Munich Personal RePEc Archive

The Gender Pay Gap: Micro Sources and Macro Consequences

Morchio, Iacopo and Moser, Christian

University of Vienna, Columbia University

11 May 2018

Online at https://mpra.ub.uni-muenchen.de/99276/
MPRA Paper No. 99276, posted 27 Mar 2020 07:56 UTC

The Gender Pay Gap: Micro Sources and Macro Consequences*

Iacopo Morchio[†] Christian Moser[‡]

March 24, 2020

Abstract

We assess the sources and consequences of the gender pay gap using a combination of theory and measurement. We start by documenting three empirical facts. First, women are more likely than men to work at low-paying employers. Second, for women as for men, pay is not the sole determinant of workers' revealed-preference rankings of employers. Third, both pay and the revealed-preference rank differ between women and men within the same employer. To interpret these facts, we develop an empirical equilibrium search model featuring endogenous gender differences in pay, amenities, and recruiting intensities across employers. The estimated model suggests that compensating differentials explain one fifth of the gender gap, that there are significant output and welfare gains from eliminating gender differences, and that an equal-pay policy fails to close the gender pay gap.

Keywords: Worker and Firm Heterogeneity, Misallocation, Compensating Differentials, Discrimination, Empirical Equilibrium Search Model, Linked Employer-Employee Data

JEL classification: E24, E25, J16, J31

*We thank Daron Acemoglu, Jim Albrecht, Jorge Alvarez, Roc Armenter, Jesper Bagger, Erling Barth, Martin Beraja, Mary Ann Bronson, Sydnee Caldwell, Rebecca Diamond, Rafael Dix-Carneiro, David Dorn, Niklas Engbom, Mikhail Golosov, Émilien Gouin-Bonenfant, Kyle Herkenhoff, Jonas Hjort, Philipp Kircher, Francis Kramarz, Kory Kroft, David Lagakos, Tzuo Hann Law, Rasmus Lentz, Ben Lester, Ilse Lindenlaub, Paolo Martellini, Guido Menzio, Magne Mogstad, Ben Moll, Giuseppe Moscarini, Andreas Mueller, Makoto Nakajima, Tommaso Porzio, Pascual Restrepo, Isaac Sorkin, Kjetil Storesletten, Coen Teulings, Gabriel Ulyssea, Gianluca Violante, and Susan Vroman for valuable comments and discussions. We also thank attendants at the 2018 GEA Conference, 2019 AEA Annual Meeting, 2019 Search and Matching Network Conference, the 2019 Bank of Italy/CEPR/IZA Conference, the 2019 SED Annual Meeting, the 2019 NBER Summer Institute, the 2019 Stanford SITE Workshop, the 3rd Dale T. Mortensen Centre Conference, the Boston Macro Juniors Conference, the 2020 Columbia Junior Micro Macro Labor Conference, and seminar participants at Columbia University, Boston University, CREST, Iowa State University, the University of Oslo, the University of Bristol, the University of Edinburgh, the University of Vienna, the Johannes Kepler University Linz, the Federal Reserve Bank of Philadelphia, and Temple University for helpful comments. Ian Ho and Rachel Williams provided outstanding research assistance. Moser gratefully acknowledges financial support from the Ewing Marion Kauffman Foundation and the Sanford C. Bernstein & Co. Center for Leadership and Ethics at Columbia Business School. Any errors are our own.

†University of Vienna. Email: iacopo.morchio@univie.ac.at.

‡Columbia University. Email: c.moser@columbia.edu.

1 Introduction

During the past decades, the introduction of gender in economic theory and empirics has had a profound impact on studies of labor markets and the macroeconomy. A common thread in these studies is the robust empirical finding of a gender pay gap that is partly explained by gender imbalances in employment across different types of jobs. The goal of this paper is to identify the microeconomic sources of the gender pay gap and to assess its macroeconomic consequences.

We focus on two competing explanations for why women work in relatively lower-paid jobs. The first explanation pertains to compensating differentials: women may self-select into jobs with low pay but attractive amenities like work-schedule flexibility and paid parental leave. The second explanation pertains to gender-specific barriers to employment in desirable jobs: women may experience more family-related labor market interruptions and certain employers may discriminate against them. The implications of the gender pay gap for output and welfare crucially depend on the relative importance of these two explanations.

In assessing the micro sources and macro consequences of the gender pay gap, we make three contributions. First, we use rich linked employer-employee data to establish novel facts on gender segregation, gender-specific pay heterogeneity, and revealed-preference ranks across employers. Second, we develop and estimate a new empirical equilibrium search model featuring endogenous gender differences in pay, amenities, and recruiting intensities across employers. Third, we use the estimated model to decompose the empirical gender pay gap, to quantify the output and welfare gains from moving to an economy with no gender differences, and to evaluate the effects of a hypothetical equal-pay policy. In doing so, we provide the first estimates of output and welfare losses from firm-level gender misallocation.

To shed light on the gender dimension of employer heterogeneity, we analyze linked employer-employee records from Brazil between 2007 and 2014. The presence of a large gender earnings gap of around 14 log points makes it interesting in its own right to study the sources of gender inequality in a nation of over 200 million people. Such a study is feasible because Brazil's remarkable data infrastructure contains detailed information on gendered labor market outcomes, including workers' educational attainment, occupation, contractual work hours, and employment histories with information on parental leaves.

We document that there is significant gender segregation across employers, with large differences in female employment shares across firms, even within sectors. To understand the link between employer segregation and the gender pay gap, we estimate an empirical specification with gender-specific employer pay components developed by Card et al. (2016), building on the seminal two-way fixed effects (FEs) framework by Abowd, Kramarz, and Margolis (1999, henceforth AKM). Controlling for unobserved worker heterogeneity, we find a gender pay gap of around 8 log points that is accounted for by gender-specific employer pay heterogeneity, with women sorting to lower-paying employers relative to men.

The extent to which empirical sorting patterns reflect compensating differentials can be inferred from revealed-preference employer rankings, which we construct separately by gender using the PageRank index (Page et al., 1998; Sorkin, 2018). The PageRank is a network centrality measure that quantifies the attractiveness of employers based on the nature of worker flows between employers. Intuitively, higher-ranked employers poach many workers from other high-ranked employers and lose few workers to low-ranked employers. We use these estimates to establish three novel facts. First, the distribution of employment across employer ranks is similar for both genders. Second, the correlation between pay and employer ranks is positive, but more so for men than for women. Third, both within and across genders, there is significant heterogeneity in employer ranks conditional on pay.

To interpret these facts, we develop an empirical equilibrium search model featuring endogenous gender differences in pay, amenities, and recruiting intensities across employers. The model remains analytically tractable while accommodating several competing explanations for the gender gap, including employer productivity differences (Burdett and Mortensen, 1998), gender-specific compensating differentials (Rosen, 1986), statistical discrimination (Arrow, 1971), and taste-based discrimination (Becker, 1971). The model gives rise to gender-specific job ladders with several notable equilibrium properties. The equilibrium wage equation is log-additively separable in a worker component and a gender-specific employer component, providing a microfoundation for the specification used in Card et al. (2016) and our own empirical investigation. Endogenous worker transitions may be associated with wage declines. Discriminatory employers may survive in equilibrium. Equilibrium spillovers of discrimination imply that even employers without regard for gender may end up offering different pay to men compared to women.

We identify four sets of gender-specific model parameters employer-by-employer using information on worker flows and employer pay across genders. We estimate gender-specific amenity values as the residual between employers' relative pay and revealed-preference ranks in a set of bilateral comparisons. We use empirical worker flows by gender to obtain estimates of labor market parameters, which map into heterogeneous degrees of statistical discrimination across employers. We compare the conditional equilibrium pay gap between coworkers of different genders within employers to identify parameters guiding a firm's payoff from employing workers of each gender. Finally, we nonparametrically estimate employers' gender-specific hiring costs from their empirical recruiting intensity among nonemployed workers.

The estimation results shed light on the microstructure of labor markets for men and women. Employer pay, ranks, and amenities are positively correlated within employers across genders. However, employer ranks depend relatively more on pay for men but on amenities for women. We find evidence of compensating differentials for both genders. Employers' preference for men over women increases with employer productivity, consistent with Becker (1971)'s idea that discrimination cannot survive among low-productivity firms with close-to-zero economic profits.

We link our structural estimates to relevant employer characteristics in the data. Women put relatively higher value on hours flexibility and parental leave benefits, while men are relatively less averse to pay fluctuations and health risks. The empirical proxies for more woman-friendly employers include higher routine-manual and nonroutine-cognitive-interpersonal task intensities at work, higher female participation in general employment and top-paid positions, and having greater accountability to major financial stakeholders. These estimates speak to different reasons why some employers are not gender blind, including taste-based discrimination (Becker, 1971) and gender-specific comparative advantages related to "brain versus brawn" (Goldin, 1992).

With the estimated equilibrium model in hand, we simulate a number of counterfactuals that shed light on the sources and consequences of the gender pay gap. We find that compensating differentials in the form of gender-specific amenities explain 1.3 log points (18 percent), employer tastes explain 5.4 log points (73 percent), and gender-specific hiring costs account for 5.6 log points (76 percent) of the gap. However, given the estimated structure of pay and nonpay characteristics across employers, closing the gender pay gap may or may not be welfare improving. We find that moving to an economy without gender differences is associated with output gains of 3.5 percent and welfare gains of 3.3 percent, reflecting the current misallocation of talent across genders (Hsieh et al., 2019). In contrast, a hypothetical equal-pay policy is mostly output- and welfare-neutral, though it has redistributive effects.

Related literature. Several macroeconomic studies have focused on the drivers of trends in female labor force participation, including structural change (Ngai and Petrongolo, 2017; Buera et al., 2019), culture (Fernández et al., 2004; Fernández and Fogli, 2009), technology (Greenwood et al., 2005; Albanesi and Olivetti, 2016), and information (Fogli and Veldkamp, 2011; Fernández, 2013). Related work has explored the implications of changes in female participation for economic growth (Heathcote et al., 2017; Hsieh et al., 2019), unemployment (Albanesi and Şahin, 2018), business cycles (Fukui et al., 2019; Albanesi, 2020), and declining dynamism (Peters and Walsh, 2019). Whereas previous work has focused on data at the aggregate, geographic, sectoral, or occupational level, our work highlights firm-level drivers of women's employment and pay.

The firm is also a natural unit of analysis for studying productivity and factor input distortions in relation to macroeconomic outcomes (Restuccia and Rogerson, 2008; Hsieh and Klenow, 2009; Lentz and Mortensen, 2008; Bagger et al., 2014). Little prior work has connected firms' gender composition to aggregate output and welfare. If gender-specific barriers impede women's relocation to higher-productivity firms, the gender pay gap may be associated with efficiency losses from misallocation of talent (Hsieh et al., 2019). By combining a rich equilibrium model with detailed microdata, we provide the first estimates of output losses from firm-level gender misallocation.

A burgeoning literature highlights firm heterogeneity in explaining empirical pay dispersion for otherwise identical workers based on AKM's seminal contribution (Card et al., 2013; Goldschmidt and Schmieder, 2017; Alvarez et al., 2018; Card et al., 2018; Gerard et al., 2018; Song et al., 2018). We build on Card et al. (2016), which estimates an empirical specification with gender-specific employer pay components on Portuguese data. Based on their specification, they decompose the gender gap in employer pay into sorting and rent sharing terms. The fundamental sources of the gender pay gap remain less well understood. By providing a microfoundation for their specification based on worker and firm optimization, our equilibrium model can rationalize gender-specific sorting and rent sharing patterns, and hence the empirical gender pay gap.

Our empirical equilibrium search model builds on the seminal framework by Burdett and Mortensen (1998). Bontemps et al. (1999) and Bontemps et al. (2000) estimate variants of this framework with heterogeneity in firm productivity. Other important extensions and empirical applications include Postel-Vinay and Robin (2002), Cahuc et al. (2006), Moscarini and Postel-Vinay (2013), Meghir et al. (2015), Engbom and Moser (2018), Heise and Porzio (2019), and Bagger and Lentz (2018). In all these models, firms are ex-ante heterogeneous in only one dimension, namely productivity. As a consequence, all workers agree on a common ranking of firms based purely on pay considerations. To rationalize our empirical facts on gender-specific employer pay and ranks, we develop and estimate a tractable framework with multiple dimensions of firm heterogeneity: productivity, gender preferences, amenity costs, and hiring costs.

Other models have addressed gender issues in the labor market. For example, Black (1995), Bowlus (1997), Bowlus and Eckstein (2002), Albanesi and Olivetti (2009), Flabbi (2010), Gayle and Golan (2011), and Amano-Patiño et al. (2019) study different forms of wage discrimination. It is well known that discrimination is hard to empirically distinguish from unobserved productivity differences or compensating differentials. In our framework, linked employer-employee data with information on worker transitions and coworker wages is necessary to separately identify employer preferences over gender from other dimensions of job heterogeneity.

Gender-specific compensating differentials à la Rosen (1986) have been the empirical subject of Goldin and Katz (2011, 2016). Goldin (2014) and Erosa et al. (2019) highlight flexibility as a job characteristic with gender-specific value, which has been underlined by recent experimental evidence (Mas and Pallais, 2017, 2019; Wiswall and Zafar, 2017). Structural models of compensating differentials have been developed and tested using survey data by Hwang et al. (1998), Lang and Majumdar (2004), Dey and Flinn (2005), Bonhomme and Jolivet (2009), and Hall and Mueller (2018). Like Taber and Vejlin (2016) and Xiao (2020), we exploit linked employer-employee data. Unlike them, we identify gender-specific parameters employer-by-employer without relying on distributional assumptions or indirect inference. Sorkin (2017), Lavetti and Schmutte (2017), and Lamadon et al. (2019) also estimate firm-level amenity values from linked employer-employee data. Our focus, in contrast to theirs, is on gender. Sorkin (2018) combines PageRanks with a partial-equilibrium framework to study gender differences in pay and amenities, concluding that gender differences in the exogenous offer distribution explain a significant share of the gender pay gap. Our equilibrium framework provides a theory of endogenous gender differences in offer distributions, which is particularly useful for our study the effects of an equal-pay policy.

Outline. The rest of the paper is structured as follows. Section 2 introduces the data. Section 3 presents empirical facts. Section 4 develops the empirical equilibrium search model. Section 5 outlines the identification strategy. Section 6 presents estimation results. Section 7 conducts model-based counterfactuals. Finally, Section 8 concludes.

2 Data

2.1 Dataset and Variables Description

Dataset. Our main data source is the *Relação Anual de Informações Sociais (RAIS)* linked employer-employee register administered by the Brazilian Ministry of Labor and Employment. Survey response by all tax-registered firms is mandatory and misreporting is deterred through threat of audits and fines. The data are available from 1985 onward, with coverage becoming near universal in 1994. Since 2007, the data contain detailed information on reasons and lengths of worker absences, including parental leaves. In 2015, the country entered a severe recession associated with a large drop in aggregate economic activity. Therefore, we focus on the eight-year period from 2007 to 2014. This leaves us with a large dataset of over 538 million employment records.

Variables. The data contain unique identifiers for workers and establishments.[1] Although reports are annual, we observe for each job spell the precise start and end dates, mean monthly earnings (henceforth "earnings"), and contractual work hours (henceforth "hours"). This allows us to avoid aggregation bias in classifying job-to-job transitions (Moscarini and Postel-Vinay, 2018). Our baseline analysis uses earnings with flexible indicator controls for hours. However, for parts of our analysis we also construct hourly wages (henceforth "wages") as earnings divided by the number of hours. Other key variables include gender in two categories, race in five categories, nationality in 37 categories, educational attainment in nine categories, worker age in years, 5-digit sector codes with 672 categories, municipality codes with 5,565 categories, 6-digit occupation codes with 2,383 categories, and tenure in years. In addition, the data contain start and end dates of any absence from work, and information on the reason for absence. We exploit the full panel dimensions of the data going back to 1985 together with the tenure variable to impute actual (not just potential) formal-sector work experience in years.[2]

2.2 Sample Selection

We first restrict attention to male and female workers between the ages of 18 and 54 who worked at least one hour per week with earnings at or above the federal minimum wage. We then keep for each worker-year combination the highest-paid among all longest employment spells. Next, we

[1] All of our analysis is at the level of the establishment, which we intercheangably refer to as "employer" or "firm."

[2] The distinction between actual and potential experience is important in general but explains little of the empirical gender pay gap—see Figure 11 in Appendix A.1.

iteratively drop singleton observations defined either by the combination of establishment identifier and gender, or by worker identifier. We also impose a minimum establishment size threshold of 10 nonsingleton workers per year on average.[3] Finally, we require that establishments appear in our sample at least four out of the eight years. Together, the last two selection criteria ensure that we are dealing with a set of reasonably large and stable establishments for which pay policies and amenity values can be credibly estimated.

To separately identify worker and employer pay components, we follow Abowd et al. (2002) in constructing the largest connected set, where connections are formed through worker mobility between establishments over time. In the language of graph theory, there are two types of connected sets. A *weakly connected set* is one in which each establishment is connected to another establishment through at least one incoming or outgoing worker. A *strongly connected set* is one in which each establishment is connected to another node through at least one incoming and one outgoing worker. For the AKM model to be identified, it is sufficient to restrict attention to weakly connected sets. However, to estimate revealed-preference ranks of employers using PageRanks requires restricting attention to strongly connected sets (Sorkin, 2018). Therefore, we restrict attention to the largest strongly connected set (henceforth "connected set").

2.3 Summary Statistics

Table 11 in Appendix A.2 presents summary statistics on observations in the connected set in 2007, in 2014, and pooled across years 2007–2014.[4] In the pooled sample, we have over 231 million worker-years, corresponding to over 55 million unique workers and over 222 thousand unique establishments. Around 38 percent of these observations are for women. The raw gender gap in earnings is around 14 log points and the gap in wages is around 6 log points. Compared to women, men are more likely to be nonwhite and hold at most a middle-school degree.[5] Men are significantly younger, work at smaller and younger establishments, work more hours, and have lower tenure compared to women.[6]

[3]Following Sorkin (2018), nonsingleton workers are those who are observed at least one more time at a future date. While the RAIS data cover only Brazil's formal sector, the employer size restriction implies that the vast majority of informal establishments would be excluded from our analysis in any case (Ulyssea, 2018; Dix-Carneiro et al., 2019).

[4]Also in Appendix A.2, we show the same summary statistics on the data before making sample selections and restricting the data to the connected set (Table 12) and comparing the two samples (Table 13).

[5]Since race information is missing for a significant share of observations, we report here the conditional mean.

[6]In Brazil, full-time employment involves either 40 or 44 hours of work per week, depending on the employer.

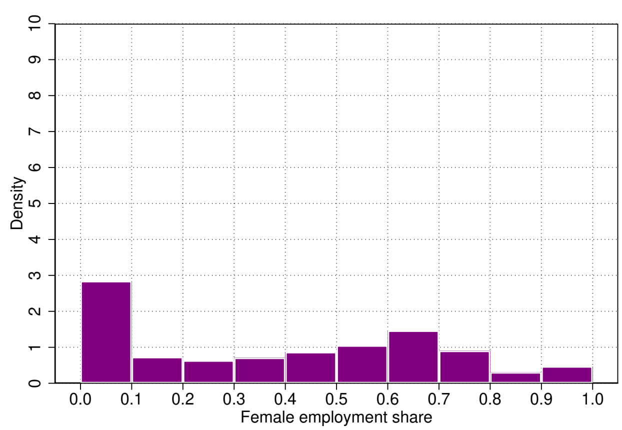
3 Employer Heterogeneity and the Gender Gap

A classical Mincerian analysis of the gender gap in pay is presented in Appendix B.1. Standard Mincerian controls only partly explain the empirical gender gap. Thus, building on AKM's seminal contribution, we investigate the role of employer heterogeneity in relation to the gender gap.

3.1 Gender Segregation Across Employers

Women make up 38 percent of Brazil's formal sector employment over the period 2007–2014. However, women are highly segregated across employers, even within industries. Figure 1 shows a histogram of female employment shares in 2014. Around 28 percent of establishments have less than 10% women among their workforce. In contrast, if women were equally distributed across employers, we would see a single bar of height 10 in the category 30–40%.

Figure 1. Histogram of female employment shares, 2014



Source: Authors' calculations based on RAIS.

We show in Appendix B.2 that gender segregation is a robust empirical phenomenon over time (Figure 12), within sectors (Figure 13), and across different employer sizes (Figure 14). Figure 15 in Appendix B.3 shows that employment levels of men and women within establishments are positively but imperfectly correlated. In Appendix B.4, we quantify the degree of gender segregation across employers by defining and estimating an employer segregation index, which we find to be higher than analogous indices estimated across industries, occupations, or states.

3.2 Quantifying Employer Pay Heterogeneity

To understand the link between employer segregation and the gender pay gap, we estimate a wage equation with gender-specific employer pay components developed by Card et al. (2016), building on the seminal two-way FEs specification by AKM. This allows for the possibility that a given employer has two pay policies—one for each gender. Formally, we model earnings of individual i in year t working at establishment $j = J(i,t)$, denoted y_{ijt}, as

$$y_{ijt} = X_{it}\beta + \alpha_i + \mathbf{1}\left[gender_i = M\right]\psi_j^M + \mathbf{1}\left[gender_i = F\right]\psi_j^F + \varepsilon_{ijt}, \tag{1}$$

where X_{it} is a vector of gender-specific worker characteristics including a set of restricted education-age dummies as well as dummies for hours, occupation, tenure, actual experience, and education-year combinations, α_i is a person FE, ψ_j^M and ψ_j^F are the male and female employer FEs, respectively, and ε_{ijt} is a residual term.[7] By including a set of person FEs, this specification controls for selection of men and women across establishments based on unobserved time-invariant worker characteristics such as ability. In estimating equation (1), our main focus is on estimates of the gender-specific employer FEs, ψ_j^M and ψ_j^F.

As is the case in all two-way fixed models, at least one normalization must be made regarding the intercept or mean of the employer FEs versus the person FEs. In our case, the model with gender-specific employer FEs requires two normalizations—one for each gender. Consistent with the theoretical model presented later, we follow Card et al. (2016) and Gerard et al. (2018) in normalizing the employer FEs of both genders to be of mean zero in the restaurant and fast-food sector, which, arguably, is populated by low-surplus employers.[8]

We now turn to our main object of interest in equation (1), namely the gender-specific employer FEs.[9] Panel (a) of Figure 2 plots the distribution of employer FEs by gender. The distribution for women has visibly lower mean and lower variance than that for men. Panel (b) of the figure shows the distribution of within-employer differences in FEs for dual-gender establishments. The distribution is relatively dispersed compared to its mean of around 2 log points.

[7]To identify age, time, and worker FEs simultaneously, we restrict the age-pay profile to be flat around ages 45–49, which is approximately consistent with empirical raw-earnings profiles. See Appendix B.5 for details.

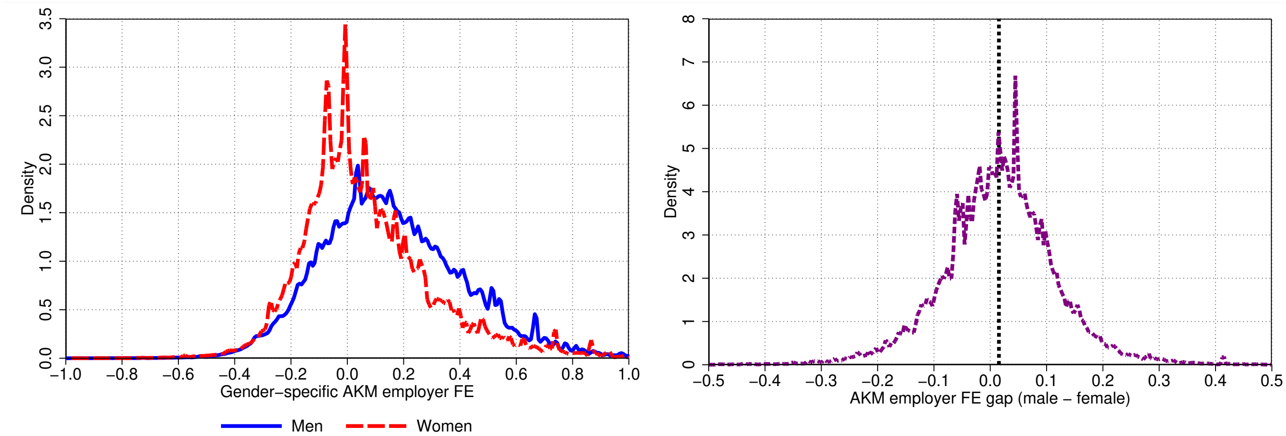
[8]For robustness, we experimented with alternative normalizations for gender-specific employer FEs. Separately, we have repeated our analysis based on a wage equation without worker FEs, making the normalization redundant.

[9]In Appendix B.5, we present auxiliary results relating to the AKM equation, including estimated gender-specific hours FEs (Figure 16), occupation FEs (Figure 17), actual-experience FEs (Figure 18), tenure FEs (Figure 19), education-year FEs (Figure 20), and education-age FEs (Figure 21).

Figure 2. Predicted AKM employer FEs for women and men

(a) Gender-specific employer FE distributions

(b) Gender-specific employer FE distributions



Source: Authors' calculations based on RAIS. *Note:* Dashed vertical line shows mean of the distribution.

Table 1 shows a log-earnings variance decomposition.[10] Men have a slightly higher variance of log earnings, with 52.4 log points compared to 51.1 log points. For both genders, the largest variance component is due to estimated worker FEs, accounting for 24 percent for men and 25 percent for women. Employer FEs account for 12 percent of the variance of earnings for men and 11 percent for women. The positive covariance terms are primarily attributed to the covariance between worker and employer FEs, education-age and employer FEs, and actual experience and employer FEs. The correlation between person and employer FEs is around 23 percent for men and 27 percent for women. For each gender, the largest connected set spans close to the full data. Finally, around 93 percent of the variation in log earnings is explained by the model.

[10]To be precise, Table 1 presents plug-in estimators of the variance components. In ongoing work, we are adapting the leave-one-out estimator by Kline et al. (2019), which implements a jackknife bias correction for limited-mobility bias (Andrews et al., 2008, 2012), to a dataset of significantly larger size like ours.

Table 1. Variance decomposition based on gender-specific employer FEs model

Variances	Men Level	Men Share	Women Level	Women Share
Variance of log earnings	0.524		0.511	
Components of variance of log earnings:				
Person FEs	0.125	0.238	0.129	0.252
Employer FEs	0.064	0.122	0.056	0.109
Education-year FEs	0.002	0.004	0.001	0.001
Education-age FEs	0.061	0.116	0.055	0.107
Hours FEs	0.000	0.001	0.001	0.002
Occupation FEs	0.013	0.025	0.012	0.023
Tenure FEs	0.003	0.006	0.005	0.010
Actual experience FEs	0.034	0.065	0.023	0.045
Covariances	0.185	0.352	0.198	0.387
Residual	0.037	0.071	0.033	0.064
Correlation person/employer FEs	0.226		0.270	
Observations	143,745,869		88,059,962	
Largest connected set	1.000		0.999	
R^2	0.929		0.936	

Source: Authors' calculations based on RAIS. *Note:* Variance components based on earnings equation (1).

3.3 Between vs. Within-Employer Pay Differences

Our focus from here on will be on differences in the gender-specific employer components (henceforth "gender gap"). Using a Oaxaca-Blinder decomposition, we can write the gender gap as

$$\begin{aligned}
\gamma^e &\equiv \mathbb{E}_{i,t}\left[\psi^M_{J(i,t)}\middle| gender_i = M\right] - \mathbb{E}_{i,t}\left[\psi^F_{J(i,t)}\middle| gender_i = F\right] \\
&= \underbrace{\mathbb{E}_{i,t}\left[\psi^M_{J(i,t)} - \psi^F_{J(i,t)}\middle| gender_i = M\right]}_{\text{within-employer gender pay gap}} + \underbrace{\left(\mathbb{E}_{i,t}\left[\psi^F_{J(i,t)}\middle| gender_i = M\right] - \mathbb{E}_{i,t}\left[\psi^F_{J(i,t)}\middle| gender_i = F\right]\right)}_{\text{between-employer gender pay gap}} \quad (2) \\
&= \underbrace{\mathbb{E}_{i,t}\left[\psi^M_{J(i,t)} - \psi^F_{J(i,t)}\middle| gender_i = F\right]}_{\text{within-employer gender pay gap}} + \underbrace{\left(\mathbb{E}_{i,t}\left[\psi^M_{J(i,t)}\middle| gender_i = M\right] - \mathbb{E}_{i,t}\left[\psi^M_{J(i,t)}\middle| gender_i = F\right]\right)}_{\text{between-employer gender pay gap}}. \quad (3)
\end{aligned}$$

Equations (2) and (3) are two alternative decompositions of the total gender gap, γ^e, into two terms. The *within-employer pay gap* or *pay-policy component* is the mean difference in gender-specific employer FEs weighted by the distribution of men and women, respectively. It reflects differences in pay between women and men at the same establishment. The *between-employer pay gap* or *sorting component* is the difference between genders in mean male-employer FEs and female-employer FEs, respectively. It reflects differences in pay between men and women due to their different allocations across establishments.[11]

Figure 22 in Appendix B.6 graphically illustrates estimates of the two components of the decompositions in equations (2) and (3). Results of the decomposition are shown in Table 2. Out of the total gender pay gap of 8.4 log points, 24 (5) percent are attributed to the pay-policy component in Decomposition 1 (2). The remainder is attributed to the sorting component. This evidence suggests that women systematically work at lower-paying employers compared to men.

Table 2. Oaxca-Blinder decompositions of the gender pay gap due to employer heterogeneity

		Pay-policy component		Sorting component	
	Gender pay gap	Level	Share	Level	Share
Decomposition 1	0.084	0.020	0.241	0.064	0.759
Decomposition 2	0.084	0.004	0.047	0.080	0.953

Source: Authors' calculations based on RAIS. *Note:* Decompositions 1 and 2 correspond to equations (2) and (3), respectively.

3.4 Life-Cycle Patterns and Event Study Analysis around Parental Leaves

An obvious candidate factor that may be behind some of the hitherto documented patterns is related to childbirth. In Appendix B.7, we study life-cycle patterns of employer pay by gender and parental status. In Appendix 24, we conduct an event study analysis around childbirth (as proxied by parental leave) following Kleven et al. (2016). While we find significant gender gaps in participation and earnings associated with childbirth, our analysis suggests that firm pay heterogeneity is *not* the only, or even a very important, factor behind these gaps.

3.5 Revealed-Preference Employer Rankings

To what extent does the gender pay gap reflect a gender utility gap? To answer this question, one must take into account both pay and nonpay characteristics of jobs for both genders. To this end, we estimate gender-specific revealed-preference rankings of employers using the PageRank index. The PageRank is a network centrality measure developed by Page et al. (1998) to rank websites for the web search engine Google and first used in an economic context by Sorkin (2018).

In a labor market context, the PageRank is defined as follows. Let $g \in \{M, F\}$ index a worker's gender, let $j \in \mathcal{J}^g = \{j_1, j_2, \ldots, j_{N^g}\}$ index a set of N^g gender-specific employers, and let $t \in \mathcal{T}$

[11] Note that the sorting component is invariant to the choice of the normalization of gender-specific employer FEs. Coincidentally, this will be the main object of interest in our study. The pay-policy component, on the other hand, depends on the normalization of men's relative to women's employer FEs.

index time. We denote by $n^g_{j,j',t}$ the number of workers of gender g transitioning from employer j to employer j' at time t, by $n^g_{j,j'} = \sum_{t\in\mathcal{T}} n^g_{j,j',t}$ the time aggregation of gender-specific flows between the two employers, and by $n^g_{j,\cdot} = \sum_{j'} n^g_{j,j'}$ the number of workers of gender g flowing out from employer j. Let $B^g(j) = \left\{ j' : n^g_{j',j} \geq 1 \right\}$ denote the set of employers who have ever lost a worker of gender g to employer j. Let $d \in [0,1]$ be a damping factor. The *PageRank index*, $s^g(j)$, is a probability distribution over all employers $j \in \mathcal{J}^g$ such that

$$s^g(j) = \frac{1-d}{N^g} + d \sum_{j' \in B^g(j)} w^g_{j',j} s^g(j'), \quad \forall j \in \mathcal{J}^g, \forall g, \tag{4}$$

where $w^g_{j',j} = n^g_{j',j} / n^g_{j',\cdot}$ is a weight equal to the share of worker flows from employer j' to employer j as a fraction of all worker flows from employer j'. Intuitively, employers with a high PageRank index poach many workers from other employers with high PageRank indices and lose few workers to other employers with low PageRank indices. The damping factor d represents the weight on the poaching term in a convex combination with equal employer weights. Based on PageRank indices, we compute gender-specific *PageRanks* $r^g(j)$ for every employer $j \in \mathcal{J}^g$ as the rank of the PageRank indices, with the lowest rank normalized to 0 and the highest rank normalized to 100.

Interestingly, the PageRank index represents the asymptotic share of time a representative worker ("random surfer") who switches jobs by following the network of empirical worker flows would spend at a given employer. Following Sorkin (2018), we choose as damping factor $d = 1$ in all our applications. By estimating PageRank indices on the strongly connected set, we avoid absorbing states ("rank sinks"), in which a worker could get indefinitely stuck at an employer. This interpretation of the PageRank index is particularly close to the definition of an employer rank in a large class of on-the-job search models, including the one we develop. Note also that an employer's PageRank does not directly depend on its pay or size. Indeed, in computing PageRanks, we did not use any information on worker wages or the number of workers at any employer.

Based on equation (4), we compute employer PageRanks separately by gender.[12] We now establish three facts relating to employer heterogeneity in pay and ranks within and across gender.[13]

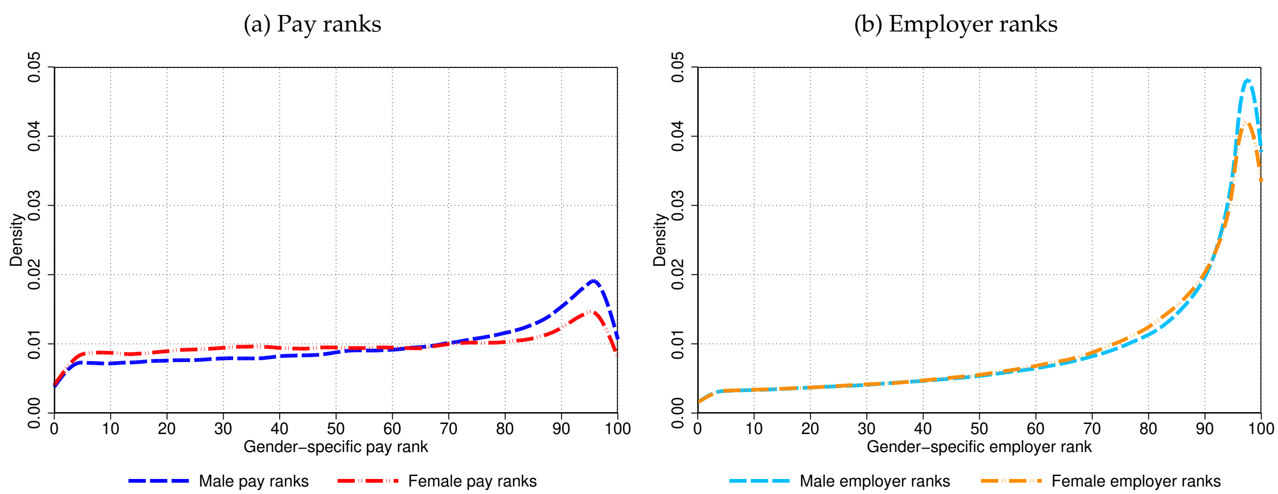
[12] In Appendix B.9, we show that the resulting PageRanks are strongly but imperfectly correlated with gender-specific employer FEs in pay. PageRank estimates are also strongly but imperfectly correlated with two other popular employer rank measures, namely the poaching rank (Moscarini and Postel-Vinay, 2008; Bagger and Lentz, 2018) and the net poaching rank (Haltiwanger et al., 2018; Moscarini and Postel-Vinay, 2018). An advantage of the PageRank over the other two rank measures is that it uses more information per worker transition in constructing an employer ranking, which reduces spurrious misclassifications of employer ranks.

[13] For the remainder of this section, we will study gender-population-weighted estimates of the unweighted PageRanks as described above. Note that PageRanks are not restricted to have any particular mean value (e.g., 50) by construction

Fact 1. *While the gender gap in pay ranks is 4.4 percentiles, that in employer ranks is 0.7 percentiles.*

Figure 3 compares the employment distributions of men and women across pay ranks and across employer ranks. Panel (a) shows employment is weakly positively related to pay for both genders. Panel (b), on the other hand, shows that employment is strongly related to employer ranks for both genders. Furthermore, the rank-based employment distribution of women looks relatively more similar to that of men than it does for the pay-based employment distribution. Women's mean employer pay rank is 53.9 while men's is 58.3, implying a gender gap in pay ranks of 4.4 percentiles. On the other hand, women's mean employer rank is 73.7 while men's is 74.4, implying a gender gap in employer ranks of 4.4 percentiles.[14]

Figure 3. Densities over pay ranks and employer ranks, by gender



Source: Authors' calculations based on RAIS.

