Endogenous lifetime, intergenerational mobility and economic development

Aso, Hiroki

Shigakukan university, Faculty of Low

25 March 2020

Endogenous lifetime, intergenerational mobility and economic development

Hiroki Aso *

Abstract

This paper analyzes the effects of endogenous lifetime on the relationship between intergenerational mobility and economic development in an overlapping generations framework. We assume that an individual's lifetime depends on health status, which improves with economic development. Increase in lifetime encourages incentives of education investment while decreasing transfer, which is the funding source for education. The dynamics of intergenerational mobility and income inequality depend crucially on lifetime. If an increase in lifetime with economic development is sufficiently small, the mobility monotonically increases while income inequality decreases. However, if lifetime increases rapidly with economic development, the mobility and income inequality exhibit cyclical, and even chaotic behavior. In fact, these various patterns of intergenerational mobility have been observed in developed countries.

JEL classifications: I15, I24, J62

Keywords: Endogenous lifetime, Intergenerational mobility, Economic development, Income inequality

Correspondence to: Hiroki Aso
Faculty of Low
Shigakukan University
1-59-1 Murasakibaru,
Kagoshima, 890-8504,
JAPAN

E-mail address: aso@shigakukan.ac.jp

* The author is deeply grateful to Tamotsu Nakamura for valuable comments. In addition, I wish to thank Hideaki Uchida, Kenichi Hashimoto, Tetsugen Haruyama and participants at the 2019 Spring meeting of Japanese Association of Applied Economics. The usual disclaimer applies.

1. Introduction

This study analyzes the effects of lifetime on intergenerational mobility and economic development and shows their interactions. As has been indicated by many studies and historical data, economic development decreases mortality by improving nutrition and sanitation. In fact, over the past few decades, developed countries have experienced a dramatic increase in lifetime with economic development. Similarly, mortality rate or lifetime is general important in determining economic growth. Many previous studies have analyzed the relationship between lifetime (or mortality) and economic development (e.g., see Galor and Moav 2002; Chakraborty 2004, 2005; Chakraborty and Das 2005; Miyazawa 2006; Chen 2010; Varvarigos and Zakaria 2013).[1]

Intergenerational mobility is also closely related to economic development since it leads to an increase in high-income or well-educated workers. The increase in the educated workforce implies that the economy has a large stock of human capital and, therefore, witnesses high growth and an increase in its workers' lifetime. Increase in lifetime affects the mobility and economic development through changes in household economic behavior, such as an increase in savings and a decrease in educational investment on children. Thus, lifetime, intergenerational mobility, and economic development have a high degree of interdependence. Lifetime is expected to play a crucial role in intergenerational mobility and income inequality if they are considered in the context of economic growth.[2] However, previous theoretical studies have overlooked it. Shedding light on the role of inequality in intergenerational mobility, the seminal work by Maoz and Moav (1999) provides a simple, but useful framework to analyze the relationship between income inequality and intergenerational mobility. They show that the economy monotonically approaches the steady state with a decrease in the wage inequality between the educated and uneducated. As Nakamura and Murayama (2011) point out, however, the monotonic behavior of both mobility and inequality depends crucially on the specific education cost function. Introducing a more general cost function, Nakamura and Murayama (2011) show that different

[1] The effects of lifetime on economic development have also been analyzed by Bhattacharya and Qiano (2007), Kunze (2014), Yakita (2001), and Zhang et al. (2001). In addition, Cigno (1998), Strulik (2004), Azarnert (2006), and Fioroni (2010) analyze the effect of child mortality on economic growth.

[2] In fact, demographic variables, such as fertility and lifetime, have a significant impact on the mobility and income inequality. Aso and Nakamura (2019) show that the fertility difference between the educated and uneducated plays a crucial role in the transitional dynamics of mobility.

dynamics of intergenerational mobility and inequality can emerge. Galor and Tsiddon (1997) analyze the effect of technological progress on mobility, income inequality, and economic growth. Iyigun (1999) and Davies et al. (2005), among others, discuss that the type of education system—public or private—is an important factor in determining upward-mobility. Fan and Zhang (2013) show the economy converges to a unique equilibrium under the private education system while multiple equilibria may exist under the public education system. As Owen and Weil (1999) also points out, parental support or self-financing, with or without a liquidity constraint, is also an issue to be analyzed. Galor and Zeira (1993) focus on the imperfect capital market. They show upward mobility is hindered by high borrowing costs; as a result, multiple equilibria emerge in the economy. Using the Maoz and Moav model, Murayama (2019) analyzes how government transfers affect intergenerational mobility and growth. He shows that larger transfers to children with higher ability foster upward mobility and growth if the economy has low income inequality.

