An Analysis of the Seasonal Cycle and the Business Cycle

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Abstract

Robert Barsky and Jeffrey Miron (1989) revealed the seasonal cycle of the U.S. economy from 1948 to 1985 was characterized by a "bubble-like" expansion in the second and fourth quarters, a "crash-like" contraction in the first quarter, and a mild contraction in the third quarter. We replicate, in part, their seasonal cycle analysis from 1946 to 2001. Our results are largely in line with theirs. Nonetheless, we find the seasonal cycle is not stable and can evolve across time. In particular, the Great Moderation affected both the business cycle and the seasonal cycle.

Robert Barsky and Jeffrey Miron also found real aggregates, like the output, move together in the seasonal cycle across broadly defined sectors, similar to a phenomenon observed under the conventional business cycle. They posed a challenge question concerning why "the seasonal and the conventional business cycles are so similar." To answer their question, we focus on a number of aggregate variables with a recursive application of the HP filter and find that aggregates, such as the GDP, consumption, the S&P 500 Index, and so forth, have a "bubble-like" expansion and a "crash-like" contraction in their cyclical trends in business cycle frequencies. Although preference shifts and production synergy are the two major forces that drive the seasonal cycle, we find the time-varying stochastic discount factor is the main cause of the business cycle and plays a more important role in macroeconomic fluctuations in business cycle frequencies than other factors.

JEL classification numbers: C53, C82, E24, E32, G12

Key words: Seasonal cycles, business cycles, jobless claims, unemployment rates, labor productivity, GDP, S&P 500

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1 Introduction

Robert Barsky and Jeffrey Miron (1989) provide a comprehensive analysis of seasonal cycles for numerous aggregate variables for the U.S. economy from 1948 to 1985 and discover that seasonal cycles have dominated short-term fluctuations of real economic activities. Furthermore, they find that the seasonal cycle of the U.S. economy can be characterized by a "bubble-like" expansion in the second and fourth quarters, a "crash-like" contraction in the first quarter, and a mild contraction in the third quarter. An intriguing result of theirs is that real aggregate variables, like the output, in the seasonal cycle move together across broadly defined sectors, similar to a phenomenon observed under the conventional business cycle. In their conclusion, they state, "The similarity of the seasonal cycle and the business cycle presents a challenge because, to paraphrase Lucas (1977, p. 10), 'it suggests the possibility of a unified explanation' of both business cycles and seasonal cycles. By trying to understand precisely why the seasonal and the conventional business cycles are so similar, we may be able to shed considerable light on all aggregate fluctuations" (Robert Barsky and Jeffrey Miron 1989, p. 529).

Their findings and remark raise a question as to whether the seasonal cycle and the business cycle have been driven by the same forces. If so, then one must explain why the two cycles with different frequencies and periodic characterizations are found with the same set of stylized facts in macroeconomic fluctuations. If not, then one must explain why the seasonal cycle has been formed with a higher and fixed frequency of one cycle per year while the business cycle has been formed with a lower and recurrent frequency. This paper provides empirical evidence to resolve this difficulty with a recursive application of the HP filter (Robert Hodrick and Edward Prescott 1997).

We agree with Robert Barsky and Jeffrey Miron that preference shifts and production synergy are the two major forces behind the formation of the seasonal cycle. However, these two forces are unlikely to be the causes for the business cycle. Our evidence has been summarized in Figure 1, using the real GDP series. Figure 1a shows the seasonal dummies estimated using their seasonal dummy model for the U.S. economy from 1946 to 2001.[1] The seasonal cycle has a fixed frequency of one cycle per year, consistent with the fact that preference shifts and production synergy at the seasonal frequency are probably

[1]The data are from BEA. Only seasonally adjusted data are available after 2001 because of a budget cut.

associated with a shift in seasons in a calender year. These dummies represent quarterly average deviations in output from the trend. The seasonal variations from 1946 to 2001, after removal of the trend and the cyclical component, are shown in Figure 1d. The cyclical trend shown in Figure 1b in business cycle frequencies is less regular than that shown in Figure 1d but the patterns of bubble-like expansions and crash-like contractions are quite similar.

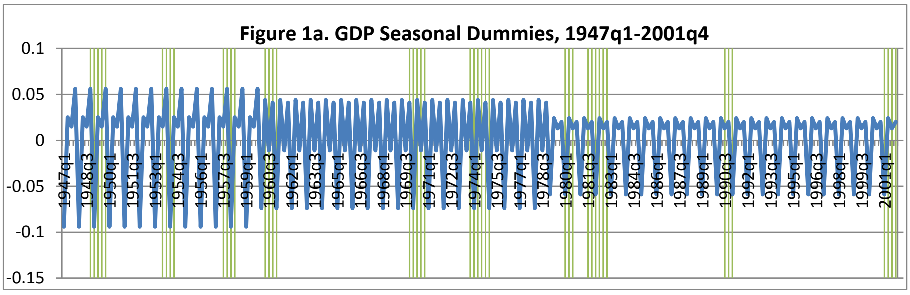


Figure 1a. GDP Seasonal Dummies, 1947q1-2001q4

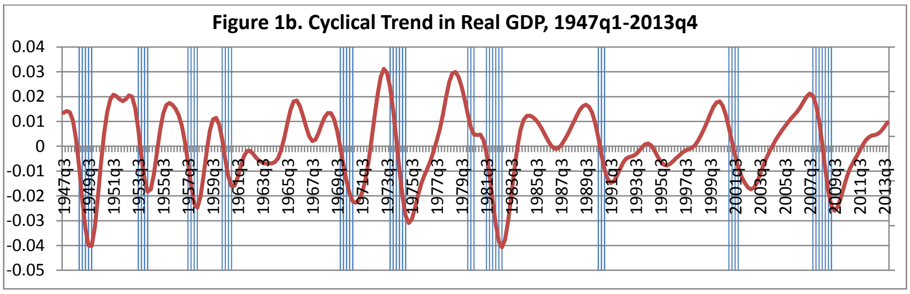


Figure 1b. Cyclical Trend in Real GDP, 1947q1-2013q4

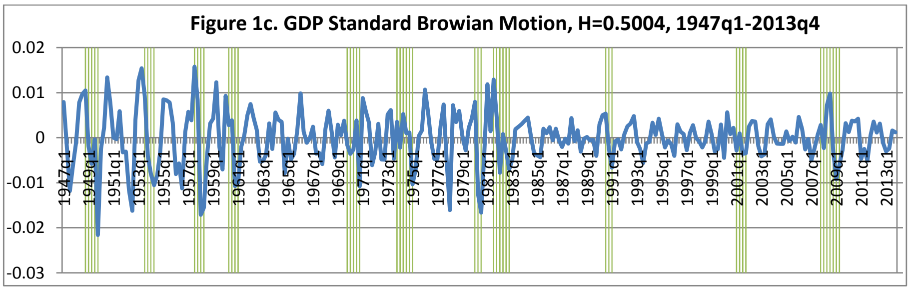


Figure 1c. GDP Standard Browian Motion, H=0.5004, 1947q1-2013q4

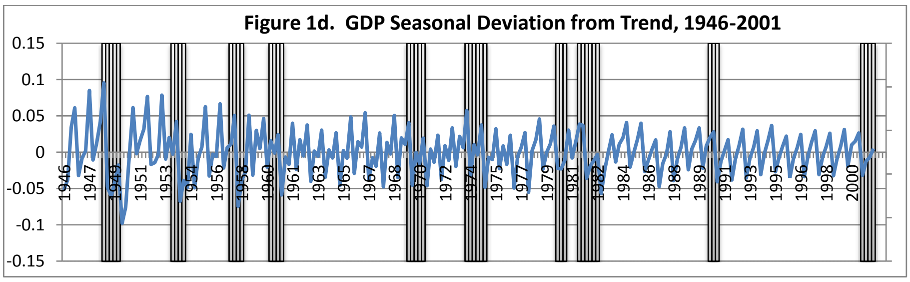


Figure 1d. GDP Seasonal Deviation from Trend, 1946-2001

In comparison with the cyclical trend in Figure 1b and the fractional Brownian motion (fBm) in Figure 1c of the cyclical component, the seasonal dummies represent a larger deviation from the trend than from the cyclical component. Such a conclusion is in line with the findings of Robert Barsky and Jeffrey Miron about the dominance of the seasonal cycle in short-term macroeconomic fluctuations. Our study of the seasonal cycle of other aggregate variables appears in detail in Section 2. In particular, we present an analysis of the seasonal pattern of various labor indicators, most of which were absent in Robert Barsky and Jeffrey Miron but are very important lately to the monetary policy of the Federal Reserve. We also find that the seasonal cycle is not stable and can evolve and change with time. For example, the Great Moderation[2] starting around 1985 affected both the business cycle and the seasonal cycle.

We focus on the cyclical components of many real aggregates. We show that the cyclical trends of many real aggregates have a bubble-like expansion and a crash-like contraction in business cycle frequencies as well, a phenomenon similar to what has been observed in the seasonal cycle in Figure 1d. We start with the cyclical component of the real GDP, using the seasonally adjusted data from 1947 to 2013. We use the HP filter to remove the trend in the GDP and then use it again numerous times to decompose the cyclical component into two parts. One part constitutes the cyclical trend, T_c, shown in Figure 1b, and the other part constitutes a fractional Brownian motion (fBm), shown in Figure 1c, which has the Hurst parameter $H \approx 0.5$,[3] estimated by the first order quadratic variations (Jacques Istas and Gabriel Lang 1997; Jean-Francois Coeurjolly 2001, 2008). The three time series in Figures 1a, 1b, and 1c have a mean of zero and represent deviations from the secular trend of the GDP.

Figures 1b and 1c both show that adversary shocks, fBm, and the endogenous weakness in the cyclical trend, T_c, are the two major causes of a recession. The fBm part dipped into

[2]The Great Moderation, identified by Olivier Blanchard and John Simon (2001) (also see James Stock and Mark Watson 2002 and references therein for detail), has affected not only the seasonal cycle, but also the fBm of the GDP. The cyclical trend of the GDP is also affected but to a smaller degree. Alternatively, one can also use the less volatile fBm of the GDP to explain the Great Moderation. See Table 6 in Section 3.3.1. for additional evidence.

[3]Equivalently, Figure 1c is the noise component of the GDP that can be modeled with a fractional ARIMA(p, d, q) process with $d = 0$ (Ton Dieker 2004). The real GDP and other aggregates, since WWII, after removal of the secular or stochastic trend by the HP filter, follow an $I(d)$ process with $\frac{1}{2} > d > 0$. We choose the equivalent fBm ($H = 0.5$) instead of a familiar ARIMA(p, d, q) process ($d = 0$) because it is easier to estimate the single Hurst parameter H. An underlying assumption is that the zero-mean cyclical component $\{X_t\}$ as a stochastic process can be modeled by $\frac{dX_t}{X_t} = \sigma dB_t^H$, using the path-wise stochastic integral, where B_t^H is a fBm process with $B_0^H = 0$, $0 < H < 1$. The model becomes the classical Black-Scholes model when $H = 0.5$ and mean $\mu = 0$.

the negative zone for each NBER recession (shown in shaded areas), indicating adversary shocks occurred in the economy for each recession. However, an adversary shock is not a sufficient condition for a recession. Adversary shocks were also observed during expansion periods. More importantly, Figure 1b shows that, before each recession, the cyclical trend started to lose its upward momentum and turned to a downward trend.

A unique feature of Figure 1b is that the cyclical trend of each expansion was quite different from the others. Indeed, no two cyclical trends were the same for two adjacent expansions. Such a difference arises likely from the difference in economic structures, which were changed by a recession. For example, substantial differences exist before and after the Great Recession in the housing and mortgage industries.

The two figures also show how a recession ends. A recession ends when the cyclical trend makes a turn from a downward momentum into an upward momentum, with positive shocks in fBm.[4] Moreover, once the economy is in an upward trend, it persists for a lengthy period, despite various adversary shocks. Nonetheless, because economic structures evolve across time, the economic persistence differs for different economic structures. These features together explain why the business cycle recurs in a lower frequency.

The cyclical trend in Figure 1b should be understood as a process of forming a bubble-like expansion and a crash-like contraction in business cycle frequencies because the long-term secular trend can be seen as the "equilibrium" of the economy, which is at mean zero in Figure 1b. Such a process is similar to the excess volatility revealed by Robert Shiller (1981) in the equity market, with the equilibrium indicated by the present value of distributed dividends of a discounted model with constant discount rate. We provide a study of the S&P 500 Index in Subsection 3.3. The S&P 500 has a cyclical trend similar to Figure 1b of the GDP. Moreover, these bubble-like expansions and crash-like contractions are commonly observed in the cyclical trends of many other aggregates. We may conclude from them that macroeconomic fluctuations in the business cycle frequencies are, in fact, the excess volatility of real economic activities. The force of the formation of such a process shown in Figure 1b is likely because of the mean reverting around the secular trend. The time-varying expected returns have been proved to be a major force behind the excess volatility in equity (see, e.g., John Cochrane 2006). Thus, we also conduct a study of

[4]The only exception is the 2001 recession. We suspect the actual recession might have lasted longer than what was declared by the NBER.

the cyclical trend in consumption in Subsection 3.4 and the stochastic discount factor in Subsection 3.5. Through these studies, we have empirical evidence to show that the intertemporal marginal rate of substitution or the stochastic discount factor has a time-varying cyclical trend, with a pattern of bubbles and crashes around its conditional mean, largely in relation to NBER recessions. We conclude from the patterns that the time-varying stochastic discount factor is a major force causing macroeconomic fluctuations in the business cycle frequencies, in line with the literature addressing excess volatility in the equity market.

We are left to explain why the seasonal cycle and the business cycle share the same set of stylized facts for macroeconomic fluctuations. The GDP is an aggregate variable. That is, an upward trend, like a bubble-like expansion in the cyclical trend of the GDP, must indicate that most sectors move in the same upward direction. Even though the downward spikes of the cyclical trend in Figure 1b and the dips in the output in the component fBm were uneven in the amplitudes across recessions, they had patterns similar to an output drop in the first quarter of the seasonal cycle. The upward spikes of the cyclical trend in Figure 1b is somewhat more complicated because the fBm part can be negative or positive. This representation is, in fact, consistent with real economic activities where economic growth during an expansion can lose momentum occasionally. Nevertheless, the output across most sectors should also follow the cyclical trend described in Figure 1b and the fBm in Figure 1c; otherwise it would be impossible for the GDP to display these same patterns. These bubble-like spikes in the cyclical trend recur in lower frequencies than in the seasonal cycle, but they are very similar to the bubble-like spikes in the second or fourth quarter of the seasonal cycle, shown in Figures 1b and 1d.

