Revenues from Financial Capital. A Formal Framework

Rohwer, Götz and Behr*, Andreas

*University Duisburg-Essen, Campus Essen, Faculty of Economics and Business Administration

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Götz Rohwer, Andreas Behr*

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Summary. The paper proposes a framework for a formal discussion of the sources of revenues which can be attributed to financial capital. The framework refers to individual units (firms, households, state institutions) and therefore allows for a representation of ownership relations. The framework distinguishes between central bank money and deposit money created by private banks and assumes an institutional setting in which the central bank is not permitted to directly finance state institutions. The paper considers a broad and a narrow definition of revenues. The broad definition includes revenues having origins in banks' expansion of the money supply (in particular, revenues from new debts). Referring to this broad notion we find that the sum of these revenues has two sources: (1) revenues which public companies and investment funds receive from participating in the real economy (activities which aim to receive revenues from selling goods, services, and labor) and (2) the expansion of the money supply. The narrow definition includes only revenues from interest and from shares in public companies and investment funds. We find that the sum of these financial gains almost completely originates from revenues which public companies and investment funds receive from participating in the real economy.

Keywords: Financial capital, revenues from financial capital, endogenous money, integrated ownership

1 Introduction

We consider revenues resulting from the possession of financial capital (dividends from public companies and investment funds, interest from loans, bonds and saving contracts, gains from speculation with derivatives) as well as gains and losses resulting from trading financial capital. The leading question concerns from where these revenues originate. We follow the view that there are mainly two sources: participation in the 'real economy' (defined as consisting of activities which aim to receive revenues from selling goods, services, and labor) and an expansion of the money supply (see, e.g., Bezemer and Hudson (2016)). This paper develops a formal framework in which this view can be precisely formulated and derived. The text is organized as follows.

- We begin with describing a formal framework which is basically a version of the 'accounting view' as described by Bezemer (2016). The framework refers to individual units (firms, households, state institutions) therefore allows for a representation of ownership relations. The framework distinguishes between central bank money and deposit money created by private banks and assumes an institutional setting in which the central bank is not permitted to directly finance state institutions.
- We then refer to ultimate units (households and state institutions) which finally receive all revenues not retained by firms and investment funds. We show that the sum of the net revenues of ultimate units consists of three components: revenues from selling labor, net revenues which firms receive from selling goods and services, and a change in the units' stock of money.
- We then consider a broad and a narrow definition of revenues which can be attributed to financial capital. The broad definition, referred to as financial revenues, includes revenues having origins in banks' expansion of the money supply (in particular, revenues from new debts). Referring to this broad notion we find that the sum of financial revenues has two sources: (1) revenues

*E-Mail: andreas.behr@vwl.uni-due.de

which public companies and investment funds receive from participating in the real economy and (2) the expansion of the money supply. The narrow definition, referred to as financial gains, includes only revenues from interest and from shares in public companies and investment funds. We find that the sum of these financial gains almost completely originates from revenues which public companies and investment funds receive from participating in the real economy.

- The final section summarizes our results and briefly hints to processes which affect the distribution of financial revenues.
- A technical appendix deals with the treatment of ownership relations between units and contains proofs of some equations.

2 The formal framework

This section describes a formal framework serving to represent a system consisting of firms, households and state institutions. For distinguishing the units we use the index sets $\mathcal{F}$ (firms), $\mathcal{H}$ (households), and $\mathcal{S}$ (state institutions), also $\mathcal{U}$ for the total set of units and $\mathcal{L}$ for ultimate units (households and state institutions). For banks, defined as companies which can generate deposit money, the index set is $\mathcal{B}$, being a subset of $\mathcal{F}$; the complementary index set of units which are not banks (briefly referred to as 'nonbanks') is $\mathcal{N}$. Investment funds are treated formally like public companies and are considered as being contained in $\mathcal{F}$. It is assumed that the system is complete, meaning that each unit which owns part of another one also belongs to the system. There is only one currency and a single central bank which, however, does not belong to the index set $\mathcal{B}$.[1]

As a temporal framework we assume a sequence of time periods indexed by t. Most of the quantities subsequently defined are flows summarizing ex post the transactions of a period. We stress that the definitions presuppose that all transactions occurring during a period are fixed.

Revenues. The revenues which a unit U_i receives in the period t will be denoted by $Y_{i,t}$.[2] They consist of the following components:

$$Y_{i,t} = Y^w_{i,t} + Y^{mn}_{i,t} + Y^{mf}_{i,t} + Y^s_{i,t} + Y^i_{i,t} + Y^r_{i,t} \tag{1}$$

$Y^w_{i,t}$ denotes wages (only received by households).

$Y^{mn}_{i,t}$ denotes revenues of a firm resulting from selling goods and services. It is assumed that only firms sell goods and services.

$Y^{mf}_{i,t}$ denotes revenues resulting from selling financial capital. They will be defined in (10).

$Y^s_{i,t}$ denotes revenues resulting from shares. These can be shares of privately owned firms, public companies, and investment funds.

$Y^i_{i,t}$ denotes the balance of receiving and paying interest:

$$Y^i_{i,t} = \sum_{j\in\mathcal{U}} (t^i_{ji,t} - t^i_{ij,t}) \tag{2}$$

where $t^i_{ij,t}$ denotes interest (for loans, emitted bonds and saving contracts) paid by U_i to U_j in the period t. The sum over all units is

$$\sum_{i\in\mathcal{U}} Y^i_{i,t} = \sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{U}} (t^i_{ji,t} - t^i_{ij,t}) = 0 \tag{3}$$

[1] The central bank is viewed as a state institution without a formal representation (no index in $\mathcal{S}$). Subsequently, when used without qualification, the word 'bank' always refers to commercial banks.

[2] The word 'revenues' will be used for all kinds of receipts of money. For example, also the money received from taking a loan will be referred to as revenues.

$Y^r_{i,t}$ denotes a balance resulting from redistribution:

$$Y^r_{i,t} = \sum_{j \in \mathcal{U}} (t^r_{ji,t} - t^r_{ij,t}) \tag{4}$$

where $t^r_{ij,t}$ is the amount of money which, on the basis of public regulations, U_i must pay to U_j in the period t. Again, the sum over all units is

$$\sum_{i \in \mathcal{U}} Y^r_{i,t} = \sum_{i \in \mathcal{U}} \sum_{j \in \mathcal{U}} (t^r_{ji,t} - t^r_{ij,t}) = 0 \tag{5}$$

Consumption and investment. Units purchase labor, goods, services and financial capital. If these purchases serve to receive revenues (which in the case of financial capital can always be assumed), they will subsequently be called investments, otherwise it will be said that they are used for consumption. For this distinction it is not relevant whether the purchased things are durable or not but only their intended use. It will be assumed that only ultimate units spend money on consumption.

$C_{i,t}$ denotes purchases, made by an ultimate unit U_i in the period t, which are intended to serve consumption. They consist of two components:

$$C_{i,t} = W_{i,t} + C^n_{i,t} \tag{6}$$

$W_{i,t}$ records payments of wages (mainly employees of state institutions, but also of households), $C^n_{i,t}$ records purchases of goods and services.