Many previous studies on intergenerational mobility focus on the education cost, education system, technology, and fiscal policy, while do not account for the effects of population dynamics on the mobility and income inequality. However, the demographic variable plays important role in intergenerational mobility (e.g. Aso and Nakamura; 2019). We study the effects on intergenerational mobility, income inequality, and economic development incorporating endogenous lifetime into the model of Maoz and Moav (1999).[3] As in Chen (2010), we assume that an individual's surviving rate depends on health status, which improves with economic development. We show that the transitional dynamics of intergenerational mobility and income inequality depend on lifetime and the average wage share of education cost. Within our framework, increase in (endogenous) lifetime with economic development encourages incentives for educational investment, while decreasing the transfer that is the funding source for acquiring education. On the other hand, as the economy develops, the average wage increases more than education cost and therefore the average wage share of education cost decreases with economic development. This decrease in education cost share with economic development allow facilitates upward-mobility, which implies that more people acquire an education. If the increase in lifetime

[3] Maoz and Moav (1999) analyze the transitional dynamics of intergenerational mobility, income inequality and economic development in simple framework. Thus, we can clearly show the effects of endogenous lifetime on the transitional dynamics of mobility, income inequality and economic development by incorporating it into Maoz and Moav (1999).

is sufficiently small and therefore the effect of decreasing education cost share is dominant, then the mobility and income inequality monotonically converge toward steady state, as in Maoz and Moav (1999). In contrast, if lifetime rapidly increases and therefore the effect of increasing lifetime is dominant, then the mobility and income inequality exhibit cyclical behavior and even chaos in the economy. This cyclical motion is not shown in Maoz and Moav (1999). In other words, they do not account for lifetime, so only the monotonous motion of mobility appears in their model. As a result, this paper shows that intergenerational mobility and income inequality depend crucially on lifetime.

In fact, various patterns of intergenerational mobility have been observed in developed countries. While Jin et al. (2019) find that mobility has non-monotonically changed in China, Bratberg et al. (2007) show the mobility has monotonically increased in Norway. China's life expectancy rose far more rapidly than that of Norway. Between 1960 and 2015, life expectancy increased from 42.4 to 74.6 in China, while Norway's life expectancy increased from 71.4 to 80.5 (Source: OECD Health Statistics 2019). Using simple but useful framework, hence, this paper indicates that one of the causes of various motions of the mobility that have been observed in developed countries may be the increase in lifetime.

The rest of this paper is organized as follows. Section 2 sets up the model. Section 3 analyzes the transitional dynamics of the economy. Section 4 examines the transitional dynamics of the economy using numerical analysis. Section 5 discusses the results and concludes the paper.

2. The model

The model is based on incorporating endogenous lifetime in Moaz and Moav (1999).[4] Consider the competitive equilibrium of an overlapping generations economy with a constant population that is normalized to one. Each individual lives potentially for three periods, that is, "*childhood*", "*young adulthood,*" and "*old adulthood.*" While most individuals live during the second period, survival into old adulthood is uncertain and depends on health status.

[4] Although our model is based on Maoz and Moav (1999), it includes the probability of surviving from young adulthood to old adulthood therefore, it can analyze the intertemporal utility maximization problem. As shown later, changes in household economic behavior with increase in lifetime play an important role in the dynamics of an economy.

2.1 Production and factor prices

Following Owen and Weil (1999), we assume that aggregate output in period t is characterized by the following production function.

$$Y_t = AK_t^{\alpha} E_t^{(1-\alpha)(1-\beta)} U_t^{(1-\alpha)\beta}, \qquad A > 0, 0 < \alpha < 1, 0 < \beta < 1, \tag{1}$$

where K_t is the physical capital, E_t is the number of educated workers, and U_t is the number of uneducated workers. The total number of workers is normalized to unity and each supplies one unit of labor. Then, $E_t + U_t = 1$; therefore, the above production function can be written as $Y_t = AK_t^{\alpha} E_t^{(1-\alpha)(1-\beta)} (1 - E_t)^{(1-\alpha)\beta}$.

To focus on human capital accumulation, as with Owen and Weil (1999), we assume that this model economy has a small, open capital market, despite labor not being internationally mobile. Hence, the marginal product of physical capital is determined by the world interest rate $\bar{r}$. Assuming that each factor receives its product in the equilibrium, the returns to an educated and uneducated worker in period t are, respectively,

$$w_t^e = (1 - \beta)\Theta A \left(\frac{1 - E_t}{E_t}\right)^{\beta}, \tag{2}$$

$$w_t^u = \beta\Theta A \left(\frac{1 - E_t}{E_t}\right)^{\beta-1}, \tag{3}$$

where $\Theta = (1 - \alpha)(\alpha/\bar{r})^{\alpha/1-\alpha}$; the subscripts e and u denote "educated" and "uneducated," respectively. Hence, the wage inequality becomes:

$$\frac{w_t^e}{w_t^u} = \frac{1 - \beta}{\beta}\left(\frac{1 - E_t}{E_t}\right). \tag{4}$$

To ensure that $w_t^e > w_t^u$, we assume that $E_t < 1 - \beta$.

2.2 Individuals

As a child, who does not work, the individual receives a transfer from her parent. It is used for consumption and possible education. When young, she works, and divides her income between consumption, savings, and a transfer to her children, regardless of the survival status during old age.[5] She faces a survival probability from young to old adulthood. If she survives to old age, she retires and only consumes.

The preference of individual i, born in period t, is expressed by the following expected lifetime utility function:

$$v_t^i = logc_t^i + logc_{t+1}^i + logx_{t+1}^i + \pi_{t+1} logc_{t+2}^i, \tag{5}$$

---

[5] The modeling of transfer follows Zhang et al. (2001) and Kunze (2014).

where $i \in \{e, u\}$; c_t^i is consumption in period t, c_{t+1}^i is consumption in period $t+1$, x_{t+1}^i is the transfer per child in period $t+1$, π_{t+1} is the survival probability in period $t+1$ and c_{t+2}^i is consumption in period $t+2$.

