Is Race to the bottom is modeled as Prisoner's dilemma?

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Abstract

The subject of this study is the modeling of Race to the bottom to verify, is really Race to the bottom is a kind of Prisoner's dilemma.

The importance of this issue is explained by the following: if Race to the bottom is a kind of Prisoner's dilemma, achieving equilibrium tax competition two or more economies leads to deterioration of their economic results As a result, governments have to weaken social, environmental, labor standards and norms.

At the same time, many statistical studies of real economies do not discover the above consequence of the tax competition, so it is concluded that there is no Race to the bottom.

On the other hand, if Race to the bottom is not a kind of PD then that there is no deterioration in standards during the tax competition does not mean that there is no Race to the bottom.

Using a game-theoretic model we consider 3 objective functions for government behavior: the investment volume, the budget revenue, and their combination. For each function, there were calculated conditions under which Race to the bottom is a kind of Prisoner's dilemma.

Introduced a concept of tax-investment equilibrium, as a situation in which all economies are equal for the investor. For the tax-investment equilibrium, there were calculated sufficient conditions under which Race to the bottom is a kind of Prisoner's dilemma.

Key words: Race to the bottom; Prisoner's dilemma; tax competition; government behavior; corporate tax rate; game theory; tax-investment equilibrium

JEL codes: C72; E62; H30

Introduction

Since the first articulation of the problem Race to the bottom (what is credited to Louis Brandeis), this issue has received much attention.

Since taxes, as the main budgetary source, affect many spheres of government activity, the topic of competition for investment by decreasing the tax burden attracts the attention not only of economists and managers but also of lawyers, political and social scientists, etc. (Tiebout, 1956, Cary, 1974, Murphy, 2004, Lindblom, 1977, Bates, 1985, Przeworski, 1988, Wallerstein, 1995, Blinder, 2006, Levy, 2005, Cohen, 2005, Freeman, 2005). Herewith the areas (the subjects) of research are global capital markets, in particular, offshore activities, global policy and regional government, environmental and social standards.

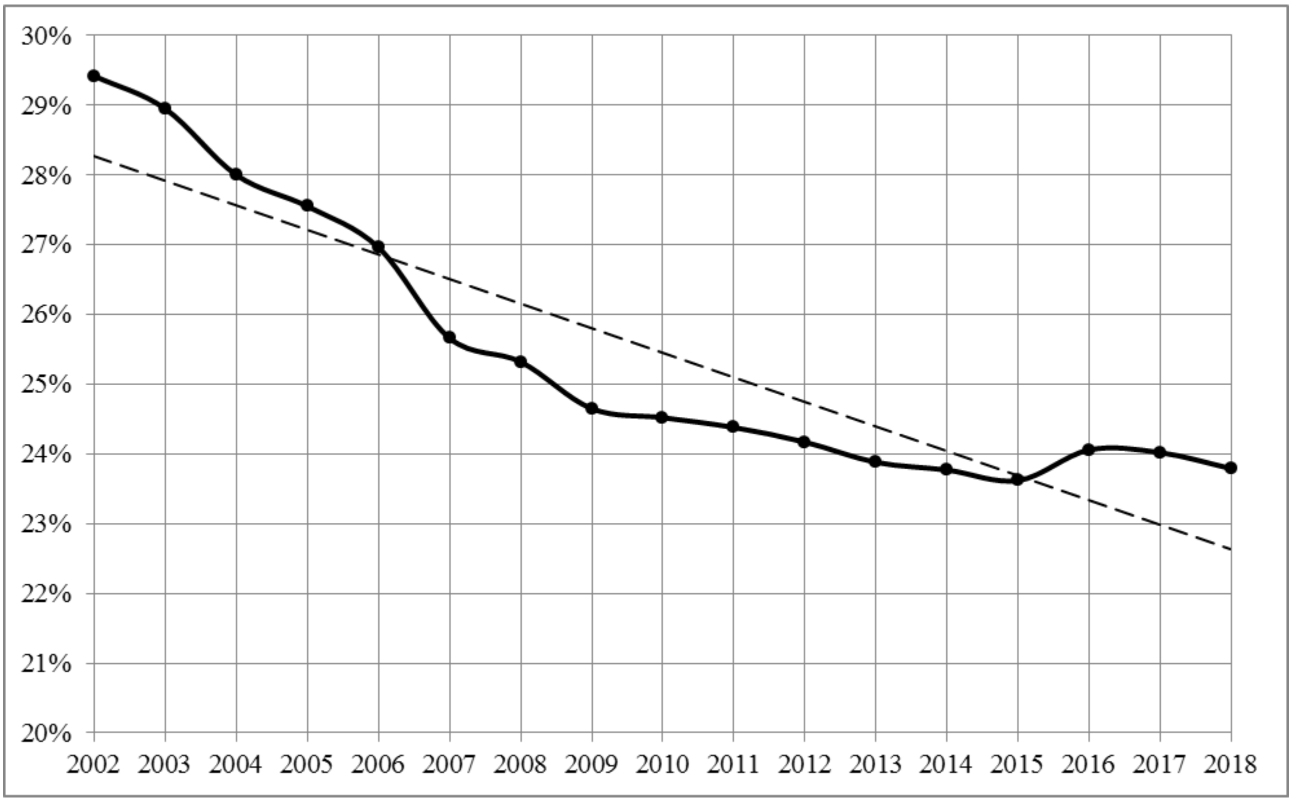
The main issues of this problem that are currently being discussed are:

1) is really there a trend in the global economy to decrease the corporate tax burden?
2) does such a decrease lead to lower social (and environmental) standards due to a lack of budget revenues?
3) do governments apply compensatory mechanisms by raising other taxes?

1) The trend to decrease the tax burden on corporations in the global economy definitely is.

The analysis of CIT rate for 171 countries for which statistics are available from 2002 till 2018 (Corporate tax rates table, 2019) shows as for this time horizon the average CIT rate reduced by 5,63%: from 29,42% to 23,79% (Fig. 1).

Fig. 1. CIT change in world countries from 2005 until 2017 years



Source: (Corporate tax rates table, 2019)

2) The rest of the issues got an active discussion and a wide range of opinions in the scientific world.

The hypothesis, that Race to the bottom almost inevitably leads to a deterioration of social, environmental, etc standards and norms, gained considerable popularity (see i.e. Polanyi (1944), pp. 57, 73, Schlesinger (1997), Kuttner (1997), Tonelson (2000), p. 15, Greider (2001)).

Within the identification of the concept Race to the bottom, many researchers make general assumptions about government economic behavior that summarized e.g. in (Kahler, 1998), namely, they assume that:

- ✓ corporations always prefer lower regulatory standards;
- ✓ the government regulation influences investors' choice of production location;
- ✓ the government responds only (or, at least, generally) to the mobile capital advantage factor.

It concludes that such a situation leads to:

- ✓ a negative correlation between the inflow of investment and the strictness of the regulatory standards of the country, because capital is looking for more profitable investment, and stricter norms and standards increase production costs, thus reducing the profit margin;
- ✓ if one government lowers its regulatory standards to attract more investment, other open economies do the same. In addition, it's assumed that states have no choice but to lower their standards in order not to face the risk of capital flight.

However numerous statistical measurements of such a paradigm show in some degree controversial results.

Kiefer and Rada (2013a, 2013b) verify based on statistics for 40 years, whether the trend of tax rates decreasing for (1) 13 EU countries; 2) OECD countries) is caused by Race to the bottom.

Based on a study of 1980s statistics by individual EU countries (Union Kingdom, France, Germaine, Italy) Mendoza and Tesar quantify macroeconomic effects of capital income tax competition (Mendoza, Tesar, 2003a), and calculate mutual influence between external factors and quantitative results of tax competition (Mendoza, Tesar, 2003b).

Trandafir, Brezeanu and Stanciu investigate how Race to the bottom affects the welfare services condition in Romania (Trandafir, Brezeanu, Stanciu, 2011).

The research subject by Wheeler (Wheeler, 2001) is to test a statement that Race to the bottom leads to a decrease in social and environmental standards and norms, lastly, to environmental conditions and living standards. The statistical basis for this test is the statistics of 50 developing countries, and 5 ASEAN countries for 1997-2006 and 1996-2012 yrs.

Abbas and Klemm (2013), and Revilla (2016) analyze the effect of the corporate income tax change on domestic and foreign investment.