$I_{i,t}$ denotes purchases, made by a unit U_i in the period t, intended to serve investments. They consist of three components:

$$I_{i,t} = W_{i,t} + I^n_{i,t} + I^f_{i,t} \tag{7}$$

$W_{i,t}$ records payments of wages, $I^n_{i,t}$ records purchases of goods and services. The sum of these two components will be called *nonfinancial investment*. The third component, $I^f_{i,t}$, records financial investment and will be defined below.

We assume that only firms make nonfinancial investments and receive revenues from selling goods and services.[3] This entails the following equation:

$$\sum_{i \in \mathcal{F}} Y^{mn}_{i,t} = \sum_{i \in \mathcal{F}} I^n_{i,t} + \sum_{i \in \mathcal{L}} C^n_{i,t} \tag{8}$$

The first summand on the right-hand side records firms' investments in goods and services, the second one records expenditures of ultimate units on goods and services (both without payment of wages).

Financial investments are defined by

$$I^f_{i,t} = \sum_{j \in \mathcal{U}} t^f_{ij,t} \tag{9}$$

where $t^f_{ij,t}$ denotes payments from U_i to U_j for purchasing (including re-purchasing) financial capital in one or more of the following forms (note that, in general, $t^f_{ij,t} \neq t^f_{ji,t}$).

- *Saving contracts* (including all kinds of term deposits). U_i buys from U_j (e.g., a bank or insurance company) a saving contract and pays $t^f_{ij,t}$. When the contract ends there is a payment $t^f_{ji,t}$ to the owner of the contract.

[3]To simplify the notation we consider payments of households and state institutions for financial services as being part of their consumption.

- *Bilateral credits*. U_i (a bank or nonbank) buys financial capital by granting a credit to U_j and pays the credit amount $t^f_{ij,t}$. The repayment consists in U_j's re-purchasing the financial capital by paying $t^f_{ji,t}$ to U_i.
- *Bonds*. U_i buys from U_j bonds and pays $t^f_{ij,t}$. U_j could be the original emitter or a later owner of the bond.[4] The repayment consists in a payment of the emitter to the current owner of the bond.
- *Shares of public companies*. U_i buys from U_j shares and pays $t^f_{ij,t}$. U_j can be the original emitter or a later owner of the shares. If shares are re-purchased by the emitter, the financial capital represented by the shares is destroyed.
- *Shares of investment funds*. U_i buys shares from an investment fund U_j and pays $t^f_{ij,t}$. If the shares are re-purchased by the fund by paying an amount $t^f_{ji,t}$, the financial capital previously represented by the shares is destroyed.
- *Bets on derivatives*. U_i pays $t^f_{ij,t}$ for participating in a betting game with derivatives. The contracting party U_j is a stock exchange or another unit participating in an OTC contract.

The financial investments defined by (9) comprise both purchases and re-purchases of financial capital (including repayments of loans and bonds).[5] The difference is that re-purchasing financial capital implies its destruction.

Corresponding to financial investments, a unit selling financial capital receives payments

$$Y^{mf}_{i,t} = \sum_{j\in\mathcal{U}} t^f_{ji,t} \tag{10}$$

These payments consist of money
- from terminating saving contracts,
- from loans and from the emission of bonds,
- from the repayment of loans and bonds,
- from selling stock both at primary and secondary markets,
- from betting with derivatives,
- which an investment fund receives from selling shares or a shareholder receives from redeeming shares.

The positive or negative cash flow which U_i receives or pays in the period t is $Y^{mf}_{i,t} - I^f_{i,t}$. Correspondingly, $I^f_{i,t} - Y^{mf}_{i,t}$ is the net financial investment. Summing over all units one obtains the equation

$$I^f_t = \sum_{i\in\mathcal{U}} I^f_{i,t} = \sum_{i\in\mathcal{U}} Y^{mf}_{i,t} = Y^{mf}_t \tag{11}$$

The total of financial investments equals the total of receipts from selling financial capital.

Notations for sums. If a symbol omits the reference to a unit, as for example I^f_t, it denotes the sum over all units. If the reference to a unit is replaced by an index set, the symbol denotes the sum over all units in the index set, e.g., $Y^w_{\mathcal{H},t} = \sum_{i\in\mathcal{H}} Y^w_{i,t}$.

Use of the revenues. Units' use of revenues depends on whether they are ultimate units or firms:

Households and state institutions $\quad Y_{i,t} = C_{i,t} + I^f_{i,t} + \Delta M_{i,t} \quad (12)$

Firms (without banks) $\quad Y_{i,t} = I_{i,t} + A_{i,t} + \Delta M_{i,t} \quad (13)$

[4] If bonds (or shares) are traded at a stock exchange a contractant is not uniquely identified. We then assume a unit which is randomly selected from the order book.

[5] The definition of investments given above is thereby extended: Investments may also serve a repayment of debt or other kinds of liability.

Ultimate units spend revenues on consumption and financial investments. Firms (both public and privately owned firms) and investment funds spend revenues on investments and on payments to shareholders, denoted by $A_{i,t}$. $\Delta M_{i,t}$ records the change in the unit's stock of money.

Changes in the quantity of money. We distinguish between banks' deposit money (which can be created by banks) and central bank money.[6] For our formal framework we make two simplifying assumptions:

- Units other than banks only use deposit money. This implies that also for all transactions between banks and nonbanks only deposit money is used.[7]
- For transactions in the banking sector, including the central bank, only central bank money is used.

The stock of deposit money owned by a unit U_i at the end of a period t will be denoted by $M_{i,t}$. The change in this stock is

$$\Delta M_{i,t} = M_{i,t} - M_{i,t-1} \tag{14}$$

Since banks can create deposit money, $M_{i,t} = 0$ if U_i is a bank, and consequently also $\Delta M_{i,t} = 0$. Summing over all units leads to

$$\Delta M_t = \text{ Change in the total quantity of deposit money}$$

Entailed by the above made assumptions, the following holds:

$$\begin{array}{l}\text{Payments of banks to nonbanks increase the stock}\\ \text{of deposit money, payments of nonbanks to banks}\\ \text{decrease the stock of deposit money.}\end{array} \tag{15}$$

With respect to transactions involving banks we need to distinguish between receipts/payments between banks and nonbanks (made with deposit money) and among banks (made with central bank money). In order to allow unified notations for transactions involving nonbanks we use the following notations for banks ($i \in \mathcal{B}$):

- $Y_{i,t}$ denotes receipts of payments from nonbanks, and $I_{i,t}$ and $A_{i,t}$ denote payments to nonbanks made for investments and payouts, respectively.[8] It then follows from (15) that the contribution of a bank U_i to the supply of deposit money is

  $$\Delta \tilde{M}_{i,t} = I_{i,t} + A_{i,t} - Y_{i,t} \tag{16}$$

  and the total increase is

  $$\Delta M_t = \Delta \tilde{M}_{\mathcal{B},t} \tag{17}$$

- $Y^*_{i,t}$ denotes receipts of payments from other banks, and $I^*_{i,t}$ and $A^*_{i,t}$ denote payments to other banks made for investments and payouts, respectively. We may then write

  $$Y^*_{i,t} = I^*_{i,t} + A^*_{i,t} + \Delta M^*_{i,t} \tag{18}$$

[6]For understanding banks' ability to create deposit money see, e.g., McLeay, Radia, and Thomas (2014), Werner (2014), Jakab and Kumhof (2019).