Let h_t^i denote the education cost of individual i, born in period t. As with Maoz and Moav (1999), we assume an imperfect capital market that a child cannot access. Hence, individual uses up all the transfers from parents during childhood. A surviving individual will receive not only own her past savings plus interest, but also the return from mutual funds since we assume a perfect annuities market in line with Chakraborty (2004) and Fanti and Gori (2014). Thus, if she acquires education, her budget constraints are

$$c_t^i + h_t^i = x_t^i; \tag{6.a}$$

$$w_{t+1}^e = c_{t+1}^i + x_{t+1}^i + s_{t+1}^i; \text{ and} \tag{6.b}$$

$$\frac{R}{\pi_{t+1}} s_{t+1}^i = c_{t+2}^i, \tag{6.c}$$

where $R = 1 + \bar{r}$. If she does not acquire education, her budget constraints become

$$c_t^i = x_t^i; \tag{7a}$$

$$w_{t+1}^u = c_{t+1}^i + x_{t+1}^i + s_{t+1}^i; \text{ and} \tag{7.b}$$

$$\frac{R}{\pi_{t+1}} s_{t+1}^i = c_{t+2}^i. \tag{7.c}$$

Since we assume an imperfect capital market, the utility maximization problem can be solved backwards in two stages. First, the individual considers optimal allocation for the periods of adulthood. Then, the individual decides whether to acquire education in childhood. The utility maximization in the periods of adulthood is formulated as follows.

$$\max_{c_{t+1}^i, x_{t+1}^i, c_{t+2}^i} \; log c_{t+1}^i + log x_{t+1}^i + \pi_{t+1} log c_{t+2}^i,$$

$$subject\ to\ c_{t+1}^i + x_{t+1}^i + \frac{\pi_{t+1}}{R} c_{t+2}^i = w_{t+1}^i,$$

The optimal consumption, transfer in period $t+1$, and optimal consumption in period $t+2$ become, respectively,

$$c_{t+1}^{i} = x_{t+1}^{i} = \frac{w_{t+1}^{i}}{2+\pi_{t+1}}, \qquad c_{t+2}^{i} = \frac{R\, w_{t+1}^{i}}{2+\pi_{t+1}}. \tag{8}$$

Hence, the indirect utility function in the periods of adulthood is:

$$z(w_{t+1}^{i}) = 2log\left[\frac{w_{t+1}^{i}}{2+\pi_{t+1}}\right] + \pi_{t+1}log\left[\frac{Rw_{t+1}^{i}}{2+\pi_{t+1}}\right]. \tag{9}$$

If the utility derived from investing in education is higher than or equal to the utility derived from not investing in education, then individual i will acquire education. Thus,

$$log(x_t^i - h_t^i) + z(w_{t+1}^e) \geq logx_t^i + z(w_{t+1}^u),$$

or,

$$h_t^i \leq x_t^i\left[1 - \frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right] = x_t^i\left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^{2+\pi_{t+1}}\right]. \tag{10}$$

From (10), we have the following critical value of education cost $\hat{h}_t^i$ for individual i :

$$\hat{h}_t^i = x_t^i\left[1 - \frac{z(w_{t+1}^u)}{z(w_{t+1}^e)}\right] = x_t^i\left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^{2+\pi_{t+1}}\right]. \tag{11}$$

As can be seen from (11), in addition to the wage inequality w_{t+1}^e/w_{t+1}^u and the transfer x_t^i, the surviving rate π_{t+1} also plays an important role in education choice. The higher the value of π_{t+1}, the larger the incentive to acquire education. It implies that, when lifetime increases, the incentive of acquiring education increases since lifetime returns to education investment increases. Hence, an increase in π_{t+1} encourages educational investment.[6]

Suppose that $\hat{h}_t^e$ ($\hat{h}_t^u$) is the critical value of educational cost for the individual born to an educated (uneducated) worker to acquire education. Then, from (11),

$$\hat{h}_t^e = \frac{w_t^e}{2+\pi_t}\left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^{2+\pi_{t+1}}\right], \qquad \hat{h}_t^u = \frac{w_t^u}{2+\pi_t}\left[1 - \left(\frac{w_{t+1}^u}{w_{t+1}^e}\right)^{2+\pi_{t+1}}\right]. \tag{12}$$

2.3 Education cost among individuals

Following Maoz and Moav (1999), the following cost is assumed to be incurred for the education of individual i in period t.

$$h_t^i = \theta^i c(\overline{w}_t) = \theta^i(a + b\overline{w}_t), \tag{13}$$

[6] Hence, increase in lifetime has the *"Ben-Porath effect"* in our model.

where $\overline{w} = E_t w_t^e + (1 - E_t) w_t^u$ is a weighted average of educated and uneducated wages, θ^i is a parameter representing individual $i's$ ability to learn; the higher the ability, the lower is the value of θ^i. We further assume that θ^i is uniformly distributed in the interval $(\underline{\theta}, \overline{\theta})$, regardless of the ability and class of the parents in any period. Hence, h_t^i is also uniformly distributed in the interval $(\underline{h}_t, \overline{h}_t)$, where $\underline{h}_t = \underline{\theta}(a + b\overline{w}_t)$ and $\overline{h}_t = \overline{\theta}(a + b\overline{w}_t)$.

