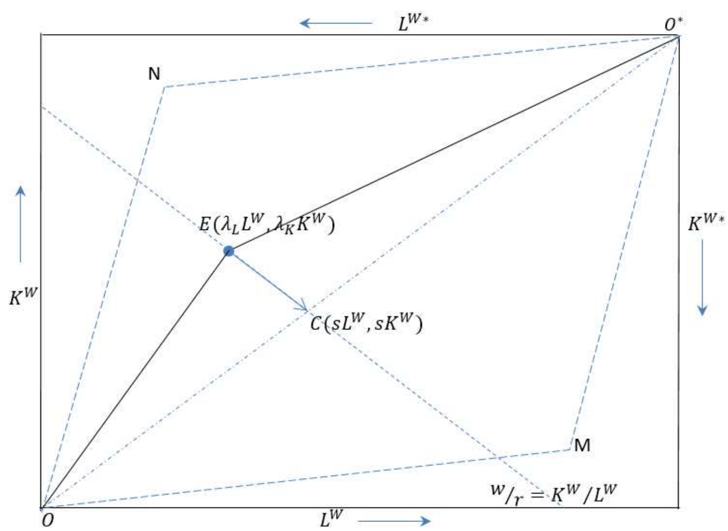

Figure 1 IWE Diagram

The factor prices and commodity prices are unchanged within the parallelogram by ONO^*M on the IWE diagram. That was proofed by Dixit and Norman (1980) and other studies. The factor price ratio (r^*/w^*) should be unchanged. Therefore, q should be constant when the allocation of factor endowments changes. Equation (2-11) illustrates that the rental/wage ratio is the function of the world factor endowments. This is why the FPE holds within the parallelogram formed by ONO^*M in the IWE diagram.

We have interesting to know what value q takes. At point $C(s^H L^W, s^H K^W)$, We see that $\lambda_L = s^H$ and $\lambda_K = s^H$. There is no trade at this point. We now suppose that allocation E is nearby to C or imagine point E moves to close to its equilibrium point C. If the allocation E is above the diagonal line OO', it means that country home is capital abundant. It also implies that $s^H - \lambda_L > 0$ and $\lambda_K - s^H > 0$. Taking limitation as $\lambda_L \to s^H$ and $\lambda_K \to s^H$ yields

$$\lim_{\substack{\lambda_L \to s \\ \lambda_k \to s}} \frac{(s-\lambda_L)}{(\lambda_K - s)} = 1 = q \tag{2-12}$$

We see that constant q equals to 1. Substituting q=1 into equation (2-10) and solving it for s yield,

$$s^H = \frac{1}{2}(\lambda_L + \lambda_K) = \frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) \tag{2-13}$$

Besides, equation (2-11) is reduced as

$$\frac{r^*}{w^*} = \frac{L^W}{K^W} \tag{2-14}$$

This is true for any allocation of factor endowments within parallelogram ONO^*M.

With the equilibrium share of GNP (2-13) and the rental/wage ratio (2-14), we now obtain the whole equilibrium solution of the Heckscher-Ohlin model as

$$r^* = \frac{L^W}{K^W} \tag{2-15}$$

$$w^* = 1 \tag{2-16}$$

$$p_1^* = a_{k1}\frac{L^W}{K^W} + a_{L1} \tag{2-17}$$

$$p_2^* = a_{k2}\frac{L^W}{K^W} + a_{L2} \tag{2-18}$$

$$F_K^h = \frac{1}{2}\frac{K^h L^W - K^W L^h}{L^W}, \qquad F_L^h = -\frac{1}{2}\frac{K^h L^W - K^W L^h}{K^W}, \quad (h = H, F) \tag{2-19}$$

$$T_1^h = x_1^h - \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_1^W, \qquad T_2^h = x_2^h - \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_2^W, \quad (h = H, F) \tag{2-20}$$

$$s^h = \frac{1}{2}\left(\frac{K^h}{K^W} + \frac{L^h}{L^W}\right), \qquad (h = H, F) \tag{2-21}$$

where p_i^* is world price for commodity i; T_i^h is the trade volume of commodity i in country h. Here, we assumed $w^* = 1$ by using Walras' equilibrium condition to drop one market clear condition.