There are a number of areas in which the hypothesis is confirmed but the main of the empirical work focuses on labor and environmental regulations. On a political level are promoted an argue that globalization causes the decrease down wage burden and raises labor and environmental standards (see for example Kapstein (1996), Newland (1999)). However, there are no convincing evidences that these predicates are true.

As empirical proofs to support the hypothesis Race to the bottom is given examples, that corporations move production to countries because of cheap labor and, implicitly, poor labor standards (see for example International Confederation of Free Trade Unions (2000)). However, these observations are different from other cases where such standards do not affect the choice of local investment (Spar, 1998). Multivariate tests do not show a correlation between FDI and labor standards (Jensen, 2006).

Numerous researches, that have tested the impact of environmental standards on export indexes in the economy for the 1970s and 1980s, did not show any significant correlation (Dean, 1992). The use of recent data establishes that environmental regulations have a statistically significant effect on the structure of exports of OECD countries in only one study (Beers, 1999). However, there are reasons not to consider this result too reliable (Harris, 2002). Intra-industry analyzes that allow more exactly to verify industries that are heavily pollutants also do not support the hypothesis Race to the bottom (Tobey, 1990, Ratnayake, 1998).

The trend to Race to the bottom is confirmed for new members, for old members there is no statistical confirmation (Dvořáková, 2013). Kiefer, Rada (2013a, 2013b) assume that the downward trend in tax rates is caused by Race to the bottom. (Trandafir, Brezeanu, Stanciu, 2011) accent that very few scientists have discovered that Race to the bottom leads to a different structure of public service delivery. Rota-Graziosi (2019) presents conditions on the marginal productivity of tax-competing economies which bind the functions of the demand for the capital of these economies and are sufficient to predict Race to the bottom. Wheeler (2001) concludes with the fallacy of the hypothesis that Race to the bottom leads to a decrease in social and environmental standards and norms, lastly to a decrease in environmental and living standards. Abbas and Klemm (2013) i Revilla (2016) note a negative correlation between tax burden and investment. Herewith, Abbas and Klemm (2013) point out that higher rates increase the economy's revenue in the short term.

3) The observer by Mendoza and Tesar (2003) shows that during tax competition countries offset the negative consequences of CTR revenue reductions by regulating other taxes, such as taxes on labor, consumption, etc.

Empirical analysis and modeling based on it by Rota-Graziosi (2019) show

if countries compete for capital taxes by regulating labor taxes to maintain fiscal solvency, there is no Race to the bottom, and Nash-equilibrium is close to the observed taxes;

instead, if consumption taxes are adjusted to support fiscal solvency, competition for capital taxes is leads to Race to the bottom.

As a result, the main observations of Race to the bottom are as follows:

- ✓ not all countries can statistically confirm Race to the bottom;
- ✓ classically accepted effects are observed on Race to the bottom not very clear and far from always;
- ✓ there are cases of balancing CTR by other taxes.

The first two points are based on the main criticism of the concept of Race to the bottom, which mainly concerns the anticipated consequences: deterioration of conditions, standards, etc.

It has caused a row of studies designed to find out based issues of Race to the bottom, namely,

- ✓ is there any Race to the bottom at all?
- ✓ what factors and actions weaken its impact?

There arises a situation that looks as paradoxical: there is a downward trend in tax rates but, at the same time, there are no convincing proofs of a deterioration in social standards.

It causes such questions:

- ✓ is such decreasing tax rates can it consider Race to the bottom?

- ✓ is it necessarily to balance decreasing CTR by increasing other tax rates, i.e., whether a decrease in CTR necessarily results in budget losses?

As for the first question, by definition, Race to the bottom is

the competition with a decrease of the tax rate,

which by itself does not demand any deterioration (or reduction). The latter is a possible consequence of Race to the bottom but is not its essence or property.

Naturally to assume that the conclusion about the unconditional deterioration of standards because of tax rate decrease is largely due to the thesis that Race to the bottom is a form of Prisoner's dilemma (Revesz, 1997, supra note 1, at 1217-1218). Really, the essence of Prisoner's dilemma is players choose Nach-equilibrium, at which they get smaller payoffs for other factors being equal. Clearly, in a real situation, governments must somehow compensate for this lack of revenue, and thus or reduce their other costs to compensate for decreasing tax rate, or, by keeping tax rate, relax the requirements for potential investors.

Thus this research main task is to verify is really Race to the bottom a form of Prisoner's dilemma.

On the other hand, the hypothesis of unaccountability in the model of certain parameters leads to increasing the number of factors and a complication of the model. Therefore, there is a desire to analyze Race to the bottom with minimal tools – only with CTR – and to show that all the above phenomena are possible even in such a case.

So there is the task to investigate the tax mutual behavior of countries and their consequences without compensatory mechanisms by other taxes.

Methodology

For researching of the problem Race to the bottom authors generally apply two based groups of methods: statistical methods (statistical methods (Kiefer, Rada, 2013a; 2013b) and regression modeling (Trandafir, Brezeanu, Stanciu, 2011), panel researches (Wheeler, 2001) what helps study actual statistics; also models of game theory, that allows us to simulate the economic behavior and economies' interaction, and determine the formal relationship between the parameters of interaction.

In particular Mendoza, Tesar (2003a) use a neoclassical dynamic general equilibrium model for 2 players, which contains 3 key externalities of tax competition: the relative price externality, the wealth distribution externality, and the fiscal solvency externality. In another, their work (Mendoza, Tesar, 2003b) these authors are modeling a tax competition as a one-shot game over time-invariant capital taxes with dynamic payoffs, which they use to analyze the competition between the 2 economies by regulating capital, labor and consumption taxes. In addition, changes in tax policies of the studied countries are consistent with the quantitative predictions of a neoclassical dynamic general equilibrium model of tax competition that contains the above basic factors of international tax policy (Mendoza, Tesar, 2005).

Lastly in the article (Mendoza, Tesar, 2014), they offer the model develop for 2 economies considering the observed elasticity of the capital tax base and using endogenous capacity and a partial depreciation rate.

The same, game-theory method for research the issue is applied by Rota-Graziosi (2019). In this case, it's supermodular games that counts r-concavity of the demand for capital, uses production functions in particular quadratic function and Cobb-Douglas function.

In summary, research methods of Race to the bottom can divide into 2 groups: statistical methods – to analyze the actual data of economic activity of countries; game methods – to model the behavior of governments and economies.

Since the above main research task is to answer the question is really Race to the bottom a form of Prisoner's dilemma, and Prisoner's dilemma is modeled by game theory, here we use also game theory.

Modeling

The purpose of tax competition is to attract investment as a result of more favorable conditions for investors. On the other hand, the state attracts investments to increase its own welfare i.e., simplistically, its own revenues in other words budget revenues.

So, the objective function of government regulation in the case of tax competition can be to maximize either the volume of investment involved in the economy or budget revenue from investors' activity or the sum of the above values.

Clearly, taxes are directly related to budget revenues, so we have such potential quantitative estimates for government behavior (strategies value), as the volume of investments, budget revenues, or certain integral quantity that combines the two above variables.

The general matrix of the interaction of 2 countries in the case of Race to the bottom is as follows (table 1):

Table 1. The general form of interaction of 2 countries in the case of Race to the bottom

Country 1 \ Country 2	discount tax rate (τ_2–$\Delta\tau_2$)	normal tax rate (τ_2)
discount tax rate (τ_1–$\Delta\tau_1$)	$\left\{p^1_{00};\ p^2_{00}\right\}$	$\left\{p^1_{11};\ p^2_{11}\right\}$
normal tax rate (τ_1)	$\left\{p^1_{01};\ p^2_{01}\right\}$	$\left\{p^1_{11};\ p^2_{11}\right\}$

In order for the above game to represent Prisoner's dilemma it's necessary that

- ✓ the above game is symmetric, i.e. the conditions: $p^1_{00} = p^2_{00} = p_{00};\ p^1_{11} = p^2_{11} = p_{00};\ p^1_{01} = p^2_{10} = p_{01};\ p^1_{10} = p^2_{01} = p_{10}$ are fulfilled;
- ✓ Nash equilibrium of the game is achieved for a mutual strategy $\left\{p_{00};\ p_{00}\right\}$ that means that inequalities: $p_{01} < p_{00} < p_{11} < p_{10}$ hold true.

