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Fanti, Lucrezia and Zamparelli, Luca

National Institute for Public Policy Analysis (INAPP), Department of Social Sciences and Economics, Sapienza University of Rome

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Lucrezia Fanti*, Luca Zamparelli†

Abstract

We analyze the paradox of thrift in the two-sector Kaleckian growth model. We consider an economy with one consumption and one investment good, and differential sectoral mark-ups. We show that when the investment function depends on aggregate capacity utilization and on the aggregate profit share (the Bhaduri-Marglin investment function) the paradox of thrift in its growth version may fail if mark-ups are higher in the investment good sector. In this case, the reduction in the saving rate produces a reallocation of economic activity towards the investment good sector; the aggregate profit share rises and its positive effect on investment may offset the reduction in average capacity utilization if investment is relatively more sensitive to profitability than to the level of activity.

Keywords: two-sector growth model, paradox of thrift, Bhaduri-Marglin investment function

JEL Classification: D33, E11, O14

*National Institute for Public Policy Analysis (INAPP), Corso d'Italia 33, Rome Italy 00198 Email: l.fanti@inapp.org.

†Department of Social Sciences and Economics, Sapienza University of Rome. P.le Aldo Moro 5, Rome Italy 00185. Email: Luca.Zamparelli@uniroma1.it.

1 Introduction

Dating back to at least Mandeville's popular *Fable of the Bees* (Mandeville , 1714[1988], I, remark Q), the paradox of thrift became a central proposition in Macroeconomics after the publication of Keynes's *General Theory of Employment Interest and Money.* It is also a distinguishing feature of the Kaleckian model of growth and distribution, where it appears both in a short- and in a long-run version. It states that an increase in the saving rate produces a reduction of production and capacity utilization in the short-run, and of the growth rate in the long-run. A remarkable feature of this result is its robustness against the specification of the investment function, a controversial issue in Kaleckian economics.

We investigate the validity of the paradox of thrift in the two-sector version of the Kaleckian growth model. To the purpose, we consider an economy with one consumption and one investment good, and differential sectoral mark-ups. We show that the paradox of thrift is confirmed in both level and growth versions when investment is a function of (average) capacity utilization or the profit rate. However, when the investment function depends on both aggregate capacity utilization and on the aggregate profit share, that is when the Bhaduri-Marglin investment function (Bhaduri and Marglin , 1990; Marglin and Bhaduri , 1990) is adopted, the paradox of thrift in its growth version may fail if mark-ups are higher in the investment good sector. In this case, the reduction in the saving rate produces a reallocation of economic activity towards the investment good sector; the aggregate profit share rises and its positive effect on investment may offset the reduction in average capacity utilization.

The analysis is based on the standard Kaleckian mark-up pricing assumption, where sectoral mark-ups are exogenous and independent of each other. We show, however, that analogous results can be obtained if firms set prices by targeting a specific return rate when capacity utilization is at its normal level; this alternative assumption is important since it captures a possible dependence of the consumption good sector mark-up on the one in the investment good sector.

First developed in seminal contributions by Dutt (1988, 1990); Park (1995); Dutt (1997a); Lavoie and Ramirez-Gaston (1997); Franke (2000), the two-sectors Keynesian-Kaleckian model growth and distribution is experiencing a recent revival. Kim and Lavoie (2017) have studied the convergence between the actual and the normal rates of capacity utilization; Fujita (2018) has considered the growth implication of shocks to sectoral mark-ups; Murakami (2018) has analyzed the effect of sectoral interactions on business cycles in a Keynesian model; Nishi (2019) has introduced sectoral endogenous labor productivity growth, and analyzed its effects on cyclical demand, growth and distribution; Beqiraj et al. (2019) have studied how changes in consumers' preferences and the saving rate may affect income distribution through changes in the composition of output. None of these contributions, however, investigate the paradox of thrift in the two-sector Kaleckian model when accumulation is based on the Bhaduri-Marglin investment function.

The rest of the paper is organized as follows. Section 2 develops the model and states the theoretical results. Section 3 generalizes the main result of the paper when firms adopt target-return, rather than mark-up, pricing. Section 4 offers some concluding remarks while the most tedious proofs can be found in Section 5.

