Differential Fertility, Intergenerational Mobility and the Process of Economic Development

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Abstract

This paper analyzes the effects of population dynamics with differential fertility between the educated and the uneducated on intergenerational mobility, income inequality and economic development in an overlapping generations framework. Population dynamics has two effects on the economy: the direct effect on the educated share through changing in population size of the economy as whole, and the indirect effect on the educated share through decreasing/increasing transfer per child. When population growth increases sufficiently, the mobility and income inequality exhibit cyclical behavior due to rapidly decreasing transfer per child and population size. In contrast, when population growth decreases sufficiently, the mobility and income inequality monotonically approach steady state and the economy has low steady state with high population growth and income inequality, and high steady state with low population growth and income inequality. As a result, population dynamics with economic development plays crucial role in the transitional dynamics of mobility.

JEL classifications: I24, I25, J13, J62

Keywords: Differential the fertility, Intergenerational mobility, Economic development

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1. Introduction

Interest in intergenerational mobility, along with income inequality, is growing in in a wide range of fields, including sociology, demographics and economics. Intergenerational mobility is important not only in considering intergenerational equity and efficiency but also in analyzing economic development. The relationship between fertility and intergenerational mobility has also long been argued in many fields: sociology, demographics, social biology and economics. If parents limit family size, i.e. parents decrease the number of children, transfers per child, educational investment and bequest for children, increases and therefore intergenerational upward-mobility will increase. This hypothesis is called "family size limitation hypothesis" in literature of intergenerational mobility in sociology.[1] Van Bavel (2006) and Van Bavel et al. (2014) find that family size limitation has strong effect on intergenerational mobility in Belgium. Lindahl also shows that increase in fertility prevents the upward-mobility from the rich (or the educated) to the poor (or the uneducated) in Sweden. Although many empirical studies have indicated the strong relationship, there are few theoretical studies explaining these interactions. This paper analyzes that the effects of population dynamics with differential fertility between the educated and the uneducated on intergenerational mobility, income inequality and economic development and shows their interactions.

The seminal work by Maoz and Moav (1999) focus on the incentive of income inequality for acquiring education and analyzes the relationship intergenerational mobility, income inequality and economic growth in simple overlapping generations framework. They show the mobility monotonically increases with decreasing income inequality as the economy grows. Hence, as empirical studies have indicated, they demonstrate that the mobility is negatively correlated with income inequality. In contrast, Nakamura and Murayama (2011) focus on the education cost in Maoz and Moav (1999) model. Assuming general cost function, they show that the behavior of education cost plays crucial role in the transitional dynamics of the mobility. Owen and Weil (1999) and Galo and Zeira (1993) analyze the dynamics of mobility in an economy with imperfect capital market. Bernasconi and Profeta (2012) Uchida (2018) analyzes the effect of private and/or public education on the mobility in the economy with the mismatch of talents. In this frame work, Bernasconi and Profeta (2012) show expanding public education decreases the mismatch and therefore encourages the mobility, while Uchida (2019) indicate that the transitional dynamics of the mobility depend on political power between the educated and the uneducated incorporating private education and voting on public education into Bernasoni and Profeta (2012). Hassler et al. (2007) and Murayama (2018) analyze the effects of fiscal policy on the mobility and income inequality. Hassler et al. (2007) show that educational subsidies make a

[1] In economics, this is known as "Quality and Quantity hypothesis" in the literature of human capital accumulation.

difference in income the mobility and income inequality around each countries. Introducing governmental cash transfers into Maoz and Moav (1999) model, Murayama (2018) show that larger transfers to children with higher ability encourage upward mobility and growth if the economy has low income inequality.

Many previous theoretical studies analyze intergenerational mobility and income inequality by focusing on education system, education cost, liquidity constraint and fiscal policies. To our knowledge, few theoretical papers analyze the relationship between population dynamics and intergenerational mobility. An exception is Aso and Nakamura (2019). Aso and Nakamura (2019) analyze the effect of population growth on the mobility. However, they assume exogenous fertility difference between workers and worker's and exogenous worker's wages and therefore do not adequately analyze the effects of population dynamics with economic development on the mobility and income inequality. In fact, we show population dynamics plays crucial role in the transitional dynamics of mobility, income inequality and economic development and therefore various transitional dynamics of the economy appears in this paper.

The purpose of this paper analyzes the effect of population dynamics on the mobility, income inequality and economic development and show their interactions. For the purpose of our analysis, we employ the (endogenous) differential fertility in Maoz and Moav (1999) model. Our results are summarized as follows. (i) differential fertility between the educated and the uneducated depend crucially on the elasticity of substitution between consumption, transfer for children and the number of children. (ii) Population dynamics has two effect on the economy: the direct effect on the educated share through changing population size of the economy as a whole, and the indirect effect on the educated share through increasing/decreasing transfer per child. (iii) When the elasticity of substitution is enough larger than unity, i.e., the fertility of the educated is enough larger than that of the uneducated and therefore population growth increases sufficiently, the mobility and income inequality exhibit cyclical behavior due to rapidly decreasing population effect and transfer per child. (iv) In contrast, when the elasticity of substitution is enough smaller than unity, i.e., the fertility of the uneducated is enough larger than that of the educated and therefore population growth decreases sufficiently, the mobility and income inequality monotonically approach steady state and the economy may have two steady state: low steady state with high population growth and income inequality, and high steady state with low population growth and income inequality.

In fact, various patterns of intergenerational mobility have been observed in developed countries. Nicoletti and Ermisch (2007) find monotonous decrease in Britain, while Bratberg et al. (2007) indicate monotonous increase in Norway. Aaronson and Mazumber (2008) show that the mobility has changed non-monotonous motion in the USA. This paper presents one contribution to their explanation.

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 analyzes the transitional dynamics of the economy. Section 4 examines the transitional dynamics of the economy using numerical analysis. Section 5 discusses the results and concludes the paper.

2. The model

Consider the competitive equilibrium of an overlapping generations economy with endogenous fertility. Each individual lives for two periods. In the first period, she does not work, receives a transfer from her parent. I t is used for consumption and possible education. In the second period, she has children, works and divides her income between consumption, child costs and transfer for her children.

2.1 Technology and factor prices

we assume that aggregate output in period t is characterized by the following constant returns to scale production function.

$$Y_t = AE_t^{1-\alpha}U_t^{\alpha}, \qquad A > 0, \quad 0 < \alpha < 1 \tag{1}$$

where E_t is the number of educated workers, and U_t is the number of uneducated workers. Adult population is in period t, $N_t = E_t + U_t$. Therefore, per capita output becomes

$$y_t = A\lambda_t^{\alpha}(1-\lambda_t)^{1-\alpha}, \tag{2}$$

where $\lambda_t = E_t/N_t$ is the population share of the educated. Similarly, $(1-\lambda_t) = U_t/N_t$ is the population share of the uneducated worker. Hence, we have following in equilibrium:

$$w_t^e = (1-\alpha)A\left(\frac{1-\lambda_t}{\lambda_t}\right)^{\alpha} \tag{3}$$

$$w_t^u = \alpha A\left(\frac{1-\lambda_t}{\lambda_t}\right)^{\alpha-1} \tag{4}$$

where subscripts e and u denote "educated" and "uneducated," respectively. Hence, the wage inequality becomes:

$$\frac{w_t^e}{w_t^u} = \frac{1-\alpha}{\alpha}\left(\frac{1-\lambda_t}{\lambda_t}\right). \tag{5}$$

To ensure that $w_t^e > w_t^u$, we assume that $\lambda_t < 1-\alpha$.