Above there are given the strict conditions of Prisoner's dilemma. In the general case, it's possible not to fulfill the symmetry condition, but then it's necessary to fulfill not one and two groups of inequalities – on personal for each player:

$$p_{01}^1 < p_{00}^1 < p_{11}^1 < p_{10}^1; \quad (1.1)$$

$$p_{10}^2 < p_{00}^2 < p_{11}^2 < p_{01}^2. \quad (1.2)$$

Let

- ✓ the investment volume in each of two economies in the normal tax burden τ_1 and τ_2 is x_1 and x_2;
- ✓ the attraction of additional investment because of a decrease tax burden by values $\Delta\tau_1$ and $\Delta\tau_2$ is respectively Δx_{01}^1, Δx_{01}^2, Δx_{10}^1, Δx_{10}^2 and Δx_{11}^1, Δx_{11}^2 – for different cases of taxation policy of countries.

Let the analyze system of economies is closed, i.e. the investment is not attracted from outside and not bring out of economies. Then investment inflow in one economy is accompanied by equivalent investment outflow in the other one. It means that $\Delta x_{01}^1 = -\Delta x_{01}^2 = \Delta x_{01}$, $\Delta x_{10}^1 = -\Delta x_{10}^2 = \Delta x_{10}$, $\Delta x_{11}^1 = -\Delta x_{11}^2 = \Delta x_{11}$. At that, the above values additional attraction of investment Δx_{01}, Δx_{10}, and Δx_{11} depend not only on $\Delta\tau_1$ and $\Delta\tau_2$, also on τ_1, x_1, τ_2, x_2, i.e. on all parameters of the current situation.

Lower we will use the somewhat idealistic assumption that the economies constantly stand in tax-investment equilibrium (indifference), specifically if the equilibrium is disturbed as a result of changes in the tax burden it is restored instantly by transferring investments from one economy to another. Let's describe the essence of tax-investment equilibrium (indifference) for the investors regarding of attractiveness of some economy and formal consequences regarding interrelationship investment values in different countries.

Tax-investment equilibrium for *n* economies

Let the profitability before taxation ordinary investor in economy i is described by the exponential function on investment value in it (saturation of economy):

$$p_i(x_i) = a_i e^{-\alpha_i x_i}, \quad (2.1)$$

where x_i – a part of the total volume of investments in economy i, i.e.,

$$\forall i: 1 \le i \le N: 0 \le x_i \le 1; \sum_{i=1}^{N} x_i \equiv 1; \qquad (2.2)$$

a_i, α_i – the parameters of the investment climate in economy i, $\forall i: 1 \le i \le N: a_i > 0,\ \alpha_i > 0$;

N – the number of interacting economies.

The declining dependence of profitability on investment value can be explained that investment increase in economy saturates it and, in accordance, decrease normalized profitability[1].

Tax-investment equilibrium (indifference state) arises is supposed to if for the investors all N economies are equal, i.e. the revenues after taxation in each of them are equal for relative alike investors, so the investments not flow from one economy in another. I.e.

$$\forall i, j: 1 \le i, j \le N: \left(1-\tau_i\right) p_i = \left(1-\tau_j\right) p_j;$$

$$\forall i, j: 1 \le i, j \le N: \left(1-\tau_i\right) a_i e^{-\alpha_i x_i} = \left(1-\tau_j\right) a_j e^{-\alpha_j x_j}, \qquad (2.3)$$

τ_i – tax rate on income in economy i, $\forall i: 1 \le i \le N: 0 \le \tau_i \le 1$.

So here it's

$$\forall i, j: 1 \le i, j \le N: \frac{1-\tau_i}{1-\tau_j} = \frac{a_j}{a_i} e^{\alpha_i x_i - \alpha_j x_j}, \qquad (2.4)$$

In the cause of equilibrium (indifference state), the relation $\frac{1-\tau_i}{1-\tau_j}$ depends not on absolute values, and on the relations $\frac{a_i}{a_j}$ and $\frac{e^{\alpha_i x_i}}{e^{\alpha_j x_j}}$.

[1] The dependence of productivity of an economy on investment value in it is not necessarily exponential, there are possible other options: sigmoid function (S-curve, e.g., Gompertz curve), power function etc.

Results

Case 1. Resulting factor "volume of attracting investment"

According to the model Race to the bottom if one economy decreases the tax burden as the other keeps it at the same level; it leads to additional investment in the first economy. It means that for case 1 occurs $p_{00}^{1} > p_{01}^{1}$, $p_{10}^{1} > p_{11}^{1}$, $p_{00}^{2} > p_{10}^{2}$, $p_{01}^{2} > p_{11}^{2}$, i.e. Nash equilibrium is really achieved if to use the mutual strategy $\{\tau_1 - \Delta\tau_1;\ \tau_2 - \Delta\tau_2\}$ at a price $\left\{p_{00}^{1};\ p_{00}^{2}\right\}$. However, in the general case there is no reason to consider that $p_{00}^{1} < p_{11}^{1}$ and/or $p_{00}^{2} < p_{11}^{2}$, i.e., the mutual strategy $\{\tau_1;\tau_2\}$ is more Pareto-efficient than mutual strategy $\{\tau_1 - \Delta\tau_1;\tau_2 - \Delta\tau_2\}$.

The matrix of the interaction of 2 economies in the course of Race to the bottom, if the resulting factor is the volume of attracted investment is as follows (Table 2):

Table 2. The matrix of the interaction of 2 economies in Race to the bottom for the resulting factor "the volume of investment"

Country 2 / Country 1	discount tax rate (τ_2–$\Delta\tau_2$)	normal tax rate (τ_2)
discount tax rate (τ_1–$\Delta\tau_1$)	$\{x_1 + \Delta x_{11};\ x_2 - \Delta x_{11}\}$	$\{x_1 + \Delta x_{10};\ x_2 - \Delta x_{10}\}$
normal tax rate (τ_1)	$\{x_1 - \Delta x_{01};\ x_2 + \Delta x_{01}\}$	$\{x_1;\ x_2\}$

Considering (1.1) Prisoner's dilemma conditions are in this case such:

$$x_1 - \Delta x_{01} < x_1 + \Delta x_{11} < x_1 < x_1 + \Delta x_{10}; \tag{1.3}$$

$$x_2 - \Delta x_{10} < x_2 - \Delta x_{11} < x_2 < x_2 + \Delta x_{01}. \tag{1.4}$$

It's seen that the conditions (1.3) and (1.4) contradict each other. Really $\Delta x_{11} > 0$ makes possible the fulfillment (1.4), in return one of conjunct (1.3) is not hold, namely $x_1 + \Delta x_{11} < x_1$. On the other hand, the negativeness of Δx_{11} opposite makes possible the fulfillment of the condition (1.3), in return does not hold (1.4) because of $x_2 - \Delta x_{11} < x_2$. So if resulting factor of interaction is investment volume, Prisoner's dilemma not arises.

Case 2. Resulting factor "volume of budgetary receipts"

The matrix of the interaction of 2 economies in the course of Race to the bottom, if the resulting factor is the volume of budget revenues is as follows (Table 3):

Table 3. The matrix of the interaction of 2 economies in Race to the bottom for resulting factor "the volume of budget revenues"

Country 2 / Country 1	discount tax rate ($\tau_2 - \Delta\tau_2$)	normal tax rate (τ_2)
discount tax rate ($\tau_1 - \Delta\tau_1$)	$\begin{Bmatrix}(\tau_1 - \Delta\tau_1)(x_1 + \Delta x_{11}); \\ (\tau_2 - \Delta\tau_2)(x_2 - \Delta x_{11})\end{Bmatrix}$	$\begin{Bmatrix}(\tau_1 - \Delta\tau_1)(x_1 + \Delta x_{10}); \\ \tau_2(x_2 - \Delta x_{10})\end{Bmatrix}$
normal tax rate (τ_1)	$\begin{Bmatrix}\tau_1(x_1 - \Delta x_{01}); \\ (\tau_2 - \Delta\tau_2)(x_2 + \Delta x_{01})\end{Bmatrix}$	$\{\tau_1 x_1;\ \tau_2 x_2\}$