[7]Actually, accounts of state institutions are located at central banks. We therefore use the following definition: *Deposit money* consists of money held by nonbanks in accounts at private banks and held by state institutions in accounts at the central bank. Using this definition greatly simplifies the formal framework and can be justified with the presupposition of an institutional setting in which the central bank is not permitted to directly finance state institutions. This entails that transactions involving state institutions are always mediated by private banks. In particular, transactions of state institutions with units other than banks generally employ banks' deposit money. A payment to one of these units ends up in an increase of the deposit money in the unit's bank account. Conversely, payments to state institutions (e.g., taxes) start from bank accounts and therefore are made with deposit money.

[8]Since banks can use deposit money also for payments to nonbank shareholders, the quantities $A_{i,t}$ cannot be determined purely by cash flows. We therefore consider these quantities as resulting from autonomous decisions of the banks (limited by legal regulations through accounting rules).

where $\Delta M^*_{i,t}$ denotes the change in the bank's stock of central bank money. The total change of reserves in the period, $\Delta M^*_{\mathcal{B},t}$, results from transactions between banks and the central bank (which has no explicit representation in our framework). Note that there is no formal relation between $\Delta M^*_{\mathcal{B},t}$ and ΔM_t.

Savings. In our framework, financial and nonfinancial investments are both considered as investments aiming to receive gains and/or to accumulate wealth. Consequently, only changes in the stock of money are considered as savings, and the basic equations defining savings are:

$$\Delta M_{i,t} = \begin{cases} Y_{i,t} - I_{i,t} - A_{i,t} & \text{nonbank firms} \\ Y_{i,t} - I^f_{i,t} - C_{i,t} & \text{ultimate units} \end{cases} \tag{19}$$

The savings of banks are defined as being zero. This entails that the sum of all savings equals the change in the money supply, ΔM_t. Note that our notion of savings refers to a flow which can be positive, negative, or zero. In contrast, a unit's accumulated savings, $M_{i,t}$, cannot be negative.

Retrospective notions. The flows defined in this section record what *has happened* during a period. Definite values only exist at the end of the period. For example, if a unit U_i receives an amount of money, say c, from a loan provided by a bank U_j, one only can conclude:[9]

$\Delta M_{i,t} \mathrel{+}= c$ $\quad$ U_i's stock of money increases by c

$Y^{mf}_{i,t} \mathrel{+}= c$ $\quad$ U_i's sales of financial capital increase by c

$I^f_{j,t} \mathrel{+}= c$ $\quad$ U_j's financial investments increase by c

In general, $\Delta M_{i,t}$, $Y^{mf}_{i,t}$ and $I^f_{i,t}$ will depend on many more transactions having taken place until the end of the period.

3 Revenues of ultimate units

In our framework all firms ($i \in \mathcal{F}$ which includes investment funds) are owned by other units. These could again be firms or ultimate units which are not owned by other units. For ultimate units we use the index set $\mathcal{L}$. This section considers their revenues.

Revenues from shares. Revenues $Y^s_{i,t}$ which a unit U_i receives from shares result from payments by firms (including investment funds). For making the relationship explicit we use the following definition:

$s_{ij,t}$ denotes U_i's share of U_j at the point in time when U_j makes the payment $A_{j,t}$.

The relationship can then be written as

$$Y^s_{i,t} = \sum_{j \in \mathcal{F}} s_{ij,t}\, A_{j,t} \tag{20}$$

Since we have assumed that the system of units is complete, the equation

$$\sum_{i \in \mathcal{U}} s_{ij,t} = 1 \tag{21}$$

holds for each firm U_j.

Total revenues from shares. As a consequence of ownership relations among firms the sum of the quantities $A_{j,t}$ ($j \in \mathcal{F}$) has no economically sensible meaning. A useful quantity can be defined, however, by referring to ultimate units. Based on our formal framework one can derive the following equation (the derivation is described in the appendix):

$$Y^s_{\mathcal{L},t} = (Y^{mn}_{\mathcal{F},t} - W_{\mathcal{F},t} - I^n_{\mathcal{F},t}) + (Y^{mf}_{\mathcal{F},t} - I^f_{\mathcal{F},t}) + Y^i_{\mathcal{F},t} + Y^r_{\mathcal{F},t} + \Delta M_{\mathcal{L},t} \tag{22}$$

The equation shows components of the sum of revenues which ultimate units receive from shares:

[9] We use '$+=$' and '$-=$' to mean, respectively, 'increased by' and 'decreased by'.

- firms' net receipts from selling goods and services,
- firms' net receipts from selling and buying financial capital,
- firms' balance of received and paid interest,
- firms' balance of redistribution due to public regulations, and
- the sum of the ultimate units' savings.

The last component equals the increase in the money supply minus the savings of firms:

$$\Delta M_{\mathcal{L},t} = \Delta M_t - \Delta M_{\mathcal{F},t} \tag{23}$$

Ultimate revenues. This term will be used for referring to the sum of the revenues of ultimate units:

$$Y_{\mathcal{L},t} = Y^w_{\mathcal{L},t} + Y^s_{\mathcal{L},t} + Y^i_{\mathcal{L},t} + Y^r_{\mathcal{L},t} + Y^{mf}_{\mathcal{L},t} \tag{24}$$

Inserting (22), and taking into account (3) and (5), this sum can also be written as

$$Y_{\mathcal{L},t} = Y^w_{\mathcal{L},t} + (\, Y^{mn}_{\mathcal{F},t} - W_{\mathcal{F},t} - I^n_{\mathcal{F},t}\,) + I^f_{\mathcal{L},t} + \Delta M_{\mathcal{L},t} \tag{25}$$

showing the components which contribute to the ultimate revenues (recall that $Y^w_{\mathcal{L},t} = W_{\mathcal{F},t} + W_{\mathcal{L},t}$). Using (8) and (11), one obtains

$$Y_{\mathcal{L},t} = C_{\mathcal{L},t} + I^f_{\mathcal{L},t} + \Delta M_{\mathcal{L},t} \tag{26}$$

which describes the usage side. Using the term 'net revenues' for referring to revenues minus investments, one finds

$$\begin{aligned} Y_{\mathcal{L},t} - I^f_{\mathcal{L},t} &= Y^w_{\mathcal{L},t} + (\, Y^{mn}_{\mathcal{F},t} - W_{\mathcal{F},t} - I^n_{\mathcal{F},t}\,) + \Delta M_{\mathcal{L},t} \\ &= C_{\mathcal{L},t} + \Delta M_{\mathcal{L},t} \end{aligned} \tag{27}$$

The first equation shows the components, the second equation shows that the sum of the ultimate units' net revenues equals their consumption plus savings.

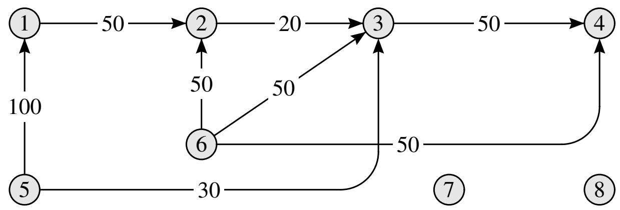
Illustration. To illustrate the formal notations, we consider a system consisting of 8 units: U_1, U_2, U_3 and U_4 (a bank) are firms, U_5, U_6 and U_7 are households, U_8 is a state institution. The upper part of Box 1 shows the ownership relations (in %). The table in the lower part of the box shows arbitrarily chosen values referring to a period t.

