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Abstract We show the existence of involuntary unemployment without assuming wage rigidity. We derive involuntary unemployment by considering utility maximization of consumers and profit maximization of firms in an overlapping generations model under monopolistic competition with increasing or constant returns to scale technology and homothetic preferences of consumers. Indivisibility of labor supply may be a ground for the existence of involuntary unemployment. However, we show that there exists involuntary unemployment even when labor supply is divisible. The existence involuntary unemployment in our model is due to that we use an overlapping generations model of consumptions and labor supply. In a two-periods overlapping generations model it is possible that a reduction of the nominal wage rate reduces unemployment. However, if we consider a three-periods overlapping generations model including a childhood period, a reduction of the nominal wage rate does not necessarily reduce unemployment.

Keywords involuntary unemployment, monopolistic competition, divisible labor supply, three-periods overlapping generations model.

JEL Classification No.: E12, E24.

1 Introduction

According to Otaki (2009) the definition of involuntary unemployment consists of two elements.

1. The nominal wage rate is set above the nominal reservation wage rate.
2. The employment level and economic welfare never improve by lowering the nominal wage rate.

Umada (1997) derived an upward-sloping labor demand curve from mark-up principle for firms under increasing returns to scale technology, and argued that such an upward-sloping

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labor demand curve leads to the existence of involuntary unemployment without wage rigidity[1]. But his model of firms' behavior is ad-hoc. In this paper we consider utility maximization of consumers and profit maximization of firms in an overlapping generations model under monopolistic competition according to Otaki (2007), Otaki (2009), Otaki (2011) and Otaki (2015) with increasing or constant returns to scale technology and homothetic preferences of consumers, and show the existence of involuntary unemployment without assuming wage rigidity. In some other papers we have shown the existence of involuntary unemployment under perfect or monopolistic competition when labor supplies by individuals are indivisible.

Indivisibility of labor supply means that labor supply of each individual can be 1 or 0. On the other hand, if labor supply is divisible, it is a variable in $[0,1]$. As discussed by Otaki (2015) (Theorem 2.3) and Otaki (2012), if labor supply is infinitely divisible, there exists no unemployment. However, if labor supply by each individual is not so small, there may exist involuntary unemployment even when labor supply is divisible. In this paper the first element of Otaki's two elements of involuntary unemployment should be

Labor supply of each individual is positive at the current real wage rate.

In the next section we analyze consumers' utility maximization in an overlapping generations model with two periods. We consider labor supplies by individuals as well as their consumptions. In Section 3 we consider profit maximization of firms under monopolistic competition. In Section 4 we show the existence of involuntary unemployment when labor supply is divisible. The main discussions are as follows.

1. The aggregate demand, the aggregate supply, the total employment (total labor demand which is necessary to produce the total supply) and the price of the good are determined by the values of the government expenditure and consumptions of the older generation consumers according to (18) and (28) given the nominal wage rate. Then, we get the real wage rate.
2. Labor supply of each individual is determined by the total employment according to (19), and the employment (number of employment) is determined. It may be smaller than the population of labor, then there exists involuntary unemployment. There exists no mechanism to reduce involuntary unemployment unless the real values of the government expenditure and consumptions of the older generation consumers are increased.

If individuals consume almost all income in his younger period and his saving is very small, the multiplier is very large. Then, unless the government expenditure is not so small, full-employment is always realized. Therefore, the reason for the existence of involuntary unemployment in our model is that we use an overlapping generations model for consumers.

In a two-periods overlapping generations model it is possible that a reduction of the nominal wage rate reduces unemployment. However, if we consider a three-periods overlapping generations model including a childhood period, a reduction of the nominal wage rate does not necessarily reduce unemployment. Please see Section 7.

2 Consumers

We consider a two-period (young and old) overlapping generations model under monopolistic competition according to Otaki (2007, 2009, 2011 and 2015). There is one factor of

[1] Lavoie (2001) presented a similar analysis.

production, labor, and there is a continuum of goods indexed by $z \in [0,1]$. Each good is monopolistically produced by Firm z. Consumers are born at continuous density $[0,1]\times[0,1]$ in each period. They supply l units of labor when they are young (the first period), $0 \le l \le 1$.

We use the following notations.

$c^i(z)$: consumption of good z at period i, $i = 1,2$.
$p^i(z)$: the price of good z at period i, $i = 1,2$.
$X^i = \left\{\int_0^1 c^i(z)^{1-\frac{1}{\eta}} dz\right\}^{\frac{1}{1-\frac{1}{\eta}}}$, $i = 1,2$, $\eta > 1$.
W: nominal wage rate.
Π: profits of firms which are equally distributed to each consumer.
l: labor supply of an individual, $0 \le l \le 1$.
L: employment of each firm and the total employment.
L_f: population of labor or employment at the full-employment state. We implicitly assume $L_f = 1$.
$y(Ll)$: labor productivity, which is increasing or constant with respect to "employment $\times$ labor supply (Ll)", $y(Ll) \ge 1$, $y' \ge 0$.

We define the elasticity of the labor productivity with respect to Ll as follows.

$$\zeta = \frac{y'}{\frac{y(Ll)}{Ll}}.$$

We assume that $0 \le \zeta < 1$ and it is constant. Increasing returns to scale means $\zeta > 0$.

η is (the inverse of) the degree of differentiation of the goods. At the limit when $\eta \to +\infty$, the goods are homogeneous. We assume

$$\left(1 - \frac{1}{\eta}\right)(1+\zeta) < 1$$

so that the profits of firms are positive.

We assume that the utility function a consumer about consumption is homothetic. This means that his utility function about consumption is a strictly monotonic transformation of a function which is homogeneous of degree one. The utility of consumers of one generation over two periods is

$$U(X^1, X^2, l) = F(u(X^1, X^2)) - G(l).$$

F is a strictly increasing and differentiable function, thus $F' > 0$. $u(X^1, X^2)$ is homogeneous of degree one. $G(l)$ is disutility of labor. It is continuous, strictly increasing, differentiable and strictly convex, thus $G' > 0$ and $G'' > 0$. Utility of consumption and disutility of labor are additively separable.