2.4 Endogenous lifetime

We assume that the probability of surviving π_t depends on health status H_t; this relationship is represented as follows:

$$\pi_t = \pi(H_t) = \frac{\underline{\pi} + \overline{\pi} H_t^\delta}{1 + H_t^\delta}, \tag{14}$$

where $\delta > 0$; $0 < \overline{\pi} \leq 1$; $0 < \underline{\pi} < \overline{\pi}$; $\pi(0) = \underline{\pi} > 0$; $\pi'(H) > 0$; $lim_{H \to \infty} \pi(H) = \overline{\pi} \leq 1$; $\pi''(H) < 0$ if $\delta \leq 1$ and $\pi''(H) \gtreqless 0$, for any $H \lesseqgtr \overline{H} \equiv [(\delta - 1)/(1 + \delta)]^{1/\delta}$ if $\delta > 1$.

The parameter δ represents how an additional unit of health investment is transformed into greater longevity through health technology. If $\delta \leq 1$, π_t is a concave function. If $\delta > 1$, π_t is a S-shaped function, that is, threshold effects exist (see Fanti and Gori, 2014). In other words, when $\delta < 1$, there is a relatively slow increase in with economic development. In contrast, if $\delta > 1$, lifetime suddenly and rapidly increases with economic development owing to the sudden effect. An increase in δ increases the speed of converges from $\underline{\pi}$ to $\overline{\pi}$. In other words, the larger δ increases rapidly lifetime. This function from captures empirical evidences of Martikainen et al. (2009) and Fioroni (2010), which show that the larger thresh hold effects (the larger δ) make the speed of increase in lifetime slow when H is relatively small, while increasing efficiently and rapidly the speed of increase in lifetime when $H > \overline{H}$.

In addition, mortality tends to fall with economic development owing to improvement in nutrition and sanitation. Similar to Chen (2010), we assume that the health status H_t is determined by economic development, i.e., per capita income $y_t = \overline{w}_t/(1 - \alpha)$.

$$H_t = H(y_t) = \phi y_t, \qquad \phi > 0, \tag{15}$$

where ϕ represents health productivity. Thus, health status improves with economic development and therefore, lifetime also increases.

3. Dynamics of the model

In this section, we show the dynamics of the economy. Intergenerational mobility can be expressed in two ways—upward-mobility (UM_t) and downward-mobility (DM_t). In our model, upward-mobility means that individuals born to an uneducated parent become educated adults, while downward-mobility means that individuals born to an educated parent become uneducated adults. The dynamics of E_t can therefore be expressed as:

$$E_{t+1} - E_t = \underbrace{(1 - E_t)\frac{\hat{h}_t^u - \underline{h}_t}{\overline{h}_t - \underline{h}_t}}_{\equiv UM_t} - \underbrace{E_t \frac{\overline{h}_t - \hat{h}_t^e}{\overline{h}_t - \underline{h}_t}}_{\equiv DM_t} \tag{16}$$

or

$$E_{t+1} = (1 - E_t)\frac{\hat{h}_t^u - \underline{h}_t}{\overline{h}_t - \underline{h}_t} + E_t \frac{\hat{h}_t^e - \underline{h}_t}{\overline{h}_t - \underline{h}_t} \tag{17}$$

Taking into account $\overline{h}_t = \overline{\theta} c(\overline{w}_t)$, $\underline{h}_t = \underline{\theta} c(\overline{w}_t)$ and Eq. (12), Eq. (17) can be written as follows:

$$E_{t+1} = \frac{1}{\underbrace{[2 + \pi(y_t)]}_{(*1)}(\overline{\theta} - \underline{\theta})} \frac{\overbrace{f(E_{t+1})}^{(*2)}}{\underbrace{s(\overline{w}_t)}_{(*3)}} - \frac{\underline{\theta}}{\overline{\theta} - \underline{\theta}}, \tag{18}$$

where $f(E_{t+1}) = \left[1 - (w_{t+1}^u / w_{t+1}^e)^{2+\pi_{t+1}(y_{t+1})}\right]$ and $s(\overline{w}_t) = c(\overline{w}_t)/\overline{w}_t$ represents the average income share of education cost.

Investigating Eq. (19), we can see the dynamic behavior of intergenerational mobility, inequality, and economic development. Totally differentiating Eq. (19),

$$G_1 dE_{t+1} = G_2 dE_t,$$

where

$$G_1 = 1 - \frac{1}{[2 + \pi(y_t)](\overline{\theta} - \underline{\theta})} \frac{f'(E_{t+1})}{s(\overline{w}_t)}, \tag{20}$$

$$G_2 = -\frac{\varepsilon_t^s + \varepsilon_t^\pi}{[2 + \pi(y_t)]^2(\overline{\theta} - \underline{\theta})} \frac{f(E_{t+1})}{s(\overline{w}_t)\, E_t / \pi(y_t)} \gtreqless 0, \tag{21}$$

and

$$\varepsilon_t^s = \frac{\partial s(\overline{w}_t)/s(\overline{w}_t)}{\partial E_t / E_t} = s'(\overline{w}_t)\, \overline{w}_t' \frac{E_t}{s(\overline{w}_t)} < 0, \tag{22}$$

$$\varepsilon_t^\pi = \frac{\partial \pi(y_t)/\pi(y_t)}{\partial E_t / E_t} = \pi'(y_t) y_t' \frac{E_t}{\pi(y_t)} > 0. \tag{23}$$

