# On treatment of interests, profits and equilibrium non-existence in general equilibrium models

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**ABSTRACT**

It is sometimes argued that one cannot criticize general equilibrium models on grounds of equilibrium non-existence. I argue that once problematic treatments of interests and profits in general equilibrium models are corrected, equilibrium non-existence issues arise again. Uniform rate of interest and zero economic profit of firms must hold after corrections, which allow equilibrium existence for only restricted circumstances. This demonstrates ongoing relevance of the Cambridge capital controversies.

**KEYWORDS**

Cambridge capital controversies, general equilibrium, uniform rate of interest, zero economic profit, equilibrium existence

**JEL CLASSIFICATION**

B22, D50, E13

## 1. Introduction

In the conventional treatment of mainstream general equilibrium models (Arrow & Debreu, 1954; McKenzie, 1954), shares of firms that individuals hold are considered to be fixed. One can see why this has to be - it really does not make sense for agents to trade shares in a model where all trades happen at a single period, or treated as such. By Arrow and Debreu (1954); McKenzie (1954), it was established under convexity assumptions that an equilibrium always exists for these general equilibrium models, sometimes called as Arrow-Debreu models.

The problem arises when we apply or extend static general equilibrium models to dynamic settings. Can we really say that the static general equilibrium framework can still be used for dynamic settings just with some tweaks?

The question involved is how we should treat interests, dividends and profits, which tend

to be dynamic questions. Their interconnections are not well-outlined in mainstream macroeconomics models, often analyzed in flawed ways. In fact, it would be accurate to say that modern dynamic stochastic general equilibrium (DSGE) models, widely used in mainstream macroeconomics, largely brush aside the above question. This paper explores how careful considerations of these issues, even in simplified settings, resurrect equilibrium non-existence issues.

We can cast the discussion of the above and below in terms of the Cambridge capital controversies. (Cohen & Harcourt, 2003) It is true that modern neoclassical economists have embraced dis-aggregated capital models. This, however, does not mean that they abandoned uniform rate of interest, as we will see - the point commonly misunderstood. Aggregation of capital is not necessary for uniform rate of interest, and modern DSGE models, such as New Keynesian models (Christiano, Eichenbaum, & Evans, 2005; Smets & Wouters, 2007), largely work under this uniform rate of interest benchmark. This holds true, regardless of whether firms, their capital or their production functions are aggregated or not. Furthermore, marginal product theory of capital is back as dis-aggregated form in DSGE models, despite no capital aggregation. Marginal product of capital forms capital demand function of firms, while uniform rate of interest comes as an optimization result of capital funding supply side.

From the above, one may form an impression that the Cambridge capital controversies are about aggregation issues summarized by the Sonnenschein-Mantel-Debreu theorem (Debreu, 1974; Mantel, 1974; Sonnenschein, 1972, 1973), and otherwise, mainstream economics remains intact. Even for heterodox responses, directions have varied. (Fratini, 2019) Is equilibrium existence the issue with general equilibrium models, and there are some faults in understanding the models such that results establishing equilibrium existence (Arrow & Debreu, 1954; McKenzie, 1954) must be overturned? Or is stability of equilibrium the issue with general equilibrium models? The focus has largely been on the latter. (Fratini, 2019) But this does not need to be.

## 2. Benchmark general equilibrium model

A representative agent general equilibrium model is described. Somewhat surprisingly, this will be sufficient to prove the point that equilibrium non-existence issues are alive. There is

no stochastic shock in this model.

### 2.1. *Representative consumer and investor*

A representative consumer and investor has the following utility function and the utility maximization problem:

$$\max_{C_t, K_{t+1}, N_t} U = \max_{C_t, K_{t+1}, N_t} \sum_{t=0}^{\infty} \beta^t u(C_t, N_t)$$

where the present time is $t = 0$ where decisions are made, $C_t$ is consumption of the only consumption good at time $t$, $N_t$ is labor utilized to get wage. The budget constraint at time $t$ is:

$$P_t C_t + P_t \left[K_{t+1} - (1-\delta)K_t\right] \leq W_t N_t + r_t P_t K_t$$

where we assume that capital good is same type as consumption good, thus sharing same price $P_t$. Investment is $I_t \equiv K_{t+1} - (1-\delta)K_t$, where $\delta$ represents capital depreciation rate. $r_t$ is the rate of interest on capital and $W_t$ is wage level.

### 2.2. *Extending to multiple capital goods: uniform rate of interest*

We can extend the above to two capital goods ($K_\alpha$,$K_\beta$). Modify the budget constraint, without changing the utility function, as:

$$P_t C_t + P_{\alpha,t}\left[K_{\alpha,t+1} - (1-\delta_\alpha)K_{\alpha,t}\right] + P_{\beta,t}\left[K_{\beta,t+1} - (1-\delta_\beta)K_{\beta,t}\right] \leq W_t N_t + r_{\alpha,t} P_{\alpha,t} K_{\alpha,t} + r_{\beta,t} P_{\beta,t} K_{\beta,t}$$

with utility maximization now done by controlling $C_t$,$K_{\alpha,t+1}$,$K_{\beta,t+1}$,$N_t$. We can solve the utility maximization problem using the Lagrangian method, assuming price vector is given. Let $\lambda_t$ be Lagrange multiplier attached to the budget constraint at time $t$ in the Lagrangian.