The budget constraint for an employed individual is

$$\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz = Wl + \Pi.$$

$p^2(z)$ is the expectation of the price of good z at period 2. The Lagrange function is

$$\mathscr{L} = F(u(X^1, X^2)) - G(l) - \lambda\left(\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz - Wl - \Pi\right).$$

λ is the Lagrange multiplier. The first order conditions are

$$F'\frac{\partial u}{\partial X^1}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^1(z)^{-\frac{1}{\eta}} = \lambda p^1(z), \tag{1}$$

and

$$F'\frac{\partial u}{\partial X^2}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^2(z)^{-\frac{1}{\eta}} = \lambda p^2(z). \tag{2}$$

They are rewritten as

$$F'\frac{\partial u}{\partial X^1}X^1\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1} c^1(z)^{1-\frac{1}{\eta}} = \lambda p^1(z)c^1(z), \tag{3}$$

$$F'\frac{\partial u}{\partial X^2}X^2\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1} c^2(z)^{1-\frac{1}{\eta}} = \lambda p^2(z)c^2(z). \tag{4}$$

Let

$$P^1 = \left(\int_0^1 p^1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}},\ P^2 = \left(\int_0^1 p^2(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}.$$

They are price indices. By some calculations we obtain (please see Appendix)

$$u(X^1,X^2) = \left(\frac{\lambda}{F'}\right)\left[\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz\right] = \left(\frac{\lambda}{F'}\right)(Wl+\Pi), \tag{5}$$

$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}}, \tag{6}$$

$$P^1X^1 + P^2X^2 = Wl + \Pi. \tag{7}$$

The indirect utility of consumers is written as follows

$$V = F\left(\frac{Wl+\Pi}{\varphi(P^1,P^2)}\right) - G(l). \tag{8}$$

$\varphi(P^1,P^2)$ is a function of P^1 and P^2. It is positive, increasing in P^1 and P^2, and homogeneous of degree one. Maximization of V with respect to l implies

$$F'W = \varphi(P^1,P^2)G'(l). \tag{9}$$

Let $\rho = \frac{P^2}{P^1}$. From (9)

$$F'\omega = F'\frac{W}{P^1} = \varphi(1,\rho)G'(l). \tag{10}$$

ω is the real wage rate. F' is a function of $\frac{Wl+\Pi}{\varphi(P^1,P^2)}$ such that

$$F' = F'\left(\frac{Wl+\Pi}{\varphi(P^1,P^2)}\right) = F'\left(\frac{\omega l+\pi}{\varphi(1,\rho)}\right),$$

where

$$\pi = \frac{\Pi}{P^1}.$$

If the value of ρ is given, l is obtained from (10) as a function of ω.

From (10)

$$\frac{dl}{d\omega}=\frac{F'+F''\frac{\omega l}{\varphi(1,\rho)}}{\varphi(1,\rho)G''-F''\frac{\omega^2}{\varphi(1,\rho)}}. \tag{11}$$

We assume

$$\varphi(1,\rho)G''-F''\frac{\omega^2}{\varphi(1,\rho)}>0, \tag{12}$$

and

$$F'+F''\frac{\omega l}{\varphi(1,\rho)}>0. \tag{13}$$

Then, $\frac{dl}{d\omega}>0$, and labor supply l is increasing in the real wage rate ω. If $F(u(X^1,X^2))$ is homogeneous of degree one, $F'=1$ and $F''=0$.

For an unemployed individual the income is only Π. Thus, his indirect utility is

$$F\left(\frac{\Pi}{\varphi(P^1,P^2)}\right).$$

Log-linear utility function

Tanaka (2013) pointed out that if one assumes log-linear utility function of consumptions in the model by Masayuki Otaki such as Otaki (2007), there exists no appropriate equilibrium solution. Although in Otaki's model it is assumed that labor supply is indivisible and the nominal wage rate is equal to the reservation nominal wage rate[2], we do not consider such a situation, and thus we can analyze involuntary unemployment in a case of log-linear utility function.

We assume the following utility function of consumers.

$$U(X^1,X^2,l)=\alpha\ln X^1+(1-\alpha)\ln X^2-G(l).$$

The meanings of X^1, X^2, $G(l)$, and the budget constraint for an employed individual are the same as those in the above case. The first order conditions are

$$\alpha\frac{1}{X^1}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}}c^1(z)^{-\frac{1}{\eta}}=\lambda p^1(z),$$

$$(1-\alpha)\frac{1}{X^2}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}}c^2(z)^{-\frac{1}{\eta}}=\lambda p^2(z).$$

They mean

$$\alpha\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\lambda\int_0^1 p^1(z)c^1(z)dz=\alpha,$$

$$(1-\alpha)\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz=\lambda\int_0^1 p^2(z)c^2(z)dz=1-\alpha,$$

[2] If the nominal wage rate is equal to the reservation nominal wage rate, employment and unemployment are indifferent for consumers, and there does not exist involuntary unemployment.

$$\left(\frac{\alpha}{X^1}\right)^{1-\eta}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\lambda^{1-\eta}\int_0^1 p^1(z)^{1-\eta}dz,$$

and

$$\left(\frac{1-\alpha}{X^2}\right)^{1-\eta}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz=\lambda^{1-\eta}\int_0^1 p^2(z)^{1-\eta}dz.$$

From the third and the fourth equations

$$\frac{\alpha}{X^1}=\lambda\left(\int_0^1 p^1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}=\lambda P^1,$$

and

$$\frac{1-\alpha}{X^2}=\lambda\left(\int_0^1 p^2(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}=\lambda P^2.$$

Therefore, we obtain

$$P^1X^1+P^2X^2=\frac{1}{\lambda}=Wl+\Pi,$$

$$X^1=\frac{\alpha(Wl+\Pi)}{P^1},X^2=\frac{(1-\alpha)(Wl+\Pi)}{P^2},$$

and the indirect utility function is

$$\begin{aligned}V=&\alpha\ln\frac{\alpha(Wl+\Pi)}{P^1}+(1-\alpha)\ln\frac{(1-\alpha)(Wl+\Pi)}{P^2}-G(l)\\=&\ln(Wl+\Pi)+\ln\frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{(P^1)^\alpha(P^2)^{1-\alpha}}-G(l).\end{aligned}$$

The condition for maximization of V with respect to l is

$$W=(Wl+\Pi)G'(l).$$

Then, we have

$$\omega=(\omega l+\pi)G'(l). \tag{14}$$

From this we obtain the labor supply as a function of ω. (14) means

$$\frac{dl}{d\omega}=\frac{1-G'(l)l}{(\omega l+\pi)G''+\omega G'(l)}. \tag{15}$$

Since $G'>0$, $G''>0$, if

$$1-G'(l)l>0$$

l is an increasing function of ω.