2.2 Individuals

Individuals derive utility from consumption c_t of an individual born in period t, consumption c_{t+1} in period $t+1$, transfer for children b_{t+1} and their number of children n_{t+1}. The preference is assumed to be represented by CES (Constant Elasticity of Substitution) type utility function:

$$u_t = c_t \left\{ \left[(c_{t+1})^{\beta} (b_{t+1})^{1-\beta} \right]^{\frac{\sigma-1}{\sigma}} + (n_{t+1})^{\frac{\sigma-1}{\sigma}} \right\}^{\frac{\sigma}{\sigma-1}}, \tag{6}$$

where $0 < \beta < 1$ and $\sigma > 0$. σ stands for the elasticity of substitution between $(c_{t+1})^{\beta}(b_{t+1})^{1-\beta}$ and n_{t+1}. To focus on the role of σ, we assume that the marginal rate of substitution between consumption and transfer is independent demand for children n_{t+1}.[2]

Let h_t^i and w_{t+1}^i denote the education cost of individual i, born in period t and her wage in period $t+1$. Assuming no capital market in the economy, if she acquires education, her budget constraints are

$$c_t^i + h_t^i = x_t^i, \qquad c_{t+1}^i + b_{t+1}^i + \gamma n_{t+1}^i w_{t+1}^e = w_{t+1}^e, \tag{7.a}$$

where $i \in \{e, u\}$, subscript e stands for "educated" and subscript u stands for "uneducated". $x_t^i = b_t^i / n_t^i$ is transfer per child and $\gamma n_{t+1}^i w_{t+1}^i$ is non-negligible cost for rearing children, where $0 < \gamma < 1$. We assume that $n_{t+1}^i < 1/\gamma$ always holds in the following. If she does not acquire education, her budget constraint become

$$c_t^i = x_t^i, \qquad c_{t+1}^i + b_{t+1}^i + \gamma n_{t+1}^i w_{t+1}^u = w_{t+1}^u, \tag{7.b}$$

Since we assume there is no capital market, the utility maximization problem can be solved backwards in two stages. First, each individual considers optimal allocation in the second period. Then, each individual decides whether to acquire education in the first period.

Suppose that the utility maximization problem of the second period. Following Nakamura (2018), we can solve the utility maximization of second period in two steps. At the first step, each individual determines optimal consumption c_{t+1}^i and transfer for children b_{t+1}^i assuming that the number of children n_{t+1}^i has been optimally chosen.

$$\max_{c_{t+1}^i, b_{t+1}^i} \left(c_{t+1}^i\right)^{\beta} \left(b_{t+1}^i\right)^{1-\beta} \qquad \textit{subject to } c_{t+1}^i + b_{t+1}^i + \gamma n_{t+1}^i w_{t+1}^i = w_{t+1}^i,$$

Hence, optimal consumption and transfer for children become

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[2] With $\sigma \to 1$, the utility function becomes the Cobb-Douglas function. This type of utility function is used in Borck (2011) and Nakamura (2018).

$$c_{t+1}^i = \beta\left(w_{t+1}^i - \gamma n_{t+1}^i w_{t+1}^i\right), \tag{8}$$

$$b_{t+1}^i = (1-\beta)\left(w_{t+1}^i - \gamma n_{t+1}^i w_{t+1}^i\right). \tag{9}$$

Substituting (8) and (9) into the utility function of the second period, we can represent it as a function of n_{t+1}^i as follows.

$$v\left(n_{t+1}^i\right) = \left[\tilde{\beta}\left(w_{t+1}^i - \gamma n_{t+1}^i w_{t+1}^i\right)^{\frac{\sigma-1}{\sigma}} + \left(n_{t+1}^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \tag{10}$$

where $\tilde{\beta} = \left[\beta^{\beta}(1-\beta)^{1-\beta}\right]^{\frac{\sigma-1}{\sigma}}$. At the second step, each individual chooses the number of children n_{t+1}^i so as to maximize (10). Hence,

$$\max_{n_{t+1}^i} \left[\tilde{\beta}\left(w_{t+1}^i - \gamma n_{t+1}^i w_{t+1}^i\right)^{\frac{\sigma-1}{\sigma}} + \left(n_{t+1}^i\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

The first-order condition is

$$n_{t+1}^i = \left(\tilde{\beta}\gamma w_{t+1}^i\right)^{-\sigma}\left[w_{t+1}^i - \gamma n_{t+1}^i w_{t+1}^i\right] \tag{11}$$

From (11), we obtain the following:

$$n_{t+1}^i = \frac{w_{t+1}^i}{\gamma w_{t+1}^i + \left(\tilde{\beta}\gamma w_{t+1}^i\right)^{\sigma}}. \tag{12}$$

From (9) and (12), transfer per child become

$$x_t^i = \frac{b_t^i}{n_t^i} = (1-\beta)\left(\tilde{\beta}\gamma w_t^i\right)^{\sigma} \tag{13}$$

Differentiating (12) with respect to w_{t+1}^i, we obtain

$$\frac{\partial n_{t+1}^i}{\partial w_{t+1}^i} = \frac{\left(\tilde{\beta}\gamma w_{t+1}^i\right)^{\sigma}(1-\sigma)}{\left[\gamma w_{t+1}^i + \left(\tilde{\beta}\gamma w_{t+1}^i\right)^{\sigma}\right]^2} \gtreqless 0 \iff \sigma \lesseqgtr 1, \tag{14}$$

When income rises, whether the number of children increases or decreases depend on the elasticity of substitution σ. Since c_{t+1}^i, b_{t+1}^i and n_{t+1}^i is normal goods, rise in income increases the number of children due to income effect. On the other hand, rise in income has substitution effect. A rise in income increases the cost of rearing children and therefore each individual substitutes the number of children by consumption and transfer for children to prevent a decline in utility.[3] Hence, we have following the proposition.

Proposition 1 Differential the fertility between the educated and the uneducated depend on the substitution elasticity σ. When $\sigma < 1$, since the income effect is dominant, the fertility of the educated is larger than that of the uneducated. When $\sigma > 1$, since the substitution effect is dominant, the fertility of the uneducated is larger than that of the educated. When $\sigma = 1$, since the income effect and the substitution effect are offset, the fertility of the educated equals that of the uneducated, that is, there is no differential the fertility between income classes.

2.3 Education choice

In this subsection, we consider individual's education choice in first period. Substituting (12) into (10), we obtain indirect utility function:

$$z(w_{t+1}^i) = \left\{\tilde{\beta}\left[w_{t+1}^i - \gamma n^i(w_{t+1}^i)\, w_{t+1}^i\right]^{\frac{\sigma-1}{\sigma}} + \left[n^i(w_{t+1}^i)\right]^{\frac{\sigma-1}{\sigma}}\right\}^{\frac{\sigma}{\sigma-1}}. \tag{15}$$

If the utility derived from investing in education is higher than or equal to the utility derived from not investing in education, then individual i will acquire education. Thus, if the following holds, then individual i acquires education.

$$\left(x_t^i - h_t^i\right) z(w_{t+1}^e) \geq x_t^i\, z(w_{t+1}^u).$$

or

$$h_t^i \leq x_t^i \left[1 - \frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right]. \tag{16}$$

Hence, we have the following critical value of education cost $\hat{h}_t^i$ for individual i :

$$\hat{h}_t^i = x_t^i \left[1 - \frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right], \tag{17}$$

where $\partial z(w_{t+1}^i)/\partial w_{t+1}^i > 0$ and $z(w_{t+1}^e) > z(w_{t+1}^u)$. In addition to transfer per child x_t^i, as can be seen from (18), the wage inequality w_{t+1}^e/w_{t+1}^u plays an important role in education choice. The higher w_{t+1}^e/w_{t+1}^u is, the higher $z(w_{t+1}^e)/z(w_{t+1}^u)$ is, and therefore the larger the incentive to acquire education.