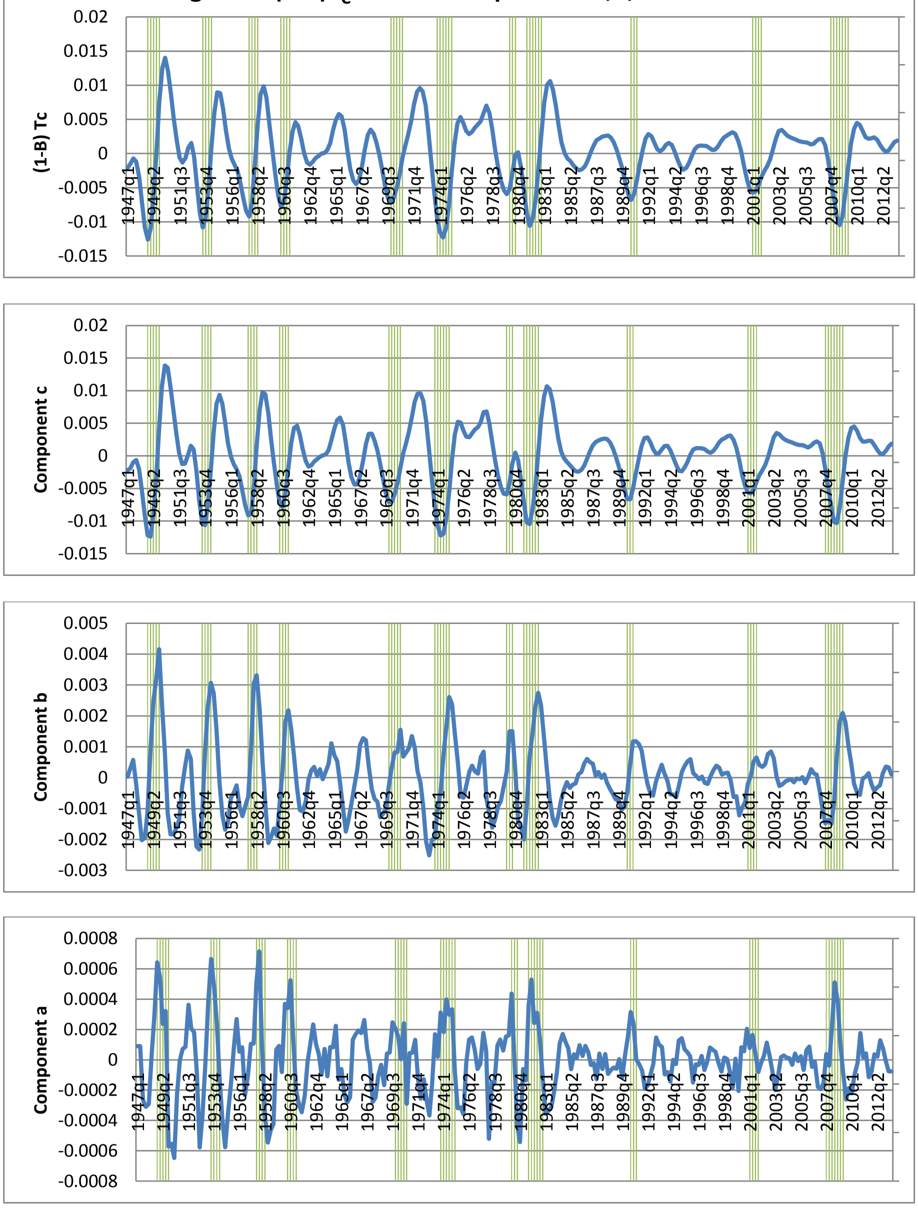
To understand better the force that shapes the cyclical trend in Figure 1b, we further decompose it into three forces. We find that the first difference of the cyclical trend has a three-part decomposition:

$$(1-B)T_c = \text{Component a} + \text{Component b} + \text{Component c}$$

with Component c as the dominant force for $(1-B)T_c$, where B is a backshift operator with $B^k x_i = x_{i-k}$. The three-part decomposition of $(1-B)T_c$ has been given in Figure 2.

This exercise indicates the cyclical trend T_c has been shaped by a major force c, together with two other minor forces, a and b. Interestingly, Component b leads Component c, but Component a operates against Component c. We provide a detailed explanation of this decomposition in the appendix.

Figure 2. $(1\text{-}B)T_C$ and its Components a,b,c



Our approach regarding the cyclical component follows closely the literature concerning detrending an aggregate time series. The unobserved component model in Peter Clark (1987) and Andrew Harvey (1985), using a restricted ARMA(p, q), and the Beveridge-Nelson approach (Stephen Beveridge and Charles Nelson 1981; Charles Nelson and Charles Plosser 1982; Mark Watson 1986; John Campbell and N. Gregory Mankiw 1987; John

Cochrane 1988), using a restriction-free ARMA(p, q), are most well known.[5] A third popular approach involves using the HP filter to decompose an aggregate time series into a smooth trend and a cyclical component. Pierre Perron and Tatsuma Wada (2009) provide a study of the three approaches and show that the HP filter can provide a trend of the real GDP more smoothly than the other two.

The application of this HP filter in this paper differs in two major ways from those in the literature. First, we find that the cyclical component of an aggregate such as the GDP obtained from the HP filter is often not close to a "random walk." Instead, it is quite similar to a factional Brownian motion that has a long-range memory and persistence or momentum trend, similar to that observed in Clive Granger (1980), Charles Nelson and Charles Plosser 1982, Mark Watson 1986, John Campbell and N. Gregory Mankiw 1987, Francis Diebold and Glenn Rudebusch (1989), and Andrew Lo (1991), among others. However, the long-range dependence that we address in this paper is in a business cycle frequency, higher than the frequency of a secular trend.

In our estimation, many cyclical components (of the post war series) revealed after using the HP filter have a Hurst parameter H greater than $\frac{1}{2}$. For example, the cyclical component of the GDP has $H = 0.7965$ after removal of the secular trend. That is, the cyclical component of the HP filter itself contains a cyclical trend, displaying "intermediate-range" dependence behavior. To separate this trend from the "noise" component at very high frequencies, we apply the HP filter recursively to the cyclical components numerous times so that the "noise" part eventually becomes an fBm with H close to $\frac{1}{2}$ ($H = 0.5004$ for the fBm in Figure 1c). Such an approach is largely motivated by John Cochrane (1988, 1991, 1994) and Robert King and Mark Watson (1994). John Cochrane (1988, 1991) demonstrates that a time series with a unit root or a stationary series displaying near unit root behavior contains a "small" random walk component. Robert King and Mark Watson (1994) decompose the post-war unemployment and inflation series into three parts—a secular trend (using a fixed low-pass filter), a business cycle trend (using a fixed band-pass filter), and an irregular component (residuals after the two filters)—and establish a relationship between the two cyclical trends of unemployment and inflation in support of the

[5]See James Stock and Mark Watson (1988) for additional literature. Mark Watson (1986) shows that the two approaches provide substantially different trends and cycles while James Morley, Charles Nelson, and Eric Zivot (2003) reveal the essence causing these differences and provide a means of unifying the two approaches. Also see Pierre Perron and Tatsuma Wada (2009) for a comparison study.

Phillips curve. John Cochrane (1994) uses the less volatile consumption as the trend and the residual as the transitory component of GNP, using the VAR identification approach, and finds the residual accounts for most variations in the GNP. He also establishes a similar relationship between stock prices and dividends. Here, we use the HP filter recursively to remove this "noise," "irregular," or transitory part from a number of aggregate variables. The noise part is identified as the standard Brownian motion, which is known to be an approximation of a random walk of i.i.d. random variables (with proper rescaling), under the Donsker's theorem (Tommi Sottinen 2001).

Second, our study is motivated by the remark of Robert Barsky and Jeffrey Miron. To understand how different forces operate in recessions and expansions, we use the spline approach to fit this cyclical trend of the cyclical component and decompose its first difference into three parts, as demonstrated in Figure 2. It appears that such a decomposition is very fruitful. Many indicators thought to be lagging in the previous literature become leading indicators. Thus, the approach extracts undisclosed information from these indicators, possibly motivating many future studies of their implications for the seasonal cycle and the business cycle.

The rest of this paper is organized as follows; Section 2 includes a detailed study of the seasonal cycle, motivated by Robert Barsky and Jeffrey Miron. Section 3 addresses the business cycle and focuses on the cyclical trends of various aggregate variables. In particular, Section 3.5 documents the evidence that the stochastic discount factor also has a pattern forming bubbles and crashes. Section 4 concludes the paper. The appendix provides technical details of the approach used in the paper.

2 Seasonal Cycles

In this section we replicate, in part, a study of the seasonal cycle in Robert Barsky and Jeffrey Miron (1989) using their seasonal dummies approach for the U.S. economy, with the not seasonally adjusted data from 1946 to 2001. We follow Robert Barsky and Jeffrey Miron in using the HP filter to remove the secular trend. Thus, the seasonal dummies we obtain represent average deviations from the trends of the studied variables.

Our study focuses on the seasonal cycle of a number of real aggregate variables, such as the GDP, consumption, investment, and so on. Our study differs from that of Robert

Barsky and Jeffrey Miron largely in that we find changes in the seasonal cycle over time, partly because we have a larger sample size. Various former studies have tested a shift of the seasonal cycle (Fabio Canova and Eric Ghysels 1994). The factors that cause such a shift are not precisely known, but it appears the shift may be somehow related to the average growth rates in the GDP and the volatility of the GDP. This possibility should be investigated further because of its implication for the social welfare of economic policy that aims at reducing seasonal or business cycle variations.

Let y_k, $k = 1, 2, \cdots, N$, be a time series. The HP filter uses the spline approach to decompose the series into a smoothed (secular) trend $\{\tau_k\}$ and a cyclical component $\{c_k\}$, that is, $y = \tau + c$, by solving the following minimization problem

$$(HP) \quad min_{\{\tau_k\}_{k=1}^{k=N}} \{\sum_{k=1}^{N}(y_k - \tau_k)^2 + \lambda \sum_{k=2}^{N-1}[(\tau_{k+1} - \tau_k) - (\tau_k - \tau_{k-1})]^2\},$$

where λ is a parameter that depends on whether the time series is quarterly or monthly data. For a quarterly data series, λ is set at 1600 while it is set at 129600 for the monthly data (Morten Ravn and Harald Uhlig 2001). For an aggregate variable investigated in this paper, $\{y_k\}$ is the logarithm of the original series. The only exception is the unemployment rate and labor productivity, where $\{y_k\}$ equals the original series.

2.1 Seasonal Fluctuations

The standard deviation of the deterministic seasonal component of output is estimated at 4.35 percent deviation from the trend, which accounts for about 84.9 percent of the deterministic fluctuations in output, as shown in Table 1. Similar magnitudes are shown for gross investment, government spending, exports, and imports. As for consumption spending, the standard deviation of the seasonal component is estimated at 6.15 percent deviation from the trend and accounts for 90.8 percent of the deterministic fluctuations in consumption. Similar magnitude is found for fixed investment where the standard deviation of the deterministic seasonal component accounts for 8.37 percent deviation from trend and for 84.6 percent of the deterministic fluctuations in fixed investment.

In line with Robert Barsky and Jeffrey Miron, the fluctuations of the deterministic component of the seasonal dummies is at its highest for consumption spending on durable goods and residential investment spending. As shown in the table, the standard deviation of the seasonal dummies reaches 13.75 percent of deviations from the trend and

Table 1. Summary Statistics for Seasonal Dummies, 1946q1-2001q4

Variables	Standard Deviation of Dummies	Standard Error of the Regression	R-squared
Gross Domestic Product	0.0435	0.0184	0.849
Consumption	0.0615	0.0196	0.908
a. Durables	0.1375	0.0527	0.872
b. Services	0.0073	0.0090	0.400
Gross Investment	0.0308	0.0828	0.125
Fixed Investment	0.0837	0.0357	0.846
a. Nonresidential	0.0562	0.0390	0.676
Structures	0.0847	0.0385	0.830
b. Residential	0.1791	0.0603	0.899
Government	0.0400	0.0379	0.528
a. Federal	0.0419	0.0701	0.264
a. State and Local	0.0468	0.2004	0.846
Exports	0.0440	0.0557	0.387
Imports	0.0302	0.0490	0.278

Note-Average deviation from trend. Data source: Quarterly data from BEA.

accounts for 87.2 percent of the deterministic fluctuations in consumption spending on durable goods. Similarly, the standard deviation of the seasonal component of residential investment accounts for 17.91 percent of deviations from trend and accounts for almost 90 percent of its deterministic fluctuations. Furthermore, in line with Robert Barsky and Jeffrey Miron, the deterministic component of the seasonal dummies for the consumption spending on services is the smallest of all macroeconomic variables. For instance, the standard deviation of seasonal dummies for deviation from the trend in consumption spending on services accounts for only 0.73 percent and explains about 40 percent of all variations.

Table 2. Summary Statistics for Seasonal Dummies, Various Periods

Periods⇒	1946q1-1959q4			1960q1-1978q4			1979q1-2001q4		
Variables ⇓	Standard Deviation of Dummies	Standard Error of the Regression	R-squared	Standard Deviation of Dummies	Standard Error of the Regression	R-squared	Standard Deviation of Dummies	Standard Error of the Regression	R-squared
Gross Domestic Product	0.0566	0.0225	0.866	0.0481	0.0103	0.958	0.0341	0.0077	0.953
Consumption	0.0855	0.0185	0.957	0.0618	0.0112	0.970	0.0475	0.0083	0.972
a. Durables	0.1509	0.0752	0.806	0.1522	0.4187	0.933	0.1188	0.0343	0.926
b . Services	0.0075	0.0089	0.422	0.0098	0.0090	0.554	0.0080	0.0063	0.624
Gross Investment	0.0922	0.1118	0.421	0.0159	0.0621	0.065	0.0343	0.0435	0.395
Fixed Investment	0.1040	0.0495	0.820	0.0836	0.0278	0.905	0.0730	0.0233	0.911
a. Nonresidential	0.0820	0.0530	0.713	0.0511	0.0325	0.722	0.0468	0.0242	0.796
Structures	0.0862	0.0446	0.793	0.0909	0.0348	0.879	0.0794	0.0367	0.830
b. Residential	0.1682	0.0683	0.863	0.1910	0.0627	0.907	0.1772	0.0486	0.933
Government	0.0461	0.0532	0.433	0.0372	0.0246	0.707	0.0498	0.0194	0.873
a. Federal	0.0543	0.0801	0.324	0.0351	0.0443	0.398	0.0780	0.0455	0.754
b. State and Local	0.0673	0.0171	0.942	0.0478	0.0192	0.867	0.0340	0.0085	0.943
Exports	0.0610	0.0787	0.392	0.0592	0.0551	0.548	0.0218	0.0254	0.434
Imports	0.0171	0.0586	0.083	0.0443	0.0516	0.438	0.0323	0.0335	0.492

Note-Average deviation from trend. Data source: Quarterly data from BEA.

Table 2 expands the analysis of Table 1 by dividing the period from 1946:1-2001:4 into three periods such that period I covers 1946:1-1959:4, period II covers 1960:1-1978:4, and period III covers 1979:1-2001:4. For the three periods, the standard deviation of the deterministic seasonal component is the largest for consumption spending on durable goods and fixed residential investment and is at the smallest for the consumption spending on services. The standard deviation of the seasonal component for the government spending is always less than 5 percent in the three periods and account for about 71 percent and 87 percent of the total fluctuations in government spending in the second and the third periods, respectively, but only 43 percent in the first period.

Labor market is a focal point of the Federal Reserve. It is of a great interest to investigate the seasonal patterns of various labor-related indicators. As shown in Table 3 the standard deviation of the deterministic seasonal component of the unemployment rate is 7.15 percent of the deviation from trend, which accounts for about 65 percent of the total deviations. The deterministic seasonal component of the hiring rate accounts for the same percentage of total deviation, however its standard deviation is larger of about 21 percent of the deviations from trend. The deterministic seasonal component accounts for the largest fluctuations in separation rate of about 91 percent of total fluctuations with a standard deviation of about 16 percent deviations from trend, which is about three quarters of 21 percent standard deviation of the hiring rate. Thus, substantial differences exist between the seasonal components of the hiring rate and the separation rate. Our study indicates that the hiring rate, similar to the unemployment rate, has been affected more than the separation rate by the cyclical fluctuations.

As is obvious from the table, the deviations from trend of the weekly initial jobless claims and the weekly continued jobless claims contain a larger stochastic component than the other measures of the labor market discussed above. For instance, the standard deviation of the deterministic seasonal component of the initial jobless claims is about 0.04 percent for the deviation from trend and it only explains 9.4 percent of the total fluctuations. Also, the standard deviation of the deterministic seasonal component of the continued jobless claims is about 2.1 percent and explains only about 15.5 percent of the total fluctuations in this variable.