2 The Model

2.1 Production and technology

The economy consists of a consumption (C) and an investment (I) good. Output in both sectors (X_i) is produced through a sector-specific Leontief production function:

$$X_i = min[u_i B_i K_i, A_i L_i], i = C, I \tag{1}$$

where B and A are capital and labor productivities, K is the capital stock, L is employment, and $u \leq 1$ is the degree of capacity utilization. When $u = 1$,output is at its full capacity level (X^p). Capital does not depreciate. Profit maximization ensures:

$$X_i = u_i B_i K_i = A_i L_i. \tag{2}$$

We normalize capital productivities $B_i = 1$.

2.2 Society and saving assumptions

There are two classes in society. Capitalists earn profits on the capital stock they own. They save the share $s > 0$ of their income. Workers earn the wage rate w, uniform across sectors, and do not save.

2.3 Mark-up prices

In standard Kaleckian fashion, firms set prices by charging an exogenous sector-specific constant mark-up (z_i) over unit labor cost. If we let p_i be the price of good i, and we choose the consumption good as the numeraire we have $p_C = 1 = (1 + z_C)w/A_C$ and $p_I = (1 + z_I)w/A_I$. Accordingly

$$w = \frac{A_C}{1 + z_C}, \tag{3}$$

$$p_I = \frac{1 + z_I}{1 + z_C}\frac{A_C}{A_I} = \frac{1 + z_I}{1 + z_C}\gamma, \tag{4}$$

where $\gamma \equiv A_C/A_I$, is the labor productivity ratio. We define the relative price as $p \equiv$

$p_I/p_C = p_I$.

2.4 Value added distribution

In each sector, value added is distributed as wages and profits to labor and capital employed in production. If we let r_i be the interest rate in sector i we have $p_i X_i = wL_i + r_i p_I K_i$, which, after using (2), (3), (4) and rearranging, yields

$$r_C = \frac{z_C}{1+z_I}\frac{1}{\gamma}u_C, \tag{5}$$

and

$$r_I = \frac{z_I}{1+z_I}u_I. \tag{6}$$

2.5 Output uses

We distinguish consumption depending on its income source. We denote consumption out of wages as C^w, and consumption out of profits as C^π, so that

$$X_C = C^w + C^\pi. \tag{7}$$

Investment good output is fully absorbed in the accumulation of capital. If we let g be the growth rate of the aggregate capital stock we have

$$X_I = gK. \tag{8}$$

2.6 Balanced growth under alternative closures

There are two possible consistent specifications of the two-sector Kaleckian growth model (see Park (1995) and Dutt (1997a) for a discussion). The first version of the model assumes that capital instantaneously moves between sectors to equate sectoral profit rates; in this framework, sectoral capital stocks (K_C and K_I) are not state variables and we can only specify aggregate investment and the growth rate, rather than the sectoral ones. In fact, K_C and K_I are defined only after profit rates are equalized. In the second version of the model, there is no sectoral capital mobility in the short run, so that K_C and K_I are given and known before firms' investment decision; we can specify sectoral investment and growth rates, and profit rates will not be equalized unless by a fluke. We investigate the functioning of the paradox of thrift under the first version of the model.

Since workers do not save, the whole wage fund is spent as consumption out of wages. Using (2) we have

$$C^w = w(L_C + L_I) = w\left(\frac{u_C K_C}{A_C} + \frac{u_I K_I}{A_I}\right).$$

Hence, substituting for the wage rate from (3) yields

$$C^w = \frac{A_C}{1+z_C}\left(\frac{u_C K_C}{A_C} + \frac{u_I K_I}{A_I}\right) = \frac{1}{1+z_C}\left(u_C K_C + \gamma u_I K_I\right). \tag{9}$$

On the other hand, capitalists' propensity to consume out of profits is $(1-s)$. Accordingly

$$C^\pi = (1-s)\left(r_I p K_I + r_C p K_C\right),$$

which, using (4),(5) and (6) implies

$$C^\pi = \frac{1-s}{1+z_C}\left(z_C u_C K_C + z_I \gamma u_I K_I\right).$$

Once we know consumption out of wages and profits, we can use equation (7) to find

$$X_C = \frac{1}{1+z_C}\left(u_C K_C(1+(1-s)z_C) + \gamma u_I K_I(1+(1-s)z_I)\right).$$