In this example we assume that firms (other than banks) finance their investments completely with bank loans and do not use retained earnings.[10] To simplify notations we also assume that the loans have a duration of less than one period. A firm U_i takes loans $Y^{mf}_{i,t}$ at the beginning of t and makes repayments within the period. The repayment is recorded as a part of the firms financial investments, $I^f_{i,t}$. In the present example we assume an interest rate of 5 %;[11] firms repay their loans completely and do not retain earnings ($\Delta M_i = 0$). This entails that the payment to shareholders is $A_i = Y_i - I_i$ (here we assume that the bank only makes financial transactions and its payment to shareholders equals the receipt of interest). The state institution gets revenues from taxes paid by households. Together, the ultimate units receive $Y_{\mathcal{L}} = 470 = 356$ (wages) + 114 (revenues from shares). The two last components in (25) are both zero. The ultimate revenues are completely used for consumption (450 for buying goods and services and 20 for paying wages).

[10]Since banks can provide credits through generating deposit money foregoing savings are not required; see, e.g., Bibow (2001), Cardim de Carvalho (2012), Lindner (2015), Jakab and Kumhof (2019).

[11]Therefore, $Y^i_{i,t} = -0.05\, Y^{mf}_{i,t}$. Note that, in general, $Y^i_{i,t}$ may also contain payments of interest for credits taken in earlier periods.

Box 1



$$
(s_{ij,t}) = \begin{bmatrix} 0 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0.3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

	I_i^f	Y_i^{mf}	I_i^n	C_i^n	W_i	Y_i^w	Y_i^{mn}	Y_i^i	A_i	Y_i^s	Y_i^r	Y_i	ΔM_i
U_1	80	80	50	0	50	0	134	−4	43.7	13.7	0	223.7	0
U_2	100	100	34	0	136	0	190	−5	27.4	12.4	0	297.4	0
U_3	100	100	10	0	150	0	220	−5	62	7	0	322	0
U_4	280	280	0	0	0	0	0	14	14	0	0	294	0
U_5	0	0	0	172.3	0	150	0	0		62.3	−40	172.3	0
U_6	0	0	0	167.7	0	166	0	0		51.7	−50	167.7	0
U_7	0	0	0	40	0	40	0	0		0	0	40	0
U_8	0	0	0	70	20	0	0	0		0	90	90	0
	560	560	94	450	356	356	544	0		(114)	0	(470)	0

Totals in brackets refer only to ultimate units.

4 Revenues from financial capital

In this section we use our framework to discuss from where the revenues which units receive from owning, or selling, financial capital originate. We begin with a broad definition of *financial revenues*:

$$Y_{i,t}^f = Y_{i,t}^i + (Y_{i,t}^{mf} - I_{i,t}^f) + Y_{i,t}^{sf} \tag{28}$$

The first component records the balance of received and paid interest. The second component records net revenues from selling financial capital. The third component records revenues from shares which can be attributed to financial capital:

$$Y_{i,t}^{sf} = \sum_{j \in \mathcal{F}} s_{ij,t} \, A_{j,t}^f \tag{29}$$

Note that $Y_{i,t}^f$ can be negative and then represents a payment of interest and/or financial investments.

In the definition of payouts which can be attributed to financial capital one has to distinguish between privately owned firms (the index set is $\mathcal{F}_{pr}$) and public companies and investment funds (the index set is $\mathcal{F}_{pu}$):

- If U_j is a privately owned firm, but not a bank, we define

$$A_{j,t}^f = Y_{j,t}^f - \alpha_j \, \Delta M_{j,t} \tag{30}$$

where α_j specifies the fraction of the savings, $\Delta M_{j,t}$, attributed to financial revenues ($0 \leq \alpha_j \leq 1$). Since empirical information is only available about the complete savings, the fraction $\alpha_j \, \Delta M_{j,t}$ must be considered as an imputed quantity. We note that $\Delta M_{j,t}$ can be negative, an example (E6) will be considered below.

- If U_j is a public company or investment fund its payouts can be considered as completely resulting from the possession of financial capital (stock and shares of investment funds):

$$A^f_{j,t} = A_{j,t} = Y_{j,t} - I_{j,t} + \Delta\tilde{M}_{j,t} \tag{31}$$

where $\Delta\tilde{M}_{j,t} = -\Delta M_{j,t}$ if U_j is not a bank (see (13) and (16)). In order to simplify notations we also assume for privately owned banks that their payouts can be completely attributed to financial capital. In this section they will therefore be considered as belonging to the index set $\mathcal{F}_{pu}$. This entails:

$$\sum_{j \in \mathcal{F}_{pu}} \Delta\tilde{M}_{j,t} = \Delta M_t - \sum_{j \in \mathcal{F}_{pu}} \Delta M_{j,t} \tag{32}$$

Note that the quantities $A^f_{j,t}$ can be negative. For example, if the unit U_3 in Box 1 would own, instead of 50 %, only 20 % of the shares of the bank, then $A^f_3 = -5 + 0.2 \cdot 14 = -2.2$. However, the quantities $A^f_{j,t}$ do not serve to represent actual payouts but to assess sources of financial revenues which can be positive or negative. (This example will be further considered below (see E5).)

Aggregation of financial revenues. Based on the above definitions one can derive the following equation (the derivation is described in the appendix):

$$\begin{aligned} &\sum_{i \in \mathcal{L}} Y^f_{i,t} + \sum_{j \in \mathcal{F}_{pr}} \alpha_j \, \Delta M_{j,t} + \sum_{j \in \mathcal{F}_{pu}} \Delta M_{j,t} = \\ &\quad \Big(\sum_{j \in \mathcal{F}_{pu}} (Y^{mn}_{j,t} - W_{j,t} - I^n_{j,t}) + (Y^s_{j,t} - Y^{sf}_{j,t}) + Y^r_{j,t} \Big) + \Delta M_t \end{aligned} \tag{33}$$

The left-hand side records the sum of all financial revenues. There are three parts: (1) financial revenues received by ultimate units, (2) the part of financial revenues used for savings by privately owned firms, (3) the savings of public companies, investment funds, and privately owned banks. The first part of the right-hand side records nonfinancial revenues of public companies and investment funds: the first term records revenues from selling goods and services, then follow revenues from shares except those which can be attributed to financial capital, and finally there is a balance of redistributions based on public regulations. The second component equals the total increase in the money supply.

Illustration. To illustrate Eq. (33) we consider some examples following the setup depicted in Box 1 with the assumption that U_1 and U_3 are privately owned firms and U_2 and U_4 are public companies. Numerical values corresponding to (33) are shown in Box 2.

E1 We begin with the example in Box 1. There are no savings, and all financial revenues are received by ultimate units. The sum of these revenues equals the nonfinancial revenues of U_2.

E2 If the public company U_2 saves $\Delta M_2 = 5$, there is a corresponding decrease in the financial revenues of ultimate units.

E3 We assume that the privately owned firm U_1 saves $\Delta M_1 = 5$. This entails that the payouts A_1 decrease by 5. The financial payouts, A^f_1, depend on α_1. In Box 2 it is assumed that $\alpha_1 = 0.5$ which entails $Y^f_5 = A^f_1 + 0.3 \cdot A^f_3 = (13.7 - 4 - 2.5) + (0.3 \cdot 2) = 7.8$ ($Y^f_6 = 21.7$ as before).