Considering (1.1) the conditions of presence of Prisoner's dilemma are in this case such:

$$\tau_1(x_1 - \Delta x_{01}) < (\tau_1 - \Delta\tau_1)(x_1 + \Delta x_{11}) < \tau_1 x_1 < (\tau_1 - \Delta\tau_1)(x_1 + \Delta x_{10}); \qquad (1.7)$$

$$\tau_2(x_2 - \Delta x_{10}) < (\tau_2 - \Delta\tau_2)(x_2 - \Delta x_{11}) < \tau_2 x_2 < (\tau_2 - \Delta\tau_2)(x_2 - \Delta x_{01}). \qquad (1.8)$$

After the transformation (see Appendix A) we get:

$$\max\left(\frac{x_1}{\Delta x_{10}};\ \frac{x_1 - \Delta x_{01}}{\Delta x_{11} + \Delta x_{01}}\right) < \frac{\tau_1}{\Delta\tau_1} - 1 < \frac{x_1}{\Delta x_{11}}; \qquad (1.9)$$

$$\frac{x_2}{\Delta x_{01}} < 1 - \frac{\tau_2}{\Delta\tau_2} < \min\left(\frac{\Delta x_{10} - x_2}{\Delta x_{10} - \Delta x_{11}};\ \frac{x_2}{\Delta x_{11}}\right). \qquad (1.10)$$

Case 3. Resulting factor “the volume of budgetary receipts with regard given of attraction of additional investment”

In this case, it can already not unequivocally assert that if one economy decreases the tax burden as another keeps it at the same level, it leads to a reduction in first-country budget revenue. There are two opposite directional vectors caused by the tax rate decrease: a decrease in fees from current investments and an increase in the total amount of investments due to attracting new ones. Race to the bottom is possible if the amount of tax collected on additional investments in both countries do not compensate the losses from existing ones, but it demands additionally to fulfill conditions

$$p_{00}^{1} < p_{11}^{1} \text{ and } p_{00}^{2} < p_{11}^{2}.$$

Clearly, if the resulting factor is the total sum of attracted investments and budgetary receipts then the general matrix of the interaction of 2 economies in Race to the bottom unites naturally two previous cases and is as follows (Table 4):

Table 4. The matrix of the interaction of 2 economies in Race to the bottom for resulting factor “the total sum of attracted investments and budgetary receipts”

Country 2 / Country 1	discount tax rate (τ_2–$\Delta\tau_2$)	normal tax rate (τ_2)
discount tax rate (τ_1–$\Delta\tau_1$)	$\begin{Bmatrix} (\tau_1 - \Delta\tau_1 + 1)(x_1 + \Delta x_{11}); \\ (\tau_2 - \Delta\tau_2 + 1)(x_2 - \Delta x_{11}) \end{Bmatrix}$	$\begin{Bmatrix} (\tau_1 - \Delta\tau_1 + 1)(x_1 + \Delta x_{10}); \\ (\tau_2 + 1)(x_2 - \Delta x_{10}) \end{Bmatrix}$
normal tax rate (τ_1)	$\begin{Bmatrix} (\tau_1 + 1)(x_1 - \Delta x_{01}); \\ (\tau_2 - \Delta\tau_2 + 1)(x_2 + \Delta x_{01}) \end{Bmatrix}$	$\{(\tau_1 + 1)x_1;\ (\tau_2 + 1)x_2\}$

Considering (1.1) the conditions of presence of Prisoner’s dilemma are in this case such:

$$(\tau_1 + 1)(x_1 - \Delta x_{01}) < (\tau_1 - \Delta\tau_1 + 1)(x_1 + \Delta x_{11}) < (\tau_1 + 1)x_1 < (\tau_1 - \Delta\tau_1 + 1)(x_1 + \Delta x_{10}); \quad (1.11)$$

$$(\tau_2+1)(x_2-\Delta x_{10})<(\tau_2-\Delta\tau_2+1)(x_2-\Delta x_{11})<(\tau_2+1)x_2<(\tau_2-\Delta\tau_2+1)(x_2-\Delta x_{01}),\quad (1.12)$$

or after the transformation (see Appendix A):

$$\max\left(\frac{x_1}{\Delta x_{10}};\ \frac{x_1-\Delta x_{01}}{\Delta x_{11}+\Delta x_{01}}\right)<\frac{\tau_1+1}{\Delta\tau_1}-1<\frac{x_1}{\Delta x_{11}}\ ;\qquad (1.13)$$

$$\frac{x_2}{\Delta x_{01}}<1-\frac{\tau_2+1}{\Delta\tau_2}<\min\left(\frac{\Delta x_{10}-x_2}{\Delta x_{10}-\Delta x_{11}};\ \frac{x_2}{\Delta x_{11}}\right).\qquad (1.14)$$

Case of 2 economies

If n=2, (2.3) and (2.4) can be simplified:

$$(1-\tau_1)a_1e^{-\alpha_1 x}=(1-\tau_2)a_2e^{-\alpha_2(1-x)};$$

$$\frac{1-\tau_1}{1-\tau_2}=\frac{a_2}{a_1}e^{\alpha_1 x-\alpha_2(1-x)},\qquad (2.5)$$

on this basis

$$x=\frac{1}{\alpha_1+\alpha_2}\left(\alpha_2+\ln\frac{a_1}{a_2}+\ln\frac{1-\tau_1}{1-\tau_2}\right);$$
$$1-x=\frac{1}{\alpha_1+\alpha_2}\left(\alpha_1-\ln\frac{a_1}{a_2}-\ln\frac{1-\tau_1}{1-\tau_2}\right).\qquad (2.6)$$

The expression (2.6) characterize dependence investment parts in each economy on relation tax burdens (specifically, their complements).

Also from (2.5) it's can deduce the dependence τ_2 on τ_1 and x:

$$\frac{1-\tau_1}{1-\tau_2}=\frac{a_2}{a_1}e^{\alpha_1 x-\alpha_2(1-x)};$$

$$1-\tau_2=\frac{a_1}{a_2}e^{\alpha_2(1-x)-\alpha_1 x}(1-\tau_1);$$

$$\tau_2=1-\frac{a_1}{a_2}e^{\alpha_2(1-x)-\alpha_1 x}(1-\tau_1).\qquad (2.7)$$

The investors' revenues in 1st and 2nd economies, if equilibrium, are equal in accordance:

$$r_1 = (1-\tau_1) a_1 e^{-\alpha_1 x} x =$$

$$= (1-\tau_1) \frac{a_1}{\alpha_1 + \alpha_2} \frac{\alpha_2 + \ln \frac{a_1}{a_2} + \ln \frac{1-\tau_1}{1-\tau_2}}{\left(e^{\alpha_2} \frac{a_1}{a_2} \cdot \frac{1-\tau_1}{1-\tau_2} \right)^{\frac{\alpha_1}{\alpha_1+\alpha_2}}}; \qquad (2.8)$$

and

$$r_2 = (1-\tau_2) a_2 e^{-\alpha_2 (1-x)} (1-x) =$$

$$= (1-\tau_2) \frac{a_2}{\alpha_1 + \alpha_2} \left(\alpha_1 - \ln \frac{a_1}{a_2} - \ln \frac{1-\tau_1}{1-\tau_2} \right) \left(\frac{\frac{a_1}{a_2} \cdot \frac{1-\tau_1}{1-\tau_2}}{e^{\alpha_1}} \right)^{\frac{\alpha_2}{\alpha_1+\alpha_2}}. \qquad (2.9)$$

Let:

$$A_1 = \frac{a_1}{\alpha_1 + \alpha_2} \left(\alpha_2 + \ln \frac{a_1}{a_2} + \ln \frac{1-\tau_1}{1-\tau_2} \right);$$

$$A_2 = \frac{a_2}{\alpha_1 + \alpha_2} \left(\alpha_1 - \ln \frac{a_1}{a_2} - \ln \frac{1-\tau_1}{1-\tau_2} \right).$$

Then (2.8) and (2.9) turn into:

$$r_1 = (1-\tau_1) \frac{A_1}{e^{A_1}}; \qquad (2.10)$$

$$r_2 = (1-\tau_2) \frac{A_2}{e^{A_2}}. \qquad (2.11)$$

The revenues of 1st and 2nd economies, if equilibrium, are equal in accordance:

$$R_1 = \tau_1 a_1 e^{-\alpha_1 x} x = \tau_1 \frac{A_1}{e^{A_1}}; \qquad (2.12)$$

$$R_2 = \tau_2 a_2 e^{-\alpha_2 (1-x)} (1-x) = \tau_2 \frac{A_2}{e^{A_2}}. \qquad (2.13)$$