Table 3. Summary Statistics for Seasonal Dummies

Variable	Standard Deviation of Dummies	Standard Error of the Regression	R-squared
1948:01-2014:03			
Unemployment Rate	0.0715	0.0521	0.649
Unemployment Level	0.0047	0.0035	0.639
2000:12-2014:01			
Hire Rate	0.2056	0.1459	0.650
Separation Rate	0.1558	0.0467	0.912
1967:w1-2014:w52			
Initial Jobless Claims	0.0378	0.1167	0.094
Continued Jobless Claims	0.0212	0.0490	0.155

Note-Average deviation from trend. Data source: Unemployment rate and level are NSA monthly data from BLS. Hire and separation rates are NSA monthly data from BLS. Initial and continued jobless data are NSA weekly data from the U.S. Department of Labor.

2.2 Seasonal Patterns

A more detailed investigation of the deterministic seasonal fluctuations is shown in Table 4, where each macroeconomic variable is regressed on the seasonal dummies. Using ordinary least squares (OLS), each macroeconomic variable is regressed on a set of quarterly seasonal dummies for the period from 1946:1 to 2001:4 and the three sub-periods.

As shown in the table, output for the whole period of 1946:1 to 2001:4 is well below the trend in the first quarter and above the trend for the other three quarters, with a smaller magnitude in the third quarter.[6] A closer look at the pattern of output during the three different periods shows that, not only the deviations from trend are similar to the ones found for the whole period, but also the peaks of the deviations from the trend are getting smaller as time goes on. For instance, the magnitude of the trough in the first quarter was 9.4 percent, 7.4 percent, and 5.9 percent for the first, second, and third

[6]This result is different from the results of Robert Barsky and Jeffrey Miron for their shorter period 1948-1985, where output is shown to be, on average, well below the trend for the first quarter, slightly below the trend in the second and third quarters, and well above the trend for the last quarter.

Table 4. Seasonal Variations of Seasonal Dummy Model, Various Periods

Periods⇒	46-01				46-59				60-78				79-01			
Variables⇓	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4	Q1	Q2	Q3	Q4
Gross Domestic Product	-0.072	0.031	0.005	0.036	-0.094	0.025	0.015	0.056	-0.074	0.044	-0.011	0.041	-0.059	0.024	0.013	0.020
Consumption	-0.097	0.041	-0.006	0.063	-0.133	0.049	-0.008	0.095	-0.097	0.048	-0.010	0.059	-0.077	0.030	-0.002	0.047
a. Durables	-0.207	0.127	-0.037	0.122	-0.226	0.130	-0.033	0.147	-0.217	0.148	-0.066	0.137	-0.187	0.108	-0.016	0.093
b. Services	0.011	-0.010	-0.0004	-0.001	0.012	-0.0004	-0.001	-0.008	0.015	-0.012	-0.002	-0.002	0.006	-0.014	0.001	0.005
Gross Investment	0.0001	-0.014	0.048	-0.035	0.050	-0.096	0.131	-0.070	-0.025	0.014	0.00004	0.015	-0.008	0.013	0.038	-0.055
Fixed Investment	-0.113	0.122	0.008	-0.013	-0.152	0.138	0.029	0.005	-0.106	0.127	0.003	-0.021	-0.097	0.107	-0.001	-0.017
a. Non-residential	-0.070	0.075	-0.032	0.030	-0.115	0.102	-0.023	0.052	-0.052	0.073	-0.043	0.025	-0.061	0.061	-0.029	0.022
Structures	-0.131	0.093	0.052	-0.014	-0.136	0.082	0.065	-0.004	-0.141	0.102	0.049	-0.009	-0.119	0.091	0.046	-0.024
b. Residential	-0.223	0.242	0.091	-0.106	-0.225	0.209	0.116	-0.071	-0.084	0.030	0.016	0.016	-0.211	0.247	0.074	-0.120
Government	-0.059	0.040	0.031	-0.017	-0.084	0.030	0.017	0.016	-0.041	0.059	0.001	-0.120	-0.060	0.031	0.064	-0.034
a. Federal	-0.069	0.029	0.037	-0.003	-0.090	0.004	-0.007	0.062	-0.028	0.058	-0.027	-0.005	-0.091	0.020	0.117	-0.041
a. State and Local	-0.053	0.057	0.035	-0.038	-0.072	0.080	0.054	-0.059	-0.055	0.062	0.030	-0.036	-0.039	0.040	0.027	-0.027
Exports	-0.018	0.043	-0.063	0.039	-0.014	0.060	-0.091	0.053	-0.031	0.066	-0.080	0.047	-0.011	0.015	-0.032	0.024
Imports	-0.014	0.059	-0.091	0.054	0.008	0.024	-0.006	-0.022	-0.023	0.069	0.003	-0.051	-0.045	0.045	0.007	-0.011

Note-Seasonal variations from the mean. Data source: Quarterly data from BEA period, respectively. In addition, the magnitude of the peaks for the fourth quarter are getting smaller as time goes on, as they reach 5.6 percent, 4.1 percent, and 2.0 percent for the first, second, and the third periods, respectively.[7]

The components of output are generally procyclical with the exception of the consumption expenditure on services. The deviations from the trend in consumption expenditure on durable goods are, on average, below the trend in the first and third quarters but above the trend in the second and fourth quarters. This indicates that durable goods generally move with the trend deviations in output with the exception of the third quarter. In addition and in line with Robert Barsky and Jeffrey Miron, for all periods, the magnitudes of these deviations are larger than that of output.

The trend deviations in residential investment are, on average, below the trend in the first quarter, above the trend in the second and third quarters, and again below the trend in the fourth quarter. The trend deviations for investment in structures are similar and support Robert Barsky and Jeffrey Miron. Similarly, with few exceptions, trend deviations in government spending and its two components, federal and state and local spending, on average, are below the trend for the first and fourth quarters but above the trend for the second and third quarters.

The deviations in export alternate between above and below the trend. For instance, it is below the trend for the first and third quarters but above the trend in the second and fourth quarters for the whole period from 1946-2011. As for imports, the trend deviations for the whole period are very similar to those of exports, where, on average, the deviations for the first and third quarters are below the trend but above the trend for the second and fourth quarters. Seasonal patterns of export change lilttle with time, but those of import do change with time.

Generally, the tendencies of output and its components are similar to the results of Robert Barsky and Jeffrey Miron. More specifically, the decline in output from the fourth to the first quarter is similar to the findings of Robert Barsky and Jeffrey Miron, which is also reflected in almost all the components of output. In addition, our results confirm the fourth quarter increase in consumption of durable goods and the first quarter peak in

[7]This result is different from the results of Robert Barsky and Jeffrey Miron for their shorter period 1948-1985, where output is shown to be, on average, well below the trend for the first quarter, slightly below the trend in the second and third quarter, and well above the trend for the last quarter.

services spending rather than a first quarter decline. Further, we find a strong growth in fixed investment in the second quarter followed by a slight increase in the third quarter and slight decrease in the fourth quarter, that is, besides the increase in structures investment in the second and third quarters. Finally, the peaks in federal and state and local spending in the second and third quarters are all in agreement with the results of Robert Barsky and Jeffrey Miron. These results seem to confirm that the trend deviations in macroeconomic variables are correlated with the season and the weather: shopping season in the fourth quarter results in an increase in spending, bad weather in the first quarter and the end of the shopping season results in a drop in spending, and good weather in the second and third quarters results in an increase in residential and structures spending. Changes in season and weather appear to be two major forces driving preference shifts and production synergy, a finding that supports Robert Barsky and Jeffrey Miron. A detailed discussion of the two forces can be found further in Jeffrey Miron (1996). For a study of other developed countries, see Joseph Beaulieu, Jeffrey MacKie-Mason and Jeffrey Miron (1992).

3 Business Cycles

We have presented our studies of the real GDP in Figures 1b, 1c, and 2. The methodology of the study is addressed in the appendix. Here, we will document our studies of other aggregate variables, such as the weekly continued claims for unemployment insurance, the unemployment rate, labor productivity, the S&P 500 Index, and personal consumption expenditures. We document the evidence that the patterns of the GDP shown in Figures 1b, 1c, and 2 are widely observed among these variables, especially in relation to recessions. That is, we show that business cycle bubbles and crashes occur in these aggregate variables. These bubbles and crashes are less regular than the seasonal bubbles and crashes and take a longer time to form. Despite the differences, these business cycle bubbles and crashes are all similar. Moreover, they are similar to those observed in the seasonal cycle.

3.1 Weekly Continued Claims

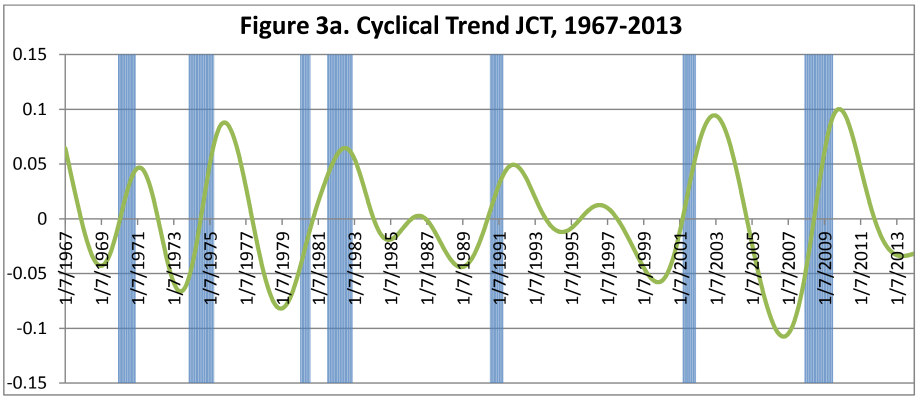
The weekly initial and continued claims for unemployment insurance have lately received more than usual attention among financial analysts and investors, partly because the Federal Reserve watches these numbers closely, among other indicators, to determine the strength of the U.S. labor market. In addition, its monetary policy is, in part, based on the performance of these indicators. Quite surprisingly though, study or use of these two weekly indicators for the business cycle among economists is quite rare. Margaret McConnell (1998) is an exception. She studied the weekly initial claims and showed the indicator provides useful information during a recession but fails to do so during an expansion. A jump in the initial claims during an expansion may not indicate the onset of a recession. This fact limits the use of this indicator to forecast a recession even though it sends out a correct signal after a recession has occurred. This present study is motivated by her work as we investigate whether the weekly continued claims as an indicator are useful in forecasting a recession. More importantly, by studying this indicator, we can determine whether the indicator has a similar pattern in the cyclical trend of the GDP as observed in Figure 1b.

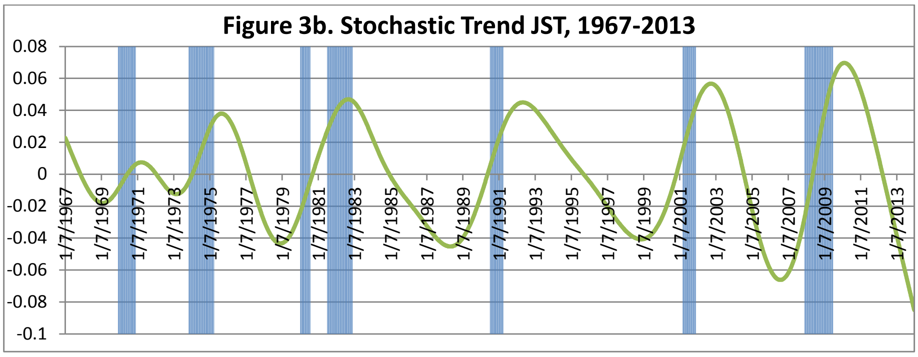
First, we apply the HP filter to the logarithm time series of the NSA weekly continued claim numbers and its seasonal factor[8] from 1967:01:7 to 2014:03:15 to remove the secular trend, with λ set at 33177600 (Morten Ravn and Harald Uhlig 2001). Second, we obtain the cyclical time series, denoted $\{J_k\}$, by subtracting the cyclical component of the seasonal factor from the cyclical component of the weekly continued claim numbers. Third, we apply the HP filter again to $\{J_k\}$ to get the stochastic trend, denoted $\{JST_k\}$, and the cyclical component, denoted $\{J_k - JST_k\}$, which is decomposed further into a cyclical trend $\{JCT_k\}$ and its fBm component $\{JC_k\}$, which has the Hurst parameter $H = 0.4969$, estimated using Patrick Flandrin's (1992) method by computing the slope of the log-log plot of the variance versus the level.

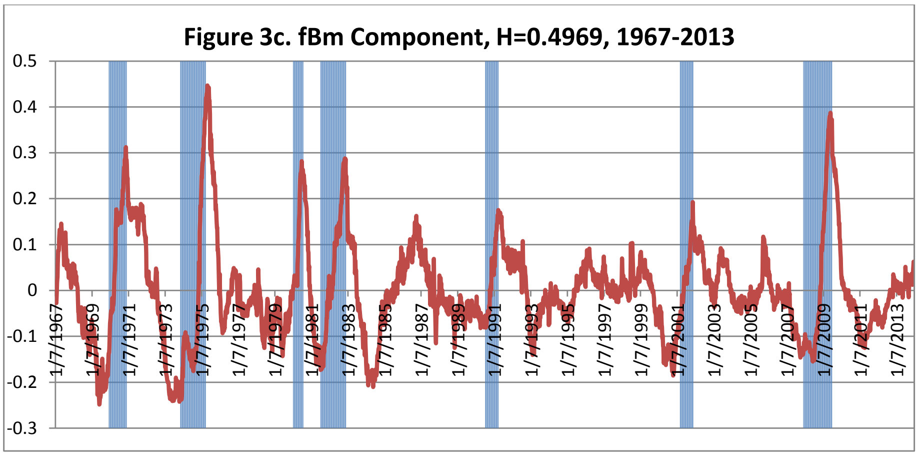
Figure 3 documents these exercises. Figure 3a presents the cyclical trend series $\{JCT_k\}$ and Figure 3b presents the stochastic trend $\{JST_k\}$, with the fBm given in Figure 3c. These figures represent the cyclical deviation from the secular trend, after adjustment for the seasonal factor. The time series JST is a leading indicator, and it starts to turn upward before a recession begins. However, both JCT and JST are lagging indicators for predicting the end of a recession. Without JST, JCT may provide a false signal, a

[8]The seasonal factor (SF) is determined by the beginning of the year and fixed for the whole year. It is typically modified in March. See William Cleveland and Stuart Scott (2007) for a detailed analysis of how the seasonal factor is determined. The cyclical deviation of the weekly continued claims is adjusted with the SF in this paper.

case similar to that documented by Margaret McConnell (1998) for the initial claims.[9] With the help of JST, JCT can provide a better timed signal of a recession. The fBm component is a reliable indicator for forecasting the end of a recession.







These figures reveal some important causes for a recession. They should shed considerable light on how recessions begin and end. First, if a recession is caused only by shocks, then one should not be able to observe the turn in JCT and JST before a recession begins.

[9] The analysis for this indicator is available upon request.

Because these two indicators lead a recession, far before a recession begins, the economy starts to lose its momentum, as shown in Figure 1b. This loss in momentum makes the economy fragile to external shocks. Second, during a recession, the fBm component can spike much higher. Many firms lay off workers in massive numbers during a recession. However, once such layoffs taper off, as fBm indicates, a recession ends. The turn in fBm is sharp and reliable. This may indicate why a recession is typically shorter than an expansion, and there is asymmetry in the business cycle, similar to that found in a number of papers on Friedman's plucking model (Milton Friedman 1993; Francisco Nadal De Simone and Sean Clarke 2007; Tingguo Zheng, Yujuan Teng, and Tao Song 2010). However, our results show that it is far from conclusive that the plucking caused by a transitory shock, as stated in these papers, is indeed the only cause of a recession.

As observed above, JST and JCT lag to predict the end of a recession. Such an issue can be overcome by extracting more signals from them after decomposing them into different components:

$$(1-B)JST = \ JST's \text{ Components } a \ +b+c$$

$$(1-B)JCT = \ JCT\text{'s Components } a \ +b+c$$

The results are shown in Figures 4 and 5. Because Component c in both cases is dominant, we have $(1-B)JST \simeq JST$'s Component c and $(1-B)JCT \simeq JCT$'s Component c, (Components c are not reported in the two figures.) The dominance of Component c does not mean that Components a and b are not important. In fact, they provide very important information about business cycles, especially with respect to recessions.