Suppose that $\hat{h}_t^e$ $(\hat{h}_t^u)$ is the critical value of educational cost for the individual born to

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[3] When $\sigma = 1$, the the fertility between the educated and the uneducated is the same $n_t^i = n_t = 1/2\gamma$ and become constant over time.

an educated (uneducated) worker to acquire education. Then, from (17),

$$\hat{h}_t^e = (1-\beta)\left(\tilde{\beta}\gamma w_t^e\right)^\sigma \left[1 - \frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right], \tag{18}$$

$$\hat{h}_t^u = (1-\beta)\left(\tilde{\beta}\gamma w_t^u\right)^\sigma \left[1 - \frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right]. \tag{19}$$

2.4 Education cost among individuals

Following Maoz and Moav (1999), the following cost is assumed to be incurred for the education of individual i in period t.

$$h_t^i = \theta^i c(\bar{w}_t) = \theta^i (a + b\bar{w}_t), \tag{20}$$

where θ^i is a parameter representing individual $i's$ ability to learn; the higher the ability, the lower is the value of θ^i. We further assume that θ^i is uniformly distributed in the interval $\left(\underline{\theta}, \overline{\theta}\right)$, regardless of the ability and class of the parents in any period. Hence, h_t^i is also uniformly distributed in the interval $\left(\underline{h}_t, \overline{h}_t\right)$, where $\underline{h}_t = \underline{\theta}(a + b\bar{w}_t)$ and $\overline{h}_t = \overline{\theta}(a + b\bar{w}_t)$.

2.5 Population growth

Adult population in period $t+1$ is $N_{t+1} = E_{t+1} + U_{t+1}$ or $N_{t+1} = n_t^e E_t + n_t^u U_t$. Therefore, the population growth as a whole in period t, $\bar{n}_t$ becomes

$$\bar{n}_t = \bar{n}(\lambda_t) = n_t^e \lambda_t + n_t^u (1-\lambda_t). \tag{21}$$

When the educated share λ_t increases, the behavior of population growth $\bar{n}_t$ can be expressed as follows.

$$\frac{\partial \bar{n}(\lambda_t)}{\partial \lambda_t} = \left[(1-\lambda_t)\frac{\partial n_t^u}{\partial w_t^u}\frac{\partial w_t^u}{\partial \lambda_t} - \lambda_t \frac{\partial n_t^e}{\partial w_t^e}\frac{\partial w_t^e}{\partial \lambda_t}\right] + (n_t^e - n_t^u) \tag{22}$$

From (14), the following holds:

$$(1-\lambda_t)\frac{\partial n_t^u}{\partial w_t^u}\frac{\partial w_t^u}{\partial \lambda_t} - \lambda_t \frac{\partial n_t^e}{\partial w_t^e}\frac{\partial w_t^e}{\partial \lambda_t} \gtreqless 0 \iff \sigma \lesseqgtr 1, \tag{23}$$

$$n_t^e - n_t^u \gtreqless 0 \iff \sigma \lesseqgtr 1.$$

Thus, the behavior of population growth $\bar{n}_t$ becomes

$$\frac{\partial \bar{n}(\lambda_t)}{\partial \lambda_t} \gtreqless 0 \iff \sigma \lesseqgtr 1 \tag{24}$$

As can be seen from (24), the elasticity of substitution σ plays crucial role in dynamics of population growth. Thus, we have the following proposition.

Proposition 2 The dynamics of population growth depends on substitution elasticity σ. When $\sigma < 1 (\sigma > 1)$, since the fertility of the educated is larger (smaller) than that of the uneducated, population growth increases (decreases) with increase in the educated share. When $\sigma = 1$, population growth is constant over since the fertility between income classes is the same.

3. The dynamics of the model

In this section, we analyze the dynamics of intergenerational mobility, income inequality and population growth. At the first step, we show intergenerational mobility in the model.

3.1 Intergenerational mobility

The dynamics of E_t can be expressed as

$$E_{t+1} = n_t^e E_t \, p_t + n_t^u U_t \, q_t \, , \tag{25}$$

where

$$p_t = \frac{\hat{h}_t^e - \underline{h}_t}{\bar{h}_t - \underline{h}_t}, \qquad\qquad q_t = \frac{\hat{h}_t^u - \underline{h}_t}{\bar{h}_t - \underline{h}_t}, \tag{26}$$

The first term in the right-hand side of (25) represents individuals born to the educated acquiring education, while the second term represents individuals born to the uneducated acquiring education. The dynamics of the educated share λ_t is obtained by dividing both sides by the total working population in period $t+1$; $N_{t+1} = \bar{n}(\lambda_t) N_t$.

$$\lambda_{t+1} = \lambda_t \, \frac{n_t^e}{\bar{n}(\lambda_t)} \, p_t + (1 - \lambda_t) \, \frac{n_t^u}{\bar{n}(\lambda_t)} \, q_t \, . \tag{27}$$

From (22), if $\lambda_t = 0$, then $\lambda_{t+1} = 0$. Eq. (27) can be rewritten as follows:

$$\lambda_{t+1} - \lambda_t = \underbrace{(1 - \lambda_t) \, \frac{n_t^u}{\bar{n}(\lambda_t)} \, q_t}_{UM_t} - \underbrace{\left[\lambda_t - \lambda_t \, \frac{n_t^e}{\bar{n}(\lambda_t)} \, p_t\right]}_{GDM_t}, \tag{28}$$

where UM_t is *upward-mobility* and GDM_t is *gross downward-mobility*. In our model, upward-mobility means that individuals born to an uneducated parent become educated workers, while gross downward-mobility means that how much individuals born to educated parent increases or decreases compared to educated workers of parents' generation. Upward-mobility equals to gross downward-mobility in steady state; $UM_t = GDM_t$. Moreover, we can rewrite (28) as follows.

$$\lambda_{t+1} - \lambda_t = \underbrace{(1-\lambda_t)\frac{n_t^u}{\bar{n}(\lambda_t)}q_t}_{UM_t} - \underbrace{\lambda_t(1-p_t)\frac{n_t^e}{\bar{n}(\lambda_t)}}_{DM_t} - \underbrace{\lambda_t\left[1-\frac{n_t^e}{\bar{n}(\lambda_t)}\right]}_{PE_t}, \tag{29}$$

where DM_t is *(net) downward-mobility* and PE_t is *population effect*. Downward-mobility means that individuals born to an educated parent become uneducated workers. On the other hand, population effect represents the effect of increase or decrease in population growth on the educated share in the economy. Increase (decrease) in population growth, regardless of the mobility, directly decreases (increases) the educated share through increasing (decreasing) in population size of the economy as a whole. We call the effect "*Population effect*" and define the sum of the downward-mobility and the population effect as the gross downward-mobility in this paper. When $\sigma < 1$ ($\sigma > 1$), i.e., the fertility of the educated is larger (smaller) than population growth; population effect is negative (positive); $PE_t < 0$ ($PE_t > 0$).[4] It implies that λ_{t+1} is larger than λ_t since $n_t^e > (<)\bar{n}(\lambda_t)$. Hence, the larger difference between the fertility of the educated and population growth facilitates larger population effect, while population effect is smaller with increase in the educated share since the difference narrows.