Fact 2. *Mean employer ranks are steeper increasing in pay ranks for men than for women.*

Figure 4 suggests that employer ranks are positively related to pay ranks for both men and women. However, the gradient is steeper for men than for women, especially in the bottom half of employer pay ranks. This means that there exist low-paying jobs that are at the same time relative attractive for women, and that this is less so the case for men. Therefore, for men compared to women, pay is relatively more important in their overall evaluation of an employer's rank.[15]

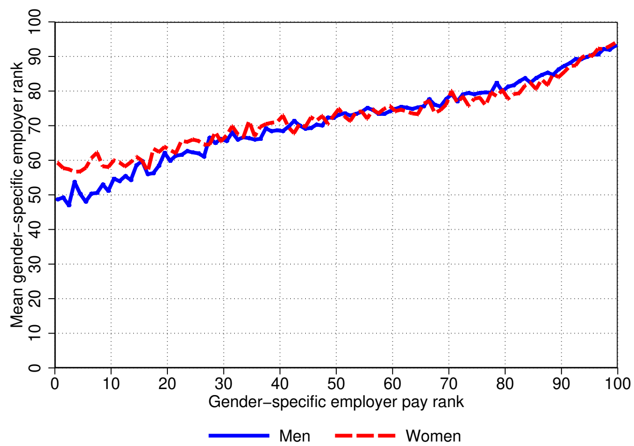
Fact 3. *There is significant heterogeneity in employer ranks conditional on pay within and across genders.*

since the PageRank estimation is independent of the cross-sectional employment distribution.

[14]Note, however, that this does not rule out a utility gap between genders.

[15]Appendix B.10 presents several robustness checks. Figure 27 shows the relationship between employer ranks and pay ranks across sectors. Table 17 shows that this fact is not driven by sectoral or geographic differences. Table 18 shows that this fact is consistent with the dynamics of pay for different worker transitions across employer ranks.

Figure 4. Employer rank and pay, by gender

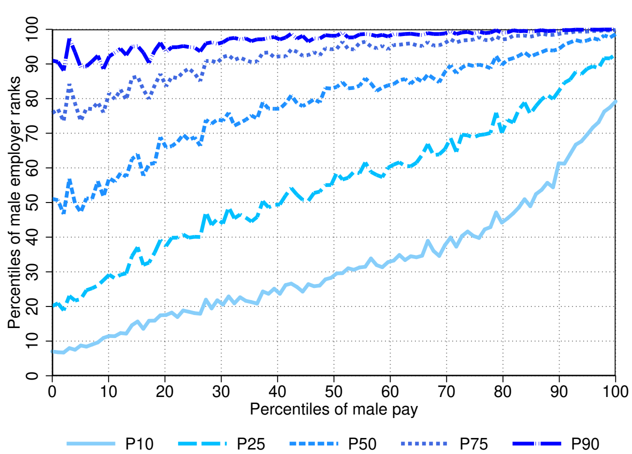


Source: Authors' calculations based on RAIS.

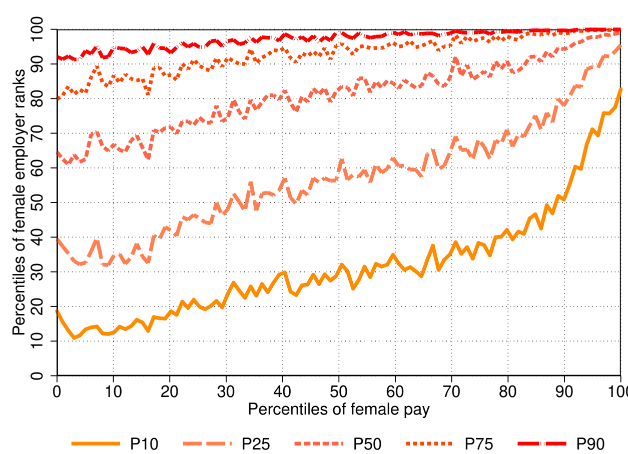
Figure 5 shows that there is significant dispersion in employer ranks conditional on pay for men in panel (a) and for women in panel (b). This suggests heterogeneity in nonpay characteristics of employers.[16] For both men and women, ranks are relatively more dispersed at low-pay employers than at high-pay employers. This suggests that establishments with high pay are also high in utility. This is consistent with either their pay being high enough to compensate for their level of (dis-)amenity, or alternatively their amenities being high on top of their high pay.[17]

Figure 5. Percentiles of employer rank distribution conditional on pay ranks, by gender

(a) Men



(b) Women



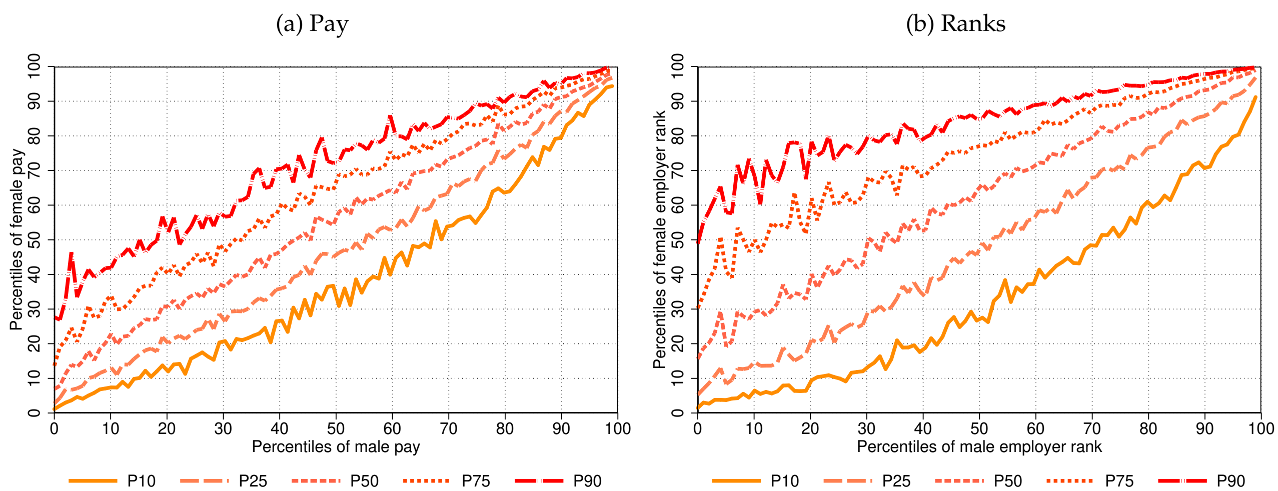
Source: Authors' calculations based on RAIS.

[16]Figure 30 in Appendix B.11 shows that the standard deviation of employer ranks is similarly decreasing in employer pay ranks for men and women.

[17]Appendix B.11 shows that the same qualitative conclusions apply when, for robustness, we compare employer ranks across pay ranks by industry for women (Figure 28) and for men (Figure 29).

Figure 6 shows that there is also significant within-employer between-gender dispersion in pay ranks in panel (a) and in employer ranks in panel (b). Pay and employer ranks are strongly positively correlated within employers across genders. This is consistent with the idea that an employer's productivity and amenities, such as its location and certain benefit policies, are partly shared by its male and female workers. Cross-gender employer ranks are also relatively more dispersed than cross-gender pay ranks. This may reflect that productivity (e.g., technology or management practices) is shared more freely across genders compared to valuations of certain amenities (e.g., hours flexibility or parental leave policies). Finally, men and women closely agree on their rankings of top employers, both in terms of pay ranks and employer ranks, but less so for lower-ranked employers.[18]

Figure 6. Female vs. male employer characteristics



Source: Authors' calculations based on RAIS.

4 Model

Addressing the above empirical facts requires a structural model with the following ingredients. First and foremost, the model must allow for an employer's revealed-preference rank to differ from its pay rank. To rationalize this, workers in the model value an employer's *amenities* in addition to pay. Second, the model must generate differences in pay and amenities across employers. To rationalize this, the labor market is modeled as frictional. Third, the model must admit gender differences in pay, revealed-preference ranks, and employment within the same employer. To rationalize this,

[18]For robustness, Appendix B.11 shows the same relation between female and male pay ranks by industry (Figure 31) and that between female and male employer ranks by industry (Figure 32). The same qualitative conclusions apply within each of the industries.

employers in the model post gender-specific wages, amenities, and job vacancies. We combine these ingredients in an equilibrium model of the labor market.

4.1 General Environment

A measure 1 of workers and measure E of firms meet in a continuous-time frictional labor market.

4.2 Workers

Workers are infinitely lived, risk neutral, and discount the future at rate ρ. They permanently differ in ability $a \in [\underline{a}, \overline{a}]$ and gender $g \in \{M, F\}$ with measure $\mu_{a,g}$ such that $\sum_{g=M,F} \int_a \mu_{a,g}\, da = 1$. At any point in time, they find themselves either employed or nonemployed.[19]

Job search. While nonemployed, workers receive flow utility $b_{a,g}$ and engage in random job search within segmented labor markets by worker type. Search is random in the sense that workers cannot direct their search to specific firms. Labor markets are segmented in the sense that workers search for jobs in a market specific to their type. While employed, workers receive flow utility $x = w + \pi$ equal to the sum of their wage, w, and job amenity value, π. Employed workers also engage in on-the-job search within the same segmented markets.

As a result of job search, workers receive regular job offers with arrival rate $\lambda^u_{a,g}$ from nonemployment and with rate $\lambda^e_{a,g}$ from employment. While regular on-the-job offers admit free disposal, workers also receive mandatory on-the-job offers (sometimes termed a "Godfather shock," or an offer one can't refuse) at rate $\lambda^G_{a,g}$ in both employment states. We think of the latter as capturing, among other things, spousal relocation problems and other idiosyncratic reasons for switching jobs. We will write $\lambda^e_{a,g} = s^e_{a,g}\lambda^u_{a,g}$ and $\lambda^G_{a,g} = s^G_{a,g}\lambda^u_{a,g}$, where $s^e_{a,g}$ and $s^G_{a,g}$ are the relative search intensities of regular and mandatory on-the-job search, respectively.

A job offer is an opportunity to work at some firm with associated wage w and amenity value π, drawn from a distribution $\tilde{F}(w, \pi)$, which workers take as exogenous but which is determined endogenously through firms' equilibrium decisions. Since a worker's flow utility $x = w + \pi$ is sufficient for summarizing their state, jobs will be ranked on a ladder according to x and we can restrict attention to the implied flow-utility offer distribution $F(x)$. A job can be terminated endogenously

[19] In mapping the model to the data, we think of the "nonemployed" in the model as capturing the pool of the unemployed, workers on temporary (parental or other) leave, workers marginally attached to the labor force, and workers in informal employment. The estimation of labor market parameters will take into account that some workers might spend longer periods outside of formal employment due to these factors.

when a worker with flow utility x in their current job accepts an offer from a higher-utility job at rate $\lambda^e_{a,g}(1 - F(x))$, or exogenously: at rate $\lambda^G_{a,g}$ the worker relocates to a randomly-drawn job, and at rate $\delta_{a,g}$ the worker becomes nonemployed.

Value functions. The value of an employed worker of type (a, g) in a job with flow utility x is summarized as follows:

$$\rho S_{a,g}(x) = x + \lambda^e_{a,g} \int\limits_{x' \geq x} \left[S_{a,g}(x') - S_{a,g}(x)\right] \, dF_{a,g}(x') + \lambda^G_{a,g} \int\limits_{x'} \left[S_{a,g}(x') - S_{a,g}(x)\right] \, dF_{a,g}(x') + \delta_{a,g}[W_{a,g} - S_{a,g}(x)] \tag{5}$$

Analogously, the value of a nonemployed worker of type (a, g) is summarized as follows:

$$\rho W_{a,g} = b_{a,g} + (\lambda^u_{a,g} + \lambda^G_{a,g}) \int\limits_{x'} \max\left\{S_{a,g}(x') - W_{a,g}, 0\right\} \, dF_{a,g}(x') \tag{6}$$

Policy function. Strict monotonicity of the value function $S_{a,g}(x)$ implies that the optimal job acceptance strategy of a nonemployed worker will be characterized by a threshold rule with reservation flow utility $\phi_{a,g}$. Thus, a nonemployed worker will accept an offer if $x \geq \phi_{a,g}$ and reject it otherwise. The reservation flow utility simply equals the sum of the flow value of nonemployment plus the forgone option value of receiving job offers while nonemployed:

$$\phi_{a,g} = b_{a,g} + (\lambda^u_{a,g} - \lambda^e_{a,g}) \int\limits_{x' \geq \phi_{a,g}} \frac{1 - F_{a,g}(x')}{\rho + \delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g}\left[1 - F_{a,g}(x')\right]} \, dx' \tag{7}$$

Employed workers in a job with flow utility x simply accept any job that delivers flow utility x' such that $x' > x$.

Nonemployment and utility dispersion. Since in equilibrium no firm will post a contract worth less than $\phi_{a,g}$ in any market (a, g), the steady-state nonemployment rate for each worker type is

$$u_{a,g} = \frac{\delta_{a,g}}{\delta_{a,g} + \lambda^u_{a,g} + \lambda^G_{a,g}}. \tag{8}$$

The cross-sectional distribution of flow utilities is given by

$$G_{a,g}(x) = \frac{F_{a,g}(x)}{1 + \kappa^e_{a,g}\left[1 - F_{a,g}(x)\right]},$$

where $\kappa^e_{a,g} = \lambda^e_{a,g}/(\delta_{a,g} + \lambda^G_{a,g})$ governs the effective speed of workers climbing the job ladder.

4.3 Firms

Firms differ in four dimensions. First, they have heterogeneous productivity $p \in [\underline{p}, \overline{p}] \subset \mathbb{R}_{++}$ as in Burdett and Mortensen (1998). Second, firms differ in a set of *employer wedges* $z_{a,g} \in [\underline{z}, \overline{z}] \subset \mathbb{R}$ representing the firm's disutility from worker type (a, g), as in Becker (1971). Third, firms are heterogeneous in a set of amenity cost shifters $c^{\pi,0}_{a,g} > 0$, as in Hwang et al. (1998). Finally, firms differ in a set of vacancy cost shifters $c^{v,0}_{a,g} > 0$. Thus, a firm's type is $j = (p, \{z_{a,g}\}_{a,g}, \{c^{v,0}_{a,g}\}_{a,g}, \{c^{\pi,0}_{a,g}\}_{a,g})$, which we assume is distributed continuously according to $\Gamma(j)$.

Wages, amenities, and job vacancies. Firms deliver value to workers through a combination of two channels. First, they post in each market a wage rate $w_{a,g}$ that is constant for the duration of the employment spell. Second, they also post a market-specific value of amenities $\pi_{a,g}$. Following Hwang et al. (1998), we assume that the cost of producing a level of amenities $\pi_{a,g}$ must be paid per worker of type (a, g) employed at the firm, and that the per-worker amenity flow cost can be written as $c^{\pi}_{a,g}(\pi_{a,g}) = c^{\pi,0}_{a,g} \times \tilde{c}^{\pi}_{a,g}(\pi_{a,g})$, where the function $\tilde{c}^{\pi}_{a,g}(\cdot)$ satisfies $\tilde{c}^{\pi}_{a,g}(0) = 0$, $\partial\tilde{c}^{\pi}_{a,g}/\partial\pi(0) = 0$, and $\partial\tilde{c}^{\pi}_{a,g}/\partial\pi(\pi), \partial^2\tilde{c}^{\pi}_{a,g}(\pi)/\partial\pi^2 > 0$ for all $\pi > 0$ and all (a, g). In order to recruit workers and produce output, firms also post $v_{a,g}$ job vacancies in each market subject to flow cost $c^v_{a,g}(v_{a,g}) = c^{v,0}_{a,g} \times \tilde{c}^v(\pi_{a,g})$, where the function $\tilde{c}^v(\cdot)$ satisfies $\tilde{c}^v(0) = 0$, $\partial\tilde{c}^v/\partial v(0) = 0$, and $\partial\tilde{c}^v/\partial v(v), \partial^2\tilde{c}^v(v)/\partial v^2 > 0$ for all $v > 0$.

Production. A firm with productivity p employing $\{l_{a,g}\}_{a,g}$ workers of each type produces output according to the following linear production technology:

$$y(p, \{l_{a,g}\}_{a,g}) = p \sum_{g=M,F} \int_a a l_{a,g}\, da$$

Employer wedges. In addition to output specified above, the model allows employers to care about employing different worker types. We model this as a set of employer wedges $\{z_{a,g}\}_{a,g}$, which may capture as two special cases taste-based discrimination as in Becker (1971) or firm-level comparative advantages in productivity across genders related to "brain versus brawn" (Goldin, 1992; Rendall, 2018). We restrict these wedges to take the form $z_{a,g} = \mathbf{1}[g = F] z_a$, where z_a guides an employer's relative preference for employing men over women among workers of ability a.

Value function. Firms post wages, amenities, and vacancies in each market to maximize steady-state flow payoff. The value $\Pi(j)$ of a firm of type $j = (p, \{z_a\}_a, \{c^{v,0}_{a,g}\}_{a,g}, \{c^{\pi,0}_{a,g}\}_{a,g})$ is given by

$$\rho\Pi(j) = \max_{\{w_{a,g},\pi_{a,g},v_{a,g}\}_{a,g}} \left\{ \sum_{g=M,F} \int_a \left[pa - w_{a,g} - c^{\pi}_{a,g}(\pi_{a,g}) - z_{a,g} \right] l_{a,g}(w_{a,g}, \pi_{a,g}, v_{a,g}) - c^{v}_{a,g}(v_{a,g})\, da \right\}. \tag{9}$$

4.4 Matching

The effective mass of job searchers in market (a, g) equals

$$U_{a,g} = \mu_{a,g} \left[u_{a,g} + s^{e}_{a,g}(1 - u_{a,g}) + s^{G}_{a,g} \right]. \tag{10}$$

The total mass of vacancies posted in market (a, g) across firm types j equals

$$V_{a,g} = E \int_j v_{a,g}(j)\, d\Gamma(j). \tag{11}$$

In the Diamond-Mortensen-Pissarides tradition, a Cobb-Douglas matching function with constant returns to scale combines the effective mass of job searchers with the total mass of job vacancies to produce a measure of matches between workers and firms, $m_{a,g}$, according to

$$m_{a,g} = \chi_{a,g} V^{\alpha}_{a,g} U^{1-\alpha}_{a,g},$$

where $\chi_{a,g} > 0$ is the matching efficiency and $\alpha \in (0, 1)$ is the matching elasticity with respect to aggregate vacancies. Define labor market tightness as

$$\theta_{a,g} = \frac{V_{a,g}}{U_{a,g}}, \quad \forall (a, g). \tag{12}$$

The job-finding rate among nonemployed workers, $\lambda^u_{a,g}$, the job-finding rate among the employed, $\lambda^e_{a,g}$, the arrival rate of mandatory offers, $\lambda^G_{a,g}$, and firms' job filling rate, $q_{a,g}$, are given by

$$\lambda^u_{a,g} = \chi_{a,g}\theta^{\alpha}_{a,g}, \qquad \lambda^e_{a,g} = s_{a,g}\lambda^u_{a,g}, \qquad \lambda^G_{a,g} = s^G_{a,g}\lambda^u_{a,g}, \text{ and} \qquad q_{a,g} = \chi_{a,g}\theta^{\alpha-1}_{a,g}. \tag{13}$$

4.5 Firm Size Distribution

The following Kolmogorov forward (or Fokker-Planck) equation describes the law of motion of firm sizes given a firm's flow-utility and vacancy policy (x, v), the market distribution of flow utilities $F_{a,g}(x)$, and market tightness $\theta_{a,g}$:

$$\dot{l}_{a,g}(x,v) = \left[-\delta_{a,g} - \lambda^e_{a,g}\left[1 - F_{a,g}(x)\right] - \lambda^G_{a,g}\right] l_{a,g}(x,v) + \left[\frac{u_{a,g} + (1-u_{a,g})s^e_{a,g}G_{a,g}(x) + s^G_{a,g}}{u_{a,g} + (1-u_{a,g})s^e_{a,g} + s^G_{a,g}}\right] v q_{a,g}.$$

Solving for the stationary firm size distribution, we find

$$l_{a,g}(x,v) = \left(\frac{1}{\delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g}\left[1 - F_{a,g}(x)\right]}\right)^2 \frac{v}{V_{a,g}} \mu_{a,g}(u_{a,g} + s^G_{a,g})\lambda^u_{a,g}(\delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g}). \tag{14}$$

4.6 Equilibrium Characterization

We define a *stationary equilibrium* of the economy in Appendix C.1. The assumed market segmentation and linearity of the production technology allow us to keep this problem tractable in spite of the many dimensions of worker and firm heterogeneity. These assumptions allow us to divide the firm's problem into separate subproblems by market. Conditional on productivity, a firm's optimal choice in each market is essentially independent of all other markets, which means that we can solve the firm's problem in each market in isolation.

For any posted wage-amenity combination, firms find themselves ranked on a market-specific ladder according to their flow-utility offer x. An argument analogous to that in Burdett and Mortensen (1998) shows that the equilibrium offer distribution $F_{a,g}(x)$ and the cross-sectional distribution $G_{a,g}(x)$ are continuous and strictly increasing for $x > \max\left\{\underline{p}a - \mathbf{1}[g=F]\overline{z}\,,\, \phi_{a,g}\right\}$ in each market (a, g) up to some maximum value. Next, we characterize firms' optimal policy functions.

Lemma 1 (Optimal Amenities)**.** *A firm's optimal amenity policy $\pi^*_{a,g}(\cdot)$ is strictly decreasing in its amenity cost shifter $c^{\pi,0}_{a,g}$ and invariant to all other parameters. Furthermore, $0 < c^{\pi}_{a,g}(\pi^*_{a,g}) < \pi^*_{a,g}$.*

Proof. See Appendix C.2.1. □

Lemma 1 extends to our setting a key result in Hwang et al. (1998), who also assume that firms are heterogeneous in their convex-increasing per-worker cost of amenities. Inuitively, firms optimally offer amenities up to the point when the marginal cost of amenities equals that wages, which equals one. That the cost-minimization problem does not depend on a firm's productivity, employer wedge, or recruiting costs follows from two assumptions: that worker utility is additively separable between wages and amenities and that the amenity cost is paid per worker. An implication of Lemma 1 is that, due to the bijection between firm-specific amenity cost shifters and optimal amenity values, we can treat $\pi^*_{a,g}$ as an exogenous firm-level parameter. Furthermore, in model counterfactuals, a firm's optimal amenity choice remains at the estimated value unless there are changes to its amenity cost function relative to its wage cost function.

Define a firm's *composite productivity* in market (a, g) as $\tilde{p}_{a,g} = pa + \pi_{a,g} - c^{\pi}_{a,g}(\pi_{a,g}) - z_{a,g}$. We can treat $\tilde{p}_{a,g}$ as an exogenous firm characteristic, allowing us to rewrite the problem of a firm as

$$\rho\Pi_{a,g}(\tilde{p}_{a,g}, c^{v,0}_{a,g}) = \max_{x,v} \left\{ \left[\tilde{p}_{a,g} - x\right] l_{a,g}(x, v) - c^{v}_{a,g}(v) \right\}, \quad \forall (a, g). \tag{15}$$

Therefore, the current model is essentially isomorphic to one without amenities or employer wedges but with two modifications.[20] First, productivity p is replaced by composite productivity $\tilde{p}$. Second, wages w are replaced by flow utility x. This isomorphism allows us to derive comparative statics with respect to the different components of $\tilde{p}_{a,g}$.

Lemma 2 (Optimal Market Selection)**.** *A firm optimally employs workers in market (a, g) if $\tilde{p}_{a,g} > \phi_{a,g}$.*

Proof. See Appendix C.2.2. □

A firm makes positive monetary profits if $pa + \pi_{a,g} - c^{\pi}_{a,g}(\pi_{a,g}) > \phi_{a,g}$.[21] However, Lemma 2 states that, due to the presence of employer wedges, this condition is neither necessary nor sufficient for a firm to select into a market. Depending on z_a in relation to the monetary surplus $pa + \pi_{a,g} - c^{\pi}_{a,g}(\pi_{a,g}) - \phi_{a,g}$ in each market, the firm may hire any combination of genders: both, either one, or none (in which case it does not operate).

Lemma 3 (Optimal Vacancy Policy)**.** *A firm's optimal vacancy policy $v^*_{a,g}(\cdot)$ is strictly increasing in productivity p, strictly decreasing in the vacancy cost shifter $c^{v,0}_{a,g}$ for all worker types, and strictly decreasing (constant) in z_a for women (men).*

[20] See Engbom and Moser (2018) for an example of such a model.

[21] We implicitly assume here that a firm's productivity is high enough to pay the minimum wage in monetary units.

Proof. See Appendix C.2.3. □

The intuition behind Lemma 3 is that more productive firms have a higher marginal payoff per contacted worker, thus they invest more into recruiting both men and women. The opposite is true with regards to female vacancies at firms with a higher employer wedge in their payoff function. Naturally, firms with a higher vacancy cost post fewer vacancies for both genders.

Lemma 4 (Optimal Flow Utility and Wages)**.** *A firm's optimal flow-utility policy $x^*_{a,g}(\cdot)$ and wage policy $w^*_{a,g}(\cdot)$ are strictly increasing in p for all worker types, constant in the vacancy cost shifter $c^{v,0}_{a,g}$ for all worker types, and strictly decreasing (constant) in the employer wedge z_a for women (men).*

Proof. See Appendix C.2.4 □

Lemma 4 extends the comparative statics results with respect to wages in Mortensen (2003) to an environment with richer employment contracts (amenities and wages, instead of just wages) and richer sources of worker mobility (Godfather shocks and heterogeneous arrival rates from nonemployment and employment, instead of just homogeneous arrival rates). Intuitively, firms with a larger payoff from employing a given worker optimally offer workers higher utility through wages in order to attract and retain a larger workforce.

Lemma 5 (Optimal Employment)**.** *A firm's optimal employment $l^*_{a,g}(\cdot)$ is strictly increasing in p for all worker types, strictly decreasing in the vacancy cost shifter $c^{v,0}_{a,g}$ for all worker types, and strictly decreasing (constant) in the employer wedge z_a for women (men).*

Proof. See Appendix C.2.5. □

Lemma 5 states that firms with higher composite productivity $\tilde{p}$ have greater steady-state employment, which is a combination of their rank in the job ladder, as guided by their flow-utility rank, and their recruitment intensity, as guided by the share of their aggregate-share of vacancies.

4.7 Equilibrium Wage Equation

The current equilibrium model provides a microfoundation for the decomposition of log wages into worker FEs and gender-firm FEs by Card et al. (2016), which is based on the seminal two-way FEs framework developed by AKM. To back up this claim, we provide a set of sufficient conditions for the log-wage decomposition to obtain as an equilibrium outcome in the model.

Assumption 1 (Vacancy cost function)**.** *Vacancy-posting costs $c_{a,g}^{v,0}$ scale linearly in worker ability a:*

$$c_{a,g}^{v,0} = ac_g^{v,0}, \quad \forall a$$

Assumption 1 could reflect that recruiting costs be paid in terms of time given to new hires for orientation and training, or in terms of the time of equally-skilled workers devoted to recruiting.

Assumption 2 (Job offer arrival and separation rates)**.** *The relative arrival rates of optional job offers $s_{a,g}^E$, that of mandatory job offers $s_{a,g}^G$, and separation rates $\delta_{a,g}$ are constant in worker ability a:*

$$s_{a,g}^E = s_g^E, \qquad s_{a,g}^G = s_g^G, \qquad \delta_{a,g} = \delta_g, \qquad \forall a$$

Assumption 2 allows for differential worker mobility across, but not within, genders.

Assumption 3 (Amenity cost function)**.** *The amenity creation cost function $c_{a,g}^\pi(\pi)$ takes on the following piece-rate form:*

$$c_{a,g}^\pi(\pi) = ac_g^{\pi,0}\tilde{c}\left(\frac{\pi}{a}\right), \qquad \forall a$$

Assumption 3 states that the cost of creating amenities is proportional to worker ability, and that amenities are paid to worker as a piece rate in their ability. A natural interpretation for this would be that some amenities involve time spent off work, such as in the context of paid parental leave. In this case, the cost of providing some units of time in amenities to a worker scales linearly in the worker's ability or foregone production due to the worker's absence from the job.

Assumption 4 (Flow values of nonemployment and employer wedges)**.** *The flow values of nonemployment $b_{a,g}$ and employer wedges z_a scale linearly in worker ability a:*

$$b_{a,g} = b_g a, \qquad z_a = za, \qquad \forall a$$

Assumption 4 ensures symmetry in participation and composite productivity across labor markets. It may be justified by higher-ability workers being also more skilled at home production, and by employers being willing to give up a fraction of workers' output to avoid interacting with them.

The following result links the structural model to the reduced-form approach in Section 3.2.

Proposition 1 (Equilibrium Wage Equation)**.** *Under Assumptions 1–4, the equilibrium wage of a worker with ability a and gender g at a firm with composite productivity* $\tilde{p}_g$ *and amenity cost shifter* $c_g^{\pi,0}$ *is*

$$\ln w_{a,g}\left(a, \tilde{p}_g, c_g^{\pi,0}\right) = \underbrace{\alpha_a}_{\text{"worker FE"}} + \underbrace{\psi_g\left(\tilde{p}_g, c_g^{\pi,0}\right)}_{\text{"gender-firm FE"}}, \tag{16}$$

where

$$\begin{aligned} \alpha_a &= \ln a, \\ \psi_g\left(\tilde{p}_g, c_g^{\pi,0}\right) &= \ln\left(\tilde{p}_g - \pi_g^*\left(c_g^{\pi,0}\right) - \int_{\tilde{p}' \geq \phi_g}^{\tilde{p}_g} \left[\frac{1+\kappa_g^e\left[1 - F_g\left(x_g^*\left(\tilde{p}_g\right)\right)\right]}{1+\kappa_g^e\left[1-F_g\left(x_g^*\left(\tilde{p}'\right)\right)\right]}\right]^2 d\tilde{p}'\right). \end{aligned} \tag{17}$$

Proof. See Appendix C.2.6. □

Proposition 1 shows that, under appropriate scaling assumptions, equilibrium wages in the model are log-additive between a worker component ("worker FE") and a gender-specific firm component ("gender-firm FE"). The worker FE α_a is a strictly monotonic transformation of worker ability. The gender-firm FE $\psi_g(\tilde{p}_g, c_g^{\pi,0})$ depends only on gender-firm-specific parameters, namely a firm's composite productivity $\tilde{p}_g$ and its amenity cost shifter $c_g^{\pi,0}$. Therefore, the equilibrium model provides a microfoundation for the wage equation with gender-specific employer pay components developed by Card et al. (2016). We will maintain Assumptions 1–4 and focus on differences in gender-firm FEs between men and women for the remainder of the analysis.

4.8 Discussion of Equilibrium Properties

Appendix C.3 discusses some of the more restrictive model assumptions and their implications.

The above model has three notable equilibrium properties. First, the model can rationalize job-to-job transitions with wage declines. On one hand, workers receive exogenous relocation shocks that result in forced transitions from wage w to $w' < w$. On the other hand, workers endogenously transition from wage-utility combination (w, x) to (w', x') with $x' > x$ but $w' < w$.

Second, "discriminatory" firms (as captured by the employer wedge z) can survive in a frictional environment.[22] A prediction of Becker (1971)'s seminal framework of taste-based discrimination is

[22] We do not want to claim that the employer wedge z only relates to discrimination. On the contrary, we think of it as capturing many different mechanisms. Among such mechanisms, taste-based discrimination is one of particular interest. All else equal, higher taste-based discrimination against female workers is associated with higher values of z.

that, in a competitive market, employers with a distaste for certain workers are driven out of the market. In contrast, in the current model, firms with nonzero employer wedges z survive in the presence of labor market frictions.

Third, even "nondiscriminatory" firms (as captured by z) may pay women less due to *statistical discrimination* based on gender-specific transition rates, due to *compensating differentials*, or due to their equilibrium response to the presence of other discriminatory employers.

5 Identification Strategy

To bridge the model and the data, we connect key model objects with their empirical counterparts. Our starting point is the special case of the model characterized in Proposition 1 of Section 4.7. Under the maintained assumptions, this allows us to pool workers of different ability types in the data and drop a from all subscripts of this section. We adopt a three-step identification strategy.

5.1 Step 1: Employer Ranks

In the first step, we estimate revealed-preference ranks of employers by gender using the PageRank index (Page et al., 1998; Sorkin, 2018) described in Section 3.5. This constitutes a set of $N_M + N_F$ estimates, where N_M and N_F are the numbers of establishments hiring men and women, respectively, in the data. The PageRank index represents the asymptotic share of time a representative worker ("random surfer") would spend at a given employer. This notion of employer rank coincides with that in the structural model of Section 4, in which workers are less likely to endogenously separate from, and more likely to accept offers at, higher-utility employers. In what follows, we conflate ranks and employer identities by indexing establishments by their rank $r_g \in \{1, 2, \ldots, R_g\}$, where 1 is the lowest and R_g is the highest rank for workers of gender g.

5.2 Step 2: Labor Market Parameters

In the second step, we estimate labor market parameters by combining employer ranks from Step 1 with monthly information on worker flows.[23] We seek gender-specific estimates of the cumulative density function (CDF) of offers F_g^r, separation rates δ_g, job finding rates from nonemployment λ_g^u, the

[23] The high-frequency nature of our data allows us obtain more precise estimates of employer ranks than has been possible in previous work. For example, Sorkin (2018) uses quarterly data to compute employer ranks based on what is effectively annual information on employment spells. We find that time aggregation bias (Moscarini and Postel-Vinay, 2018) can be substantial when repeating our estimates using aggregated data at the quarterly or annual level.

relative arrival rate of mandatory on-the-job offers s_g^G, and relative arrival rates of voluntary on-the-job offers s_g^e. This constitutes a set of $N_M + N_F + 8$ parameters. To this end, we exploit the model's job-ladder property that worker transitions depend only on ordinal employer ranks.

Job offer distributions. After ordering employers by their revealed-preference rank r, we compute the share of hires from nonemployment of each employer j out of total hires from nonemployment to estimate the gender-specific offer CDF $F_g^r = F_g(x_g^r)$.

Exogenous separation rates. We identify δ_g off separation rates into nonemployment:

$$\widehat{\delta_i} = \mathbb{E}_i \mathbf{1} \left[\text{nonemployed}_{i,t+1} \middle| \text{employed}_{i,t}, \text{gender}_i = g \right].$$

Offer rates from nonemployment. We identify λ_g^u off a log-hazard model for the time it takes for a worker to return to the data from nonemployment:

$$\widehat{\lambda}_g^u = 1 - \exp\left(\frac{\ln\left(\mathbb{E}_i \mathbf{1}\left[\text{nonemployment duration}_i \geq t \middle| \text{gender}_i = g\right]\right)}{t} \right)$$

Mandatory on-the-job offer rates. Two insights allow us to identify λ_g^G using information on worker transitions between employers. First, we focus on transitions in rank, not pay, space. Second, the share of rank-increasing transitions due to mandatory on-the-job offers declines in F_g^r. Formally, the total number of job-to-job transitions from employer rank r is

$$J2J_g^r = n_g^r[\lambda_g^e(1 - F_g^r) + \lambda_g^G], \tag{18}$$

where n_g^r is the number of workers of gender g at r. Rearranging and taking expectations, we have

$$\widehat{\lambda}_g^G = \mathbb{E}_i \left[\frac{J2J_g^{r\downarrow}}{n_g^r \widehat{F}_g^r} \right],$$

where $J2J_g^{r\downarrow} = J2J_g^r - n_g^r(\lambda_g^e + \lambda_g^G)(1 - F_g^r)$ is the number of job-to-job transitions to lower ranks.

Voluntary on-the-job offer rates. On-the-job offers not associated with mandatory transitions must have been voluntary. Hence, once we know λ_g^G, we can use equation (18) to estimate λ_g^e as

$$\widehat{\lambda}_g^e = \frac{J2J_g^r / n_g^r - \widehat{\lambda}_g^G}{1 - \widehat{F}_g^r}.$$

5.3 Step 3: Employer-Level Parameters and Values of Nonemployment

In the third step, we estimate employer-level parameters—productivity, amenity cost shifters, employer wedges, and vacancy cost shifters—together with workers' flow values of nonemployment using information on gender-specific employer ranks, pay, and labor market parameters. This constitutes a set of $3(N_M + N_F) + 2$ parameters. The insight that amenity values act as a residual between employers' rank and pay allows us to set-identify amenity values for each employer.[24] We further narrow down the identified amenity sets using equilibrium restrictions from the structural model. Within the narrowed-down set of amenities, we pick the minimal amenity values needed to rationalize the empirical employer rank-pay distribution. Appendix D.1 presents an illustrative example of the identification routine with three employers. We now delineate the general case.