In this method, we used the Dixit and Norman's conclusion that world prices remain the same for any allocations of factor endowment within the parallelogram.

Method 2

Helpman and Krugman (1985, pp23) showed that a line paralleled to the diagonal line in the IWE diagram is an equal trade volume line. They derived that there are some (γ_L , γ_K) for all equal trade volumes lines, which satisfy the following relationships:

$$VT = \gamma_L L^H + \gamma_K K^H \quad (2\text{-}22)$$

$$-\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} \quad (2\text{-}23)$$

They defined VT as the trade volume[3]. They identified that one of γ_L, γ_K is negative. They characterized the equilibrium relationships from trade volumes. It did not catch much attention to other scholars. However, it is an insight derivation. It showed price-trade equilibrium from the view of the factor contents of trade. It is more abstract in logic. It can either serve as a reference for the price-trade equilibrium or be a way to solve the equilibrium independently.

Those two variables γ_L and γ_K can be just interpreted as factor contents of trade

$$\gamma_L = -F_K^H = -\left(K^H - s^H K^W\right) \quad (2\text{-}24)$$

$$\gamma_K = -F_L^H = -\left(L^H - s^H L^W\right) \quad (2\text{-}25)$$

The two equations above define the two variables by the imports of factor content of trade (Helpman and Krugman defined factor content of trade as imports. See page 18 in their book).

Substituting (2-24) and (2-25) into (2-23) yields

$$-\frac{K^H - s^H K^W}{L^H - s^H L^W} = \frac{K^W}{L^W} \quad (2\text{-}26)$$

Solving it, we get

$$s^H = \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} \quad (2\text{-}27)$$

It is just the same result (2-21). With it, we can get the same solutions of (2-15) through (2-11). The (2-26) implies, by the balance of factor content of trade (2-9),

$$-\frac{\gamma_L}{\gamma_K} = \frac{K^W}{L^W} = \frac{w^*}{r^*} = -\frac{F_K^H}{F_L^H} \quad (2\text{-}28)$$

This relationship can also fit equation (2-22). Substituting (2-24) and (2-25) into (2-22) yields

$$VT = -\left(K^H - s^H K^W\right)L^H - \left(L^H - s^H L^W\right)K^H \quad (2\text{-}29)$$

Substituting (2-27) into it yields

$$VT = \frac{1}{2}\frac{(K^H L^W - K^W L^H)^2}{L^W K^W} \quad (2\text{-}30)$$

We now check its correctness. The definition of the trade volume of factor contents is

$$VT = 2\left(K^H - s^H K^W\right)r^* = 2F_K^H r^* \quad (2\text{-}31)$$

By (2-28), (2-24) and (2-25), we can write

$$w^* = F_K^H \quad (2\text{-}32)$$

$$r^* = -F_L^H \quad (2\text{-}33)$$

In equation (2-9), we dropped one market clear condition by the assumption of $w^* = 1$. Now we do it in a different way as (2-31). We defined $w^* = F_K^H$ in (2-32). All prices are relative prices in economics and in the real world.
Substituting (2-33) into (2-31) yields

[3] The trade volume of commodities is not as same as the trade volume of factor contents VT. We take VT here as world trade volume of net factor contents: $VT = 2r^*\left(K^H - s^H K^W\right) = -2w^*\left(L^H - s^H L^W\right)$ when country H is capital abundant.

$$VT = -2F_K^H * F_L^H \quad (2\text{-}34)$$

Substituting (2-19) into the above yields

$$VT = \frac{1}{2}\frac{(K^H L^W - K^W L^H)^2}{L^w K^w} \quad (2\text{-}35)$$

The trade volumes of net factor contents by (2-30) and (2-35) are the same. It shows that Helpman and Krugman's analytical analyses are right.

By equations (2-32), (2-33), (2-24) and (2-25), we had supposed

$$w^* = -\gamma_L \quad (2\text{-}36)$$

$$r^* = \gamma_K \quad (2\text{-}37)$$

Substituting them into (2-22) yields

$$VT = -w^* L^H + r^* K^H \quad (2\text{-}38)$$

It shows a logic that the difference between total capital cost and total labor cost in a country equals to the world trade volume of net factor content. This conclusion is true for all allocations by the parallelogram by ONO^*M.