Figures 4 and 5 also show that the cyclical patterns revealed here should come from the data, not from the HP filter, which is often criticized as generating spurious cycles. No randomly generated data can produce such a nice pattern in Figures 4 and 5 under the HP filter.[10]

[10]Simulation results are out of the scope and not reported here.

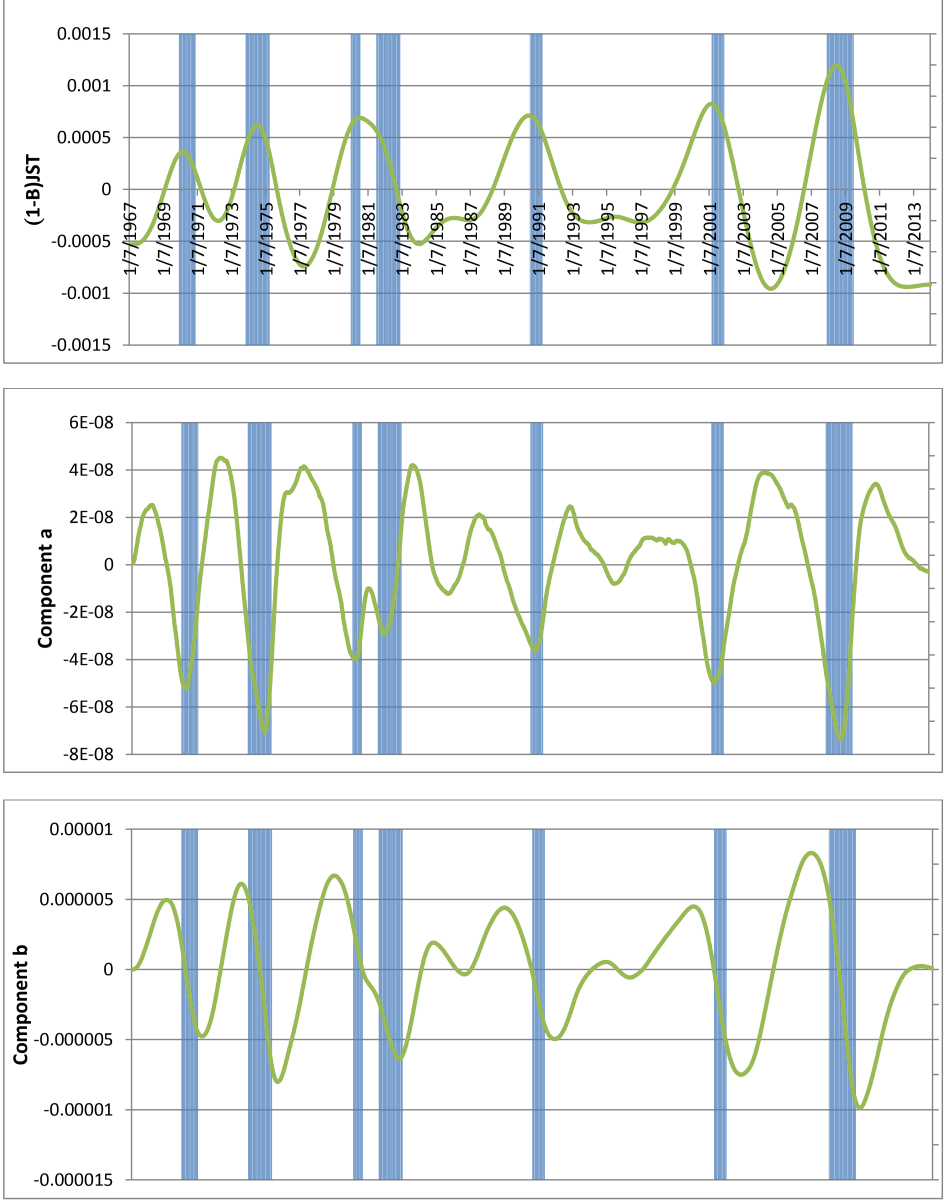
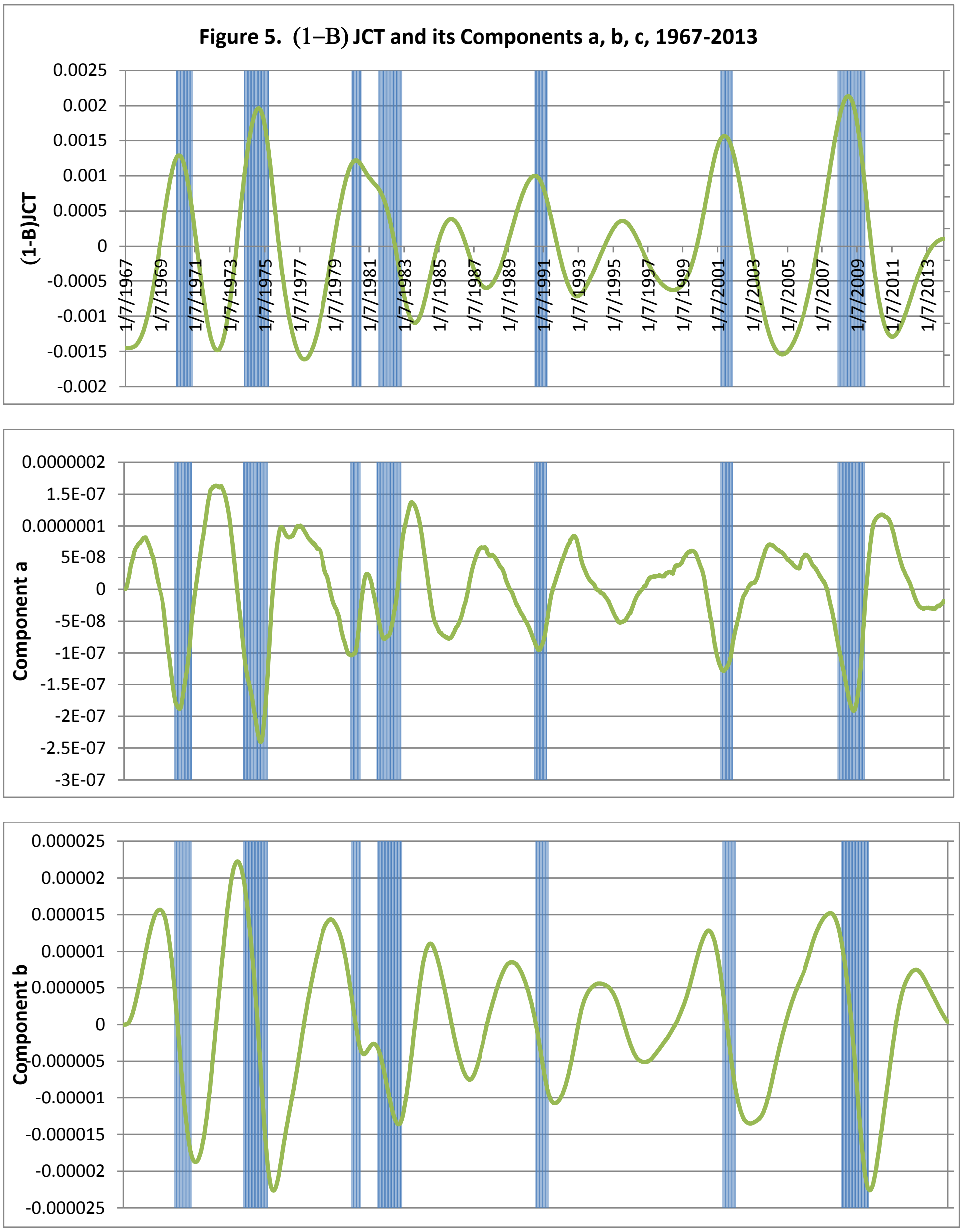


Figure 4. $(1-B)$ **JST and its Components a, b, c, 1967-2013**



As shown in Figures 4 and 5, Component b is a leading indicator in predicting the beginning of a recession. Note that Component a operates against Component c. Thus, both Components c and a become leading indicators in predicting the end of recessions. $(1-B)JST$ and $(1-B)JCT$ are leading indicators as well. They reach their local peaks before the end of recessions. Two "false" signals around 1987 and 1996 with respect to the NBER recessions occur using $(1-B)JST$ and $(1-B)JCT$.[11] Such an issue can be eliminated by using JST.

We can conclude from Figures 3, 4 and 5 that excess volatility exists in the cyclical trend of the continued claims numbers.

3.2 Unemployment Rate and Labor Productivity

The unemployment rate and labor productivity are the two most important measures for the labor market. Numerous articles have been written on these indicators. The following summarizes our understanding of the unemployment rate as leading economists describe it:

> The unemployment rate is a trendless indicator that moves in the opposite direction from most other cyclical indicators. [...] The NBER business-cycle chronology considers economic activity, which grows along an upward trend. As a result, the unemployment rate often rises before the peak of economic activity, when activity is still rising but below its normal trend rate of increase. Thus, the unemployment rate is often a leading indicator of the business-cycle peak. [...] On the other hand, the unemployment rate often continues to rise after activity has reached its trough. In this respect, the unemployment rate is a lagging indicator. (`http://nber.org/cycles/recessions_faq.html`)

Nonetheless, controversies abound concerning these two indicators. For example, to answer the question as to why the Phillips curve and the Beveridge curve models have both lost their forecasting capability recently, Alan Krueger, Judd Cramer, and David Cho (2014) declared that the existing unemployment rate is an insufficient indicator and does not reflect well the current strength or weakness of the labor market. A solution suggested by these authors is to replace the existing one with the unemployment rate for workers unemployed for less than 26 weeks. Such an alternative would fail to account for a great number of unemployed workers, those unemployed for durations longer than 26 weeks. The authors claimed that these workers are on the margins and become irrelevant

[11] There was some truth behind the two "false" signals if one recalled the two minicrashes in equity market in these years. Many still question the two crashes because they were not supported by the fundamentals, an understanding our figures contradict. Fundamentals did show substantial weakness during the minicrashes.

for increasing wages. A question left unanswered is the cause of the sharply increasing long-term unemployment rate, especially from the Great Recession.

Labor productivity is procyclical: it is higher during booms and lower during slumps (Susanto Basu and John Fernald 2001). However, Ellen McGrattan and Edward Prescott (2012) provide a new puzzle: Measured in level, labor productivity is procyclical and positively correlated with the GDP by 54% for the period from 1960 to 1985; since then, it has become much less procyclical, and its correlation with the GDP has dramatically dropped to only 5% (see Figure 1 in Ellen McGrattan and Edward Prescott 2012).

Motivated by these studies and Robert Barsky and Jeffrey Miron (1989), we investigate how these two indicators perform in relation to the business cycle and whether similar business cycle bubble and crash spikes conform with those observed in the GDP and the continued claim numbers. Labor productivity in our paper is measured by percentage change from previous of the output per hour for business sectors, downloaded from BLS. We use the rate of labor productivity rather than the level because we want to compare it with the unemployment rate.

Our procedure is as follows. First, we apply the HP filter, as in Ellen McGrattan and Edward Prescott (2012), to obtain the stochastic trends[12] of the two indicators (without taking the logarithm). These stochastic trends represent the trends over long-term horizons. Figure 6 shows the stochastic trend UST for the unemployment rate, and Figure 7a shows the stochastic trend for labor productivity. Second, we apply the HP filter to the cyclical part of labor productivity recursively[13] and decompose labor productivity into a cyclical trend component (Figure 7b) and an $AR(1)$ process (Figure 7c and Table 5).

Our decompositions are given as follows:

$$\text{Unemployment rate} = UST \text{ in Figure 6} + CU \text{ in Figure 8}$$

$$\text{Labor productivity} = \text{ Figure 7a} + \text{Figures 7b} + AR(1) \text{ in Figure 7c}$$

[12]The two indicators have no secular trends.

[13]The exact number is 100 times. However, one can choose other numbers near 100 without changing the essence of what we intend to obtain from the cyclical trend.

Figure 6. Stochastic Trend in Unemployment Rates (%)

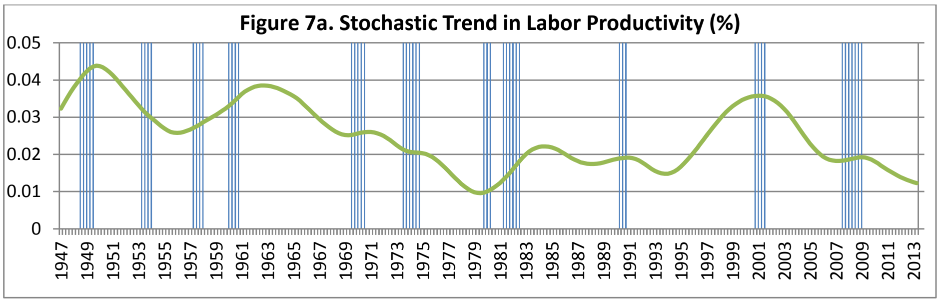


Figure 7a. Stochastic Trend in Labor Productivity (%)

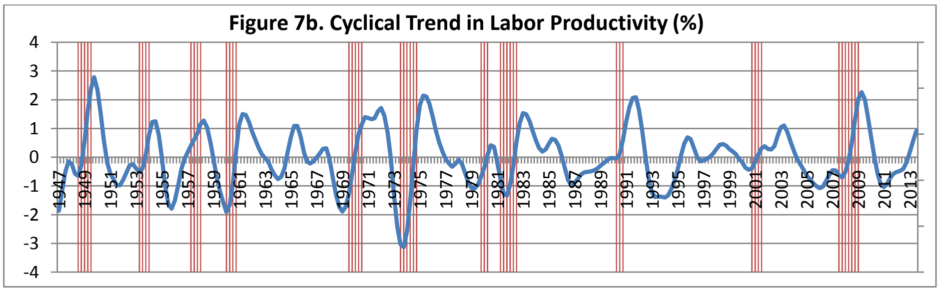


Figure 7b. Cyclical Trend in Labor Productivity (%)

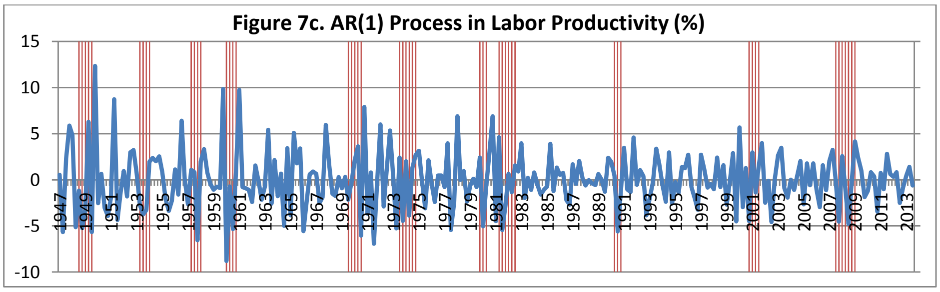


Figure 7c. AR(1) Process in Labor Productivity (%)

The stochastic trend of the unemployment rate in Figure 6 is closely related to the stochastic trend of labor productivity in Figure 7a. The stochastic trend of labor productivity has three major segments: the first is from the post-war period to 1979, with a gradual declining trend starting around 1963; the second is from 1979 to 2001, with a gradual upward trend; and the third begins in 2001, with a declining trend at a faster

Table 5. $AR(1)$ process of labor productivity

Conditional Probability Distribution: Gaussian		Standard	t
Parameter Value		Error	Statistic
Constant	-1.9089e-05	0.19045	-0.0001
AR(1)	-0.263626	0.056552	-4.6617
Variance	8.74395	0.687041	12.727

declining pace than the decline in the 1970s. The end of the third declining trend has not been shown, but it will probably end at next recession. A good sign is that the long term unemployment trend starts to show a down turn, which is often accompanied with an upward trend in long-term labor productivity.