From (2.6) it follows that constraint $0 \le x \le 1$ is equivalent to constraint:

$$0 \le \frac{1}{\alpha_1 + \alpha_2}\left(\alpha_2 + \ln\frac{a_1}{a_2} + \ln\frac{1-\tau_1}{1-\tau_2}\right) \le 1;$$

$$0 \le \alpha_2 + \ln\frac{a_1}{a_2} + \ln\frac{1-\tau_1}{1-\tau_2} \le \alpha_1 + \alpha_2;$$

$$-\alpha_2 \le \ln\frac{a_1}{a_2} + \ln\frac{1-\tau_1}{1-\tau_2} \le \alpha_1;$$

$$-\alpha_2 \le \ln\frac{a_1}{a_2} \cdot \frac{1-\tau_1}{1-\tau_2} \le \alpha_1;$$

$$e^{-\alpha_2} \le \frac{1-\tau_1}{1-\tau_2} \cdot \frac{a_1}{a_2} \le e^{\alpha_1};$$

$$\frac{a_2}{a_1} e^{-\alpha_2} \le \frac{1-\tau_1}{1-\tau_2} \le \frac{a_2}{a_1} e^{\alpha_1}. \qquad (2.18)$$

If the equilibrium is formed for the tax rates τ_1, τ_2, then in order to maintain the equilibrium the quantities of their decreasing $\Delta\tau_1$, $\Delta\tau_2$ are calculated as follows: let $\Theta = \frac{1-\tau_1}{1-\tau_2}$, then must fulfill $\Theta = \frac{1-\tau_1-\Delta\tau_1}{1-\tau_2-\Delta\tau_2}$, it follows that:

$$\Theta = \frac{1-\tau_1}{1-\tau_2} = \frac{1-\tau_1-\Delta\tau_1}{1-\tau_2-\Delta\tau_2};$$

$$\frac{\Delta\tau_1}{\Delta\tau_2} = \frac{1-\tau_1}{1-\tau_2} = \Theta, \qquad (2.19)$$

i.e. the equilibrium is maintained if the ratio of the reduction of rates is equal to the ratio of the profits of the parts remaining to investors.

Prisoner's dilemma conditions at tax-investment equilibrium

So, when the system of 2 economies at tax burden, respectively, $\{\tau_1;\ \tau_2\}$ (one of pure game strategy) is in the tax-invest equilibrium, the symmetrical to it pure strategy $\{\tau_1 - \Delta\tau_1;\ \tau_2 - \Delta\tau_2\}$ naturally is also a tax-invest equilibrium, at which is no investment movement, i.e. to consider parameters of tax burden decreasing $\Delta\tau_1$

and $\Delta\tau_2$ those that don't upset the equilibrium (2.19). In other words, the relation of the revenue parts at the disposal of investors after-tax must not change. From (2.19) can deduce the dependence of $\Delta\tau_2$ on $\Delta\tau_1$, τ_1 and τ_2:

$$\frac{1-\tau_2}{1-\tau_1}\Delta\tau_1 = \Delta\tau_2, \tag{3.1}$$

The other two symmetrical one-to-one strategies are convenient to consider such situations when one government decreases tax burden and another keeps it at the previous level. In this case, there is a flow of investments from the economy with a fixed tax burden to the one where the burden has been decreased – to the extent necessary to restore the tax-investment equilibrium. Clearly, without loss of generality, such a situation is also the case when the second economy decreases the tax rate but less than it's necessary to restore a tax-invest equilibrium.

The main importance of the tax-invest equilibrium is that in this situation both in case 2 and in case 3 $p_{00}^1 < p_{11}^1$ and $p_{00}^2 < p_{11}^2$, as the volume of investment in each country remains fixed, and tax burden and as result budget revenues decrease. As a consequence the implementation of inequalities $p_{01}^1 < p_{00}^1$, $p_{11}^1 < p_{10}^1$, $p_{10}^2 < p_{00}^2$ and $p_{11}^2 < p_{01}^2$ is a sufficient condition for Prisoner's dilemma. Formally, in the case of a tax-invest equilibrium, the expressions in Tables 2-4 and formulas (1.3)-(1.14) are simplified. In particular $\Delta x_{11} \equiv 0$, $\Delta x_{01} > 0$, $\Delta x_{10} > 0$. From what it follows that in cases 2 & 3 (Tables 3 and 4) the middle inequality is fulfilled always that creates necessary preconditions for Prisoner's dilemma.

Consider what the values and the ratio of the parameters create sufficient conditions for Prisoner's dilemma. Clearly for this, it's necessary to fulfill 4 inequalities in each case.

Case 2

(1.9) and (1.10) turn to

$$\frac{x_1}{\Delta x_{01}} < \frac{\tau_1}{\Delta \tau_1};$$
$$\frac{x_1}{\Delta x_{10}} + 1 < \frac{\tau_1}{\Delta \tau_1}; \qquad (3.2)$$

$$\frac{x_2}{\Delta x_{10}} < \frac{\tau_2}{\Delta \tau_2};$$
$$\frac{\tau_2}{\Delta \tau_2} < 1 - \frac{x_2}{\Delta x_{01}}. \qquad (3.3)$$

Based on (2.6) can be written for exponential function:

$$x_1 = x = \frac{1}{\alpha_1 + \alpha_2}\left(\alpha_2 + \ln\frac{a_1}{a_2} + \ln\frac{1-\tau_1}{1-\tau_2}\right);$$
$$x_2 = 1 - x = \frac{1}{\alpha_1 + \alpha_2}\left(\alpha_1 - \ln\frac{a_1}{a_2} - \ln\frac{1-\tau_1}{1-\tau_2}\right).$$

Then (3.2) can be written as

$$1 + \frac{\alpha_2 + \ln\frac{a_1}{a_2} + \ln\frac{1-\tau_1}{1-\tau_2}}{(\alpha_1 + \alpha_2)\Delta x_{10}} < \frac{\tau_1}{\Delta \tau_1};$$
$$\frac{\alpha_2 + \ln\frac{a_1}{a_2} + \ln\frac{1-\tau_1}{1-\tau_2}}{(\alpha_1 + \alpha_2)\Delta x_{01}} < \frac{\tau_1}{\Delta \tau_1}; \qquad (3.4)$$

and (3.3), counting (3.1), as

$$\frac{\alpha_1 - \ln\frac{a_1}{a_2} - \ln\frac{1-\tau_1}{1-\tau_2}}{(\alpha_1 + \alpha_2)\Delta x_{10}} < \frac{\tau_2}{\Delta \tau_2};$$
$$\frac{\tau_2}{\Delta \tau_2} < 1 - \frac{\alpha_1 - \ln\frac{a_1}{a_2} - \ln\frac{1-\tau_1}{1-\tau_2}}{(\alpha_1 + \alpha_2)\Delta x_{01}};$$

$$\frac{\alpha_1 - \ln\frac{a_1}{a_2} - \ln\frac{1-\tau_1}{1-\tau_2}}{(\alpha_1+\alpha_2)\Delta x_{10}} < \frac{\tau_2}{\frac{1-\tau_2}{1-\tau_1}\Delta\tau_1};$$

$$\frac{\tau_2}{\frac{1-\tau_2}{1-\tau_1}\Delta\tau_1} < 1 - \frac{\alpha_1 - \ln\frac{a_1}{a_2} - \ln\frac{1-\tau_1}{1-\tau_2}}{(\alpha_1+\alpha_2)\Delta x_{01}};$$

$$\begin{gathered}\frac{\alpha_1 - \ln\frac{a_1}{a_2} - \ln\frac{1-\tau_1}{1-\tau_2}}{(\alpha_1+\alpha_2)\Delta x_{10}} < \frac{1-\tau_1}{1-\tau_2}\cdot\frac{\tau_2}{\Delta\tau_1};\\ \frac{1-\tau_1}{1-\tau_2}\cdot\frac{\tau_2}{\Delta\tau_1} < 1 - \frac{\alpha_1 - \ln\frac{a_1}{a_2} - \ln\frac{1-\tau_1}{1-\tau_2}}{(\alpha_1+\alpha_2)\Delta x_{01}}.\end{gathered} \qquad \textit{(3.5)}$$