In this method, we used Helpman and Krugman's analytical condition (2-22) and (2-23).

Method 3

We view the equilibrium from the angle of trade competition by a trade box in the IWE diagram. Fisher (2011) proposed an insight concept of "goods price diversification cone". It is the counterpart of the diversification cone of factor endowments. The output prices should lie between the rays of goods price diversification cone in algebra as,

$$\frac{a_{K1}}{a_{K2}} > \frac{p_1^*}{p_2^*} > \frac{a_{L1}}{a_{L2}} \quad (2\text{-}39)$$

This condition will make sure that the factor prices from unit cost equation (2-2) are positive. The boundaries of the share of GNP, corresponding the rays of the goods price diversification cone (2-39), can be calculated as

$$s_b^H(p) = s\left(p\left(\frac{a_{K1}}{a_{K2}}, 1\right)\right) = \frac{a_{K1}x_1 + a_{K2}x_2}{a_{K1}x_1^W + a_{K2}x_2^W} = \frac{K^H}{K^W} = \lambda_K \quad (2\text{-}40)$$

$$s_a^H(p) = s\left(p\left(\frac{a_{L1}}{a_{L2}}, 1\right)\right) = \frac{a_{L1}x_1 + a_{L2}x_2}{a_{L1}x_1^W + a_{L2}^H x_2^W} = \frac{L^H}{L^W} = \lambda_L \quad (2\text{-}41)$$

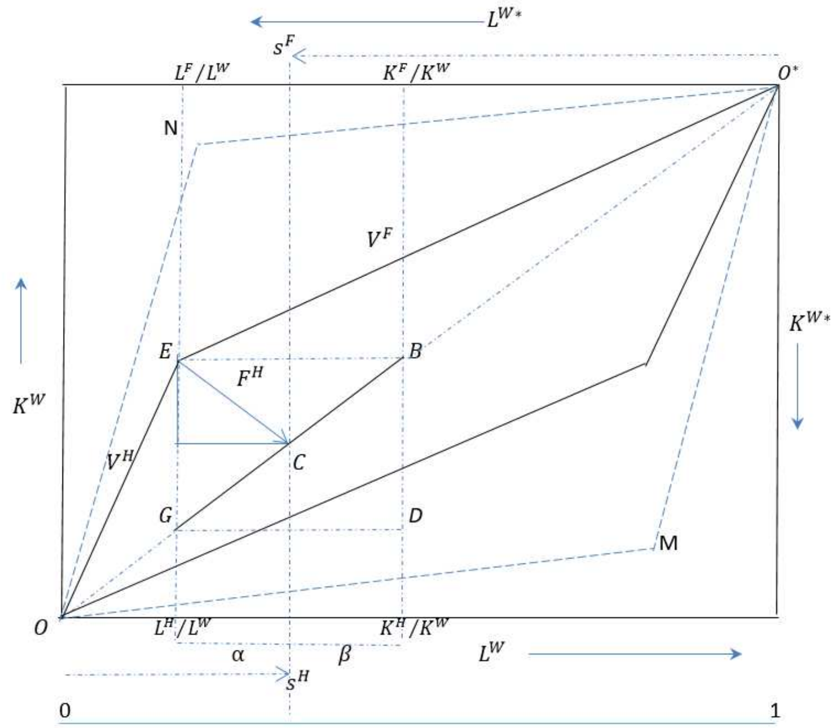

Figure 2 IWE Diagram with Trade Box

Figure 2 is an IWE diagram added with a trade box. The shares of GNP by (2-40) and (2-41) identify the trade box $EBDG$ in Figure 2. If a output price lies in the price diversification cone, the share of GNP lies in the trade box.