Because log-linear utility function is also homothetic, let us consider the relation between (10) and (14). If the utility function is log-linear, F' is

$$F'=\frac{1}{(X^1)^\alpha(X^2)^{1-\alpha}}=\frac{(P^1)^\alpha(P^2)^{1-\alpha}}{\alpha^\alpha(1-\alpha)^{1-\alpha}(Wl+\Pi)}=\frac{\rho^{1-\alpha}}{\alpha^\alpha(1-\alpha)^{1-\alpha}(\omega l+\pi)}.$$

Since

$$\varphi(P^1,P^2)=\frac{(P^1)^\alpha(P^2)^{1-\alpha}}{\alpha^\alpha(1-\alpha)^{1-\alpha}},$$

$$\varphi(1,\rho)=\frac{\rho^{1-\alpha}}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}}=F'(\omega l+\pi). \tag{16}$$

By (10)

$$F'\omega=F'(\omega l+\pi)G'(l)$$

Thus, we obtain (14).

Consider the relation between (11) and (15). For the log-linear utility function F'' is

$$-(F')^2.$$

The numerator of (11) is

$$F'+F''\frac{\omega l}{\varphi(1,\rho)}=F'\left(1-F'\frac{\omega l}{\varphi(1,\rho)}\right).$$

By (10) this is equal to

$$F'\left(1-G'(l)l\right).$$

On the other hand, from (16) the denominator of (11) is rewritten as

$$\varphi(1,\rho)G''-F''\frac{\omega^2}{\varphi(1,\rho)}=F'(\omega l+\pi)G''+(F')^2\frac{\omega^2}{\varphi(1,\rho)}=F'\left[(\omega l+\pi)G''+F'\frac{\omega^2}{\varphi(1,\rho)}\right].$$

By (10) this is equal to

$$F'\left[(\omega l+\pi)G''+\omega G'(l)\right].$$

Therefore, (11) and (15) are equivalent.

3 Firms

Consider an employed individual. Let

$$\alpha=\frac{P^1X^1}{P^1X^1+P^2X^2}=\frac{X^1}{X^1+\rho X^2},\ 0<\alpha<1.$$

From (3) $\sim$ (7),

$$\alpha(Wl+\Pi)\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c^1(z)^{-\frac{1}{\eta}}=p^1(z).$$

Since

$$X^1=\frac{\alpha(Wl+\Pi)}{P^1},$$

we have

$$(X^1)^{\frac{1}{\eta}-1}=\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}=\left(\frac{\alpha(Wl+\Pi)}{P^1}\right)^{\frac{1}{\eta}-1}.$$

Therefore,

$$\alpha(Wl+\Pi)\left(\frac{\alpha(Wl+\Pi)}{P^1}\right)^{\frac{1}{\eta}-1}c^1(z)^{-\frac{1}{\eta}}=\left(\frac{\alpha(Wl+\Pi)}{P^1}\right)^{\frac{1}{\eta}}P^1c^1(z)^{-\frac{1}{\eta}}=p^1(z).$$

Thus,

$$c^1(z)^{\frac{1}{\eta}} = \left(\frac{\alpha(Wl+\Pi)}{P^1}\right)^{\frac{1}{\eta}} P^1 \left(p^1(z)\right)^{-1}.$$

Hence,

$$c^1(z) = \frac{\alpha(Wl+\Pi)}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

This is demand for good z of an individual of younger generation. Similarly, his demand for good z in the second period is

$$c^2(z) = \frac{(1-\alpha)(Wl+\Pi)}{P^2}\left(\frac{p^2(z)}{P^2}\right)^{-\eta}.$$

Let $\bar{c}_2(z)$, $\bar{l}$, be demand for good z and labor supply of an older generation consumer, $\bar{W}$ and $\bar{\Pi}$ be the nominal wage rate and the profit in his first period. Then

$$\bar{c}^2(z) = \frac{(1-\alpha)(\bar{W}\bar{l}+\bar{\Pi})}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

$(1-\alpha)(\bar{W}\bar{l}+\bar{\Pi})$ is his saving carried over from his first period. Let M be the saving. Then, his demand for good z is

$$\frac{M}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

The government expenditure constitutes the national income as well as consumptions of younger and older generations. For an unemployed individual we replace $Wl+\Pi$ by Π. Then, the total demand for good z is written as

$$c(z) = \frac{Y}{P^1}\left(\frac{p^1(z)}{P^1}\right)^{-\eta}.$$

Y is the effective demand defined by

$$Y = \alpha(WLl + L_f\Pi) + G + M.$$

G is the government expenditure (about this demand function please see Otaki (2007), Otaki (2009)). The total employment, the total profits, the total government expenditure and the total consumption of the older generation are

$$\int_0^1 Ldz = L,\ \int_0^1 \Pi dz = \Pi,\ \int_0^1 Gdz = G,\ \int_0^1 Mdz = M.$$

We have

$$\frac{\partial c(z)}{\partial p^1(z)} = -\eta\frac{Y}{P^1}\frac{p^1(z)^{-1-\eta}}{(P^1)^{-\eta}} = -\eta\frac{c(z)}{p^1(z)}.$$

From $c(z) = Lly(Ll)$,

$$\frac{\partial(Ll)}{\partial p^1(z)} = \frac{1}{y(Ll)+Lly'}\frac{\partial c(z)}{\partial p^1(z)}.$$

The profit of Firm z is

$$\pi(z) = p^1(z)c(z) - \frac{W}{y(Ll)}c(z).$$

P^1 is given for Firm z. $y(Ll)$ is the productivity of labor, which is increasing with respect to Ll.