If there is no upward-mobility and downward-mobility ($q_t \leq 1$ and $p_t \geq 1$) in the economy, then the educated share in period $t+1$; λ_{t+1} depends on only the population effect.[5]

$$\lambda_{t+1} = \lambda_t \frac{n_t^e}{\bar{n}(\lambda_t)}. \tag{30}$$

Hence, whether the educated share in period $t+1$; λ_{t+1} increases or decreases compared to the educated share in period t; λ_t depends on whether the fertility of the educated is larger than population growth. If $q_t \leq 0$ *and* $p_t \geq 1$, then

$$\lambda_{t+1} \gtreqless \lambda_t \iff n_t^e \gtreqless \bar{n}(\lambda_t) \iff \sigma \lesseqgtr 1. \tag{31}$$

Thus, when $\sigma > 1$, if there is no upward-mobility in the economy, the economy falls into a poverty trap due to population effect. In this paper, poverty trap refers to a state in which the

[4] If $n_t^e \gtreqless \bar{n}(\lambda_t)$, then $n_t^e/\bar{n}(\lambda_t) \gtreqless 1$ and therefore $PE_t \lesseqgtr 0$.
[5] If there is no population effect, (30) is $\lambda_{t+1} = \lambda_t$.

educated share decreases due to population effect and approaches $\lambda_t = 0$ in the long run.[6]

Proposition 3 When $\sigma < 1$, since the fertility of the educated is larger than population growth, the economy does not fall into poverty trap due to population growth even without upward-mobility. In contrast, when $\sigma > 1$ and there is no upward-mobility, since the fertility of the population growth is larger than the fertility of the educated, the economy falls into poverty trap due to population growth.

Taking into account $\overline{h}_t = \overline{\theta} c(\overline{w}_t)$, $\underline{h}_t = \underline{\theta} c(\overline{w}_t)$, (18) and (19), Eq. (27) can be written as follows:

$$\lambda_{t+1} = \frac{f(\lambda_{t+1})}{(\overline{\theta} - \underline{\theta})} \frac{\overline{b}(\lambda_t)}{c(y_t)\overline{n}(\lambda_t)} - \frac{\underline{\theta}}{\overline{\theta} - \underline{\theta}}, \tag{32}$$

where $f(\lambda_{t+1}) = [1 - z(w_{t+1}^u)/z(w_{t+1}^e)]$ and $\overline{b}(\lambda_t) = \lambda_t b_t^e + (1 - \lambda_t) b_t^u$ is average of transfer per child.

3.2 Transitional dynamics

The dynamics of λ_t is given by (32). We can express Eq. (32) as an implicit function.

$$G(\lambda_{t+1}, \lambda_t) = \lambda_{t+1} - \frac{f(\lambda_{t+1})}{(\overline{\theta} - \underline{\theta})} \frac{\overline{b}(\lambda_t)}{c(y_t)\overline{n}(\lambda_t)} + \frac{\underline{\theta}}{\overline{\theta} - \underline{\theta}} = 0 \tag{33}$$

Investigating Eq. (33), we can see the dynamic behavior of intergenerational mobility, inequality, population dynamics and the process of economic development. Totally differentiating Eq. (33),

$$G_1 d\lambda_{t+1} = G_2 d\lambda_t, \tag{34}$$

where

$$G_1 = 1 - \frac{f'(\lambda_{t+1})}{(\overline{\theta} - \underline{\theta})} \frac{\overline{b}(\lambda_t)}{c(y_t)\, \overline{n}(\lambda_t)} > 0, \tag{35}$$

and $f'(\lambda_{t+1}) < 0$, and therefore $G_1 > 0$,

$$G_2 = \frac{f(\lambda_{t+1})}{(\overline{\theta} - \underline{\theta})} \frac{\varepsilon_t^{\overline{b}} - \varepsilon_t^c - \varepsilon_t^{\overline{n}}}{c(y_t)\, \overline{n}(\lambda_t)\, \lambda_t / \, \overline{b}(\lambda_t)}, \tag{36}$$

and

[6] In section 4, we illustrate an example of poverty trap using numerical analysis.

$$\varepsilon_t^{\bar{b}} = \frac{\partial \bar{b}(\lambda_t)/\bar{b}(\lambda_t)}{\partial \lambda_t/\lambda_t} = \bar{b}'(\lambda_t)\,\frac{\lambda_t}{\bar{b}(\lambda_t)} > 0, \tag{37}$$

$$\varepsilon_t^{c} = \frac{\partial c(y_t)/c(y_t)}{\partial \lambda_t/\lambda_t} = c'(y_t)\,y_t'\,\frac{\lambda_t}{c(y_t)} > 0, \tag{38}$$

$$\varepsilon_t^{\bar{n}} = \frac{\partial \bar{n}(\lambda_t)/\bar{n}(\lambda_t)}{\partial \lambda_t/\lambda_t} = \bar{n}'(\lambda_t)\,\frac{\lambda_t}{\bar{n}(\lambda_t)} \gtreqless 0 \quad \Leftrightarrow \quad \sigma \lesseqgtr 1. \tag{39}$$

$\varepsilon_t^{\bar{b}}$ represents the elasticity of average of transfer per child with respect to the share of the educated in period t, ε_t^{c} represents the elasticity of education cost with respect to the share of the educated in period t and $\varepsilon_t^{\bar{n}}$ represents the elasticity of population growth with respect to the share of the educated in period t. Since $G_1 > 0$, the behavior of λ_t depend on sign of G_2, i.e., the sum of each elasticity.

$$sign\left[\frac{d\lambda_{t+1}}{d\lambda_t}\right] = sign\left[\frac{G_2}{G_1}\right] = sign\left[\varepsilon_t^{\bar{b}} - \varepsilon_t^{c} - \varepsilon_t^{\bar{n}}\right]. \tag{40}$$

In particular, the elasticity of population growth with respect to the share of the educated $\varepsilon_t^{\bar{n}}$ plays an important role in the transitional dynamics of λ_t. Population growth directly influences the educated share through changing in population size of the economy as a whole. On the other hand, increasing in population growth decreases transfer per child and therefore discourages the mobility. When population growth decreases, the opposite holds. If population growth decreases sufficiently , then $G_2/G_1 = \left[\varepsilon_t^{\bar{b}} - \varepsilon_t^{c} - \varepsilon_t^{\bar{n}}\right] > 0$; and hence Eq. (29) is upwards-sloping in the $(\lambda_t, \lambda_{t+1})$ plane. In contrast, if population growth increases sufficiently, then $G_2/G_1 = \left[\varepsilon_t^{\bar{b}} - \varepsilon_t^{c} - \varepsilon_t^{\bar{n}}\right] < 0$; and hence, Eq. (30) is downwards-sloping in the $(\lambda_t, \lambda_{t+1})$ plane. As a result, the following proposition holds about the transitional dynamics of intergenerational mobility, income inequality and population growth.[7]

Proposition 4 The transitional dynamics of intergenerational mobility depends on the sum of each elasticities, i.e., the elasticity of average of transfer per child with respect to the share of the educated, the elasticity of education cost with respect to the share of the educated and the elasticity of population growth with respect to the share of the educated. In particular, when the fertility of the uneducated is greatly larger than that of the educated, that is, population growth decreases

[7] When the fertility between income classes is the same, population growth does not influence the basic dynamics of the economy. Then, the transitional dynamics of the mobility depends on income share of education cost.

sufficiently, the mobility monotonically increases and therefore income inequality decreases. In contrast, when the fertility of the educated is greatly larger than that of the uneducated, that is, population growth increases sufficiently, the mobility and income inequality exhibit cyclical behavior.