The stochastic trend of the unemployment rate also has three major periods: the first is from the post-war period to 1982, with a gradual uptrend, increasing from the lowest point, 4 percent, in 1952 and 1967 to more than 8 percent in 1982. In addition, in this period long-term labor productivity shows a declining trend. The second is from 1982 to 1999, with a downward trend. The long-term unemployment rate declined from more than 8 percent to about 4.5 percent. This period had surging long-term labor productivity. The unemployment rate surged from the low of 4.5 percent in 1999 to more than 8 percent again in 2011. This period also had a sharp decline in long-term labor productivity. The reason long-term labor productivity affects the long-term unemployment rate may be quite obvious: Higher labor productivity increases the demand for labor. What is not so clear is the relationship between the declining trend in the long-term unemployment rate and the volatility in the cyclical trend and the AR(1) process of labor productivity. The two periods of 1961-1968 and 1982-1999 both have a downward trend in the long-term unemployment rates. Figures 7b and 7c show the cyclical trends and the AR(1) processes of labor productivity for both periods have lower volatilities than other periods have.

The Great Moderation, starting around 1985 (Olivier Blanchard and John Simon 2001; James Stock and Mark Watson 2002), affected the seasonal cycle and the business cycle. James Stock and Mark Watson (2002) provided many causes for the Great Moderation. The lower volatility identified above indicates the Great Moderation should have something to do with the declining trend in the long-term unemployment rate and the uptrend in long-term labor productivity.

Joseph Beaulieu, Jeffrey MacKie-Mason, and Jeffrey Miron (1992) found an economy with a large seasonal cycle also has a large business cycle. In our study of the seasonal cycle in Section 2, we divided the U.S. economy into three subperiods. We can now see from Figure 1 to 7 the same phenomenon as observed by Joseph Beaulieu, Jeffrey MacKie-Mason, and Jeffrey Miron. A subperiod that has a large seasonal cycle also has a large business cycle (see Table 6 below). Moreover, such a phenomenon is related to the fact that long-term labor productivity is in a declining trend and the long-term unemployment rate is in an upward trend. This phenomenon may explain why the Great Recession happened in 2008, when the long-term unemployment rate was near its peak. At another point in the 1980s, that recession was the worst since the Great Depression. In another words, these two worst recessions since World War II occurred when long-term labor productivity was near its lowest point and the long-term unemployment rate was near its peak. This phenomenon might be explained as follows: First, when long-term labor productivity is near the lowest point, there must exist more workers who have lost their skills. Second, when long-term labor productivity is in a declining trend, it should be harder for firms to raise labor productivity during a recession because it is going against the trend. Thus, firms may need to lay off more workers at such points to raise labor productivity.

Considering all eleven recessions, long-term labor productivity (Figure 7b) declined in only one, in 1954, when long-term labor productivity was in a downtrend. For the other recessions, long-term labor productivity either remained flat or moved higher, indicating a recession is helpful for raising long-term labor productivity. The recessionary effect on labor productivity can be seen in the cyclical trend shown in Figure 7b as well. During recessions, cyclical labor productivity is higher.

Whether the level of labor productivity (output per hour) is procyclical depends on whether the AR(1) process and the cyclical trend stay in the positive zone during expansions and in the negative zone during recessions. As shown in Figure 7c, labor productivity in the AR(1) component declined rather sharply into negative zones during recession periods. Interestingly enough, by the end of each recession, a surge in labor productivity to the positive occurs even during the AR(1) process. One key observation here is that, before a recession, there is a substantial decline, often into the negative zone, in the cyclical trend of labor productivity. These observations support our claim that recessions are likely caused by firms' demands to overcome declining cyclical labor productivity. Ironically, such a practice will cause a decline in labor productivity in the AR(1) part first. Thus, asserting the level of labor productivity is procyclical is not very precise considering the evolution of labor productivity during recessions and expansions.

Our analysis appears to be consistent with the idea of creative destruction initiated by Joseph Schumpeter (1942) and further developed by Philippe Aghion and Peter Howitt (1992). However, creative destruction can go well beyond innovations in technology and formation of human capital. For example, it can extend to other important aspects of the economy, such as firm-customer relationships (Erik Canton and Harald Uhlig 1999).

Table 8 in Subsection 3.3.1 shows that labor productivity is negatively related to the GDP and consumption in the cyclical trend by two unexpected large numbers, -0.556 and -0.387, respectively, for the period from 1950 to 2013, and the correlations remain steady for the period from 1950 to 1985. The negative relationships are even higher for the period from 1986 to 2013. The negative relation arises because labor productivity increases in the cyclical trend during recession periods and starts to fall after a recession (Figure 7b). This feature shows that a recession is a period in which firms use layoffs of less productive workers to battle falling productivity. Massive layoffs across broad sectors send the economy into a recession. The sharp increases in the continued claim numbers during recessions appear to support such a theory. Thus, the force behind a recession is very different from the seasonal cycle, in which labor productivity in the seasonal cycle has been found procyclical by Robert Barsky and Jeffrey Miron (1989) and Jeffrey Miron (1996).

We find that the relationship between labor productivity (measured in percent change) and the GDP or consumption does not change substantially for the two different periods of 1950-1985 and 1986-2013, in contrast to what has been documented in Ellen McGrattan and Edward Prescott (2012). The lower volatility in the AR(1) of labor productivity for the period 1986-2013 is likely affected by the Great Moderation.

As with the GDP, we can decompose the first differencing $(1-B)UST$ of the stochastic trend UST for the unemployment rate into three components:

$$(1-B)UST = \text{UST's Components } a+b+c.$$

The result is shown in Figure 8a, together with the cyclical unemployment rate (CU) in Figure 8b, which is obtained by removing UST from the original unemployment rate time series. From Figure 8a, we can see that Component b is a leading indicator in predicting the beginning of a recession while Component a is a reliable leading indicator in predicting the end of a recession. The cyclical component (CU) of Figure 8b is also a reliable indicator of recessions. These observations provide some new understanding of the unemployment rate.

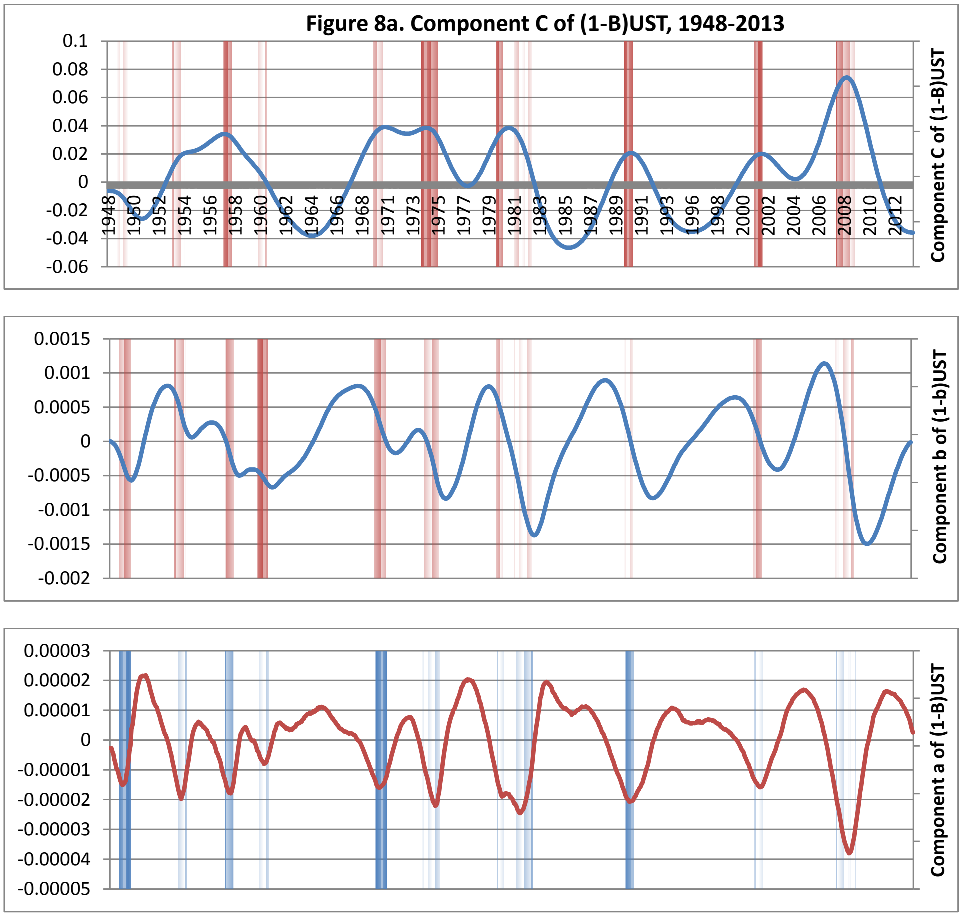


Figure 8a. Component C of (1-B)UST, 1948-2013

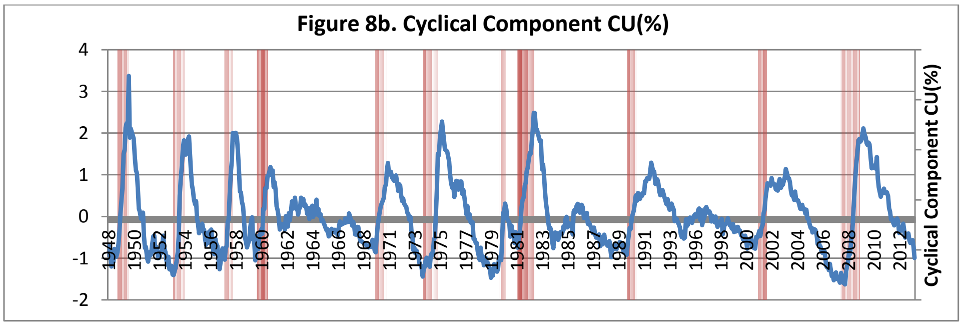


Figure 8b. Cyclical Component CU(%)

3.3 S&P 500 Index

The efficient market hypothesis claims that the equity market follows the fundamentals of the economy. It may deviate from the fundamentals occasionally but not for a very long period. That is, the equity market must be closely affected by the business cycle. In particular, the expected returns should be lower during an economic boom and higher during an economic slump. Thus, the equity market should move higher during booms and lower during slumps. On the other hand, the excess volatility discovered by Robert Shiller (1981) shows that the equity market, using the real S&P 500 Index as a proxy, may often form a bubble (or crash), with prices that are well above (or below) a level supported by the fundamentals in real terms. The formation of a bubble or crash in Shiller (1981) takes a longer time than a typical business cycle duration. Robert Shiller (1981) uses the present value of distributed dividends as the level supported by the fundamentals adjusted by the inflation. Even though Robert Shiller (1981) does not provide a precise time when a bubble or crash is formed in an equity market, a juncture of the efficient market hypothesis and his excess volatility indicates that a bubble should be formed more often during an economic boom and a crash should be formed more often during an economic slump.[14]

In this section, we provide a study of the S&P 500 Index quarterly close prices adjusted with dividends from 1950Q1 to 2014Q2. The data were downloaded from Yahoo. We use the same methodology as was used to analyze the GDP data. Figure 9 shows the cyclical trend SPT and the fBm of the S&P 500 Index in such an exercise. Figure 9a shows the efficient market hypothesis, in fact, holds very well in the sense that those price spikes near recessions of the cyclical trend of the S&P 500 are very much like those in the GDP shown in Figure 1b.

The bubbles and crashes shown in Figure 9a arise from the deviations from the secular trend while Robert Shiller (1981) identifies a bubble or crash as deviations from the fundamentals computed by a discounted model with a constant discount rate. Our bubbles and crashes appear to be much smaller in amplitude and occur more often than those

[14] This explanation does not mean that the equity market will not form a bubble or crash at other times. For example, Black Monday, October 19, 1987, and a minicrash that occurred on Friday, October 13, 1989, were both mysterious. These two minicrashes occurred when the economy was in an NBER expansion. Nonetheless, the weakness in the GDP during these two periods, which Figure 1 shows, may have contributed to these two minicrashes.

identified by Shiller. That is, a major bubble or crash (in annual data) in Shiller consists of a sequence of bubbles or crashes (in quarterly or monthly data) identified in this paper.

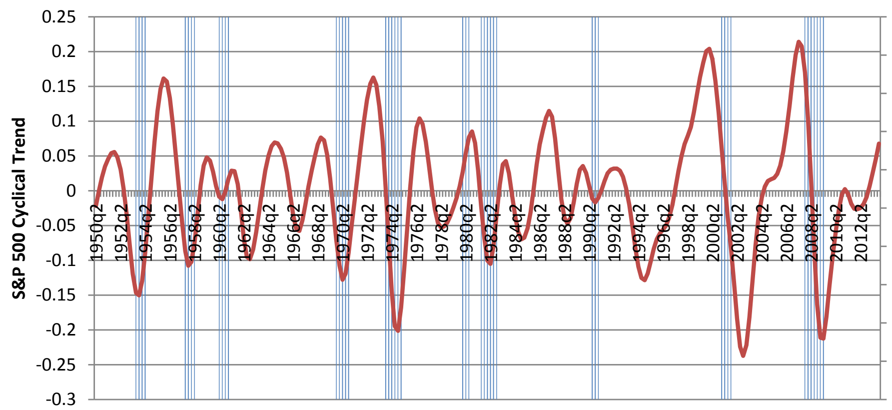


Figure 9a. S&P 500 Cyclical Trend *SPT*, 1950q2-2014q2

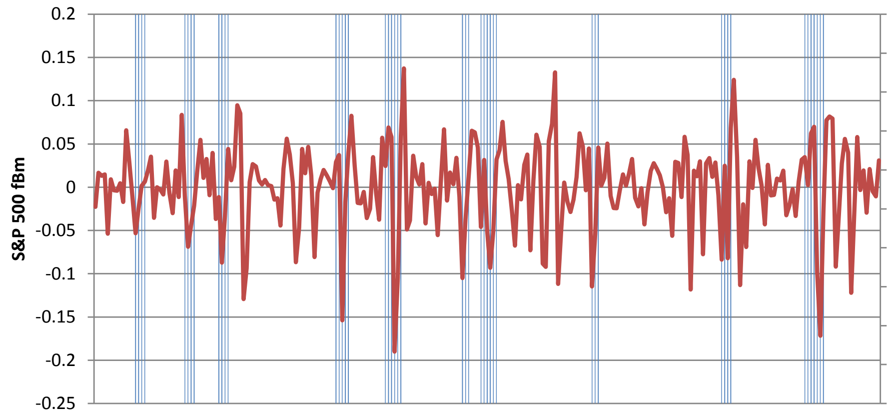


Figure 9b. S&P 500 fBm, *H*=0.5058

3.3.1 Key Statistics

We provide some key statistics in tables in this section. Two types of excess volatility occur in the cyclical trend and the fBm of the S&P 500. The first is based on Figure 9a in which the cyclical trend of the Index deviates from its secular trend to form bubbles and crashes. These bubbles and crashes are similar to those observed in real aggregates, such as the GDP, the unemployment rate, and so on. The standard deviations of the cyclical trend of the S&P 500 reached about 16 percent for the period of 1950q1-2001q4 while the standard deviations of the fBm were about 10 percent. The standard deviations of the cyclical trend and the fBm of the Index were the highest in the period from 1960q1 to

1978q4. The Great Moderation seems to affect excess volatility of the S&P 500.

The second is the difference in the standard deviations of the S&p 500 from those of the GDP, consumption, and other aggregates. The cyclical trend of the S&P 500 appears to have cycles with much larger amplitudes, as shown in Tables 6 and 7. For instance, the standard deviation of the cyclical trend of the GDP over the period 1947:q1 to 2001:q4 reached only 3% while that of the S&P 500 reached about 16 percent over roughly the same period. Furthermore, the period from 1960:q1 to 1978:q4 had the largest gap between the standard deviations of these two variables and accounted for 20.3 percent with the S&P 500 superseding the GDP. The first type of excess volatility is closely related to the original finding in Robert Shiller (1981) while the second is more related to the equity premium puzzle in Rajnish Mehra and Edward Prescott (1985).