The elasticity of the labor productivity with respect to Ll is

$$\zeta = \frac{y'}{\frac{y(Ll)}{Ll}}.$$

The condition for profit maximization with respect to $p^1(z)$ is

$$\begin{aligned}
&c(z) + \left[p^1(z) - \frac{y(Ll) - c(z)y'\frac{1}{y(Ll)+Lly'}}{y(Ll)^2}W\right]\frac{\partial c(z)}{\partial p^1(z)}\\
=&c(z) + \left[p^1(z) - \frac{1 - Lly'\frac{1}{y(Ll)+Lly'}}{y(Ll)}W\right]\frac{\partial c(z)}{\partial p^1(z)}\\
=&c(z) + \left[p^1(z) - \frac{W}{y(Ll)+Lly'}\right]\frac{\partial c(z)}{\partial p^1(z)} = 0.
\end{aligned}$$

From this

$$p^1(z) = \frac{W}{y(Ll)+Lly'} - \frac{c(z)}{\frac{\partial c(z)}{\partial p^1(z)}} = \frac{W}{(1+\zeta)y(Ll)} + \frac{1}{\eta}p^1(z).$$

Therefore, we obtain

$$p^1(z) = \frac{W}{\left(1 - \frac{1}{\eta}\right)(1+\zeta)y(Ll)}.$$

With increasing returns to scale, since $\zeta > 0$, $p^1(z)$ is lower than that in a case of constant returns to scale given the value of W.

4 Involuntary unemployment

Since the model is symmetric, the prices of all goods are equal. Then,

$$P^1 = p^1(z).$$

Hence

$$P^1 = \frac{W}{\left(1 - \frac{1}{\eta}\right)(1+\zeta)y(Ll)}. \tag{17}$$

The real wage rate is

$$\omega = \frac{W}{P^1} = \left(1 - \frac{1}{\eta}\right)(1+\zeta)y(Ll). \tag{18}$$

It is determined by firms' behavior. Under increasing (constant) returns to scale, since ζ is constant, ω is increasing (constant) with respect to Ll.

From (10) and (18) we get

$$F'\left(1 - \frac{1}{\eta}\right)(1+\zeta)y(Ll) = \varphi(1,\rho)G'(l). \tag{19}$$

If the utility function of consumers is log-linear, from (14) and (18), we obtain

$$\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)=(\omega l+\pi)G'(l). \tag{20}$$

By (16)

$$\varphi(1,\rho)=F'(\omega l+\pi).$$

Thus, (20) and (19) are equivalent. From (18) F' is written as

$$F'=F'\left(\frac{\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)l+\pi}{\varphi(1,\rho)}\right).$$

From (19) labor supply of an individual is obtained as a function of L. Denote it by $l(L)$. We assume

$$\begin{aligned}\Theta =&\varphi(1,\rho)\Gamma''(l)-F'\left(1-\frac{1}{\eta}\right)(1+\zeta)y'L \\ &-F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)}{\varphi(1,\rho)} \\ &-F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{\left(1-\frac{1}{\eta}\right)(1+\zeta)y'Ll}{\varphi(1,\rho)}>0,\end{aligned} \tag{21}$$

$$F'+F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{l(L)}{\varphi(1,\rho)}>0, \tag{22}$$

and

$$\varphi(1,\rho)\Gamma''(l)-F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)}{\varphi(1,\rho)}>0. \tag{23}$$

(21), (22) and (23) guarantee that $l(L)$ is increasing and $Ll(L)$ is strictly increasing with respect to L because

$$\frac{dl(L)}{dL}=\frac{\left(1-\frac{1}{\eta}\right)(1+\zeta)y'l(L)\left[F'+F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{l(L)}{\varphi(1,\rho)}\right]}{\Theta}\geq 0,$$

$$\begin{aligned}\frac{d(Ll(L))}{dL}&=l(L)+L\frac{dl(L)}{dL} \\ &=\frac{\left[\varphi(1,\rho)\Gamma''(l)-F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)}{\varphi(1,\rho)}\right]l(L)}{\Theta}>0.\end{aligned}$$

By (18) we find that (22) and (23) are the same conditions as, respectively, (13) and (12).

The real wage rate ω is increasing in L because $y'\geq 0$.

Alternatively, from (19) l is obtained as a function of Ll. Denote it by $l(Ll)$. Then,

$$\frac{dl(Ll)}{d(Ll)}=\frac{\left(1-\frac{1}{\eta}\right)(1+\zeta)y'\left[F'+F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{l(L)}{\varphi(1,\rho)}\right]}{\varphi(1,\rho)G''-F''\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll)\frac{(1-\frac{1}{\eta})(1+\zeta)y(Ll)}{\varphi(1,\rho)}}\geq 0.$$

The aggregate supply of the good is equal to

$$(WLl + L_f\Pi) = P^1 Lly(Ll).$$

Ll is an abbreviation of $Ll(L)$ or $Ll(Ll)$. The aggregate demand is

$$\alpha(WLl + L_f\Pi) + G + M = \alpha P^1 Lly(Ll) + G + M.$$

Since they are equal,

$$P^1 Lly(Ll) = \alpha P^1 Lly(Ll) + G + M, \tag{24}$$

or[3]

$$P^1 Ll = \frac{G+M}{(1-\alpha)y(Ll)}, \text{ or } P^1 Lly(Ll) = \frac{G+M}{1-\alpha}. \tag{25}$$