4. Numerical analysis

As the analysis in the previous section shows, the mobility and income inequality depend on population dynamics. In this section, we use numerical analysis to illustrate the Proposition 4 and to show complex the transitional dynamics of the economy with population dynamics. We take the parameter values to satisfy $n_t^i < 1/\gamma$. Except for $A = 50$ and $\beta = 0.5$, they are given by $\alpha = 0.5$, $\gamma = 0.3$, $\overline{\theta} = 5$, $\underline{\theta} = 1$ following Maoz and Moav (1999) and Nakamura (2018).

4.1 Constant population growth $(\boldsymbol{\sigma = 1})$

When $\sigma = 1$, the fertility between income classes is the same, that is, population growth is constant. We analyze it as the benchmark case. Fig.1 demonstrates the transitional dynamics of λ_t in the case of constant population growth. When the fertility between income classes, the population growth does not influence the basic dynamics of λ_t, and therefore economy has not population effect in Fig.1.

In Fig.1(a), the educated share monotonically increases with upward mobility and therefore income inequality decreases. If initial educated share is small and education cost is large sufficiently, as can be shown in Fig.1(b), the educated share does not change in the long run since there is no population effect. However, if initial the educated share is large sufficiently, the educated share increases with upward mobility. As a result, the mobility approaches toward steady state.

Numerical result 1 When $\sigma = 1$, the economy has no population effect. If the education cost is high and hence upward-mobility does not occur at an early stage of the economy, the economy does not develop at all since there is no population effect. In contrast, if the upward-mobility occurs, the mobility and income inequality monotonically approach steady state.

[Insert Fig.1 about here]

4.2 Increasing population growth $(\boldsymbol{\sigma < 1})$

When $\sigma < 1$, the fertility of the educated is larger than that of the uneducated, and therefore population growth increases with increase in the educated share. Increase in population growth discourages the mobility due to decreasing (positive) population effect for the educated share and transfer per child.

Fig.2 illustrates the transitional dynamics of mobility when $\sigma < 1$. As can be shown in Fig.2(a), when the elasticity of substitution is small sufficiently, that is, population growth greatly increases with increase in the educates share, the mobility exhibits cyclical behavior. The intuition behind the cyclical behavior is as follows. When λ_t is small, education cost is also small. Although the educated have many children, many children born to the educates acquire education since educated worker's wages is high and therefore children born to the educated receive a large enough transfer. The gross downward-mobility is negative due to population effect. It implies that the educated increases due to population effect. In addition, since the elasticity of substitution is small, the fertility of the uneducated is also small. This encourages upward-mobility since each individual born to the uneducated receives larger transfer per child. In summary, when λ_t is small, the educated share greatly increases due to population effect and upward-mobility. This sharp increase in λ_t, in turn, increases education cost and population growth. Increase in population discourages the mobility growth due to decreasing average transfer and population effect. As a result, λ_t in next period decreases due to increase in population growth and education cost. These observations explain the cyclical behavior.

Fig.2(b) and Fig.2(c) demonstrates the dynamics of $\sigma = 0.75$ and $\sigma = 0.95$, respectively. As the elasticity of substitution approaches unity, the direct and indirect effect of population growth on the mobility is smaller. Thus, when $\sigma = 0.75$, the fluctuations of economy are smaller. When value of the elasticity of substitution is almost unity ($\sigma = 0.95$), the mobility monotonically increases since differential fertility between income classes is quite small.

Finally, as shown in Proposition 3, we illustrate that the economy does not fall into poverty trap when $\sigma < 1$. Fig.2(d) demonstrates that education cost is high sufficiently and hence there is no upward mobility at an early stage of economy. In contrast to Fig.1 (b) in the case of $\sigma = 1$, the economy develops due to population effect without falling into the poverty trap. We summarize the results below.

Numerical result 2 When the elasticity of substitution is smaller than unity, as the elasticity approaches toward unity, the transitional dynamics of mobility changes from non-monotonous behavior to monotonous behavior and the educated share in steady state is larger since the effect of population dynamics on the mobility is smaller. In addition, even if the education cost is high and hence upward-mobility does not occur at an early stage of the economy, the economy does not fall into poverty trap due to population effect.

[Insert Fig.2 about here]

4.3 Decreasing population growth ($\sigma > 1$)

When $\sigma > 1$, the fertility of the uneducated is larger than that of the educated, and therefore population growth decreases with increase in the educated share. Decrease in population growth encourages the mobility due to decreasing (negative) population effect for the educated share and increasing transfer per child.

Fig.3 illustrates the transitional dynamics of mobility when $\sigma = 1.3$ and $\sigma = 1.6$, respectively. When $\sigma > 1$, the gross downward-mobility is positive value due to population effect. Thus, if the upward-mobility does not occur at an early stage of the economy, in other words, if $\lambda_t < \lambda_1^*$, then the economy falls into poverty trap. If $\lambda_t > \lambda_1^*$, the educated share increases and hence the economy converges to λ_2^* with decrease in income inequality and population growth.

When $\sigma > 1$, since increase in σ increases the fertility of the uneducated and (negative) population effect and hence discourages the upward mobility and facilitates downward-mobility, it promotes the economy into poverty trap as can be shown in Fig.3(b). In addition, when the larger $\sigma > 1$, the fertility of uneducated rapidly decreases with increase in uneducated worker's wage due to increase in the educated share and hence the upward-mobility sharply increases. Hence, when the larger $\sigma > 1$, the educated share increases in both high-equilibrium and low-equilibrium and therefore population growth and income inequality is lower in both equilibriums. In summary, we have the following the result.

Numerical result 3 When the elasticity of substitution is larger than unity, the economy has three equilibrium: the stable high-equilibrium with low population growth and income inequality, the unstable low-equilibrium with high population growth and income inequality and poverty trap equilibrium, where the educated share is zero in the long run. As the elasticity of substitution increases, when the elasticity of substitution is larger than unity, an increase in the elasticity of substitution promotes the economy into poverty trap, while the educated share in the equilibrium increases due to rapid decrease in the fertility of the uneducated with income, and therefore population growth and income inequality decrease in the equilibrium.

[Insert Fig.3 about here]

Then, suppose that the upward mobility occurs at an early of the economy, that is, the economy does not fall into poverty trap. A policy to get out of poverty trap is for the government to reduce the education cost for individuals with high ability. To simplify the analysis, we assume that the government reduce the education cost for individuals with high ability $\underline{\theta}$ to encourage the upward-mobility.

As can be shown in Fig.4 compared to Fig.3, Upward-mobility occurs at an early of the economy and therefore the economy develops without falling into poverty trap. Hence, it is important that the government supports individual with high ability so that the economy does not fall into the poverty trap. However, even if the government implements such a policy, the economy may converge to stable low-equilibrium, that is, the economy may fall into development trap, when σ is large significantly as can be illustrated in Fig.5. Hence, multiple steady states appear in the economy when σ is large significantly: the high steady state with low population growth and small income inequality and the low steady state with high population growth and income inequality.