Define $\delta \equiv K_C/K \in (0,1)$ as the share of the capital stock employed in the consumption good sector. Dividing both sides of the previous equation by K and rearranging yields

$$\delta u_C = (1-\delta)u_I\gamma\frac{(1+(1-s)z_I)}{sz_C} \equiv (1-\delta)u_I\gamma\Gamma(s), \tag{10}$$

with $\Gamma'(s) < 0$. Let us now turn to the equilibrium in the investment sector. Using factors demands found in (2), and dividing both sides of equation (8) by K, we find

$$u_I(1-\delta) = g. \tag{11}$$

Next, we impose the equalization of profit rates across sectors, so that

$$r_C = r_I = r. \tag{12}$$

Using (5) and (6), the equalization yields:

$$u_C = \gamma\frac{z_I}{z_C}u_I. \tag{13}$$

We close the model with three alternative investment functions that generalize to the two-sector case the standard assumptions of one-sector Kaleckian growth model, where investment depends either on the rate of capacity utilization, on the profit rate, on the profit share or on some combination of them. We have already mentioned that the instantaneous profit rates equalization implies that sectoral capital stocks are not state variables, and that only aggregate investment and growth can be defined. This hypothesis also implies that firms will have to look at average, rather than sectoral, utilization rates, profit rates and profit shares when making their investment decision. In fact, only once profit rates are equalized, given total investment, they do know what share of the capital stock and investment is employed in either sector.

If we let the average degree of capacity utilization in the economy be $\bar{u}$, and the aggregate profit share be π, we take into account the following investment functions:

- the first one extends to the two-sector case the early Kaleckian models that had capacity utilization as determinant of investment (Amadeo , 1986a; Dutt , 1997b)

$$g_1 = g(\bar{u}); \tag{14}$$

- the second one assumes investment to depend on the profit rate, the 'stagnationist' investment function (Taylor , 1985; Amadeo , 1986b)

$$g_2 = g(r); \tag{15}$$

- the third one generalizes the Bhaduri-Marglin investment function (Bhaduri and Marglin , 1990; Marglin and Bhaduri , 1990) by positing that growth depends on both aggregate capacity utilization and profit share

$$g_3 = g(\bar{u}, \pi). \tag{16}$$

Under the first and third specifications, the model consists of four equations, (10),(11), (13), and either (14) or (16), for the four unknowns δ, u_I, u_C, g. When the investment function is (15), the unknowns are δ, u_I, u_C, r, g in the five equations (10), (11), (12), (13), and (15). In all three cases we can plug (13) into (10) to find the equilibrium share of capital employed in the consumption goods sector[1]

$$\delta^*(s) = \frac{\Gamma(s)}{\Gamma(s) + z_I/z_C} \in (0, 1). \tag{17}$$

[1]In what follows, we will denote with x^* the balanced growth value of variable x.

The aggregate profit share, that is the ratio between the value of total profits and value added, is also independent on the investment function adopted. Its balanced growth value is

$$\pi^*(s) = \frac{r_C p K_C + r_I p K_I}{X_C + p X_I} = \frac{rp}{\delta u_C + (1-\delta) p u_I} =$$

$$= \frac{z_I}{(1-\delta)\left((1+z_C)\,\Gamma(s) + 1 + z_I\right)} = \frac{z_C \Gamma(s) + z_I}{(1+z_C)\Gamma(s) + (1+z_I)}, \tag{18}$$

where we used (2), (4), (5), (6), (10), (12) and (17). Inspection of (18) shows that π^* is economically meaningful being bounded between zero and one. It is a function of sectoral mark-ups and the saving rate.