Using Lemma 1, we can search for amenity values rather than amenity cost shifters, since the two are isomorphic. Given wages $(w_g^1, w_g^2, \ldots, w_g^{R_g}) \in \mathbb{R}_{++}^{R_g}$, the problem is to find separately by gender g a vector of amenity values $(\pi_g^1, \pi_g^2, \ldots, \pi_g^{R_g}) \in \mathbb{R}_{+}^{R_g}$ subject to a sequence of flow-utility monotonicity constraints dictated by Lemma 4:

$$w_g^r + \pi_g^r \leq w_g^{r+1} + \pi_g^{r+1}, \quad \forall r < R_g \tag{19}$$

In partial equilibrium, one would pick an amenity vector from the identified set, for example by minimizing the sum of squared differences between rank-adjacent utilities defined as[25]

$$\sum_r \left[\left(w_g^{r+1} + \pi_g^{r+1} \right) - \left(w_g^r + \pi_g^r \right) \right]^2. \tag{20}$$

However, in general equilibrium we can do better by taking into account additional model re-

[24] The reason for set identification (as opposed to point identification) is that it is impossible to deduce cardinal utility measures from just ordinal employer rank and pay information absent additional strong restrictions on the environment. For example, the utility offered by the highest-ranked employer can be bounded from below because it must exceed that of the second-highest-ranked employer, but it cannot be bounded from above.

[25] We tried several alternative ways of choosing amenities from the identified set in Monte Carlo simulations and found that choosing the utility-distance-minimizing performed best across different data generating processes.

strictions on the amenity vector imposed by Lemma 4:

$$\tilde{p}_g^r \leq \tilde{p}_g^{r+1}, \quad \forall r < R_g \tag{21}$$

Rewriting firms' first-order condition (FOC) with respect to flow utility x in equation (15) yields[26]

$$\tilde{p}^r = w_g^r + \pi_g^r + \frac{1 + \kappa_g^e(1 - F_g(x_g^r))}{2\kappa_g^e f_g(x_g^r)}. \tag{22}$$

To summarize, given wages $(w_g^1, w_g^2, \ldots, w_g^{R_g})$, estimates of the offer distribution $\widehat{F}_g^r$, and estimates of labor market parameters $\widehat{\kappa}_g^e$, we find gender-specific amenity values $(\pi_g^1, \pi_g^2, \ldots, \pi_g^{R_g})$ that minimize equation (20) subject to the constraints in equations (19), (21), and (22):

$$\begin{aligned}
(\widehat{\pi}_g^1, \widehat{\pi}_g^2, \ldots, \widehat{\pi}_g^{R_g}) = & \underset{(\pi_g^1, \pi_g^2, \ldots, \pi_g^{R_g}) \in \mathbb{R}_+^{R_g}}{\arg\min} \sum_r \left[\left(w_g^{r+1} + \pi_g^{r+1} \right) - \left(w_g^r + \pi_g^r \right) \right]^2 \\
\text{s.t.} \quad & w_g^r + \pi_g^r \leq w_g^{r+1} + \pi_g^{r+1}, \quad \forall r < R_g \\
& w_g^r + \pi_g^r + \frac{1 + \widehat{\kappa}_g^e(1 - \widehat{F}_g^r)}{2\widehat{\kappa}_g^e \widehat{f}_g^r} \leq w_g^{r+1} + \pi_g^{r+1} + \frac{1 + \widehat{\kappa}_g^e(1 - \widehat{F}_g^{r+1})}{2\widehat{\kappa}_g^e \widehat{f}_g^{r+1}}, \quad \forall r < R_g
\end{aligned} \tag{23}$$

Given amenity estimates, we back out amenity cost shifters $\{\widehat{c}_g^{\pi,0,r}\}$ given the functional form of the amenity cost function $\tilde{c}(\cdot)$. Next, we combine estimates of amenity values, wages, and labor market parameters to back out composite productivites using equation (22). The definition of composite productivity for men yields employer productivity $\widehat{p}^r = \widehat{\tilde{p}}_M^r - \widehat{\pi}_M^r + c_M^{\pi,r}(\widehat{\pi}_M^r)$. For dual-gender employers, we can estimate the employer wedge as $z^r = \widehat{p}^r - \widehat{\tilde{p}}_F^r + \widehat{\pi}_F^r - c_F^{\pi,r}(\widehat{\pi}_F^r)$. By the definition of the offer distribution, $v_g^r = f_g^r V_g$. Rearranging the FOCs for optimal vacancies $\{v_g^r\}_r$, we estimate vacancy cost shifters as

$$\widehat{c}_g^{v,0,r} = \frac{T_g(\widehat{\tilde{p}}_g^r - \widehat{x}_g^r) \left(\widehat{\delta}_g + \widehat{\lambda}_g^G + \widehat{\lambda}_g^e(1 - \widehat{F}_g^r) \right)^{-2}}{\partial \tilde{c}^v(\widehat{f}_g^r V_g) / \partial v_g^r}, \quad \forall r, \tag{24}$$

where $\widehat{x}_g^r = \widehat{w}_g^r + \widehat{\pi}_g^r$ and $T_g = \mu_g[(u_g + s_g^G)\lambda_g^u(\delta_g + \lambda_g^G + \lambda_g^e)]/V_g$. Equation (24) relates the vacancy cost shifter $c_g^{v,0,r}$ to the aggregate mass of vacancies V_g. Given that the latter is of no independent interest and the invariance of all else, we normalize $V_g = 1$. Finally, gender-specific outside option values are estimated as $\widehat{\phi}_g = \min_r \{w_g^r + \widehat{\pi}_g^r\}$. Together with a value of the exogenous discount rate

[26]See Appendix D.2 for further details on the change of variables from x to r.

ρ, equation (7) yields estimates of the gender-specific flow values of nonemployment, $\widehat{b}_g$.

To summarize, we have estimated amenity and vacancy cost shifters $\{\widehat{c}_g^{\pi,0,r}, \widehat{c}_g^{\pi,0,r}\}_r$ for each gender at every employer, productivities and employer wedges $\{\widehat{p}^r, \widehat{z}^r\}_r$ for dual-gender firms, a set of productivities $\{\widehat{p}^r\}_r$ for firms employing only men, a set of female composite productivites $\{\widehat{\tilde{p}}_F^r\}_r$ for firms employing only women, and gender-specific flow values of nonemployment $\widehat{b}_g$.

Two final comments are in order. First, although we estimate relative amenity values across employers, we are unable to identify the mean of amenities, and hence the level of utility, for either gender. The reason is, simply, the invariance of revealed preferences to a level shift in utilities. Thus, we normalize amenities to be weakly positive for both genders. Second, we can obviously not (point-)identify parameters relating both genders within an employer with workers of only one gender. We still use single-gender employers in the estimation, since they add to the identification of all other parameters. We assume that the parameters of single-gender employers with no workers of gender g are such that their composite productivity falls short of the outside option value for that gender, $\tilde{p}_g < \phi_g$. We keep these employers unchanged in counterfactuals involving gender-specific parameters.[27]

6 Estimation Results

6.1 Exogenous Parameters and Functional Form Assumptions

We assume that the cost functions for amenities and vacancies are of the power form, $\tilde{c}_g^{\pi}(\pi) = \pi^{\eta_\pi}/\eta_\pi$ with $\eta_\pi = 2$ and $\tilde{c}_g^{v}(v) = v^{\eta_v}/\eta_v$ with $\eta_v = 2$. Neither of these assumptions is relevant for model fit as we can match the distributions of π_g^r and f_g^r establishment by establishment in the data regardless of functional forms or parameter values.[28] Finally, we assume a discount factor of $\rho = 0.051$, which corresponds to an annual compound real interest rate of 5.3%. See Table 3.

Table 3. Exogenous parameters

Parameter	Description	Value
η_π	Amenity cost elasticity	2
η_v	Vacancy cost elasticity	2
ρ	Discount rate	0.051

Source: Authors' calculations based on RAIS. *Note:* Parameter values are externally set and treated as fixed.

[27] We have repeated all of our results for the subset of dual-gender employers.

[28] We have experimented with different elasticities of the amenity cost and vacancy cost functions for counterfactuals.

6.2 Labor Market Parameters and Flow Values of Nonemployment

Estimated labor market parameters are shown in Table 4. Women exhibit lower transition rates in general, both between employment states and between jobs. The implied nonemployment rates are $u_M = 0.243$ and $u_F = 0.244$, reflecting the presence of a large informal sector in Brazil, but similar across genders. While women are more likely to be permanently employed in the informal sector, men and women are similarly attached to the formal sector conditional on ever participating. For both men and women, mandatory on-the-job offers are about twice as frequent as voluntary ones. Finally, the flow value of nonemployment is higher for men than for women.

Table 4. Job offer arrival rates, job destruction rates, and flow values of nonemployment

Parameter	Description	Value	Implied rate
λ_M^u	Offer arrival rate from nonemployment (M)	0.100	0.100
λ_F^u	Offer arrival rate from nonemployment (F)	0.087	0.087
δ_M	Job destruction rate (M)	0.036	0.036
δ_F	Job destruction rate (F)	0.031	0.031
s_M^e	Relative arrival rate of voluntary on-the-job offers (M)	0.057	0.006
s_F^e	Relative arrival rate of voluntary on-the-job offers (F)	0.061	0.005
s_M^G	Relative arrival rate of mandatory on-the-job offers (M)	0.119	0.012
s_F^G	Relative arrival rate of mandatory on-the-job offers (F)	0.107	0.009
b_M	Flow value of nonemployment (M)	1.357	
b_F	Flow value of nonemployment (F)	1.267	

Source: Authors' calculations based on RAIS. *Note:* "M" denotes parameter for men, "F" denotes parameter for women. Implied rates are monthly.

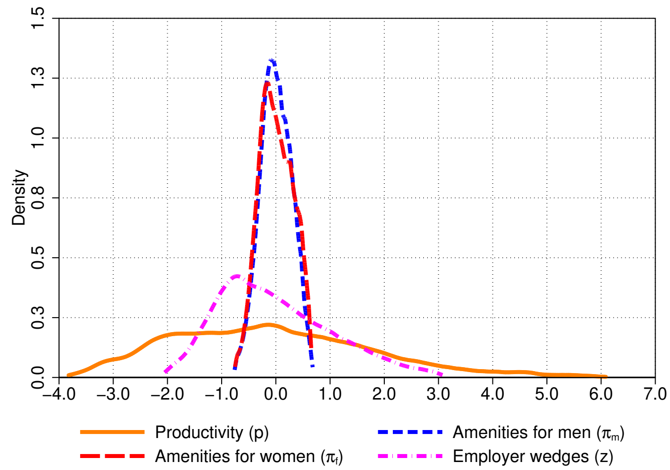
6.3 Distributions of Productivity, Amenities, and Employer Wedges

Figure 7 shows the marginal distributions of estimated productivity, amenity values, and employer wedges. Productivity dispersion is substantially larger than that in amenities or employer wedges. Productivity and employer wedges are positively skewed. Male and female amenities are similarly dispersed, left-skewed, and have thinner tails than a normal distribution.

Table 5 reports employment-weighted pairwise correlations between gender-specific pay (ψ_g), gender-specific PageRanks (r_g), productivity (p), gender-specific amenity values (π_g), employer wedges (z), and vacancy cost shifters ($c_g^{v,0}$).[29] A few points are worth noting. First, we find strong positive correlations between pay (0.900), ranks (0.651), amenities (0.662), and vacancy costs (0.672) within

[29] In Appendix E.1, we report for robustness the relation between our estimates using PageRanks and analogous estimates using the poaching rank (Moscarini and Postel-Vinay, 2008; Bagger and Lentz, 2018) and the net poaching rank (Haltiwanger et al., 2018; Moscarini and Postel-Vinay, 2018).

Figure 7. Marginal distributions of estimated employer parameters



Source: Authors' calculations based on RAIS. *Note:* Figure shows de-meaned marginal distributions of estimated productivity (p), gender-specific amenity values (π_g), and employer wedges (z).

employers across genders, indicating establishment-specific factors shared by men and women. Second, productivity is strongly positively correlated with ranks for men (0.847) but less so for women (0.586). This is because productivity is similarly positively correlated with pay for men (0.546) and women (0.582), but more positively correlated with amenities for men (0.556) than for women (0.247). In contrast, benchmark job-ladder models à la Burdett and Mortensen (1998) predict that productivity ranks are perfectly positively correlated with pay ranks. Third, the correlation between pay and ranks is more positive for men (0.414) than for women (0.349), while that between amenities and ranks is less positive for women (0.666) than for men (0.602). Fourth, amenities are similarly negative correlated to pay for men (-0.331) and for women (-0.343), suggesting compensating differentials for both genders. Finally, employer wedges are correlated positively with ranks for men (0.376) but negatively for women (-0.281), and correlated positively with productivity (0.507). This is consistent with Becker (1971)'s idea that taste-based discrimination cannot survive among employers with close-to-zero economic profits.

6.4 Relating Amenity Estimates to Observable Employer Characteristics

Our amenity estimates are residuals that rationalize employer ranks given their pay. To find out what economic factors these residuals capture, we relate our amenity estimates to a rich set of employer

Table 5. Correlation table for estimated employer parameters

	ψ_M	ψ_F	r_M	r_F	p	π_M	π_F	z	$c_M^{v,0}$	$c_F^{v,0}$
ψ_M	1.000									
ψ_F	0.900	1.000								
r_M	0.414	0.428	1.000							
r_F	0.277	0.349	0.651	1.000						
p	0.546	0.582	0.847	0.586	1.000					
π_M	−0.331	−0.245	0.602	0.420	0.556	1.000				
π_F	−0.341	−0.343	0.332	0.666	0.247	0.662	1.000			
z	0.363	0.238	0.376	−0.281	0.507	0.183	−0.403	1.000		
$c_M^{v,0}$	0.219	0.214	−0.170	−0.031	−0.174	−0.425	−0.226	−0.208	1.000	
$c_F^{v,0}$	0.361	0.334	0.025	−0.069	0.016	−0.340	−0.399	0.144	0.672	1.000

Source: Authors' calculations based on RAIS. *Note:* Table reports employment-weighted pairwise correlations between gender-specific pay (ψ_g), gender-specific PageRanks (r_g), productivity (p), gender-specific amenity values (π_g), employer wedges (z), and vacancy cost shifters ($c_g^{v,0}$) across employers.

characteristics not used in the estimation through the following regression:

$$\widehat{\pi}_{g,j} = E_{g,j}\eta_g + \iota_{g,j}, \tag{25}$$

where $\widehat{\pi}_{g,j}$ is the estimated amenity value for employer j, $E_{g,j}$ is a vector of employer covariates, and $\iota_{g,j}$ is an error term. We include as covariates in $E_{g,j}$ twelve variables, which we construct using the RAIS data—see Appendix E.2 for details.

Results from estimating equation (25) are presented in Table 6.[30] Reassuringly, unmeasured income in the form of food stamps loads similarly positively onto men's and women's amenity values. Both genders value positively a number of employer amenities related to job flexibility and paid leave. Both value negatively attributes related to earnings fluctuations, workplace conflict (as proxied by the share of unjust firings), and workplace risk (as proxied by worker death rates). Compared to men, women put relatively greater value on employer amenities such as hours flexibility and parental leave.[31] Women put relatively greater negative value on employer disamenities such as unpaid leave, earnings risk, and workplace risk. Altogether, we explain around 32 percent of the variation in estimated amenities for men and 47 percent of the variation for women.

[30] Results shown here are for all (dual- and single-gender) employers. For robustness, we have repeated the analysis for only dual-gender employers.

[31] Note that while we find a large negative coefficient on parental leave incidence for men, men are empirically two orders of magnitude less likely to ever take parental leave compared to women.

Table 6. Regression of estimated amenity values on employer characteristics, by gender

	Men	Women
Indicator: employer provides food stamps	0.089*** (0.000)	0.083*** (0.000)
Share of workers with part-time contract	0.033*** (0.000)	0.096*** (0.000)
Share of workers with hours change since previous year	0.034*** (0.001)	0.123*** (0.001)
Share of workers with paid sick leave	0.175*** (0.001)	0.144*** (0.001)
Share of workers with parental leave	−4.969*** (0.036)	0.065*** (0.005)
Share of workers with unpaid leave	−0.085*** (0.004)	−0.125*** (0.005)
Share of workers with earnings cut since previous year	−0.165*** (0.001)	−0.219*** (0.001)
Share of workers with noncontractual earnings fluctuations	−0.045*** (0.001)	−0.218*** (0.001)
Share of workers with work-related accident	−0.334*** (0.007)	−0.534*** (0.012)
Share of workers with commute-related accident	−0.792*** (0.026)	−0.311*** (0.044)
Share of worker separations due to firing for unjust reasons	−0.162*** (0.000)	−0.188*** (0.000)
Share of worker separations due to worker death	−0.627*** (0.003)	−0.786*** (0.004)
Industry FEs	✓	✓
Municipality FEs	✓	✓
Number of unique establishments	272,549	168,862
Observations	17,407,809	9,760,711
R^2	0.320	0.471

Source: Authors' calculations based on RAIS. *Note:* ***, **, * denote significance at 1%, 5%, 10% levels.

6.5 Relating Employer-Wedge Estimates to Observable Employer Characteristics

Analogous to our analysis of amenities, our employer wedge estimates are residuals that rationalize equilibrium pay for men and women within an employer. Such employer wedges could capture taste-based discrimination (Becker, 1971) or employer-level comparative advantages across genders (Goldin, 1992). To unpack this black box, we relate our employer wedge estimates to a rich set of employer characteristics not used in the estimation through the following regression:

$$\widehat{z}_j = E_j \eta + \iota_j, \tag{26}$$

where $\widehat{z}_j$ is the estimated employer wedge for establishment j, E_j is a vector of employer covariates, and ι_j is an error term. We include as covariates in E_j twelve variables, which we construct using the RAIS data—see Appendix E.2 for details.

Results from estimating equation (25) are presented in Table 7. We group the independent variables into two categories. The first group pertains to proxies for employer-level comparative advantages across genders. It includes task content measures and proxies for risks of physical harm and strength requirements. Our results indicate mixed support for the employer-level-comparative-advantage story. On one hand, we find that employer wedges are positively related to nonroutine-manual task intensity and the share of workers with work-related accidents, suggesting that women are perceived as less productive in physical and risky jobs.[32] On the other hand, we find that employer wedges are negatively related to routine-manual task intensity and the share of worker separations due to worker death, suggesting that women are not perceived as less productive in physically demanding jobs.[33] All results are robust across specifications with cumulatively added controls for employers' industry and municipality, which may themselves be correlated with gender-based comparative advantage.

The second group pertains to proxies for taste-based discrimination. It includes measures of gender composition and a measure of financial independence. Our results lend support to the interpretation of employer wedges as being related to taste-based discrimination. Employer wedges are strongly negatively related to the female employment share, consistent with more female-friendly workplaces attracting and retaining more female workers. Similarly, establishments with a woman in the highest-paid position on average have a lower employer wedge, suggesting that female metnors may affect the female friendliness of a workplace. Finally, small firms with little financial dependence on average have a significantly higher wedge, possibly due to discrimination being more likely to survive under lower accountability to investors and other stakeholders. Results are robust across different sets of controls.

Overall, while we cannot definitively pin down the factors behind the intra-employer wedge, our analysis sheds light on proxies related to potential economic explanations. Altogether, we explain around 76 percent of the variation in estimated employer wedges. Most of this variation is due to variables related to gender-based comparative advantage and taste-based discrimination rather than due to the inclusion of controls for industry and municipality. Thus, while our findings are suggestive of some economic explanations behind the estimated employer wedges, they also leave room for other factors.

6.6 Model Fit

We solve for the equilibrium of the model given the above parameter estimates. We first use the model to match exactly, employer by employer, the empirical offer distribution of pay and amenities F_g^r. We then ask the model to predict the cross-sectional equilibrium pay and amenity distribution,

[32] *Nonroutine-manual tasks* include jobs that are physical but involve a wide set of tasks and are not rule-based, such as "janitor, home health aide, and personal care aide." (Siu and Jaimovich, 2015).

[33] *Routine-manual tasks* include "jobs that are both rule based and emphasize physical", such as "factory workers who operate welding, fitting, and metal press machines [and] forklift operators." (Siu and Jaimovich, 2015).

Table 7. Regression of estimated employer wedges on empl. characteristics, by gender

	(1)	(2)	(3)
Routine-manual task intensity	−0.107*** (0.000)	−0.059*** (0.001)	−0.057*** (0.001)
Nonroutine-manual task intensity	0.278*** (0.001)	0.176*** (0.001)	0.155*** (0.001)
Routine cognitive task intensity	−0.013*** (0.000)	−0.005*** (0.001)	0.003*** (0.001)
Nonroutine cognitive interpersonal task intensity	−0.123*** (0.001)	−0.029*** (0.001)	−0.030*** (0.001)
Nonroutine cognitive analytical task intensity	0.089*** (0.001)	0.055*** (0.001)	0.034*** (0.001)
Share of worker separations due to worker death	−0.753*** (0.005)	−0.471*** (0.005)	−0.395*** (0.006)
Share of workers with work-related accidents	2.229*** (0.021)	1.500*** (0.021)	0.295*** (0.020)
Female employment share	−4.206*** (0.001)	−3.645*** (0.001)	−3.835*** (0.001)
Indicator: highest-paid worker is a woman	−0.239*** (0.001)	−0.166*** (0.001)	−0.121*** (0.001)
Indicator: no major financial stakeholders	0.048*** (0.001)	0.031*** (0.001)	0.034*** (0.001)
Industry FEs		✓	✓
Municipality FEs			✓
Number of unique establishments	96,065	96,065	96,065
Observations	17,287,101	17,287,101	17,287,101
R^2	0.693	0.730	0.764

Source: Authors' calculations based on RAIS. *Note:* ***, **, * denote significance at 1%, 5%, 10% levels.

which we compare to the data.[34] Table 8 shows the model fit vis-à-vis a set of salient empirical moments. We find that the model fits the data well. The model somewhat understates the magnitude of the gender pay gap and the variance of employer pay compared to the data. The model closely matches the empirical variance of the gender pay gap, empirical job-to-job transition rates for both genders, and the correlation between men's and women's pay within employers.

Table 8. Model fit

Moment	Description	Data	Model
$\mathbb{E}\left[\psi_M - \psi_F\right]$	Gender pay gap	0.084	0.074
$\mathbb{E}\left[\psi^M - \psi^F \middle\| g = M\right]$	Gender pay gap between employers	0.074	0.055
$\mathbb{E}\left[\psi^F \middle\| g = M\right] - \mathbb{E}\left[\psi^F \middle\| g = F\right]$	Gender pay gap within employers	0.009	0.018
$Var(\psi_M)$	Variance of men's pay	0.051	0.040
$Var(\psi_F)$	Variance of women's pay	0.046	0.032
$Var(\psi_M - \psi_F)$	Variance of gender pay gap	0.010	0.009
$\mathbb{E}\left[\lambda_M^e\left(1 - F\left(x\right)\right) + \lambda_M^G\right]$	Job-to-job transition rate for men	0.016	0.015
$\mathbb{E}\left[\lambda_F^e\left(1 - F\left(x\right)\right) + \lambda_F^G\right]$	Job-to-job transition rate for women	0.012	0.012
$Corr(\psi_M, \psi_F)$	Correlation between men's and women's pay	0.926	0.932

Source: Authors' calculations based on RAIS. Rates are monthly.

[34] The numerical solution algorithm we use is discussed in Appendix E.4.

7 Equilibrium Counterfactuals

7.1 Sources of the Gender Pay Gap

With the estimated model in hand, we recompute equilibria while shutting down various gender differences. We consider four counterfactuals. First, we set the amenity cost shifters of women equal to those of men, $c_F^{\pi,0} = c_M^{\pi,0}$.[35] Second, we shut down employer wedges, $z = 0$. Third, we only shut down differences in vacancy creation cost shifters $c_g^{v,0}$ across genders. Fourth and finally, we shut down all of the above gender differences simultaneously.[36] Table 9 describes our baseline economy (column 0) and results from the four counterfactuals (columns 1–4).

Table 9. Results from simulating equilibrium counterfactuals

	Baseline	Counterfactuals			
Gender differences in...	(0)	(1)	(2)	(3)	(4)
amenities	✓		✓	✓	
employer wedges	✓	✓		✓	
vacancy posting costs	✓	✓	✓		
Gender pay gap...	0.074	0.061	0.020	0.018	0.000
between employers	0.055	0.056	0.047	0.016	0.000
within employers	0.018	0.005	-0.026	0.002	0.000
Output	1.000	1.001	1.012	1.033	1.035
Worker welfare from...	0.000	0.004	0.015	-0.004	0.027
payroll for women	0.000	0.010	0.026	0.019	0.029
amenity value for women	0.000	-0.006	-0.010	-0.022	-0.002
Payroll-equivalent welfare change	–	0.005	0.019	-0.004	0.033
Employer welfare from...	1.000	0.997	1.011	0.986	1.039
profits	1.004	1.002	1.011	1.039	1.039
employer wedges	-0.004	-0.006	-0.000	-0.053	0.000
Total employment for women	0.760	0.759	0.762	0.760	0.757

Source: Authors' calculations based on RAIS. *Note:* Table reports simulation results from model-based counterfactuals. Baseline results (column 0) are compared against counterfactuals without gender differences in amenities (column 1), in employer wedges (column 2), in vacancy posting costs (column 3), and without any gender differences (column 4).

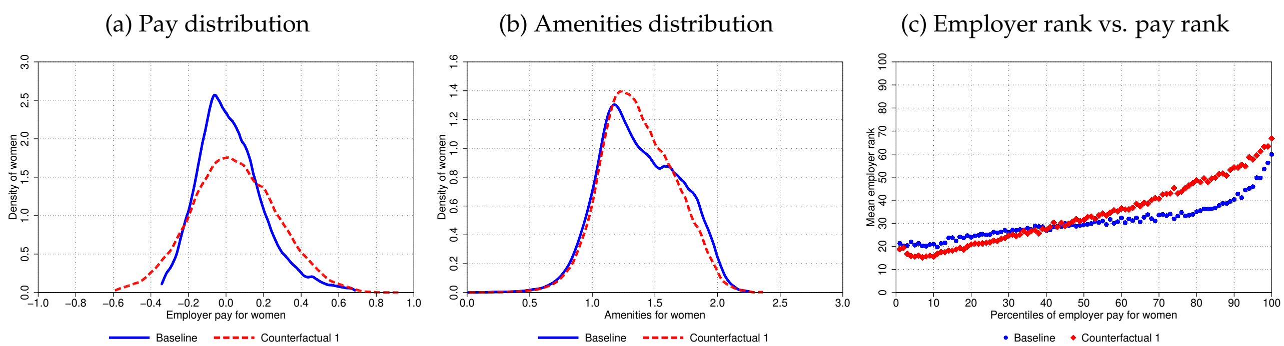
Counterfactual 1: Shutting down gender differences in amenity costs. Gender differences in amenities account for around 1.3 log points (18 percent) of the gender gap. Worker welfare in-

[35] For robustness, we have simulated counterfactuals under a range of amenity cost elasticities. In Appendix F, we show a variant of the counterfactuals with exogenous amenities, i.e., $\eta_\pi = \infty$. Our insights remain substantially unchanged.

[36] In all counterfactuals but the last, we keep single-gender employers unchanged. Single-gender employers are important because they make up a significant share of all employers mediate the response of other firms to counterfactual parameter changes. For the gender-neutral economy, we equalize the flow utility of unemployment across geneders and change single-gender firms so that men-only firms now also hire women, and women-only firms cease to exist. In essence, the gender-neutral economy is an economy that consists of two copies of the current market for men.

creases by 0.5% in payroll-equivalent units. The increase in pay for women (panel (a) of Figure 8) is partly offset by a corresponding decrease in mean amenity values (panel (b)). Output increases slightly because high-productivity firms increase their amenities, thus attracting more women. As a consequence, pay inequality for women increases. However, employer welfare decreases: profits decline because firms now pay higher wages, and wedges increase because more women work at high-wedge firms. Finally, women's employer rank-pay rank relationship becomes steeper (panel (c)), more like that of men documented in Section 3.5. See column 1 of Table 9.

Figure 8. Effects of counterfactual 1 on amenities and pay for women



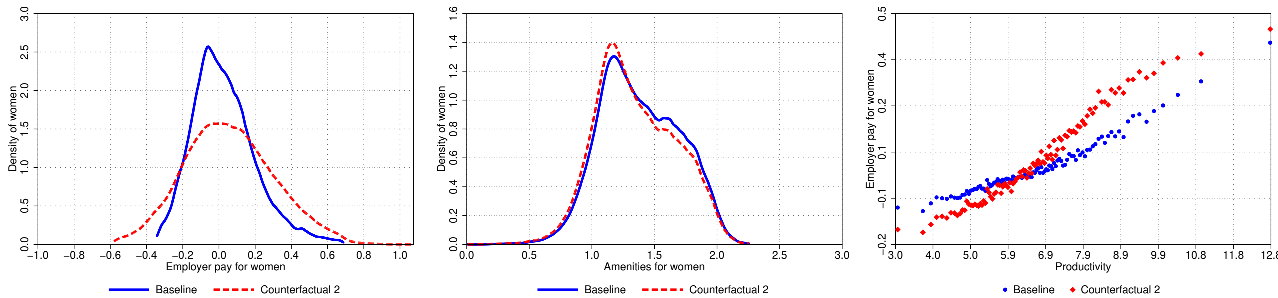
Source: Authors' calculations based on RAIS. *Note:* Counterfactual corresponding to column 1 of Table 9.

Counterfactual 2: Shutting down employer wedges. Employer wedges account for around 5.4 log points (73 percent) of the gender gap. Most of the decrease in the gender gap is within firms, but the between-firm gap also decreases due to equilibrium reallocation of women. The mean and dispersion of women's wages increase (panel (a) of Figure 9). Since wedges were positively correlated with productivity, removing them increases women's dispersion in composite productivity and hence pay. Output increases by 1.2 percent, worker welfare by 1.9 percent, and profits by 0.7 percent. Output increases due to women relocating to high-productivity, formerly high-wedge firms. The increase in worker welfare is accounted for by a large rise in women's payroll and partly offset by a decline in amenity values due to worker relocation (panel (b)). That employer profits also increase reflects the fact that employers with a nonzero wedge were not maximizing monetary profits. Another result is a steepening of the productivity-pay relationship for women (panel (c)) that resembles an increase in women's "bargaining power" (Card et al., 2016). See column 2 of Table 9.

Counterfactual 3: Shutting down gender differences in vacancy costs. Gender differences in vacancy costs account for around 5.6 log points (76 percent) of the gender gap. Again, most of this

Figure 9. Effects of counterfactual 2 on pay, amenities and offer distribution for women

(a) Pay distribution (b) Amenities distribution (c) Pay vs. productivity

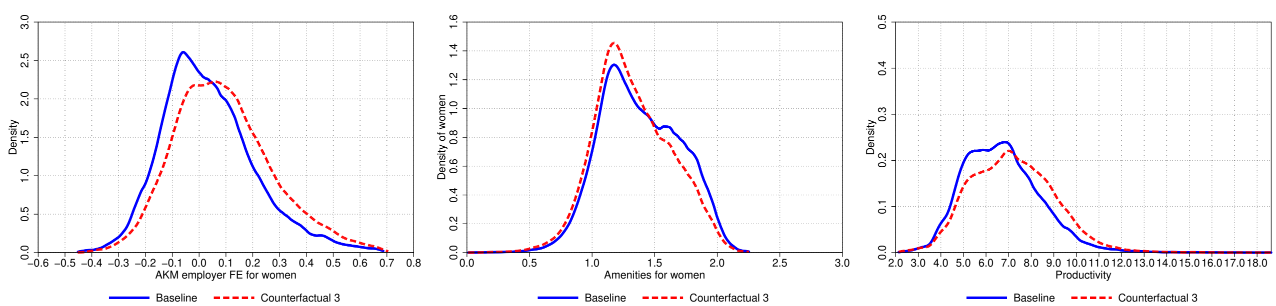


Source: Authors' calculations based on RAIS. *Note:* Counterfactual corresponding to column 2 of Table 9.

decline is accounted for by a decline in the between-employer pay gap. More women become employed at high-paying firms (panel (a) of Figure 10). There is a significant increase in output of around 3.3 percent due to the relocation of women to more productive firms (panel (c)). However, the impact on worker welfare is net negative, around -0.4 percent. The reason for this is that especially high-wedge, high-productivity, but low-amenity employers increase their employment of women (panel (c)). This also means that in spite of an increase in monetary profits of around 3.5 percent, employers are worse off on average. A key take-away from this simulation is that gender differences in employer allocation are not necessarily inefficient. See column 3 of Table 9.

Figure 10. Effects of counterfactual 3 on the distribution of women's employment.

(a) Women's distribution over pay for women (b) Women's distribution over amenities (c) Women's distribution over productivity



Source: Authors' calculations based on RAIS. *Note:* Counterfactual corresponding to column 3 of Table 9.

Counterfactual 4: Moving to a gender-neutral economy. By construction, moving to a gender-neutral economy eliminates the gender gap. Put differently, women's mean pay increases by 7.4 percent. Interestingly, this is also associated with large gains in output (3.5 percent), worker welfare (3.3 percent), and employer welfare (3.9 percent). Output increases because women relocate to higher-productivity employers. Most of the increase in worker welfare is due to an increase in pay, not amenities. And employer welfare increases due to a combination of higher profits and lower wedges. See column 4 of Table 9.

Interaction effects and the distinction between output versus welfare. One important insight from our simulations is that the different structural gender differences interact nonlinearly. While removing all gender differences simultaneously leads to large output and welfare gains, addressing only one at a time may actually result in welfare losses. Another important insight is that output and welfare are fundamentally different, and that the two can move in opposite directions.

7.2 The (Unintended) Consequences of an Equal-Pay Policy

The salient gender gap suggests an interesting thought experiment: What would be the effects of a mandated equal-pay policy that requires men and women of identical ability to be paid the same within employers? We implement this experiment within our model as follows: firms may choose only one wage, which must be paid to both men and women, while still being allowed to produce gender-specific amenities and post different amounts of vacancies for men and women.

We summarize the effects of the equal-pay policy in Table 10.[37] By construction, the within-employer gender pay gap disappears under the policy. More surprisingly, the between-employer gender gap actually increases by 0.2 log points. This happens because firms with positive wedges find it especially costly to pay a single wage to workers of both genders. Consequently, they hire relatively fewer women. Because employer wedges are higher at high-productivity firms, the equal-pay policy reduces women's employment at relatively high-paying firms, thereby increasing the between-employer gap.

The policy also has subtle redistributive effects. The policy is associated with a significant increas in pay for women of 1.5 log points. At the same time, men's pay and employment each decrease by 0.2 log points. Men lose relatively less pay than women gain, but experience higher unemployment, because of men's relatively higher outside option value. In addition, employers compensate a substantial share of the pay changes for each gender with changes in amenities in the opposite directions.

On aggregate, the equal-pay policy has little effect on output and welfare. Worker welfare slightly

[37] The numerical solution algorithm used to solve for the equilibrium under the equal-pay policy departs significantly from that used for the previous results and is detailed in Appendix F.1.

increases, as the increase in women's payroll exceeds the decreases in women's amenities and men's payroll net of amenities. Employer welfare slightly decreases, mostly due to an increase in the mean employer wedge. A key take-away is that the (unintended) consequences of forcing equal pay on employers are quite different from moving to a gender-neutral economy.

Table 10. Results from simulating effects of an equal-pay policy

	Baseline	Equal-pay policy
	(0)	(1)
Mean pay for men	1.000	0.998
Mean pay for women	1.000	1.015
Gender pay gap...	0.074	0.057
between employers	0.055	0.057
within employers	0.018	-0.000
Output	1.000	1.000
Worker welfare from...	0.000	0.000
total payroll...	0.000	0.001
for men	0.000	-0.001
for women	0.000	0.003
total amenity value...	0.000	-0.001
for men	0.000	0.001
for women	0.000	-0.002
Payroll-equivalent welfare change	0.000	0.001
Employer welfare from...	1.000	0.999
profits	1.004	1.004
employer wedges	-0.004	-0.005
Total employment for men	0.757	0.755
Total employment for women	0.760	0.760

Note: Table reports simulation results of an equal-pay policy. Baseline results (column 0) are compared against the economy under an equal-pay policy (column 1).

8 Conclusion

Our analysis sheds light on the sources and consequences of the gender pay gap. We document that a large share of the gender pay gap in Brazil is accounted for by women working at lower-paying employers compared to men. At the same time, pay is not the sole determinant of workers' revealed-preference rankings of employers. We find significant differences in employer pay and ranks across genders. To interpret these facts, we develop an empirical equilibrium search model that can rationalize the gender differences in pay and revealed-preference ranks in the data. We use the estimated model to simulate a series of equilibrium counterfactuals.

The fact that women are paid less than men may or may not reflect output and welfare losses, depending on the microstructure of their labor markets. In our specific context, we demonstrate that some ways of closing the gender gap can increase the utility of women, while others leave women worse off. This does not mean that gender gaps should not be addressed. We find that there are both output and welfare gains from moving to a gender-neutral economy, reflecting the current misallocation of talent across genders (Hsieh et al., 2019). Nevertheless, achieving a gender-neutral economy may not be a easily achieved. Our counterfactual simulations suggest that an equal-pay policy fails to close the gender pay gap, though it has redistributive effects.