In real terms

$$Lly(Ll) = \frac{1}{1-\alpha}(g+m), \tag{26}$$

or

$$Ll = \frac{1}{(1-\alpha)y(Ll)}(g+m), \tag{27}$$

where

$$g = \frac{G}{P^1},\ m = \frac{M}{P^1}.$$

By (18) and (25) we get

$$Ll = \frac{(1-\frac{1}{\eta})(1+\zeta)(G+M)}{(1-\alpha)W}. \tag{28}$$

From (19) we obtain the value of $l(Ll)$, and the value of L is determined by $L = \frac{Ll}{l(Ll)}$. L can not be larger than L_f. However, it may be strictly smaller than L_f. Then, there exists *involuntary umemployment* due to demand deficiency. Then, $Ll < L_f l(L_f)$ because Ll is strictly increasing in L. The relation between L and Ll is obtained as follows.

$$\frac{dL}{d(Ll)} = \frac{\varphi(1,\rho)\Gamma''(l) - F'(1+\zeta)y'L - F''(1+\zeta)y(Ll)\frac{(1+\zeta)y(Ll)}{\varphi(1,\rho)} - F''(1+\zeta)y(Ll)\frac{(1+\zeta)y'Ll}{\varphi(1,\rho)}}{\left[\varphi(1,\rho)\Gamma''(l) - F''(1+\zeta)y(Ll)\frac{(1+\zeta)y(Ll)}{\varphi(1,\rho)}\right]l(L)} > 0.$$

If we consider the following budget constraint for the government with a lump-sum tax T on the younger generation consumers,

$$G = T,$$

the aggregate demand and the aggregate supply are

$$\alpha[(WLl + L_f\Pi) - G] + G + M = \alpha(P^1 Lly(Ll) - G) + G + M = P^1 Lly(Ll).$$

Then, we get[4]

$$Ll = \frac{1}{(1-\alpha)y(Ll)}[(1-\alpha)g + m],$$

3 $\frac{1}{1-\alpha}$ is a multiplier.

4 This equation means that the balanced budget multiplier is 1.

If labor supply of each individual is small, there exists no unemployment. If it is not so small, however, it is likely that there exists involuntary unemployment without sufficiently large value of $g+m$.

If

$$F'\left(1-\frac{1}{\eta}\right)(1+\zeta)y(Ll) > \varphi(1,\rho)G'(l) \text{ for any } 0<l<1, \text{given } L,$$

individuals choose $l=1$, and then the labor supply is indivisible.

On the other hand, if

$$F'\left(1-\frac{1}{\eta}\right)(1+\zeta)\lim_{Ll\to 0} y(Ll) < \varphi(1,\rho)G'(0),$$

individuals choose $l=0$. However, if $G'(0)$ is sufficiently small, $l>0$.

Summary of discussions

1. The "employment $\times$ labor supply (Ll)" and the price P^1 are determined by the value of $G+M$ according to (18) and (28) given the nominal wage rate W, and then the real wage rate ω is determined. $Lly(Ll)$ is the aggregate supply of the goods which is equal to the aggregate demand, and Ll is labor demand which is necessary to produce the aggregate supply.
2. Labor supply of each individual is determined by Ll according to (19).
3. The employment L is determined by

   $$L=\frac{Ll}{l(Ll)}.$$

   The employment may be smaller than the population of labor, then there exists involuntary unemployment.
4. The real wage rate is determined by Ll according to (18).

There exists no mechanism to reduce involuntary unemployment unless $g+m$ is increased.

Steady state

At the steady state where $\rho=1$. If $g+m$ is constant, the employment is constant. Let T be the tax revenue which is not necessarily equal to G. Then, we have

$$\alpha(P^1Lly(Ll)-T)+G+M=P^1Lly(Ll).$$

The savings of the consumers of the younger generation is

$$(1-\alpha)(P^1Lly(Ll)-T)=G-T+M.$$

Since at the steady state this is equal to M, which is the consumption of the older generation, we need $G=T$. Denote the initial values of L, G and M by L^0, G^0, M^0. Then, we get

$$L^0l=\frac{M^0}{(1-\alpha)P^1y(Ll)}+\frac{G^0}{P^1y(Ll)}.$$

Comment on the nominal wage rate

The reduction of the nominal wage rate induces a proportionate reduction of the price even when there exists involuntary unemployment (please see 17), and it does not rescue involuntary unemployment. Proposition 2.1 in Otaki (2016) says

> Suppose that the nominal wage sags. Then, as far as its indirect effects on the aggregate demand are negligible, this only results in causing a proportionate reduction of the price level. In other words, the reduction of the nominal wage never rescues workers who are involuntarily unemployed.

There may exist indirect effects on the aggregate demand of a reduction of the nominal wage rate. If the price of the good falls, the real value of consumptions of the older generation may increase and unemployment may be reduced. However, if we consider a three-periods overlapping generations model including a childhood period, a reduction of the nominal wage rate does not necessarily reduce unemployment. Please see Section 7.

In our model no mechanism determines the nominal wage rate. When the nominal value of $G+M$ increases, the nominal aggregate demand and supply increase. If the nominal wage rate rises, the price also rises. If the rate of an increase in the nominal wage rate is smaller than the rate of an increase in $G+M$, the real aggregate supply and the employment increase. Partition of the effects by an increase in $G+M$ into a rise in the nominal wage rate (and the price) and an increase in the employment may be determined by bargaining between labor and firm[5].

System of equations and variables

If we consider that (10), (18), (28) constitute a system of equations. The variables are ω, l and Ll; or ω, l and L. We can consider that (10) and (28) constitute a system of equations with l and Ll (or l and L) as variables. The parameters are G, M and W. M is determined by the choice of labor supplies and consumptions if the older generation consumers. The solution of Ll is not necessarily equal to $L_f l(L_f)$.

Full-employment case

If $L = L_f$, full-employment is realized. Then, (25) is written as

$$L_f l(L_f) y(L_f l(L_f)) = \frac{1}{1-\alpha}(g+m). \tag{29}$$

$l(L_f)$ is obtained from

$$(1+\zeta)y(L_f l) = \varphi(1,\rho)G'(l).$$

$L_f l(L_f) > Ll(L)$ for any $L < L_f$ because $Ll(L)$ is strictly increasing in L. Since $L_f l(L_f)$ is constant, (29) is an identity not an equation. On the other hand, (26) is an equation not an identity. (29) should be written as

$$L_f l(L_f) y(L_f l(L_f)) \equiv \frac{1}{1-\alpha}(g+m). \tag{30}$$

[5] Otaki (2009) has shown the existence of involuntary unemployment using efficient wage bargaining according to McDonald and Solow (1981). The arguments of this paper, however, do not depend on bargaining.