We can state:

Proposition 1. *an increase in the saving rate raises the equilibrium profit share if and only if $z_I > z_C$.*

Proof. $\pi'(s) = \frac{d}{ds}\left(\frac{z_C\Gamma(s)+z_I}{(1+z_C)\Gamma(s)+(1+z_I)}\right) = \frac{\Gamma'(s)(z_C - z_I)}{((1+z_C)\Gamma(s)+(1+z_I))^2} > 0 \Leftrightarrow z_I > z_C$. □

A rise in the saving rate reduces capitalists' consumption, so that the composition of output changes in favor of the investment sector. If mark-ups are higher in the investment rather than in the consumption sector ($z_I > z_C$), the reallocation generates a rise in the aggregate profit share.

Our next step is to verify whether the paradox of thrift holds under the alternative investment functions we have proposed. In order to obtain closed-form solutions for the growth rate, and in line with most of the Kaleckian tradition, we assume linear functional forms.

Let us start with the accelerator version of investment:

$$g_1 = \beta_0 + \beta_1 \bar{u}, \tag{19}$$

where

$$\bar{u} = \frac{X_C + pX_I}{pK} = \frac{1+z_I}{1+z_C}\frac{u_C \delta}{\gamma} + u_I(1-\delta).$$

We can use (10) to find:

$$\bar{u} = \frac{X_C + pX_I}{pK} = u_I(1-\delta)\left(\frac{1+z_I}{1+z_C}\Gamma(s) + 1\right). \tag{20}$$

Hence, using (11),

$$u_I(1-\delta) = g = \tag{21}$$

$$= \beta_0 + \beta_1 u_I(1-\delta)\left(\frac{1+z_I}{1+z_C}\Gamma(s) + 1\right),$$

which, by factoring $u_I(1-\delta)$, solves for the steady state growth rate of the first model as a function of the saving rate

$$g_1^*(s) = \frac{\beta_0}{1 - \beta_1\left(\frac{1+z_I}{1+z_C}\Gamma(s) + 1\right)}. \tag{22}$$

Since $\Gamma'(s) < 0$, the growth rate is a negative function of the saving rate and the paradox of thrift holds in its growth version. From (11) and (20) we can write aggregate capacity utilization as

$$\bar{u}_1^*(s) = g_1^*\left(\frac{1+z_I}{1+z_C}\Gamma(s) + 1\right). \tag{23}$$

It is the product of two negative functions of the saving rate so that the paradox of thrift in its level form also holds.

The second investment function makes investment dependent on the profit rate:

$$g_2 = \lambda_0 + \lambda_1 r. \tag{24}$$

Hence,

$$u_I(1-\delta) = g = \tag{25}$$

$$= \lambda_0 + \lambda_1 \frac{z_I}{1+z_I} u_I,$$

and, by factorizing u_I, using (17) and rearranging

$$g_2^*(s) = \frac{\lambda_0 \frac{z_I/z_C}{\Gamma(s)+z_I/z_C}}{\frac{z_I/z_C}{\Gamma(s)+z_I/z_C} - \lambda_1 \frac{z_I}{1+z_I}}. \tag{26}$$

We show in the Appendix (section 5.1) that $dg_2^*/ds < 0$. The paradox of thrift in its growth version is confirmed also under the second type of investment function. Similarly to (23), we can find the aggregate utilization rate as

$$\bar{u}_2^*(s) = g_2^*(s) \left(\frac{1+z_I}{1+z_C} \Gamma(s) + 1 \right),$$

so that, given $dg_2^*/ds < 0$ and $\Gamma'(s) < 0$, the paradox of thrift in level form also holds.

We now turn to the main result of our paper, and we investigate how growth responds to changes in the saving rate under a Bhaduri-Marglin investment function. In linearized terms, we can specify the function as

$$g_3 = \mu_0 + \mu_1 \bar{u}(s) + \mu_2 \pi(s). \tag{27}$$

Hence,

$$u_I(1-\delta) = g = \tag{28}$$

$$= \mu_0 + \mu_1 u_I (1-\delta) \left(\frac{1+z_I}{1+z_C} \Gamma(s) + 1 \right) + \mu_2 \pi(s),$$

and

$$g_3^*(s) = \frac{\mu_0 + \mu_2 \pi(s)}{1 - \mu_1 \left(\frac{1+z_I}{1+z_C} \Gamma(s) + 1 \right)}. \tag{29}$$

We are now able to state:

Proposition 2. *an increase in the saving rate raises the growth rate if and only if* $z_I > z_C$ *and* $\frac{\mu_2}{\mu_1} > \frac{1}{z_I - z_C} \frac{1+z_I}{1+z_C} \frac{\mu_0 + \mu_2 \pi(s)}{1 - \mu_1 \left(\frac{1+z_I}{1+z_C} \Gamma(s) + 1 \right)} \left((1+z_C)\Gamma(s) + (1+z_I) \right)^2$.