Table 8 shows the correlation between various aggregate variables in the periods 1950-2013, 1950-1985, and 1986-2013. This table reveals that the fBm component of the S&P 500 is positively related to those in the GDP and consumption, but the magnitudes of correlation are very small.[15] That is, the HP filter extracted much information from the original data in terms of the cyclical trends. The cyclical trend of the S&P 500 is positively related to the cyclical trends in the GDP and consumption by about 50 percent and the correlation becomes higher for the period of 1986-2013. Thus, volatility in the noise part of the equity market does not follow that in the fundamentals of the economy, especially during the period 1950-1985. That is, different forces drive the noise parts of the equity market and the real economy. We conclude that the data in Tables 7 and 8 support Shiller's excess volatility (also see Table 9) in both the short and the long terms.

The low correlation at about 20 percent between the equity market and the macroeconomic aggregates, such as the GDP and consumption, has been called *the correlation puzzle* (John Cochrane and Lars Peter Hansen 1992; John Campbell and John Cochrane 1999, 2000) and can be seen in part as a factor that contributes to the equity premium puzzle (Rajnish Mehra and Edward Prescott 1985). Our result reveals that such a low level of correlation is caused by the noise parts of the fBms. The correlation between the cyclical trends of these variables can reach a reasonable level of 50-75%. Of interest is that the equity market appears to be more correlated with the aggregates since the start (around 1985) of the Great Moderation.

[15]The only exception is the period 1950-1985 for which there is a negative correlation of -0.01.

Table 6. Standard Deviations, annualized(%)

		1946q1-2001q4	46q1-59q4	60q1-78q4	79q1-01q4
Seasonal Dummies					
of GDP		8.70	11.32	9.62	6.82
		1947q1-2001q4	47q1-59q4	60q1-78q4	79q1-01q4
GDP	Cyclical Trend	3.00	3.50	3.01	2.66
	fBm	1.20	1.79	1.09	0.84
Growth Rate	Average*	3.44	3.58	3.91	2.97
		(2.01)	(2.71)	(1.97)	(1.53)
Personal	Cyclical Trend	2.25	1.97	2.51	2.19
Expenditure	fBm	1.04	1.66	0.82	0.72
Total Goods	Cyclical Trend	3.55	3.16	3.95	3.42
	fBm	1.73	2.54	1.47	1.35
Durabale Goods	Cyclical Trend	8.39	10.12	8.44	7.31
	fBm	4.84	7.53	3.84	3.58
Services	Cyclical Trend	1.34	1.35	1.18	1.46
	fBm	0.63	0.92	0.48	0.56
		1947q2-2001q4	47q2-59q4	60q1-78q4	79q1-01q4
Labor	Cyclical Trend	2.04	2.14	2.24	1.60
Productivity(%)	AR(1)	6.47	7.64	7.35	4.87
		1950q1-2001q4	50q1-59q4	60q1-78q4	79q1-01q4
S&P500	Cyclical Trend	15.69	16.68	23.31	15.16
	fBm	9.63	6.31	13.90	9.81

*Annualized average and standard deviations in parentheses. Consumption is studied in the next section.

Table 7. Excess Volatility: Ratios of Standard Deviations.

		1950q1-2001q4	50q1-59q4	60q1-78q4	79q1-01q4
GDP	Cyclical Trend	5.23	4.77	7.74	5.70
	fBm	8.03	3.53	12.75	11.68
Personal	Cyclical Trend	6.96	8.45	9.28	6.91
Expenditure	fBm	9.26	3.80	17.00	13.50
Seasonal Dummies	Cyclical Trend	1.80	1.47	2.42	2.22
of GDP($*$)	fBm	1.11	0.56	1.44	1.44

Note-Derived from Table 6 by dividing the standard deviations of the cyclical trend and fBm of the S&P 500 by the standard deviations of other aggregates.

($*$) We divide the standard deviations of the cyclical trend and fBm of the S&P 500 by the standard deviations of the seasonal dummies. As indicated in Table 6, the two periods 1950q1-2001q4 and 50q1-59q4 in Table 7 are not quite the same as the GDP in Table 6. We expect no major differentials are caused by the inconsistency.

Table 8. Correlation between aggregate variables

Correlation between fBms	1950q1-2013q4				1950q1-1985q4				1986q1-2013q4			
	GDP	Prod.	Con.	S&P500	GDP	Prod.	Con.	S&P500	GDP	Prod.	Con.	S&P500
GDP	1				1				1			
Productivity	0.31	1			0.31	1			0.32	1		
Consumption	0.62	0.16	1		0.64	0.16	1		0.45	0.14	1	
S&P500	0.03	0.11	0.08	1	-0.01	0.09	0.01	1	0.14	0.15	0.04	1
Correlation between Cyclical Trends	1950q1-2013q4				1950q1-1985q4				1986q1-2013q4			
GDP	1				1				1			
Productivity	-0.56	1			-0.55	1			-0.58	1		
Consumption	0.83	-0.39	1		0.79	-0.27	1		0.94	-0.66	1	
S&P500	0.47	-0.12	0.54	1	0.33	-0.01	0.49	1	0.75	-0.29	0.67	1

Note-Prod. denotes labor productivity. Con. denotes personal consumption expenditures.

Table 9 shows the regression relationship of the cyclical trend of the S&P 500 Index and that of the GDP or the services. The two regressions show that one percentage deviation in the cyclical trend from the secular trend in the GDP (services) results in about three (six and a half) percentage deviation in the cyclical trend from its secular trend in the S&P 500.

Table 9. Regression of Figure 9a on Figure 1b (GDP) or Figure 10(C), 1950q1-2013q4.

GDP		SE	t statistics	p value	R^2_{adj}
Constant	-0.0017	0.0048	-0.3492	0.7272	0.2209
Coefficient	2.9257	0.3418	8.5607	0.0000	
Services					
Constant	-0.0007	0.0048	-0.1545	0.8773	0.2412
Coefficient	6.4883	0.7163	9.0582	0.0000	

Table 10. Variances and Covariances, annualized, 1950q1-2013q4

fBm	GDP	Labor Productivity	Services	S&P 500
GDP	1.18			
Labor Productivity(%)	2.02	35.56		
Services	0.16	0.48	0.29	
S&P 500	0.37	6.2	0.30	98.74
Cyclical Trend				
GDP	8.02			
Labor Productivity(%)	-3.09	35.57		
Services	3.27	-1.19	1.78	
S&P 500	23.45	-4.24	11.53	306.39

Covariance matrices of the fbm and the cyclical trends of the S&P 500 and others are shown in Table 10. For example, in a well-diversified portfolio whose only risk comes from the macroeconomic risk described by the cyclical trend of the GDP or services, the coefficients in two regressions above are the betas of the S&P 500 in its cyclical trend with respect to the two well-diversified portfolios, respectively. Thus, Tables 9 and 10 provide additional evidence to support the excess volatility of Robert Shiller (1981). The equity market in its cyclical trend is far more volatile than its underlying fundamentals. The low covariance of 0.37 between the fBms of the GDP and the S&P 500 should be evaluated in light of the fact that prices in the equity market change by seconds while prices in the real economy may not change for weeks or even months. Nonetheless, the covariance between the two cyclical trends can reach a level of 23.45, indicating the real economy and the equity market do move together in lower frequencies.

3.4 Consumption

Personal consumption expenditures (C) include expenditures on nondurable goods, durable goods, and services. We address the personal consumption expenditures, total expenditures on goods, total expenditures on durable goods, and total expenditures on services. The data in our study are quantity indexes for the real personal consumption expenditures, seasonally adjusted (index numbers, 2009=100) from BEA. In the estimation of the Hurst parameters H for the four series, we find only the service expenditures have an H higher than $\frac{1}{2}$; the cyclical trends of the four series as shown in Figure 10 are based on the service series to obtain the Hurst parameter H close to $\frac{1}{2}$. Thus, the fBm parts of personal consumption expenditures, total goods, and durable goods in Figure 11 will have H smaller than $\frac{1}{2}$, indicating they are short memory-dependent processes, not a standard Brownian motion such as the fBm of the service expenditures.

The cyclical trends in Figure 10 are similar to the pattern in Figure 1b, the cyclical trend in the GDP, not surprising, personal consumption expenditures in the U.S. economy are about $\frac{2}{3}$ of the GDP. The correlation between the GDP and consumption in cyclical trends in Table 8 reaches more than 94 percent for the period 1986-2013. According to Table 6, consumption and services are less volatile than the GDP, but durable goods and total goods are, in general, more volatile than the GDP.

Figure 10. Cyclical Trends of C, Gd, Dgd and S, 1947q1-2014q2

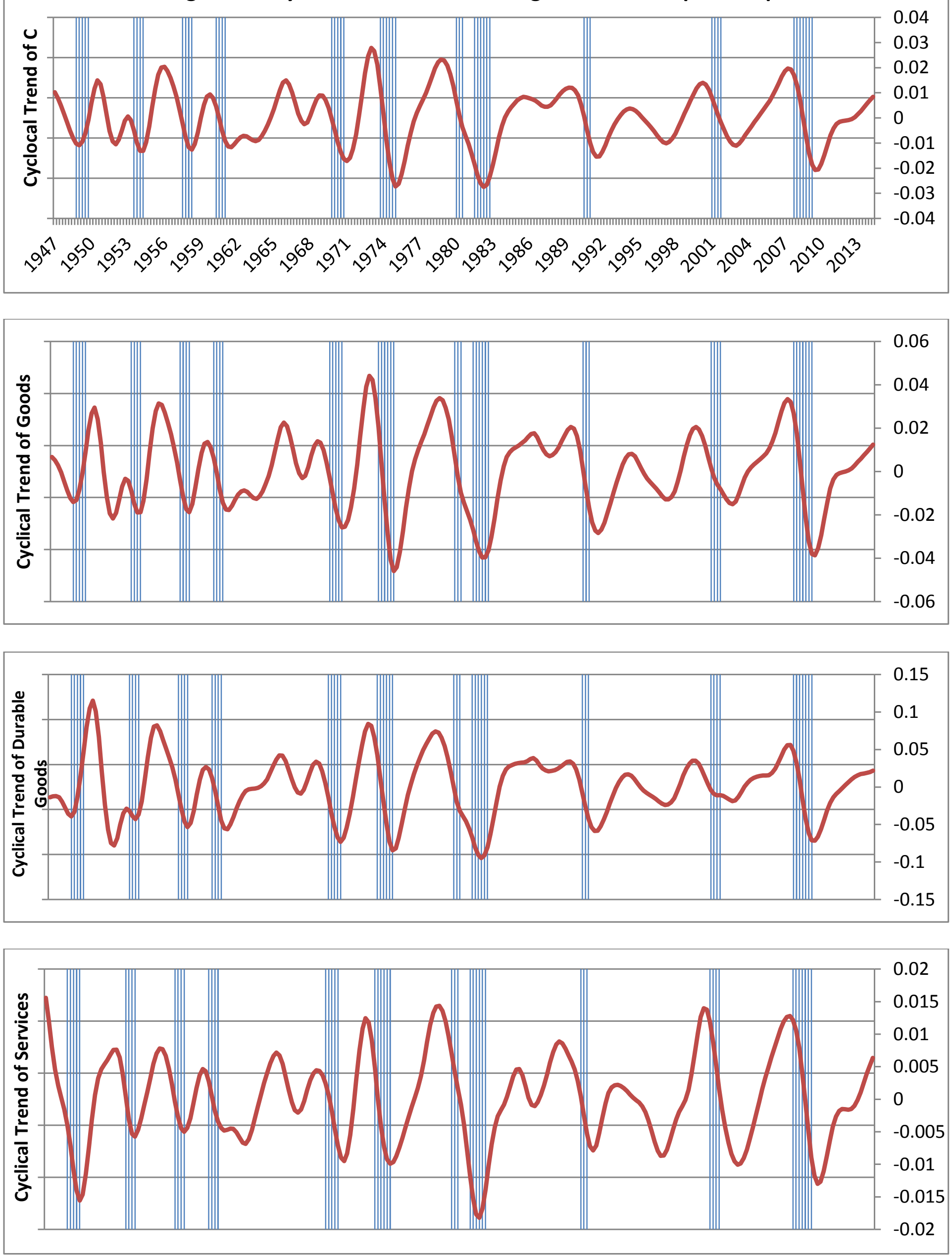
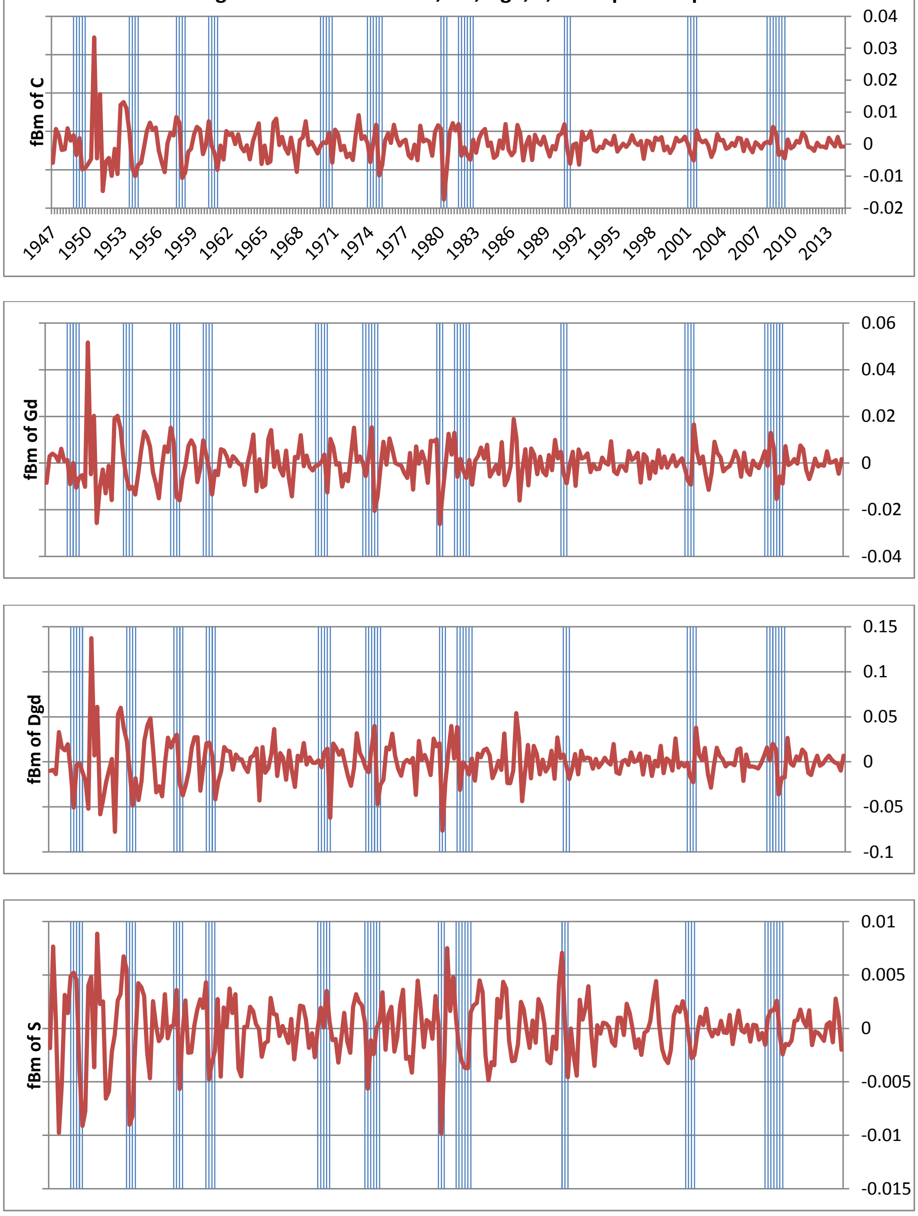


Figure 11. fBm Parts of C, Gd, Dgd, S, 1947q1-2014q2



The relationship between the volatility in the cyclical component and the growth rate of the GDP is rather complicated (Table 6). Even though higher volatility in the GDP does not imply a higher growth rate, it appears from Table 6 that a low volatility in the GDP does not imply a high growth rate either. Thus, a policy that aims at reduction in macroeconomic fluctuations does not always result in an increase in the social welfare. A higher volatile economy can mean more workers who are less productive and less skilled become unemployed at a faster pace. This results in better chances for productive and skillful workers to enter the labor force during the next recovery and expansion phase. When the economy has a lack of volatility for a lengthy period, the process of replacing less productive workers is slower, possibly hurting future labor productivity and economic growth. Seasonal variations accomplish this replacement process in a higher and more regular frequency. Business cycle variations accomplish the process on a slower and less regular basis. Because Table 6 and Figure 1 indicate the seasonal cycle and the business cycle may be related through certain channels (see Irma Hindrayanto, Jan Jacobs, and Denis Osborn 2014), a policy that greatly reduces seasonal variations may not always improve social welfare in the long run (Jeffrey Miron 1996). This does not mean that monetary policy to help stabilize the economy during a depressed time is bad. In addition, seasonal and business cycle variations are not always bad. The cyclical trends of consumption in Figure 10 appear to reveal more fluctuations, but the fBm components in Figure 11 are still weak, indicating the effect of the Great Moderation may persist to the near future.