This defines the value of $g+m$ which realizes the full-employment state.

From (30) we have

$$P^1 = \frac{1}{(1-\alpha)L_f l(L_f)y(L_f l(L_f))}(G+M),$$

where

$$g = \frac{G}{P^1},\ m = \frac{M}{P^1}.$$

Therefore, the price level P^1 is determined by $G+M$, which is the sum of nominal values of government expenditure and consumption of the older generation. Also the nominal wage rate is determined by

$$W = \left(1-\frac{1}{\eta}\right)(1+\zeta)y(L_f l(L_f))P^1.$$

A case where α is very large

If α is very large and close to 1, the savings of consumers and the value of M are very small. On the other hand, the multiplier $\left(\frac{1}{1-\alpha}\right)$ is very large. Then, unless the government expenditure G is not so small, we can consider that full-employment is always realized. Therefore, the reason for the existence of involuntary unemployment in our model is that we use an overlapping generations model for consumers.

The relation between our model and a textbook macroeconomic model

Let Y be national income, $C=\alpha Y$ be a consumption function (we abbreviate a constant), I be investment and G be government expenditure. A textbook macroeconomic model is written as

$$Y = \alpha Y + I + G.$$

This yields

$$Y = \frac{I+G}{1-\alpha}.$$

We obtain a multiplier $\frac{1}{1-\alpha}$. In our model we have no capital, thus no investment. Instead we have consumption by the older generation consumers. Replacing I by M and Y by P^1Lly, we get (25).

5 Graphical representation

In Fig. 1 we present a graphical representation under increasing returns to scale. Line I in the first quadrant expresses the relation between Ll and ω in (18), Line II in the second quadrant expresses labor supply obtained from (10), and Line III in the fourth quadrant represents the relation between Ll and $l(Ll)$.

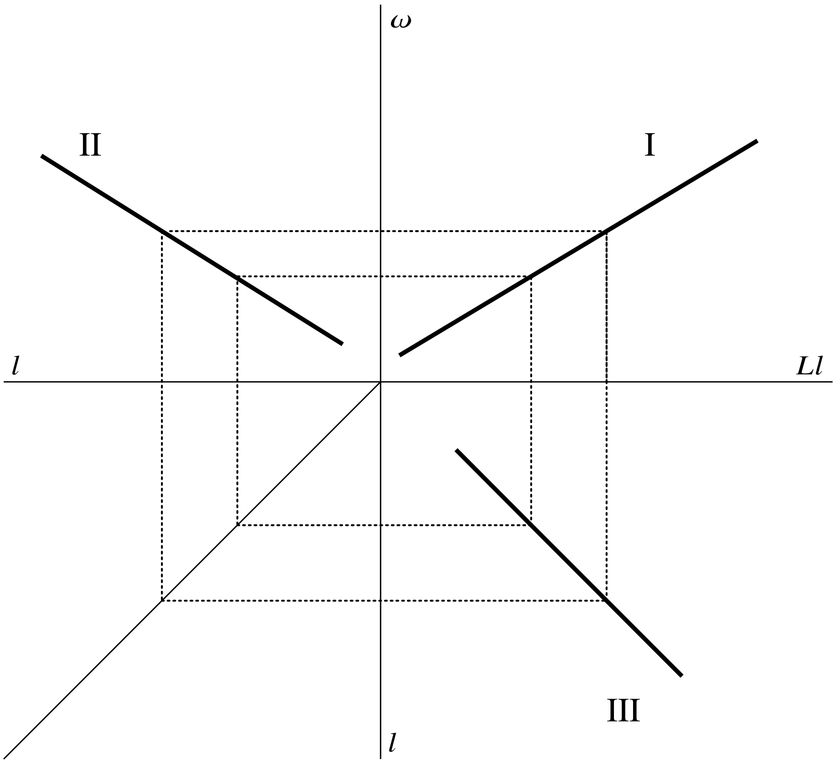

Fig. 1 Graphical representation

6 Analysis by a static model

According to Tanaka (2013) we present an analysis by a static model in which the utility function of a consumer depends on his consumption and saving. We consider the following utility function of a consumer.

$$U(X^1, m, l) = F(u(X^1, m)) - \Gamma(l),\ X^1 = \left\{ \int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right\}^{\frac{1}{1-\frac{1}{\eta}}}.$$

m is the real value of his saving. u is homogeneous of degree one. The budget constraint is

$$\int_0^1 p^1(z)c^1(z)dz + P^1 m = Wl + \Pi.$$

Let λ be the Lagrange multiplier. The conditions for utility maximization are

$$F' \frac{\partial u}{\partial X^1} \left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}} dz \right)^{\frac{\frac{1}{\eta}}{1-\frac{1}{\eta}}} c^1(z)^{-\frac{1}{\eta}} = \lambda p^1(z),$$

and

$$F' \frac{\partial u}{\partial m} = \lambda P^1.$$

They, with (34), (35) and (36) in the appendix, mean

$$F'\frac{\partial u}{\partial X^1}X^1=\lambda P^1X^1,$$

and

$$F'\frac{\partial u}{\partial m}m=\lambda P^1m$$

Since u is homogeneous of degree one,

$$u(X^1,m)=\frac{\lambda}{F'}P^1(X^1+m)=\frac{\lambda}{F'}(Wl+\Pi).$$

Therefore, we obtain the following indirect utility function.

$$V=F\left(\frac{Wl+\Pi}{\psi(P^1)}\right)-\Gamma(l).$$

$\psi(P^1)$ is a function of P^1. It is homogeneous of degree one.