Proof. see Appendix (section 5.2).

□

A rise in the saving rate has two opposing effects on growth. On the one hand, there is the standard depressing effect due to the reduction in capitalists' consumption and, in turn, in aggregate demand and capacity utilization. On the other hand, though, the higher propensity to save entails a shift in the composition of output away from consumption goods. When the profit share is higher in the investment goods sector, the aggregate profit share rises thus producing a positive incentive to invest. When investment is sufficiently more sensitive to profitability than to economic activity, that is when μ_2/μ_1 is 'high enough', the paradox of thrift in its growth version fails.

This mechanism can be better understood by focusing on the function $\Gamma(s)$. In fact, the propensity to save only enters the system through $\Gamma(s)$. First, notice that from (2) $L_i = u_i K_i / A_i$, so that $L_C/L_I = u_C K_C A_I / [u_I K_I A_C] = \delta u_C / [(1-\delta) u_I \gamma]$. Next, (10) shows that $\Gamma(s) = \delta u_C / [(1-\delta) u_I \gamma] = L_C/L_I$, that is $\Gamma(s)$ equals the employment ratio in the two sectors. Therefore, a rise in the saving rate raises employment and output in the investment sector relative to the consumption one. The change in the composition of output towards the sector with the highest mark-up raises the aggregate profit share (our Proposition 1). Second, $\Gamma(s)$ enters the definition of aggregate capacity utilization through (20). A higher propensity to save reduces capitalists' consumption, thus depressing the aggregate capacity utilization. The negative shock to capacity utilization tends to depress the equilibrium growth rate; this effect can only be offset if investment reacts to the profit share as assumed in the Bhaduri-Marglin investment function (our Proposition 2).

It is not obvious that in this setting aggregate capacity utilization $\bar{u}_3^*(s) = g_3^*(s)\left(\frac{1+z_I}{1+z_C}\Gamma(s) + 1\right)$ declines with the propensity to save. In principle, when the paradox of thrift in its growth version is violated, the positive effect on growth could offset the negative effect on consumption and produce an overall rise in utilization. However, it turns out that the negative consumption effect uniformly dominates the (possibly positive) growth effect, so that the paradox on thrift in the level form always holds. We prove this result in the Appendix (section 5.2).

3 A Generalization: the Model with Target-Return Pricing

We developed our results under the standard mark-up pricing assumption that characterizes one- and two-sector Kaleckian growth models. The assumption, however, is controversial. According to Steedman (1992), mark-up pricing is problematic in that it does not take into account input-output relations. This critique would be particularly relevant for our two-sector model: since the investment good is a basic good, the mark-up in the consumption sector cannot be taken as exogenous and independent of the mark-up in the investment sector.

Without entering this discussion, we show that the logic of our main result is robust to a generalization of the model that does not suffer from the critique. To the purpose, we adopt the target-return pricing assumption that Lavoie and Ramirez-Gaston (1997) have applied to the two-sector Kaleckian model in order to take the interdependence between mark-ups into account. As we are about to show, under this different priceing rule mark-ups are not exogenous variables anymore.

Let us start by introducing the sectoral normal degree of capacity utilization, u_i^n. Next, we define the sectoral target rate of return (r_i^n) as the return rate that firms target when output and sales correspond to the normal degree of capacity utilization, that is when $X_i = u_i^n K_i \equiv X_i^n$. Given the target rate and normal output, the normal flow of profits (Π_i^n) can be written both as $\Pi_i^n = z_i(w/A_i)X_i^n$ and $\Pi_i^n = r_i^n pK_i = r_i^n pX_i^n/u_i^n$. The equalization of the two profit flows expressions, while using (3) and (4), yields

$$z_i = r_i^n pA_i/(wu_i^n) = r_i^n(1 + z_I)A_i/(A_I u_i^n). \tag{30}$$

Equation (30) shows that mark-ups in the investment good sector, i.e. the basic good sector, depend only on its own econonomic features:

$$z_I = \frac{r_I^n/u_I^n}{1 - r_I^n/u_I^n} = \frac{r_I^n}{u_I^n - r_I^n}. \tag{31}$$