[Insert Fig.4 about here]

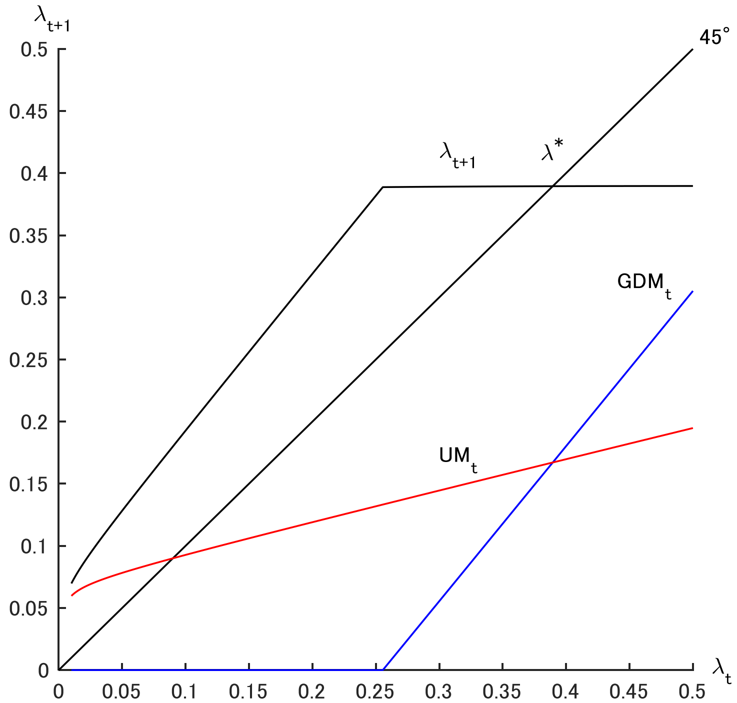
The intuition behind the multiple steady states appear as follows. Although high ability children born to the uneducated become the educated since $\underline{h}_t$ is low, the larger fertility of the uneducated discourages the upward-mobility due to larger σ. In addition, larger population effect increases significantly gross downward-mobility due to low the fertility of the educated and high population growth. As a result, the economy falls into development trap since gross downward mobility is larger than upward-mobility. When the economy develops sufficiently, the fertility of the uneducated sharply decreases with increase in wage and therefore the transfer per child sharply increases. Increase in them encourages upward-mobility. In contrast, since the difference between the fertility of the educated and population growth with economic development, population growth effect decreases and therefore gross downward-mobility also decreases. As a result, the economy develops and converges the high steady state since upward-mobility is larger than gross downward-mobility. Hence, the rapid change in population dynamics with economic development generates the multiple steady states. This result implies that population dynamics plays a crucial role in the mobility and economic development. In summary, we have the following the result.

Numerical result 4 If the government implements a policy to support individuals with high ability, then the economy can get out of the poverty trap. However, when the elasticity of substitution is larger significantly, the economy has low steady state with higher population growth and income inequality and high steady state with lower population growth and income inequality due to rapid change in population dynamics.

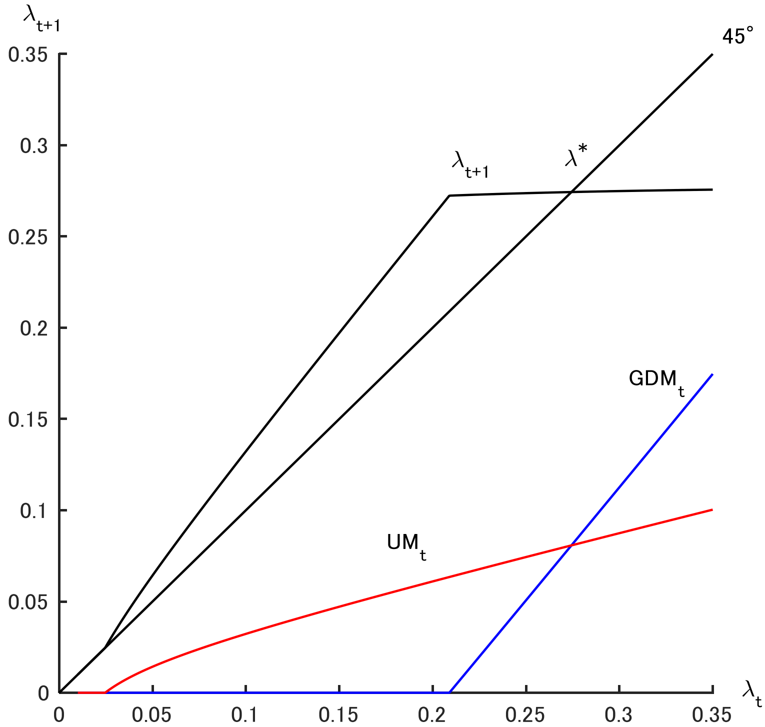
[Insert Fig.5 about here]

5. Conclusions

As many studies indicate, the fertility influences significantly intergenerational mobility. Incorporating (endogenous) differential fertility into overlapping generations model, this paper analyzes the effect of population dynamics on the mobility, income inequality and economic development and shows their interactions. Population dynamics has two effect on the economy: the direct effect on the educated share through changing population size of the economy as a whole, and the indirect effect on the educated share through increasing/decreasing transfer per child. When the elasticity of substitution is enough larger than unity, i.e., the fertility of the educated is enough larger than that of the uneducated and therefore population growth increases sufficiently, the mobility and income inequality exhibit cyclical behavior due to rapidly decreasing population effect and transfer per child. In contrast, when the elasticity of substitution is enough smaller than unity, i.e., the fertility of the uneducated is enough larger than that of the educated and therefore population growth decreases sufficiently, the mobility and income inequality monotonically approach steady state and the economy may have two steady state: low steady state with high population growth and income inequality, and high steady state with low population growth and income inequality. As a result, this show that population dynamics with differential fertility plays crucial role in transitional dynamics of mobility and economic development.

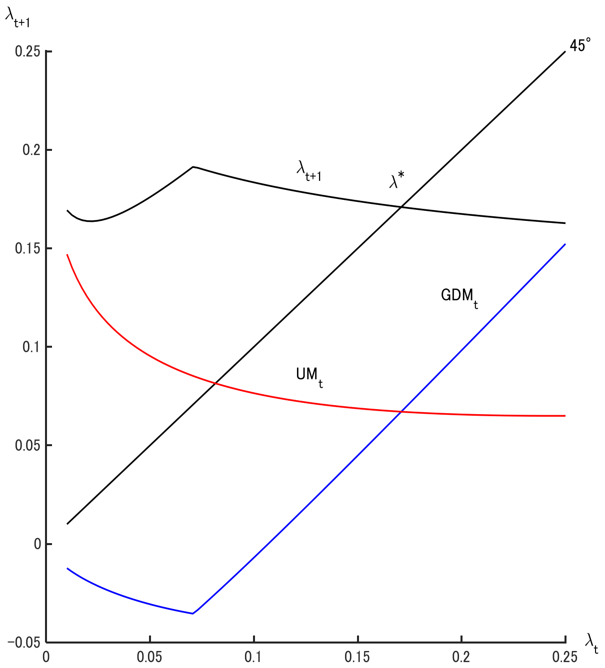


(a) $a = 0.03$, $b = 0.02$, ($\lambda^* = 0.3895$, $n^* = 1.667$)