We had assumed the home country is capital abundant. The range of the share of GNP for country H is

$$\frac{K^H}{K^W} > s^H > \frac{L^H}{L^W} \quad (2\text{-}42)$$

The range of the share of GNP for country F is

$$\frac{L^F}{L^W} > s^F > \frac{K^F}{K^W} \quad (2\text{-}43)$$

The HOV studies had explored (2-42) and (2-43) many years before. The home country's share of GNP, s, divides the trade box into two parts in Figure 2. Their lengths are α and β respectively as

$$\alpha = \left(\frac{K^H}{K^W} - s\right), \qquad \beta = \left(s - \frac{L^H}{L^W}\right) \quad (2\text{-}44)$$

When α increases, the home country's share of GNP increases and the foreign country's share of GNP decreases, and vice versa. In trade competitions, both countries want to reach their maximum GNP share in free trade.

For reaching a competitive price-trade equilibrium of the model, we set a utility function as the product of redistributable shares of GNP of the two countries as

$$u = \alpha\beta \quad (2\text{-}45)$$

This simple utility function reflects the trade competition that each country is trying to reach its larger share of GNP. The share of GNP is the function of commodity outputs and commodity prices. The utility function (2-45) reflects that one country cannot obtain gains without trade-off from another country.

Substituting (2-44) into (2-45) yields

$$u = (s^H - \frac{L^H}{L^W})(\frac{K^H}{K^W} - s^H) \quad (2\text{-}46)$$

The market adjusts prices directly. Assume

$$w^* = 1 \quad (2\text{-}47)$$

the share of GNP is the function of rental,

$$s^H = \frac{K^H r^* + L^H}{K^W r^* + L^W} \quad (2\text{-}48)$$

Substituting it into (2-46) yields

$$u = (\frac{K^H r^* + L^H}{K^W r^* + L^W} - \frac{L^H}{L^W})(\frac{K^H}{K^W} - \frac{K^H r^* + L^H}{K^W r^* + L^W}) \quad (2\text{-}49)$$

We are interested in maximizing the utility function u, so we take differential of (2-49) with respect to r^* yields

$$\frac{du}{dr^*} = \left(-2\frac{K^H r^* + L^H}{K^W r^* + L^W} + \left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right)\right)\left(\frac{K^H(K^W r^* + L^W) - K^W(K^H r^* + L^H)}{(K^W r^* + L^W)^2}\right) \quad (2\text{-}50)$$

By the first-order condition, we obtain

$$s^H = \frac{K^H r^* + L^H}{K^W r^* + L^W} = \frac{1}{2}\left(\frac{K^H}{K^W} + \frac{L^H}{L^W}\right) \quad (2\text{-}51)$$

Solving it, we have

$$r^* = \frac{L^W}{K^W} \quad (2\text{-}52)$$

The rental/wage ratio (2-52) is as same as the result in method 1 and method 2. we can get the same result of the general trade equilibrium (2-15) through (2-20). The optimal factor price maximizes the redistributable shares of GNP for both countries.

This method does not set any pre-conditions in its derivations.

From the factor content of trade (2-19), we see that when $\frac{K^H}{L^H} > \frac{K^W}{L^W}$, then $F_L^H > 0$ and $F_K^H > 0$. This just states the Heckscher-Ohlin theorem.

3. Autarky Price and Comparative Advantage

The new logic from the last section is that world factor resource determines world prices. We now apply it to a country with an isolated market. Its “autarky” prices can be determined by its “autarky” factor endowments.

The IWE diagram itself supports the logic that autarky factor resources determine autarky prices analytically. Assuming that the factor endowments of country H shrinks to very small, the factor enwoments of country F will close to be world factor endowments. Country F's autarky prices are then the world prices after the trade. Mathematically, when $V^H \to 0$, inside the IWE box, then $V^F \to V^W$ and the world relative factor price r^* after trade will close to the relative autarky factor price of country H.

Rewrite raltive rental price as

$$r^* = \frac{L^W}{K^W} = \frac{L^H + L^F}{K^H + K^F} \tag{3-1}$$

Seeking the limit above yields

$$\lim_{\substack{L^H \to 0 \\ K^H \to 0}} \frac{L^H + L^F}{K^H + K^F} = \frac{L^F}{K^F} = r^{Fa} \tag{3-2}$$

Moreover, the world output prices will close to the autarky output prices of country F. Therefore, we proved the autarky price formation mathematically. Samuelson (1949) argued this idea very clearly. Something he mentioned is that the autarky price of a country is the world prices if the country is divided into two countries geographically, supposing that all other things are unchanged.