On the contrary, mark-ups in the consumption good sector are affected by the fundamentals of both sectors:

$$z_C = \frac{r_C^n}{u_C^n}\gamma(1 + z_I) = \gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n - r_I^n}. \tag{32}$$

Now, in our exercise we only consider a saving shock. No changes to mark-ups are analyzed and, in turn, the interdepence between the two sectors' mark-ups never comes into play. We can, however, substitute (31) and (32) into (18) to find the aggegate wage share as

$$\pi^*(s) = \frac{\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n}\widetilde{\Gamma}(s) + \frac{r_I^n}{u_I^n-r_I^n}}{(1+\gamma\frac{r_C^n}{u_C^n}\frac{u_I^n}{u_I^n-r_I^n})\widetilde{\Gamma}(s) + 1 + \frac{r_I^n}{u_I^n-r_I^n}},$$

where $\widetilde{\Gamma}(s) = \frac{1+(1-s)r_I^n/(u_I^n-r_I^n)}{s\gamma r_C^n u_I^n/[u_C^n(u_I^n-r_I^n)]}$. We can now restate Proposition 1 as

Proposition 3. *an increase in the saving rate raises the equilibrium profit share if and only* *if* $\frac{r_I^n}{u_I^n} > \gamma \frac{r_C^n}{u_C^n}$.

Proof. Notice first that $\widetilde{\Gamma}'(s) < 0$. Then, $\pi'(s) = \frac{\widetilde{\Gamma}'(s)(\gamma \frac{r_C^n}{u_C^n} \frac{u_I^n}{u_I^n - r_I^n} - \frac{r_I^n}{u_I^n - r_I^n})}{\left((1+\gamma \frac{r_C^n}{u_C^n} \frac{u_I^n}{u_I^n - r_I^n})\widetilde{\Gamma}(s) + \frac{r_I^n}{u_I^n - r_I^n}\right)^2} > 0 \Leftrightarrow \frac{r_I^n}{u_I^n} > \gamma \frac{r_C^n}{u_C^n}$.

□

In order to interpret the emended condition for the positive relation between the saving rate and the profit share, let us define the sectoral normal profit shares as $\pi_i^n \equiv \Pi_i^n / X_i^n = r_i^n p / u_i^n$. We can see that the condition found in Proposition 3 is equivalent to $\pi_I^n > \gamma \pi_C^n$, or $A_I \pi_I^n > A_C \pi_C^n$. We thus see that the necessary condition for the violation of the paradox of thrift requires that the normal profit share (weighted by labor productivity) in the investment good sector be higher than in the consumption sector. In order to produce a rise in the profit share, the rise in the saving rate must be associated to a reallocation of resources towards the relatively more profitable sector. From a qualitative point of view, the result confirms what found without target-return pricing.

Once the possibility that $\pi'(s) < 0$ is established, it will always be possible to find a threshold for the relative weights of the profit share and capacity utilization in the investment function (μ_2/μ_1) such that the paradox of thrift is violated. With respect to the result found in Proposition 2, the threshold will be a function of $\gamma, r_C^n, u_C^n, u_I^n$ and r_I^n rather than z_C and z_I; but the economic content will be analogous.

4 Conclusions

Our theoretical note shows that the paradox of thrift may not work in the Kaleckian growth and distribution framework, once it is generalized to a two-sector economy. This possibility arises because the saving rate affects not only the level of aggregate demand, but also its composition. In particular, a rise in the saving rate, besides depressing aggregate demand, shifts the sectoral composition of output towards the investment goods sector. If this sector is characterized by relatively high mark-ups the aggregate profit share rises; and such an increase in profitability may have a positive effect on growth if, as assumed in Bhaduri and Marglin (1990) and Marglin and Bhaduri (1990), investment reacts to the profit share.