The condition for maximization of V with respect to l is

$$F'W=\psi(P^1)\Gamma'(l).$$

From this we get

$$F'\omega=\psi(1)\Gamma'(l).$$

Thus, l is obtained as a function of ω. This equation is the same as (10) replacing $\varphi(1,\rho)$ by $\psi(1)$. If we assume

$$\alpha=\frac{X^1}{X^1+m},\ 0<\alpha<1,$$

and

$$Y=\alpha(WLl+L_f\Pi)+G,$$

we can proceed analyses of firms' behavior and involuntary unemployment. The difference between this static model and the overlapping generations model is that in the static model the savings of consumers do not yield consumptions of the older generation consumers.

7 Three-periods overlapping generations model

We add a childhood period (period 0) to a overlapping generations model with two periods, younger period (period 1, working period) and older period (period 2, retired period). In a childhood period people consume the good by borrowing money from their parents generation (the younger generation) and repay the debt in the next period. The savings of the younger generation may be insufficient for the consumption of the childhood generation. Thus, we assume that childhood generation consumers can borrow scholarship from the government. They must repay the scholarship in their period 1 (when they belong to the younger generation). Therefore, in period 1 the consumers of the younger generation have to save money for their consumptions in period 2 (when they belong to the older generation) and repay their debt and scholarship. Since the consumers make their consumption plans at the beginning of period 1 (working period), their consumptions in the childhood period are constant.

We consider the following utility function of a consumer who is employed

$$F(u(X^1, X^2, D)) - G(l),$$

where

$$D = \left\{ \int_0^1 \hat{c}(z)^{1-\frac{1}{\eta}} dz \right\}^{\frac{1}{1-\frac{1}{\eta}}}.$$

$\hat{c}(z)$ is consumption of good z in the childhood period. It is constant. Thus, D is constant.

If a consumer is not employed in his period 1, he can not repay his debt. Therefore, we assume that unemployed consumers receive unemployment benefits from the government. They are covered by taxes on employed consumers of the younger generation. Let R be the unemployment benefit, Θ be the tax for the unemployment benefit. Then, the budget constraint for an employed consumer is

$$\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz = Wl - D - \Theta + \Pi.$$

$D + \Theta$ is the sum his own debt repayment and the tax for repayment of the debt of unemployed consumers. Since Θ satisfies

$$D(L_f - L) = L\Theta,$$

we have

$$D + \Theta = \frac{L_f D}{L}.$$

The value of the right-hand side of this equation is given for an employed consumer. The budget constraint of an unemployed consumer is

$$\int_0^1 p^1(z)c^1(z)dz + \int_0^1 p^2(z)c^2(z)dz = R - D + \Pi = \Pi.$$

R is not used for consumption of an unemployed consumer in period 1. If the government aids consumptions of unemployed consumers, it is another policy.

Analyses of consumptions in the younger generation and the older generation are similar to those in the previous case (two-periods overlapping generations model). Let

$$\alpha = \frac{P^1X^1}{P^1X^1 + P^2X^2}.$$

Denote the savings of the older generation by M. Then, the effective demand is

$$Y = \alpha[(Wl - D - \Theta)L + L_f\Pi] + L_f D' + G + M. \tag{31}$$

D' is the consumption in the childhood period of consumers of the next generation. It is constant. The difference between the two-periods overlapping generations model and the three-periods overlapping generations model exists in the effective demand.

Profit maximization of firms implies

$$P^1 = \frac{W}{\left(1 - \frac{1}{\eta}\right)(1+\zeta)y(Ll)}.$$

Using the above effective demand and this condition we can analyze involuntary unemployment. Let us compare (31) with the effective demand in a two-period overlapping generations model,

$$Y = \alpha(WLl + L_f\Pi) + G + M.$$

The difference between them is

$$L_f D' - \alpha(D+\Theta)L.$$

In the case of three-periods overlapping generations model (24), (25) and (27) are written as

$$\begin{aligned} P^1 Lly(Ll) =& \alpha P^1 Lly(Ll) - \alpha(D+\Theta)L + L_f D' + G + M \\ =& \alpha P^1 Lly(Ll) - \alpha L_f D + L_f D' + G + M, \end{aligned} \tag{32}$$

$$P^1 Ll = \frac{L_f D' + G + M - \alpha L_f D}{(1-\alpha)y(Ll)},$$

and

$$Ll = \frac{L_f d' + g + m - \alpha L_f d}{(1-\alpha)y(Ll)},$$

where

$$g = \frac{G}{P^1}, m = \frac{M}{P^1}, d' = \frac{D'}{P^1}, d = \frac{D}{P^1}.$$

If the value of L obtained from this equation is smaller than L_f, there exists involuntary unemployment.

Steady state

Let T be the tax revenue for the government expenditure, G, then (32) is written as

$$\alpha(P^1 Lly(Ll) - T - L_f D) + L_f D' + G + M = P^1 Lly(Ll).$$

G does not include scholarship. Since at the steady state where $\rho = 1$ we have $D = D'$, the savings of the consumers of the younger generation is

$$(1-\alpha)(P^1 Lly(Ll) - T - L_f D) = G - T + L_f D' - L_f D + M = G - T + M.$$

Since at the steady state this is equal to M, which is the consumption of the older generation, we need $G = T$. Denote the initial values of L, G, M and D by L^0, G^0, M^0, D^0. Then, we get

$$L^0 l = \frac{M^0}{(1-\alpha)P^1 y(Ll)} + \frac{G^0}{P^1 y(Ll)} + \frac{L_f D^0}{P^1 y(Ll)}.$$

Money demand and supply at the steady state

The demand for money is the sum of

1. savings of the younger generation,
2. tax payment,
3. repayment of scholarship,
4. repayment of other debt.

The supply of money is the sum of

1. lending of the younger generation,
2. consumption of the older generation,
3. government expenditure,
4. supply of scholarship

At the steady state where the price of the good is constant, we have

savings of the younger generation=consumption of the older generation,
repayment of debt other than scholarship=lending of the younger generation,
repayment of scholarship=supply of scholarship,
tax payment=government expenditure.