Case 3

It's easily seen that in this case τ_1 and τ_2 are replaced by τ_1+1 and τ_2+1. Similarly (3.2), (3.3) should be replaced by

$$\begin{gathered}\frac{x_1}{\Delta x_{01}} < \frac{\tau_1+1}{\Delta\tau_1};\\ \frac{x_1}{\Delta x_{10}} + 1 < \frac{\tau_1+1}{\Delta\tau_1};\end{gathered} \qquad \textit{(3.6)}$$

$$\begin{gathered}\frac{x_2}{\Delta x_{10}} < \frac{\tau_2+1}{\Delta\tau_2};\\ \frac{\tau_2}{\Delta\tau_2} < 1 - \frac{x_2+1}{\Delta x_{01}},\end{gathered} \qquad \textit{(3.7)}$$

and (3.4), (3.5) – for exponential function – by

$$1+\frac{\alpha_2+\ln\frac{a_1}{a_2}+\ln\frac{2-\tau_1}{2-\tau_2}}{(\alpha_1+\alpha_2)\Delta x_{10}}<\frac{\tau_1+1}{\Delta\tau_1};$$
$$\frac{\alpha_2+\ln\frac{a_1}{a_2}+\ln\frac{2-\tau_1}{2-\tau_2}}{(\alpha_1+\alpha_2)\Delta x_{01}}<\frac{\tau_1+1}{\Delta\tau_1}; \tag{3.8}$$

$$\frac{\alpha_1-\ln\frac{a_1}{a_2}-\ln\frac{2-\tau_1}{2-\tau_2}}{(\alpha_1+\alpha_2)\Delta x_{10}}<\frac{2-\tau_1}{2-\tau_2}\cdot\frac{\tau_2+1}{\Delta\tau_1};$$
$$\frac{2-\tau_1}{2-\tau_2}\cdot\frac{\tau_2+1}{\Delta\tau_1}<1-\frac{\alpha_1-\ln\frac{a_1}{a_2}-\ln\frac{2-\tau_1}{2-\tau_2}}{(\alpha_1+\alpha_2)\Delta x_{01}}. \tag{3.9}$$

Conclusions

1. As strictly scientific well as more popular economic thought connect the fact of Race to the bottom Race to the bottom with the unconditional deterioration of economic, social, environmental, etc. standards. This conclusion is largely caused by the thesis that Race to the bottom is a form of Prisoner's dilemma.

On the other hand, the conclusion that Race to the bottom is equal to Prisoner's dilemma not always, so the tax-competition equilibrium can be to achieve in Pareto optimal point, casts doubt on the obligation of governments to compensate for revenue shortfalls by reducing costs or weakening standards.

Firstly, it lets separate the issues of Race to the bottom and compensation mechanisms;
secondly, it removes the controversy many authors write about when in conditions of tax competition, there is no reduction in standards.

Acceptance of this thesis lets analyzes Race to the bottom with minimal tools, highlighting the quintessence of economic interaction between economies.

2. Since taxes are directly related to budget revenues, we have such potential objective functions for government behavior, as investment volume, budget revenues, or their combination.

3. An analysis of the above 3 objective functions shows the strategy of Race to the bottom does not always lead to Prisoner's dilemma. If the objective function for government behavior is "the volume of investment", there is no Prisoner dilemma at all.

If the objective function is "budget revenue" or the sum of the volume of investment and budget revenue, we can get Prisoner's dilemma, but with certain values of economic interaction parameters.

4. Introduced a concept of tax-investment equilibrium, namely, as a state in which all economies are equal for the potential investor in terms of productivity of investments and therefore no investment movement takes place between these economies.

For the tax-investment equilibrium, there were calculated sufficient conditions under which Race to the bottom is a kind of Prisoner's dilemma.

Bibliography

Abbas, S. and Klemm, A., 2013. A partial race to the bottom: corporate tax developments in emerging and developing economies. International Tax and Public Finance, 20(4), pp. 596–617.

Bates, R. & Lien, D.-H. D., 1985. A Note on Taxation, Development, and Representative Government. Politics & Society 14(1), pp. 53-70.

Beers van, C. and Bergh van der, J. C. J. M., 1999. An Empirical Multi-country Analysis of the Impact of Environmental Regulations on Foreign Trade Flows. Kyklos 50(February), pp. 29-46.

Blinder, A., 2006. Offshoring: The Next Industrial Revolution? Foreign Affairs 85(March/April): pp. 113-128.

Cary, W. L., 1974. Federalism and Corporate Law: Reflections upon Delaware. Yale Law Journal 83(4), pp. 663-705.

Cho, H. J., 2002. Political Risk, Labor Standards and Foreign Direct Investment. Paper presented at UCLA's CIBER Doctoral Research Seminar, Ventura, CA, June.

Cohen, S. S. & DeLong, J. B., 2005. Shaken and Stirred. The Atlantic Monthly 295(January/February), pp. 112-117.

Corporate tax rates table, 2019. *KMPG*. [online] Available at: https://home.kpmg/xx/en/home/services/tax/tax-tools-and-resources/tax-rates-online/corporate-tax-rates-table.html. [Accessed 31/03/2020].

Dean, J. M., 1992. Trade and the Environment: A Survey of Literature. In Patrick Low, ed. International Trade and the Environment, Washington, DC: World Bank.

Dvořáková, V., 2013. Are the tendencies of the race to the bottom real in the European Union? Acta Universitatis Agriculturae et Silviculturae Mendelianae Brunensis, 61(4), pp. 893–898.

Freeman, R., 2005. What Really Ails Europe (And America): The Doubling of the Global Workforce. The Globalist, May. [online] Available at: http://www.theglobalist.com/StoryId.aspx?StoryId=4542. [Accessed 31/03/2020].

Greider, W., 2001. A new giant sucking sound. The Nation, December 31.

Harris, M. N., Kónya, L. and Mátyás, L., 2002. Modelling the Impact of Environmental Regulations on Bilateral Trade Flows. The World Economy 25(March), pp. 387-405.

International Confederation of Free Trade Unions, 2000. Behind the wire: anti-union repression in the export processing zones. [online] Available at: http://www.icftu.org/english/tncs/etnexpzo.html. [Accessed 31/03/2020].

Jaffe, A. B., Peterson, S. R., Portney, P. R., & Stavins, R. N., 1995. Environmental Regulation and the Competitiveness of U.S. Manufacturing. Journal of Economic Literature 33(March), pp. 132-163.

Jensen, N., 2006. Nation-Sates and the Multinational Corporation: A Political Economy of Foreign Direct Investment, Princeton, NJ: Princeton University Press.

Kahler, M., 1998. Modeling Races to the Bottom," paper presented at the 1998 meeting of the American Political Science Association, Boston, MA. See also Hirst and Thompson, Globalization in Question, chapter one.

Kapstein, E., 1996. Workers and the world economy. Foreign Affairs, 75(3), pp. 16-24.

Kiefer, D. and Rada, C., 2013. Profit maximizing goes global: the race to the bottom. Working Paper Series, Department of Economics, University of Utah 2013_05, University of Utah, Department of Economics.

Kiefer, D. and Rada, C., 2013. A note on the wage share in OECD countries: Is there a race to the bottom? Working Paper Series, Department of Economics, University of Utah 2013_13, University of Utah, Department of Economics.

Kuttner, R., 1997. Rules That Liberate. The American Prospect, May.

Levy, D. J., 2005. Offshoring in the New Global Political Economy. Journal of Management Studies 42(May), pp. 685-693.

Lindblom, C., 1977. Politics and Markets, New York: Basic Books.

Mendoza, E. G. and Tesar, L. L., 2003. Winners and Losers of Tax Competition in the European Union. NBER Working Papers 508, Research Seminar in International Economics, University of Michigan.

Mendoza, E. G. and Tesar, L. L., 2003. A Quantitative Analysis of Tax Competition v. Tax Coordination under Perfect Capital Mobility. NBER Working Papers 9746, National Bureau of Economic Research, Inc.

Mendoza, E. G. and Tesar, L. L., 2005. Why hasn't tax competition triggered a race to the bottom? Some quantitative lessons from the EU. Journal of Monetary Economics, 52(1), pp. 163–204.

Mendoza, E. G. and Tesar, L. L., Zhang, J., 2014. Saving Europe?: The Unpleasant Arithmetic of Fiscal Austerity in Integrated Economies. Working Paper Series WP-2014-13, Federal Reserve Bank of Chicago.

Murphy, D., 2004. The Structure of Regulatory Competition: Corporations and Public Policies in a Global Economy, New York: Oxford University Press.