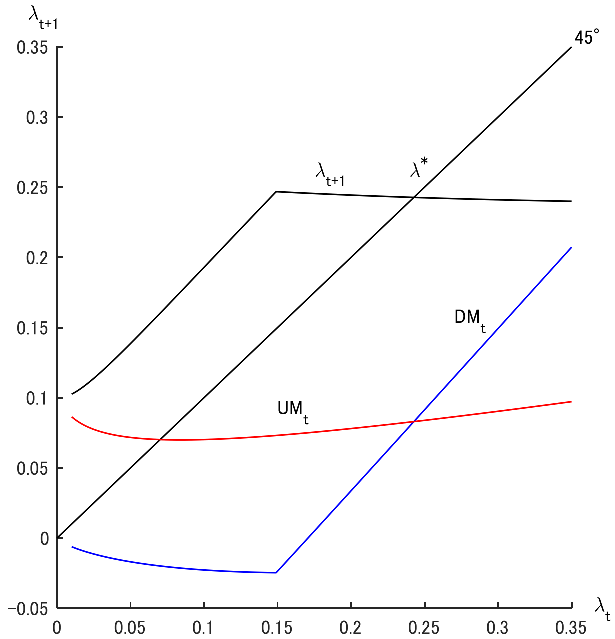


(b) $a = 0.1$, $b = 0.03$, ($\lambda^* = 0.2743$, $n^* = 1.667$)

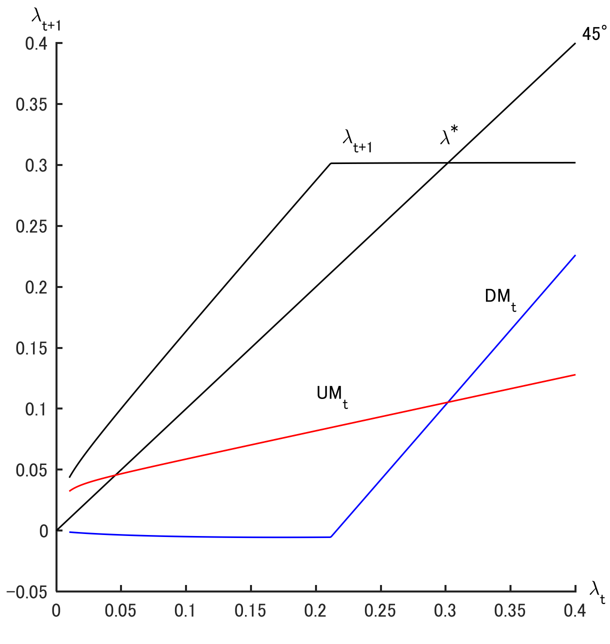
Fig.1 The transitional dynamics of λ_t when $\sigma = 1$



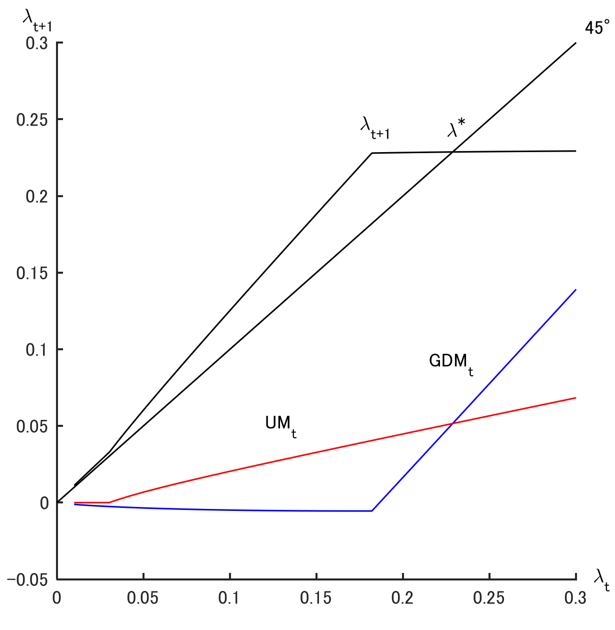
(a) $\sigma = 0.5$, $(\lambda^* = 0.17092,\ \bar{n}^* = 1.9872)$



(b) $\sigma = 0.75$, $(\lambda^* = 0.2427,\ \bar{n}^* = 1.8790)$

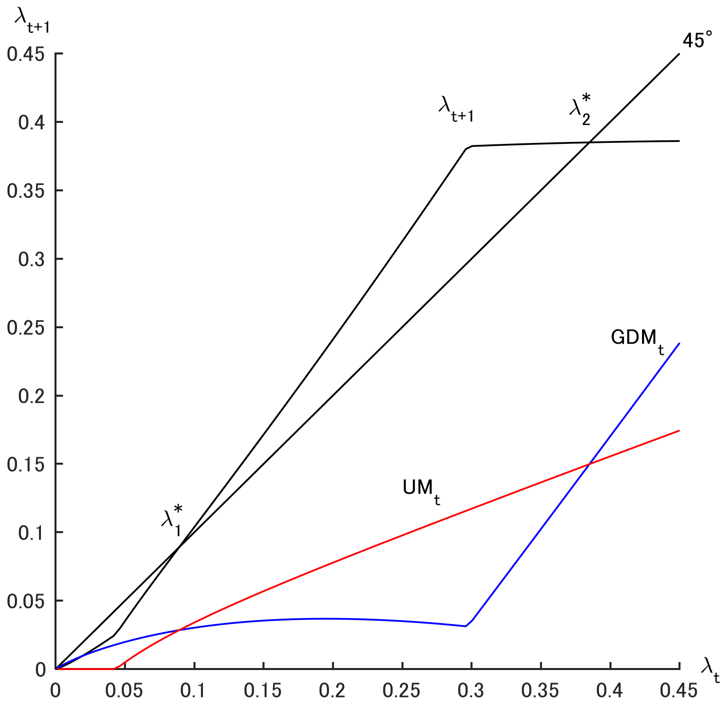


(c) $\sigma = 0.95$, $(\lambda^* = 0.3017\ \ \bar{n}^* = 1.7148)$



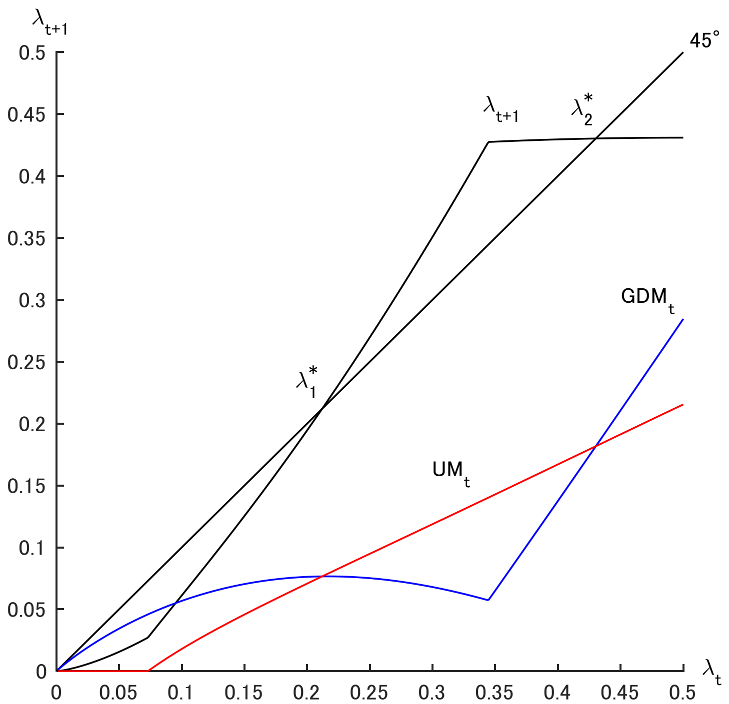
(d) $\sigma = 0.95$, $(\lambda^* = 0.3017\ \ \bar{n}^* = 1.7148)$

Fig.2 The transitional dynamics of λ_t when $\sigma < 1$



(a) $\sigma = 1.3$,

($\lambda_1^* = 0.0891$, $\overline{n}_1^* = 1.5775$, $\lambda_2^* = 0.0891$, $\overline{n}_2^* = 1.5775$)