Therefore, the demand for money is equal to the supply of money. The taxes for repayment of the debt of unemployed consumers are included in the repayment of scholarship and the repayment of debt other than scholarship, not ”the tax revenue”.

On reduction of the nominal wage rate

If the nominal wage rate reduces, the price of the good proportionately reduces. Without any special policy even if the price of the good reduces, we can consider that the real values of the government expenditure, g, and the consumption in the childhood period of the next generation, d', are maintained. On the other hand, the nominal values of the consumption of the older generation, M, the debt (including the scholarship) of the younger generation, D, and the tax for repayment of the debt, Θ, are maintained even if the price of the good reduces. Therefore, a reduction of the nominal wage rate increases or decreases the effective demand and employment whether

$$M - \alpha(D+\Theta)L = M - \alpha L_f D$$

is positive or negative. Since at the steady state

$$M = (1-\alpha)(P^1 Lly - T - L_f D),$$

we obtain

$$M - \alpha L_f D = (1-\alpha)(P^1 Lly - T) - L_f D. \tag{33}$$

Whether this is positive or negative is not clear. It depends on whether savings for the retirement stage is larger, or consumption in the childhood stage is large. In the former case (33) is likely to be positive, and in the latter case it is likely to be negative. Also, the relation between L and L_f, that is, whether the situation is close to full employment or not, or L is large or not affects the sign of (33). In the former case it is likely to be positive, and in the latter case it is likely to be negative. Thus, a reduction of the nominal wage rate does not necessarily reduces involuntary unemployment.

8 Concluding Remark

In this paper we have examined the existence of involuntary umemployment using a monopolistic competition model with increasing or constant returns to scale technology and homothetic preferences of consumers. It is a limited assumption that the goods are produced by only labor. The analysis of a case where the goods are produced by capital and labor is one of themes of future researches.

Appendix: Derivations of (5), (6), (7) and (8)

From (3) and (4)

$$\frac{\partial u}{\partial X^1}X^1\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\frac{\partial u}{\partial X^1}X^1=\frac{\lambda}{F'}\int_0^1 p^1(z)c^1(z)dz,$$

$$\frac{\partial u}{\partial X^2}X^2\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz=\frac{\partial u}{\partial X^2}X^2=\frac{\lambda}{F'}\int_0^1 p^2(z)c^2(z)dz.$$

Since $u(X^1,X^2)$ is homogeneous of degree one,

$$u(X^1,X^2)=\frac{\partial u}{\partial X^1}X^1+\frac{\partial u}{\partial X^2}X^2.$$

Thus, we obtain

$$\frac{\int_0^1 p^1(z)c^1(z)dz}{\int_0^1 p^2(z)c^2(z)dz}=\frac{\frac{\partial u}{\partial X^1}X^1}{\frac{\partial u}{\partial X^2}X^2},$$

and

$$u(X^1,X^2)=\frac{\lambda}{F'}\left[\int_0^1 p^1(z)c^1(z)dz+\int_0^1 p^2(z)c^2(z)dz\right]=\frac{\lambda}{F'}(Wl+\Pi). \tag{5}$$

From (1) and (2), we have

$$\left(\frac{\partial u}{\partial X^1}\right)^{1-\eta}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c^1(z)^{1-\frac{1}{\eta}}=\left(\frac{\lambda}{F'}\right)^{1-\eta}p^1(z)^{1-\eta}, \tag{34}$$

and

$$\left(\frac{\partial u}{\partial X^2}\right)^{1-\eta}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}c^2(z)^{1-\frac{1}{\eta}}=\left(\frac{\lambda}{F'}\right)^{1-\eta}p^2(z)^{1-\eta}.$$

They mean

$$\left(\frac{\partial u}{\partial X^1}\right)^{1-\eta}\left(\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz=\left(\frac{\lambda}{F'}\right)^{1-\eta}\int_0^1 p^1(z)^{1-\eta}dz, \tag{35}$$

and

$$\left(\frac{\partial u}{\partial X^2}\right)^{1-\eta}\left(\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right)^{-1}\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz=\left(\frac{\lambda}{F'}\right)^{1-\eta}\int_0^1 p^2(z)^{1-\eta}dz.$$

Note that

$$F'=F'(u(X^1,X^2)),$$

and

$$X^1=\left\{\int_0^1 c^1(z)^{1-\frac{1}{\eta}}dz\right\}^{\frac{1}{1-\frac{1}{\eta}}},\ X^2=\left\{\int_0^1 c^2(z)^{1-\frac{1}{\eta}}dz\right\}^{\frac{1}{1-\frac{1}{\eta}}}.$$

Then, we obtain

$$\frac{\partial u}{\partial X^1}=\left(\frac{\lambda}{F'}\right)\left(\int_0^1 p^1(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}}=\left(\frac{\lambda}{F'}\right)P^1, \tag{36}$$

and
$$\frac{\partial u}{\partial X^2} = \left(\frac{\lambda}{F'}\right)\left(\int_0^1 p^2(z)^{1-\eta}dz\right)^{\frac{1}{1-\eta}} = \left(\frac{\lambda}{F'}\right)P^2.$$
From them we get
$$u(X^1,X^2) = \left(\frac{\lambda}{F'}\right)(P^1X^1 + P^2X^2),$$
$$\frac{P^2}{P^1} = \frac{\frac{\partial u}{\partial X^2}}{\frac{\partial u}{\partial X^1}}, \tag{6}$$
and
$$P^1X^1 + P^2X^2 = Wl + \Pi. \tag{7}$$
Since $u(X^1,X^2)$ is homogeneous of degree one, $\frac{\lambda}{F'}$ is a function of P^1 and P^2, and $\frac{F'}{\lambda}$ is homogeneous of degree one because proportional increases in P^1 and P^2 reduce X^1 and X^2 at the same rate given $Wl+\Pi$. We obtain the following indirect utility function.
$$V = F\left(\frac{Wl+\Pi}{\varphi(P^1,P^2)}\right) - G(l). \tag{8}$$
$\varphi(P^1,P^2)$ is a function which is homogenous of degree one.

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