ε_t^s and ε_t^π represent the elasticity of education cost share with respect to the share of the educated in period t and the elasticity of surviving rate with respect to the share of the educated in period t, respectively. An increase in surviving rate π_t decreases the transfer that is the funding source of education investment, and, therefore, discourages intergenerational mobility. Unfortunately, since the sign of $f'(E_{t+1})$ is ambiguous, the sign of G_1 is also ambiguous. However, to see determinants of the dynamics of economy, we assume that $G_1 > 0$ holds as follows.

Suppose that $G_1 > 0$. As is evidently from Eqs. (20) and (21), the transitional dynamics of intergenerational mobility depends on ε_t^s and ε_t^π. Hence, we have

$$sign\left[\frac{dE_{t+1}}{dE_t}\right] = sign\left[\frac{G_2}{G_1}\right] = -sign[\varepsilon_t^s + \varepsilon_t^\pi]. \tag{24}$$

As educated workers increases, that is, the economy grows, the education cost share decreases, and $\varepsilon_t^s < 0$. In other words, the average wage increases more than the education cost. This reduction in education cost share encourages the mobility. On the other hand, lifetime increases with economic development, and $\varepsilon_t^\pi > 0$. This increase in lifetime decreases transfer from parents to children caused by higher savings, and, therefore discourages mobility. If increase in lifetime is sufficiently small, then $G_2/G_1 = -[\varepsilon_t^s + \varepsilon_t^\pi] > 0$; hence Eq. (18) is upwards-sloping in the (E_t, E_{t+1}) plane. In contrast, if increase in lifetime is sufficiently large, then $G_2/G_1 = -[\varepsilon_t^s + \varepsilon_t^\pi] < 0$; hence, Eq. (18) is downwards-sloping in the (E_t, E_{t+1}) plane. Thus, we have following proposition.[7]

Proposition Suppose that $G_1 > 0$. the transitional dynamics of intergenerational mobility depends on two effects, i.e. the positive effect of a decrease in education cost share and the negative effect of an increase in lifetime. When the former is dominant, that is, increase in lifetime is sufficiently small, the mobility and income inequality monotonically approach the steady state, as in Maoz and Moav (1999). In contrast, when the latter is dominant, that is, the increase in lifetime is sufficiently large, the mobility and income inequality exhibit cyclical behavior.

[7] If surviving rate does not depend on economic development, that is, it has an exogenous value, the transitional dynamics of mobility depends only on the behavior of education cost share, and, therefore, the mobility monotonically increases, as in Maoz and Moav (1999).

4. Numerical analysis

In this section, we use numerical analysis to illustrate the Proposition. We take the parameter values $A = 12$, $\alpha = 0.5$, $\beta = 0.5$, $\bar{r} = 0.05$, $\bar{\theta} = 5$, $\underline{\theta} = 1$, $\bar{\pi} = 0.95$, $\underline{\pi} = 0.3$, and $\phi = 0.1$.[8]

Fig.1 shows the transitional dynamics of intergenerational mobility. As was shown in the previous subsection, the dynamics of mobility depend on both ε_t^s and ε_t^π. Since increase in lifetime is sufficiently small, Eq. (18) is upwards-sloping in the (E_t, E_{t+1}) plane in Fig. 1(a). This implies that the positive effect of a decrease in the education cost share on mobility is larger than the negative effect on it of an increase in lifetime. Thus, the mobility and lifetime monotonically increase toward the steady state, and, therefore, income inequality decreases with economic development.

In contrast, Fig. 1 (b) shows that Eq. (18) is downwards-sloping in the (E_t, E_{t+1}) plane because the increase in lifetime is much larger than the decrease in education cost share. Hence, the mobility, lifetime, and income inequality monotonically approach economic development initially, and then exhibit cyclical behavior around the steady state.

This behavior of the mobility can be interpreted as follows. Suppose that the educated E_t is low, and, hence, education cost and lifetime is also low. Then, as the number of those who are educated increases, the wage of uneducated worker increases and upward-mobility occurs. Hence, the educated monotonically increases with economic development owing to upward-mobility. When E_t exceeds a certain threshold, that is, economy is sufficiently developed, the lifetime increases rapidly. This sharp increase in lifetime increases the incentive for educational investment, even as it greatly decreases transfer, which is the funding source for acquiring education. Since this decrease in transfer is dominant, an increase in lifetime impedes mobility. Whether E_t increases or decreases in the next period depends on the positive effect of the decline in education cost share and the negative effect of increase in lifetime with economic development. Because the latter is larger than the former, E_t decreases in the next period in Fig. 1 (b); this, in turn, decreases the lifetime and education cost, and, therefore, E_t increases in following period.

[8] Except for A and ϕ, the parameters follow Fanti and Gori (2014), Maoz and Moav (1999), and Owen and Weil (1999).