Newland, K., 1999. Workers of the world, now what? Foreign Policy, 114, pp. 52-65.

Polanyi, K., 1944. The great transformation, Boston: Beacon Press.

Przeworski, A. & Wallerstein, M., 1988. Structural Dependence of the State on Capital. American Political Science Review 82(1), pp. 11-29.

Ratnayake, R., 1998. Do Stringent Environmental Regulations Reduce International Competitiveness? International Journal of the Economics of Business 5(February), pp. 97-118.

Revesz, R. L., 1997. Federalism and environmental regulation: A normative critique. In The new federalism: Can the states be trusted? John Ferejohn and Barry R. Weingast eds. Hoover institution press publication, Hoover Institution Press, Stanford University.

Revilla, M. and Laarni D., 2016. Cross-Country Econometric Study on the Impact of Fiscal Incentives on Foreign Direct Investment. Discussion Papers DP 2016-17, Philippine Institute for Development Studies.

Rodrik, D., 1999. Globalization and Labor," in Richard Baldwin et al eds., Market Integration, Regionalism, and the Global Economy, Cambridge: Cambridge University Press.

Rota-Graziosi, G., 2019. The supermodularity of the tax competition game. Journal of Mathematical Economics, 83(C), pp. 25–35.

Schlesinger jr., A., 1997. Has democracy a future? Foreign Affairs, New York, September/October, pp. 7-8.

Spar, D., 1998. Attracting High Technology Investment: Intel's Costa Rican Plant. FIAS Occasional Paper No. 11, World Bank, Washington DC, April.

Tiebout,C., 1956. A Pure Theory of Local Expenditures. Journal of Political Economy 64(5), pp. 416-424.

Tobey, J., 1990. The Impact of Domestic Environmental Policies on Patterns of World Trade. Kyklos 43(Ma), pp. 191-209.

Tonelson, A., 2000. The race to the bottom, Boulder, Westview Press.

Trandafir, A., Brezeanu, P. and Stanciu A., 2011. The Race to the Bottom and its Impact on the Provision of Public Services in Romania. Annals of the University of Petrosani, Economics, 11(1), pp. 261–266.

Wallerstein, V. & Przeworski, A., 1995. Capital Taxation with Open Borders. Review of International Political Economy 2(3), pp. 425-445.

Wheeler, D., 2001. Racing to the bottom: foreign investment and air pollution in developing countries. Policy Research Working Paper Series 2524, The World Bank.

Appendix A

Algebraic transformations

Case 2. Resulting factor "value of budgetary receipts"

$$\tau_1\left(x_1-\Delta x_{01}\right)<\left(\tau_1-\Delta\tau_1\right)\left(x_1+\Delta x_{11}\right)<\tau_1 x_1<\left(\tau_1-\Delta\tau_1\right)\left(x_1+\Delta x_{10}\right);$$

$$\begin{aligned}&\tau_1 x_1-\tau_1\Delta x_{01}<\tau_1 x_1-\Delta\tau_1 x_1+\tau_1\Delta x_{11}-\Delta\tau_1\Delta x_{11}\ \wedge\\ &\wedge\ \tau_1 x_1-\Delta\tau_1 x_1+\tau_1\Delta x_{11}-\Delta\tau_1\Delta x_{11}<\tau_1 x_1\ \wedge\\ &\wedge\ \tau_1 x_1<\tau_1 x_1-\Delta\tau_1 x_1+\tau_1\Delta x_{10}-\Delta\tau_1\Delta x_{10};\end{aligned}$$

$$\begin{aligned}&\Delta\tau_1 x_1+\Delta\tau_1\Delta x_{11}<\tau_1\Delta x_{01}+\tau_1\Delta x_{11}\ \wedge\\ &\wedge\ \tau_1\Delta x_{11}<\Delta\tau_1 x_1+\Delta\tau_1\Delta x_{11}\ \wedge\\ &\wedge\ \Delta\tau_1 x_1+\Delta\tau_1\Delta x_{10}<\tau_1\Delta x_{10};\end{aligned}$$

$$\begin{aligned}&\Delta\tau_1\left(x_1+\Delta x_{11}\right)<\tau_1\left(\Delta x_{01}+\Delta x_{11}\right)\wedge\\ &\wedge\ \tau_1\Delta x_{11}<\Delta\tau_1\left(x_1+\Delta x_{11}\right)\ \wedge\\ &\wedge\ \Delta\tau_1\left(x_1+\Delta x_{10}\right)<\tau_1\Delta x_{10};\end{aligned}$$

$$\frac{x_1+\Delta x_{11}}{\Delta x_{11}}>\frac{\tau_1}{\Delta\tau_1}>\frac{x_1+\Delta x_{11}}{\Delta x_{01}+\Delta x_{11}}\ \wedge\ \frac{\tau_1}{\Delta\tau_1}>\frac{x_1+\Delta x_{10}}{\Delta x_{10}};$$

$$\max\left(\frac{x_1}{\Delta x_{10}};\ \frac{x_1-\Delta x_{01}}{\Delta x_{11}+\Delta x_{01}}\right)<\frac{\tau_1}{\Delta\tau_1}-1<\frac{x_1}{\Delta x_{11}}\ .$$

$$\tau_2\left(x_2-\Delta x_{10}\right)<\left(\tau_2-\Delta\tau_2\right)\left(x_2-\Delta x_{11}\right)<\tau_2 x_2<\left(\tau_2-\Delta\tau_2\right)\left(x_2-\Delta x_{01}\right);$$

$$\begin{aligned}&\tau_2 x_2-\tau_2\Delta x_{10}<\tau_2 x_2-\Delta\tau_2 x_2-\tau_2\Delta x_{11}+\Delta\tau_2\Delta x_{11}\ \wedge\\ &\wedge\ \tau_2 x_2-\Delta\tau_2 x_2-\tau_2\Delta x_{11}+\Delta\tau_2\Delta x_{11}<\tau_2 x_2\ \wedge\\ &\wedge\ \tau_2 x_2<\tau_2 x_2-\Delta\tau_2 x_2-\tau_2\Delta x_{01}+\Delta\tau_2\Delta x_{01};\end{aligned}$$

$$\begin{aligned}&\Delta\tau_2 x_2-\Delta\tau_2\Delta x_{11}<\tau_2\Delta x_{10}-\tau_2\Delta x_{11}\ \wedge\\ &\wedge\ \Delta\tau_2\Delta x_{11}-\Delta\tau_2 x_2<\tau_2\Delta x_{11}\ \wedge\\ &\wedge\ \tau_2\Delta x_{01}<\Delta\tau_2\Delta x_{01}-\Delta\tau_2 x_2;\end{aligned}$$

$$\Delta\tau_2\left(x_2-\Delta x_{11}\right)<\tau_2\left(\Delta x_{10}-\Delta x_{11}\right)\wedge$$
$$\wedge\ \Delta\tau_2\left(\Delta x_{11}-x_2\right)<\tau_2\Delta x_{11}\ \wedge$$
$$\wedge\ \tau_2\Delta x_{01}<\Delta\tau_2\left(\Delta x_{01}-x_2\right);$$

$$\frac{x_2-\Delta x_{11}}{\Delta x_{10}-\Delta x_{11}}<\frac{\tau_2}{\Delta\tau_2}\wedge\frac{\Delta x_{11}-x_2}{\Delta x_{11}}<\frac{\tau_2}{\Delta\tau_2}<\frac{\Delta x_{01}-x_2}{\Delta x_{01}};$$

$$\frac{x_2}{\Delta x_{01}}<1-\frac{\tau_2}{\Delta\tau_2}<\min\left(\frac{\Delta x_{10}-x_2}{\Delta x_{10}-\Delta x_{11}};\ \frac{x_2}{\Delta x_{11}}\right).$$