Our work opens up several avenues for future research. We have been mostly agnostic about the factors underlying gender differences in various labor market margins. Additional evidence is needed to shed light on these margins and help address them with policies. The applicability of our framework is not limited to gender; the theoretical framework and estimation routine we develop could help assess the sources and consequences of other dimensions of inequality. Preference heterogeneity may exist not just across gender but across other population subgroups. By allowing for separate job ladders for men and women, our work takes but a first step in the direction of integrating richer heterogeneity in models of the labor market.

References

Abowd, John M., Francis Kramarz, and David N. Margolis, "High Wage Workers and High Wage Firms," *Econometrica*, 1999, *67* (2), 251–333.

__ , Robert H. Creecy, and Francis Kramarz, "Computing Person and Firm Effects Using Linked Longitudinal Employer-Employee Data," 2002.

Acemoglu, Daron and David Autor, "Skills, Tasks and Technologies: Implications for Employment and Earnings," in "Handbook of Labor Economics, Volume 4. Amsterdam: Elsevier-North" 2011, pp. 1043–1171.

Albanesi, Stefania, "Changing Business Cycles: The Role of Women's Employment," *Working Paper*, 2020.

__ and Ayşegül Şahin, "The Gender Unemployment Gap," *Review of Economic Dynamics*, 2018, *30*, 47–67.

__ and Claudia Olivetti, "Home Production, Market Production and the Gender Wage Gap: Incentives and Expectations," *Review of Economic Dynamics*, 2009, *12* (1), 80–107.

__ and __ , "Gender Roles and Medical Progress," *Journal of Political Economy*, 2016, *124* (3), 650–695.

Alvarez, Jorge, Felipe Benguria, Niklas Engbom, and Christian Moser, "Firms and the Decline of Earnings Inequality in Brazil," *American Economic Journal: Macroeconomics*, 2018, *10* (1), 149–189.

Amano-Patiño, Noriko, Tatiana Baron, and Pengpeng Xiao, "Equilibrium Wage-Setting and the Life-Cycle Gender Pay Gap," *Working Paper*, 2019.

Andrews, Martyn J., Leonard Gill, Thorsten Schank, and Richard Upward, "High Wage Workers and Low Wage Firms: Negative Assortative Matching or Limited Mobility Bias?," *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 2008, *171* (3), 673–697.

__ , __ , __ , and __ , "High Wage Workers Match with High Wage Firms: Clear Evidence of the Effects of Limited Mobility Bias," *Economics Letters*, 2012, *117* (3), 824–827.

Arrow, Kenneth J., "Some Models of Racial Discrimination in the Labor Market," Technical Report, Rand Corporation, Santa Monica, CA 1971.

Autor, David and David Dorn, "This Job Is "Getting Old": Measuring Changes in Job Opportunities Using Occupational Age Structure," *American Economic Review*, 2009, *99* (2), 45–51.

Bagger, Jesper and Rasmus Lentz, "An Empirical Model of Wage Dispersion with Sorting," *The Review of Economic Studies*, 2018, *86* (1), 153–190.

_ , Bent Jesper Christensen, and Dale T. Mortensen, "Productivity and Wage Dispersion: Heterogeneity or Misallocation?," *Working Paper*, 2014, pp. 1–40.

Becker, Gary S., *The Economics of Discrimination*, Economic Research Studies, 1971.

Black, Dan A., "Discrimination in an Equilibrium Search Model," *Journal of Labor Economics*, 1995, *13* (2), 309–334.

Bonhomme, Stéphane and Grégory Jolivet, "The pervasive absence of compensating differentials," *Journal of Applied Econometrics*, 2009, *24* (5), 763–795.

Bontemps, Christian, Jean marc Robin, and Gerard J. van den Berg, "An Empirical Equilibrium Job Search Model with Search on the Job and Heterogeneous Workers and Firms," *International Economic Review*, 1999, *40* (4), 1039–1074.

_ , Jean-Marc Robin, and Gerard J. van den Berg, "Equilibrium Search with Continuous Productivity Dispersion: Theory and Nonparametric Estimation," *International Economic Review*, 2000, *41* (2), 305–358.

Bowlus, Audra J., "A Search Interpretation of Male-Female Wage Differentials," *Journal of Labor Economics*, 1997, *15* (4), 625–657.

_ and Zvi Eckstein, "Discrimination and Skill Differences in an Equilibrium Search Model," *International Economic Review*, 2002, *43* (4), 1309–1345.

Buera, Francisco J., Joseph P. Kaboski, and Min Qiang Zhao, "The rise of services: the role of skills, scale, and female labor supply," *Journal of Human Capital*, 2019, *13* (2), 157–187.

Burdett, Kenneth and Dale T. Mortensen, "Wage Differentials, Employer Size, and Unemployment," *International Economic Review*, 1998, *39* (2), 257–273.

Cahuc, Pierre, Fabien Postel-Vinay, and Jean-Marc Robin, "Wage Bargaining with On-the-Job Search: Theory and Evidence," *Econometrica*, 2006, *74* (2), 323–364.

Card, David, Ana Rute Cardoso, and Patrick Kline, "Bargaining, Sorting, and the Gender Wage Gap: Quantifying the Impact of Firms on the Relative Pay of Women," *Quarterly Journal of Economics*, 2016, *131* (2), 633–686.

_ , _ , Jörg Heining, and Patrick Kline, "Firms and Labor Market Inequality: Evidence and Some Theory," *Journal of Labor Economics*, 2018, *36* (S1), S13–S70.

_ , Jörg Heining, and Patrick Kline, "Workplace Heterogeneity and the Rise of West German Wage Inequality," *Quarterly Journal of Economics*, 2013, *128* (3), 967–1015.

Dey, Matthew S. and Christopher J. Flinn, "An Equilibrium Model of Health Insurance Provision and Wage Determination," *Econometrica*, 2005, *73* (2), 571–627.

Dix-Carneiro, Rafael, Pinelopi Goldberg, Costas Meghir, and Gabriel Ulyssea, "Trade and Informality in the Presence of Labor Market Frictions and Regulations," *Working Paper*, 2019.

Engbom, Niklas and Christian Moser, "Earnings Inequality and the Minimum Wage: Evidence from Brazil," *Federal Reserve Bank of Minneapolis OIGI Working Paper 7*, 2018.

Erosa, Andrés, Luisa Fuster, Gueorgui Kambourov, and Richard Rogerson, "Hours, Occupations, and Gender Differences in Labor Market Outcomes," *Working Paper*, 2019.

Fernández, Raquel, "Cultural Change as Learning: The Evolution of Female Labor Force Participation over a Century," *American Economic Review*, 2013, *103* (1), 472–500.

_ , Alessandra Fogli, and Claudia Olivetti, "Mothers and Sons: Preference Formation and Female Labor Force Dynamics," *The Quarterly Journal of Economics*, 2004, *119* (4), 1249–1299.

_ and _ , "Culture: An Empirical Investigation of Beliefs, Work, and Fertility," *American Economic Journal: Macroeconomics*, 2009, *1* (1), 146–77.

Flabbi, Luca, "Gender Discrimination Estimation in a Search Model with Matching and Bargaining," *International Economic Review*, 2010, *51* (3), 745–783.

Fogli, Alessandra and Laura Veldkamp, "Nature or Nurture? Learning and the Geography of Female Labor Force Participation," *Econometrica*, 2011, *79* (4), 1103–1138.

Fukui, Masao, Emi Nakamura, and Jón Steinsson, "Women, Wealth Effects, and Slow Recoveries," *Working Paper*, 2019.

Gayle, George-Levi and Limor Golan, "Estimating a Dynamic Adverse-Selection Model: Labour-Force Experience and the Changing Gender Earnings Gap 1968-1997," *The Review of Economic Studies*, 2011, *79* (1), 227–267.

Gerard, François, Lorenzo Lagos, Edson Severnini, and David Card, "Assortative Matching or Exclusionary Hiring? The Impact of Firm Policies on Racial Wage Differences in Brazil," *NBER Working Paper No. 25176*, 2018.

Goldin, Claudia, *Understanding the Gender Gap: An Economic History of American Women*, Oxford University Press, 1992.

_ , "A Grand Gender Convergence: Its Last Chapter," *American Economic Review*, 2014, *104* (4), 1091–1119.

_ **and Lawrence F. Katz**, "The Cost of Workplace Flexibility for High-Powered Professionals," *Annals of the American Academy of Political and Social Science*, 2011, *638* (1), 45–67.

_ **and** _ , "A Most Egalitarian Profession: Pharmacy and the Evolution of a Family-Friendly Occupation," *Journal of Labor Economics*, 2016, *34* (3), 705–746.

Goldschmidt, Deborah and Johannes F. Schmieder, "The Rise of Domestic Outsourcing and the Evolution of the German Wage Structure," *The Quarterly Journal of Economics*, 2017, *132* (3), 1165–1217.

Greenwood, Jeremy, Ananth Seshadri, and Mehmet Yorukoglu, "Engines of Liberation," *Review of Economic Studies*, 2005, *72* (1), 109–133.

Hall, Robert E. and Andreas I. Mueller, "Wage Dispersion and Search Behavior: The Importance of Nonwage Job Values," *Journal of Political Economy*, 2018, *126* (4), 1594–1637.

Haltiwanger, John C., Henry R. Hyatt, Lisa B. Kahn, and Erika McEntarfer, "Cyclical Job Ladders by Firm Size and Firm Wage," *American Economic Journal: Macroeconomics*, 2018, *10* (2), 52–85.

Heathcote, Jonathan, Kjetil Storesletten, and Giovanni L. Violante, "The Macroeconomics of the Quiet Revolution: Understanding the Implications of the Rise in Women's Participation for Economic Growth and Inequality," *Research in Economics*, 2017, *71* (3), 521–539.

Heckman, James J., Lance J. Lochner, and Petra E. Todd, "Earnings Functions, Rates of Return and Treatment Effects: The Mincer Equation and Beyond," in Erik Hanushek and F. Welch, eds., *Handbook of the Economics of Education*, Vol. 1 of *Handbook of the Economics of Education*, Elsevier, 2006, chapter 7, pp. 307–458.

Heise, Sebastian and Tommaso Porzio, "Spatial Wage Gaps in Frictional Labor Markets," *Working Paper*, 2019.

Hsieh, Chang-Tai and Peter J. Klenow, "Misallocation and Manufacturing TFP in China and India," *The Quarterly Journal of Economics*, 2009, *124* (4), 1403–1448.

_ **, Erik Hurst, Charles I. Jones, and Peter J. Klenow**, "The Allocation of Talent and U.S. Economic Growth," *Econometrica*, 2019, *87* (5), 1439–1474.

Hwang, Hae-shin, Dale T. Mortensen, and W. Robert Reed, "Hedonic Wages and Labor Market Search," *Journal of Labor Economics*, 1998, *16* (4), 815–847.

Kleven, Henrik Jacobsen, Camille Landais, and Jakob Egholt Søgaard, "Children and Gender Inequality: Evidence from Denmark," *Working Paper*, 2016.

Kline, Patrick, Raffaele Saggio, and Mikkel Sølvsten, "Leave-Out Estimation of Variance Components," Technical Report, National Bureau of Economic Research 2019.

Lamadon, Thibaut, Magne Mogstad, and Bradley Setzler, "Imperfect Competition, Compensating Differentials and Rent Sharing in the U.S. Labor Market," *Working Paper*, 2019.

Lang, Kevin and Sumon Majumdar, "The Pricing Of Job Characteristics When Markets Do Not Clear: Theory And Policy Implications," *International Economic Review*, 2004, *45* (4), 1111–1128.

Lavetti, Kurt and Ian M. Schmutte, "Estimating Compensating Wage Differentials with Endogenous Job Mobility," Working Paper 29, Labor Dynamics Institute, Cornell University 2017.

Lentz, Rasmus and Dale T. Mortensen, "An Empirical Model of Growth Through Product Innovation," *Econometrica*, 2008, *76* (6), 1317–1373.

Mas, Alexandre and Amanda Pallais, "Valuing Alternative Work Arrangements," *American Economic Review*, 2017, *107* (12), 3722–59.

_ **and** _ , "Labor Supply and the Value of Non-work Time: Experimental Estimates from the Field," *American Economic Review: Insights*, 2019, *1* (1), 111–26.

Meghir, Costas, Renata Narita, and Jean-Marc Robin, "Wages and Informality in Developing Countries," *American Economic Review*, 2015, *105* (4), 1509–1546.

Mincer, Jacob, *Schooling, Experience, and Earnings*, New York: NBER Press, 1974.

Mortensen, Dale T., *Wage Dispersion: Why are Similar Workers Paid Differently?*, 1 ed., MIT Press Books, The MIT Press, 2003.

Moscarini, Giuseppe and Fabien Postel-Vinay, "The Timing of Labor Market Expansions: New Facts and a New Hypothesis," *NBER Macroeconomics Annual*, 2008, *23*, 1–52.

_ **and** _ , "Stochastic Search Equilibrium," *Review of Economic Studies*, 2013, *80* (4), 1545–1581.

_ **and** _ , "The Cyclical Job Ladder," *Annual Review of Economics*, 2018, *10*, 165–188.

Ngai, Rachel and Barbara Petrongolo, "Gender Gaps and the Rise of the Service Economy," *American Economic Journal: Macroeconomics*, 2017, *9* (4), 1–44.

Page, Lawrence, Sergey Brin, Rajeev Motwani, and Terry Winograd, "The PageRank Citation Ranking: Bringing Order to the Web," *Working Paper*, 1998.

Peters, Michael and Conor Walsh, "Declining Dynamism, Increasing Markups and Missing Growth: The Role of the Labor Force," *Working Paper*, 2019.

Postel-Vinay, Fabien and Jean-Marc Robin, "Equilibrium Wage Dispersion with Worker and Employer Heterogeneity," *Econometrica*, 2002, *70* (6), 2295–2350.

Rendall, Michelle, "Brain versus Brawn: The Realization of Women's Comparative Advantage," *Working Paper*, 2018.

Restuccia, Diego and Richard Rogerson, "Policy Distortions and Aggregate Productivity with Heterogeneous Plants," *Review of Economic Dynamics*, 2008, *11* (4), 707–720.

Rosen, Sherwin, "The Theory of Equalizing Differences," in "Handbook of Labor Economics," Vol. 1 1986, pp. 641–692.

Siu, Henry and Nir Jaimovich, "Jobless Recoveries," Report, Washington DC 2015.

Song, Jae, David J. Price, Fatih Guvenen, Nicholas Bloom, and Till Von Wachter, "Firming Up Inequality," *The Quarterly Journal of Economics*, 2018, *134* (1), 1–50.

Sorkin, Isaac, "The Role of Firms in Gender Earnings Inequality: Evidence from the United States," *American Economic Review*, 2017, *107* (5), 384–87.

_ , "Ranking Firms Using Revealed Preference," *The Quarterly Journal of Economics*, 2018, *133* (3), 1331–1393.

Taber, Christopher and Rune Vejlin, "Estimation of a Roy/Search/Compensating Differential Model of the Labor Market," *NBER Working Paper No. 22439*, 2016.

Ulyssea, Gabriel, "Firms, Informality, and Development: Theory and Evidence from Brazil," *American Economic Review*, 2018, *108* (8), 2015–2047.

Wiswall, Matthew and Basit Zafar, "Preference for the Workplace, Investment in Human Capital, and Gender*," *The Quarterly Journal of Economics*, 2017, *133* (1), 457–507.

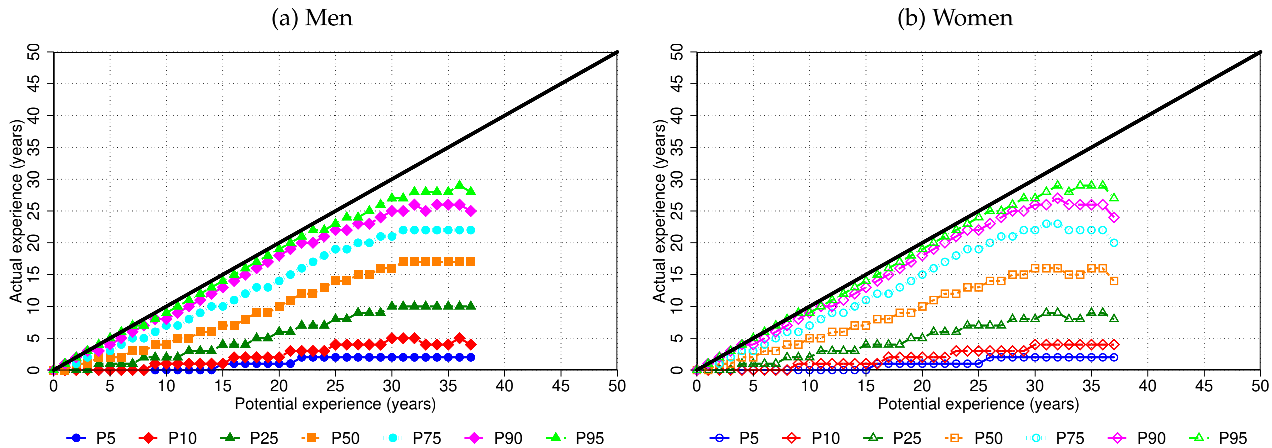
Xiao, Pengpeng, "Wage and Employment Discrimination by Gender in Labor Market Equilibrium," *Working Paper*, 2020.

Online Appendix—Not for Publication

A Data Appendix

A.1 Comparison of Actual versus Potential Experience

Figure 11. Percentiles of actual experience conditional on potential experience



Note: Actual experience is constructed from panel data for 1985–2014. We compute potential experience = age − years of education + 6. *Source:* RAIS.

A.2 Summary Statistics

Table 11. Summary statistics after sample selection and restriction to connected set

	2007			2014			Pooled 2007–2014		
	Overall	Men	Women	Overall	Men	Women	Overall	Men	Women
Share nonwhite	0.355	0.385	0.305	0.407	0.438	0.360	0.384	0.416	0.333
Share primary school	0.113	0.146	0.057	0.073	0.097	0.037	0.092	0.121	0.045
Share middle school	0.220	0.264	0.146	0.186	0.226	0.126	0.204	0.246	0.136
Share high school	0.475	0.460	0.500	0.554	0.548	0.563	0.520	0.508	0.540
Share college	0.192	0.130	0.296	0.186	0.129	0.274	0.183	0.125	0.278
Mean years of education	10.8	10.1	12.0	11.3	10.7	12.1	11.0	10.4	12.1
(std. dev.)	(3.5)	(3.5)	(3.2)	(3.1)	(3.2)	(2.8)	(3.3)	(3.4)	(3.0)
Mean age	33.5	33.2	34.0	34.5	34.3	34.9	33.9	33.6	34.2
(std. dev.)	(9.5)	(9.5)	(9.5)	(9.5)	(9.5)	(9.5)	(9.5)	(9.5)	(9.5)
Mean establishment size	3,477	1,877	6,179	3,627	2,540	5,284	25,078	14,878	41,728
(std. dev.)	(19,946)	(12,956)	(27,831)	(18,287)	(13,743)	(23,485)	(141,762)	(97,283)	(192,368)
Mean gender-establishment size	2,182	850	4,431	2,267	1,420	3,558	15,542	7,259	29,062
(std. dev.)	(12,989)	(3,518)	(20,607)	(11,217)	(5,959)	(16,144)	(88,805)	(31,395)	(137,318)
Mean establishment age	30.6	27.8	35.5	33.5	30.8	37.7	31.0	28.2	35.6
(std. dev.)	(22.5)	(21.2)	(23.8)	(23.6)	(22.4)	(24.7)	(23.1)	(21.9)	(24.4)
Mean months employed in year	9.9	9.7	10.1	10.1	10.0	10.3	9.9	9.8	10.1
(std. dev.)	(3.2)	(3.3)	(3.2)	(3.0)	(3.0)	(3.0)	(3.2)	(3.2)	(3.1)
Mean contractual work hours	41.3	42.4	39.5	41.3	42.3	39.8	41.5	42.5	39.8
(std. dev.)	(5.6)	(4.2)	(7.1)	(5.4)	(4.1)	(6.6)	(5.4)	(4.0)	(6.7)
Mean tenure (years)	4.3	3.7	5.2	4.4	4.1	4.8	4.1	3.7	4.8
(std. dev.)	(6.0)	(5.5)	(6.6)	(6.0)	(5.7)	(6.4)	(5.9)	(5.5)	(6.4)
Mean log real monthly earnings	7.108	7.156	7.028	7.367	7.430	7.272	7.237	7.291	7.150
(std. dev.)	(0.728)	(0.732)	(0.714)	(0.691)	(0.692)	(0.677)	(0.709)	(0.712)	(0.695)
Mean log real hourly wage	3.402	3.416	3.377	3.659	3.692	3.609	3.525	3.548	3.487
(std. dev.)	(0.780)	(0.769)	(0.797)	(0.742)	(0.730)	(0.757)	(0.758)	(0.747)	(0.774)
Number of worker-years	24,348,192	15,292,100	9,056,092	29,881,399	18,039,128	11,842,271	231,805,831	143,745,869	88,059,962
Number of unique workers	24,348,192	15,292,100	9,056,092	29,881,399	18,039,128	11,842,271	55,078,455	33,197,634	21,880,821
Number of unique establishments	184,168	135,346	48,822	191,504	138,171	53,333	222,695	153,081	69,614
Share female	0.372			0.396			0.380		
Mean log gender earnings gap	0.128			0.158			0.141		
Mean log gender wage gap	0.039			0.083			0.062		

Source: Authors' calculations based on RAIS.

Table 12. Summary statistics before sample selection and restriction to connected set

	2007			2014			Pooled 2007–2014		
	Overall	Men	Women	Overall	Men	Women	Overall	Men	Women
Share nonwhite	0.339	0.368	0.294	0.388	0.416	0.349	0.365	0.394	0.321
Share primary school	0.113	0.149	0.056	0.072	0.099	0.035	0.090	0.122	0.044
Share middle school	0.232	0.278	0.160	0.189	0.233	0.129	0.209	0.255	0.143
Share high school	0.493	0.462	0.543	0.586	0.560	0.621	0.545	0.515	0.588
Share college	0.162	0.111	0.241	0.153	0.108	0.214	0.156	0.109	0.225
Mean years of education	10.7	10.0	11.7	11.1	10.5	11.9	10.9	10.3	11.8
(std. dev.)	(3.4)	(3.5)	(3.0)	(3.0)	(3.1)	(2.6)	(3.2)	(3.3)	(2.8)
Mean age	33.0	33.0	33.0	33.7	33.8	33.7	33.3	33.4	33.3
(std. dev.)	(9.5)	(9.5)	(9.4)	(9.5)	(9.6)	(9.5)	(9.5)	(9.5)	(9.5)
Mean establishment size	2,137	1,188	3,610	2,055	1,503	2,817	15,300	9,429	23,897
(std. dev.)	(15,571)	(10,183)	(21,317)	(13,672)	(10,416)	(17,144)	(110,234)	(76,567)	(145,728)
Mean gender-establishment size	1,347	542	2,596	1,291	849	1,900	9,522	4,622	16,697
(std. dev.)	(10,196)	(2,759)	(15,843)	(8,451)	(4,587)	(11,839)	(69,490)	(24,776)	(104,473)
Mean establishment age	23.5	22.0	25.8	23.8	22.6	25.6	23.5	22.2	25.4
(std. dev.)	(21.3)	(19.9)	(23.0)	(22.3)	(21.1)	(23.7)	(21.8)	(20.5)	(23.3)
Mean months employed in year	9.6	9.5	9.7	9.7	9.7	9.8	9.6	9.6	9.7
(std. dev.)	(3.4)	(3.4)	(3.4)	(3.3)	(3.3)	(3.3)	(3.3)	(3.3)	(3.4)
Mean contractual work hours	42.0	42.8	40.9	42.1	42.8	41.2	42.1	42.8	41.2
(std. dev.)	(5.0)	(3.8)	(6.2)	(4.7)	(3.7)	(5.7)	(4.7)	(3.7)	(5.8)
Mean tenure (years)	3.8	3.5	4.2	3.6	3.5	3.8	3.6	3.4	3.9
(std. dev.)	(5.3)	(5.0)	(5.7)	(5.2)	(5.0)	(5.4)	(5.2)	(5.0)	(5.5)
Mean log real monthly earnings	6.981	7.034	6.898	7.232	7.299	7.139	7.116	7.176	7.028
(std. dev.)	(0.683)	(0.692)	(0.661)	(0.639)	(0.648)	(0.614)	(0.663)	(0.673)	(0.638)
Mean log real hourly wage	3.254	3.285	3.205	3.501	3.549	3.434	3.385	3.425	3.326
(std. dev.)	(0.733)	(0.727)	(0.739)	(0.686)	(0.683)	(0.685)	(0.710)	(0.706)	(0.710)
Number of worker-years	38,401,131	23,359,048	15,042,083	50,798,080	29,427,684	21,370,396	364,776,727	216,756,022	148,020,705
Number of unique workers	38,401,131	23,359,048	15,042,083	50,798,080	29,427,684	21,370,396	77,297,426	44,401,043	32,896,383
Number of unique establishments	2,716,661	1,664,469	1,052,192	3,583,804	2,062,313	1,521,491	5,927,621	3,329,980	2,597,641
Share female	0.392			0.421			0.406		
Mean log gender earnings gap	0.137			0.161			0.148		
Mean log gender wage gap	0.080			0.115			0.099		

Source: RAIS.

Table 13. Comparison of summary statistics before and after sample selection and restriction to connected set

	Pooled 2007–2014, connected set			Pooled 2007–2014, all			Ratio: connected set to all		
	Overall	Men	Women	Overall	Men	Women	Overall	Men	Women
Share nonwhite	0.384	0.416	0.333	0.365	0.394	0.321	1.052	1.056	1.037
Share primary school	0.092	0.121	0.045	0.090	0.122	0.044	1.022	0.992	1.023
Share middle school	0.204	0.246	0.136	0.209	0.255	0.143	0.976	0.965	0.951
Share high school	0.520	0.508	0.540	0.545	0.515	0.588	0.954	0.986	0.918
Share college	0.183	0.125	0.278	0.156	0.109	0.225	1.173	1.147	1.236
Mean years of education	11.0	10.4	12.1	10.9	10.3	11.8	1.009	1.010	1.025
(std. dev.)	(3.3)	(3.4)	(3.0)	(3.2)	(3.3)	(2.8)	1.031	1.030	1.071
Mean age	33.9	33.6	34.2	33.3	33.4	33.3	1.018	1.006	1.027
(std. dev.)	(9.5)	(9.5)	(9.5)	(9.5)	(9.5)	(9.5)	1.000	1.000	1.000
Mean establishment size	25,078	14,878	41,728	15,300	9,429	23,897	1.639	1.578	1.746
(std. dev.)	(141,762)	(97,283)	(192,368)	(110,234)	(76,567)	(145,728)	1.286	1.271	1.320
Mean gender-establishment size	15,542	7,259	29,062	9,522	4,622	16,697	1.632	1.571	1.741
(std. dev.)	(88,805)	(31,395)	(137,318)	(69,490)	(24,776)	(104,473)	1.278	1.267	1.314
Mean establishment age	31.0	28.2	35.6	23.5	22.2	25.4	1.319	1.270	1.402
(std. dev.)	(23.1)	(21.9)	(24.4)	(21.8)	(20.5)	(23.3)	1.060	1.068	1.047
Mean months employed in year	9.9	9.8	10.1	9.6	9.6	9.7	1.031	1.021	1.041
(std. dev.)	(3.2)	(3.2)	(3.1)	(3.3)	(3.3)	(3.4)	0.970	0.970	0.912
Mean contractual work hours	41.5	42.5	39.8	42.1	42.8	41.2	0.986	0.993	0.966
(std. dev.)	(5.4)	(4.0)	(6.7)	(4.7)	(3.7)	(5.8)	1.149	1.081	1.155
Mean tenure (years)	4.1	3.7	4.8	3.6	3.4	3.9	1.139	1.088	1.231
(std. dev.)	(5.9)	(5.5)	(6.4)	(5.2)	(5.0)	(5.5)	1.135	1.100	1.164
Mean log real monthly earnings	7.237	7.291	7.150	7.116	7.176	7.028	1.017	1.016	1.017
(std. dev.)	(0.709)	(0.712)	(0.695)	(0.663)	(0.673)	(0.638)	1.069	1.058	1.089
Mean log real hourly wage	3.525	3.548	3.487	3.385	3.425	3.326	1.041	1.036	1.048
(std. dev.)	(0.758)	(0.747)	(0.774)	(0.710)	(0.706)	(0.710)	1.068	1.058	1.090
Number of worker-years	231,805,831	143,745,869	88,059,962	364,776,727	216,756,022	148,020,705	0.635	0.663	0.595
Number of unique workers	55,078,455	33,197,634	21,880,821	77,297,426	44,401,043	32,896,383	0.713	0.748	0.665
Number of unique establishments	222,695	153,081	69,614	5,927,621	3,329,980	2,597,641	0.038	0.046	0.027
Share female	0.380			0.406			0.936		
Mean log gender earnings gap	0.141			0.148			0.953		
Mean log gender wage gap	0.062			0.099			0.626		

Source: RAIS.

B Empirical Appendix

B.1 A Mincerian Approach To Measuring the Gender Gap

As a starting point in our pursuit of understanding the sources of the gender pay gap, we run a series of classical Mincer regressions without controls for employer identity (Mincer, 1974; Heckman et al., 2006). The goal is twofold. First, to deliver a set of estimates that are directly comparable to the large existing literature that has studied gender gaps using household surveys or similar datasetes. Second, to understand the part of the gender gap that is explained by worker and job characteristics associated with labor supply factors, which we think of as orthogonal to employer characteristics and other labor demand factors.

A classical Mincerian specification for income (i.e., either earnings or wages) of individual i in year t, denoted y_{it}, is simply

$$y_{it} = X_{it}\beta + \mathbf{1}\left[gender_i = M\right]\alpha^M + \mathbf{1}\left[gender_i = F\right]\alpha^F + \varepsilon_{it}, \tag{27}$$

where X_{it} is a vector of observable worker and job characteristics discussed below, $\mathbf{1}\left[gender_i = M\right]$ and $\mathbf{1}\left[gender_i = F\right]$ are indicator functions that equal 1 if the gender of individual i is male or female, respetively, and 0 otherwise, α^M and α^F are gender-specific intercepts, and ε_{it} is a residual term. We estimate this equation via ordinary least squares (OLS) under the usual strict exogeneity assumption that $\mathbb{E}\left[\varepsilon_{it}|X_{it}, gender_i\right] = 0$. The main object of interest resulting from equation (27) is the (conditional) gender pay gap $\gamma \equiv \alpha^M - \alpha^F$, which captures the mean pay difference between female versus male workers who are otherwise observationally identical.

Table 14 shows the (conditional) gender gap in four different specifications: the earnings gap without any controls in column 1; the earnings gap controlling for a linear term in years of education and a second-order polynomial in actual experience in column 2; the wage gap with the same controls in column 3; and the earnings gap with an additional set of dummies for education, actual experience, age, hours, nationality, municipality, industry, occupation, and tenure in column 4.[38]

Our preferred specification is reportsin column 4, with a conditional gender pay gap of around 12 log points. By including a rich set of observable worker and job characteristics as controls, this specification purges the raw data from various labor supply-related gender differences highlighted in the previous literature—see, for example, Goldin (2014) and Erosa et al. (2019). This specification flexibly controls for hours dummies with earnings as the dependent variable. If we restricted hours to enter linearly with coefficient one in this regression, then this would be identical to using the wage rate as the depdendent variable. More generally, a complete set of hours fixed effects (FEs) controls for nonconstant wage rates as a function of hours worked. Finally, it is worth noting that our preferred specification yields a high R^2 value of close to 70 percent, which suggests that we are controlling for a set of gender-pay-relevant characteristics with considerable explanatory power.[39]

We conclude that a large gender pay gap remains within narrowly defined population subgroups defined by a rich set of covariates related to labor supply. With a large gap left unexplained by labor supply, we next turn to factors related to labor demand and ask: what is the role of gender-specific employer heterogeneity in explaining the gender pay gap?

[38] Table 15 repeats the same set of mincer reg regressions for the year 2007. Between 2007 and 2014, the raw gender pay gap (corresponding to column 1 of Table 14) increased by a little less than 3 log points, while the conditional gap with our full set of controls (corresponding to column 4 of Table 14) shows a decline of a little over 1 log point.

[39] In univariate regressions and when gradually introducing controls in specification (27), we find that occupation and hours dummies account for a significant share of empirical pay variation in general and the gender gap in particular.

Table 14. Estimates from Mincer regressions, 2014

	(1)	(2)	(3)	(4)
Gender gap	0.150	0.246	0.192	0.119
Income concept	Earnings	Earnings	Wage	Earnings
Education (linear)		✓	✓	
Education FEs				✓
Actual experience (polynomial)		✓	✓	
Actual experience FEs				✓
Age FEs				✓
Contractual work hours FEs				✓
Nationality FEs				✓
Municipality FEs				✓
Industry FEs				✓
Occupation FEs				✓
Tenure FEs				✓
Observations	31,830,960	31,830,960	31,830,960	31,830,960
R^2	0.012	0.372	0.389	0.698

Source: RAIS.

Table 15. Estimates from Mincerian regressions, 2007

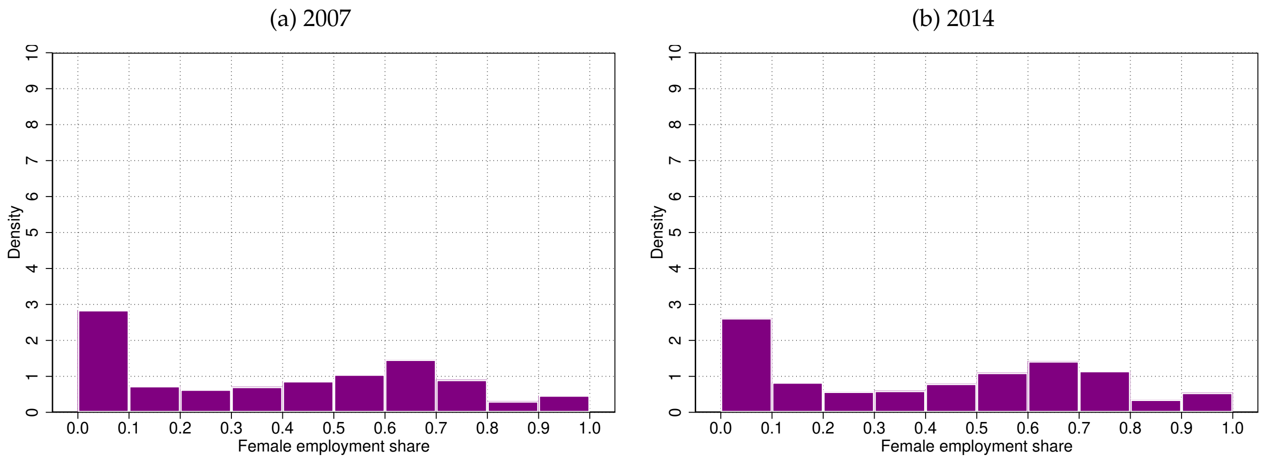
	(1)	(2)	(3)	(4)
Gender gap	0.125	0.266	0.201	0.132
Income concept	Earnings	Earnings	Wage	Earnings
Education (linear)		✓	✓	
Education FEs				✓
Actual experience (polynomial)		✓	✓	
Actual experience FEs				✓
Age FEs				✓
Contractual work hours FEs				✓
Nationality FEs				✓
Municipality FEs				✓
Industry FEs				✓
Occupation FEs				✓
Tenure FEs				✓
Observations	25,208,660	25,208,660	25,208,660	25,208,632
R^2	0.007	0.367	0.385	0.682

Source: RAIS.

B.2 Robustness for Female Employment Share Distribution

Figure 12 repeats the same histogram shown in Figure 1 for the year 2014 and also that for the year 2007, which yields the same qualitative conclusion as before.

Figure 12. Histogram of female employment shares



Source: RAIS.

Figure 13 splits the histogram shown in Figure 1 into 9 industries, which yields the same qualitative conclusion as before.

To show that the unequal distribution of women across employers is robust to different employer size selection criteria, Figure 14 shows percentiles of the female employment share distribution across firms for various employment size cutoffs. This addresses the concern that at small firms employment of women that is representative of the population may not be attainable due to the indivisibility of bodies in the data. The figure shows that empirically we are quite far away from the equal-allocation benchmark, even at very large firms with more than 10,000 employees. Even among firms with at least 1,000 employees in the data, the female employment share varies vastly between 5 and 81 percent going from the 5th percentile to the 95th percentile of the female employment share distribution.