Based on the above discussion, we present the autarky prices of countries that participate in free trade as

$$r^{ha} = \frac{L^h}{K^h} \qquad (h = H, F) \tag{3-3}$$

$$w^{ha} = 1 \qquad (h = H, F) \tag{3-4}$$

$$p_1^{ha} = a_{k1}\frac{L^h}{K^h} + a_{L1} \qquad (h = H, F) \tag{3-5}$$

$$p_2^{ha} = a_{k2}\frac{L^h}{K^h} + a_{L2} \qquad (h = H, F) \tag{3-6}$$

where superscript ha indicates the autarky price of country h.
The gains from trade are measured by

$$-W^{ha\prime}F^h > 0 \qquad (h = H, F) \tag{3-7}$$

$$-P^{ha\prime}T^h > 0 \qquad (h = H, F) \tag{3-8}$$

We add a negative sign in inequalities above since we expressed factor trade by net export, T^h . In most other works of literature, they denoted factor trade by net import. We denoted factor trade by net export. Appendix A is proof of the gain from trade by inequality (3-7). It implies that the world prices at the equilibrium will ensure the gains from trade for both countries. derived.

We summarize the content of this section as a theorem in the following.

Theorem – The comparative advantage theorem

The factor price equalized when price-trade equilibrium reached. At the equilibrium, each country exports the good that has a comparative advantage. The ratio of world commodity prices at the equilibrium lies between the ratios of autarky commodity prices of two countries. The world factor endowments, fully employed, determine world prices, which assure the gains from trade for countries participating in trade. The equilibrium demonstrated the Heckscher-Ohlin theorem.

Proof

The solution (2-15) through (2-18) shows how the world prices are formed and why it remains the same with mobile factor endowments in the IWE box. The relative factor price w/r presents an angle in Figure 1. The angle is unique for a given IWE. Therefore, the solution is unique. The FPE is true and unchanged within FPE solution set with given world factor endowments. From those two points, the equilibrium by the equalized factor prices is right. Appendix A proved the gains from trade as inequality (3-7). It is a mathematical solution of price-trade equilibrium confirmed by economic principles.

End Proof

The equilibrium shows the unification of the Heckscher-Ohlin theorem, The FPE theorem, gains from trade, and Dixit-Norman mobile equalized prices. Each of them means others. That consolidate the Heckscher-Ohlin theories.

4. General equilibrium of trade for the case of two factors, two commodities, and multiple countries

We generalize the equilibrium solution above to the model of two factors, two commodities, and multiple countries in this section.

In a two-country system, country H and country F are trade partners with each other. In a multi-country system, who is the trade partner with whom? We specify that trades are one that a country trades with the rest of the world. The trade relations are very simple by this speciication. It just likes the scenario of the two-country system from the analysis view.

Figure 3 draws an IWE diagram for three countries. The dimensions of the box represent world factor endowments. The vector $V^h(L^h, K^h)$ represents the factor endowments of country h, h=1, 2, and 3. The factor endowment vector V^1 of country 1 is arranged to start at origin point O. The rest of the world factor endowment is $V^2 + V^3$. It starts at the origin point O^*.