The actual relevance of our result should be established empirically. We have provided testable conditions on sectoral mark-ups and on the sensitivity of investment to demand conditions and profitability. Investigating this matter is left for future research.

5 Appendix

5.1 The paradox of thrift in the stagnationist version of the model

From (26) we have $dg_2^*/ds = \frac{\lambda_0\lambda_1}{\left(\frac{z_I/z_C}{\Gamma(s)+z_I/z_C} - \lambda_1 \frac{z_I}{1+z_I}\right)^2} \frac{z_I}{1+z_I} \frac{(z_I/z_C)\Gamma'(s)}{(\Gamma'(s)+z_I/z_C)^2} < 0.$

5.2 The paradox of thrift in the Bhaduri-Marglin model

Let $1 - \mu_1 \left(\frac{1+z_I}{1+z_C}\Gamma(s) + 1\right) \equiv D_0$, from (29)

$$dg_3^*/ds = \frac{1}{D_0^2}\left[\mu_2\pi'(s)\left(1 - \mu_1\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right)\right) + \mu_1\frac{1+z_I}{1+z_C}\Gamma'(s)\left(\mu_0 + \mu_2\pi(s)\right)\right].$$

Therefore

$$sign\left(dg_3^*/ds\right) = sign\left[\mu_2\pi'(s)\left(1 - \mu_1\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right)\right) + \mu_1\frac{1+z_I}{1+z_C}\Gamma'(s)\left(\mu_0 + \mu_2\pi(s)\right)\right].$$

Then, $dg_3^*/ds > 0 \Leftrightarrow \frac{\mu_2}{\mu_1} > -\frac{\frac{1+z_I}{1+z_C}\Gamma'(s)(\mu_0+\mu_2\pi(s))}{\pi'(s)\left(1-\mu_1\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right)\right)} = \frac{1}{z_I - z_C}\frac{1+z_I}{1+z_C}\frac{\mu_0+\mu_2\pi(s)}{1-\mu_1\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right)}\left((1+z_C)\Gamma(s) + (1+z_I)\right)^2.$

Let us now turn to the utilization rate. Given $\bar{u}_3^*(s) = \frac{\mu_0+\mu_2\pi(s)}{1-\mu_1\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right)}\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right) =$
$= \frac{\mu_0+\mu_2\frac{z_C\Gamma(s)+z_I}{(1+z_C)\Gamma(s)+(1+z_I)}}{1-\mu_1\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right)}\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right)$, let $D_1 = (1+z_C)\Gamma(s)+(1+z_I)$. Then, $d\bar{u}_3^*/ds =$
$\frac{\mu_2\Gamma'(s)(z_C-z_I)}{D_0 D_1^2}\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right)+$
$+\frac{1+z_I}{1+z_C}\Gamma'(s)\frac{\mu_0[(1+z_C)\Gamma(s)+(1+z_I)]+\mu_2(z_C\Gamma(s)+z_I)}{D_0^2 D_1} =$
$= -\frac{\Gamma'(s)}{D_0 D_1}\left\{\frac{\mu_2(z_I-z_C)}{D_1}\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right)+\right.$
$\left.-\frac{1+z_I}{1+z_C}\left[\mu_0\left[(1+z_C)\Gamma(s)+(1+z_I)\right]+\mu_2(z_C\Gamma(s)+z_I)\right]/D_0\right\}$. Hence, $d\bar{u}_3^*/ds < 0 \Leftrightarrow \mu_2(z_I-$
$z_C)\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right)D_0 <$
$\frac{1+z_I}{1+z_C}\left[\left[(1+z_C)\Gamma(s)+(1+z_I)\right]\left[(1+\mu_0)+\mu_2(z_C\Gamma(s)+z_I)\right]\right]$. Since $D_0 < 1$ and $\mu_2 z_I\left(\frac{1+z_I}{1+z_C}\Gamma(s)+1\right) <$
$\mu_2(z_C\Gamma(s)+z_I)\frac{1+z_I}{1+z_C}\left[(1+z_C)\Gamma(s)+(1+z_I)\right]$,then $d\bar{u}_3^*/ds < 0$ always.

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