B.3 Comparison of Gender-Specific Employment Distributions

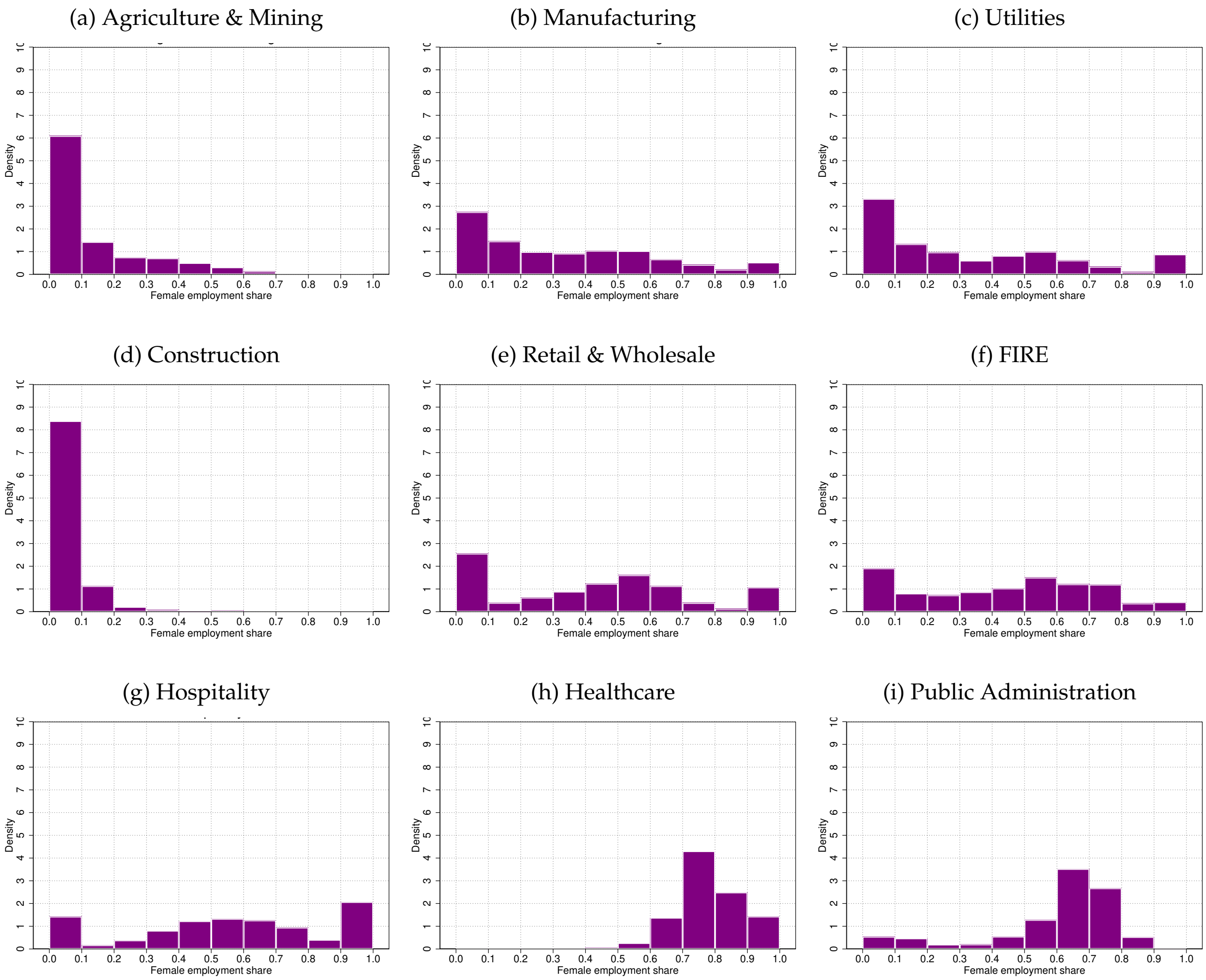
To illustrate the nonuniform distribution of women relative to that of men across employers, Figure 15 shows a histogram of male and female employment shares across firms ranked by their employment of the other gender. If the distribution of women (men) were a symmetric scaled version of that of men (women), one would see a strictly monotonic increasing density of density bins of one gender across employment ranks of the other gender. In contrast, the histograms in panels (a)–(d) show pronounced nonmonotonicities, suggesting that employers that have a relatively large mass of men do not necessarily also have a relatively large mass of women.

B.4 Gender Segregation Index

To quantify the extent to which women are nonuniformly distributed across employers, we define the following *employer segregation index*, S_t:

$$S_t = \frac{\sum_{i=1}^{N_m+N_f} \Big((\text{firm-level female share})_{J(i,t)} - (\text{population female share}) \Big)^2}{N_m \times ((\text{population female share}))^2 + N_f \times (1 - (\text{population female share}))^2}$$

Figure 13. Histogram of female employment shares, by sector



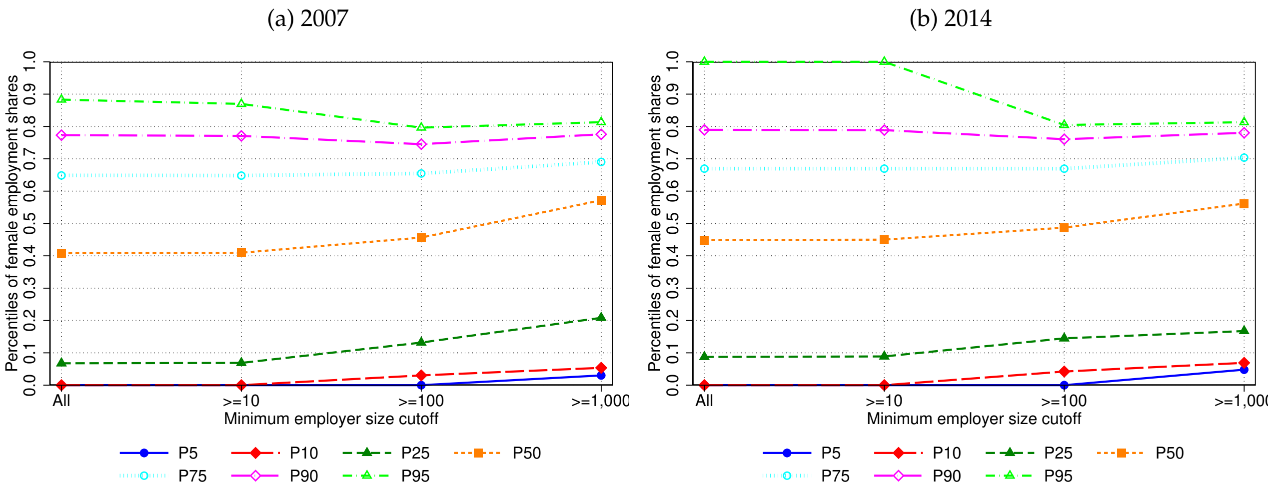
Note: FIRE stands for the Finance, Insurance, and Real Estate industry. *Source:* RAIS.

where i indexes individual workers, N_m and N_f are the number of male and female workers, respectively, and $J(i,t)$ is a function that gives the index of the employer of individual i in year t. Note that the employer segregation index S_t lies between 0 and 1, with 0 meaning that each employer has a representative share of women and 1 meaning that all women work at employers where only women work (and, hence, similarly for men).

We find that the employer segregation index, S_t, takes on a value of 0.349 in 2007. To assess whether this is an economically meaningful deviation from uniform female shares, note that an index value of 0.349 corresponds to an equivalent absolute value difference in employer gender shares of ± 0.288 around the population female share. In comparison, we find that the same index is significantly smaller when computed across industries (0.109), occupations (0.142), or states (0.002) in our data. Furthermore, the index value is relatively stable when we restrict attention to employers above minimum size thresholds of between 10 and 1,000 employees.

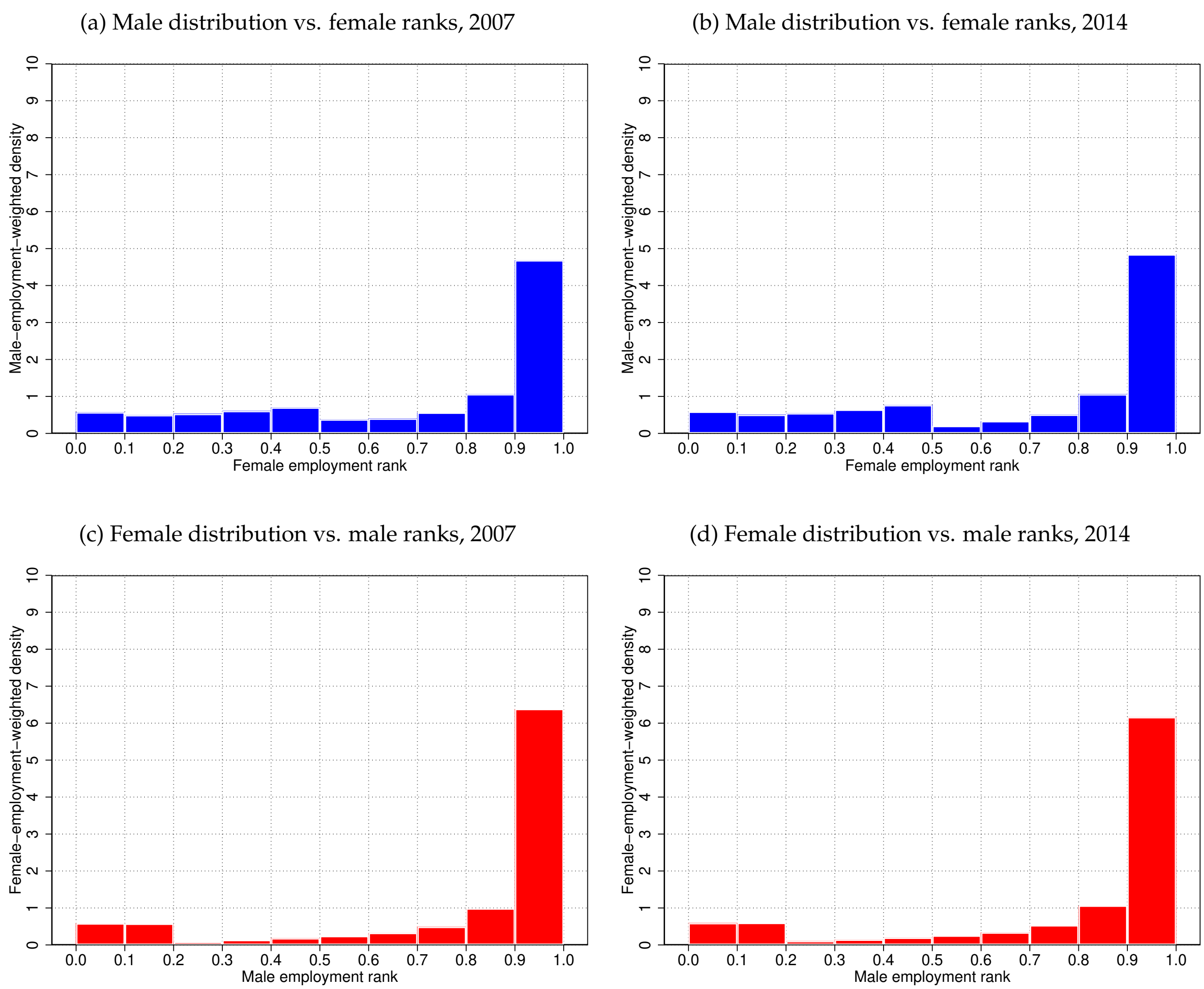
We conclude that there is a significant amount of gender segregation, with women distributed far from uniformly across employers.

Figure 14. Percentiles of female employment share distribution



Source: RAIS.

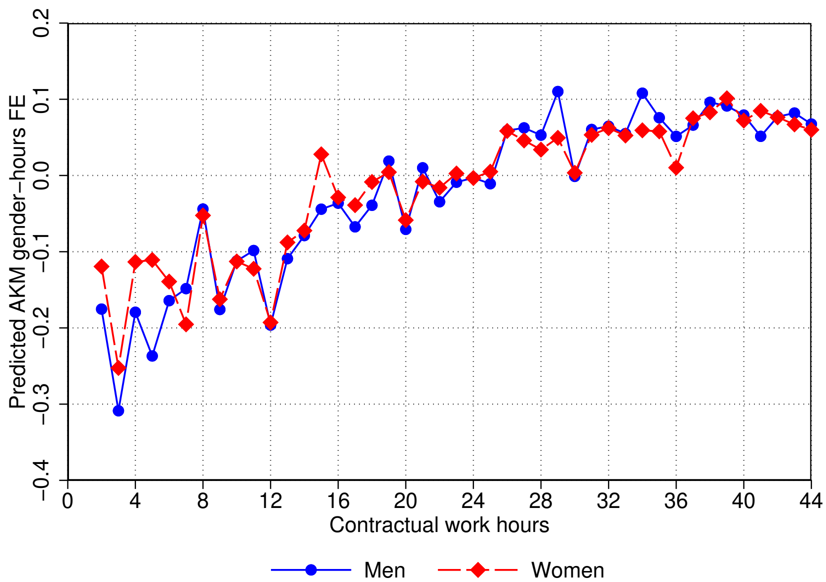
Figure 15. Comparison of gender-specific employment distributions



Source: RAIS.

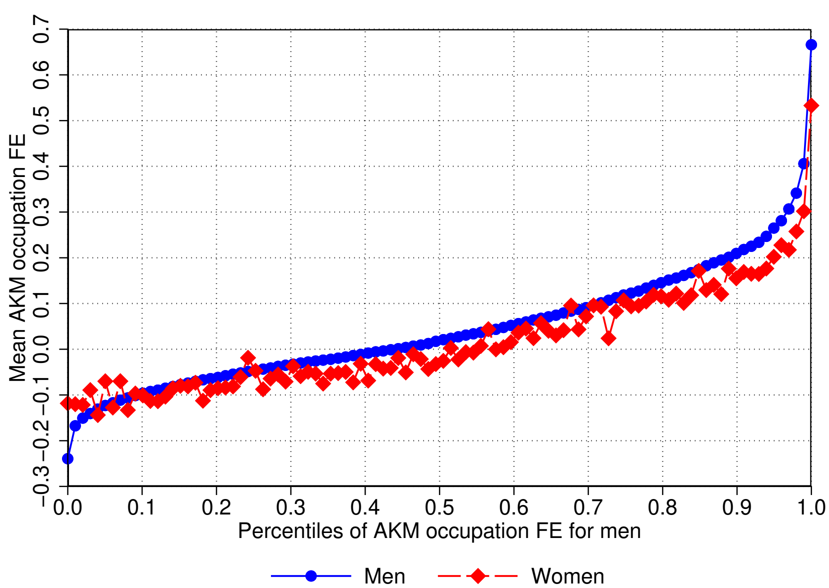
B.5 Further Details on AKM Estimation Results

Figure 16. Predicted AKM hours fixed effects, by gender



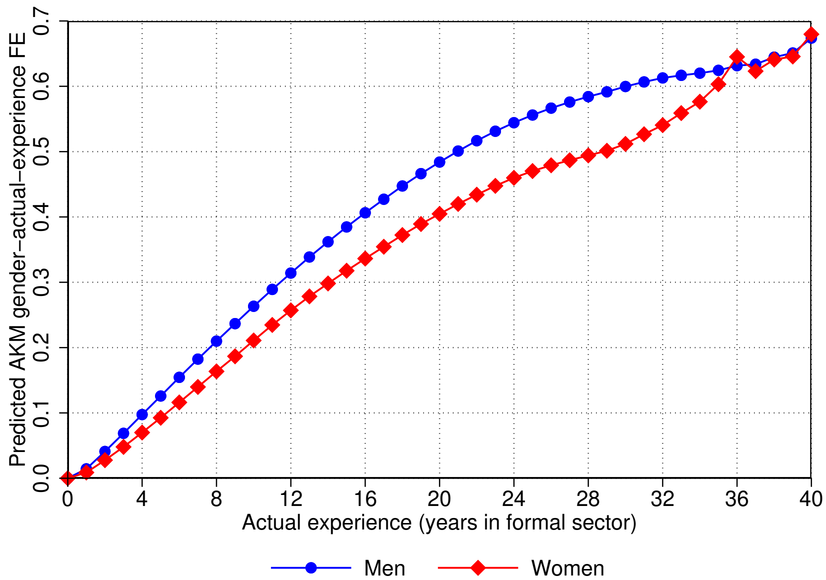
Note: The omitted category is 1 hour, for which the FE value is normalized to 0. *Source:* RAIS.

Figure 17. Predicted AKM occupation fixed effects, by gender



Source: RAIS.

Figure 18. Predicted AKM actual-experience fixed effects, by gender

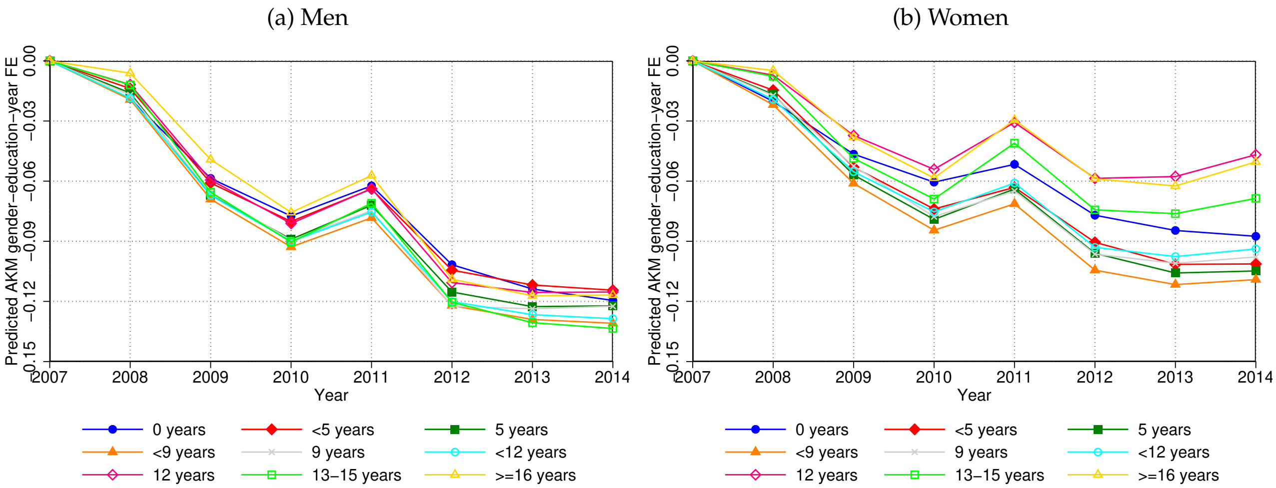


Source: RAIS.

Figure 19. Predicted AKM tenure fixed effects, by gender

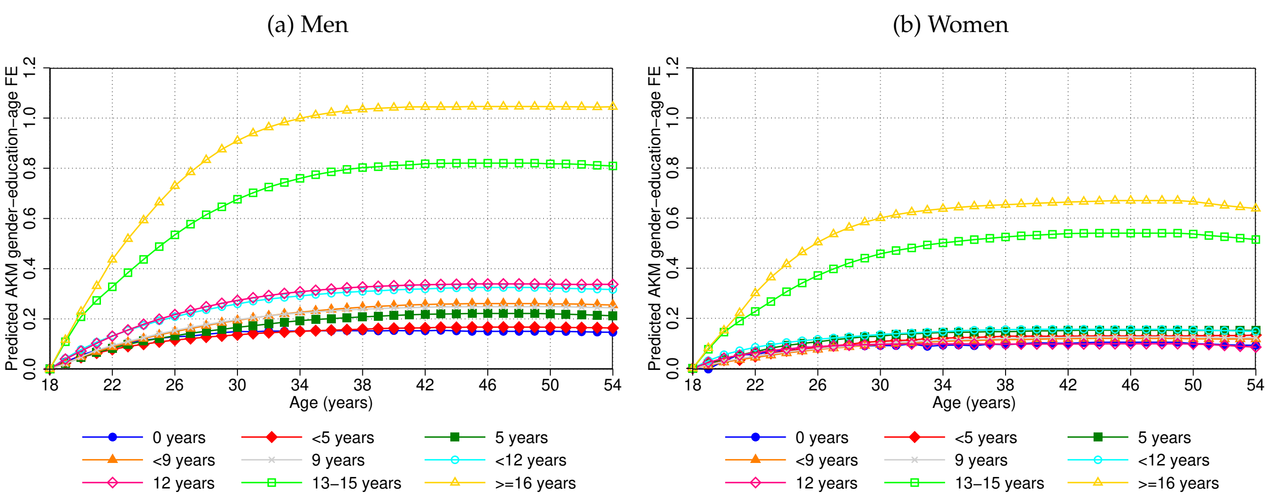
Source: RAIS.

Figure 20. Predicted AKM education-year fixed effects, by gender



Source: RAIS.

Figure 21. Predicted AKM education-age fixed effects, by gender

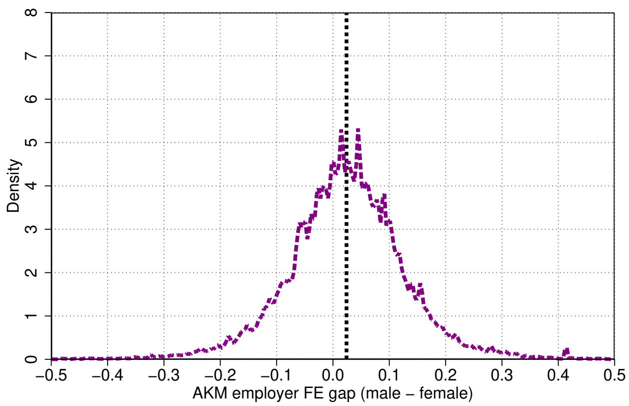


Source: RAIS.

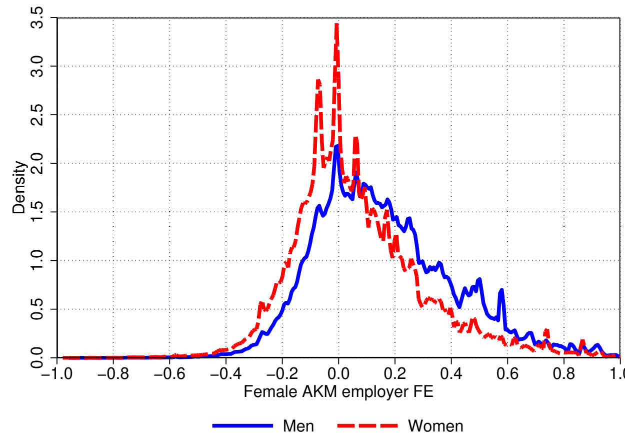
B.6 Further Details on Between vs. Within-Employer Pay Differences

Figure 22. Components of Oaxaca-Blinder decompositions

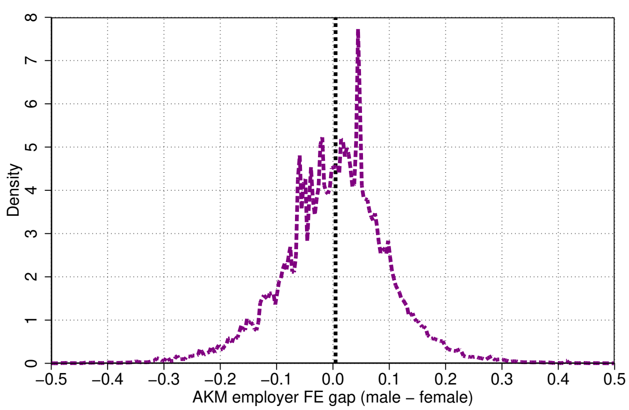
(a) Decomposition 1: Within-gap using male weights



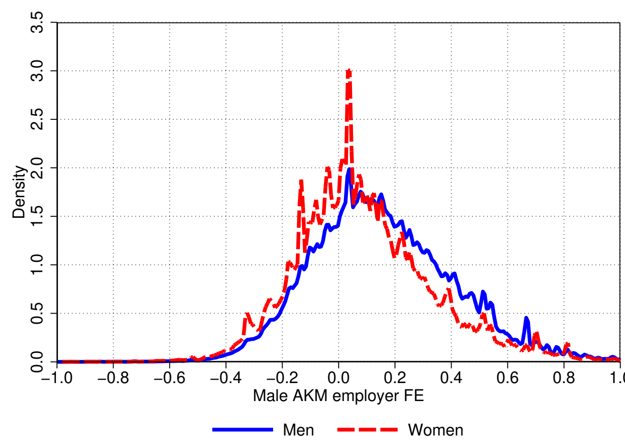
(b) Decomposition 1: Between-gap using female FEs



(c) Decomposition 2: Within-gap using female weights



(d) Decomposition 2: Between-gap using male FEs



Note: Decompositions 1 and 2 correspond to equations (2) and (3), respectively. Dashed vertical line shows mean of the distribution. *Source:* RAIS.

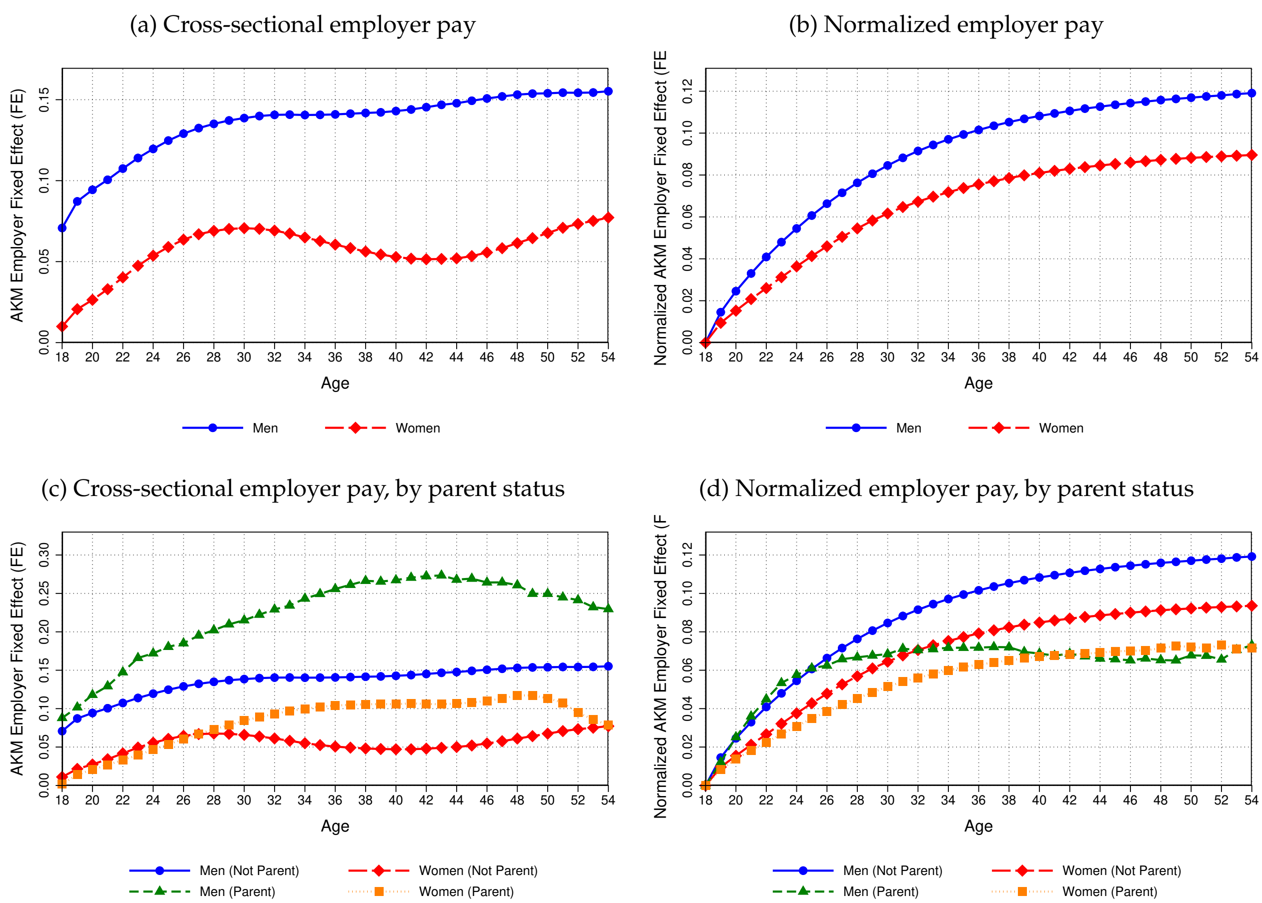
B.7 Life-Cycle Profiles by Gender and Parent Status

In this section, we are interested in life-cycle patterns in employer heterogeneity and how they differ by gender and parental status.[40] We compute two types of life-cycle statistics. The first set of statistics comprises raw, cross-sectional binned means. The second set of statistics comprises binned means of differenced variables, which we normalize to 0 at age 18.

Figure 23 shows estimated gender-specific employer FEs by gender and parent status. A few things are worth noting. First, cross-sectional life cycles (panels (a) and (c)) can be quite different from the normalized life cycles (panels (b) and (d)), plausibly due to cohort effects and other dimensions of permanent individual heterogeneity that is differenced out in the normalized statistics. Second, both men and women see marked growth in employer FEs over their life-cycle, although men significantly more so than women (panel (b)). Third, parent men look more similar to women in general, and to women with children in particular, compared to nonparent men, although nonparent women still look quite different from nonparent men (panel (d)).

Altogether, these life-cycle patterns suggest that childbirth could play some role in explaining some part of the gender pay gap.

Figure 23. Life-cycle mean gender-specific employer FEs, by gender and parent status



Source: Authors' calculations based on RAIS. *Note:* Cross-sectional estimates are simple binned means. Normalized estimates are binned means of differenced variable, normalized to 0 at age 18.

[40] We classify individuals as "parent" if they ever had a child during the sample period 2007–2014, and as "not parent" if they did not.

B.8 Event Study Analysis around Parental Leaves by Gender

Following Kleven et al. (2016), we estimate the following event-study regression for individual i of gender g in year s and at event time t:

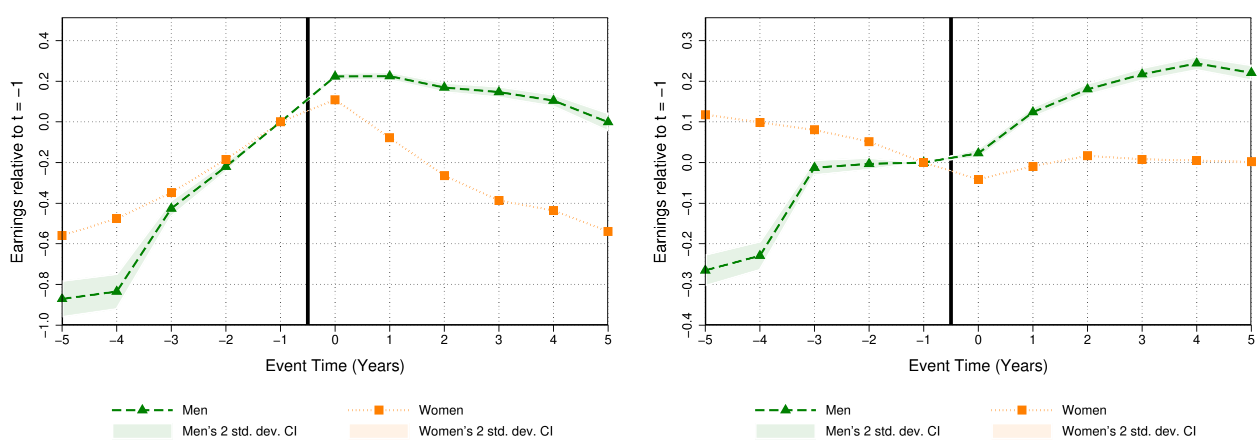
$$y_{ist} = \sum_{t' \neq 1} \alpha^g_{t'} \mathbf{1}[t' = t] + \sum_a \beta^g_a \mathbf{1}[a = \text{age}_{is}] + \sum_{s'} \gamma^g_{s'} \mathbf{1}[s' = s] + \nu^g_{ist}, \tag{28}$$

where y_{ist} is the outcome variable of interest, $\alpha^g_{t'}$ denotes a set of gender-specific event time controls, β^g_a denotes a set of gender-specific age controls, $\gamma^g_{s'}$ denotes a set of gender-specific time controls, and ν^g_{ist} is an error term. As dependent variables, we will use the level of earnings (filling in zero earnings for missing observations) or, alternatively, log earnings (dropping missing observations). Our focus will be on estimates of the coefficients $\alpha^g_{t'}$, based on equation (28), for men and women in an 11-year window around individuals' first child birth.

Figure 24 plots the resulting event study graph, including gender-specific point estimates and confidence intervals. Panel (a) of the figure shows the event study for earnings in levels. Men and women are on comparable earnings paths leading up to the time of first childbirth, marked by the vertical black solid line. After childbirth, women's earnings markedly decline, both in absolute value and compared to men's earnings, which remain relatively more stable. Panel (b) shows the event study for earnings in logarithms. Women's earnings show a declining pretrend in the five years leading up to first childbirth, both in absolute terms and compared to men, whose earnings increase over the same preperiod. After childbirth, women's earnings take a one-year dip and then remain relatively constant over the next five years. In contrast, men's earnings grow over the five years following child birth.

Figure 24. Event-study plot of earnings relative to year before first childbirth

(a) Levels (b) Logarithms

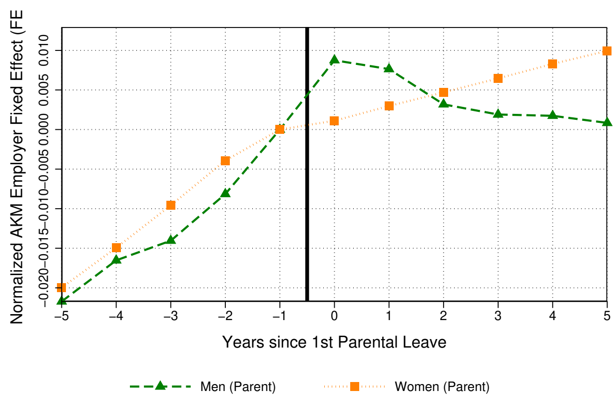


Source: Authors' calculations based on RAIS. *Note:* Vertical solid black line separates years before and after first childbirth.

Taken at face value, these results suggest that child birth has an effect on the earnings and participation of women relative to men. However, it seems that women's earnings losses around childbirth are not systematically related to changes in the employer component of earnings. Figure 25 illustrates this point by plotting an analogous event study with estimated gender-specific employer FEs from equation (1) as the dependent variable. Men and women follow a similar trend before childbirth. In the first two years after childbirth, women's gender-specific employer FE falls behind that for men but the pattern reverses during years 3 through 5. At any time in the event study, the gender gap in gender-specific employer FEs is less than 1 log point.

Altogether, this suggests that firm pay heterogeneity is *not* the only, or even a very important, factor behind women's childbirth pay penalty or the overall gender gap.

Figure 25. Event-study plot of gender-specific employer FEs rel. to year before first childbirth



Source: Authors' calculations based on RAIS. *Note:* Vertical solid black line separates years before and after first childbirth.

B.9 Details on Construction and Comparison of Employer Rank Measures

In this section, we define and implement alternative employer rank measures, which we then use to compare to the PageRank used in the main section of the paper. All three employer rank measures are consistent with a large class of on-the-job search models, including the structural framework that we will develop later on. For notational convenience, we will denote in this section the PageRank of an employer j by $r^{g,Page}(j)$.