E4 We assume that the bank buys government bonds from U_8: I^f_4 += 20 (see Box 3) which entails an equal increase in the financial revenues. The state pays interest, $Y^i_8 = -1$, and uses the rest of the borrowed money for purchasing goods and services. We also assume that the households U_5 and U_6 reduce their consumption so that the total amount of consumption does not change.

Box 2

	$E1$	$E2$	$E3$	$E4$	$E5$	$E6$
$\sum_{i\in\mathcal{L}} Y_i^f$	32	27	29.5	51.5	32	34.2
$\sum_{i\in\mathcal{F}_{pr}} \alpha_j\,\Delta M_j$	0	0	2.5	0.5	0	-2.2
$\sum_{i\in\mathcal{F}_{pu}} \Delta M_j$	0	5	0	0	0	0
	32	32	32	52	32	32
$\sum_{j\in\mathcal{F}_{pu}} Y_j^{mn} - W_j - I_j^n +$	20	20	20	20	20	20
$\sum_{j\in\mathcal{F}_{pu}} Y_j^s -$	12.4	12.4	12.4	12.4	15.76	16.2
$\sum_{j\in\mathcal{F}_{pu}} Y_j^{sf} +$	0.4	0.4	0.4	0.4	3.76	4.2
$\sum_{j\in\mathcal{F}_{pu}} Y_j^r +$	0	0	0	0	0	0
ΔM	0	0	0	20	0	0
	32	32	32	52	32	32

Box 3

	I_i^f	Y_i^{mf}	I_i^n	C_i^n	W_i	Y_i^w	Y_i^{mn}	Y_i^i	A_i	Y_i^s	Y_i^r	Y_i	ΔM_i
U_1	80	80	50	0	50	0	134	-4	43.7	13.7	0	223.7	0
U_2	100	100	34	0	136	0	190	-5	27.4	12.4	0	297.4	0
U_3	100	100	10	0	150	0	220	-5	62	7.5	0	322.5	0.5
U_4	300	280	0	0	0	0	0	15	15	0	0	295	0
U_5	0	0	0	162.8	0	150	0	0		62.3	-40	172.3	9.5
U_6	0	0	0	157.7	0	166	0	0		52.2	-50	168.2	10
U_7	0	0	0	40	0	40	0	0		0	0	40	0
U_8	0	20	0	89	20	0	0	-1		0	90	110	0
	580	580	94	450	356	356	544	0		(114)	0	(490)	20

The reduction of consumption then shows up as savings: $\Delta M_5 \mathrel{+}= 9.5$ and $\Delta M_6 \mathrel{+}= 9.5$. In addition, U_6 saves the additional payout received from the bank, $\Delta M_6 \mathrel{+}= 0.5$, which equals half of the interest paid by the state. The other half is saved by U_3.

E5 We now consider an example with negative payouts. We refer to Box 1, but assume that U_3 owns 20 % of the bank and the remaining 30 % are owned by U_2. This entails $Y_3^f = -5+0.2\cdot 14 = -2.2$. While the total payouts are positive, $A_3 = 57.8$, the part attributed to financial capital is negative: $A_3^f = -2.2$. This implies: $Y_2^s = 0.3\cdot 14+0.2\cdot 57.8 = 15.76$ and $Y_2^{sf} = 0.3\cdot 14-0.2\cdot 2.2 = 3.76$. The difference which contributes to the sum of financial revenues is still 12 (see Box 2).

E6 We continue with the previous example and assume that U_3 increases its payouts by 2.2, financed by dissaving $\Delta M_3 = -2.2$. The financial payouts depend on α_3. In Box 2 it is assumed that $\alpha_3 = 1$, entailing that the financial payouts are zero. Since the dissaving is attributed to financial capital, $-\alpha_3\,\Delta M_3$ is considered as a financial revenue.

Contribution of the money supply. The change of the money supply which occurs on the right-hand side of Eq. (33) results from transactions involving banks. For a bank U_i, starting from (1) and using (16), one can write

$$Y_{i,t} = Y_{i,t}^{mn} + Y_{i,t}^{mf} + Y_{i,t}^{s} + Y_{i,t}^{i} + Y_{i,t}^{r} = I_{i,t} + A_{i,t} - \Delta\tilde{M}_{i,t} \tag{34}$$

$\Delta\tilde{M}_{i,t}$ records the bank's contribution to the change in the money supply. Rearranging terms and recalling $I_{i,t} = W_{i,t} + I^n_{i,t} + I^f_{i,t}$, one obtains

$$\underbrace{Y^{mn}_{i,t} - W_{i,t} - I^n_{i,t} + Y^s_{i,t} + Y^i_{i,t} + Y^r_{i,t}}_{A^*_{i,t}} = A_{i,t} + I^f_{i,t} - Y^{mf}_{i,t} - \Delta\tilde{M}_{i,t} \tag{35}$$

and finally

$$\Delta\tilde{M}_{i,t} = (I^f_{i,t} - Y^{mf}_{i,t}) + (A_{i,t} - A^*_{i,t}) \tag{36}$$

$A_{i,t}$ records the actual payouts and $A^*_{i,t}$ defines a quantity of payouts which could be financed with gains from selling goods and services and from shares and interest (taking into account the redistribution balance $Y^r_{i,t}$). Summing over all banks, one finds

$$\Delta M_t = \Delta\tilde{M}_{\mathcal{B},t} = (I^f_{\mathcal{B},t} - Y^{mf}_{\mathcal{B},t}) + (A_{\mathcal{B},t} - A^*_{\mathcal{B},t}) \tag{37}$$

showing that one can safely assume that an increase in the money supply mainly results from net financial investments of banks. As follows from (11), these investments equal the net revenues which nonbank units receive from selling financial capital:

$$I^f_{\mathcal{B},t} - Y^{mf}_{\mathcal{B},t} = Y^{mf}_{\mathcal{N},t} - I^f_{\mathcal{N},t} \tag{38}$$

This can be further simplified by using the notation $t^f_{i,t}$ introduced in (9):

$$Y^{mf}_{\mathcal{N},t} - I^f_{\mathcal{N},t} = \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{U}} (t^f_{ji,t} - t^f_{ij,t}) = \sum_{i\in\mathcal{N}}\sum_{j\in\mathcal{B}} (t^f_{ji,t} - t^f_{ij,t}) \tag{39}$$

showing that the increase in the money supply mainly equals the net revenues which nonbank units receive from selling financial capital to banks. There are two kinds of transactions:

- Selling debt contracts by taking loans or selling newly emitted bonds. This is illustrated by the example E4.
- Selling financial capital which a unit already owns, e.g., selling securities, redeeming term deposits. To illustrate, one can assume that U_1 sells 20 % of the shares in U_2 to the bank and thereby receives a payment $\Delta M_1 \mathrel{+}= 20$. The payment is made with newly created deposit money which entails $\Delta M \mathrel{+}= 20$. Referring to the example in Box 1, $\Delta M_1 = 20$ at the end of the period, and since these savings result from selling financial capital one should assume $\alpha_1 = 1$ (according to Eq. 33).

In both cases revenues are different from gains which can be attributed to a continued ownership of financial capital.