3.5 Stochastic Discount Factor

In this section, we demonstrate a vital link between macroeconomic fluctuations and the intertemporal marginal rate of substitution (MRS) and document the empirical evidence that the time-varying stochastic discount factor is likely to be the major force causing macroeconomic fluctuations in the business cycle frequencies.

For a utility function of the constant relative risk aversion (CRRA) class

$$U(c) = \frac{c^{1-\alpha}}{1-\alpha}, \qquad 0 < \alpha < \infty,$$

the logarithmic stochastic discount factor $ln(m_{t+1})$ satisfies

$$ln(m_{t+1}) = ln\beta - \alpha ln(\frac{c_{t+1}}{c_t}),$$

where β is the time discount rate and c_t is consumption per capita, as in Rajnish Mehra and Edward Prescott (1985). Let $\overline{\frac{c_{t+1}}{c_t}}$ be the stochastic trend valued at $t+1$ of the consumption growth $\frac{c_{t+1}}{c_t}$. Using a Taylor expansion at $\overline{\frac{c_{t+1}}{c_t}}$, we obtain

$$ln(m_{t+1}) \approx ln\beta - \alpha ln\overline{\frac{c_{t+1}}{c_t}} - \alpha\frac{1}{\overline{\frac{c_{t+1}}{c_t}}}(\frac{c_{t+1}}{c_t} - \overline{\frac{c_{t+1}}{c_t}}) + \alpha\frac{1}{(\overline{\frac{c_{t+1}}{c_t}})^2}\frac{(\frac{c_{t+1}}{c_t} - \overline{\frac{c_{t+1}}{c_t}})^2}{2}.$$

Let $ln(\bar{m}_{t+1}) = ln\beta - \alpha ln\overline{\frac{c_{t+1}}{c_t}}$, which may be seen as the stochastic trend of the logarithmic stochastic discount factor $ln(m_{t+1})$ in the long-term, and

$$z_{t+1} = -\frac{1}{\overline{\frac{c_{t+1}}{c_t}}}\left(\frac{c_{t+1}}{c_t} - \overline{\frac{c_{t+1}}{c_t}}\right) + \frac{1}{(\overline{\frac{c_{t+1}}{c_t}})^2}\frac{(\frac{c_{t+1}}{c_t} - \overline{\frac{c_{t+1}}{c_t}})^2}{2}.$$

Then

$$\frac{1}{\alpha}(ln(m_{t+1}) - ln(\bar{m}_{t+1})) \approx z_{t+1}.$$

We may apply the HP filter to obtain the stochastic trend $\overline{\frac{c_{t+1}}{c_t}}$ of consumption growth and apply the filter recursively to z_{t+1} to obtain the cyclical trend of the deviations $\frac{1}{\alpha}(ln(m_{t+1}) - ln(\bar{m}_{t+1}))$. Because m_{t+1} is a random variable that has incorporated all types of behaviors, expectations, public and private information, and so on, for tradable and non-tradable assets, it is difficult to know precisely how it behaves. Under the basic consumption-based asset pricing model, the exercise above provides an important variable, z_{t+1}, that can be used to find the cyclical trend of $ln(m_{t+1})$. It makes it possible to know how the stochastic discount factor affects macroeconomic fluctuations, even without knowing the detail of the two parameters α and β.

The question is how many times the HP filter should be used recursively to obtain the cyclical trend of deviations $\frac{1}{\alpha}(ln(m_{t+1}) - ln(\bar{m}_{t+1}))$. We know that the consumption growth $\frac{c_{t+1}}{c_t}$ is close to i.i.d..[16] Thus, we cannot use the Hurst parameter $H = \frac{1}{2}$ directly, as in the case of the GDP. However, in the recursive application of the HP filter to obtain the cyclical trend of the GDP, as in Figure 1b, we recorded 189 of recursive applications of the HP filter during the procedure. This number has been used to obtain the cyclical trend of deviations $\frac{1}{\alpha}(ln(m_{t+1}) - ln(\bar{m}_{t+1}))$.

Our empirical results are shown in Figure 12. Figure 12a is the cyclical trend of the GDP growth rate for comparison purpose. Figures 12b and 12c are the cyclical trends of z_{t+1} based on aggregate consumption and services,[17] respectively. The cyclical trend of the logarithmic stochastic discount factor $ln(m)$ is α-multiple of that in Figure 12b (based on consumption) or 12c (based on services). We can conclude from Figure 12 that the cyclical trend of the GDP growth has been driven by the cyclical trend of the logarithmic

[16]See Ravi Bansal and Amir Yaron (2004) and references therein for a different view. In particular, Bansal and Yason provide a theoretical model under which the consumption growth rate is decomposed into a small persistent and predictable component and a "noise" component that reflects economic uncertainty or consumption volatility. Our work here may be considered an empirical counterpart to such an idea.

[17]We may assume that quarterly population growth rate is near zero.

stochastic discount factor, which is clearly a time-varying process with bubbles and crashes around its conditional mean, especially around the NBER recessions.

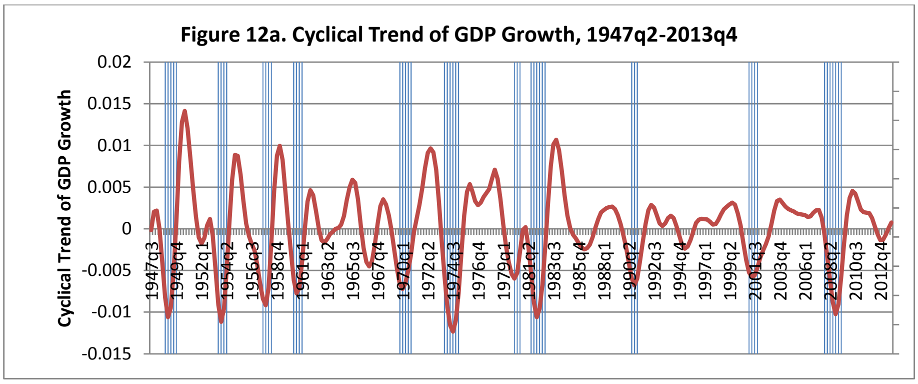


Figure 12a. Cyclical Trend of GDP Growth, 1947q2-2013q4

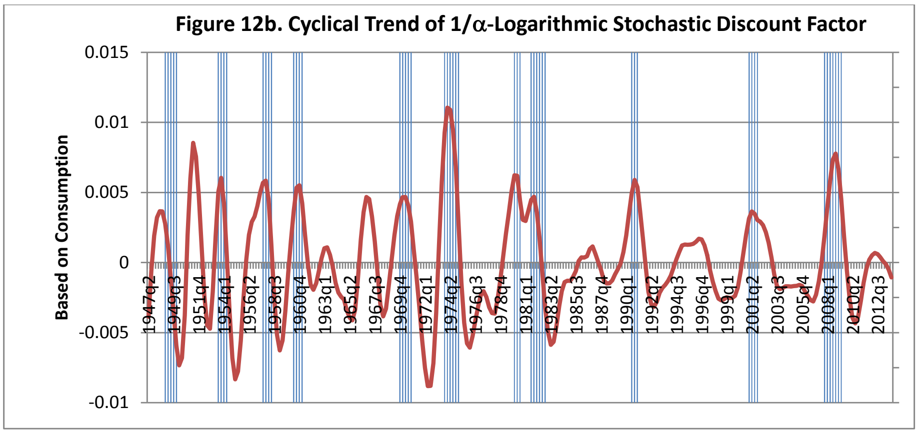


Figure 12b. Cyclical Trend of 1/α-Logarithmic Stochastic Discount Factor

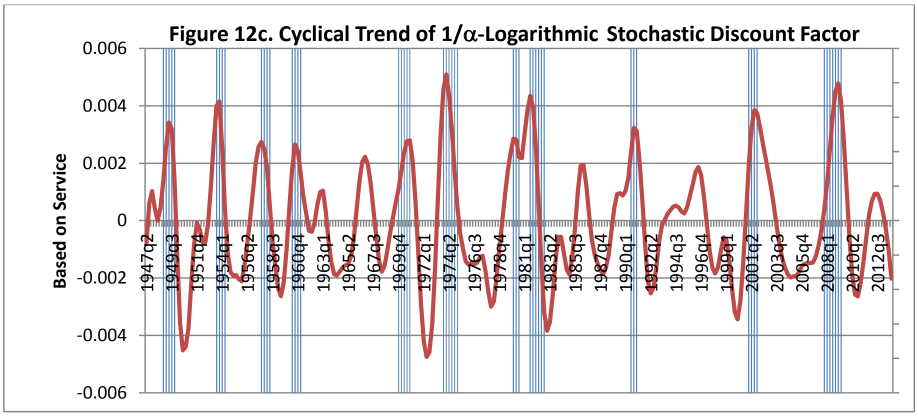


Figure 12c. Cyclical Trend of 1/α-Logarithmic Stochastic Discount Factor

In our analysis above, there is no preference shift. Thus, we conclude that preference shift is unlikely to be a major factor causing macroeconomic fluctuations, as in the seasonal cycle. This conclusion does not mean that preference shift is not important for the business cycle and that preferences always stay the same. What we have documented is that the stochastic discount factor likely plays a more important role in macroeconomic fluctuations in business cycle frequencies than do other factors.

Table 11a. Correlation between excess return, GDP growth, and $\frac{1}{\alpha}-$logarithmic stochastic discount factors z, quarterly data 1947q2 to 2013q4

Stochastic Discount	Cyclical	Trend			Cyclical	Noise		
Factor z based on	C	Gd	Dgd	S	C	Gd	Dgd	S
Excess Return R^e	-0.214	-0.219	-0.188	-0.197	-0.031	-0.029	-0.021	-0.030
Secular Trend R^e	-0.112	-0.110	-0.073	-0.119	-0.001	-0.001	-0.000	-0.001
Cyclical Trend R^e	-0.459	-0.469	-0.414	-0.418	-0.028	-0.030	-0.022	-0.019
Noise fBm R^e	-0.021	-0.023	-0.020	-0.016	-0.024	-0.021	0.036	-0.028
GDP Growth Rate	-0.531	-0.503	-0.456	-0.517	-0.345	-0.350	-0.361	-0.142

Note. Excess return equals Fama-French benchmark rate of return minus one month T-bill.

Table 11 shows a comparison analysis of the relationship between the excess return and GDP growth with the logarithmic stochastic discount factor $ln(m)$. Table 11a is the correlation matrix and Table 11b shows the (conditional) mean and standard deviations. The excess return[18] quarterly data are the Fama-French benchmark portfolio rate of return minus the one month treasury T-bill rate downloaded from Ken French data library at Dartmouth. The results in Table 11a show the cyclical trend of the logarithmic stochastic discount factor $ln(m)$ based on consumption or services can explain more than 50% of the fluctuations in the GDP growth and the noise part of the logarithmic stochastic discount factor based on the consumption (services) can explain an additional 35% (14%) of the fluctuations in the GDP growth.

In contrast, the cyclical trend of the logarithmic stochastic discount factor based on consumption and services can explain about 21 percent and 20percent, respectively, of the fluctuations in the excess return. On the other hand, the cyclical trend of the logarithmic stochastic discount factor based on consumption (services) can explain about twice that number, with more than 45% (41%) of the fluctuations in the cyclical trend of the excess return. The logarithmic stochastic discount factor, either in its cyclical trend or in its noise part, based on consumption or services, can explain the noise (fBm) part of the excess return by no more than 3%. Thus, 13.58% standard deviation of the excess return

[18]It took 202 recursive applications of the HP filter to obtain the fBm with H close to 0.5 for the S&P 500 Index. This number is used to obtain the cyclical trend and noise part of the excess return under the HP filter.

in its noise (fBm) part, of 16.41% in total, is left largely unexplained by the logarithmic stochastic discount factor $ln(m)$ under the consumption-based asset pricing model (Table 11b). This result may explain the low correlation (around 11 percent, as shown in Table 11a) between the secular trend of the excess return and the cyclical trends of the stochastic discount factors.

Table 11b. Annualized mean and standard deviation (STD) of cyclical trend, cyclical noise, excess return, and GDP growth rate, quarterly data 1947q2 to 2013q4

		mean	STD			mean	STD
Cyclical	R^e	0.04	6.33	Cyclical	R^e	0.05	13.58
Trend	C	0.00	0.73	Noise	C	0.00	1.26
	Gd	0.00	1.18		Gd	0.00	2.21
	Dgd	0.00	2.73		Dgd	0.00	6.16
	S	0.00	0.41		S	0.00	0.69
Excess Return	R^e	7.99	16.41	Secular Trend	R^e	7.92	2.90
GDP Growth	Rate	3.23	1.94				

Furthermore, Table 11 provides evidence for why the consumption-based asset pricing model

$$0 = E_t[R^e_{t+1}\beta(\frac{c_{t+1}}{c_t})^{-\alpha}]$$

fails empirically in the long run: The stochastic discount factor based on consumption or services has volatility (with a small α; Table 11b) and correlation with the excess return (Table 11a) that are too low. Because the cyclical trend and the noise component of the excess returns both have means close to zero, the positive excess returns for those who conduct a buy-and-hold strategy solely driven by the long-term or secular trend, which awards those investors who can tolerate the macroeconomic risk and the risk associated with the noise part. Such an investor has been awarded with quarterly excess returns for holding equity from 1926q3 to 2014q2, at an annualized rate of 8.56% and standard deviation of 3.22% (not included in Table 11b). An investor using the buy-and-hold strategy for the period 1947q2-2013q4 has been awarded with excess return at mean of 7.92% and standard deviation of 2.90% (Table 11b). To earn those returns, investors often must have an unbinding budget constraint because they must hold cash or be capable of borrowing beyond the Fama-French benchmark portfolio to prepare for smoothing their consumption. Otherwise, investors must sell assets in the portfolio to smooth their consumption during economic slumps like the Great Depression and recessions, and then they can no longer use their buy-and-hold strategy. Thus, the above Euler equation may not apply to those investors aiming at a long-term return with the passive buy-and-hold strategy.

4 Conclusions

Using the HP filter recursively to extract the cyclical component of real aggregates, we find the cyclical trends of many real aggregates have a bubble-like expansion and a crash-like contraction in the business cycle frequencies as well, a phenomenon similar to that observed in the seasonal cycle, which itself is not stable and evolves across time.