Poaching rank. According to the poaching rank (Moscarini and Postel-Vinay, 2008; Bagger and Lentz, 2018), higher-ranked employers hire relatively more workers from employment than from unemployment. Formally, the *poaching index* is defined as the share of all new hires that are due to poached workers from other employers:

$$s^{g,poach}(j) = \frac{n^g(.,j)}{n^g(0,j) + n^g(.,j)} \tag{29}$$

where $n^g(.,j)$ is the number of gender-specific hires that employer j makes from employment at other establishments and $n^g(0,j)$ is the number of gender-specific hires that employer j makes from unemployment. Intuitively, if the underlying employer rank of an establishment is higher, then it poaches more workers from its competitors, so the poaching index is increasing in the underlying employer rank. Finally, we construct the *poaching rank* of an employer as its rank among the set of poaching indices constructed as in equation (29), with the lowest rank normalized to 0 and the highest rank normalized to 100:

$$r^{g,poach}(j) = 100\frac{\sum_{j' \in \mathcal{J}^g} \mathbf{1}\left[s^{g,poach}(j') \leq s^{g,poach}(j)\right]}{N^g} \tag{30}$$

Net poaching rank. According to the net poaching rank (Haltiwanger et al., 2018; Moscarini and Postel-Vinay, 2018), higher-ranked employers hire relatively more workers from other competitors and lose relatively fewer workers to other competitors. Formally, the *net poaching index* is defined as the net growth rate of an establishment's employment due to job-to-job transitions into and out of it:

$$s^{g,net}(j) = \frac{n^g(.,j) - n^g(j,.)}{E^g(j)} \tag{31}$$

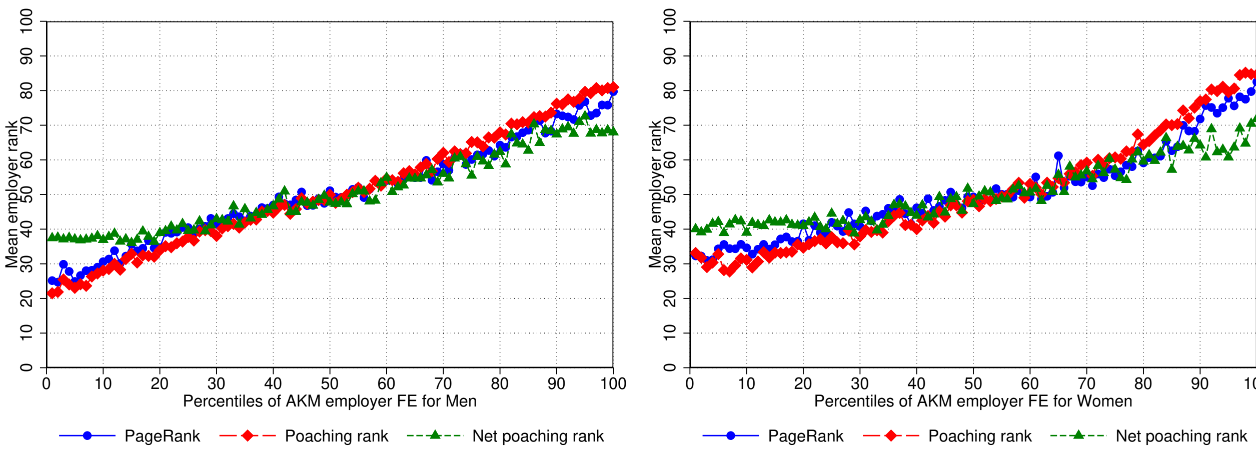
where $n^g(.,j)$ is the number of hires that employer j makes from employment at other establishments, $n^g(j,.)$ is the number of workers that employer j loses to other establishments through job-to-job transitions, and $E^g(j)$ is the gender-specific size of the workforce of employer j. Intuitively, if the underlying employer rank of an establishment is higher, then it poaches more workers from its competitors and retains more of its own workers, so the net poaching index is increasing in the underlying employer rank. Finally, we construct the *net poaching rank* of an employer as its rank among the set of net poaching indices constructed as in equation (31), with the lowest rank normalized to 0 and the highest rank normalized to 100:

$$r^{g,net}(j) = 100\frac{\sum_{j' \in \mathcal{J}^g} \mathbf{1}\left[s^{g,net}(j') \leq s^{g,net}(j)\right]}{N^g} \tag{32}$$

Comparison of alternative employer rank measures. To compare the PageRank from the main text with the poaching rank from equation (30) and the net poaching rank from equation (32), Figure 26 shows the relationship of the means of the three employer rank measures with the estimated employer pay FEs by gender. There is a strong positive correlation between all three indices across genders. Although they are not perfectly correlated, particularly in the tails, the overall shape and slope across employer FE ranks is remarkably similar.

Figure 26. Comparison of employer rank measures

(a) Men (b) Women



Source: RAIS.

While all three employer rank measures are strongly related on average, there are also some important discrepancies between them. Table 16 shows rank correlations between the three employer rank measures and also their pay rank by gender. The correlation between PageRank and poaching rank is 0.549 for men and 0.552 for women. That between between PageRank and net poaching rank is 0.226 for men and 0.236 for women. All three measures are positively related to pay rank, with correlations between 0.256 and 0.469 for men and between 0.242 and 0.448 for women. Note that the rank correlation between the net poaching rank and other employment ranks as well as pay rank is relatively weak and suggesting a larger role for nonpay employer characteristics in explaining the data.

Table 16. Rank correlations of various employer rank measures, by gender

	Men				Women			
	PageRank	Poaching rank	Net poaching rank	Pay rank	PageRank	Poaching rank	Net poaching rank	Pay rank
PageRank	1.000				1.000			
Poaching rank	0.549	1.000			0.552	1.000		
Net poaching rank	0.226	0.425	1.000		0.236	0.430	1.000	
Pay rank	0.400	0.469	0.256	1.000	0.359	0.448	0.242	1.000

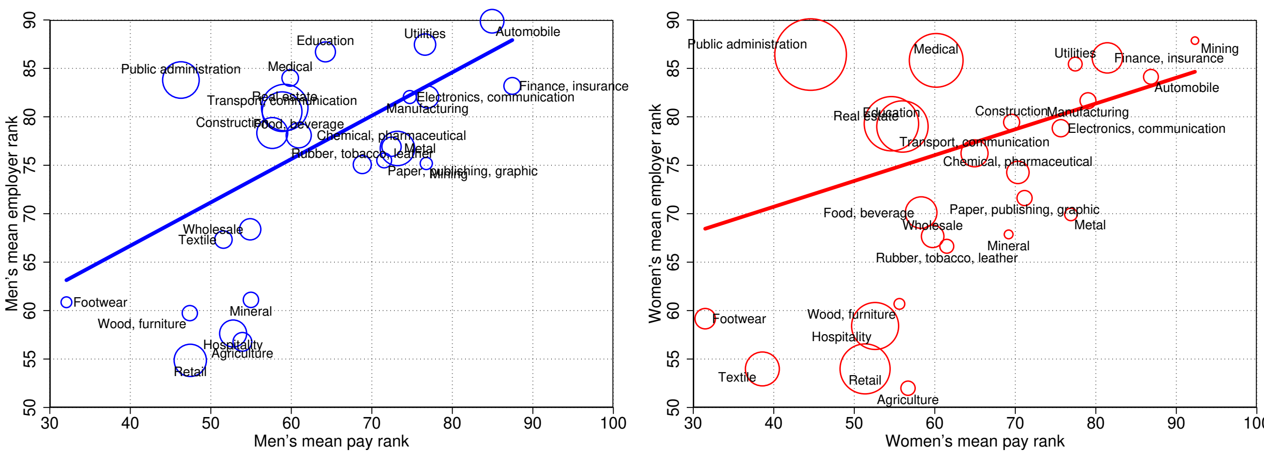
Source: RAIS.

B.10 Further Details on Fact 2

Employer ranks versus pay by industry. We find interesting heterogeneity in pay and employer ranks across industries for both genders. Figure 27 shows the mean pay ranks and mean employer ranks across 25 industries, with circle sizes representing employment shares and the solid line showing the weighted linear best fit. Comparing panel (a) and panel (b), the aggregate positive correlation between employer ranks and pay ranks is again evident, as is the lower gradient for women compared to men. There are many similarities between industry-level mean pay ranks and mean employer ranks across genders. For example, Footwear is the lowest-paying of all setors, while Retail and also Agriculture are the lowest-ranked sectors. Sectors that are high-paying and attractive for both genders include Finance and Insurance, Utilities, and the Automobile sector. However, there are also interesting differences across genders. For example, Agriculture, Metal, and the Rubber, Tobacco, and Leather sector are relatively preferred by men, while the Medical sector and Public Administration are relatively preferred by women.

Figure 27. Employer ranks versus pay across industries, by gender

(a) Men (b) Women



Source: Authors' calculations based on RAIS. *Note:* Circle size is proportional to employment share. Solid line is weighted linear best fit.

Regression analysis of employer rank-pay relationship. Table 17 shows the results of regressions of employer rank on employer pay rank with various controls. Columns (1) and (4) repeat the raw employer rank-pay rank relationship from above, which shows a gradient of 0.401 for men and 0.323 for women. In columns (2) and (5), 5-digit industry FEs are added as controls, which reduces the gradient for both genders and somewhat more so for men, resulting in gradients of 0.364 for men and 0.316 for women. Finally, columns (3) and (6) add municipality FEs as controls. In this richest specification, the gradients are reduced further and by approximately the same absolutely amount for both genders, resulting in a gradient of 0.314 for men and 0.255 for women. We conclude that there remains a positive correlation between employer rank and pay rank that is steeper for men than for women, even within narrowly defined industries and geographic units.

Changes in pay by type of employer rank transition. In line with a job-ladder view of the world, Table 18 shows the conditional changes in earnings and in establishment FEs upon making an employment-to-employment (henceforth "E-to-E") transition between two consecutive years. We see that both men and women on average see an increase in earnings (employer FEs) upon making an E-to-E transition, and disproportionately so when moving up the employer rank distribution. However, the share of workers who see an increase in their earnings (employer FEs) upon transitioning is far below 1, namely around 0.591 (0.630) for men and 0.586 (0.619) for women. Furthermore, the absolute change and also the share of transitions with positive changes in earnings (employer FEs) is higher for men than for women. From this we conclude that pay is positively but imperfectly correlated

Table 17. Employer rank-pay rank gradient, various controls

	Men			Women		
	(1)	(2)	(3)	(4)	(5)	(6)
Employer pay rank	0.401***	0.364***	0.314***	0.323***	0.316***	0.255***
	(0.001)	(0.001)	(0.002)	(0.002)	(0.002)	(0.002)
Industry FEs		✓	✓		✓	✓
Municipality FEs			✓			✓
Observations	143,745,8690	143,745,8690	143,745,8690	88,059,962	88,059,962	88,059,962
R^2	0.191	0.396	0.461	0.124	0.438	0.529

Source: Authors' calculations based on RAIS. *Note:* ***, **, * denote significance at 1%, 5%, 10% levels.

with employer ranks, and that for men pay is more closely aligned with employer ranks than it is for women.

Table 18. Changes in earnings and employer FEs upon E-to-E transitions, by gender

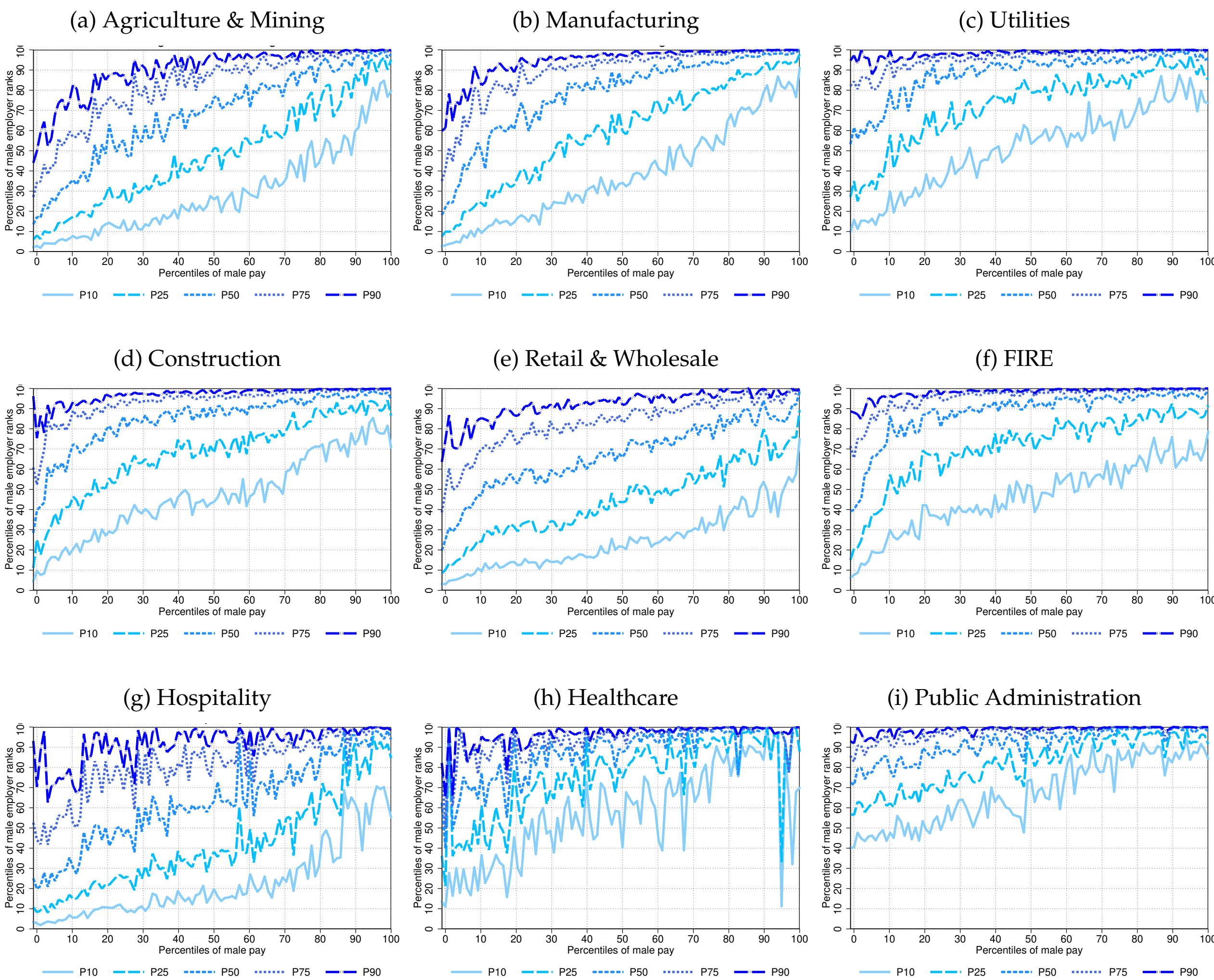
	Men			Women		
	Overall	To lower rank	To higher rank	Overall	To lower rank	To higher rank
$\mathbb{E}\left[\Delta y_{ijt}\right]$	0.031	-0.018	0.072	0.033	-0.008	0.068
$\mathbb{E}\left[\mathbf{1}\left[\Delta y_{ijt} > 0\right]\right]$	0.549	0.500	0.591	0.552	0.512	0.586
$\mathbb{E}\left[\Delta \psi_j\right]$	0.021	-0.033	0.066	0.022	-0.024	0.060
$\mathbb{E}\left[\mathbf{1}\left[\Delta \psi_j > 0\right]\right]$	0.540	0.435	0.630	0.541	0.449	0.619
Share	0.206	0.095	0.112	0.128	0.058	0.070
Share of E-to-E transitions	1.000	0.459	0.541	1.000	0.456	0.544

Source: RAIS.

B.11 Further Details on Fact 3

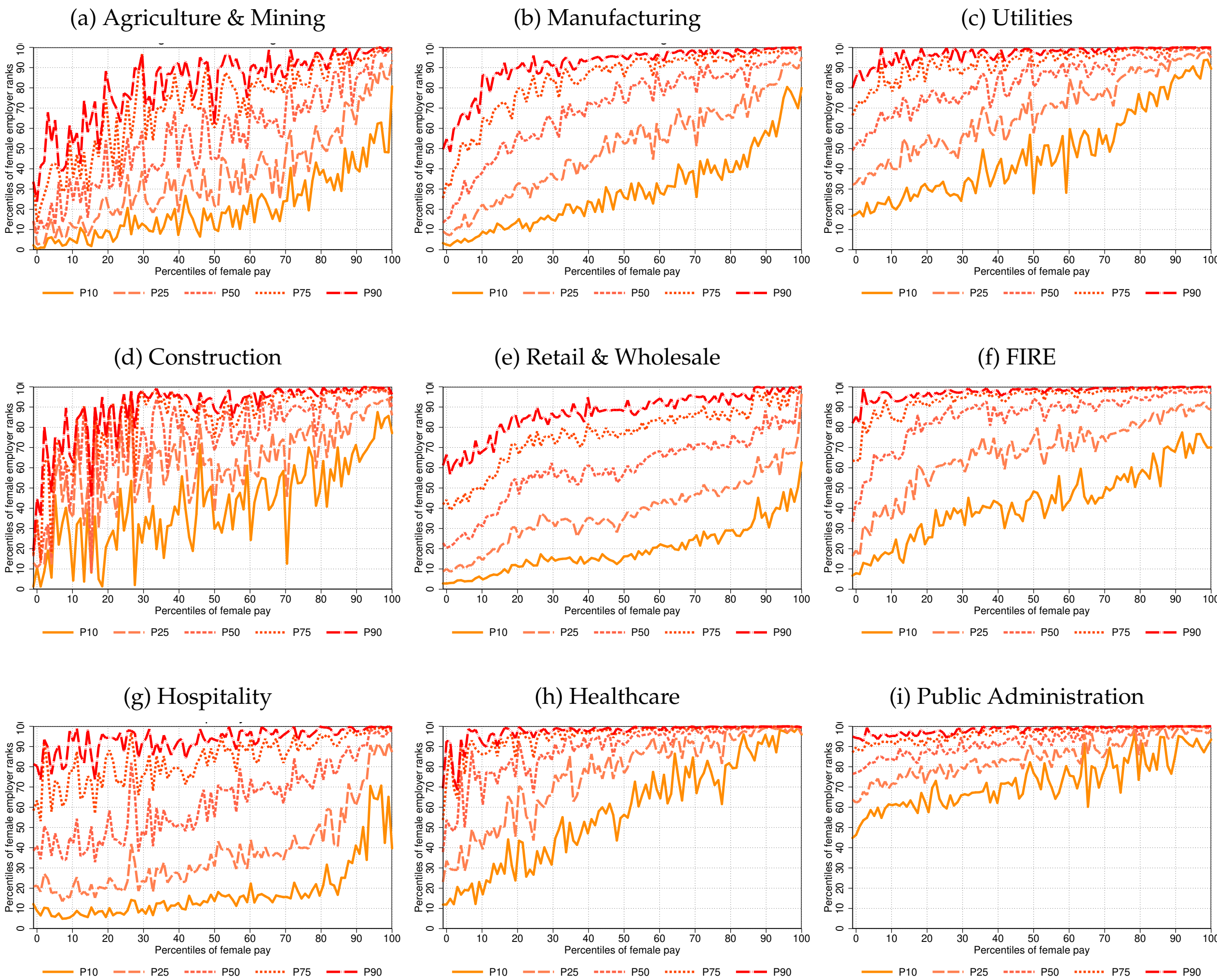
Percentiles of employer rank distribution conditional on pay.

Figure 28. Percentiles of employer ranks conditional on pay ranks for men, by industry



Notes: FIRE stands for the Finance, Insurance, and Real Estate industry. *Source:* RAIS.

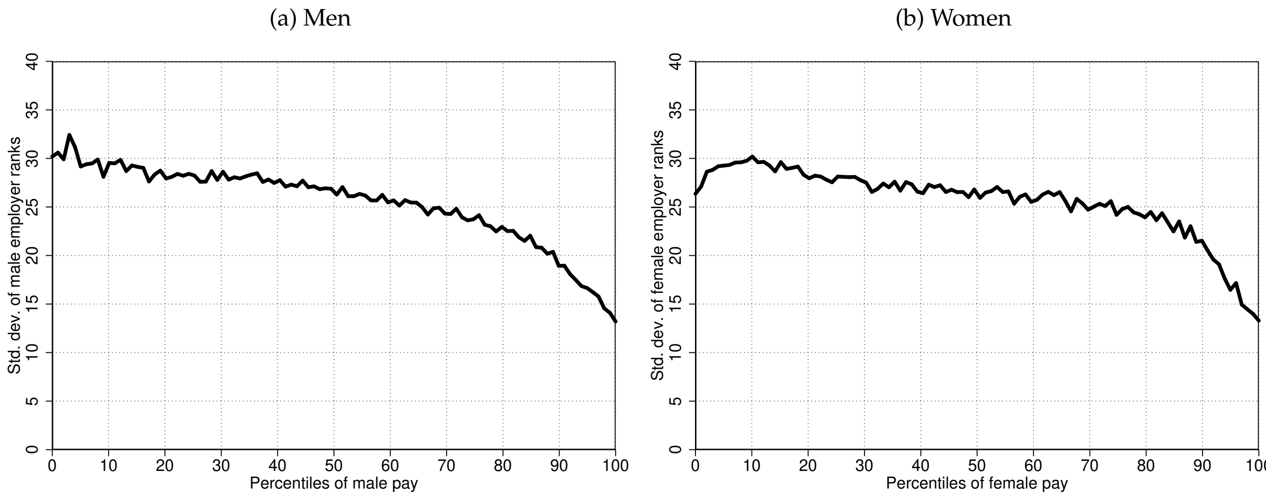
Figure 29. Percentiles of employer ranks conditional on pay ranks for women, by industry



Notes: FIRE stands for the Finance, Insurance, and Real Estate industry. *Source:* RAIS.

Dispersion in employer ranks conditional on pay.

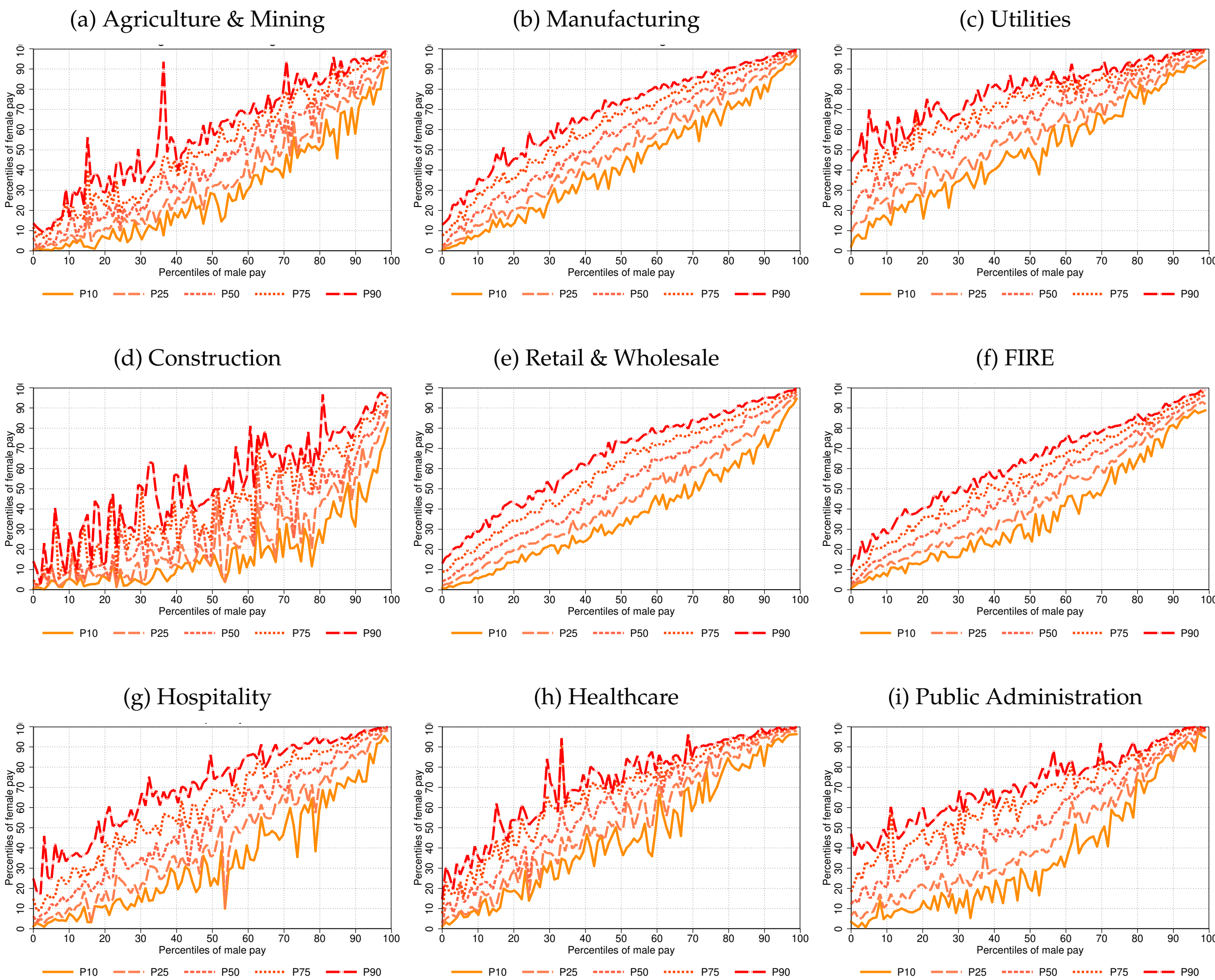
Figure 30. Standard deviation of employer ranks conditional on pay rank, by gender



Source: RAIS.

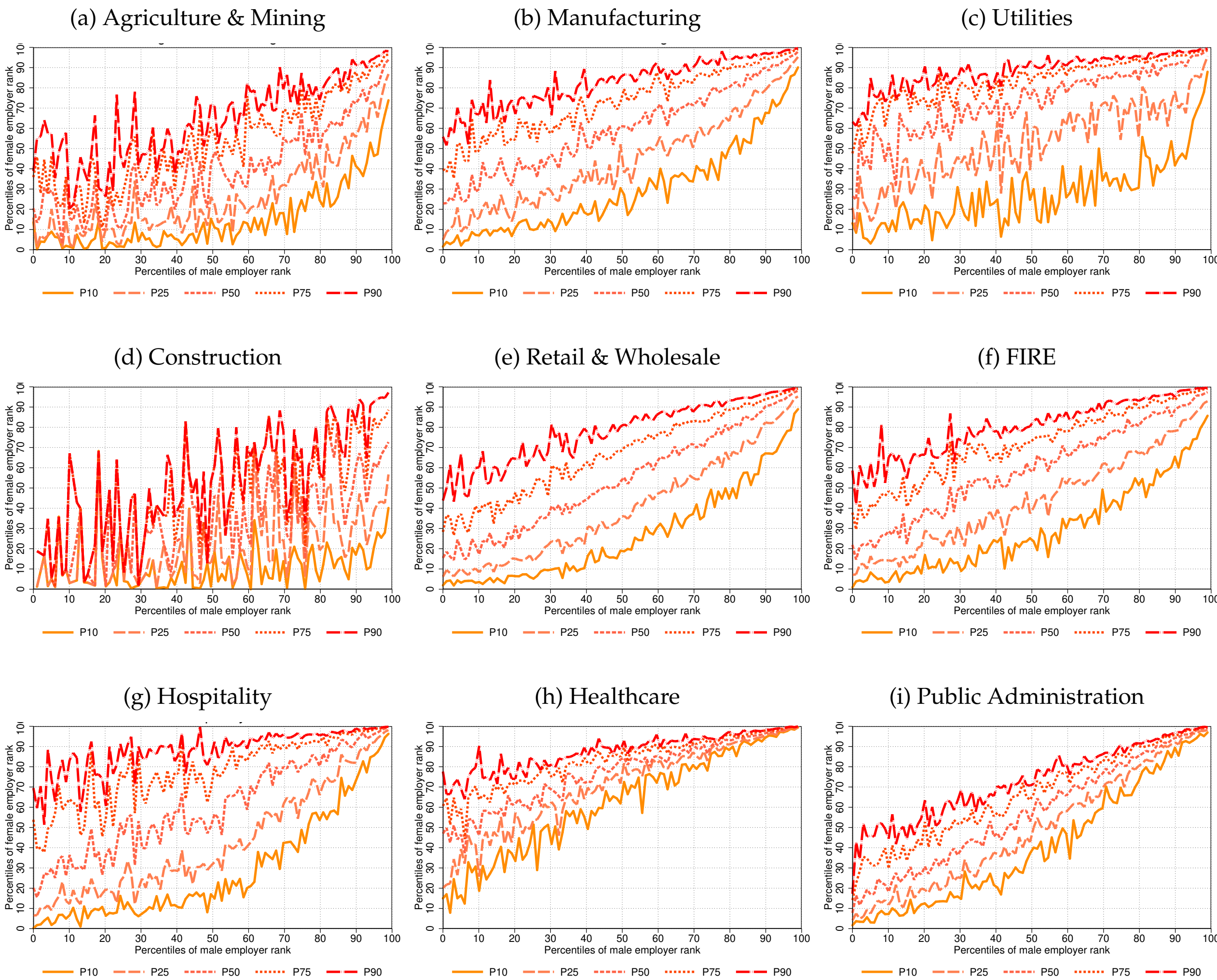
Cross-gender comparisons of pay and employer ranks.

Figure 31. Female vs. male pay, by industry



Notes: FIRE stands for the Finance, Insurance, and Real Estate industry. *Source:* RAIS.

Figure 32. Female vs. male employer ranks, by industry



Notes: FIRE stands for the Finance, Insurance, and Real Estate industry. *Source:* RAIS.

C Model Appendix

C.1 Definition of a Stationary Equilibrium

We are now ready to define a *stationary search equilibrium* for this economy.

Definition. *A stationary search equilibrium is a set of worker value functions $\{S_{a,g}, W_{a,g}\}_{a,g}$ and policy functions $\{\phi_{a,g}\}_{a,g}$; firm value function Π and policy functions $\{w_{a,g}, \pi_{a,g}, v_{a,g}\}_{a,g}$; flow-utility offer distributions $\{F_{a,g}(x)\}_{a,g}$; measures of unemployed workers $\{u_{a,g}\}_{a,g}$, aggregate job searchers $\{U_{a,g}\}_{a,g}$, aggregate vacancies $\{V_{a,g}\}_{a,g}$, and labor market tightnesses $\{\theta_{a,g}\}_{a,g}$; job offer arrival rates $\{\lambda^u_{a,g}, \lambda^e_{a,g}, \lambda^G_{a,g}\}_{a,g}$; and firm sizes $\{l_{a,g}\}_{a,g}$ such that for all (a, g):*

- *Given $F_{a,g}(x)$ and $\{\lambda^u_{a,g}, \lambda^e_{a,g}, \lambda^G_{a,g}\}$, the value functions $S_{a,g}$ and $W_{a,g}$ satisfy equations (5) and (6);*
- *Unemployed workers' job acceptance policy follows a threshold rule $\phi_{a,g}$ given by equation (7) and employed workers with flow utility x accept any job x' such that $x' > x$;*
- *Given $l_{a,g}$, firms' value function Π is given by equation (9);*
- *Firms policy functions $\{w_{a,g}, \pi_{a,g}, v_{a,g}\}$ solve the problem in equation (9);*
- *Measures of unemployed workers are given by equation (8), aggregate job searchers $U_{a,g}$ are given by equation (10), aggregate vacancies $V_{a,g}$ are given by equation (11), and labor market tightness $\theta_{a,g}$ is given by equation (12).*
- *Given $\theta_{a,g}$, the job offer arrival rates $\{\lambda^u_{a,g}, \lambda^e_{a,g}, \lambda^G_{a,g}\}$ satisfy equation (13);*
- *Given $F_{a,g}(x)$, $\{\lambda^u_{a,g}, \lambda^e_{a,g}\}_{a,g}$, $\lambda^G_{a,g}$, and $V_{a,g}$, firm sizes satisfy equation (14);*
- *The offer distribution satisfies $F_{a,g}(x) = \int_j v_{a,g}(j)\mathbf{1}[x_{a,g}(j) \le x]\, d\Gamma(j)/V_{a,g}$.*

C.2 Additional Proofs

C.2.1 Proof of Lemma 1 (Optimal Amenities)

Proof. Based on the insight that workers care only about the flow utility of a job, we can rewrite the problem of a firm in equation (9) as one of choosing in each market a flow utility x and vacancies v that solve the following problem:

$$\max_{x,v} \left\{ \left[pa - c^x_{a,g}(x) - z_{a,g}\right] l_{a,g}(x,v) - c^v_{a,g}(v) \right\}, \quad \forall (a,g),$$

where $c^x_{a,g}(x)$ is the solution to the following cost-minimization subproblem in each market:

$$c^x_{a,g}(x) = \min_{w,\pi} \left\{ w + c^\pi_{a,g}(\pi) \right\} \quad \text{s.t.} \quad w + \pi = x \tag{33}$$

Once written in this way, it is evident that an interior solution to the firm's cost-minimization problem in equation (33) is characterized by the following optimality conditions:

$$\begin{gathered} c^{\pi,0}_{a,g} \times \frac{\partial \tilde{c}^\pi_a(\pi^*)}{\partial \pi} = 1 \\ w^* = x - \pi^*, \quad \forall (a,g) \end{gathered} \tag{34}$$

Equation (34) uniquely pins down a firm's optimal amenity choice $\pi^*_{a,g}(c^{\pi,0}_{a,g})$ for every market a as a function of only the heterogeneous amenity cost shifter $c^{\pi,0}_{a,g}$. Obviously, $\partial\pi^*/\partial c^{\pi,0}_{a,g} < 0$ by the chain rule. The optimal wage is then chosen to deliver the remainder of flow utility x.[41]

Since $\tilde{c}^{\pi}_{a}(0) = 0$ and $\partial\tilde{c}^{\pi}_{a}/\partial\pi(0) = 0$, a firm will always create some amount of amenities $\pi_{a,g} > 0$. Finally, we have

$$c^{\pi}_{a,g}(\pi^*) = \int_0^{\pi^*} \frac{\partial\tilde{c}^{\pi}_{a}(\pi)}{\partial\pi}\, d\pi \;<\; \int_0^{\pi^*} 1\, d\pi \;=\; \pi^*.$$

□

C.2.2 Proof of Lemma 2 (Optimal Market Selection)

Proof. Recall that composite productivity is defined as output value plus amenity value net of amenity production costs minus employer distaste, $\tilde{p}_{a,g} = pa + \pi_{a,g} - c^{\pi}_{a,g}(\pi_{a,g}) - z_{a,g}$. Since the vacancy cost function satisfies $c^{v}_{a,g}(0) = 0$ and $\partial c^{v}_{a,g}(0)/\partial v = 0$, a firm makes positive profits if and only if it makes positive profits per worker: $\tilde{p}_{a,g} - x > 0$. To attract workers in a market, a firm has to offer flow utility equal to or higher than the outside option of workers through a combination of wages and amenities: $x \geq \phi_{a,g}$. Therefore, a firm is profitably active in a market if and only if $\tilde{p}_{a,g} > \phi_{a,g}$. □

C.2.3 Proof of Lemma 3 (Optimal Vacancies)

Proof. We first reformulate the firm's problem. Expected profits per worker contacted by a firm is

$$\pi_{a,g}(\tilde{p}, x) = h_{a,g}(x) J_{a,g}(\tilde{p}, x),$$

where $h_{a,g}(x)$ is the acceptance probability and $J_{a,g}(\tilde{p}, x)$ is the value of employing a worker to a firm with composite productivity $\tilde{p}$ providing flow utility x. Under the assumption that firms maximize long-run profits, the value of employing a worker is simply

$$\begin{aligned} J_{a,g}(\tilde{p}, x) &= \frac{\tilde{p} - x}{\delta_{a,g} + \lambda^{e}_{a,g}(1 - F_{a,g}(x)) + \lambda^{G}_{a,g}} \\ &= \frac{(\tilde{p} - x) \,/ \left(\delta_{a,g} + \lambda^{G}_{a,g}\right)}{1 + \kappa^{e}_{a,g}\left(1 - F_{a,g}\left(x\right)\right)}, \end{aligned}$$

[41] Taking into account possible corner solutions, the optimal wage-amenity combination takes the following form:

$$\pi^{**}_{a,g}\left(x, c^{\pi,0}_{a,g}\right) = \begin{cases} x & \text{if } x < \overline{x}\left(c^{\pi,0}_{a,g}\right) \\ \pi^{*}_{a,g}\left(c^{\pi,0}_{a,g}\right) & \text{if } x \geq \overline{x}\left(c^{\pi,0}_{a,g}\right) \end{cases}, \quad w^{**}_{a,g}\left(x, c^{\pi,0}_{a,g}\right) = \begin{cases} 0 & \text{if } x < \overline{x}\left(c^{\pi,0}_{a,g}\right) \\ x - \pi^{**}_{a,g}\left(c^{\pi,0}_{a,g}, x\right) & \text{if } x \geq \overline{x}\left(c^{\pi,0}_{a,g}\right) \end{cases},$$

where $\overline{x}\left(c^{\pi,0}_{a,g}\right)$ solves $\partial c^{\pi}_{a,g}(\overline{x}\left(c^{\pi,0}_{a,g}\right))/\partial\pi = 1$. Note, however, that in such corner solutions the optimal wage is $w^{**} = 0$, which is empirically not relevant. Naturally, going forward we focus on the case of an interior solution.