Gains from owning financial capital. In order to understand the sources of revenues which can be attributed to the ownership of financial capital, we use a definition of *financial gains* which only include revenues from interest and shares in public companies and investment funds:

$$Y^{f*}_{i,t} = Y^i_{i,t} + Y^{sf}_{i,t} \tag{40}$$

Completely analogous to the derivation of Eq. (33), and taking into account (37), one obtains the following equation:

$$\begin{aligned} &\sum_{i\in\mathcal{L}} Y^{f*}_{i,t} + \sum_{j\in\mathcal{F}_{pr}} \alpha_j\, \Delta M_{j,t} + \sum_{j\in\mathcal{F}_{pu}\setminus\mathcal{B}} (\Delta M_{j,t} + I^f_{j,t} - Y^{mf}_{j,t}) = \\ &\quad \Big(\sum_{j\in\mathcal{F}_{pu}} (Y^{mn}_{j,t} - W_{j,t} - I^n_{j,t}) + (Y^s_{j,t} - Y^{sf}_{j,t}) + Y^r_{j,t}\Big) + (A_{\mathcal{B},t} - A^*_{\mathcal{B},t}) \end{aligned} \tag{41}$$

Compared with (33), there are two differences.

1) The change in the money supply, ΔM_t, no longer occurs on the right-hand side. In addition to revenues which public companies and investment funds receive from participating in the real economy there only remains a term reflecting a change in the money supply through nonfinancial investments and payouts of banks.

2) On the left-hand side, the term referring to public companies and investment funds, without public and private banks, now contains net financial investments in addition to monetary savings. However, these transactions do not change the sum $\Delta M_{j,t} + I^f_{j,t} - Y^{mf}_{j,t}$. For example, if U_2 sells stock to the bank, say $\Delta M_2 \mathrel{+}= c$, there is an equal negative investment $Y^{mf}_2 \mathrel{+}= c$. This is different for privately owned firms. If U_1 sells the stock, $\Delta M_1 \mathrel{+}= c$, but then $\alpha_1 = 0$ because it is not a financial gain.

One can conclude that financial gains which can be attributed to the possession of financial capital are limited by revenues from the real economy, that is, from selling goods, services, and labor.

5 Conclusion

We have developed a formal framework to serve a discussion of the sources of revenues from financial capital. We have distinguished between a broad and a narrow definition of these revenues. The broad definition ($Y^f_{i,t}$), referred to as financial revenues, includes revenues having origins in banks' expansion of the money supply (in particular, revenues from new debts). Referring to this broad notion we find that the sum of financial revenues has two sources: (1) revenues which public companies and investment funds receive from participating in the real economy and (2) the expansion of the money supply.

The narrow definition ($Y^{f*}_{i,t}$), referred to as financial gains, includes only revenues from interest and from shares in public companies and investment funds. We find that the sum of these financial gains almost completely originates from revenues which public companies and investment funds receive from participating in the real economy.

Both statements refer to a system-wide sum of revenues and therefore abstract from processes affecting the distribution of financial revenues, in particular: paying and receiving interest, selling and purchasing financial capital, participating in betting games with derivatives. However, except when involving an increase in the money supply, these processes cannot increase the sum of financial revenues. To illustrate the argument we consider trading. In order that a nonbank unit U_i can sell some piece of financial capital there must be another unit buying it. Then, whatever the market price, U_i's receipt from selling, say $Y^{mf}_i \mathrel{+}= c$, equals U_j's investment, $I^f_j \mathrel{+}= c$. If U_j is not a bank no change in the sum of financial revenues occurs. An increase only occurs if U_j is a bank which pays with newly created deposit money. However, the increase in the financial revenues then originates from the increase in the money supply, completely independent of the development of market prices.

We finally note that the focus on revenues also suggests a critique of the belief that rising prices of financial capital signify rising wealth. In order to realize some revenues a unit would need to sell a part of its financial capital. This requires other units willing to buy and to pay the actual price. Any gains of the seller originate from payments of buyers. If such transactions lead to an increase in the sum of financial revenues it does not result from rising asset prices but from an increase in the money supply used for buying financial capital.

6 Appendix

Integrated ownership. The coefficients s_{ij}, introduced in Section 3, denote the share of a corporation or investment fund U_j owned by a unit U_i. Integrated coefficients, denoted by h_{ij}, intend to capture shares which are owned both directly and indirectly. Following Baldone et al. (1997), they can be defined by

$$h_{ij} = s_{ij} + \sum_{k \in \mathcal{F}, k \neq i} h_{ik}\, s_{kj} \tag{42}$$

In order to solve this system of equations it is helpful to use matrix notations. Let n denote the number of units, and let the coefficients be given, respectively, by (n,n)-matrices $\mathbf{S} = (s_{ij})$ and

$\mathbf{H} = (h_{ij})$. To begin with, the system (42) can be written as follows:

$$h_{ij} = s_{ij} + \sum_{k \neq i} h_{ik}\, s_{kj} = (1 - h_{ii})\, s_{ij} + \sum_{k=1}^{n} h_{ik}\, s_{kj} \tag{43}$$

Using $\mathbf{I}$ to denote the unit matrix of order n, these equations can be written as

$$\mathbf{H} = (\mathbf{I} - \mathrm{diag}(\mathbf{H}))\, \mathbf{S} + \mathbf{HS} \tag{44}$$

where $\mathrm{diag}(\mathbf{H})$ is a matrix containing only the elements on the main diagonal of $\mathbf{H}$. This immediately leads to

$$\mathbf{H}\,(\mathbf{I} - \mathbf{S}) = (\mathbf{I} - \mathrm{diag}(\mathbf{H}))\, \mathbf{S} \tag{45}$$

Multiplying with $(\mathbf{I} - \mathbf{S})^{-1}$ and $(\mathbf{I} - \mathrm{diag}(\mathbf{H}))^{-1}$ one obtains

$$(\mathbf{I} - \mathrm{diag}(\mathbf{H}))^{-1}\mathbf{H} = \mathbf{S}\,(\mathbf{I} - \mathbf{S})^{-1} \tag{46}$$

and from this, one derives:[12]

$$\begin{aligned}
\mathrm{diag}\{\mathbf{S}(\mathbf{I} - \mathbf{S})^{-1}\} &= \mathrm{diag}\{(\mathbf{I} - \mathrm{diag}(\mathbf{H}))^{-1}\mathbf{H}\} \\
&= \mathrm{diag}\{(\mathbf{I} + \mathrm{diag}(\mathbf{H}) + \mathrm{diag}(\mathbf{H})^2 + \ldots)\mathbf{H}\} \\
&= \mathrm{diag}(\mathbf{H}) + \mathrm{diag}(\mathbf{H})^2 + \ldots \\
&= (\mathbf{I} - \mathrm{diag}(\mathbf{H}))^{-1} - \mathbf{I}
\end{aligned}$$

which then gives

$$(\mathbf{I} - \mathrm{diag}(\mathbf{H}))^{-1} = \mathbf{I} + \mathrm{diag}(\mathbf{S}(\mathbf{I} - \mathbf{S})^{-1})$$

Inserting this into (46), one finds

$$\begin{aligned}
\mathbf{S}(\mathbf{I} - \mathbf{S})^{-1} &= \mathrm{diag}(\mathbf{I} + \mathbf{S}(\mathbf{I} - \mathbf{S})^{-1})\mathbf{H} \\
&= \mathrm{diag}(\mathbf{I} + \mathbf{S} + \mathbf{S}^2 + \ldots)\mathbf{H} \\
&= \mathrm{diag}((\mathbf{I} - \mathbf{S})^{-1})\mathbf{H}
\end{aligned}$$

Finally, one obtains the result

$$\mathbf{H} = \mathrm{diag}((\mathbf{I} - \mathbf{S})^{-1})^{-1}\mathbf{S}(\mathbf{I} - \mathbf{S})^{-1} \tag{47}$$

showing how to derive $\mathbf{H}$ from $\mathbf{S}$.