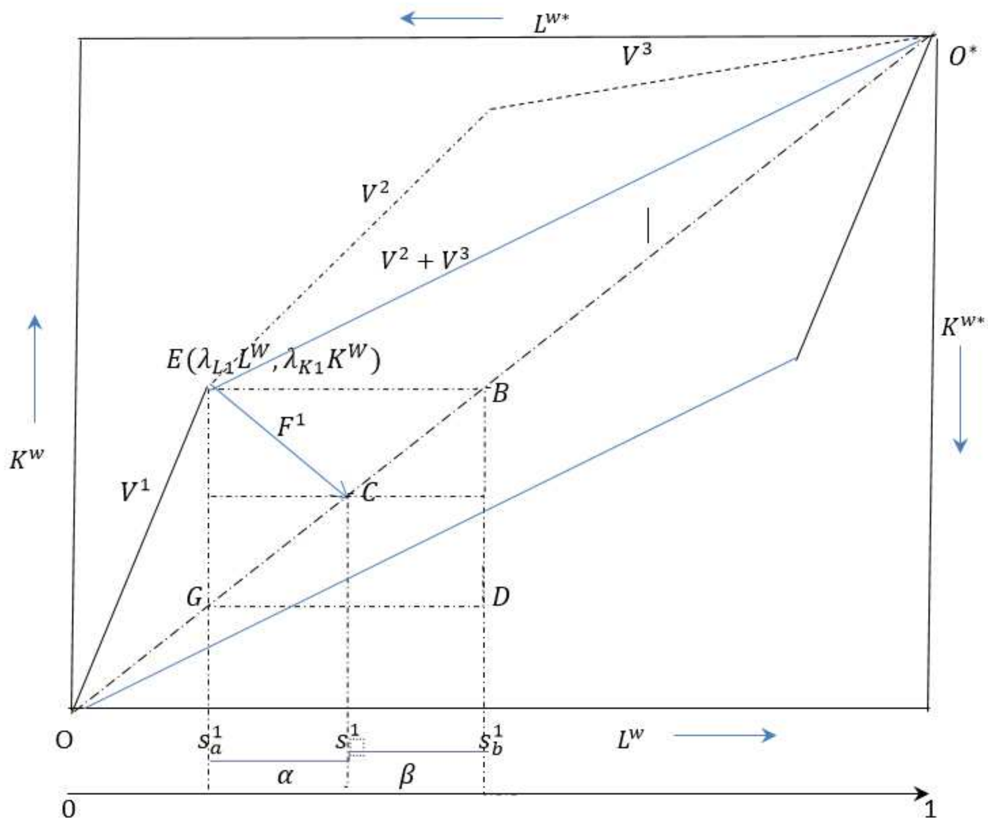

Figure 4 IWE diagram with GNP Share Box for 2 × 2 × 3 model

The system notation for the 2 x 2 x M model is as same as equation (2-1) and (2-2); the only difference is the country number. The country number now goes from 1 to M (In Figure 4, we only present 3 countries for illustration).

We now introduce two lists of parameters, which are the shares of factor endowments of country h to their world factor endowments respectively as

$$0 \leq \lambda_{Lh} \leq 1 \ , \quad 0 \leq \lambda_{Kh} \leq 1 \qquad (h = 1,2,\ldots,M) \tag{4-1}$$

$$\sum_{h=1}^{M} \lambda_{Lh} = 1 \ , \qquad \sum_{h=1}^{M} \lambda_{Kh} = 1 \tag{4-2}$$

The factor endowments of country h can be denoted as

$$L^h = \lambda_{Lh} L^W \qquad (h = 1,2,\ldots,M) \tag{4-3}$$

$$K^h = \lambda_{Kh} K^W \qquad (h = 1,2,\ldots,M) \tag{4-4}$$

The allocation of factor endowments of country 1 in Figure 3 is $E(\lambda_{L1}L^W, \lambda_{K1}K^W)$. It shows how a country trades with the rest of the world.

The factor contents of trade of country h are

$$F_K^h = K^h - s^h K^W = (\lambda_{Kh} - s^h) K^W \qquad (h = 1,2,\ldots,M) \tag{4-5}$$

$$F_L^h = L^h - s^h L^W = (\lambda_{Lh} - s^h) L^W \qquad (h = 1,2,\ldots,M) \tag{4-6}$$

The trade balance of factor contents for country h is

$$\frac{r^{*h}}{w^{*h}} = \frac{(s^h - \lambda_{Lh}) L^W}{(\lambda_{Kh} - s^h) K^W} \qquad (h = 1,2,\ldots,M) \tag{4-7}$$

where r^{*h} is the equalized rental in country h, w^{*h} is the equalized wage in country h. It displays the trade balance between country h and the rest world. Extending the result (2-12) in the last section to the equation above, we have

$$\frac{(s^h-\lambda_{Lh})}{(\lambda_{Kh}-s^h)} = 1 \qquad (h = 1,2,\dots,M) \qquad (4\text{-}8)$$

$$\frac{r^{*h}}{w^{*h}} = \frac{L^W}{K^W} \qquad (h = 1,2,\dots,M) \qquad (4\text{-}9)$$