The resulting first-order conditions obtained for optimization over $K_{\alpha,t+1}$ and $K_{\beta,t+1}$ are:

$$\lambda_{t+1} P_{\alpha,t+1} \left[ (1 - \delta_\alpha) + r_{\alpha,t+1} \right] = \lambda_t P_{\alpha,t}$$

$$\lambda_{t+1} P_{\beta,t+1} \left[ (1 - \delta_\beta) + r_{\beta,t+1} \right] = \lambda_t P_{\beta,t}$$

Re-arranging,

$$\frac{\lambda_t}{\lambda_{t+1}} = \frac{P_{\alpha,t+1}}{P_{\alpha,t}} \left[ (1 - \delta_\alpha) + r_{\alpha,t+1} \right] = \frac{P_{\beta,t+1}}{P_{\beta,t}} \left[ (1 - \delta_\beta) + r_{\beta,t+1} \right]$$

Thus, factoring in price change and depreciation rate, there is uniform rate of interest. It is easy to see that this generalizes to any $n$ capital goods circumstances.

From now on, I will again assume single capital good that is homogeneous with consumption good.

### 2.3. *Interpreting profits: argument for zero economic profit*

The representative firm is now modeled. It maximizes economic profit:

$$\max_{K_t, N_t} \pi_t \equiv \max_{K_t, N_t} \left[ P_t f(K_t, N_t) - W_t N_t - r_t P_t K_t \right]$$

where $Y_t \equiv C_t + I_t \equiv f(K_t, N_t)$ is production function. Optimization results in marginal product theory:

$$\frac{W_t}{P_t} = f'(N_t), \;\; r_t = f'(K_t)$$

Now the problem. It initially seems that the budget constraint for the representative consumer should rather be, as standard in DSGE and real business cycle literature:

$$P_t C_t + P_t \left[ K_{t+1} - (1 - \delta) K_t \right] \leq W_t N_t + r_t P_t K_t + \pi_t$$

Notice the added profit term. But whether as an agent funding purchase of capital by a firm or as an agent directly providing capital goods to firms, there is no reason to distinguish profits and payments due to use of capital. Both cases are rewards of use of capital or funding of used capital. Thus, the budget constraint should remain as:

$$P_t C_t + P_t \left[K_{t+1} - (1-\delta)K_t\right] \leq W_t N_t + r_t P_t K_t$$

In such a case, what exactly is economic profit $\pi_t$ of the representative firm? Why is it maximizing such economic profit? Shouldn't it maximize $r_t P_t K_t$ instead?

The answer is that a firm follows the following line of thoughts: "While market rate of interest is $r_t$, shareholders (or capital provider) seek to obtain more interests. The goal of firms is to generate as much interests as possible. Thus, a firm seeks to maximize what is left after paying, as market price dictates, wage $W_t N_t$ and $r_t K_t$. This remainder is paid back to shareholders."

Since this is competitive economy, providing non-zero economic profit fails - otherwise, market rate of interest would not be $r_t$. Therefore, $\pi_t = 0$.

But that comes with a problem. We have same first-order conditions rising out of optimization problems regardless of how $\pi_t$ is handled in the budget constraint of the representative consumer (investor) - as in mainstream DSGE literature or as described here.

Suppose this benchmark general equilibrium model produces a unique equilibrium, with additional specifications, when the budget constraint of the representative consumer includes $\pi_t$. Then this benchmark general equilibrium model should generally have no equilibrium when the budget constraint does not include $\pi_t$.

When production function $f(K_t, N_t)$ is $f(K_t, N_t) = A_t K_t^{\alpha} N_t^{1-\alpha}$, then there indeed is zero economic profit, and we see no problem, regardless of how the budget constraint is considered. However, for other types of production function, we generally do not get zero economic profit when the budget constraint includes $\pi_t$. Thus, when the budget constraint is corrected, there is no equilibrium.

## 3. Conclusion

Thus, even in a representative agent general equilibrium model, equilibrium non-existence issues remain, when we properly treat interests and profits. It is through muddling of logic about interests and profits or choice of production function that ensures equilibrium existence in general equilibrium models. We should expect much worse when we move onto general equilibrium models of multiple heterogeneous consumers (investors) and multiple capital goods.

Therefore, what remain of the Cambridge capital controversies are not just about instability of general equilibrium models, where different opinions are possible - the Cambridge capital controversies can instead be resurrected as being about equilibrium existence. Joan Robinson and later heterodox economists asked the question of history versus equilibrium in analyzing economies (Lang & Setterfield, 2006), and it becomes more apparent when the general equilibrium framework is internally difficult to maintain due to prevalence of equilibrium non-existence.

The Cambridge capital controversies are not resolved - they are only brushed aside, and revisits of these controversies are bound to occur. (Cohen & Harcourt, 2003)

## Conflicts of interest

Authors report no conflict of interest.

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