Elimination of transitory revenues. The index set of all units, $\mathcal{U}$, consists of two subsets: $\mathcal{L}$ refers to ultimate units (households and state institutions) which are not owned by other units, $\mathcal{F}$ refers to firms which are owned by other units (firms and/or ultimate units). s_{ij} denotes the share of U_j owned by U_i, and it is assumed that

$$\sum_{i \in \mathcal{U}} s_{ij} = \begin{cases} 1 & \text{if } j \in \mathcal{F} \\ 0 & \text{otherwise} \end{cases}$$

The revenues y_i of a unit U_i consist of two parts: one part, x_i, results from U_i's own business, the remainder from payments of other units. So one can set-up the equation

$$y_i = x_i + \sum_{j \in \mathcal{F}} s_{ij}\,(y_j - z_j) \tag{48}$$

[12] Here (and later once more) one uses the equation $(\mathbf{I} - \mathbf{A})^{-1} = \sum_{k=0}^{\infty} \mathbf{A}^k$, where $\mathbf{A}$ is a square matrix and it is presupposed that $(\mathbf{I} - \mathbf{A})$ is invertible.

where $y_j - z_j$ is the part of y_j distributed to share holders of U_j. Defining $x_i^* = x_i - \sum_{j\in\mathcal{F}} s_{ij} z_j$, the equation can also be written as

$$y_i = x_i^* + \sum_{j\in\mathcal{F}} s_{ij}\, y_j \tag{49}$$

Then, for ultimate units ($i \in \mathcal{L}$), one derives

$$y_i = x_i^* + \sum_{j\in\mathcal{F}} s_{ij}\, y_j = x_i^* + \sum_{j\in\mathcal{F}} h_{ij}\, x_j^* \tag{50}$$

where h_{ij} denotes the direct and indirect shares owned by U_i defined above. The following matrix notations help to understand the derivation.

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{bmatrix} \quad \mathbf{x}^* = \begin{bmatrix} \mathbf{x}_1^* \\ \mathbf{x}_2^* \end{bmatrix} \quad \mathbf{S} = \begin{bmatrix} \mathbf{S}_1 & 0 \\ \mathbf{S}_2 & 0 \end{bmatrix} \quad \mathbf{H} = \begin{bmatrix} \mathbf{H}_1 & 0 \\ \mathbf{H}_2 & 0 \end{bmatrix}$$

The upper parts refer to companies, the lower parts to ultimate units. From Eq. (47) one obtains[13]

$$\mathbf{H}_2 = \mathbf{S}_2\,(\mathbf{I}_1 - \mathbf{S}_1)^{-1} \tag{51}$$

which entails (50).[14] Furthermore, one can derive for the column sums of $\mathbf{H}_2$:[15]

$$\sum_{i\in\mathcal{L}} h_{ij} = 1 \tag{52}$$

Consequently,

$$\sum_{i\in\mathcal{L}} y_i = \sum_{i\in\mathcal{L}} x_i^* + \sum_{i\in\mathcal{L}}\sum_{j\in\mathcal{F}} h_{ij}\, x_j^* = \sum_{i\in\mathcal{U}} x_i^* \tag{53}$$

Using the definition of x_i^*, one finally finds

$$\sum_{i\in\mathcal{L}} y_i = \sum_{i\in\mathcal{U}} x_i^* = \sum_{i\in\mathcal{U}} x_i - \sum_{i\in\mathcal{F}} z_i \tag{54}$$

The sum of the revenues of the ultimate units equals the sum of all directly generated revenues minus the part retained by companies.

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[13] Using the block matrices, Eq. (47) can be written as:

$$\left[\begin{array}{c|c} \mathbf{H}_1 & 0 \\ \mathbf{H}_2 & 0 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{D} & 0 \\ 0 & \mathbf{I}_2 \end{array}\right] \cdot \left[\begin{array}{c|c} \mathbf{S}_1 & 0 \\ \mathbf{S}_2 & 0 \end{array}\right] \cdot \left[\begin{array}{c|c} \mathbf{I}_1 - \mathbf{S}_1 & 0 \\ -\mathbf{S}_2 & \mathbf{I}_2 \end{array}\right]^{-1}$$

$$= \left[\begin{array}{c|c} \mathbf{D} & 0 \\ 0 & \mathbf{I}_2 \end{array}\right] \cdot \left[\begin{array}{c|c} \mathbf{S}_1 & 0 \\ \mathbf{S}_2 & 0 \end{array}\right] \cdot \left[\begin{array}{c|c} (\mathbf{I}_1 - \mathbf{S}_1)^{-1} & 0 \\ \mathbf{S}_2(\mathbf{I}_1 - \mathbf{S}_1)^{-1} & \mathbf{I}_2 \end{array}\right]$$

where $\mathbf{I}_1$ and $\mathbf{I}_2$ are identity matrices and $\mathbf{D}$ is a diagonal matrix (not required for $\mathbf{H}_2$).

[14] Equation (49) is equivalent with $\mathbf{y}_1 = \mathbf{x}_1^* + \mathbf{S}_1\mathbf{y}_1$ and $\mathbf{y}_2 = \mathbf{x}_2^* + \mathbf{S}_2\mathbf{y}_1$, and therefore: $\mathbf{x}_2^* + \mathbf{H}_2\,\mathbf{x}_1^* = \mathbf{x}_2^* + \mathbf{S}_2\,(\mathbf{I}_1 - \mathbf{S}_1)^{-1}\,(\mathbf{I}_1 - \mathbf{S}_1)\,\mathbf{y}_1 = \mathbf{x}_2^* + \mathbf{S}_2\,\mathbf{y}_1 = \mathbf{y}_2$.

[15] Assume that $\mathbf{S}_1$ is an (n, n) and $\mathbf{S}_2$ is an (m, n) matrix and let 1_n and 1_m denote vectors consisting, respectively, of n and m ones. From (51) one obtains:

$$1_m'\mathbf{H}_2(\mathbf{I}_1 - \mathbf{S}_1) = 1_m'\mathbf{S}_2$$

From (21) one obtains:

$$1_m'\mathbf{S}_2 = 1_n' - 1_n'\mathbf{S}_1 = 1_n'(\mathbf{I}_1 - \mathbf{S}_1)$$

Taken together,

$$1_m'\mathbf{H}_2(\mathbf{I}_1 - \mathbf{S}_1) = 1_n'(\mathbf{I}_1 - \mathbf{S}_1)$$

and consequently $1_m'\mathbf{H}_2 = 1_n'$.