This means that the relative factor price is the same for all countries.

$$\frac{r^{*h}}{w^{*h}} = \frac{L^W}{K^W} = \frac{r^*}{w^*} \qquad (4\text{-}10)$$

By assuming $w^* = 1$ to drop one market-clearing condition by Walras's equilibrium, we obtain

$$s^h = \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} \qquad (h = 1,2,\dots,M) \qquad (4\text{-}11)$$

$$\frac{r^*}{w^*} = \frac{L^W}{K^W} \qquad (4\text{-}12)$$

$$w^* = 1 \qquad (4\text{-}13)$$

$$p_1^* = a_{k1}\frac{L^W}{K^W} + a_{L1} \qquad (4\text{-}14)$$

$$p_2^* = a_{k2}\frac{L^W}{K^W} + a_{L2} \qquad (4\text{-}15)$$

$$F_K^h = \frac{1}{2}\frac{K^h L^W - K^W L^h}{L^W} \qquad (h = 1,2,\dots,M) \qquad (4\text{-}16)$$

$$F_L^h = -\frac{1}{2}\frac{K^h L^W - K^W L^h}{K^W} \qquad (h = 1,2,\dots,M) \qquad (4\text{-}17)$$

$$x_1^h = x_1^h - \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_1^W \qquad (h = 1,2,\dots,M) \qquad (4\text{-}18)$$

$$x_2^h = x_1^h - \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} x_2^W \qquad (h = 1,2,\dots,M) \qquad (4\text{-}19)$$

We see that

$$\sum_{h=1}^{H} s^h = \sum_{h=1}^{H} \frac{1}{2}\frac{K^h L^W + K^W L^h}{K^W L^W} = 1 \qquad (4\text{-}20)$$

Those are the equilibrium solution for the $2 \times 2 \times M$ model. We can demonstrate that all countries participating in trade gain from trade. It showed that world factor endowments determine world prices in the multi-country economy.

5. Related Discussions

The price-trade equilibrium above displayed the origin of the FPE in the IWE. The trade box illustrates how the redistributable shares of GNP are divided into each country in trade competition. It is a Pareto optimal solution since the trade box shows how social trade-off played. It is a balanced trade that the share of a country in world spending equals to its share in world income.

Dixit (2010) mentioned, "The Stolper-Samuelson and factor price equalization papers did not actually produce the Heckscher-Ohlin theorem, namely the prediction that the pattern of trade will correspond to relative factor abundance, although the idea was implicit there. As Jones (1983, 89) says, 'it was left to the next generation to explore this 2×2 model in more detail for the effect of differences in factor endowments and growth in endowments on trade and production patterns.' That, plus the Rybczynski theorem which arose independently, completed the famous four theorems." The equalized factor prices at the equilibrium of this study presented the Heckscher-Ohlin theorem.

The multiple-country equilibrium is more intricate in economic logic. The equation (4-21) shows that the sum of the shares of GNP of all countries equals to 1. It confirms that both the solution and the approach of this study are right mathematically.

Conclusion

The paper attained the equalized factor prices and the general equilibrium of trade in the Heckscher-Ohlin model. The equilibrium presents the Heckscher-Ohlin theorem with trade volume, the factor-price equalization theorem with price structure, and comparative advantage with gains from trade.

The study illustrates the economic logic that world factor resources determine world prices. Its first application is to identify autarky prices.

The solution of equalized prices is ascertained by Dixit and Norman's mobile-factor FPE that the prices remain the same when the allocation of factor endowments changes. It also confirmed by Helpman and Krugman' equilibrium relationship of factor content of trade.

The result of gains from trade is a good side effect of the trade equilibrium of this paper. It is an important property of the equilibrium and the FPE. It is what we expected.

The equalized factor prices provide the theoretical background for further analyses of factor price none-equalization when countries have different productivities.