This observation explains the cyclical behavior of intergenerational mobility, lifetime, and income inequality.

Numerical Result 1 Increase in lifetime with economic development plays a crucial role in the transitional dynamics of mobility. If the increase in lifetime with economic development is quite small, the mobility approaches toward steady state, as in Moaz and Moav (1999). In contrast, if increase in lifetime with economic development is sufficiently large, the mobility exhibits cyclical behavior around the steady state.

[Insert Fig.1 about here]

In particular, if the decrease in education cost share is quite small and lifetime increases more suddenly and rapidly, the fluctuation is greater and even chaotic. We now show an example of chaotic equilibrium in Fig. 2. Since δ is greater, the lifetime increases more rapidly, and, then, greatly decreases the transfer. As a result, a chaotic equilibrium appears in the economy. This chaotic equilibrium implies that intergenerational mobility, income inequality, and economic growth continue to fluctuate over time.

Numerical Result 2 If the decrease in education cost share is sufficiently small and lifetime increases rapidly, that is, the negative effect of an increase in lifetime is much larger than the positive effect of a decrease in the income share of education cost, the mobility, income inequality, and lifetime exhibit a chaotic equilibrium.

[Insert Fig.2 about here]

5. Conclusions and remarks

Many studies and historical data indicate that lifetime has increased with economic development in developed countries. Increase in lifetime affects the mobility and economic development through changes in household economic behavior. However, previous theoretical studies on intergenerational mobility have overlooked this. This study shows that the effects of an increase in lifetime on the mobility and economic development and analyzes their interactions.

We show that an increase in lifetime with economic development increases the incentive for acquiring education, and decrease in the transfer, which is the funding source for acquiring education. In particular, a decrease in the transfer with an increase in lifetime plays a crucial role in the transitional dynamics of mobility.

The transitional dynamics of mobility depends on two effects: the positive effect of decreasing education cost share and the negative effect of increasing lifetime. When the former is dominant, that is, an increase in lifetime is quite small, the mobility, lifetime, and income inequality monotonically approach the steady state with economic development, as in Maoz and Moav (1999). In contrast, when the latter is dominant, that is, an increase in lifetime is sufficiently large, the mobility, lifetime, and income inequality exhibit cyclical behavior. In particular, when the decrease in education cost share is quite small and lifetime increases rapidly, a chaotic equilibrium appears in the economy. Hence, the mobility and economic development depend on lifetime, and, therefore, various patterns of transitional dynamics emerge. In fact, various patterns of intergenerational mobility have been observed in developed countries. An increase in lifetime with economic development may be one of the factors generating various dynamics of the economy.

In future research, we extend the model in several ways. For example, to simplify the analysis and explanation, we adopt extreme assumptions regarding the capital market: imperfect, small, and open. Relaxing these assumptions is an issue for future study. Further, we can analyze the positive effect of an increase in lifetime on economic development by incorporating health capital into the model since health capital encourages economic development. Considering them in future studies will make it more interesting to explore interactions between endogenous life time, intergenerational mobility, and economic development.

Compliance with Ethical Standards:

Funding: This research did not receive only specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Conflict of Interest: The authors declare that they have no conflict of interest.

References

Aso H and Nakamura T (2019) Population growth and intergenerational mobility. Appl Econ Lett, forthcoming.

Azarnert LV (2006) “Child mortality, fertility and human capital accumulation”. J of Popul Econ, 19(2): 285-297.

Bhattacharya J and Qiao X (2007) Public and Private expenditures on health in a growth model. J Econ Dyna Cont, 31(8): 2519-2535.

Chakraborty S (2004) Endogenous lifetime and economic growth. J Econ Theory 116(1)**:** 119-137.

Chakraborty S (2005) Mortality, human capital and persistent inequality. J Econ Growth 10(2): 159-192.

Chakraborty S and Das M (2005) Mortality, fertility and child labor. Econ Lett 86(2): 273-278.

Cigno A (1998) Fertility decisions when infant survival is endogenous. J Popul Econ 11(1): 21-28.

Davies JB, Zhang J, Zeng J (2005) Intergenerational mobility under private vs public education. Scand J Econ 107(3): 399-417.

Fan CS, Zhang J (2013) Differential fertility and intergenerational mobility under the private versus public education. J Popul Econ, 26(3):907-41.

Fanti L, Gori L (2014) Endogenous fertility, endogenous lifetime and economic growth: the role of child policies. J of Popul Econ 27(2): 529-564.

Fioroni T (2010) Child mortality and fertility: public vs private education. J Popul Econ, 23(1): 73- 97.

Galor O (2005) From stagnation to growth: unified growth theory. In Aghion P, Durlauf S (eds) Handbook of economic growth, Elsevier, Amsterdam, pp 171-293.

Galor O, Moav O (2002) Natural selection and the origin of economic growth. Q J Econ 117(4):1133-1192.

Galor O, Tsiddon D (1997) Technological progress, mobility, and economic growth. Am Econ Rev 87(3): 363-382.

Galor O, Weil DN (2000) Population, technology, and growth: from Malthusian stagnation to the demographic transition and beyond. Am Econ Rev 90(4):806-828.