The acceptance probability for a firm offering x is

$$\begin{aligned}
h_{a,g}(x) &= \frac{u_{a,g} + s^e_{a,g}\left(1-u_{a,g}\right) G_{a,g}\left(x\right) + s^G_{a,g}}{u_{a,g} + s^e_{a,g}\left(1-u_{a,g}\right) + s^G_{a,g}} \\
&= \frac{\delta_{a,g} + s^e_{a,g}\left(\lambda^u_{a,g} + \lambda^G_{a,g}\right) G_{a,g}\left(x\right) + s^G_{a,g}\left(\delta_{a,g} + \lambda^u_{a,g} + \lambda^G_{a,g}\right)}{\delta_{a,g} + s^e_{a,g}\left(\lambda^u_{a,g} + \lambda^G_{a,g}\right) + s^G_{a,g}\left(\delta_{a,g} + \lambda^u_{a,g} + \lambda^G_{a,g}\right)} \\
&= \frac{1 + s^e_{a,g}\kappa^u_{a,g} G_{a,g}\left(x\right) + s^G_{a,g}\left(1+\kappa^u_{a,g}\right)}{1 + s^e_{a,g}\kappa^u_{a,g} + s^G_{a,g}\left(1+\kappa^u_{a,g}\right)} \\
&= \frac{1 + s^e_{a,g}\kappa^u_{a,g}\left[\frac{F_{a,g}(x)}{1+\kappa^e_{a,g}\left[1-F_{a,g}(x)\right]}\right] + s^G_{a,g}\left(1+\kappa^u_{a,g}\right)}{1 + s^e_{a,g}\kappa^u_{a,g} + s^G_{a,g}\left(1+\kappa^u_{a,g}\right)} \\
&= \frac{1 + \kappa^e_{a,g}\left[1-F_{a,g}\left(x\right)\right] + s^e_{a,g}\kappa^u_{a,g} F_{a,g}\left(x\right) + s^G_{a,g}\left(1+\kappa^u_{a,g}\right)\left[1 + \kappa^e_{a,g}\left[1-F_{a,g}\left(x\right)\right]\right]}{\left[1 + s^e_{a,g}\kappa^u_{a,g} + s^G_{a,g}\left(1+\kappa^u_{a,g}\right)\right]\left[1 + \kappa^e_{a,g}\left[1-F_{a,g}\left(x\right)\right]\right]},
\end{aligned}$$

where $\kappa^u_{a,g} = (\lambda^u_{a,g} + \lambda^G_{a,g})/\delta_{a,g}$. Combining expressions, expected profits per contacted worker are

$$\begin{aligned}
\pi\left(\tilde{p}, x\right) &= h\left(x\right) J\left(\tilde{p}, x\right) \\
&= \frac{\left\{1 + \kappa^e_{a,g}\left[1-F_{a,g}\left(x\right)\right] + s^e_{a,g}\kappa^u_{a,g} F_{a,g}\left(x\right) + s^G_{a,g}\left(1+\kappa^u_{a,g}\right)\left[1 + \kappa^e_{a,g}\left[1-F_{a,g}\left(x\right)\right]\right]\right\}\left(\tilde{p} - x\right)}{\left[1 + s^e_{a,g}\kappa^u_{a,g} + s^G_{a,g}\left(1+\kappa^u_{a,g}\right)\right]\left[1 + \kappa^e_{a,g}\left(1-F_{a,g}\left(x\right)\right)\right]^2\left(\delta_{a,g} + \lambda^G_{a,g}\right)}.
\end{aligned} \tag{35}$$

Then the firm's problem becomes

$$\max_{x,v}\left\{\pi_{a,g}\left(\tilde{p}, x\right) v q_{a,g} - c^v_{a,g}\left(v\right)\right\}.$$

Therefore, the optimal flow-utility and vacancy policy functions satisfy

$$\begin{aligned}
x^*_{a,g}\left(\tilde{p}, \cdot\right) &= \arg\max_x \pi_{a,g}\left(\tilde{p}, x\right) \\
\frac{\partial c^v_{a,g}\left(v^*\left(\tilde{p}, \cdot\right)\right)}{\partial v} &= \max_x \pi_{a,g}\left(\tilde{p}, x\right).
\end{aligned} \tag{36}$$

Since the vacancy cost function $c^v\left(\cdot\right)$ is convex, and $\pi\left(\tilde{p}, x\right)$ in equation (35) is strictly increasing in $\tilde{p}$, then it follows from an application of the envelope theorem to equation (36) that $v^*\left(\tilde{p}, \cdot\right)$ is strictly increasing in $\tilde{p}$. Therefore, $v^*_{a,g}(\cdot)$ is strictly increasing in productivity p and strictly decreasing (constant) in z_a for women (men). Since $c^v_{a,g}(v_{a,g}) = c^{v,0}_{a,g} \times \tilde{c}^v(\pi_{a,g})$, equation (36) also yields that optimal mass of vacancies is strictly decreasing in the vacancy cost shifter $c^{v,0}_{a,g}$. □

C.2.4 Proof of Lemma 4 (Optimal Flow Utility and Wages)

Proof. We proceed in two steps.

Step 1. In the first step, we prove monotonicity of $x^*_{a,g}$ in components of $\tilde{p}_{a,g}$. Lemma 1 implies that, at the optimum, amenities can be equivalently considered exogenous. Thus, we rewrite the FOCs as functions of exogenous parameters, the endogenous offer distribution, and $x_{a,g}$:

$$[\partial x_{a,g}]: \quad 1 = (\tilde{p}_{a,g} - x_{a,g}) \frac{2\lambda^e_{a,g} f_{a,g}(x_{a,g})}{\delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g}(1 - F_{a,g}(x_{a,g}))} \tag{37}$$

$$[\partial v_{a,g}]: \quad c^{v,0}_{a,g} \frac{\partial \tilde{c}^v(v_{a,g})}{\partial v_{a,g}} = T_{a,g}(\tilde{p}_{a,g} - x_{a,g}) \left(\frac{1}{\delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g}(1 - F_{a,g}(x_{a,g}))} \right)^2, \tag{38}$$

where $T_{a,g} = \mu_{a,g}[(u_{a,g} + s^G_{a,g})\lambda^u_{a,g}(\delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g})]/V_{a,g}$. Equation (37) already shows that the optimal flow utility $x_{a,g}$ is independent of the cost of posting vacancies, proving the first statement. Now consider equation (38); because the term on the right-hand side is always positive for $\tilde{p}_{a,g} > \phi_{a,g}$, it follows that optimal vacancies $v^*_{a,g}(\tilde{p}_{a,g}, c^{v,0}_{a,g})$ are always strictly positive.

We now show that the derivative of wages with respect to $\tilde{p}_{a,g}$ is always positive. Define $h_{a,g}(\tilde{p}_{a,g}) = F_{a,g}(x^*_{a,g}(\tilde{p}_{a,g}))$. Thus:

$$\begin{aligned} h_{a,g}(\tilde{p}_{a,g}) =& \frac{\int_{\tilde{p}' \geq \phi_{a,g}}^{\tilde{p}_{a,g}} \overline{v}^*_{a,g}(\tilde{p}_{a,g}) \gamma_{a,g}(\tilde{p}_{a,g})}{V_{a,g}} \, d\,\tilde{p}' & (39) \\ h'_{a,g}(\tilde{p}_{a,g}) =& f_{a,g}(x^*_{a,g}(\tilde{p}_{a,g}))\, {x^*_{a,g}}'(\tilde{p}_{a,g}) & (40) \\ f_{a,g}(x^*_{a,g}(\tilde{p}_{a,g})) =& h'_{a,g}(\tilde{p}_{a,g}) / {x^*_{a,g}}'(\tilde{p}_{a,g}), & (41) \end{aligned}$$

where $\overline{v}^*_{a,g}(\tilde{p}_{a,g}) = \int v^*_{a,g}(\tilde{p}_{a,g}, c')\gamma^c_{a,g}(c'|\tilde{p}_{a,g})\, dc'$ is the integral of optimal vacancies conditional on $\tilde{p}_{a,g}$ and $\gamma^c_{a,g}(c|\tilde{p}_{a,g})$ is the density of vacancy posting costs $c^{v,0}_{a,g}$ conditional on $\tilde{p}_{a,g}$, $\gamma_{a,g}(\tilde{p}_{a,g})$ is the marginal density of composite productivity $\tilde{p}_{a,g}$ and $\partial x^*_{a,g}(\tilde{p}_{a,g})/\partial \tilde{p}_{a,g} = {x^*_{a,g}}'(\tilde{p}_{a,g})$ is the derivative of equilibrium flow utility with respect to $\tilde{p}_{a,g}$. Thus, we can rewrite $h'_{a,g}(\tilde{p}_{a,g}) = \frac{\overline{v}^*_{a,g}(\tilde{p}_{a,g})}{V_{a,g}} \gamma(\tilde{p}_{a,g})$ by differentiating equation (39) using Leibniz's integral rule.

Using these identities, we can write $f_{a,g}(x^*_{a,g}(\tilde{p}_{a,g})) = \frac{\overline{v}^*_{a,g}(\tilde{p}_{a,g})}{V_{a,g}} \gamma_{a,g}(\tilde{p}_{a,g}) \partial \tilde{p}_{a,g} / \partial x^*_{a,g}(\tilde{p}_{a,g})$. Thus, we can rewrite equation (37) as

$$\frac{\partial x^*_{a,g}(\tilde{p}_{a,g})}{\partial \tilde{p}_{a,g}} = (\tilde{p}_{a,g} - x^*_{a,g}) \frac{2\lambda^e_{a,g}}{\delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g}(1 - h_{a,g}(\tilde{p}_{a,g}))} \frac{\overline{v}^*_{a,g}(\tilde{p}_{a,g})}{V_{a,g}} \gamma_{a,g}(\tilde{p}_{a,g}). \tag{42}$$

Because the right-hand side of this expression is always positive for $\tilde{p}_{a,g} > \phi_{a,g}$, it follows that $\partial x^*_{a,g}(\tilde{p}_{a,g})/\partial \tilde{p}_{a,g} > 0$, thus proving that equilibrium flow utility is increasing in $\tilde{p}_{a,g}$.

Since $\tilde{p}_{a,g}$ is increasing in p and decreasing (constant) in z_a for women (men), it follows that optimal flow utility is increasing in p and decreasing (constant) in z_a for women (men).

Step 2. In the second step, we prove monotonicity of $w_{a,g}$ in components of $\tilde{p}_{a,g}$. The characterization of $w_{a,g} = x_{a,g} - \pi_{a,g}$ follows from combining Lemmas 1 and 4. □

C.2.5 Proof of Lemma 5 (Optimal Employment)

Proof. Consider two otherwise identical employers with composite productivities $\tilde{p}_2 > \tilde{p}_1$ and optimal flow-utility and amenity choices (x_2, v_2) and (x_1, v_1), respectively. Using the notation from equation (15), we can write

$$\begin{aligned}[\tilde{p}_2 - x_2]\, l_{a,g}(x_2, v_2) - c^v_{a,g}(v_2) &> [\tilde{p}_2 - x_1]\, l_{a,g}(x_1, v_1) - c^v_{a,g}(v_1)\\ > [\tilde{p}_1 - x_1]\, l_{a,g}(x_1, v_1) - c^v_{a,g}(v_1) &> [\tilde{p}_1 - x_2]\, l_{a,g}(x_2, v_2) - c^v_{a,g}(v_2),\end{aligned}$$

where the first and third strict inequalities follow from uniqueness of the profit-maximizing wage choice given that firm types are distributed continuously, while the second strict inequality follows trivially. Subtracting the fourth term from the first and the third term from the second, we have $l_{a,g}(x_2, v_2) > l_{a,g}(x_1, v_1)$. This proves the comparative statics with respect to firm productivity p and the intra-employer wedge $z_{a,g}$. The proof for the comparative statics with respect to the vacancy cost shifter $c^{v,0}_{a,g}$ is a direct consequence of the two results that the vacancy policy $v_{a,g}(\cdot)$ is strictly decreasing in $c^{v,0}_{a,g}$ (Lemma 3), while the flow-utility policy $x_{a,g}(\cdot)$ is constant in $c^{v,0}_{a,g}$ (Lemma 4). □

C.2.6 Proof of Proposition 1 (Equilibrium Wage Equation)

Proof. We proceed in two steps. We first prove the proposition under exogenous firm-level vacancies that are constant within but may differ across genders. We then prove that under the maintained assumptions the propositions also holds under endogenous vacancy posting.

Step 1. Suppose firms differ in their exogenous number of vacancies for each gender, $\{v_g\}_g$. Define $T_{a,g} = \mu_{a,g}[(u_{a,g} + s^G_{a,g})\lambda^u_{a,g}(\delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g})]/V_{a,g}$. First of all, Assumption 2 implies that $T_{a,g} = T_g$ for all a. Second, under exogenous vacancies a firm's type is defined by its composite productivity $\tilde{p}_{a,g}$ and its exogenous vacancies v_g, which are constant across ability markets. As a consequence, $V_{a,g} = V_g$ in all a-markets. Using equation (14), the firm's problem can be written as

$$x^*_{a,g}(\tilde{p}_{a,g}) = \arg\max_x (\tilde{p}_{a,g} - x)\left(\frac{1}{\delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g}(1 - F_{a,g}(x))}\right)^2 v_g T_g$$

Thus, given fixed vacancies, equilibrium firm profits in equation 15 can be written as

$$\Pi_{a,g}(\tilde{p}_{a,g}, v_g) = (\tilde{p}_{a,g} - x^*(\tilde{p}_{a,g}))\left(\frac{1}{\delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g}(1 - F_{a,g}(x^*(\tilde{p}_{a,g})))}\right)^2 v_g T_g \tag{43}$$

We can write the offer distribution as

$$F_{a,g}(x^*(\tilde{p}_{a,g})) = h_{a,g}(\tilde{p}_{a,g}) = \frac{1}{V_{a,g}} \int_{p' > \phi_{a,g}}^{\tilde{p}_{a,g}} \int v_g \gamma(p', v')\, dp'\, dv',$$

where $\gamma_{a,g}(p', v')$ is the joint density function of $\tilde{p}_{a,g}$ and v_g, and $h_{a,g}(\tilde{p}_{a,g})$ is the CDF of the marginal distribution of $\tilde{p}_{a,g}$, in which values are weighted by vacancies posted by firms with each particular $\tilde{p}_{a,g}$. The expression for $h_{a,g}$ is equivalent to equation (39) in Lemma 4, except that here we are integrating over exogenous rather than endogenous vacancies. Applying the Envelope Theorem yields

$$\frac{\partial \Pi_{a,g}(\tilde{p}_{a,g}, v_g)}{\partial \tilde{p}_{a,g}} = \left(\frac{1}{\delta_{a,g} + \lambda^G_{a,g} + \lambda^e_{a,g}(1 - h_{a,g}(\tilde{p}))}\right)^2 v_g T_{a,g} \tag{44}$$

When $\tilde{p}_{a,g} = \phi_{a,g}$, $\Pi(\phi_{a,g}, v_g) = 0$ for all v_g, which gives us a boundary condition to solve the differential equation for profits. Rearranging (43) and integrating equation (44) yields

$$x_{a,g}\left(\tilde{p}_{a,g}\right) = \tilde{p}_{a,g} - \int\limits_{y \geq \phi_{a,g}}^{\tilde{p}_{a,g}} \left[\frac{1+\kappa^e_{a,g}(1-h_{a,g}(\tilde{p}_{a,g}))}{1+\kappa^e_{a,g}(1-h_{a,g}(y))}\right]^2 dy, \tag{45}$$

where $\kappa^e_{a,g} = \lambda^e_{a,g}/(\delta_{a,g} + \lambda^G_{a,g})$. This equation parallels equation (47) in Burdett and Mortensen (1998), where composite productivity $\tilde{p}_{a,g}$ in the current model plays the role of job productivity differentials in their model.

Equation (34) and Assumption 3 imply that optimal amenities satisfy

$$c^{\pi,0}_{a,g} \times \frac{\partial \tilde{c}^{\pi}_{a}(\pi^*_{a,g})}{\partial \pi} = 1$$
$$\frac{a c^{\pi,0}_{g}}{a} \frac{\partial \tilde{c}^{\pi}(\frac{\pi^*_{a,g}}{a})}{\partial \pi} = 1$$

which obviously implies that $\pi^*_g = \pi^*_{a,g}/a$ is constant for all ability types a. Thus, amenities $\pi^*_{a,g}$ are proportional to ability. Also, the cost function implies that $c^{\pi}_{a,g}(\pi^*_{a,g}) = a\tilde{c}^{\pi}_{g}(\pi_g)$, so that also the equilibrium amenity cost is proportional to ability.

Summing up, under Assumptions 1 and 3, it follows that $\pi_{a,g} = a\pi_g$, $c^{\pi}_{a,g}(\pi_{a,g}) = ac^{\pi}_{g}(\pi_g)$ and $z_a = az$. Therefore, composite productivity $\tilde{p}_{a,g} = p\,a + \pi_{a,g} - c^{\pi}_{a,g}(\pi_{a,g}) - \mathbf{1}(g = F)z_a$ is proportional to a, and we can write $\tilde{p}_{a,g} = a\tilde{p}_g$, where $\tilde{p}_g = p + \pi_g - c^{\pi}_{g}(\pi_g) - \mathbf{1}(g = F)z$ is distributed according to $h_g(\tilde{p}_g)$. By definition, $h_{a,g}(\tilde{p}_{a,g}) = h_{a,g}(a\tilde{p}_g) = h_g(\tilde{p}_g)$. Due to Assumption 2, $\kappa_{a,g} = \kappa_g$ for all a. Thus, with a change of variables and using that vacancies of each firm are constant across ability markets, we can rewrite equation (45) as

$$x_g\left(a, \tilde{p}_g\right) = a\tilde{p}_g - \int\limits_{y \geq \phi_{a,g}}^{\tilde{p}_g} a\left[\frac{1+\kappa^e_{g}(1-h_{g}(\tilde{p}_{g}))}{1+\kappa^e_{g}(1-h_{g}(y))}\right]^2 dy, \tag{46}$$

We still need to prove that $\phi_{a,g}$ is also proportional to a under the assumption that $b_{a,g} = ab_g$. We use a guess-and-verify approach: we guess that the case in which $\phi_{a,g}$ and equilibrium flow utility $x\left(\tilde{p}_g, v_g\right)$ are proportional to a is an equilibrium of the model and we verify it below. From equation (7) and Assumptions 1–4, we have

$$\phi_{a,g} = ab_g + (\lambda^u_g - \lambda^e_g) \int\limits_{x' \geq \phi_{a,g}} \frac{1 - F_{a,g}(x')}{\rho + \delta_g + \lambda^G_g + \lambda^e_g(1 - F_{a,g}(x'))}\, dx'\,.$$

We proceed to show that, if $\phi_{a,g} = a\phi_g$, then $x\left(\tilde{p}_g\right)$ is also proportional to a. The proof follows trivially from equation (46): if $\phi_{a,g} = a\phi_g$,

$$x_g\left(a, \tilde{p}_g\right) = a\tilde{p}_g - a\int\limits_{y \geq \phi_{g}}^{\tilde{p}_g} \left[\frac{1+\kappa^e_{g}(1-h_{g}(\tilde{p}_{g}))}{1+\kappa^e_{g}(1-h_{g}(y))}\right]^2 dy\,.$$

Next we show that, if $x\left(a, \tilde{p}_g\right)$ is proportional to a, then $\phi_{a,g}$ must be proportional to a. Consider the bijective mapping $\tilde{p}_g(x, a) = [x^*(a, \tilde{p}_g)]^{-1}$. We can rewrite the outside option as

$$\begin{aligned}
\phi_{a,g} =& ab_g + (\lambda_g^u - \lambda_g^e) \int\limits_{x' \geq \phi_{a,g}} \frac{1 - h_{a,g}([x'(a, \tilde{p}_g)]^{-1})}{\rho + \delta_g + \lambda_g^G + \lambda_g^e(1 - h_{a,g}([x'(a, \tilde{p}_g)]^{-1}))} \, dx' \\
=& ab_g + a(\lambda_g^u - \lambda_g^e) \int\limits_{x' \geq \phi_{a,g}} \frac{1 - h_g([x'(1, \tilde{p}_g)]^{-1})}{\rho + \delta_g + \lambda_g^G + \lambda_g^e(1 - h_{a,g}([x'(1, \tilde{p}_g)]^{-1}))} \, dx' \,,
\end{aligned}$$

which implies that the only solution to this equation satisfies $\phi_{a,g} = a\phi_g$.

Finally, recalling that $\tilde{p}_g = p + \pi_g - c_g^{\pi}(\pi_g) - z$ and that $w = x - \pi$, we can write monetary wages as

$$w\left(a, \tilde{p}_g, c_g^{\pi,0}\right) = a\left[\tilde{p}_g - \pi_g(c_g^{\pi,0}) - \int\limits_{\tilde{p}' \geq \phi_g}^{\tilde{p}_g} \left[\frac{1 + \kappa_g^e(1 - h_g(\tilde{p}_g))}{1 + \kappa_g^e(1 - h_g(\tilde{p}'))}\right]^2 d\tilde{p}'\right], \tag{47}$$

which completes the proof that the desired equilibrium wage equation holds under exogenous vacancies that are constant across ability levels.

Step 2. All that remains to be shown for the desired result to follow is that in the model with endogenous vacancy posting we have $v_{a,g}^* = v_g^*$ for all a, so that the offer distribution $h_{a,g}$ is the same across all ability markets. Under Assumption 1 we have $c_{a,g}^{v,0} = ac_g^{v,0}$. Next, we follow a guess-and-verify approach. Suppose that $x_{a,g}^*(\tilde{p}_g)$ is proportional to ability a. Using that $F_{a,g}(x_{a,g}^*(\tilde{p}_{a,g})) = h_{a,g}(\tilde{p}_{a,g})$, we can write the first-order condition for vacancy creation in equation (38) as

$$\begin{aligned}
ac_g^{v,0}\frac{\partial \tilde{c}_g^v(v_{a,g})}{\partial v_{a,g}} &= T_{a,g}(\tilde{p}_{a,g} - x_{a,g}^*(\tilde{p}_{a,g}) \left(\frac{1}{\delta_{a,g} + \lambda_{a,g}^G + \lambda_{a,g}^e(1 - h_{a,g}(\tilde{p}_{a,g}))}\right)^2 \\
c_g^{v,0}\frac{\partial \tilde{c}_g^v(v_{a,g})}{\partial v_{a,g}} &= T_g(\tilde{p}_g - x_g^*(\tilde{p}_g) \left(\frac{1}{\delta_g + \lambda_g^G + \lambda_g^e(1 - h_g(\tilde{p}_g))}\right)^2,
\end{aligned}$$

immediately proving that $v_{a,g} = v_g$ for all a. Equation (11) thus implies that aggregate vacancies satisfy $V_{a,g} = V_g$ which, together with Assumption 2, also implies that in equilibrium $u_{a,g} = u_g$ and $\lambda_{a,g}^u = \lambda_g^u$. As a consequence, all terms in the wage equation (47) scale linearly in ability. Therefore, log wages take the form of the desired equilibrium wage equation. □

C.3 Discussion of Modeling Assumptions

We now turn to a brief discussion of some of the more restrictive modeling assumptions and their implications. A first set of assumptions made in the model is that output is additively separable across, and linear within, worker types. These assumptions allow for considerable analytical tractability but we also think they are not unreasonable.

That output is linear within worker types is not particularly restrictive since a firm's net payoff function is already concave due to the convex vacancy cost. Conceptually, there is no reason not to simultaneously allow for curvature in the ability-weighted number of workers of each type. In practice, however, if this curvature were such that the marginal product tended to infinity as the number of workers of a given type tends to zero—as would be the case with standard constant-elasticity-of-substitution specifications—then every firm would employ a strictly positive mass of each worker type, including men and women. This would be at odds with the presence of single-gender firms in the data.

That output is additively separable across worker types ultimately allows the model to admit a log-linear wage equation, which is a requirement for us to be able to take the model to the data. Assuming complementarities between genders would lead to the counterfactual implication that no single-gender firms could exist. Therefore, it seems like a natural starting point to think of men and women as perfect substitutes in production.

A second set of assumption is that labor markets are segmented by worker types, and that firms can direct wages, amenities, and vacancies to each market. These assumptions significantly simplify the analysis of the firm's problem. That firms can direct wages and vacancies towards certain worker types may seem like an extreme assumption that is at odds with national nondiscrimination laws. But, of course, a firm need not publicly post a lower wage or invest less recruitment effort when hiring workers in order to discriminate. Such differences may naturally arise in more subtle ways when screening résumés, at the interview stage, and at the negotiation table.

There are also good empirical reasons to adopt market segmentation. In particular, three "natural" modeling alternatives have clearly counterfactual predictions. First, a model in which firms offer only one wage for workers of both genders conditional on ability would fail to account for the empirical within-firm pay differences documented in Sections 3.2 and 3.3.[42] Second, a model in which amenities are shared across workers of both genders within a firm would counterfactually predict no dispersion in firm ranks conditional on gender-specific pay. Third, a model in which vacancy costs are not additively separable across genders or one in which vacancies are undirected would clearly fail to account for the empirical dispersion of female employment shares in the data.[43]

Finally, the simplifying assumptions underlying Proposition 1 allow us to make significant progress in bringing the model to the data. One may expect that some of these assumptions do not hold exactly in the data. For example, when labor market parameters differ across ability levels, then the decomposition in equation (16) will not hold exactly. However, Engbom and Moser (2018) show that an AKM decomposition of log wage predicts around 99% of the variance of log earnings in data simulated from a model that allows for flexible variation in labor market parameters across ability types estimated to similar data from Brazil.

C.4 Alternative Modeling Assumptions on Vacancy Posting

C.4.1 Model Alternative 1: Directed Vacancy Posting with Joint Cost Function

As a first alternative to the benchmark model, suppose that, instead of the vacancy cost being separable across genders, we assume that the vacancy cost is a function of the total number of vacancies posted. This model has the strong prediction that any firm will employ either only men, or only women, except in knife-edge cases.

Setup. Each firm posts a number $v_{a,M}$ of vacancies targeted at male workers and $v_{a,F}$ vacancies targeted at women. The total cost of posting $(v_{a,M}, v_{a,F})$ vacancies for men and women is given by $c_a^v(v_{a,M} + v_{a,F})$, where the function c_a^v retains the properties laid out in the main text: $c_a^v(0) = 0$, $\partial c_a^v(\cdot)/\partial v > 0$, $\partial^2 c_a^v(\cdot)/\partial v^2 > 0$.

[42] We solve the model with gender-neutral wage offers later when we consider the equilibrium effects of an equal-pay policy.

[43] We solve both of these models in Appendix C.4. The model with directed vacancy posting and a joint cost function in Appendix C.4.1 predicts that, with the exception of knife-edge cases, there exist no dual-gender firms. The model with undirected vacancy posting in Appendix C.4.2 predicts that, quantitatively, there is far too little dispersion in female employment shares compared to the empirical distribution we see in the data. We conclude that the benchmark model with targeted vacancies and separate cost functions is a good starting point for our investigation.

Equilibrium characterization. To see that this setup implies gender segregation except in knife-edge cases, note that the firm's problem can now be written as

$$\max_{x_{a,M},x_{a,F},v_{a,M},v_{a,F}} \left\{ \sum_{g=M,F} (\tilde{p}_{a,g} - x_{a,g}) l_{a,g}(x_{a,g}, v_{a,g}) - c_a^v(v_{a,M} + v_{a,F}) \right\}$$

The FOCs with respect to vacancy posting now read

$$[\partial v_{a,M}]: \quad c_a^{v\prime}(v_{a,M} + v_{a,F}) = T_{a,M}(\tilde{p}_{a,M} - x_{a,M}) \left(\frac{1}{\delta_{a,M} + s_{a,M}\lambda^u_{a,M}(1 - F_{a,M}(x_{a,M}))} \right)^2, \tag{48}$$

$$[\partial v_{a,F}]: \quad c_a^{v\prime}(v_{a,M} + v_{a,F}) = T_{a,F}(\tilde{p}_{a,F} - x_{a,F}) \left(\frac{1}{\delta_{a,F} + s_{a,F}\lambda^u_{a,F}(1 - F_{a,F}(x_{a,F}))} \right)^2. \tag{49}$$

Putting equations (48) and (49) into simple economic terms, the marginal cost of an additional vacancy (the left-hand side) is equated to the marginal benefit of an additional vacancy (the right-hand side). The latter consists of an increase in the employment of that worker type multiplied by the profits made per worker of that type, which is independent of the amount of vacancies posted. This is because wages are set according to other first-order conditions, which do not depend on the amount of vacancies posted by that firm.

Since the right-hand sides in equations (48) and (49) are generically not equal, except in knife-edge cases, it follows that not both FOCs can hold. This means that the firm will be at a corner solution with regards to one of the two genders, and this must invove posting zero vacancies for that gender.

Empirical shortcomings. According to the above analysis, except for knife-edge cases, firms would hire only men or only women—whichever gives the highest marginal benefit to the firm. This model implication is empirically counterfactual since the vast majority of firms in the real world employ a mix of men and women.

C.4.2 Model Alternative 2: Undirected Vacancy Posting

As a second alternative to the benchmark model, suppose that, instead of vacancies being directed to men and women separately, we assume that firms cannot discriminate between genders in their recruiting. While qualitatively such a model can account for dual-gender firms it turns out that, quantitatively, such a model clearly fails to replicate the empirical distribution of female employment shares across firms that we documented in Section 3.2.

Setup. Each firm posts a number v_a of gender-neutral vacancies for workers of each ability level at cost $c_a^v(v_a)$. In such a model, a firm's problem can be written as

$$\max_{x_{a,M},x_{a,F},v_a} \left\{ \sum_{g=M,F} (\tilde{p}_{a,g} - x_{a,g}) l_{a,g}(x_{a,g}, v_a) - c_a^v(v_a) \right\}$$

Notice that we do not impose that firms hire both genders in each submarket: it is always possible for a firm to offer flow utility $x_{a,g} < \phi_{a,g}$ such that no worker of gender g will accept it. Consequently, while a total of V_a vacancies are posted in each submarket in the aggregate, only $V_{a,g} \leq V_a = \int v_a(\tilde{p}_{a,g}, c^{v,0}_{a,g})\, d\Gamma_{a,g}(\tilde{p}_{a,g}, c^{v,0}_{a,g})$ vacancies are accepted in equilibrium by workers of type

(a, g). This implies that the number of matches produced in the labor market is given by

$$m_{a,g} = A^g[\mu_{a,g}(u_{a,g} + s^e_{a,g}(1 - u_{a,g}) + s^G_{a,g})]^\alpha V_a^{1-\alpha} \frac{V_{a,g}}{V_a},$$

which already incorporates the probability that a worker of gender g will meet a vacancy that is associated with a wage below the reservation threshold, leading to a rejection. It is straightforward to show that this matching function exhibits all the properties of standard matching functions, and that in particular $f_{a,g}/q_{a,g} = V/[u_{a,g} + s^e_{a,g}(1 - u_{a,g}) + s^G_{a,g}]$, where $f_{a,g} = m_{a,g}/[u_{a,g} + s^e_{a,g}(1 - u_{a,g}) + s^G_{a,g}]$ is the job finding rate per effective job searcher and $q_{a,g} = m_{a,g}/V$ is the vacancy yield rate.

Equilibrium characterization. The following equation represents the law of motion of firm sizes:

$$\begin{aligned} \dot{l}_{a,g}(x, v) =& - \delta_{a,g} l^g_a(x, v) - s^g_a \lambda^e_{a,g}(1 - F_{a,g}(x)) l^g_a(x, v) + \\ & v q_{a,g} \left[\frac{u_{a,g}}{u_{a,g} + (1 - u_{a,g}) s_{a,g}} + \frac{(1 - u_{a,g}) s_{a,g}}{u_{a,g} + (1 - u_{a,g}) s_{a,g})} G_{a,g}(x) \right] \end{aligned}$$

Solving for the stationary solution:

$$l_{a,g}(x_{a,g}, v_a) = \left(\frac{1}{\delta_{a,g} + s_{a,g} \lambda^u_{a,g}(1 - F_{a,g}(x_{a,g}))} \right)^2 \frac{v_a}{V_a} \mu_{a,g} u_{a,g} \lambda^u_{a,g}(\delta_{a,g} + s_{a,g} \lambda^u_{a,g}) \tag{50}$$

To find the firm's policy functions, define $T_{a,g} = \mu_{a,g}[u_{a,g} \lambda^u_{a,g}(\delta_{a,g} + s_{a,g} \lambda^u_{a,g})]/V_a$. we rewrite the firm's problem as a function of the steady state mass of employed workers as follows:

$$\begin{aligned} \max_{x_{a,M}, x_{a,F}, v_a} \Bigg\{ & T_{a,M} v_a (ap + \pi_m - x_{a,M}) \left(\frac{1}{\delta_{a,M} + s_{a,M} \lambda^{M,u}_a (1 - F_{a,M}(x_{a,M}))} \right)^2 \\ & + T_{a,F} v_a (ap + \pi_f - z - x_{a,F}) \left(\frac{1}{\delta_{a,F} + s_{a,F} \lambda^{W,u}_a (1 - F_{a,F}(x_{a,F}))} \right)^2 - c_a(v_a) \Bigg\} \end{aligned}$$

The associated FOCs read

$$\begin{aligned} c'(v_a) =& T_{a,M}(ap + \pi_m - x_{a,M}) \left(\frac{1}{\delta_{a,M} + s_{a,M} \lambda^{m,u}_a (1 - F_{a,M}(x_{a,M}))} \right)^2 \\ & + T_{a,F}(ap + \pi_f - z - x_{a,F}) \left(\frac{1}{\delta_{a,F} + s_{a,F} \lambda^{f,u}_a (1 - F_{a,F}(x_{a,F}))} \right)^2 \\ 1 =& (ap + \pi_m - x_{a,M}) \frac{2 s_{a,M} \lambda^{m,u}_a f_{a,M}(x_{a,M})}{\delta_{a,M} + s_{a,M} \lambda^{m,u}_a (1 - F_{a,M}(x_{a,M}))} \\ 1 =& (ap + \pi_f - z - x_{a,F}) \frac{2 s_{a,F} \lambda^{f,u}_a f_{a,F}(x_{a,F})}{\delta_{a,F} + s_{a,F} \lambda^{f,u}_a (1 - F_{a,F}(x_{a,F}))}. \end{aligned}$$

Empirical shortcomings. Recall from Section 3.2 that firm-level female employment shares are dispersed, ranging from almost 0 to almost 1 in the data. It is this salient feature of the data that the undirected-vacancy-posting model fails to replicate. To demonstrate this, we show that analytically-derived expressions for the lowest and highest female employment shares are inconsistent with the data for realistic calibrations of the labor market parameters guiding worker flows.

Using equation (50), we can write the female share of a firm as

$$\begin{aligned}
s_f =& \frac{l_{a,F}(x_{a,F}, v_a)}{l_{a,F}(x_{a,F}, v_a) + l_{a,M}(x_{a,M}, v_a)} \\
=& \frac{\left(\frac{1}{\delta_{a,F}+s_{a,F}\lambda_a^{f,u}(1-F_{a,F}(x_{a,F}))}\right)^2 \frac{v_a}{V_a} u_{a,F}\lambda_a^{f,u}(\delta_{a,F}+s_{a,F}\lambda_a^{f,u})}{\left(\frac{1}{\delta_{a,F}+s_{a,F}\lambda_a^{f,u}(1-F_{a,F}(x_{a,F}))}\right)^2 \frac{v_a}{V_a} u_{a,F}\lambda_a^{f,u}(\delta_{a,F}+s_{a,F}\lambda_a^{f,u}) + \left(\frac{1}{\delta_{a,M}+s_{a,M}\lambda_a^{m,u}(1-F_{a,M}(x_{a,M}))}\right)^2 \frac{v_a}{V_a} u_{a,M}\lambda_a^{m,u}(\delta_{a,M}+s_{a,M}\lambda_a^{m,u})} \\
=& \frac{1}{1+\frac{\left(\frac{1}{\delta_{a,M}+s_{a,M}\lambda_a^{m,u}(1-F_{a,M}(x_{a,M}))}\right)^2 u_{a,M}\lambda_a^{m,u}(\delta_{a,M}+s_{a,M}\lambda_a^{m,u})}{\left(\frac{1}{\delta_{a,F}+s_{a,F}\lambda_a^{f,u}(1-F_{a,F}(x_{a,F}))}\right)^2 u_{a,F}\lambda_a^{f,u}(\delta_{a,F}+s_{a,F}\lambda_a^{f,u})}}
\end{aligned}$$

However, in the data we find that the E-to-U transition rates and U-to-E transition rates are almost identical between men and women. That is, $\delta_{a,F} \approx \delta_{a,M} \equiv \delta$ and $\lambda_a^{f,u} \approx \lambda_a^{m,u} \equiv \lambda_a^u$. Therefore unemployment rates are also identical across genders, so that the expression for the female share simplifies to

$$s_f = \frac{1}{1+\frac{\left(\frac{1}{\delta+s_{a,M}\lambda_a^{m,u}(1-F_{a,M}(x_{a,M}))}\right)^2 (\delta+s_{a,M}\lambda_a^u)}{\left(\frac{1}{\delta+s_{a,F}\lambda_a^u(1-F_{a,F}(x_{a,F}))}\right)^2 (\delta+s_{a,F}\lambda_a^u)}}.$$

Since firm sizes are monotonically increasing in flow utility x offered by the firm, we can obtain expressions for the minimum female employment share $\underline{s}_f$ and the maximum female employment share $\bar{s}_f$ by focusing on employers that are at the very top of the job ladder for one gender and simultaneously at the very bottom of the job ladder for the other gender. Specifically, among all dual-gender firms, the firm with the highest female employment share has $F_{a,F} = 1$ and $F_{a,M} = 0$. Conversely, the firm with the lowest female employment share has $F_{a,F} = 0$ and $F_{a,M} = 1$.