Appendix A

We express the gains from trade for the home country as

$$-(W^{Ha})'F^H > 0 \tag{A-1}$$

Adding trade balance condition $W^{*'}F^H = 0$ on (A-1) yields

$$-((W^{Ha})' - W^{*'})F^H > 0 \tag{A-2}$$

We see

$$W^{Ha} = \begin{bmatrix} \frac{L^H}{K^H} \\ 1 \end{bmatrix} \quad , \qquad W^* = \begin{bmatrix} \frac{L^W}{K^W} \\ 1 \end{bmatrix} \tag{A-3}$$

Substituting them into (A-2) yields,

$$-\begin{bmatrix} \frac{L^H}{K^H} - \frac{L^W}{K^W} & 0 \end{bmatrix} \begin{bmatrix} \frac{1}{2}\frac{K^H L^W - K^W L^H}{L^W} \\ -\frac{1}{2}\frac{K^H L^W - K^W L^H}{K^W} \end{bmatrix} > 0 \tag{A-4}$$

It can be rewritten to

$$-(\frac{L^H}{K^H}-\frac{L^W}{K^W})\times\frac{1}{2}\frac{K^H L^W-K^W L^H}{L^W}>0 \quad \text{(A-5)}$$

Simplify the above to

$$\frac{(K^H L^W-K^W L^H)^2}{2L^W}K^W K^H>0 \quad \text{(A-6)}$$

It is true. So that (A-1) holds. Similarly, we can obtain

$$-W^{Fa'}F^F=\frac{(K^H L^W-K^W L^H)^2}{2L^W}K^W K^F>0 \quad \text{(A-7)}$$

Reference

Deardorff, A. V. (1982), “General Validity of the Heckscher-Ohlin Theorem”, The American Economic Review, Vol. 72, No. 4 (Sep. 1982), 683-694.

Deardorff, A. V. (1994), “The possibility of factor price equalization revisited”, Journal of International Economics, XXXVI, 167-75.

Dixit, A.K. and V. Norman (1980), Theory of International Trade, J. Nisbert/Cambridge University Press.

Dixit, A.K (2012), “Paul Samuelson's Legacy”, Annual Reviews of Economics, Vol. 4, 2012

Fisher, E. O’N. (2011), “Heckscher-Ohlin Theory When Countries Have Different Technologies.” *International Review of Economics and Finance* 20, 202-210.

Fisher, E. O’N. and K. G. Marshall (2011), “The Structure of the American Economy.” *Review of International Economics* 19 (February 2011), 15-31.

Helpman, E. and P. Krugman (1985), Market Structure and Foreign Trade, Cambridge, MIT Press. pp 12.

Guo, B. (2005), **“**Endogenous Factor-Commodity Price Structure by Factor Endowments”, International Advances in Economic Research, November 2005, Volume 11, Issue 4, p 484

Guo, B. (2019), “Trade Effects Based on Trade Equilibrium”, Theoretical and Applied Economics, Asociatia Generala a Economistilor din Romania - AGER, vol. 0(1(618), S), pages 159-168,

Jones, RW. (1965), “The Structure of Simple General Equilibrium Models,” Journal of Political Economy 73 (1965): 557-572.

Jones, RW. (1983). "International Trade Theory." In eds. EC Brown and RM Solow (1983), 69-103.

McKenzie, L.W. (1955), “Equality of factor prices in world trade”, Econometrica 23, 239-257.

Samuelson, P. A. (1948), “International Trade and the Equalization of Factor Prices”, Economic Journal, June, pp. 163-184.

Samuelson, P. A. (1949), “International Factor-Price Equalization Once Again.” Economic Journal 59:181-197. Reprinted in ed. JE Stiglitz (1966), 869-885.

Trefler, D. (1993), "International Factor Price Differences: Leontief Was Right!" Journal of Political Economy 101 (1993), 961-87.

Woodland, A. (2013), General Equilibrium Trade Theory, Chp. 3, Palgrave Handbook of International Trade, Edited by Bernhofen, D., Falvey, R., Greenaway, D. and U. Kreickemeier, Palgrave Macmillan.