Case 3. Resulting factor "value of budgetary receipts with regard given the dynamics of attraction of additional investment"

$$(\tau_1+1)(x_1-\Delta x_{01})<(\tau_1-\Delta\tau_1+1)(x_1+\Delta x_{11})<(\tau_1+1)x_1<(\tau_1-\Delta\tau_1+1)(x_1+\Delta x_{10});$$

$$\tau_1x_1+x_1-\tau_1\Delta x_{01}-\Delta x_{01}<\tau_1x_1+x_1-\Delta\tau_1x_1+\tau_1\Delta x_{11}+\Delta x_{11}-\Delta\tau_1\Delta x_{11}\ \wedge$$
$$\wedge\ \tau_1x_1+x_1-\Delta\tau_1x_1+\tau_1\Delta x_{11}+\Delta x_{11}-\Delta\tau_1\Delta x_{11}<\tau_1x_1+x_1\ \wedge$$
$$\wedge\ \tau_1x_1+x_1<\tau_1x_1+x_1-\Delta\tau_1x_1+\tau_1\Delta x_{10}+\Delta x_{10}-\Delta\tau_1\Delta x_{10};$$

$$\Delta\tau_1x_1+\Delta\tau_1\Delta x_{11}<\tau_1\Delta x_{01}+\Delta x_{01}+\tau_1\Delta x_{11}+\Delta x_{11}\ \wedge$$
$$\wedge\ \tau_1\Delta x_{11}+\Delta x_{11}<\Delta\tau_1x_1+\Delta\tau_1\Delta x_{11}\ \wedge$$
$$\wedge\ \Delta\tau_1x_1+\Delta\tau_1\Delta x_{10}<\tau_1\Delta x_{10}+\Delta x_{10};$$

$$\Delta\tau_1\left(x_1+\Delta x_{11}\right)<\left(\tau_1+1\right)\left(\Delta x_{01}+\Delta x_{11}\right)\wedge$$
$$\wedge\ \left(\tau_1+1\right)\Delta x_{11}<\Delta\tau_1\left(x_1+\Delta x_{11}\right)\ \wedge$$
$$\wedge\ \Delta\tau_1\left(x_1+\Delta x_{10}\right)<\left(\tau_1+1\right)\Delta x_{10};$$

$$\frac{x_1+\Delta x_{11}}{\Delta x_{11}}>\frac{\tau_1+1}{\Delta\tau_1}>\frac{x_1+\Delta x_{11}}{\Delta x_{01}+\Delta x_{11}}\wedge\frac{\tau_1+1}{\Delta\tau_1}>\frac{x_1+\Delta x_{10}}{\Delta x_{10}};$$

$$(\tau_2+1)(x_2-\Delta x_{10})<(\tau_2-\Delta\tau_2+1)(x_2-\Delta x_{11})<(\tau_2+1)x_2<(\tau_2-\Delta\tau_2+1)(x_2-\Delta x_{01});$$

$$\tau_2x_2+x_2-\tau_2\Delta x_{10}-\Delta x_{10}<\tau_2x_2+x_2-\Delta\tau_2x_2-\tau_2\Delta x_{11}-\Delta x_{11}+\Delta\tau_2\Delta x_{11}\ \wedge$$
$$\wedge\ \tau_2x_2+x_2-\Delta\tau_2x_2-\tau_2\Delta x_{11}-\Delta x_{11}+\Delta\tau_2\Delta x_{11}<\tau_2x_2+x_2\ \wedge$$
$$\wedge\ \tau_2x_2+x_2<\tau_2x_2+x_2-\Delta\tau_2x_2-\tau_2\Delta x_{01}-\Delta x_{01}+\Delta\tau_2\Delta x_{01};$$

$$\Delta\tau_2 x_2 - \Delta\tau_2 \Delta x_{11} < \tau_2 \Delta x_{10} + \Delta x_{10} - \tau_2 \Delta x_{11} - \Delta x_{11} \wedge$$
$$\wedge \Delta\tau_2 \Delta x_{11} - \Delta\tau_2 x_2 < \tau_2 \Delta x_{11} + \Delta x_{11} \wedge$$
$$\wedge \tau_2 \Delta x_{01} + \Delta x_{01} < \Delta\tau_2 \Delta x_{01} - \Delta\tau_2 x_2;$$

$$\Delta\tau_2 \left(x_2 - \Delta x_{11}\right) < \left(\tau_2 + 1\right)\left(\Delta x_{10} - \Delta x_{11}\right) \wedge$$
$$\wedge \Delta\tau_2 \left(\Delta x_{11} - x_2\right) < \left(\tau_2 + 1\right)\Delta x_{11} \wedge$$
$$\wedge \left(\tau_2 + 1\right)\Delta x_{01} < \Delta\tau_2 \left(\Delta x_{01} - x_2\right);$$

$$\frac{x_2 - \Delta x_{11}}{\Delta x_{10} - \Delta x_{11}} < \frac{\tau_2 + 1}{\Delta\tau_2} \wedge \frac{\Delta x_{11} - x_2}{\Delta x_{11}} < \frac{\tau_2 + 1}{\Delta\tau_2} < \frac{\Delta x_{01} - x_2}{\Delta x_{01}};$$

The investors' revenue for the tax-investment equilibrium

The investors' revenue in the 1st economy:

$$r_1 = \left(1 - \tau_1\right) a_1 e^{-\alpha_1 x} x =$$

$$= \left(1 - \tau_1\right) a_1 \frac{1}{\alpha_1 + \alpha_2}\left(\alpha_2 + \ln\frac{a_1}{a_2} + \ln\frac{1 - \tau_1}{1 - \tau_2}\right) e^{-\alpha_1 \frac{1}{\alpha_1 + \alpha_2}\left(\alpha_2 + \ln\frac{a_1}{a_2} + \ln\frac{1-\tau_1}{1-\tau_2}\right)} =$$

$$= \left(1 - \tau_1\right)\frac{a_1}{\alpha_1 + \alpha_2}\left(\alpha_2 + \ln\frac{a_1}{a_2} + \ln\frac{1 - \tau_1}{1 - \tau_2}\right) e^{\left(\alpha_2 + \ln\left(\frac{a_1}{a_2}\frac{1-\tau_1}{1-\tau_2}\right)\right)\left(-\frac{\alpha_1}{\alpha_1 + \alpha_2}\right)} =$$

$$= \left(1 - \tau_1\right)\frac{a_1}{\alpha_1 + \alpha_2}\left(\alpha_2 + \ln\frac{a_1}{a_2} + \ln\frac{1 - \tau_1}{1 - \tau_2}\right)\left(\frac{a_1}{a_2} e^{\alpha_2}\frac{1 - \tau_1}{1 - \tau_2}\right)^{-\frac{\alpha_1}{\alpha_1 + \alpha_2}};$$

$$r_1 = \left(1 - \tau_1\right)\frac{a_1}{\alpha_1 + \alpha_2}\frac{\alpha_2 + \ln\frac{a_1}{a_2} + \ln\frac{1 - \tau_1}{1 - \tau_2}}{\left(e^{\alpha_2}\frac{a_1}{a_2}\cdot\frac{1 - \tau_1}{1 - \tau_2}\right)^{\frac{\alpha_1}{\alpha_1 + \alpha_2}}}.$$

The investors' revenue in the 2nd economy:

$$r_2 = \left(1 - \tau_2\right) a_2 e^{-\alpha_2 (1 - x)}\left(1 - x\right) =$$

$$= \left(1 - \tau_2\right) a_2 \frac{1}{\alpha_1 + \alpha_2}\left(\alpha_1 - \ln\frac{a_1}{a_2} - \ln\frac{1 - \tau_1}{1 - \tau_2}\right) e^{-\alpha_2 \frac{1}{\alpha_1 + \alpha_2}\left(\alpha_1 - \ln\frac{a_1}{a_2} - \ln\frac{1-\tau_1}{1-\tau_2}\right)} =$$

$$=\left(1-\tau_2\right)\frac{a_2}{\alpha_1+\alpha_2}\left(\alpha_1-\ln\frac{a_1}{a_2}-\ln\frac{1-\tau_1}{1-\tau_2}\right)\left(e^{\alpha_1}e^{-\left(\ln\frac{a_1}{a_2}+\ln\frac{1-\tau_1}{1-\tau_2}\right)}\right)^{\left(-\alpha_2\frac{1}{\alpha_1+\alpha_2}\right)};$$

$$r_2=\left(1-\tau_2\right)\frac{a_2}{\alpha_1+\alpha_2}\left(\alpha_1-\ln\frac{a_1}{a_2}-\ln\frac{1-\tau_1}{1-\tau_2}\right)\left(\frac{\frac{a_1}{a_2}\cdot\frac{1-\tau_1}{1-\tau_2}}{e^{\alpha_1}}\right)^{\frac{\alpha_2}{\alpha_1+\alpha_2}}.$$