To see this, note that we can write the minimum female share $\underline{s}_f$ in the model as

$$\begin{aligned}
\underline{s}_f =& \frac{1}{1+\frac{\left(\frac{1}{\delta}\right)^2 (\delta+s_{a,M}\lambda_a^u)}{\left(\frac{1}{\delta+s_{a,F}\lambda_a^u}\right)^2 (\delta+s_{a,F}\lambda_a^u)}} \\
=& \frac{1}{1+\frac{\frac{1}{\delta^2}(\delta+s_{a,M}\lambda_a^u)}{\frac{1}{\delta+s_{a,F}\lambda_a^u}}} \\
=& \frac{1}{1+\frac{(\delta+s_{a,M}\lambda_a^u)(\delta+s_{a,F}\lambda_a^u)}{\delta^2}} \\
=& \frac{1}{2+\frac{s_{a,F}\lambda_a^u}{\delta}+\frac{s_{a,M}\lambda_a^u}{\delta}+\frac{s_{a,M}s_{a,F}(\lambda_a^u)^2}{\delta^2}}.
\end{aligned}$$

In our data we find roughly that $s_{a,M}\lambda_a^u \approx 0.4\delta$ and $s_{a,F}\lambda_a^u \approx 0.27\delta$. If we apply these numbers, we find that the minimum female employment share in the model is

$$\underline{s}_f = \frac{1}{2 + 0.27 + 0.4 + 0.108} \approx 0.36,$$

which is inconsistent with the minimum female employment share being close to 0 in the data.

Analogously, we can write the maximum female share $\overline{s}_f$ in the model as

$$\begin{aligned}
\overline{s}_f =& \frac{1}{1 + \frac{\left(\frac{1}{\delta + s_{a,M}\lambda_a^u}\right)^2 (\delta + s_{a,M}\lambda_a^u)}{\left(\frac{1}{\delta}\right)^2 (\delta + s_{a,F}\lambda_a^u)}} \\
=& \frac{1}{1 + \frac{\frac{1}{\delta + s_{a,M}\lambda_a^u}}{\frac{(\delta + s_{a,F}\lambda_a^u)}{\delta^2}}} \\
=& \frac{1}{1 + \frac{\delta^2}{(\delta + s_{a,M}\lambda_a^u)(\delta + s_{a,F}\lambda_a^u)}} \\
=& \frac{1}{1 + \frac{\delta^2}{\delta^2 + \delta s_{a,M}\lambda_a^u + \delta s_{a,F}\lambda_a^u + s_{a,M}s_{a,F}(\lambda_a^u)^2}} \\
=& \frac{1}{1 + \frac{\delta^2}{\delta^2 + 0.4\delta^2 + 0.27\delta^2 + 0.108\delta^2}} \approx 0.64,
\end{aligned}$$

which is inconsistent with the maximum female employment share being close to 1 in the data. We further scrutinize the properties of the undirected-vacancy-posting model using the numerical solution algorithm proposed in Appendix C.5.

C.5 Undirected-Vacancy-Posting Model Solution Algorithm

The undirected-vacancy-posting model is challenging because it does not allow us to solve men and women as two separate differential equations. Thus, we rely on a different algorithm to solve the undirected-vacancy-posting vacancy posting model. Define (firm-specific) composite productivities, amenities, and the intra-employer wedge for each gender as:

$$\begin{aligned}
p_{a,M} =& ap + \pi_m \\
p_{a,F} =& ap + \pi_f - z
\end{aligned} \tag{51}$$

Assume $c(v_a) = c\frac{v_a^2}{2}$. Then,

$$\begin{aligned}
v_a =& \frac{T_{a,M}}{c}(p_{a,M} - \tilde{w}_{a,M})\left(\frac{1}{\delta_{a,M} + s_{a,M}\lambda_a^{m,u}(1 - F_{a,M}(\tilde{w}_{a,M}))}\right)^2 \\
&+ \frac{T_{a,F}}{c}(p_{a,F} - \tilde{w}_{a,F})\left(\frac{1}{\delta_{a,F} + s_{a,F}\lambda_a^{f,u}(1 - F_{a,F}(\tilde{w}_{a,F}))}\right)^2
\end{aligned} \tag{52}$$

By definition:

$$
\begin{aligned}
V_{a,M} &= \int\int v_a \, \mathbb{I}[p_{a,M} > \phi_{a,M}] \, \gamma(p_{a,M}, p_{a,F}) \, dp_{a,M} \, dp_{a,F} \\
V_{a,F} &= \int\int v_a \, \mathbb{I}[p_{a,F} > \phi_{a,F}] \, \gamma(p_{a,M}, p_{a,F}) \, dp_{a,M} \, dp_{a,F} \\
V_a &= \int\int v_a \, \mathbb{I}[p_{a,M} > \phi_{a,M} \ \text{ OR } \ p_{a,F} > \phi_{a,F}] \, \gamma(p_{a,M}, p_{a,F}) \, dp_{a,M} \, dp_{a,F}
\end{aligned} \tag{53}
$$

Start by defining $\gamma(p_{a,M})$ as the marginal distribution of $p_{a,M}$. Also, define

$$
\bar{v}_a(p_{a,M}) = \int \frac{v(p_{a,M}, p_{a,F})\gamma(p_{a,M}, p_{a,F})}{\gamma(p_{a,M})} \, dp_{a,F}
$$

as the average vacancies posted by firms with male productivity $p_{a,M}$. Then we can write

$$
\begin{aligned}
h(p) &= F(\tilde{w}(p)) \\
\implies \quad h'(p) &= f(w(p)) \, w'(p) \\
\implies \quad f(w(p)) &= h'(p)/w'(p) \\
\bar{v}_a(p_{a,M}) &= \frac{V_{a,M} h'(p_{a,M})}{\gamma(p_{a,M})} \\
\implies \quad h'(p_{a,M}) &= \frac{\bar{v}_a(p_{a,M})}{V_{a,M}} \gamma(p_{a,M})
\end{aligned} \tag{54}
$$

Thus, the wage FOC can be rewritten as follows:

$$
\begin{aligned}
1 &= (p_{a,g} - \tilde{w}_{a,M}) \frac{2 s_{a,g} \lambda^u_{a,g} f_{a,g}(\tilde{w}_{a,g})}{\delta_{a,g} + s_{a,g} \lambda^u_{a,g} (1 - F_{a,g}(\tilde{w}_{a,g}))} \\
\tilde{w}'(p_{a,g}) &= (p_{a,g} - \tilde{w}_{a,g}) \frac{2 s_{a,g} \lambda^u_{a,g} h'(p_{a,g})}{\delta_{a,g} + s_{a,g} \lambda^u_{a,g} (1 - h(p_{a,g}))}
\end{aligned}
$$

We start from the case in which $\gamma(p_{a,M}, p_{a,F})$ is an analytical function and all marginal and conditional distributions associated are easy to compute (therefore also the marginals $\gamma(p_{a,g})$ are known). Using the previous intuitions, the algorithm works as follows:

1. Start with a guess for $\tilde{w}(p_{a,g})$, $F(\tilde{w}(p_{a,g})$ and $V_{a,g}$ for each gender.
2. Calculate transition rates $\lambda^u_{a,g}$, $\lambda^e_{a,g}$ using the guess for $V_{a,g}$.
3. Using the guess for the wage function, compute $v_a(p_{a,M}, p_{a,F})$ on a large grid over values of $p_{a,M}$ and $p_{a,F}$.
4. Normalize v_a to get $V_{a,g}$. Use the function v_a to compute $h'(p_{a,g})$ for both genders as in 54.
5. Use $h'(p_{a,g})$ to compute the new function $h(p_{a,g}) = F(\tilde{w}(p_{a,g})$.
6. Use h' and h to solve the ODE in C.5.
7. Update the wage function $\tilde{w}(p_{a,g})$ and the CDF $F(\tilde{w}(p_{a,g})$.
8. Go back to step 2. Repeat until convergence.

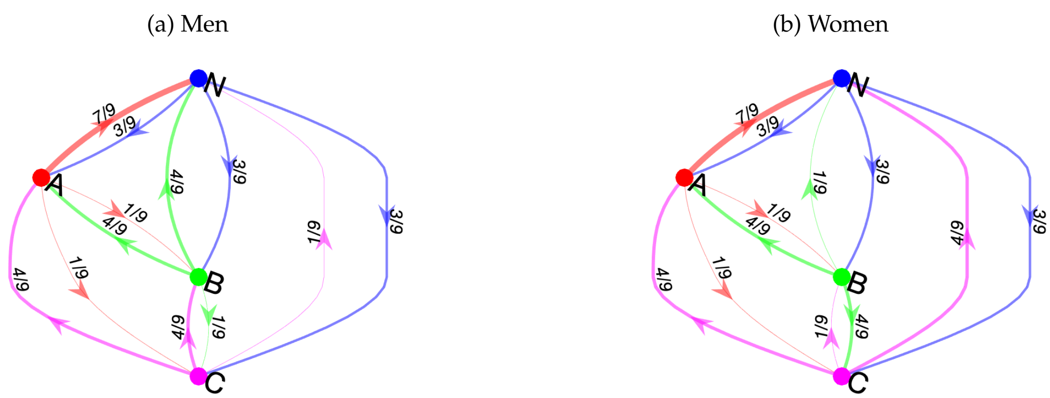
D Identification Appendix

D.1 Illustrative Identification Example

To illustrate how we estimate employer ranks, productivity, amenities, and employer wedges from data on worker flows and pay across establishments, we use a simple example. For the purpose of this simple example, we abstract from endogenous vacancy and amenity creation, and heterogeneity in the offer densities and labor market parameters, all of which will be present in the general estimation routine.

Consider three employers A, B, and C and a pool of nonemployed workers N. Because the PageRank does not depend on employer size, we can think of all three employers as having a large number of male and female workers. To simplify the example, we assume that each employer hires a fixed number of male and female workers but from different sources. Employer A hires men in equal proportions by poaching from B, C, and nonemployment; employer B hires from C and nonemployment most of the time but rarely from A; while employer C hires from nonemployment most of the time but rarely from A and B. Female worker flows are identical except that employer B hires women from nonemployment most of the time but rarely from A and C; while employer C hires from B and nonemployment most of the time but rarely from A. Figure 33 summarizes the labor markets for men and women graphically.

Figure 33. Example worker flows between nonemployment and employers, by gender



Source: Authors' calculations. *Note:* Nodes A, B, and C represent employers. Node N represents nonemployment. Arrows represent worker flows. Numbers above arrows represent share of all worker flows from a given node.

Estimating PageRanks based on equation (4) of Section 3.5 for this labor market yields separate employer rankings by gender. Intuitively, employers that poach a lot of workers of a given gender from other high-ranked employers are themselves highly ranked according to the PageRank. For men, employer A is ranked highest (PageRank index 0.423), employer B is middle-ranked (PageRank index 0.326), and employer C is ranked lowest (PageRank index 0.251). For women, employer A is also ranked highest (PageRank index 0.423), employer B is ranked in lowest (PageRank index 0.251), and employer C is middle-ranked (PageRank index 0.326). Men and women agree on employer A being ranked highest but disagree on the ranking of the remaining two employers B and C. The resulting PageRanks for men and women are in columns (1)–(2) of Table 19.

Suppose that pay at employers (A, B, C) is $(8.0, 7.0, 4.0)$ for men and $(8.0, 6.0, 3.8)$ for women, as in columns (3)–(4) of Table 19. Suppose also that the underlying amenity values at those employers are $(1.0, 0.0, 0.0)$ for men and $(0.0, 0.0, 2.3)$ for women, as in columns (5)–(6) of Table 19. Finally, suppose that the underlying employer productivities are $(15.9, 12.3, 4.0)$ as in column (13) and employer wedges are $(5.3, 6.3, 0.0)$ as in column (15) of Table 19.

Table 19. Example pay, PageRanks, amenities, utilities, productivities, employer wedges

	PageRanks		Pay		Amenities				Utilities				Prod.		Wedge	
	(1)	(2)	(1)	(2)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
Emp.	r_M	r_F	w_M	w_F	π_M	π_F	$\widehat{\pi}_M$	$\widehat{\pi}_F$	x_M	x_F	$\widehat{x}_M$	$\widehat{x}_F$	p	$\widehat{p}$	z	$\widehat{z}$
N	–	–	–	–	–	–	–	–	4.0	6.0	4.0	6.0	–	–	–	–
A	3	3	8.0	8.0	1.0	0.0	0.0	0.0	9.0	8.0	8.0	8.0	15.9	14.6	5.3	4.0
B	2	1	7.0	6.0	0.0	0.0	0.0	0.0	7.0	6.0	7.0	6.0	12.3	12.3	6.3	6.3
C	1	2	4.0	3.8	0.0	2.3	0.0	2.3	4.0	6.1	4.0	6.1	4.0	4.0	0.0	0.0

Source: Authors' calculations. *Notes:* "Emp." stands for employer A, B, C, or nonemployment status N. For each gender g, w_g is wage, r_g is Pagerank, π_g is amenity value, p is employer productivity ("Prod."), z is employer wedges ("Wedge"). Hats denote estimates.

How can the information on (estimated) employer ranks and pay be used to infer (unobserved) employer productivity, amenities, and employer wedges? We proceed in five steps.

First, we pick gender-specific employer amenities to make employer ranks consistent with pay by gender. Without loss of generality, assume amenity values are weakly positive. For men it must be that $\pi_M^A > -1.0 + \pi_M^B$ and $\pi_M^A > -4.0 + \pi_M^C$ for A to be highest-ranked, and $\pi_M^B > -3.0 + \pi_M^C$ for B to be middle-ranked. The amenities-minimizing estimate that satisfies these inequalities is $(\widehat{\pi}_M^A, \widehat{\pi}_M^B, \widehat{\pi}_M^C) = (0.0, 0.0, 0.0)$. Similarly, for women it must be that $\pi_F^A > -2.0 + \pi_F^B$ and $\pi_F^A > -4.2 + \pi_F^C$ for A to be highest-ranked, and $\pi_F^B < -2.2 + \pi_F^C$ for B to be lowest-ranked. The amenities-minimizing estimate that satisfies these inequalities is $(\widehat{\pi}_F^A, \widehat{\pi}_F^B, \widehat{\pi}_F^C) = (0.0, 0.0, 2.3)$. These estimates are summarized in columns (7)–(8) of Table 19.

Second, we derive an estimate of the outside option value by gender. To this end, we first compute utility $\widehat{x}_g^r = w_g^r + \widehat{\pi}_g^r$ as the sum of pay and estimated amenity values for each employer. Based on the above estimates, we have $(\widehat{x}_M^A, \widehat{x}_M^B, \widehat{x}_M^C) = (8.0, 7.0, 4.0)$ and $(\widehat{x}_F^A, \widehat{x}_F^B, \widehat{x}_F^C) = (8.0, 6.0, 6.1)$, shown in columns (11)–(12) of Table 19. Defining the outside option value as the lowest utility among employed workers for each gender, $\phi_g = \min_r x_g^r$, we have $\widehat{\phi}_M = \widehat{\pi}_M^N = 4.0$ and $\widehat{\phi}_F = \widehat{\pi}_F^N = 6.0$, as shown in the top row of columns (9)–(10) of Table 19.

Third, we deduce composite productivities $\tilde{p}_g^r$ for $g = M, F$ based on the equilibrium wage equation (17), which relates w_g^r to $\tilde{p}_g^r$, π_g^r, κ_g^e, and $F_g(x_g^r)$. Since we have already estimated $\kappa_g^e = \lambda_g^e/(\delta_g + \lambda_g^G)$ in Step 2 above, we assume $\kappa_M^e = \kappa_F^e = 1$ for this example. Approximating the integral in equation (17) by use of the lower Riemann (Darboux) sum, we have

$$\tilde{p}_g^r \approx x_g^r + \sum_{r' \leq r} \left(\frac{1 + \kappa_g^e \left[1 - F_g^r\right]}{1 + \kappa_g^e \left[1 - F_g^{r'-1}\right]} \right)^2 \left(\tilde{p}_g^r - \tilde{p}_g^{r-1} \right) \tag{55}$$

We use equation (55) recursively with $(F_M^A, F_M^B, F_M^C) = (1, 2/3, 1/3)$, $(F_F^A, F_F^B, F_F^C) = (1, 1/3, 2/3)$, and $\tilde{p}_g^1 = \phi_g$ to estimate $(\widehat{\tilde{p}}_M^A, \widehat{\tilde{p}}_M^B, \widehat{\tilde{p}}_M^C) = (14.6, 12.3, 4.0)$ and $(\widehat{\tilde{p}}_F^A, \widehat{\tilde{p}}_F^B, \widehat{\tilde{p}}_F^C) = (10.6, 6.0, 6.3)$. Note that the estimated composite productivities satisfy monotonicity with respect to estimated utilities $\widehat{x}_g^r$ and PageRanks.

Fourth, we turn to men only in order to derive an estimate of employer productivity from the estimated composite productivity and amenity values. Since $z_M = 0$ by normalization, the definition of composite productivity for men yields $p = \tilde{p}_M^r - \pi_M^r$. The resulting productivity estimates are $(\widehat{p}^A, \widehat{p}^B, \widehat{p}^C) = (14.6, 12.3, 4.0)$, shown in column (14) of Table 19.

Finally, we turn to women only in order to estimate the employer wedge from the estimated composite productivity, amenity values, and productivity. The definition of composite productivity for women yields $z^r = p + \pi_F^r - \tilde{p}_F^r$. The resulting employer-wedge estimates are $(\widehat{z}^A, \widehat{z}^B, \widehat{z}^C) = (4.0, 6.3, 0.0)$, shown in column (16) of Table 19.

How do we interpret these results? Our estimates confirm that pay gaps are not utility gaps and that higher utilities are associated with higher composite productivity (Lemma 4). Focusing on employer A, we learn that equal pay (or, hypothetically, equal utility) across genders within an employer does not imply a zero employer wedge. This is because the employer wedge captures the degree to which an employer under- or overpays relative to the competitive benchmark described by the equilibrium wage equation. In this case, women at employer A are paid lower relative to the value of their outside option compared to men. Focusing on employer B, we see that the employer wedge may be nonmonotonic across ranks based on revealed preference, pay, or productivity. This is because differently-ranked employers may either under- or over-pay relative to the competitive benchmark. Focusing on employer C, we note that even employers with a zero employer wedge may deliver different pay and utility to men compared to women. This is because differences in the outside option value, due to either gender differences in the flow values of nonemployment or the presence of other employers with nonzero employer wedges, are priced into wage and utility offers in equilibrium.

It is worth noting that the parameter estimates in columns (7)–(8), (11)–(12), (14) and (16) of Table 19, imperfectly approximate the underlying parameter values in columns (5)–(6), (9)–(10), (13) and (15). Naturally, the approximation is more precise in the middle of the employer rank distribution and becomes more accurate as we increase the number of employers in the data.[44]

D.2 Further Details on Identification

A challenge in estimating productivity is that $f_g(x_g^r)$ is unknown, because it is the density function in the space of flow utilities x, rather than the change in the offer distribution f_g^r across ranks that we estimate. We begin by substituting f_g^r with the kernel density estimate $\hat{f}_g^r$, for computational stability. In other words, we need to transform the density through a change of variables: $\hat{f}_g(x_g^r) = \hat{f}_g^r \, \partial x / \partial r$. To perform the change of variables, we approximate the derivative by inserting the constraints implicitly in our algorithm. By definition, from one rank to the next $\partial r = 1$. Then, we approximate $\partial x = x_g^{r+1} - x_g^r$ and rewrite the problem as follows:

$$
\begin{aligned}
\min_{\{\pi_g^1, \dots \pi_g^{R^g}\}} & \sum_r \left[(w_g^{r+1} + \pi_g^{r+1}) - (w_g^r + \pi_g^r) \right]^2 \\
\text{s.t.} \quad & w_g^r + \pi_g^r \le w_g^{r+1} + \pi_g^{r+1}, \quad \forall r < R^g \\
& w_g^r + \pi_g^r + \frac{1 + \kappa_g^e (1 - F_g(x_g^r))}{2 \kappa_g^e \hat{f}_g^r} \left[(w_g^{r+1} + \pi_g^{r+1}) - (w_g^r - \pi_g^r) \right] \\
& \quad \le w_g^{r+1} + \pi_g^{r+1} + \frac{1 + \kappa_g^e (1 - F_g(x_g^{r+1}))}{2 \kappa_g^e \hat{f}_g^{r+1}} \left[(w_g^{r+2} + \pi_g^{r+2}) - (w_g^{r+1} - \pi_g^{r+1}) \right], \quad \forall r < R^g \, .
\end{aligned}
$$

After this substitution, the problem is written only as a function of known data inputs w_g^r, F_g^r, f_g^r, λ_g^e, λ_g^G, δ_g and of the unknowns of the problem $\{\pi_g^1, ..., \pi_g^{R^g}\}$.

[44] In the real data, we work with hundreds of thousands of employers for each gender. In Appendix D.3, we report results from Monte Carlo simulations for comparable sample sizes, which support the accuracy our estimation routine.

D.3 Identifying Productivity and Amenities in Monte Carlo Simulations

We solve our model and simulate firm-level data on wages, amenities, ranks and vacancies. We use this data to construct our estimates of rank r, density f_r and CDF F_r, and use them to estimate amenities and productivity at the firm level to test whether our algorithm is successful at uncovering the true firm-specific parameters. Our results are summarized in Table 20, which shows moments of the distribution of recovered estimates under different parametrizations of the underlying amenities distribution.

Table 20. Monte Carlo Simulations.

	Differences in the Amenity Distribution				
	(1)	(2)	(3)	(4)	(5)
Properties of true π					
Variance(π)	0.050	0.100	0.150	0.100	0.100
Corr(π,w)	-0.603	-0.668	-0.686	-0.654	-0.501
Corr(π,rank)	0.452	0.553	0.611	0.497	0.790
Corr(w,rank)	0.408	0.229	0.138	0.314	0.094
Properties of true p					
Variance(p)	0.538	0.551	0.551	0.538	0.539
Corr(p,w)	0.724	0.667	0.648	0.728	0.472
Corr(p,rank)	0.714	0.675	0.648	0.668	0.726
Corr(p,π)	-0.002	-0.002	-0.002	-0.086	0.416
Properties of estimates					
Variance($\hat{\pi}$)	0.075	0.083	0.135	0.168	0.061
Corr($\hat{\pi}$,w)	-0.277	-0.720	-0.697	-0.264	-0.710
Corr($\hat{\pi}$,rank)	0.717	0.456	0.581	0.773	0.608
Variance($\hat{p}$)	1.567	1.053	1.039	3.418	0.328
Corr($\hat{p}$,w)	0.563	0.598	0.511	0.590	0.477
Corr($\hat{p}$,rank)	0.743	0.867	0.871	0.849	0.849
Corr($\hat{p}$,$\hat{\pi}$)	0.430	0.111	0.225	0.596	0.240
Goodness of fit					
Corr($\hat{\pi}$, π)	0.933	0.989	0.997	0.893	0.965
Mean Error	0.001	0.001	0.001	0.001	0.001
Mean Squared Error	0.011	0.003	0.001	0.036	0.010
Corr($\hat{p}$, p)	0.908	0.933	0.931	0.942	0.960
Mean Error	-0.730	0.450	0.332	-1.361	0.932
Mean Squared Error	0.724	0.294	0.242	2.399	0.528

Note: Table reports estimation results using simulated data, under different parametrizations of the initial underlying amenities distribution.

E Estimation Appendix

E.1 Sensitivity Analysis Across Different Employer Rank Measures

To check how sensitive our results are to the choice of ranking, we re-estimate employer-specific parameters—productivity p, amenity values π_M and π_F, and employer wedges z—by applying our estimation routine to three alternative ranking measures: the Pagerank (as in the main text), the poaching index (Moscarini and Postel-Vinay, 2008; Bagger and Lentz, 2018), and the net poaching index (Haltiwanger et al., 2018; Moscarini and Postel-Vinay, 2018). Table 21 presents employment-weighted correlations between various estimation objects across different employer rank measures. Estimates across rank measures are significantly positively, albeit not perfectly, correlated. For our baseline analysis in the main text, we use the Pagerank index because it utilizes the most information per observed worker transition.

Table 21. Counterfactual simulations, shutting down differences across gender

Panel A. Productivity estimates			
	Pagerank	Poaching index	Net poaching index
Pagerank	1.000		
Poaching index	0.338	1.000	
Net poaching index	0.457	0.379	1.000
Panel B. Male amenity estimates			
	Pagerank	Poaching index	Net poaching index
Pagerank	1.000		
Poaching index	0.442	1.000	
Net poaching index	0.437	0.648	1.000
Panel C. Female amenity estimates			
	Pagerank	Poaching index	Net poaching index
Pagerank	1.0000		
Poaching index	0.420	1.000	
Net poaching index	0.446	0.864	1.000

Source: Authors' calculations based on RAIS. *Note:* Table reports pairwise correlations between estimates of productivity, male amenities, and female amenities based on different employer rank measures.

E.2 Details on Covariates for Analysis of Amenity Estimates

We include as covariates in $E_{g,j}$ in equation (25) the following fourteen variables that we construct using the RAIS data: an indicator for whether the employer provides in-kind remuneration in the form of food stamps; the shares of workers with part-time contracts, with hours changes since the previous year, with paid sick leave, with parental leave, with unpaid leave, with earnings cuts since the previous year, with noncontractual earnings fluctuations, with work-related accidents, with commute-related accidents; the shares of worker separations due to firing for unjust reasons and due to worker death; 5-digit industry dummies, and municipality dummies.

E.3 Details on Covariates for Analysis of Employer Wedge Estimates

We include as covariates in the vector E_j in equation (26) the following twelve variables, which we construct using the RAIS data: the mean intensity for routine-manual tasks, nonroutine-manual tasks, routine-cognitive tasks, nonroutine cognitive tasks involving interpersonal skills, nonroutine cognitive tasks involving analytical skills;[45] the share of worker separations due to worker death, the share of workers with work-related accidents; the female employment share, an indicator for whether the highest-paid worker is a woman; an indicator for whether an employer has no major financial stakeholder (as proxied by their participation in the small-business tax regime *Simples Nacional*);[46] 5-digit industry dummies, and municipality dummies.

E.4 Basic Solution Algorithm

We start by feeding to the model the estimated labor market parameters $\{\lambda^u_m, \lambda^u_f, s^e_m, s^e_f, s^G_m, s^G_f, \delta_m, \delta_f\}$ and the firm-level estimates of $\{p, \pi_m, \pi_f, z, c^{v,0}_m, c^{v,0}_f\}$. Then, we rank firms according to "composite productivity" $\tilde{p}_g$ for each gender. This is useful because, as stated in Lemma 4, firms that have higher "composite productivity" will pay higher effective wages.

We must first find the equilibrium level of aggregate vacancies V_g. We invert the equation for the offer arrival rate from unemployment in (13) to obtain:

$$V_{a,g} = U_{a,g}\left(\frac{\lambda^u_g}{\chi_{a,g}}\right)^{1/\alpha}.$$

Now consider the firm's first order conditions for vacancies in equation (38) and the rewritten first-order condition with respect to flow utility in equation (42). We use the same transformations that we have used to prove Lemma 4. Define $h_g(\tilde{p}_g) = F(x^*_g(p_g))$. Thus, $h'_g(\tilde{p}_g) = f_g(x^*_g(\tilde{p}_g))\, x'_g(\tilde{p}_g)$, therefore $f(x^*_g(\tilde{p}_g)) = h'_g(\tilde{p}_g)/x'_g(\tilde{p}_g)$. Also, $v(\tilde{p}_g) = \frac{V_g h'_g(\tilde{p}_g)}{\gamma(\tilde{p}_g)}$, so we can rewrite $h'_g(\tilde{p}_g) = \frac{v_g(p_g)}{V_g}\gamma(\tilde{p}_g)$. We assume $c^v_g(v) = c^{v,0}_g \frac{v^2}{2}$. Thus, we rewrite the first-order conditions as

$$\begin{aligned} h'_g(p_g) &= \frac{T_g(\tilde{p}_g - x_g(\tilde{p}_g))}{V_g c^{v,0}_g(p_g)}\left(\frac{1}{\delta + \lambda^G_g + \lambda^e_g(1 - h_g(\tilde{p}_g))}\right)^2 \gamma_g(\tilde{p}_g) \\ x'_g(\tilde{p}_g) &= 2\lambda^e_g \frac{T_g(\tilde{p}_g - x_g(\tilde{p}_g))^2}{V_g c^{v,0}_g(\tilde{p}_g)}\left(\frac{1}{\delta + \lambda^G_g + \lambda^e_g(1 - h_g(\tilde{p}_g))}\right)^3 \gamma_g(\tilde{p}_g) \end{aligned} \tag{56}$$

where $\gamma_g(\tilde{p}_g)$ is the density function of $\tilde{p}_g$. Then, after ranking all firms, we solve for wages and total

[45] We define task intensity as the mean z-score of a given task measures across occupations of workers at a given establishment. We obtain task measures for the Brazilian *Classificação Brasileira de Ocupaçoes (CBO)* occupation codes by hand-matching them to US Census occupation codes, which are then linked to the Occupational Information Network (O*NET) task scales constructed by Autor and Dorn (2009) and Acemoglu and Autor (2011).

[46] Eligibility for the Simples Nacional tax regime requires that the enterprise is a micro- or small business with annual revenues below BRL 1,200,00 (around USD 200,000), that it has no other companies as stakeholders, that it is not internationally owned, that it has no shareholder or partner with significant financial stakes in other companies, and that the enterprise itself has no stake in other companies.

vacancies posted by integrating the functions above by gender and firm-by-firm. For every firm j:

$$\begin{aligned}
h'_{g,j} &= \frac{T_g(\tilde{p}_{g,j} - x_{g,j})}{V_g c^{v,0}_{g,j}} \left(\frac{1}{\delta + \lambda^G_g + \lambda^e_g(1 - h_{g,j})} \right)^2 \gamma_g(\tilde{p}_{g,j}) \\
x'_{g,j} &= \frac{2\lambda^e_g T_g(\tilde{p}_{g,j} - x_{g,j})^2}{V_g c^{v,0}_{g,j}} \left(\frac{1}{\delta + \lambda^G_g + \lambda^e_g(1 - h_g g, j)} \right)^3 \gamma_g(\tilde{p}_{g,j}) \\
h_{g,j+1} &= h_{g,j} + h'_{g,j}(\tilde{p}_{g,j+1} - \tilde{p}_{g,j}) \\
x_{g,j+1} &= x_{g,j} + x'_{g,j}(\tilde{p}_{g,j+1} - \tilde{p}_{g,j})
\end{aligned} \tag{57}$$

and we calculate total vacancies obtained in equilibrium as $V^*_g = \sum_j v_{g,j}\gamma_g(\tilde{p}_{g,j})(\tilde{p}_{g,j} - \tilde{p}_{g,j-1})$. We solve the algorithm setting the initial conditions $x_{g,0} = \phi_g$ and $h_{g,0} = 0$, and we loop over a multiplier of all vacancy cost shifters $c^{v,0}_{g,j}$ until we obtain that $V^*_g = V_g$. Our solution algorithm produces gender-specific firm-level flow utility $x_{g,j}$, easily converted to wages $w_{g,j} = x_{g,j} - \pi_{g,j}$, gender-specific firm-level recruiting intensities $v_{g,j}$ and gender-specific firm-level ranks in the offer distribution $F_{g,j}$ that are exactly identical to those observed in the data, except for small rounding errors: the correlation between the data and the model-generated data is larger than 0.99999 for all variables, and it's equal to 1 by definition for ranks.

When we perform counterfactuals that involve equalizing female amenities to male amenities, we assume that $c^{\pi}_{j,g}(\pi_g) = c^{\pi,0}_{j,g}\frac{\pi_g^2}{2}$. We estimate the cost shifter $c^{\pi,0}_{j,g}$ by solving this equation for $c^{\pi,0}_{j,g}$ given our estimates of $\pi_{j,g}$. Thus, when a firm j was previously posting amenities $\pi_{j,f}$ for women in equilibrium, and now this level is equalized to the level of men $\pi_{j,m}$, we readjust composite productivity of firm j as follows:

$$\begin{aligned}
\tilde{p}^{baseline}_{j,f} &= \tilde{p} - z - c^{\pi}_{j,f}\frac{\pi^2_{j,f}}{2} \\
\tilde{p}^{counterfactual}_{j,f} &= \tilde{p} - z - c^{\pi}_{j,m}\frac{\pi^2_{j,m}}{2} \\
&= \tilde{p}^{baseline}_{j,f} + c^{\pi}_{j,f}\frac{\pi^2_{j,f,a}}{2} - c^{\pi}_{j,m}\frac{\pi^2_{j,m}}{2}
\end{aligned}$$

F Simulation Appendix

F.1 Numerical Solution Algorithm for Equal-Pay Policy

When we simulate our equal-pay policy, we cannot rely anymore on the fact that firms with higher composite productivity $p + \pi_g - c_g^\pi(\pi_g) - \mathbf{1}[g = F]z$ will post higher flow utility. The reason is that now both male and female effective productivity matter for the wage posted for either gender, because a firm has to find a single wage that maximizes total profits, which in turn depend on the profits extracted from both genders. Instead, we maximize directly the profit function firm-by-firm:

$$\max_{w,\pi_m,\pi_f,v_m,v_f} \Bigg\{ T_m v_m (p - w - c_m^\pi(\pi_m)) \left(\frac{1}{\delta_m + \lambda_m^G + \lambda_m^e (1 - F_m(w + \pi_m))} \right)^2 + T_f v_f (p - z - w - c_f^\pi(\pi_f)) \left(\frac{1}{\delta_f + \lambda_f^G + \lambda_g^e (1 - F_f(w + \pi_f))} \right)^2 - c_m^v(v_m) - c_f^v(v_f) \Bigg\}$$

where the definitions of T_g, F_g and V_g are as in the standard solution algorithm. The only unknowns in this problem are F_m and F_f, two endogenous objects to be determined in the equal-pay policy equilibrium. Firms can still hire both genders, only one gender or none.

Denote by $\mathcal{F}_\}$ the mapping from $\{F_m, F_f\}$ to the offer distributions implied by firms' behaviour. We solve the system of functional equations

$$\mathcal{F}_m(F_m^*, F_f^*) = F_m^*;$$
$$\mathcal{F}_f(F_m^*, F_f^*) = F_f^*,$$

where $\mathcal{F}_g(F_m^*, F_f^*)$ represents the offer distributions implied by the optimal choices of firms, that are a function of the offer distributions in the economy. It's worth noticing that the offer distributions of both genders implicitly depend on the offer distributions of both men and women, because when firms decide which wage to set, they have to take into account the effects this will have for attracting both genders with respect to the competition they face in the ladder.

Therefore, we solve for the equilibrium offer distributions F_m and F_f as follows:

1. Start with a guess for F_m and F_f; compute the firm's policy functions for optimal wages, amenities and vacancies taking F_m and F_f as given.

2. Aggregate optimal vacancies of firms to calculate V_g using equation (11).

3. Compute the offer distributions F_m and F_f that are implied by the firms' policy functions.

4. Find F_m and F_f such that the offer distributions taken as given by firms and the offer distributions implied by the firms' behavior are identical.

F.2 Counterfactual simulations when amenities are exogenous

We perform robustness checks in which we consider amenities as exogenous, so that firms are born with them and post them at zero cost (that is, $c_{j,g}^\pi(\pi_{j,g}) = 0$ for all j, g). Our results remain substantially unchanged for all counterfactuals, because we find that the cost of posting amenities is relatively small for most firms. Results for counterfactuals under the exogenous amenities assumption can be found in Table 22 below.

Table 22. Results from counterfactual simulations when amenities are exogenous.

	Baseline	Counterfactuals			
Gender differences in...	(0)	(1)	(2)	(3)	(4)
Amenities	✓		✓	✓	
Employer Tastes	✓	✓		✓	
Vac. Post. Costs	✓	✓	✓		
Gender pay gap	0.074	0.060	0.023	0.018	0.000
between employers	0.055	0.056	0.046	0.016	0.000
within employers	0.018	0.004	-0.023	0.002	0.000
Output	1.000	1.001	1.010	1.029	1.034
Worker welfare from...	0.000	0.005	0.015	-0.004	0.027
total payroll	0.000	0.010	0.024	0.019	0.029
payroll for men	0.000	0.000	0.000	0.000	0.000
payroll for women	0.000	0.010	0.024	0.019	0.029
total amenity value	0.000	-0.005	-0.009	-0.022	-0.002
amenity value for men	0.000	0.000	0.000	0.000	0.000
amenity value for women	0.000	-0.005	-0.009	-0.022	-0.002
Payroll-equivalent welfare change	0.000	0.006	0.018	-0.004	0.033
Employer welfare from...	1.000	0.995	1.010	0.986	1.039
profits	1.005	1.003	1.010	1.035	1.039
wedges	-0.005	-0.008	-0.000	-0.049	-0.000
Total employment for men	0.757	0.757	0.757	0.757	0.757
Total employment for women	0.760	0.760	0.762	0.760	0.757
Offer rate from nonemployment for men	0.100	0.100	0.100	0.100	0.100
Offer rate from nonemployment for women	0.087	0.087	0.088	0.087	0.100
Job-to-job transition rate for men	0.015	0.015	0.015	0.015	0.015
Job-to-job transition rate for women	0.012	0.012	0.012	0.012	0.015

Source: Authors' calculations based on RAIS. *Note:* Table reports simulation results from model-based counterfactuals. Baseline results (column 1) are compared against counterfactuals without gender differences in amenities (column 2), in employer wedges (column 3), in vacancy posting costs (column 4), and without any gender differences (column 5).