(b) $\sigma = 1.6$,

($\lambda_1^* = 0.2129$, $\overline{n}_1^* = 1.2185$, $\lambda_2^* = 0.4303$, $\overline{n}_2^*$

Fig.3 The transitional dynamics of λ_t when $\sigma > 1$

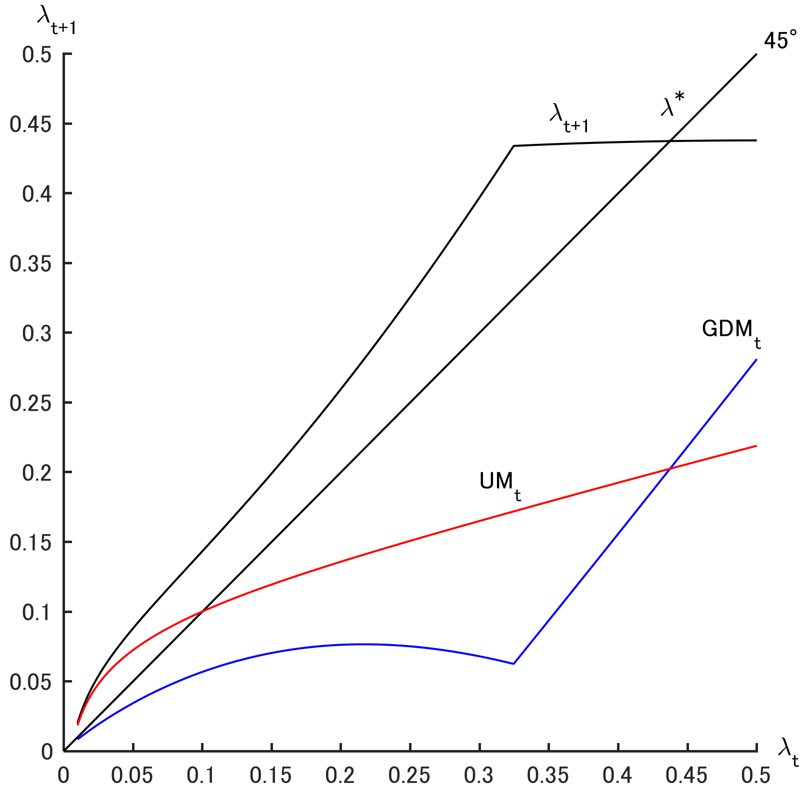
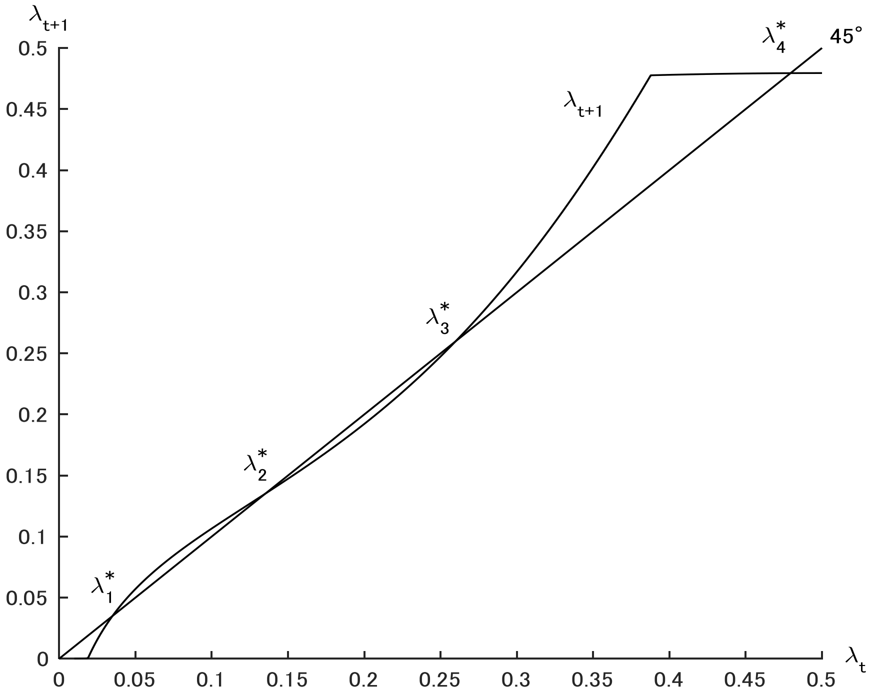
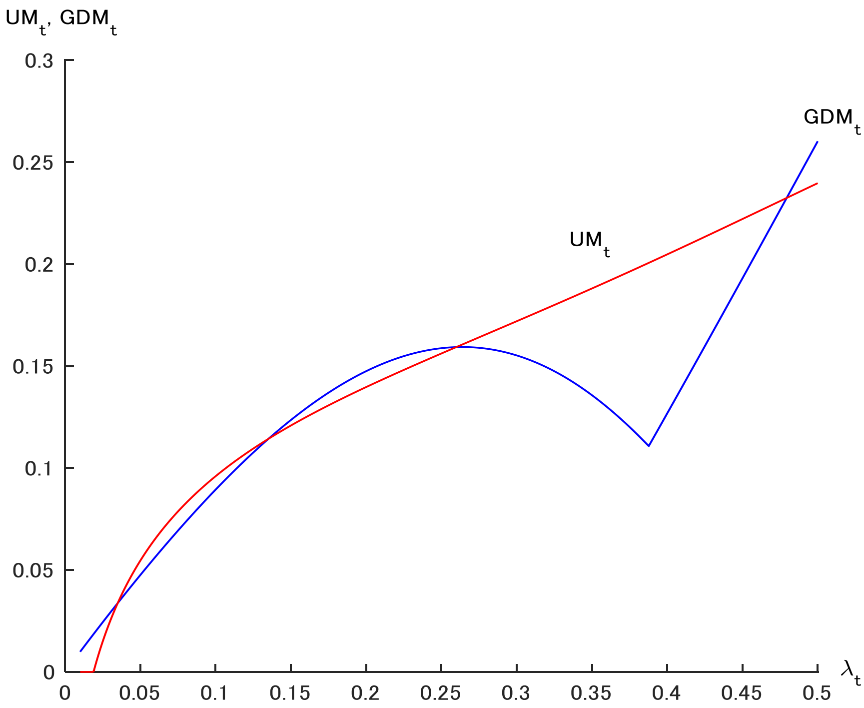


Fig.4　The transitional dynamics of λ_t

when $\sigma = 1.6$ and $\underline{\theta} = 0.5$, $a = 0.03$, $b = 0.02$, $(\lambda^* = 0.4374\,,\ \bar{n}^* = 1.0409)$



(a) Transitional dynamics of λ_t



(b) UM_t, GDM_t

Fig.5 The transitional dynamics of λ_t
when $\sigma = 2.3$ and $\underline{\theta} = 0.5$, $a = 0.03$, $b = 0.02$

Table.1 The values of $\boldsymbol{\lambda_t}$ and $\boldsymbol{\bar{n}_t}$ in steady state in Fig.5

λ_t in steady state	$\bar{n}_t$ in steady state
$\lambda_1^* = 0.0350$	$\bar{n}_1^* = 1.9571$
$\lambda_2^* = 0.1351$	$\bar{n}_2^* = 1.1036$
$\lambda_3^* = 0.2598$	$\bar{n}_3^* = 0.7175$
$\lambda_4^* = 0.4794$	$\bar{n}_4^* = 0.5084$

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