Galor O, Zeira J (1993) Income distribution and macroeconomics Rev Econ Stud 60(1): 35-52

Hassler J, Rodriguez JV, Zeira J (2007) Inequality and Mobility. J Econ Growth 12(3); 235-259

Iyigun MF(1999) Public education and intergenerational economic mobility. Int Econ Rev 40(3): 697-710.

Jin M, Bai X, Li XK , Shi W (2019) Are we born equal: a study of intergenerational income mobility in China. J Demo Econ 85(1):1-19.

Kunze L (2014) Life expectancy and economic growth. J Macro, 39: 54-65.

Maoz YD, Moav O (1999) Intergenerational mobility and the process of development. Econ J. 109(458): 677-697.

Miyazawa K (2006) Growth and inequality: a demographic explanation. J Popul Econ 19(3): 559-578.

Murayama Y (2019) CASH TRANSFERS, INTERGENERATIONAL MOBILITY, AND THE PROCESS OF DEVELOPMENT. Bull Econ Rese, 71(3): 209-218.

Nakamura T, Murayama Y (2011) Education cost, intergenerational mobility and income inequality, Econ Lett 112(3):266-269.

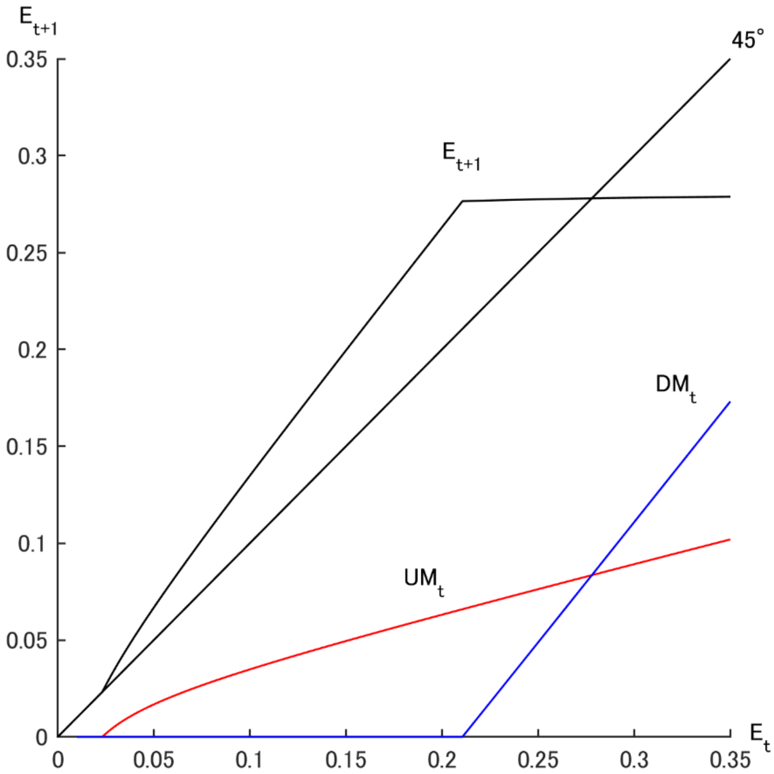
Owen AL, Weil DN (1998) Intergenerational earnings mobility, inequality and growth. J Monet Econ. 41(1):71-104.

Strulik H (2004) Child Mortality, Child Labour and Economic Development. Econ J, 114(497): 547-568.

Varvarigos D, Zakaria IZ (2013) Endogenous fertility in a growth model with public and private health expenditures. J Popul Econ 26(1):67-85

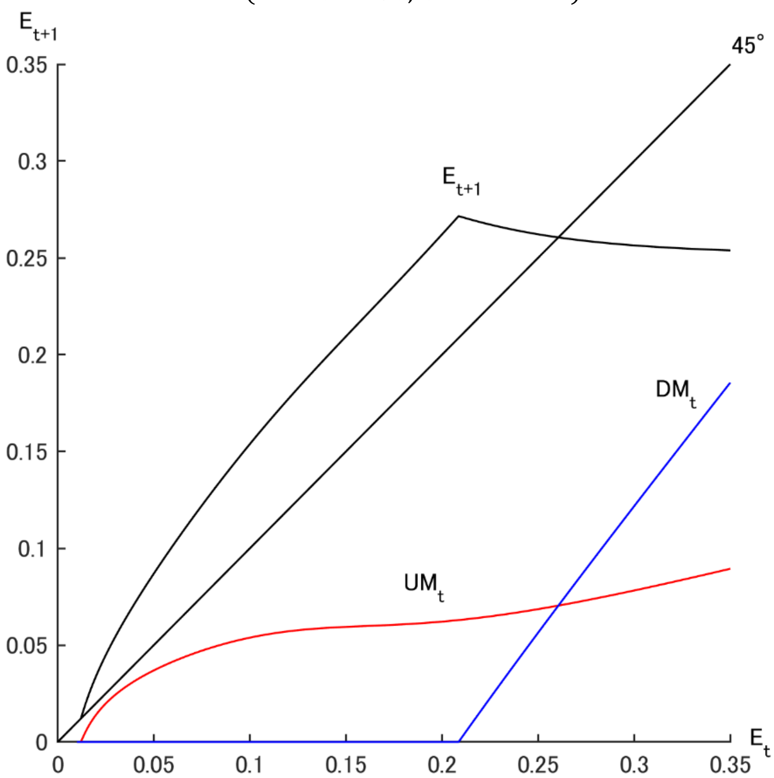
Yakita A (2001) Uncertain lifetime, fertility and social security. J Popul Econ 14(4):635-640

Zhang J, Zhang J, Lee R (2001) Mortality decline and long-run economic growth. J Pub Econ, 80(3): 485-507.