In line with Robert Barsky and Jeffrey Miron (1989), we find preference shifts and production synergy appear to be two major forces for the seasonal cycle. On the other hand, we find the time-varying stochastic discount factor is the main cause of the business cycle and plays a more important role in macroeconomic fluctuations in the business cycle frequencies than other factors. These macroeconomic fluctuations are, in fact, the excess volatility of real economic activities. The force of the formation of such a process is likely because of the mean-reverting process around the secular trend.

By analyzing the relationship between the cyclical trend in consumption and the stochastic discount factor, we show the intertemporal marginal rate of substitution or the stochastic discount factor has a time-varying cyclical trend, with a pattern of bubbles and crashes around its conditional mean, largely occurring near NBER recessions. We conclude from the pattern that the time-varying stochastic discount factor should be a major force causing macroeconomic fluctuations in business cycle frequencies, in line with the excess volatility literature on the equity market.

By studying the relationship between the equity market and the GDP, we show that the equity market in its cyclical trend is far more volatile than its underlying fundamentals are. Furthermore, the low covariance between the fractional Brownian motion of the GDP and the S&P 500 should be evaluated in light of the fact that prices in equity market change by seconds while prices in the real economy may not change for weeks or even months. Nonetheless, the relatively high covariance between the two variables in their cyclical trends indicates the real economy and the equity market move together at lower frequencies. Our study provides a reply to Morgan Housel (2014).

As a policy implication of this study, reducing macroeconomic fluctuations does not always cause an increase in the social welfare in the long run. A more volatile economy implies a faster replacement of less productive workers with more productive ones. Seasonal variations accomplish this replacement process at a higher and more regular frequency than does the business cycle. Accordingly, seasonal and business cycle variations are not always bad for the economy in the long run. An interesting issue not explored in this paper is the precise balance between seasonal and business cycle variations and long-term economic growth.

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5 Appendix (for referees and web version, not for publication)

In this appendix, we introduce the methodology used to study a number of aggregate variables in this paper. We first introduce a popular estimation method for Hurst parameter H of a fBm. Then, we introduce how we get the cyclical trend by recursively using the HP filter. Finally, we introduce the cubic spline interpolation and explain why we can use the approach to decompose a cyclical trend into three components. Such a decomposition is useful because a cyclical trend that appears to be lagging can become a leading indicator, as shown numerous times in the present paper.

5.1 Estimation of Hurst Parameter H

Let $[0, T]$ be a time interval for some $T \in (0, \infty)$. An aggregate variable $\{Y_t\}_{t\in[0,T]}$ of an economy is a stochastic process on a probability space $(\Sigma, \mathcal{A}, P)$ such that the following decomposition holds,

$$Y_t = \tau_t + c_t$$

where $\{\tau_t\}$ is the smooth trend and $\{c_t\}$ is the cyclical component. One common way to model $\{c_t\}$ is to use a fractional ARIMA(p, d, q) process. For a long-range dependence process, it is equivalent and easier to model the process with a fBm for $H = \frac{1}{2} + d$ (Ton Dieker 2004). Thus, $\{c_t\}$ can be expressed[19] by $c_t = \sigma B_t^H$, where $\{B_t^H\}$ is a factional Brownian motion (fBm) with Hurst parameter $0 < H < 1$, a continuous Gaussian process $\{B_t^H\}_{t\in(0,\infty)}$ that has expectation zero and whose covariance satisfies

$$Cov(B_t^H, B_s^H) = \frac{1}{2}(|t|^{2H} + |s|^{2H} - |t-s|^{2H}), \quad t, s \in R.$$

For $H = \frac{1}{2}$, $\{B_t^H\}$ is a standard Brownian motion. A fBm has a positive (negative) correlation of two non-overlapping increments if $H > \frac{1}{2}$ ($H < \frac{1}{2}$). For $H > \frac{1}{2}$ ($\frac{1}{2} > H$), a

[19]The path-wise solution to $\frac{dX_t}{X_t} = \sigma dB_t^H$ is given by $ln(X_t) - ln(X_0) = \sigma B_t^H$, where $B_0^H = 0$ (Tommi Sottinen 2001). The generalized solution is given by $ln(X_t) - ln(X_0) = -\frac{\sigma^2}{2}t^{2H} + \sigma B_t^H$. For the S&P500 index, as revealed in the paper, it is possible to do arbitrage using the cyclical trend. An explicit strategy of doing arbitrage is given by Tommi Sottinen (2001). But the generalized solution is arbitrage-free and complete, indicating the path-wise solution may be a better choice for modeling $\{c_t\}$ when it is a long-range dependence process. Post-war aggregates are found to have long-range dependence characterizations in their cyclical components.

fBm is a long-range dependence (short-term dependence) process. A fBm has stationary increments and is $H-$self-similar: For all $a > 0$, $\{B^H_{at}\}$ and $\{a^H B^H_t\}$ have the same distribution.

There are many ways to estimate Hurst parameter H. We introduce a popular discrete variations method to estimate H. Our introduction follows Jean-Francois Coeurjolly (2001) closely. We start with the standard fBm $\{B^H_t\}_{t\in[0,1]}$, with $B^H_0 = 0$. Let $\{B^H(k)\}$ be a sample of size N at $k \in \{0, \frac{1}{N}, \cdots, \frac{N-1}{N}\}$. A filter a, of length $l+1$ and order $p \geq 1$, is an $l+1$-dimensional vector with real components a_j, $j = 0, 1, \cdots, l$, such that

$$\sum_{j=0}^{l} j^r a_j = 0 \qquad \sum_{j=0}^{l} j^p a_j \neq 0, \quad \text{for all } 0 \leq r < p.$$

To any such filter a, define a discrete time series $V^a(j)$, $j = 0, \cdots, 1 - \frac{1}{N}$, by

$$V^a(j) = \sum_{q=0}^{l} a_q B^H(j - \frac{q}{N}), \quad j = 0, 1, \cdots, 1 - \frac{1}{N}$$

where $B^H(i), i = 0, 1, \cdots, 1 - \frac{1}{N}$ is a sample path of B^H at $i = 0, 1, \cdots, 1 - \frac{1}{N}$ in discrete time.

Let π^a_H denote the covariance function of the series $\{V^a\}$. Then, for $i \in \mathbb{Z}$

$$\pi^a_H(i) = \mathbb{E}(V^a(\frac{i}{N})V^a(\frac{j+i}{N})) = -\frac{1}{2}\sum_{q,r=0}^{l} a_q a_r |q - r + i|^{2H}.$$

By the stationarity, for the second moment under a discrete variations method, we have

$$\mathbb{E}(S^a_N) = \frac{1}{N^{2H}}\pi^a_H(0)E_2$$

where

$$S^a_N = \frac{1}{N-l}\sum_{j=l}^{N-1}(V^a(\frac{j}{N}))^2$$

and $E_2 = 2\Gamma(\frac{5}{2})\Gamma(\frac{1}{2})$.

Let $g^a_N(t) = \frac{1}{N^{2t}}\pi^a_H(0)E_2$. Consider the estimator

$$\hat{H}^a_N = g^{-1}(S^a_N).$$

Then $\hat{H}^a_N \to H$ a.s. as $N \to \infty$ for any filter a with $p \geq 1$ (Proposition 2, Jean-Francois Coeurjolly 2001).

Because c_t is not a pure fBm, a scale coefficient σ matters. We need to define the data vector by $D(j) = \sigma B^H(\frac{j}{N})$ for $j = 0, 1, \cdots, N-1$. Define a new filter a^m by $a_j^m = a_i$ for $j = im$ and $a_j^m = 0$, otherwise. The second moment is given by

$$S_N^H(a^m) = \frac{1}{N - ml} \sum_{j=l}^{N-1} |V^{a^m}(\frac{j}{N})|^2$$

where $V^{a^m} = a^m * D$. Considering that $\mathbb{E}S_N^H(a^m) = m^{2H}\mathbb{E}(S_N^H(a))$, the logarithm of $\mathbb{E}(S_N^H(a^m))$ is linear in H. Then we can run a regression to obtain H (Jean-Francois Coeurjolly 2001).

During the process to decompose the cyclical component c_t into a cyclical trend and a fBm with $H = 0.5$, we have to estimate H many times. We choose to model c_t as a fBm process because it is easier to estimate H than d.

5.2 Recursive Applications of HP filter

In general, the cyclical component $\{c_t\}$ is a long-memory stochastic process with Hurst parameter $H > \frac{1}{2}$ for post-war aggregates, such as the GDP and the S&P 500 Index (i.e., the cyclical component is an $I(d)$ process with $d > 0$). We provide a simple application of the HP filter to decompose $\{c_t\}$ into a cyclical trend $\{\eta_t\}$ and a fBm with the Hurst parameter $H \simeq \frac{1}{2}$. The procedure is to apply the HP filter recursively to the cyclical part for a number of rounds, k, so that the cyclical part at k has Hurst parameter $H \simeq \frac{1}{2}$ with some small error bound close to zero. That is, we can run the HP filter against c_t to get $c = \tau(1) + c(1)$, $c(1) = \tau(2) + c(2)$, $\cdots$, $c(k-1) = \tau(k) + c(k)$, with some finite k (Figure 13). The procedure stops with $c(k)$ which has the Hurst parameter closest to $\frac{1}{2}$. At each step, we need to estimate Hurst parameter H, as given above. Then, we define the cyclical trend η of c by $\eta = c - c(k) = \sum_{q=1}^{k} \tau(q)$. Note that if this procedure is operated as $k \to \infty$, then $\eta = c$.

This recursive procedure extracts the low frequency signal of each cyclical component. The stochastic trend is the sum of all these low frequency signals that have contributed to the long-range dependence of the process c. Figures 14a and 14b show how this procedure has changed the periodogram of the cyclical component of the quarterly S&P 500 after four rounds. The cyclical trend η obtained by the recursive procedure cannot be found by choosing a single λ for the HP filter no matter what $\lambda > 0$ is set at.

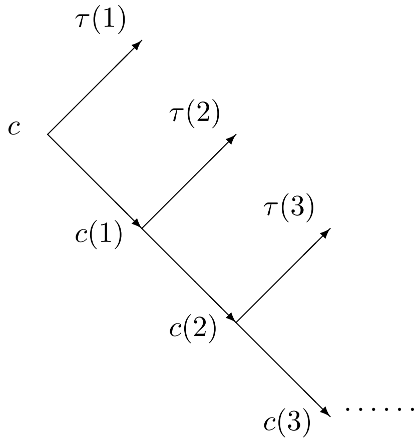
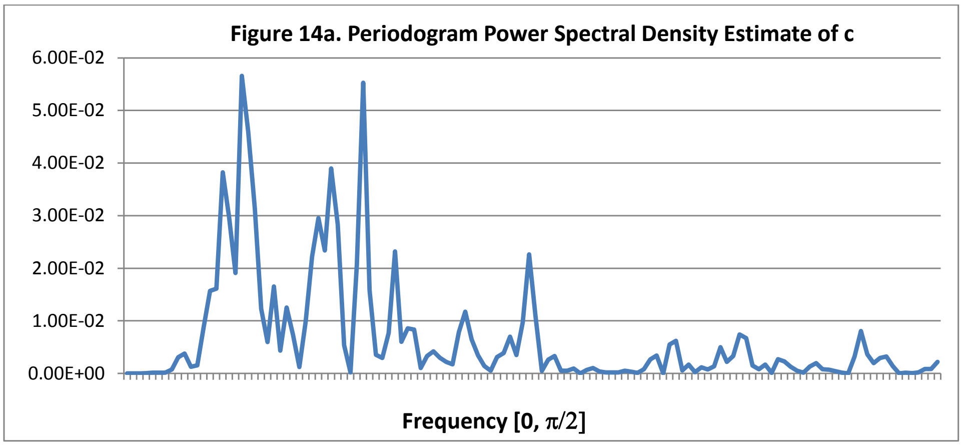
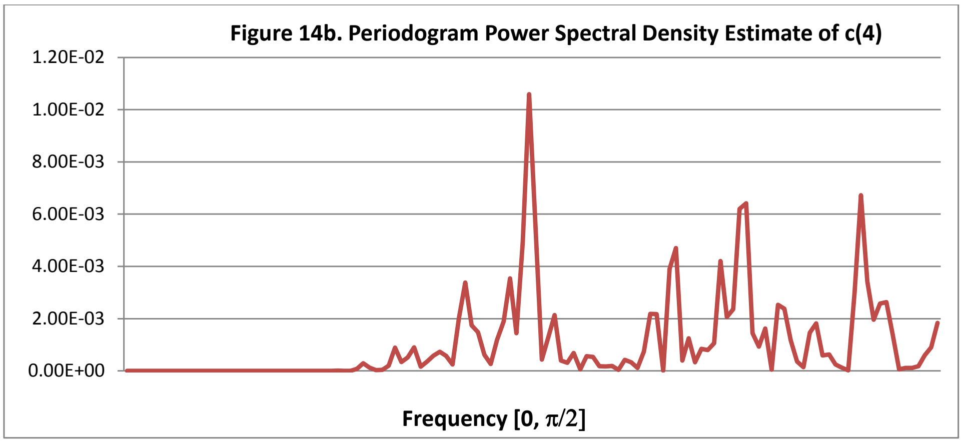


Figure 13. Recursive Applications of HP Filter





5.3 Three-part Decomposition

Let $[x_i, y_i]$ be a table of observations for $i = 0, 1, \cdots, N$, with $x_i = \frac{i}{N}$ and $h = \frac{1}{N}$. We want to find a curve $y = f(x)$ to fit these points in discrete time, which has been normalized to $[0, 1]$. The cubic spline interpolation is a piecewise continuous curve, passing through these points (Curtis Gerald and Patrick Wheatley 1994; Sky McKinley and Megan Levine 1998). The polynomial segments are denoted by $S(x)$:

$$S_i(x) = a_i(x-x_i)^3+b_i(x-x_i)^2+c_i(x-x_i)+d_i, \quad \text{for } x \in [x_i, x_{i+1}] \text{ and } i = 1, 2 \cdots, N-1.$$

These coefficients a to d satisfy

$$\begin{aligned} a_i &= \frac{M_{i+1} - M_i}{6h} && (5.1) \\ b_i &= = \frac{M_i}{2} && (5.2) \\ c_i &= \frac{y_{i+1} - y_i}{h} - h\frac{(2M_i + M_{i+1})}{6} && (5.3) \\ d_i &= y_i && (5.4) \end{aligned}$$

where M_i satisfies the following

$$\begin{pmatrix} 1 & 4 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 1 & 4 & 1 & \cdots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 4 & \cdots & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 4 & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 4 & 1 \end{pmatrix} \begin{bmatrix} M_1 \\ M_2 \\ M_3 \\ \vdots \\ M_{N-1} \\ M_N \end{bmatrix} = \frac{6}{h^2} \begin{bmatrix} y_3 - 2y_2 + y_1 \\ y_4 - 2y_3 + y_2 \\ \vdots \\ \vdots \\ y_{N-1} - 2y_{N-2} + y_{N-3} \\ y_N - 2y_{N-1} + y_{N-2} \end{bmatrix}$$

Our major observation is that the cubic spline interpolation fits our cyclical trend η very well, with errors about 10^{-14}. Thus, we have

$$y_{i+1} = a_i h^3 + b_i h^2 + c_i h + y_i, \quad i = 1, 2, \cdots, N-1.$$

Therefore, with the help of this spline interpolation, we get three-part decomposition of $(1 - B)y$:

$$(1 - B)y = \text{Component a} + \text{Component b} + \text{Component c}.$$

This result should not be a surprise because the HP filter itself follows a spline approach. What is surprising is that, across all cyclical trends investigated in this paper, we find Component c dominates the other two components. Moreover, Component b is a leading indicator, and Component a operates against Component c. Even though y may be lagging, $(1 - B)y$ is a leading indicator because Component c is. Notice that this three-part decomposition is not reported in the present paper for some indicators.