Derivation of Equation (22). From (20), (13) and (16) one obtains

$$Y_i^s = \sum_{j\in\mathcal{F}} s_{ij}\, A_j = \sum_{j\in\mathcal{F}} s_{ij}\,(Y_j - I_j + \Delta\tilde{M}_j) \tag{55}$$

where $\Delta\tilde{M}_i = -\Delta M_i$ if U_i is a nonbank (this allows to use formally the same expression for banks and nonbanks). Inserting (1), one obtains

$$Y_i^s = \underbrace{\sum_{j\in\mathcal{F}} s_{ij}\,(Y_j^{mn} + Y_j^{mf} + Y_j^i + Y_j^r - I_j + \Delta\tilde{M}_j)}_{x_i^*} + \sum_{j\in\mathcal{F}} s_{ij} Y_j^s$$

By using Eq. (50), for ultimate units ($i \in \mathcal{L}$) one derives

$$Y_i^s = x_i^* + \sum_{j\in\mathcal{F}} s_{ij}\, Y_j^s = x_i^* + \sum_{j\in\mathcal{F}} h_{ij}\, x_j^* \tag{56}$$

Summing up and recognizing $\sum_{i\in\mathcal{U}} s_{ij} = \sum_{i\in\mathcal{L}} h_{ij} = 1$, one finds

$$\begin{aligned}\sum_{i\in\mathcal{L}} Y_i^s = &\sum_{i\in\mathcal{L}}\sum_{j\in\mathcal{F}} s_{ij}\,(Y_j^{mn} + Y_j^{mf} + Y_j^i + Y_j^r - I_j + \Delta\tilde{M}_j) + \\ &\sum_{i\in\mathcal{F}}\sum_{j\in\mathcal{F}} s_{ij}\,(Y_j^{mn} + Y_j^{mf} + Y_j^i + Y_j^r - I_j + \Delta\tilde{M}_j) = \\ &\sum_{i\in\mathcal{U}}\sum_{j\in\mathcal{F}} s_{ij}\,(Y_j^{mn} + Y_j^{mf} + Y_j^i + Y_j^r - I_j + \Delta\tilde{M}_j) = \\ &\sum_{j\in\mathcal{F}} (Y_j^{mn} + Y_j^{mf} + Y_j^i + Y_j^r - I_j + \Delta\tilde{M}_j) = \\ &Y_{\mathcal{F}}^{mn} + Y_{\mathcal{F}}^{mf} + Y_{\mathcal{F}}^i + Y_{\mathcal{F}}^r - I_{\mathcal{F}} + \Delta\tilde{M}_{\mathcal{F}}\end{aligned} \tag{57}$$

Since $\Delta\tilde{M}_i = -\Delta M_i$ if U_i is a nonbank, the last summand can be written

$$\Delta\tilde{M}_{\mathcal{F}} = \Delta\tilde{M}_{\mathcal{B}} + \Delta\tilde{M}_{\mathcal{F}\setminus\mathcal{B}} = \Delta M - \Delta M_{\mathcal{F}} = \Delta M_{\mathcal{L}} \tag{58}$$

Replacing $\Delta\tilde{M}_{\mathcal{F}}$ by $\Delta M_{\mathcal{L}}$ in (57), one obtains (22).

Derivation of Equation (33). In parallel with (30), for $j \in \mathcal{F}_{pu}$ we use the formulation

$$A_j^f = Y_j^f - \alpha_j\, \Delta M_j + R_j \tag{59}$$

As will be seen below, in this case α_j eventually vanishes. Starting from (28) and using the abbreviation $N_i = Y_i^{mf} - I_i^f$, we may write:

$$\begin{aligned}Y_i^f &= Y_i^i + N_i + \sum_{j\in\mathcal{F}} s_{ij}\, A_j^f \\ &= Y_i^i + N_i + \sum_{j\in\mathcal{F}_{pu}} s_{ij}\, R_j + \sum_{j\in\mathcal{F}} s_{ij}\,(Y_j^f - \alpha_j \Delta M_j) \\ &= \underbrace{Y_i^i + N_i + \sum_{j\in\mathcal{F}_{pu}} s_{ij}\, R_j - \sum_{j\in\mathcal{F}} s_{ij}\, \alpha_j \Delta M_j}_{x_i^*} + \sum_{j\in\mathcal{F}} s_{ij}\, Y_j^f\end{aligned}$$

By using Eq. (50), for ultimate units ($i \in \mathcal{L}$) one derives

$$Y_i^f = x_i^* + \sum_{j\in\mathcal{F}} s_{ij}\, Y_j^f = x_i^* + \sum_{j\in\mathcal{F}} h_{ij}\, x_j^* \tag{60}$$

Summing up and recognizing $\sum_{i\in\mathcal{L}} h_{ij} = 1$, one obtains

$$\sum_{i\in\mathcal{L}} Y_i^f = \sum_{i\in\mathcal{L}} x_i^* + \sum_{i\in\mathcal{L}}\sum_{j\in\mathcal{F}} h_{ij}\, x_j^* = \sum_{i\in\mathcal{L}} x_i^* + \sum_{j\in\mathcal{F}} x_j^* = \sum_{i\in\mathcal{U}} x_i^* \tag{61}$$

Since $\sum_{i\in\mathcal{U}} s_{ij} = 1$ and $\sum_{i\in\mathcal{U}} Y_i^i = \sum_{i\in\mathcal{U}} N_i = 0$ (see (3) and (11)), one finds

$$\sum_{i\in\mathcal{L}} Y_i^f = \sum_{j\in\mathcal{F}_{pu}} R_j - \sum_{j\in\mathcal{F}} \alpha_j \Delta M_j \tag{62}$$

Then,

$$\begin{aligned}
&\sum_{j\in\mathcal{F}_{pu}} R_j = \sum_{j\in\mathcal{F}_{pu}} A_j - Y_j^f + \alpha_j \Delta M_j = \\
&\sum_{j\in\mathcal{F}_{pu}} Y_j - I_j - Y_j^f + \alpha_j \Delta M_j + \Delta\tilde{M}_j = \\
&\sum_{j\in\mathcal{F}_{pu}} (Y_j^{mn} - W_j - I_j^n) + Y_j^i + N_j + Y_j^r + Y_j^s - Y_j^f + \alpha_j \Delta M_j + \Delta\tilde{M}_j = \\
&\sum_{j\in\mathcal{F}_{pu}} (Y_j^{mn} - W_j - I_j^n) + Y_j^r + (Y_j^s - Y_j^{sf}) + \alpha_j \Delta M_j + \Delta\tilde{M}_j
\end{aligned}$$

Taking into account (32), one obtains $\sum_{j\in\mathcal{F}_{pu}} R_j =$

$$\sum_{j\in\mathcal{F}_{pu}} (Y_j^{mn} - W_j - I_j^n) + Y_j^r + (Y_j^s - Y_j^{sf}) + \Delta M - \sum_{j\in\mathcal{F}_{pu}} (1-\alpha_j)\, \Delta M_j$$

Combining this with (62), one arrives at

$$\begin{aligned}
&\sum_{i\in\mathcal{L}} Y_i^f + \sum_{j\in\mathcal{F}} \alpha_j \Delta M_j + \sum_{j\in\mathcal{F}_{pu}} (1-\alpha_j)\Delta M_j = \\
&\qquad \sum_{j\in\mathcal{F}_{pu}} (Y_j^{mn} - W_j - I_j^n) + Y_j^r + (Y_j^s - Y_j^{sf}) + \Delta M
\end{aligned} \tag{63}$$

from which (33) immediately follows.

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