(a) $\boldsymbol{\delta = 0.75}$, $\boldsymbol{\varepsilon_t^s + \varepsilon_t^\pi < 0}$

$(\boldsymbol{E^* = 0.2780, \pi^* = 0.6429})$

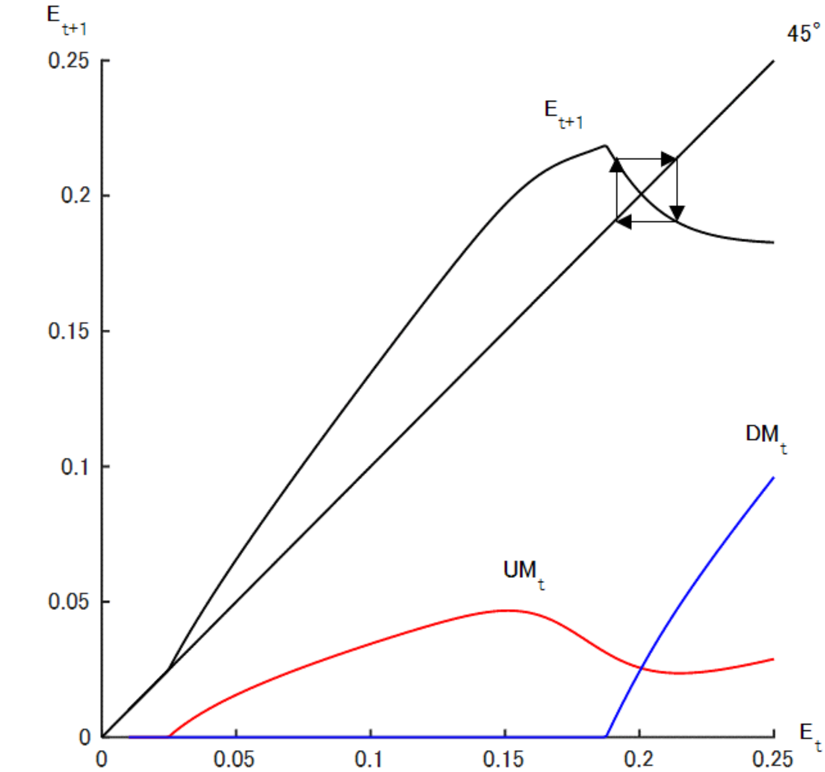


(b) $\boldsymbol{\delta = 10}$, $\boldsymbol{\varepsilon_t^s + \varepsilon_t^\pi > 0}$

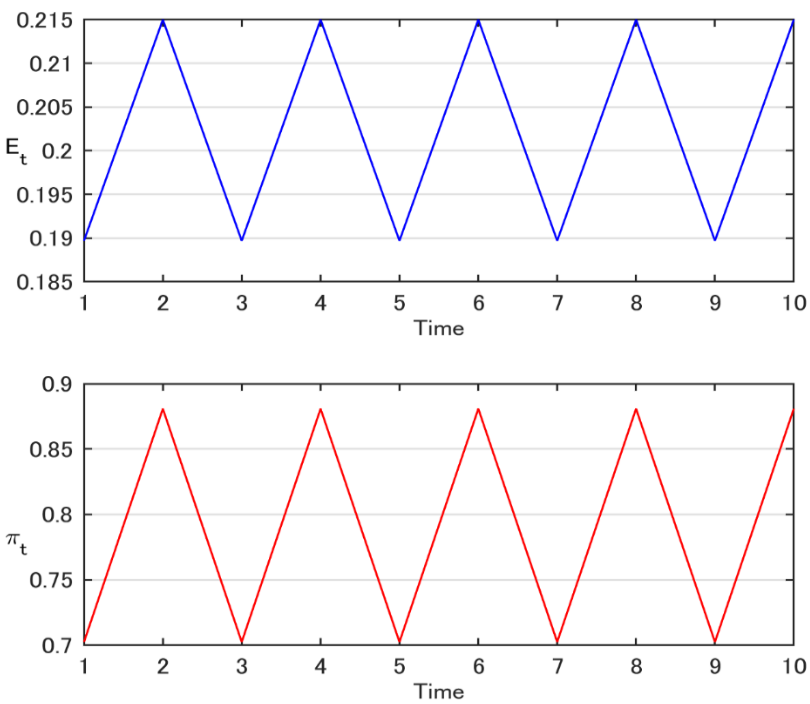
$(\boldsymbol{E^* = 0.2605, \pi^* = 0.8072})$

Fig.1 Transitional dynamics of $\boldsymbol{E_t}$

$\boldsymbol{a = 0.12, b = 0.15}$



(a) Transitional dynamics of $\boldsymbol{E_t}$



(b) Fluctuations in $\boldsymbol{E_t}$ and $\boldsymbol{\pi_t}$

Fig.2 Chaos equilibrium

$\boldsymbol{\delta = 35;\ a = 0.1; b = 0.18}$